GOVERNMENT SPENDING EFFECTS IN LOW-INCOME COUNTRIES

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ABSTRACT. A dynamic stochastic general equilibrium model is constructed for studying government consumption and public investment effects in low-income countries (LICs). The model incorporates several LIC features important for government spending effects, including a large share of hand-to-mouth households, limited international capital mobility, a reliance on external financing sources, and public investment inefficiency. Limited capital mobility in the private sector makes government financing sources—domestic vs. external—matter for the spending effects. External financing through borrowing or aid relieves the crowding-out effect in private consumption and investment, but traded output can respond quite negatively due to a much appreciated real exchange rate. Also, with limited capital mobility, whether the twin-deficit hypothesis holds depends on the extent to which fiscal deficits are financed externally. Capital scarcity in LICs implies a high rate of return to public capital, but the low investment efficiency can substantially decrease the public investment multipliers.

Keywords: government spending effects; public investment; low-income countries; government spending multipliers; small open DSGE models

JEL Codes: E62; O23; F41; O55

1. INTRODUCTION

Government spending is an important countercyclical tool for countries of all income levels. In low-income countries (LICs), pressing capital needs give government spending another role to promote long-term economic growth. As few efforts have been devoted to fiscal policy

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effects in LICs in the literature,\(^1\) this paper constructs a dynamic stochastic general equilibrium (DSGE) model to study low-income governments’ spending effects, distinguishing between government consumption and productive public investment.

The backbone of the model is a two-sector, New Keynesian framework, commonly used for analyzing macroeconomic policy effects in advanced economies (Erceg et al. (2005), Adolfson et al. (2007), and Ratto et al. (2009)). To make the model suitable for LICs, the framework is modified to include:

- a large share of hand-to-mouth (or liquidity constrained) households,
- limited international capital mobility in the private sector,
- external financing sources of government commercial borrowing and aid, and
- public investment inefficiency.

The presence of a large share of hand-to-mouth households—who do not have access to financial and capital markets—are motivated by two considerations. Satisfying the subsistence needs forces many households to devote all of their disposable income to consumption in LICs. Also, underdeveloped financial systems prevent them from accessing formal saving devices. Aside from a large share of the hand-to-mouth, limited capital mobility as observed in most LICs is also captured by high costs associated with external borrowing.\(^2\) Due to restricted capacity in mobilizing revenue (International Monetary Fund (2011)), governments in LICs can also be liquidity constrained, and external funds such as aid and external borrowing provide important financing for government spending. Finally, pervasive low efficiency in government spending in LICs has been found to be important for debt sustainability for LICs.

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\(^1\)Berg et al. (2010), Buffie et al. (2012), and Kraay (2012) are among the few that study the macroeconomic effects of government spending in LICs.

\(^2\)On a scale of −1.86 to 2.44 (the least to most financially open), the median 2011 Chinn-Ito index (based on Chinn and Ito (2008)) for low-income and lower-middle income countries is −1.17 and for high-income countries is 2.44. Also, empirical evidence (e.g., Gorodnichenko and Schnitzer (forthcoming)) find that financial constraints in terms of high costs to access external finance restrict the ability of domestically owned firms in developing countries to innovate and catch up to technology frontiers.
a debt-financed investment scaling-up (Buffie et al. (2012)). We focus on how investment efficiency affects the public investment multipliers.

Given the baseline calibration with highly limited capital mobility, we find that financing sources affect the government consumption and public investment multipliers for output and its various components. External financing, on the one hand, increases resources available for the domestic economy at least in the short run, alleviating the crowding-out effects of government spending. Private consumption and investment are less negative or can even turn positive. On the other hand, external financing appreciates the real exchange rate more than domestic financing, which reduces the competitiveness of the traded good sector and traded output more, raising the Dutch disease concern. Overall, the cumulative output multiplier is bigger when debt is externally financed. Between the two external financing methods, aid financing produces a higher cumulative output multiplier than debt financing in the longer horizon. Since aid financing does not require paying back to donors, the expansionary effects from an initial spending increase are not offset by the negative effects of fiscal adjustments, such as higher tax rates.

