Redistributive Innovation Policy, Inequality and Growth

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Abstract

In this paper, we develop a simple heterogeneous-agent growth model with in-house R&D and imperfect capital market where both growth and inequality are endogenously determined. Using simple functional forms for preferences, technical progress, and income distribution, we examine the efficiency and equity effects of different public R&D policies that target high-tech and low-tech sectors. When calibrating the model to the US economy, a regressive public innovation policy that targets the high-tech sector aggravates inequality. On the other hand, with the same model calibrated to Sub-Saharan African countries, we find that a progressive R&D policy that targets the low-tech sector could lower inequality. The impact on growth is rather ambiguous due to the inverse inequality-growth relationship. A progressive public policy could hinder growth via a negative distortionary effects on individual saving and effort while it could promote growth via mitigating economic inequality. However, a properly designed investment subsidy financed by consumption tax could restore the first best investment rate and labour supply. Our model helps us understand the contrasting effects of public R&D spending on inequality and growth of the US and African economies.

Key words:
Disproportional public innovation policy, distributional dynamics and equilibrium, growth, R&D
JEL Classification: D24, D31, E13, O41

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1. Introduction

The role of public and private R&D investment in economic growth is a widely debated topic.[1] However, the distributional effect of public R&D investment has received little attention.[2] In the extant literature, the focus is more on public education in determining economic inequality (see, for instance, Glomm and Ravikumar, 1992, 2003, among many other) and infrastructure and taxes (e.g., Garcia and Turnovsky, 2007, Getachew, 2010, Chatterjee and Turnovsky, 2012, Getachew and Turnovsky, 2015). One can find ample evidence and intuition that public R&D policy has uneven impacts on the economy. For instance, in many developing countries, the generation of drought resistant varieties and improved intercropping techniques might benefit marginal farmers more than proportionately. On the contrary, R&D policy that focuses on the development of high yielding variety, fertilizers, large machinery and chemicals may tend to favour large commercialized farmers. The aim of this paper is to investigate the inequality and growth effects of such disproportional effects of public research and development (R&D) policy.

More recent empirical studies also reveal the important role played by various types of public R&D in determining a country's inequality in various dimensions. For instance, using provincial data in China for more than four decades, Fan et al. (2004) argue that government spending on agricultural R&D besides other factors contribute to agricultural productivity growth and reduce regional inequality. Cozzi and Impullitti (2010) find that government policy in R&D procurement plays a significant role in explaining the rising inequality in recent decades in the U.S. They argue that a increase in public R&D investment in high-tech sectors of the economy in the early 1980s substantially boosted the relative wage of skilled workers.

[1] Particularly, in early 90s, there was an influx of R&D based growth theories, following the seminal works by Romer, (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992) that emphasize the role of R&D to economic growth, through influencing technological progress. R&D policies are also widely debated in terms of whether public R&D investment complements private R&D investment or crowds i out (e.g., Mamuneas and Nadiri, 1996).

[2] Chu (2010) is an exception in this regards. He argues that strengthening patent policy increases income inequality by raising the return on assets.

While innovation policies may widely vary among nations, in general two different R&D policy strategies dominate in developing and advanced economies. Most R&D investments in developed countries are rather within high-tech industries such as information technology, biology, communication, and environment industries.[3] In contrast, in most of the developing world, a significant amount of public R&D investment is made in agriculture. Beintema et al. (2012) report that there is an accelerated public investment in agricultural R&D in the developing countries during the period 2000 and 2008. The ratio of agricultural R&D spending to the global public spending increased from 39% to 46% in the developing world that includes middle and low income countries[4]. On the other hand in high income countries this proportion fell from 58% to 51%.

This paper argues that public R&D investment could aggravate or mitigate inequality depending on its regressivity or progressivity respectively. A regressive (progressive) R&D policy is biased in favour of the rich (poor) section of the population. This is consistent with the recent trend in inequality and public R&D investment in different countries. Figures 1 and 2 feature contrasting relationships between Gini index and the share of the public R&D spending in the US and SSA. While there is a positive relationship between these two variables for the U.S. (correlation coefficient is 0.81 significant at the 5% level), the scatter plot for the SSA shows a negative association (correlation coefficient is -0.25 which is statistically insignificant).[5] Dropping South Africa as an outlier (which has the highest Gini of 0.62) raises the correlation

[3]Cozzi and Impullitti, (2010) argue that public investment in equipment and software increased from 20% in 1980 to 50% in 2001. Kim, Chun and Kim (2013) argue that R&D in Korea concentrates more on high-tech sectors that results in an equity-efficiency trade off.

[4]According the same report, the average annual agricultural R&D sending growth in SSA countries increased from 0.3% during 1981-1990 to 2.8% during 2000-2008 except for a small dip of .01% during 1990-2000. In Asia and Pacific countries, it increased from 4.9% to 5.8% and in Latin American countries it increased from 1.5% to 2.1%. (Source: ASTI Global Assessment of Agricultural R&D Spending, October 2012)

[5]The data came from World Development Indicators. Time average of Gini index and the share of public R&D are computed. Twenty two 22 SSA countries, for which both inequality and R&D data are available, are included: Burundi, Burkina Faso, Congo, Ethiopia, Gambia, Kenya, Lesotho, Madagascar, Mali, Mozambique, Namibia, Nigeria, Sudan, Senegal, Tanzania,Uganda, Zambia, Cape, Verde, Botswana,Mauritius, South Africa, Seychelles.

coefficient between Gini and R&D spending ratio to -0.41 which is significant at the 5% level.[6]

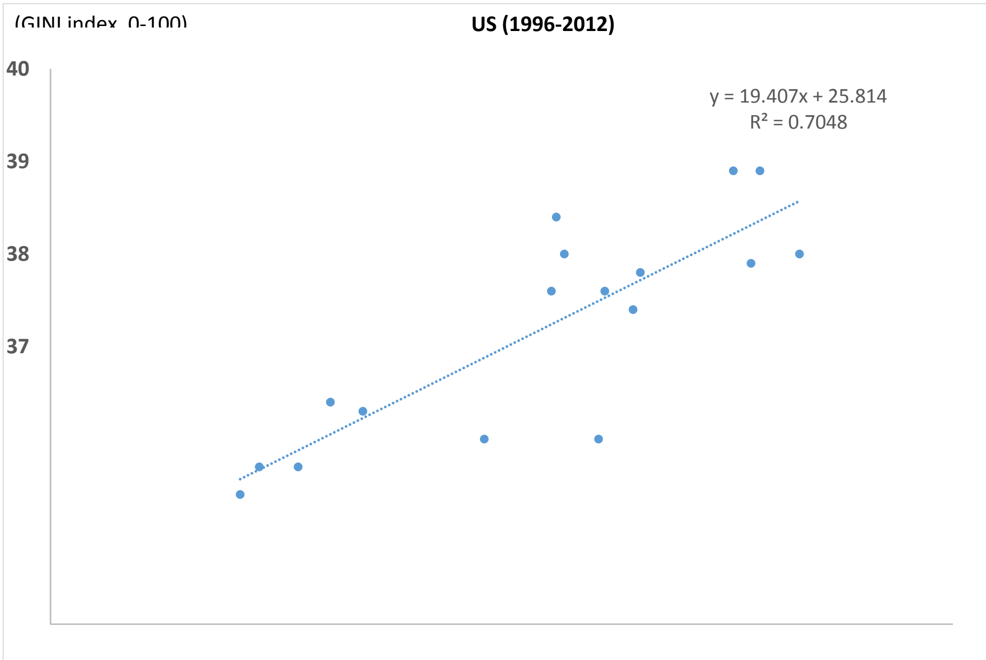

Figure 1

[6] Dropping South Africa from the sample raises the correlation to 0.34.