In an environment of limited capital mobility, our analysis provides one explanation as to why the twin-deficit hypothesis does not always hold. When fiscal deficits are financed by external debt, reduced crowding-out effects and a much negative response of traded output drive up current account deficits, generating the co-movements between fiscal and current account deficits. In a sample of ten developing countries (including three LICs), Easterly and Schmidt-Hebbel (1993) find strong evidence that fiscal deficits spill over into trade deficits, and the real exchange rate appreciates, as found in our analysis with external financing.

For public investment effects, we find that investment efficiency—defined as the ratio of the change in public capital to an increase in investment expenditures—plays a crucial role in determining the growth effects of public investment. To capture capital scarcity and implied high return to capital, we calibrate the initial steady state to have a reasonably
high rate of return at 25 percent annually. Yet, low investment efficiency can reduce the multiplier even though public capital raises the productivity of private production inputs in our model. The baseline calibration assumes an investment efficiency at 0.4 (in line with the estimate obtained by Pritchett (2000) for sub-Saharan African economies), which yields the cumulative output multiplier with external debt financing below one. If the marginal efficiency (the efficiency applies to the increases of public investment above the steady-state level) doubles to 0.8, the longer-horizon multiplier for output is also doubled.

Another factor important for the public investment effects is the degree of home bias in government purchases. Public investment in LICs largely relies on imports of machinery and skilled labor and can have a low degree of home bias. Relative to the case with more of domestically produced goods such as in higher-income countries, the short-run stimulative effects induced by more government purchases can be relatively weak in LICs.

2. Model Setup

The framework, adapted from Berg et al. (2010), is a small open New Keynesian model with nontraded (N) and traded (T) good sectors.

2.1. Households. The economy is populated by two types of households with a fraction \( f \) are savers \((a)\) and \( 1 - f \) are hand-to-mouth \((h)\). Only savers have access to financial and capital markets, and the hand-to-mouth are liquidity constrained.

2.1.1. Savers. A representative saver chooses consumption \( (c^a_t) \), the real money balance \( (m^a_t) \), labor \( (l^a_t) \), investment \( (i^N,a_t \text{ and } i^T,a_t) \), capital \( (k^{N,a}_t \text{ and } k^{T,a}_t) \), domestic government debt \( (b^{d,a}_t) \), and external debt \( (b^{hs,a}_t) \) to maximize the expected utility,

\[
E_t \sum_{t=0}^{\infty} \beta^t \left[ \frac{(c^a_t)^{1-\sigma}}{1-\sigma} - \frac{(l^a_t)^{1+\psi}}{1+\psi} + \frac{(m^a_t)^{1-\xi}}{1-\xi} \right],
\]  

(1)
where $\beta$ is the discount factor, $\sigma$, $\psi$, and $\xi$ are the inverse of the elasticity of intertemporal substitution for consumption, labor, and money. The saver’s budget constraint is

$$c_t^a + m_t^a + i_t^{N,a} + iT^{a} + b_t^{d,a} + s_tR_T \frac{b^{h,a}_{t-1}}{\pi_t} + ac_t^{b,a} + ac_t^{i,a}$$

$$= (1 - \tau_t) \left( w_t^{l,a} + r_t^{N,k_{t-1}}T^{a} + r_t^{T,k_{t-1}} \right) + \frac{R_t^{b,a}_{t-1}}{\pi_t} + s_tR_t^{b,a} + \frac{m_t^{a}}{\pi_t} + s_trm^* + z_t + \Omega_t. \quad (2)$$