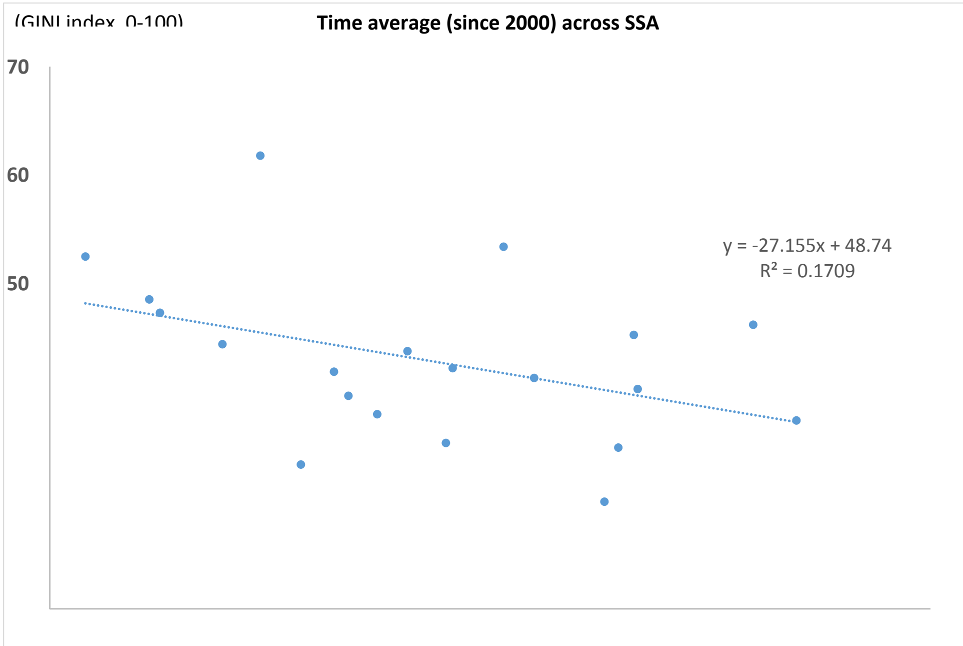

Figure 2

Our paper develops a heterogenous-agent growth model with in-house R&D where both inequality and growth are endogenously determined. We analyze the effects of regressive and progressive public R&D policies on inequality and growth. Agents are different in terms of their initial endowments of knowledge and their ability to generate knowledge. The source of endogenous growth (technological progress) is in-house R&D investment using private and public resources. Endogenous inequality is generated due to the credit and insurance market imperfection, as in Loury (1981) and Benabou (2000, 2002). The dynamics of aggregate variables and inequality are jointly determined in the model that admits a closed form analytical solution.

The key quantitative predictions based on our calibrated model to US and SSA data are as follows. A regressive public R&D policy could boost growth via a positive effect on individual saving and effort. It escalates the economic inequality and through this channel it hurts long run welfare. The relationship between degree of regressivity in the R&D spending and long run welfare is, therefore, nonlinear hump shaped admitting an optimal degree of regressivity in public R&D spending. In contrast, a progressive R&D policy could be growth and welfare improving if it is designed by a combination of consumption tax and investment subsidy. Higher investment subsidy could promote long run growth and lower economic inequality. Our model's predictions about the effect of R&D policy on inequality are in line with the stylized facts. A more regressive R&D policy escalates the economic inequality while a progressive R&D lowers inequality as it happened in the US and SSA regressions reported in Figures 1 and 2.

The paper connects to a wider literature on inequality and social mobility. First, it relates to the literature that analyzes the growth-inequality trade-offs under imperfect credit markets (see, for e.g., Loury, 1981, Galor and Zeira, 1993, Aghion and Bolton, 1997, Aghion, et al., 1999, Benabou, 2000, 2002, 2005).[7] Our paper also has implications of public investment in human capital on social mobility as in Basu and Getachew (2015) Second, our paper indirectly connects to a literature that deals with the relationship between public education and inequality.[8] Third, the paper also connects to a growing literature that studies the relationship between infrastructure and inequality via the distributional effects of a productive public good in growth models, (García-Peñalosa and Turnovsky, 2007, Getachew, 2010, 2012, and Chatterjee and Turnovsky, 2012, Getachew and Turnovsky, 2015). In general, only few of the papers in this strand of literature focus on inequality and mobility effects of public R&D investment while virtually the bulk of the literature abstracts from disproportional R&D public spending. Finally, our paper also has implications for the recent rise in wage inequality in many advanced economies due to skill biased technical change (see, for instance, Acemoglu, 2002, Aghion, 2002, and Hornstein et al., 2005). However, this literature does not explain the effects of progressive R&D policy on economic inequality.

The paper is organized as follows: The following section develops the model.

[7] This literature mainly abstracts from productive public spending issue. For instance, Benabou (2005) focuses on the distributional and growth impact of progressive taxation.

[8] See, for example, Glomm and Ravikumar (1992; 2003), Saint-Paul and Verdier (1993) and Eckstein and Zilcha (1994). While the focus of these papers is purely on public education spending, our focus is on public R&D spending, Both kinds of spending, however, contribute to the formation of human capital.

Section 3 characterizes the transitional dynamics of the economy. Section 4 deals with the steady-state. Section 5 reports the results of a quantitative analysis of our growth model calibrated to US and SSA data. Section 6 concludes.

2. The Model

We assume that the economy is populated with a continuum of heterogenous agents, $i \in (0,1)$. There is no population growth in the economy. The first generation of the ith agent is endowed with h_{i0} levels of knowledge. Initial distribution is given and assumed to take log-normal, $\ln h_{i0} \sim N(\mu_0, \sigma_0^2)$, which evolves endogenously at equilibrium. Agents also differ in their respective productivity and creativity to generate income and knowledge, respectively, where both are assumed to be i.i.d. and log-normally distributed. Combined with labour, knowledge is used to produce intermediate goods, which are, in turn, used for the production of final goods.

There are three sectors in the economy, namely the final goods, the intermediate goods and the knowledge production sector. Using a CES production function, a competitive firm transforms intermediate inputs into a final good. These differentiated intermediate inputs are produced by competitive firms using a convex technology. Each firm in this sector invests in an in-house R&D to expand a specialized know-how that is required to produce a specialized input. The production of knowledge requires both the use of public and private resources, and a backlog of knowledge stock. The government levies a fixed flat rate tax on the income of individual agents to finance the 'public good'. This public good is provided *disproportionately* among rich and poor agents to supplement private R&D investment. The extent of disproportionately in the allocation of public R&D depends on the redistributive stand of the benevolent government.

2.1. Final goods

In the spirit of Benabou (1996), the final goods and services are produced by a continuum of intermediate goods firms (indexed by i) over a unit interval using a CES aggregator. The final goods production function is given by:

$$y_t = a_1 \left(\int_0^1 \phi_{it} x_{it}^{(\varepsilon-1)/\varepsilon} di \right)^{\varepsilon/(\varepsilon-1)} ; \varepsilon > 1 \tag{1}$$

where x_{it} is the intermediate input supplied by the ith intermediate goods firm, a_1 is a deterministic TFP parameter; ϕ_{it} is idiosyncratic productivity shocks, which are i.i.d. with mean one and a constant non-zero variance. In other words,

$$\ln \phi_{it} \sim N \left(-\varkappa^2/2, \varkappa^2 \right)$$

$\varepsilon > 1$ is the elasticity of substitution between the intermediate inputs, which determines the firms' monopoly power, in the spirit of Dixit and Stiglitz (1977).

Profit maximization by the perfectly competitive firm,[9] given a unit price of the final goods, leads to the downward slopping demand function:

$$x_{it} = \phi_{it}^{\varepsilon} a_1^{1-\varepsilon} y_t \left(\frac{1}{p_{it}} \right)^{\varepsilon} \tag{2}$$

where p_{it} denotes the price of the ith intermediate good and $-\varepsilon$ is the price elasticity of demand.

2.2. Intermediate goods

The differentiated intermediate goods firms are characterized with certain features. First is the presence of specialization. Knowledge is firm-specific, and hence the production of intermediate goods. Thus each intermediate goods firm has some monopoly power of fixing its price. Consequently, the rate of returns and earnings are different among firms in this sector. Second, a firm in this sector engages in an in-house R&D investment, which is the only way of technological progress, to expand its specialized knowledge stock.

[9] The optimization problem is defined as:

$$\max_{x_{it}} \left(y_t - \textstyle\sum_i p_{it} x_{it} \right)$$

The ith firm in the intermediate sector needs $1/h_{it}$ units of labour to produce one unit of its variety:

$$x_{it} = h_{it} l_{it} \tag{3}$$

where h_{it} represents the stock of the firm specific knowledge, generated through in-house R&D activity, which is specified below; and, l_{it} is the raw labour input. Each period, the firm's profit consists of revenue from the sale of the intermediate good, x_{it}, net of the total labor cost $(l_{it}w_{it})$ where w_{it} is the wage rate per unit of labor. Thus, the firm has the following static optimization problem,

$$\max_{\{l_{it},p_{it}\}} \pi_{it} = p_{it}\left(x_{it}, .\right) x_{it} - w_{it} l_{it}$$

subject to the demand function (2). The first order condition leads to the following pricing:

$$p_{it} = \frac{w_{it}}{h_{it}} \frac{\varepsilon}{\varepsilon - 1} \tag{4}$$

While w_{it}/h_{it} is the marginal cost of producing a unit of the intermediate input, the elasticity of substitution, ε, determines the mark-up over this cost.

The ith agent's income is given by $y_{it} = p_{it} x_{it}$, which comprises wage and profit income. Substituting , (2), (3) and (4) into this, one obtains:

$$y_{it} = a\phi_{it}\left(l_{it}h_{it}\right)^{\alpha} y_t^{1-\alpha} \tag{5}$$

where $a \equiv a_1^{-\alpha}$ and $\alpha \equiv (\varepsilon - 1)/\varepsilon$.