Domestic government debt $b^{d,a}_t$ pays a nominal rate of $R_t$ at $t + 1$, and $\pi_t$ is the domestic inflation of consumption. Savers can borrow (lend) externally by issuing $b^{h,a}_t > 0$ ($< 0$) in units of foreign goods, denoted by $\ast$. The real exchange rate $s_t$ is in the units of domestic consumption per unit of foreign good. $R^*$ in the nominal interest rate demanded by foreign creditors, and $\pi^*$ is foreign inflation, both assumed to be constant. Following Schmitt-Grohé and Uribe (2003), changing foreign liabilities is subject to portfolio adjustment costs $ac_t^{b,a} \equiv s_t^{v^2} \left( \frac{b^{h,a}_t}{\pi_t} - 1 \right)^2$, where $v$ governs capital account openness in the private sector. These costs, together with a risk premium included in $R^*$, represent the financial costs that prevent LIC households from engaging in a higher degree of consumption smoothing through borrowing and lending in international financial markets. A tax rate $\tau_t$ is levied on labor and capital income, and $w_t$ is the real wage rate. Foreign remittance $rm^*$ is assumed to be constant. $^3 z_t$ is government transfers to households, and $\Omega_t$ is dividends from firms.

We assume that capital is sector specific, and $r_t^{N}$ and $r_t^{T}$ are the returns to capital in each sector. $ac_t^{i,a} \equiv \kappa \left[ \frac{(r_t^{N,k_{t-1}} - \delta)^2 r_{t-1}^{N,a}}{(r_t^{N,k_{t-1}} - \delta)^2 r_{t-1}^{N,a}} \right]$ is investment adjustment costs. The law of motion for capital is

$$k_{t-1}^j = (1 - \delta)k_{t-1}^j + i_t^j, \quad j \in \{N, T\}, \quad (3)$$

where $\delta$ is the depreciation rate. Total investment made by savers is $i_t^a = i_t^{N,a} + i_t^{T,a}$. Consumption and investment are constant-elasticity-to-scale (CES) aggregates of nontraded and traded goods with the intratemporal elasticity of substitution $\chi$ and the degree of home

$^3$The presence of remittance helps back out the holding of foreign debt by savers in the steady state.
bias $\varphi$; e.g.,

$$c_t = \left[ \varphi^N \left( c_t^N \right)^{\frac{1}{\chi}} + (1 - \varphi)^\frac{1}{\chi} \left( c_t^T \right)^{\frac{1}{\chi}} \right]^{\frac{1}{1 - \chi}}. \quad (4)$$

Nontraded goods are produced by a continuum of monopolistically competitive firms indexed by $i \in [0, 1]$. Nontraded consumption varieties are aggregated by

$$c_t^N = \left[ \int_0^1 c_t^N(i)^{\frac{1}{\chi}} \, di \right]^{\frac{1}{\chi}}, \quad (5)$$

where $\theta$ is the elasticity of substitution between varieties. The CES basket implies that the unit price of $c_t$ (CPI) is

$$1 = \left[ \varphi \left( p_t^N \right)^{1-\chi} + (1 - \varphi)(s_t)^{1-\chi} \right]^{\frac{1}{1-\chi}}, \quad (6)$$

where $p_t^N$ is the relative price of nontraded goods to the CPI. We assume the law of one price holds for traded goods. Hence, the real exchange rate $s_t$ is also the relative price of traded goods to the CPI.

Households supply labor to both sectors. Savers’ total labor supply is

$$l_t^a = \left[ (\varphi^l)^{-\frac{1}{\chi}} \left( l_t^{a,N} \right)^{\frac{1+\chi^l}{1-\chi}} + (1 - \varphi^l)^{-\frac{1}{\chi}} \left( l_t^{a,T} \right)^{\frac{1+\chi^l}{1-\chi}} \right]^{\frac{1}{1+\chi^l}}, \quad (7)$$

where $\varphi^l$ is the steady-state share of labor in the nontraded good sector. Labor can move across the sectors, and $\chi^l > 0$ is the elasticity of substitution between the labor used in each sector. From the cost minimization problem, the aggregate real wage index is derived as

$$w_t = \left[ \varphi^l \left( w_t^N \right)^{1+\chi^l} + (1 - \varphi^l) \left( w_t^T \right)^{1+\chi^l} \right]^{\frac{1}{1+\chi^l}}, \quad (8)$$

where $w_t^N$ and $w_t^T$ are the real wage rate of each sector.