Eq. (5) matches individual income to output production, characterized by constant returns to scale at individual (h_{it}) and aggregate accumulative factors (h_t) in total.[10] However, there is diminishing returns to individual factor. This shows that the model is basically in the spirit of the Arrow (1962) and Romer (1986) learning-by-doing endogenous growth models.

[10] As we see later, y_t is a linear function of h_t and $l_{it} = l$, which is constant.

2.3. In-house R&D

Each intermediate goods firm invests in an in-house R&D to produce the know-how using the following knowledge production function:

$$h_{it+1} = a_2 \zeta_{it+1} h_{it}^{\theta} s_{it}^{\upsilon} g_{it}^{\lambda} \tag{6}$$

Government intervenes in the R&D process by investing in public R&D production (g_{it}) that uses to complement the private sector, but with a redistributive intent. According to (6), knowledge is a product of both public and private investment (g_{it} and s_{it}, respectively), past knowledge stock of the firm (h_{it}) and idiosyncratic shocks (ζ_{it+1}). In addition, there is an exogenous deterministic technological parameter (a_2), in the knowledge production sector, which is common to all firms. $\{\lambda, \upsilon, \theta\} \in (0, 1)$ determine respective knowledge elasticities. ζ_{it+1} are i.i.d. and follow a log-normal distribution with mean one and a constant variance,

$$\ln \zeta_{it+1} \sim N\left(-\varrho/2, \varrho^2\right)$$

The production function (6) exhibits constant-returns to scale:

$$\theta + \upsilon + \lambda = 1 \tag{7}$$

2.4. Government budget

Public R&D investment is financed using a proportional income tax (τ), which is levied in the final goods. The government balances budget as in the growth and public investment literature (e.g., Barro, 1990):

$$g_t = \tau \textstyle\int_0^1 y_{it} di = \tau y_t \tag{8a}$$

where g_t denotes the total public investment in R&D and τ is the public expenditure GDP ratio.

The *key* feature of this paper lies in the relationship between the production of knowledge and public expenditure. We abstract from a blanket public investment provision in R&D. Rather the government expenditure in R&D has a redistributive component. Public R&D investment does not necessarily benefit individual firms proportionally. Small firms may benefit disproportionately from low-tech technologies as large firms do from high-tech. For instance an innovation of a pedal powered tractor is more beneficial to small-scale farmers,as a high-power tractors for large commercial farms:

$$g_{it} = (h_{it}/h_t)^{\omega}\, g_t \tag{9}$$

The key redistributive R&D policy parameter is ω. Its magnitude and direction determine the nature and weight of redistribution. If $\omega = 0$, for instance, $g_{it} = g_t$ is a pure public good where all firms equiproportionately benefit from public R&D. A positive ω implies merit based public expenditure. R&D firms with relatively high level of initial knowledge stock compared to the average human capital (meaning a higher h_{it}/h_t) benefit more than proportionately from public R&D spending (g_t). A negative ω, on the other hand, makes small firms with a relatively lower level of initial knowledge stock (meaning a lower h_{it}/h_t) benefit more from public spending on R&D. Hereafter, we refer to negative ω and positive ω as progressive and regressive public expenditure, respectively.

Combining (6), (8a) and (9) one obtains:

$$h_{it+1} = a_2 \zeta_{it+1} h_{it}^{\theta+\omega\lambda} s_{it}^{\upsilon} \left(g_t/h_t^{\omega}\right)^{\lambda} \tag{10}$$

where

$$\theta + \omega\lambda > 0 \tag{A2}$$

TFP and public expenditure GDP ratio have influences over individual knowledge accumulation via the effect on public R&D expenditure.

The parameters θ and λ are *ex ante* knowledge elasticities whereas $\theta + \omega\lambda$ and $\lambda - \omega\lambda$ capture *ex post* intergenerational linkages associated with firm level knowledge production that account for individual and aggregate factors in the economy, respectively.[11] Redistribution thus puts pressure on private and public knowledge elasticities. The impact of redistribution in the economy are then determined based on the resultant effects it has on these elasticities. Although some of these effects offset in the aggregate, most remain important given that individual optimal decision is crucially dependent on $\theta + \omega\lambda$, which is also the main determinant of the evolution of inequality where the later determines other macroeconomic dynamics. To ensure a positive relationship between existing knowledge and innovation, we impose the restriction $\theta + \omega\lambda > 0$, as stated in the assumption (A2). Whereas, the term $\omega\lambda$ captures the redistributive nature of the public variable and its implication for individual knowledge accumulation.

2.5. Household

There is a continuum of households indexed between $(0, 1)$. Each firm in the intermediate sector is owned by some household.[12] The credit and insurance markets are missing,[13] as children cannot be held responsible for their parents debts . We also assume members of the households are endowed with units of labour that they supply elastically. Agents maximize their utility in accordance to the following function:

$$\max_{\{c_{it}, h_{it+1}, l_{it}\}_0^\infty} \mathrm{E}_t \sum_{t=0}^{\infty} \rho^t \left(\ln c_{it} - l_{it}^{\eta}\right) \tag{12}$$

where $\eta > 1$; E_t is an individual's expectation given information at t. The budget constraint is given by:

$$c_{it} + s_{it} = (1 - \tau)\, y_{it} \tag{13}$$

where τ represents income tax, respectively.

[11] As we shall soon see g_t is a linear function of h_t.

[12] Other models that use similar type of individual enterpreneurship include Benabou (2000, 2002, 2005) and Angeletos and Calvet (2005, 2006).

[13] Although the absence of any capital market could be quite extreme, which is the price to pay for analytical tractability of the model, but what really matters is that there be some imperfections (see also Benabou, 1996, 2000).

Applying standard methods, individual household decision rules can be derived as follows[14]:

$$
\begin{aligned}
s_{it} &= b\,(1-\tau)\,y_{it} \qquad (14)\\
l_{it} &= l = \left(\frac{\alpha}{\eta}\frac{1}{1-b}\right)^{1/\eta} \qquad (15)\\
c_{it} &= (1-\tau)\,(1-b)\,y_{it} \qquad (16)
\end{aligned}
$$

where

$$b \equiv \frac{\rho\alpha\upsilon}{1-\rho\,(\theta+\omega\lambda)}$$

Eqs. (14), (15) and (16) are standard forms from the view point of household optimization. Households supply a constant unit of labour, and saving rate is independent of rate of returns, as a consequence of log utility function. Both saving rate and effort increase with the discount factor (ρ), elasticity of substitution (ε), intergenerational spillover (θ), and the elasticity of private investment (υ). On the other hand, greater regressivity in the R&D policy (higher ω) subsidizes saving and taxes consumption because b is increasing in ω. Since there is complementarity between firms' previous knowledge stock and their investment (see eq (6)). A higher ω (more regressive R&D public policy) favours bigger more advantaged firms. This provides greater incentive to bigger and advantaged firms to undertake larger investment in knowledge production. In summary, we have the following proposition.

Proposition 1. *Saving rate and labour supply increase (decrease) with regressive (progressive) public R&D fund provision.*

2.5.1. Aggregate consumption, investment and income:

Aggregate consumption and saving are given by:

[14]Detailed derivations are the decision rules are relegated to the appendix.

$$c_t = (1-\tau)(1-b)\, y_t \tag{17}$$

$$s_t = (1-\tau)\, by_t \tag{18}$$

Aggregate income is derived from aggregating (5), after substituting (15):

$$y_t = la^{1/\alpha} h_t \exp(d_t) \tag{19}$$

where d_t is a composite parameter, which captures the relationship between aggregate income and inequality:

$$d_t \equiv 0.5\,(\alpha - 1)\,\sigma_t^2 \tag{20}$$

In this case, the government budget constraint is given by, from (8a) and (19):

$$g_t \equiv z_t h_t = \tau la^{1/\alpha} h_t \exp(d_t) \tag{21}$$

where l is given by (15). Given that individuals' income is determined by their optimal labor supply, aggregate labor is an important component of aggregate income and hence aggregate public R&D expenditure. In addition, considering $d_t < 0$, the existence of diminishing returns in individual income implies that aggregate income, and hence public R&D decrease in inequality.