2.1.2. Hand-to-Mouth Households. We assume that the hand-to-mouth have an inelastic labor supply ($l_t^h = l^h \forall t$) and consume all the disposable income every period as determined by the budget constraint

$$c_t^h = (1 - \tau_t) w_t l_t^h + s_t rm^* + z_t. \quad (9)$$
2.2. Firms. The two production sectors have different market structures. Nontraded good firms are assumed to be monopolistically competitive, because nontraded goods can only be produced domestically. Since manufacturing in LICs often concentrate on resource-based and low-technology production, traded good firms are assumed to be perfectly competitive.\(^4\)

2.2.1. Nontraded Sector. The monopolistically competitive intermediate goods producer \(i \in [0, 1]\) produces with technology

\[
y_t^N(i) = z^N \left[k_{t-1}^N(i)\right]^{1-\alpha^N} \left[l_t^N(i)\right]^{\alpha^N} \left(k_{t-1}^G\right)^{\alpha^G},
\]

where \(z^N\) is total factor productivity (TFP) of nontraded good production, and \(k_{t-1}^G\) is public capital with an output elasticity \(\alpha^G\). Aggregating all nontraded goods \(y_t^N = \int_0^1 y_t^N(i) \frac{\theta-1}{\theta} di\) and solving the profit maximization problem yield the demand function for good \(i\),

\[
y_t^N(i) = \left[p_t^N(i) \frac{\theta-1}{\theta} y_t^N\right]^{-\theta} y_t^N.
\]

A nontraded good producer \(i\) chooses price, labor and capital to maximize its net present-value profits weighted by savers’ (firm owners’) utility \(\lambda_t^e\),

\[
E_t \sum_{t=0}^{\infty} \beta^t \lambda_t^e \left[(1-\tau) p_t^N(i)y_t^N(i) - ac_t(i) - w_tN l_t^N(i) - r_tN k_{t-1}^N(i) + \tau p_t^N y_t^N\right],
\]

subject to the production function (10) and the demand function (11). Price rigidity is introduced by adjustment costs \(ac_t(i) \equiv \xi_2 \left[\frac{\pi_t^N(i)}{\pi_{t-1}^N(i)} - 1\right]^2 p_t^N y_t^N\), a la Rotemberg (1982). \(\pi_t^N \equiv \frac{p_t^N}{p_{t-1}^N}\) is nontraded good inflation. Total price adjustment costs, nontraded output, and dividends are \(ac_t = \int_0^1 ac_t(i)di\), \(y_t^N = \int_0^1 y_t^N(i)di\), and \(\Omega_t = \int_0^1 \Omega_t(i)di\). To capture additional distortions in production (other than the explicit tax \(\tau\) on factor income), we introduce an implicit cost (tax) \(\iota\) in production. Unlike income taxes, the revenue collected by \(\iota\) does not enter the government budget and remains in the private sector. For simplicity, we assume the implicit cost is rebated back to firms (hence savers) in a lump-sum fashion.\(^5\)

\(^4\)Based on data from 2000 to 2009, 74.5 percent of value added in total manufacturing is resource-based and low-technology production in Africa (UNIDO and UNCTAD (2011)).

\(^5\)The implicit cost is a short cut for the model to rationalize why given the high marginal return to capital implied by capital scarcity, we do not observe a higher investment to output ratio in the steady state.
2.2.2. Traded Good Sector. A representative traded good firm $i$ produces with technology

$$y(i)_t^T = z^T \left[ k(i)_{t-1} \right]^{1-\alpha^T} \left[ I(i)_t^T \right]^\alpha T \left( k_{t-1}^G \right)^{\alpha^G}. \quad (13)$$

It chooses labor and capital to maximize periodic profits

$$(1 - \iota) s_t y(i)_t^T - w_t^T l(i)_t^T - r_t^T k(i)_{t-1}^T + \nu y_t^T,$$

where $y_t^T = \int_0^1 y_t^T(i)di$.

Total output produced in the economy at period $t$ is $y_t = p_t^N y_t^N + s_t y_t^T$.

2.3. Government. In each period, the government receives taxes and foreign aid ($a_t^*$), issues domestic and foreign debt ($b_t^d$ and $b_t^{bg*}$), and generates seigniorage revenue. Total expenditures include government consumption ($g_t^C$), public investment ($g_t^I$), transfers to households ($z$), and debt services. The flow budget constraint is

$$tax_t + b_t^d + s_t b_t^{bg*} + s_t a_t^* + m_t = p_t^G (g_t^C + g_t^I) + z + \frac{R_{t-1} b_{t-1}^d}{\pi_t} + s_t \frac{R^* b_{t-1}^{bg*}}{\pi^*} + m_t,$$

where $tax_t = \tau_t \left( w_t^l + r_t^N k_{t-1}^N + r_t^T k_{t-1}^T \right)$. We assume that the government faces the same nominal interest rate $R^*$ as savers when borrowing externally.\(^7\)

Government purchases $g_t = g_t^C + g_t^I$ are CES baskets of traded and nontraded goods with a degree of home bias $\varphi G$ and the elasticity of substitution $\chi$. The relative price of government purchases to the CPI is

$$p_t^G = \left[ \varphi G \left( p_t^N \right)^{1-\chi} + (1 - \varphi G) \left( s_t \right)^{1-\chi} \right]^{1-\chi}. \quad (16)$$

To capture low investment efficiency in LICs, we assume that one dollar of investment expenditure can deliver less than one dollar of public capital:

$$k_t^G = \left( 1 - \delta^G \right) k_{t-1}^G + \epsilon g_t^I \quad 0 \leq \epsilon \leq 1. \quad (17)$$

\(^6\)Transfers are kept constant in the model; it helps calibrate the government budget constraint in the steady state to match sample averages of other fiscal variables.

\(^7\)In reality, LICs often face a high sovereign risk premium when borrowing abroad commercially. Our specification assumes that this risk premium is constant. Empirically, the premium on average rises when a government becomes more indebted (e.g., Akitoby and Stratmann (2008)), but the relationship tends to be non-linear. See e.g., García-Cicco et al. (2010) and Buffie et al. (2012) for specifications, where risk premia rise exogenously with debt.
We assume government consumption and investment follow exogenous rules:

$$\log \frac{g^j_t}{g^j} = \rho \log \frac{g^j_{t-1}}{g^j} + \varepsilon^G_t , \quad j \in \{C, I\} . \quad (18)$$

Three financing methods are considered: domestic debt, external debt, and aid.\(^8\) When one of three is employed to finance a spending increase, the other two are set to their steady-state levels. For example, when domestic debt is used for financing, \(b^d_t\) is endogenously determined by the government budget (15), and \(b^d_t = b^{gr}, a^*_t = a^* \forall t\). Under domestic or external debt financing, total government debt rises, which triggers gradual fiscal adjustments with one quarter delay. We assume the tax rate adjusts to maintain debt sustainability.

$$\log \frac{\tau_t}{\tau} = \rho \log \frac{\tau_{t-1}}{\tau} + \gamma \log \frac{s^b_{t-1}}{s^b}, \quad \gamma \geq 0 \quad (19)$$

where \(s^b_{t-1} = \frac{b^d_{t-1} + s_{t-1}^{gr}}{y_{t-1}}\). \(^9\)

Given a less developed money and financial markets, many central banks in LICs target money in practice. To conduct monetary policy, the central bank follows a money growth rule with nominal reserve money grows at a constant rate \(\mu\); real money balance follows a process

$$m_t = \mu \frac{m_{t-1}}{\pi_t} . \quad (20)$$

2.4. Aggregation and market clearing. With two types of households, aggregate consumption, and labor are computed as follows.

$$x_t = f x^a_t + (1 - f)x^h_t , \quad x \in \{c, c^N, c^T, l, l^N, l^T\} . \quad (21)$$

\(^8\)One common financing source in LICs is external concessional borrowing, which is similar to aid but with a subsidized interest rate and a long maturity period. While not analyzed separately here, the effects of government spending financed by concessional borrowing would fall between the results with external (commercial) debt and aid financing.