2.5.2. Optimal knowledge dynamics and intergenerational mobility

The optimal dynamics of knowledge stock associated to the ith firm is derived from (5), (14), (19) and (10):

$$h_{it+1} = a_3 \psi \chi \zeta_{it+1} \phi_{it}^{\upsilon} h_{it}^{\beta} h_t^{\kappa} \exp\left(\left(\lambda + (1-\alpha)\,\upsilon\right) d_t\right) \tag{22a}$$

where $a_3 \equiv a_2 (\alpha/\eta)^{(\upsilon+\lambda)/\eta} a^{(\upsilon+\lambda)/\alpha}$ and

$$\psi \equiv b^{\upsilon} (1-b)^{-(\upsilon+\lambda)/\eta} \tag{23a}$$

$$\chi \equiv \tau^{\lambda} (1-\tau)^{\upsilon} \tag{23b}$$

$$\beta \equiv \theta + \omega\lambda + \alpha\upsilon \tag{23c}$$

$$\kappa \equiv \lambda + (1-\alpha)\upsilon - \omega\lambda \tag{23d}$$

Eq. (22) captures the optimal dynamics of knowledge at a firm level. τ and ω in (23) are policy parameters while the rest are structural parameters. The government adjusts the size of investment through its choice of τ, whereas the sign of ω determines the redistributive nature of the public fund. Such policy variables impact the TFP of individual knowledge production function via their effects on individual savings, efforts and public R&D investment. These are in particular reflected in ψ and χ. For instance, as shown in χ, there is a positive effect from income tax through its financing of public R&D expenditure; but, a negative effect in its distortionary effects in individual savings. The resultant effect is determined by the weight of the respective elasticities. From ψ, redistribution (ω) affects individual knowledge production through its effects on individual saving rate and individual and aggregate efforts. Redistributions also impact the elasticities of individual and aggregate past knowledge with an important implication to inequality dynamics.

The dynamics of individual knowledge also depends on the current individual and aggregate knowledge variables, idiosyncratic risks both in the final goods (ϕ_{it}) and R&D sectors (ζ_{it+1}) and current inequality. Risks in the final goods sectors affect individual savings and investment indirectly via individual income whereas those in the knowledge sector have a direct impact. The last two terms in (22a) reflect on the relationship between inequality and individual knowledge dynamics. $d_t < 0$ reflects the negative effects of inequality on knowledge production at firm level. Through aggregate and subsequent individual savings and investment impacts, inequality negatively impacts individual knowledge accumulation.

3. Transitional dynamics and mobility

3.1. Intergenerational mobility

One interesting aspect of eq. (22) is its direct implication for intergenerational mobility. The intergenerational elasticity (IGE) of human capital (β) is derived from (22), first by taking logs from both sides of the equation, and then computing the partial derivative of the next-period human capital with respect to the current human capital:

$$\beta \equiv \frac{\partial \ln h_{it+1}}{\partial \ln h_{it}} = \theta + \omega\lambda + \upsilon\alpha \tag{24}$$

$1 - \beta$ is a measure of intergenerational mobility.

Note that we are measuring mobility here in terms of knowledge, in contrast to the majority of the intergenerational mobility literature, where income mobility is rather central in the analysis (see, for instance, Solon, 1992 and Mazumder, 2005, among others). On the other hand, because, from (14), individual investment in knowledge and income evolves similarly, β is also a measure of the persistence of income across generations. This should not be a surprise, considering that knowledge is the only factor input in the model.

According to (24), intergenerational mobility is independent of the idiosyncratic and common shocks, but it depends crucially on the structure of goods and knowledge production and human capital accumulation technologies at the individual household level.

Proposition 2. *Intergenerational mobility increases in progressive public R&D expenditure ($\omega < 0$) whereas it decreases in firms' monopoly power, ε.*

Thus, to the extent public investment is provided progressively, the negative effect of previously acquired knowledge on mobility decreases. It increases, however, in case of a regressivity of public fund.

3.2. Inequality dynamics

The dynamics of inequality is also derived from (22a), by taking the log and variance,

$$\sigma_{t+1}^2 = \upsilon^2 \varkappa^2 + \varrho^2 + \beta^2 \sigma_t^2 \tag{25a}$$

Given $\beta \in (0,1)$, (25) is a stable dynamics that converges to a steady state inequality. The variance of the idiosyncratic shocks ($\varkappa^2$ and ϱ^2) will determine the long run property of the model. Income volatility affects inequality via its effect on individual saving while volatility in the R&D sector directly impacts inequality dynamics. The root of the dynamics of inequality is determined by β, which in turn is a function of policy and structural parameters, ε, λ, ω, υ and θ. Higher intergenerational linkage (higher θ) associate to higher transitional inequality. Strong monopoly power (higher ε) and better investment technology (higher υ) also imply slower convergence in inequality. However, the sign of ω determines the impact of public R&D investment on inequality. Private R&D investment elasticity (υ) also impact inequality through individual response to luck, with a strong implication to long-run inequality. The effect of the public variables on the dynamics of inequality rather depends on its redistributive feature (the sign of ω). If $\omega < 0$, higher elasticity of public R&D investment (higher λ) leads to faster convergence of inequality, and conversely. If $\omega = 0$, i.e. public investment in R&D is proportionally provided, however, the elasticity λ has a neutral effect in inequality. Note that β is increasing in ω and α which is $(\varepsilon - 1)/\varepsilon$. We thus have the following proposition:

Proposition 3. *A regressive (progressive) R&D investment aggravates (mitigates) transitory inequality. In other words, if $\omega > 0$ ($\omega < 0$), given σ_t^2, σ_{t+1}^2 increases (decreases) in ω, and conversely.*

Proof. From (25), if $\omega < 0$, for given σ_t^2 then σ_{t+1}^2 decreases in $|\omega|$, and conversely. ■

Note also that slower mobility (higher β) also implies slow convergence of the inequality dynamics. That is, the greater β is, the more persistent inequality becomes. However, since the coefficient in the inequality dynamics (β^2) is smaller than the mobility coefficient (β), intergenerational immobility is much more persistent than inequality.

3.3. Aggregate wealth and growth dynamics

From (17), (19) and(21), all aggregate variables except aggregate knowledge grow at the same rate:

$$\gamma_t + 1 \equiv \frac{y_{t+1}}{y_t} = \frac{g_{t+1}}{g_t} = \frac{c_{t+1}}{c_t} = \Omega_t \frac{h_{t+1}}{h_t} \tag{26}$$

where

$$\Omega_t \equiv \exp\left(0.5\,(1-\alpha)\left(\sigma_t^2 - \sigma_{t+1}^2\right)\right)$$

In the steady state where $\sigma_{t+1}^2 = \sigma_t^2 = \sigma^2$, the economy will be in a balanced growth path (BGP) where $\gamma_t = \gamma$ becomes the growth rate of the economy in transition. During the transition period, $\sigma_{t+1}^2 \neq \sigma_t^2$, the dynamics of aggregate knowledge trails or leads the dynamics of other aggregate variables depending on whether the economy starts above or below its steady state, respectively. If $\sigma_{t+1}^2 > \sigma_t^2$, the exponential term in (26) is less than one, which implies the growth rate of h is higher than that of c, g and y, and conversely. Therefore, inequality dynamics is the source of transitional dynamics in this economy.

One derives the growth rate of aggregate knowledge during the transition first by aggregating (22a), to get the dynamics of aggregate knowledge,

$$h_{t+1} = a_3 \psi \chi h_t^{\theta+\upsilon+\lambda} \exp\left(\pi + \digamma_t + q_t\right) \tag{27a}$$

$$\pi \equiv \left(0.5\upsilon\,(\upsilon - 1)\,\varkappa^2\right) < 0 \tag{27b}$$

$$\digamma_t \equiv 0.5\beta\,(\beta - 1)\,\sigma_t^2 < 0 \text{ if } \omega < 1 \tag{27c}$$

$$q_t \equiv 0.5\,(\lambda + (1-\alpha)\,\upsilon)\,(\alpha - 1)\,\sigma_t^2 < 0 \tag{27d}$$

Then, from (27), the growth rate of aggregate knowledge is easily obtained,

$$\gamma_t + 1 = \psi\chi \exp\left(\pi + \digamma_t + q_t\right) \tag{28}$$

given constant returns to scale in knowledge production sector (7).

The last two terms capture the growth-inequality trade-offs given that $\sigma_0^2 \neq 0$,

$\varkappa^2 \neq 0$ and $\varrho^2 \neq 0$. The term π is the result of individual heterogeneity in terms of idiosyncratic productivity shocks (ϕ_{it}). They impact the dynamics of knowledge in the economy at the firm level (22) through their effect on individual income (5) and savings (14). This, eventually, will have a negative impact on the aggregate economy as (given diminishing return to private investment or $\upsilon \in (0,1)$), as poorer individual have higher marginal productivity. $\digamma_t$ captures inequality effects as a result of initial differences in wealth. The first term links intergenerational mobility, to inequality and growth, because β reflects the intergenerational link of knowledge creation through public and private investment. The second is a result of the redistributive nature of the public good. Given $\beta \in (0,1)$ and $\omega < 0$, $\digamma_t \in (0,1)$ and hence inequality in terms of individual differences in initial wealth negatively impacts the evolution of knowledge in the economy. However the relationship between intergenerational mobility and growth is not that direct:

Proposition 4. *Given the current inequality σ_t^2, the growth rate of aggregate knowledge increases (decreases) in intergenerational mobility if IGE is too high (too low).*

Proof. $\partial \exp(\digamma_t) / \partial (1-\beta) = 0.5\sigma_t^2 \exp(\digamma_t)(-2\beta+1) > 0$ iff $\beta < 1/2$. ■

The term q_t reflects the micro effects of inequality. As shown in (22a), inequality also have a direct impact on the dynamics of knowledge at the firm level. This is because aggregate variables such as y_t and g_t play important role on individual income (5) and knowledge (10). Because inequality have impact on these variables (see for e.g. (19)) it will have indirectly affect the micro variables, which in turn affect aggregate productivity.

Whether inequality has a positive or negative effects on growth will be determined on the values of ω. In many cases and for more plausible variables inequality will have a negative impact on growth. If $\omega < 0$, then $\digamma_t < 1$, inequality will have a definite negative impact on growth. For range of values of positive ω, the relationship between σ_t^2 and γ_t is also negative. This is particularly in line to the literature that studies the relationship between inequality and growth under capital market imperfection (see, for e.g., Loury, 1981, Galor and Zeira, 1993, Benabou, 1996, 2000, 2002, among other). The intuition is that greater inequality corresponds to lower growth when the credit and insurance markets are missing since these prevent the efficient amount of investment to be undertaken in the economy when some poor households miss productive investment opportunities. It is, nevertheless, possible for σ_t^2 to have a positive impact on γ_t for some regressive public investment, $\omega > 0$.

4. Steady-state

Note that given $\beta \in (0,1)$, which is the sufficient condition for the stability of the distributional dynamics, (25a) converges to a unique inequality level. But, with constant-returns to scale in knowledge production, inequality is the only source of dynamics in the economy. As inequality converges to its equilibrium level, growth also converges to its steady-state level. In this case, long-run inequality and growth are given by, from (25a) and (28), respectively:

$$\sigma^2 = \left(\upsilon^2 \varkappa^2 + \varrho^2\right) / \left(1 - \beta^2\right) \tag{29}$$

$$\gamma + 1 = a_3 \psi \chi \exp\left(\pi + \digamma + q\right) \tag{30}$$

where

$$\begin{aligned} q &\equiv 0.5\sigma^2 \left(\alpha - 1\right)\left(\lambda + \left(1 - \alpha\right)\upsilon\right) < 0 \\ \digamma &\equiv 0.5\beta\left(\beta - 1\right)\sigma^2 < 0 \text{ if } \omega < 0 \end{aligned}$$

Steady-state inequality increases in IGE and volatility. γ is the steady-state growth rate of the economy. Therefore, the long-run equilibrium of the economy is a balanced growth path, with a constant non-zero level of inequality. Many of the results for the transitional periods also holds for the steady-state. From (29), Proposition 3 holds in the steady-state. Particularly, σ^2 decreases in λ if $\omega < 0$. Inequality have a negative impact on long-run growth for some reasonable parametric values. In addition, policy impacts long-run growth directly via its effect on individual and aggregate productivity, savings and effort; indirectly, via the growth-inequality trade-offs.

4.1. Growth maximizing policies

Growth maximizing policies of the government through the choice of ω and τ to impact long-run growth are easily obtained from (30). First, with respect to τ, the growth maximizing tax rate (τ^*) is given by $\lambda/(\lambda+\upsilon)$, which is independent of redistribution, ω. τ^* is the maximum when $\upsilon = 0$, that is when there is no or little private investment is made in R&D. With respect to ω, one may look at two cases: with and without inequality. For a homogeneous economy, the choice is straightforward. Regressive public policy ($\omega > 0$) favors growth. In order to maximize growth, $\omega > 0$ and take the maximum attainable value as growth rate increases in ω. This relationship holds both at the transition and steady-state.

For the heterogenous case, $\sigma_0^2 \neq 0$, $\varkappa^2 \neq 0$ and $\varrho^2 \neq 0$, however, a regressive policy has a positive effect on growth via its effect on investment and a negative effect on growth because it escalates the steady state inequality σ^2. Thus a growth maximizing ω potentially exists.

The following proposition summarizes the above discussion:

Proposition 5. *(i) When the role of private investment in R&D is greater, the need for government involvement becomes smaller. (ii) Given $\sigma_0^2 = 0$, $\varkappa^2 = 0$ and $\varrho^2 = 0$, then γ increases in ω. (iii) However, for the heterogenous case, $\sigma_0^2 \neq 0$, $\varkappa^2 \neq 0$ and $\varrho^2 \neq 0$, the relation between ω and γ is ambiguous.*

4.2. Towards welfare maximizing redistributive R&D policy

How does the degree of progressivity of R&D policy impact welfare? To analyze this we first compute the steady state aggregate welfare of all citizens at date zero.

Proposition 6. *The steady state welfare function is given by*

$$W_0 = \frac{\rho}{(1-\rho)^2}\ln(1+\gamma) + \frac{\ln c_0 - 0.5\sigma_c^2 - l^\eta}{1-\rho} \tag{31}$$

where

$$c_0 = (1-\tau)(1-b)la^{1/\alpha}h_0\exp(d_0) \text{ and } \sigma_c^2 = \chi^2 + \alpha^2\sigma^2 \tag{32}$$

Proof. *Appendix* ■

A change in ω has a nonlinear effect on the steady state welfare W_0. The nonlinearity arises due to conflicting effects of a change in ω on W_0. A higher ω unambiguously raises the steady state inequality, σ^2 as seen from (29) and through this channel it lowers growth. On the other hand, it promotes investment and thus it raises growth. A higher ω raises labour supply l via boosting b but it also has an offsetting effect on the initial consumption, c_0. On the other hand, a higher labour supply has a direct negative effect on the steady state welfare via the second term in (31).

4.3. Case for an optimal progressive R&D policy: consumption tax and investment subsidy

As seen in Proposition 1, a negative ω depresses individual saving and effort. Thus, it is distortionary in the sense that it discourages aggregate efficiency via its negative effects on private investment and labour supply. However, a consumption tax can be designed to correct for these two distortions.[15] Let the government subsidize individual saving (at a rate of ϑ) using a consumption tax. In this case, (6) becomes:

$$h_{it+1} = a_2 \zeta_{it+1} h_{it}^{\theta} \left((1+\vartheta) s_{it} \right)^{\upsilon} g_{it}^{\lambda} \tag{33}$$

where ϑ is the subsidy rate. Therefore, the individual receives an additional amount of ϑs_{it} subsidy for s_{it} level of investment. If the government chooses to finance this with a consumption tax at a rate of τ_c then, individual and (the balanced) government budget constraints become respectively,

$$(1+\tau_c) c_{it} + s_{it} = (1-\tau) y_{it} \tag{34}$$

[15] Strictly speaking the first best value for private investment rate is given by the optimal individual saving rate when $\tau = 0$ and $\omega = 0$. But here we only consider the case where the government corrects the distortions in individual saving and labour supply caused by a negative ω using consumption tax. Note also that consumption tax may not be always non-distortionary, particularly, when applies to a more general utility function.

and

$$\vartheta s_t = \tau_c c_t \tag{35}$$

The rest of the government budget is given separately by (8a). With consumption tax, only individual optimal consumption will be affected.

$$c_{it} = (1-b)(1-\tau) / (1+\tau_c)\, y_{it} \tag{36}$$

From the individual optimal solution, consumption decreases by a factor of $1/(1+\tau_c)$ whereas individual saving rate and labour supply remain the same, and given by (14) and (15) respectively.

Given that individual's *effective* saving is $\widetilde{s}_{it} = (1+\vartheta)\, s_{it}$ with the subsidy, the government could restore the distorted saving rate, b, to its optimal level, $\widetilde{b} = \rho\alpha\upsilon/(1-\rho\theta)$. To do so, first note that

$$\widetilde{s}_{it} = (1+\vartheta)\, s_{it} = \widetilde{b}\,(1-\tau)\, y_{it} \tag{37}$$

which implies that the effective saving is equal to targeted (non-distorted) saving. Then substituting eq. (14), b and $\widetilde{b}$ into the above and solving for ϑ leads to[16]:

$$\vartheta = \frac{-\rho\omega\lambda}{1-\rho\theta} \tag{38}$$

where $\vartheta > 0$ for $\omega < 0$. From (14) and (36), the optimal consumption tax is given by[17]:

[16] Detailed derivations of the key equations in this section including (37) and (38) are relegated to the appendix.

[17] To get the optimal consumption tax.use the government budget constraint (35), $s_t/c_t = \tau_c/\vartheta$ to get:

$$\begin{aligned} \frac{\tau_c}{\vartheta} &= \frac{b(1+\tau_c)}{1-b} \\ \tau_c &= \frac{\vartheta b}{1-b-\vartheta b} \end{aligned}$$

$$\tau_c = \frac{\vartheta b}{1 - b - \vartheta b} \tag{39}$$

Therefore, consumption and labour supply remain distorted while saving is restored to the second best.