\(^9\)In principle, other fiscal variables such as transfers can also adjust. In an environment with a large share of the hand-to-mouth, the effects of transfer adjustments are similar to those by the income tax rate. In LICs, because government transfers to households are minimal (the average social benefits to output ratio in SSA low-income countries (that have data from 2005 to 2012 in the WEO database) are quite small. Hence, we do not model the effects of transfer adjustments here.

\(^10\)An alternative rule to allow the money growth targets inflation can also be specified. For our purpose of studying government spending effects, this alternative rule with a reasonable response magnitude to inflation does not change materially the results presented here.
Lump-sum transfers and remittance are identical for each household. Hence,

\[ x^a_t = x^h_t = x_t, \quad x \in \{z, rm^*\}. \tag{22} \]

Since only savers have access to asset and capital markets, aggregate real money balance, investment, capital, debt, and dividends are computed as

\[ x_t = f x^a_t, \quad x \in \{m, i^N, i^T, k^N, k^T, b^d, b^{hs}, \Omega, ac^b, ac^i\}. \tag{23} \]

Finally, the market clearing condition of nontraded goods is

\[ y_t^N = (p_t^N)^{-\chi} \left[ \varphi \left( c_t + i_t + ac_t^i + ac_t^b + ac_t^p \right) + \varphi^G (p_t^G)^x g_t \right]. \tag{24} \]

The balance of payment condition is

\[
\begin{aligned}
&c_t + i_t + p^G g_t + ac_t^i + ac_t^b + ac_t^p + s_t \left( R^* - 1 \right) \frac{b^{hs}_t + b^{qs}_t}{\pi^*} - y_t - s_t rm^* \\
= &s_t \left( a^* + b^{hs}_t + b^{qs}_t - \frac{b^{hs}_{t-1} + b^{qs}_{t-1}}{\pi^*} \right). 
\end{aligned}
\tag{25}
\]

### 3. Solution and Calibration

To solve the model, we log-linearize the equilibrium system and use Sims’s (2001) algorithm for solving linear rational expectations models. Appendix A contains the optimality conditions for savers’ and firms’ maximization problems. The model is at quarterly frequency and calibrated to an average LIC, based on the average of 45 SSA country data from 2005 to 2012.\(^{11}\) Table 1 summarizes parameter values in the baseline calibration and some steady-state values.

Based on the estimate by Ogaki et al. (1996) for developing countries, the intertemporal elasticity of substitution is set to 0.34, implying \(\sigma = 2.94\). This suggests that consumption

\(^{11}\) SSA countries are mostly low-income but also include some middle-income countries. We exclude South Sudan for non-exist data before 2011. The data used to calibrate the initial steady state are the ratios of the following variables to GDP: private consumption, private investment, government consumption, public debt, public external debt, CPI inflation, international grants and aid, tax revenue, and remittance. All the data, except remittance, are downloaded from the database of World Economic Output, International Monetary Fund; remittance data are downloaded from World Bank’s databank of World Development Indicators.
decisions are less on intertemporal smoothing considerations relative to households in developed countries with the typical values $\sigma = 1$ or 2. Without empirical evidence for the Frisch labor elasticity for SSA economies, we calibrate $\psi = 1$ for savers. Together with hand-to-mouth households’ inelastic labor supply, the average Frisch labor elasticity is 0.25.\textsuperscript{12} The real money balance elasticity with respect to the nominal interest rate is set to 0.18, roughly consistent with the estimated semi-elasticity estimated using Uganda’s data (Berg et al. (forthcoming)). Together with a steady-state quarterly nominal interest rate of 4.3 percent, it implies a tiny intertemporal elasticity of substitution for the real money balance 0.008 (or $\xi = 130$, see (A.6)).