Proposition 7. *(i) With the subsidy rate (38), the effective saving rate could be restored to its second-best levels ,* $\widetilde{b} = \rho\alpha\upsilon / (1 - \rho\theta)$ *whereas consumption and labour supply remain distorted as shown in (36).*

5. Quantitative Analysis

In this section, we explore the quantitative implications of our growth model with regressive and progressive R&D policies. We construct a baseline calibrated model targeting the US economy. In the next step, we perform a sensitivity analysis by changing the key redistributive parameters to gain insights about the implications of regressive and progressive R&D policies for growth, inequality and societal welfare . The subjective discount factor ρ is fixed at 0.99 as in numerous studies. The human capital technology parameter θ reflects the adjustment cost of changing human capital that is explored in Basu and Getachew (2015). Following their estimate, we fix θ at 0.8. The markup parameter ε is set at 6.0 following Kollmann (2005). There is no consensus on η value in the literature. We fix η equal to unity in line with Kollmann (2005) as well. The initial human capital (h_0) and the initial distribution of human capital (σ_0^2) and the other shock variance parameters χ^2 and ϱ^2 are normalized at unity. For the baseline calibration, we set $\omega = 0$ which means that there no distortion beyond the second best level due to the presence of the aggregate income tax rate τ.

The parameter ν represents the elasticity of knowledge production with respect to private R&D spending as seen in (6). This could show cross country variation. In the absence of any known estimate of ν, we calibrate this as follows. We first calibrate the private saving rate rate, b for the baseline case of $\omega = 0$. Using the Word Development Indicators (WDI) database for the period 2005-2014, we find that the average proportion of R&D spending (including private and public) to GDP is 2.81% for the US and 0.55% for the SSA region. The public R&D spending ratios for these two regions are 0.548% and 0.22% respectively.[18] By subtracting the public R&D spending ratio from total R&D spending ratio, we get an estimate of the private R&D spending ratio for the US as 2.26% and for the SSA as 0.33%. Based on these, we set b equal to 0.0226 for the US and 0.0033 for SSA countries. Given that there is no distortion in the baseline model, we can then back out ν using the equation of the private saving b in the baseline case which means ν is 0.0058 for the US and 0.0011 for the SSA. Since all public R&D spending is tax financed in our model, we calibrate the tax parameter τ as 0.0055 for the US and 0.0022 for the SSA which are the average ratio of public R&D spending to GDP in these two continents during 2000-2014.

The productivity parameters a_1 and a_2 are assumed to be the same and calibrated to reproduce the average annual growth rates of GDP for each continent. For the US, we target a 2% growth rate. For the SSA the growth rates show major volatility during the late 90s due to ebola outbreak based on the WDI report. The WDI outlook for a stable growth rate of SSA is around 4.5% which we take as our baseline target. This gives us an estimate of a_1 and a_2 as 2.99 for the US and 3.58 for the SSA. Table 1 summarizes the baseline parameter values.

[18] These are computed using the WDI data. The data for public R&D are sparse for SSA. The data mostly range from 2000 with missing observations. We compute a simple average of each SSA country and arrive at a grand average of all SSA countries listed in footnote 7. Details of the calculations are available from the authors upon request.

Table 1: Benchmark values

ρ	0.99
θ	0.8 (Basu and Getachew, 2015)
ν	.0011 for SSA and .0058 for US
λ	$1 - \theta - \nu$
η	1
χ	1
ϱ	1
τ	0.00548 for the US and 0.0022 for SSA
a_1	2.99 for US and 3.58 for SSA
a_2	2.99 for US and 3.58 for SSA
ε	6 (Kollmann, 2005)
ω	0 as baseline
α	$(\varepsilon - 1)/\varepsilon$

Starting from this baseline model where $\omega = 0$, we vary ω for our two model economies within admissible range (subject to the steady state convergence condition) for two economies, namely (i) the economy with a regressive R&D policy, and (ii) the economy with a progressive R&D policy. Figures 3 through 6 present the effect of a rise in ω on long run inequality, growth and welfare in (i). Higher value of ω means more regressive R&D. This unambiguously raises steady state inequality. It lowers growth and welfare. The negative growth effect arises primarily due to a small value of ν which is the private R&D spending knowledge elasticity parameter. A higher value of ν can potentially reverse these results. Figures 6, 7 and 8 reflect such a scenario when ν is set counterfactually at 0.06. Growth now rises accompanied by higher inequality. The conflicting effects of a regressive R&D policy on growth and inequality give rise to nonlinearity in its effect on welfare. This shows up as a hump shape effect of an increase in ω on the long run welfare. An optimal R&D policy is attained when ω is around 0.35.

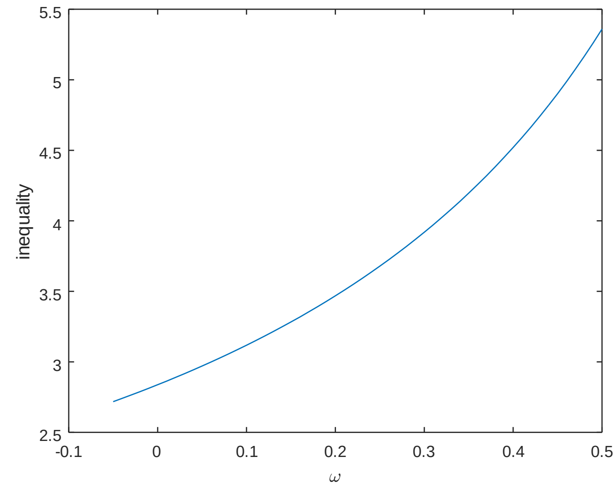

Figure 3: Regressive R&D and Inequality

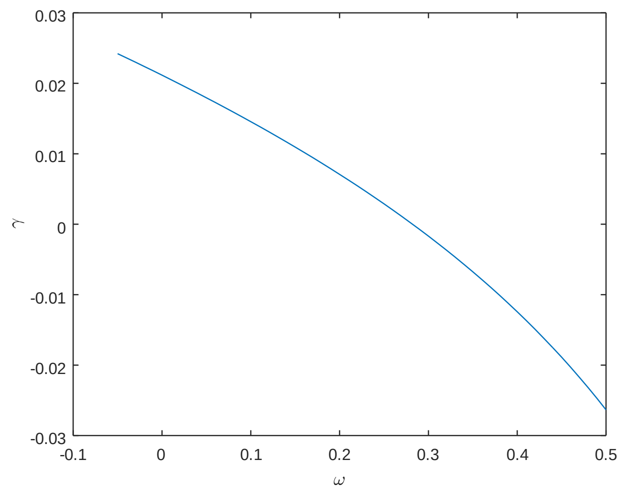

Figure 4: Regressive R&D and Growth

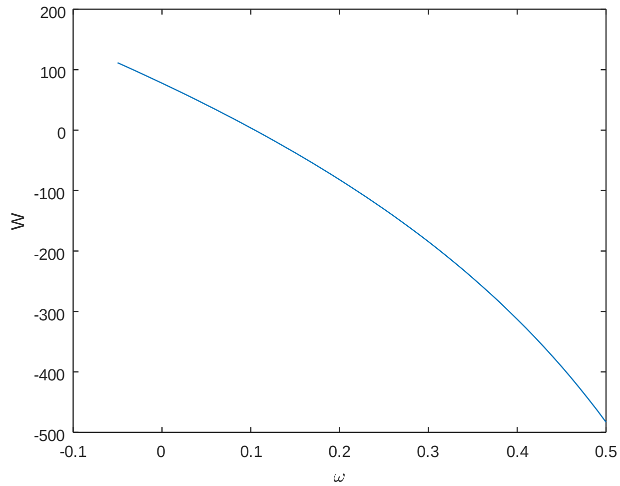

Figure 5: Regressive R&D and Welfare

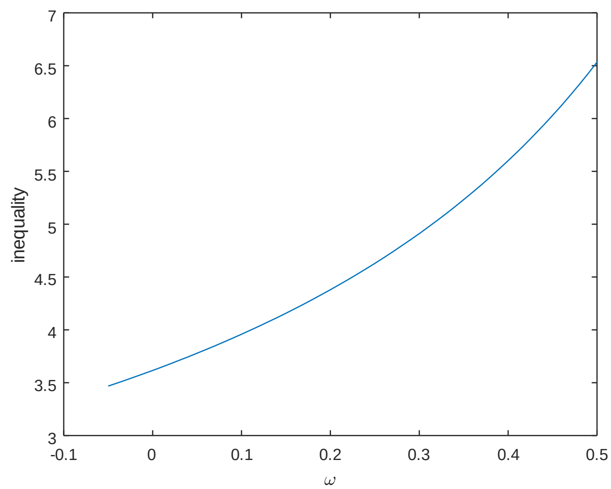

Figure 6: Regressive R&D and Inequality when omega=0.07

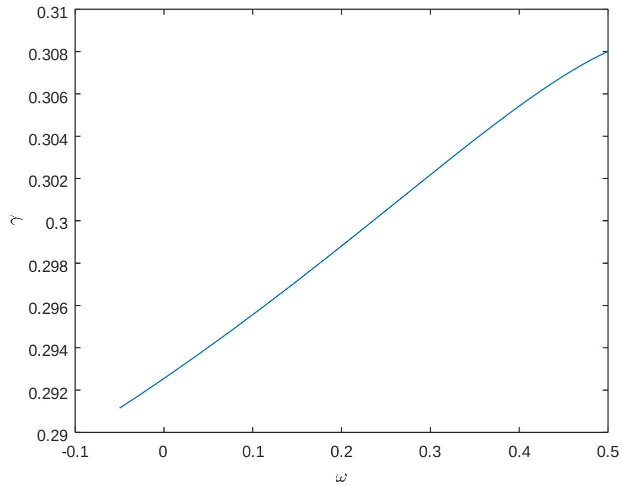

Figure 7: Regressive R&D and Growth when omega=0.06

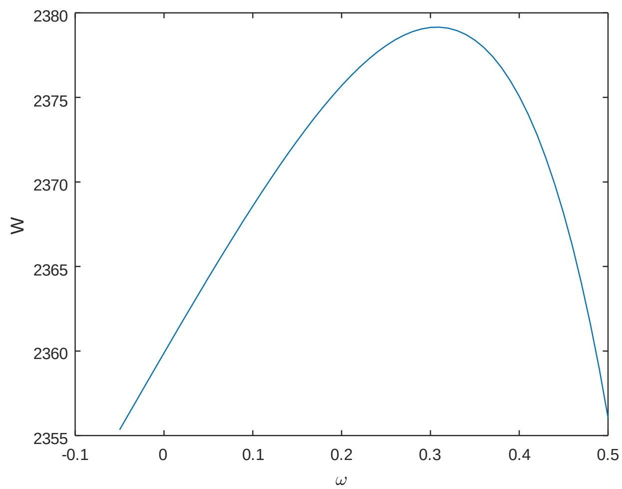

Figure 8: Regressive R&D and Welfare when omega=0.07

We next turn to the case of a progressive R&D spending scenario (ii). The progressive R&D policy ($\omega < 0$) alone has a detrimental effect on investment and growth as seen in earlier propositions. However, as seen in the earlier section, if this R&D policy is combined with a consumption tax and investment subsidy programme, it could be potentially growth and welfare improving. We calibrate the progressive

R&D scenario to SSA countries. Figures 7 through 9 plot the effects of a higher progressivity of R&D policy on inequality, growth and welfare. Higher progressiveness is reflected by a greater investment subsidy (a higher ϑ)) financed by consumption tax as explained in the earlier. A higher investment subsidy now lowers inequality promotes long run growth and welfare.

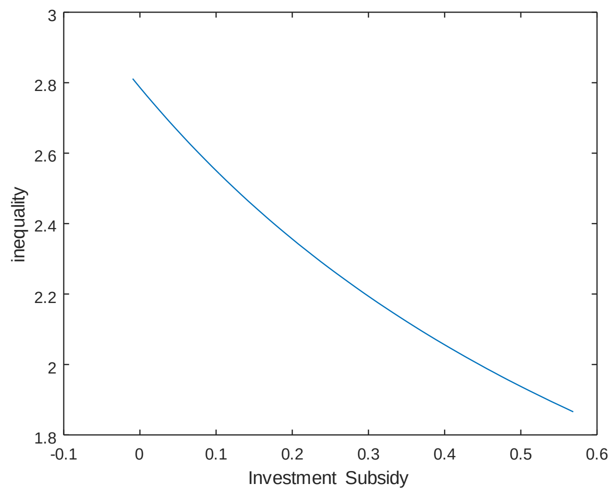

Figure 9: Progressive R&D and Inequality

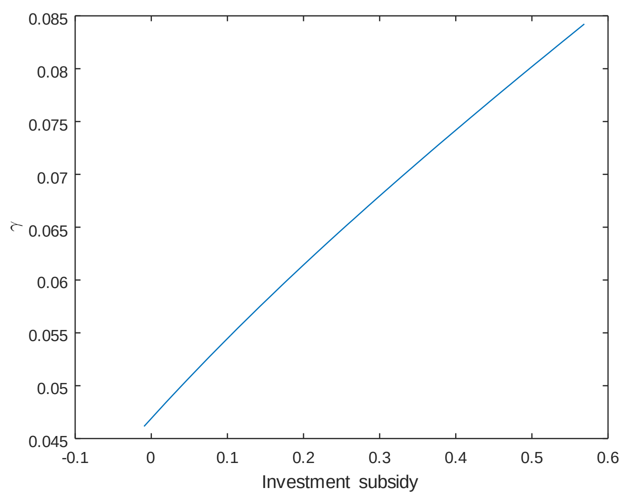

Figure 10: Progressive R&D and Growth

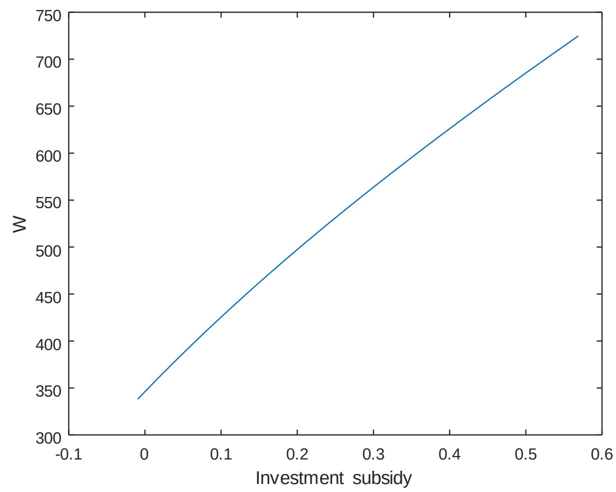

Figure 11: Progressive R&D and Welfare

The upshot of this quantitative analysis is that for empirically plausible values of the structural parameters, a regressive R&D policy elevates economic inequality and hurts long run growth and societal welfare. On the other hand, a progressive R&D policy financed by a consumption tax could promote growth and welfare and lower the inequality. These quantitative predictions from our calibrated growth model accord well with the stylized facts of inequality and growth of US and SSA countries.

6. Conclusion

This paper argues that innovation policies could play an important role in explaining high-growth low-inequality observed recently in many developing countries, particularly African economies, and mediocre-growth and high-inequality performances of the US. We develop a heterogenous-agent growth model with in-house R&D where both growth and inequality are endogenously determined. A regressive R&D policy unambiguously escalates economic inequality while the effect on growth is nonlinear. The latter nonlinearity stems from the inverse relation between growth and inequality due to the incompleteness of the credit market. In contrast, a progressive R&D policy delivered by an investment subsidy financed by a consumption tax could correct the distortion in saving and promote growth and ameliorate inequality. These results hold both during the transitional path and in the steady-state. Our calibrated model to the US and SSA predicts that a progressive (regressive) public investment aggravates (lowers) inequality while it lowers (promotes) growth which are in line with the stylized facts.

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Appendix

A. Derivation of the Optimal Decision Rules

$$\max_{\{c_{it},h_{it+1},l_{it}\}_0^\infty} \mathrm{E}_t \sum_{t=0}^{\infty} \rho^t \left(\ln c_{it} - l_{it}^{\eta}\right) \tag{40}$$

where $\eta > 1$; E_t is an individual's expectation given information at t. The budget constraint is given by:

$$c_{it} + s_{it} = (1-\tau)\, y_{it} \tag{41}$$

$$h_{it+1} = z_t h_{it}^{\theta+\alpha\omega\lambda} s_{it}^{\upsilon} \tag{42}$$

where

$$z_t \equiv a_2 \zeta_{it+1} s_{it}^{\upsilon} (\tau y_t)^{\lambda}$$

$$y_{it} = a\phi_{it} (l_{it} h_{it})^{\alpha}\, y_t^{1-\alpha} \tag{43}$$

The Lagrangnian is:

$$L = \sum_{t=0}^{\infty} \rho^t \left\{ \ln c_{it} - l_{it}^{\eta} + \chi_t \left((1-\tau)\, y_{it} - c_{it} - \frac{h_{it+1}^{1/\upsilon}}{\left(z_t h_{it}^{\theta+\omega\lambda}\right)^{1/\upsilon}} \right) \right\}$$

FOC:

$$c_{it} \;:\; \frac{1}{c_{it}} = \chi_t \tag{44}$$

$$l_{it} \;:\; -\eta l_{it}^{\eta-1} + \chi_t (1-\tau)\, \alpha \frac{y_{it}}{l_{it}} = 0 \tag{45}$$

$$h_{it+1} \;:\; \rho\chi_{t+1} (1-\tau)\, \alpha \frac{y_{it+1}}{h_{it+1}} = \left(\chi_t 1/\upsilon \frac{s_{it}}{h_{it+1}} - \rho\chi_{t+1} (\theta+\omega\lambda)\, /\upsilon \frac{s_{it+1}}{h_{it+1}} \right) \tag{46}$$

From (25):

$$\eta l_{it}^{\eta} = (1-\tau)\,\alpha \frac{y_{it}}{c_{it}} \tag{47}$$

Using undetermined coefficient: $c_{it} = a y_{it}$ and $s_{it} = d y_{it}$. Substituting these into (44) and (46)

$$\begin{aligned}
&: \quad \rho \frac{1}{a y_{it+1}} (1-\tau)\,\alpha \frac{y_{it+1}}{h_{it+1}} - \left(\frac{1}{a y_{it}} 1/\upsilon \frac{d y_{it}}{h_{it+1}} - \rho \frac{1}{a y_{it+1}} (\theta + \omega\lambda)\,/\upsilon \frac{d y_{it+1}}{h_{it+1}} \right) = 0 \\
&\Rightarrow \quad \rho \frac{1}{a} (1-\tau)\,\alpha - \left(\frac{1}{\upsilon a} d - \rho \frac{1}{\upsilon a} (\theta + \omega\lambda)\, d \right) = 0 && (48) \\
&\Rightarrow \quad d = \frac{\upsilon \rho\,(1-\tau)\,\alpha}{1 - \rho\,(\theta + \omega\lambda)} && (49)
\end{aligned}$$

Substituting $c_{it} = a y_{it}$ into (47):

$$\eta l_{it}^{\eta} = (1-\tau)\,\alpha \frac{1}{a} \tag{50}$$

Note that $a = 1 - \tau - d$. Plugging (49) and simplifying

$$a = (1-\tau)\left[1 - \frac{\alpha \nu \rho}{1 - \rho(\theta + \omega\lambda)}\right]$$

which means

$$l_{it} = \left[\frac{\alpha}{\eta} \frac{1}{1-b}\right]^{1/\eta} \tag{51}$$

where

$$b = \frac{\alpha \nu \rho}{1 - \rho(\theta + \omega\lambda)}$$

and thus

$$c_{it} = (1-\tau)(1-b) y_{it}$$

and

$$s_{it} = (1-\tau) b y_{it}$$

B. Derivation of the key equations in the case of progressive R&D policy financed by consumption tax

The lagrange changes to

$$L = \sum_{t=0}^{\infty} \rho^t \left\{ \ln c_{it} - l_{it}^{\eta} + \chi_t \left((1-\tau)\, y_{it} - (1+\tau_c) c_{it} - \frac{h_{it+1}^{1/\upsilon}}{\left(z_t h_{it}^{\theta+\omega\lambda}\right)^{1/\upsilon}} \right) \right\}$$

and z_t should be redefined as

$$z_t \equiv a_2 (1+\vartheta)^{\nu} \zeta_{it+1} s_{it}^{\upsilon} \left(\tau y_t\right)^{\lambda}$$

FOC

$$c_{it} \quad : \quad \frac{1}{c_{it}} = \chi_t (1+\tau_c) \tag{52}$$

$$l_{it} \quad : \quad -\eta l_{it}^{\eta-1} + \chi_t (1-\tau)\, \alpha \frac{y_{it}}{l_{it}} = 0 \tag{53}$$

$$h_{it+1} \quad : \quad \rho \chi_{t+1} (1-\tau)\, \alpha \frac{y_{it+1}}{h_{it+1}} = \left(\chi_t 1/\upsilon \frac{s_{it}}{h_{it+1}} - \rho \chi_{t+1} (\theta + \omega\lambda) / \upsilon \frac{s_{it+1}}{h_{it+1}} \right) \tag{54}$$

From (16) and (15),

$$\eta l_{it}^{\eta} = \frac{(1-\tau)}{(1+\tau_c)} \alpha \frac{y_{it}}{c_{it}} \tag{55}$$

The solution for d stays the same (49) because eq. (54) is free from τ_c. In other words.

$$d = \frac{\upsilon \rho (1-\tau)\, \alpha}{1 - \rho (\theta + \omega\lambda)}$$

Using the flow budget constraint

$$(1+\tau_c) c_{it} + d y_{it} = (1-\tau) y_{it}$$

which implies

$$\begin{aligned} c_{it} &= \frac{(1-\tau-d)}{(1+\tau_c)} y_{it} \\ &= \frac{(1-\tau)\left[1-\frac{\alpha\nu\rho}{1-\rho(\theta+\omega\lambda)}\right]}{(1+\tau_c)} y_{it} \end{aligned}$$

In other words, written in compact form,

$$c_{it} = \frac{(1-\tau)(1-b)}{(1+\tau_c)} y_{it} \tag{56}$$

which upon substitution in (55) yields

$$\begin{aligned} \eta l_{it}^{\eta} &= \frac{(1-\tau)}{(1+\tau_c)} \alpha \frac{y_{it}}{\frac{(1-\tau)(1-b)}{(1+\tau_c)} y_{it}} \\ &= \frac{\alpha}{1-b} \end{aligned}$$

which means

$$l_i = \left[\frac{\alpha}{\eta}\frac{1}{1-b}\right]^{1/\eta} \tag{57}$$

Thus the labour supply is exactly the same as (51) as in the case of no government tax-subsidy intervention.

Now the government chooses investment subsidy ϑ such that b is elevated to the second best level (when $\omega = 0$). It is not first best because the government cannot set $\tau = 0$ becuase then $g_t = 0$.

$$\widetilde{b} = \frac{\alpha\nu\rho}{1-\rho\theta} = \frac{\alpha\nu\rho(1+\vartheta)}{1-\rho(\theta+\omega\lambda)}$$

This means that

$$\vartheta = \frac{-\rho\omega\lambda}{1-\rho\theta}$$

which proves (38).

The government can now set a tax rate that satisfies the governement budget constraint (35) and this yields

$$\tau_c = \frac{\vartheta b}{1 - b - \vartheta b}$$

which proves (39).

This operation of the government has no effect on household's optimal saving and labour supply. Thus only saving can be elevated to the second best level whiich promotes growth but consumption remains distorted. Labour supply is ditorted because the undistorted labour supply should be

$$\tilde{l}_i = \left[\frac{\alpha}{\eta}\frac{1}{1-\tilde{b}}\right]^{1/\eta} \text{ which exceeds } l_i = \left[\frac{\alpha}{\eta}\frac{1}{1-b}\right]^{1/\eta}$$

C. Derivation of the steady state welfare

$$\begin{aligned} W_0 &= E_i E_0 \sum_{t=0}^{\infty} \rho^t \left(\ln c_{it} - l_{it}^{\eta}\right) \\ &= E_0 \sum_{t=0}^{\infty} \rho^t E_i \left(\ln c_{it} - l_{it}^{\eta}\right) \\ &= E_0 \sum_{t=0}^{\infty} \rho^t \left(\ln c_t - 0.5\sigma_{t,c}^2 - l^{\eta}\right) \end{aligned}$$

In the steady state:

$$
\begin{aligned}
W_0 &= E_0 \sum_{t=0}^{\infty} \rho^t \left(\ln\left(c_0 (1+\gamma)^t\right) - 0.5\sigma_c^2 - l^\eta\right) \\
&= E_0 \sum_{t=0}^{\infty} \rho^t \left(\ln c_0 + t\ln(1+\gamma) - 0.5\sigma_c^2 - l^\eta\right) \\
&= E_0 \sum_{t=0}^{\infty} \rho^t t \ln(1+\gamma) + \frac{\ln c_0 - 0.5\sigma_c^2 - l^\eta}{1-\rho} \\
&= \ln(1+\gamma)\, E_0 \sum_{t=0}^{\infty} \rho^t t + \frac{\ln c_0 - 0.5\sigma_c^2 - l^\eta}{1-\rho} \\
&= E_0 \frac{\rho}{(1-\rho)^2} \ln(1+\gamma) + \frac{\ln c_0 - 0.5\sigma_c^2 - l^\eta}{1-\rho} \\
&= \frac{\rho}{(1-\rho)^2} \ln(1+\gamma) + \frac{\ln c_0 - 0.5\sigma_c^2 - l^\eta}{1-\rho}
\end{aligned}
$$

Using (17), (19)and (20), write c_0 as

$$c_0 = (1-\tau)(1-b)la^{1/\alpha}h_0 \exp(d_0)$$

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