The discount factor $\beta = 0.98$ is consistent with an annual real interest rate of 8 percent. The share of savers is set to $f = 0.25$. Ardic et al. (2013) report that about 25 percent of the poor population has a bank account. Based on data collected in 2011, Demirguc-Kunt and Klapper (2012) compute that on average about 24 percent of adults in SSA and 23 percent of people living under $\$2$ a day has an account in a formal financial institution.\textsuperscript{13}

The degree of home bias in private consumption and investment is set to $\varphi = 0.6$ and in government purchases is $\varphi^G = 0.7$. Since distribution costs can be high in rural Africa, we assume a slightly higher share than the typical value of 0.5 (Burstein et al. (2005)). We follow the convention to assume a higher degree of home bias in government purchases because a large part of government spending goes to pay for civil service. Together with calibrated private consumption and investment shares to output in data (see Table 1), the model implies that almost 60 percent of labor works in the nontraded good sector, and the value added by traded output in the steady state is 35 percent of GDP.

\textsuperscript{12}Goldberg (2013) estimates that the intertemporal elasticity of working probability in a daily labor market in rural Malawi is 0.15-0.17. The concept of her estimated elasticity is different from the Frisch labor elasticity, though.

\textsuperscript{13}A wide variation, however, exists across SSA countries. Based on the estimates using the Global Findex Database, the percentage of adults with an account in a formal financial institution in SSA is 45 percent in the richest quintile countries and only 12 percent in the poorest quintile (Demirguc-Kunt and Klapper (2012)).
For the elasticity of substitution between traded and nontraded goods, we set $\chi = 0.44$, following Stockman and Tesar’s Stockman and Tesar (1995) estimate based on a sample of 30 countries including developing and developed countries. The labor income shares in nontraded and traded production are set to $\alpha^N = 0.45$ and $\alpha^T = 0.6$, following Buffie et al. (2012) for calibrating an average African economy. Consistent with the common value used for the depreciation rate of private capital, $\delta = 0.025$ implies an annual depreciation rate of 10 percent. The elasticity of substitution between variety of goods is set to $\theta = 6$, so a steady-state markup in the goods market is 20 percent, as calibrated in Galí and Monacelli (2005) for a small open economy. The intra-temporal elasticity of substitution between labor of the two sectors is set to $\chi^l = 0.6$. Horvath (2000) estimates this elasticity to be 1 using the U.S. sectoral data. Artuc et al. (2013) estimate that on average labor mobility costs are 4.26 times of annual wages in SSA countries, and only 2.41 times in developed countries. Thus, we assume less mobility in our model, compared to developed countries. The investment adjustment cost parameter is set to $\kappa = 1.4$, based on the only estimate we could locate for a developing country with the same specification (Mexico, Aguiar and Gopinath (2007)).

Since public capital consists mostly infrastructure, which has a lower depreciation rate than equipments, we assume $\delta^G = 0.012$ or annual rate of 5 percent. To calibrate public investment efficiency, we resort to the estimates by Pritchett (2000) for SSA economies and Hurlin and Arestoff (2010) for Colombia and Mexico. When the TFP growth rate is assumed to be zero, the former obtains an efficiency of 0.49 for SSA economies. The latter obtains an estimate around 0.4 for the two developing countries. Our baseline calibration assumes $\epsilon = 0.4$. To see the effects of investment efficiency on multipliers, analysis is also performed under a marginal efficiency of 0.8 and 1. The public investment to output ratio in the steady state is calibrated to be 0.045 to yield a public capital to output ratio of 0.37 in the initial steady state. The output elasticity with respect to public capital is selected to be $\alpha^G = 0.11$ such that in the steady state the annual net rate of return to public capital is 25 percent,
close to the median rate of return of World Bank projects in 2008 at 24 percent (International Bank for Reconstruction and Development and the World Bank (2010)).

The model has the income tax rate as a fiscal adjustment instrument. The adjustment magnitude, \( \gamma = 0.03 \), is chosen such that a minimal sufficient adjustment is implemented to ensure debt sustainability. For capital account openness in the private sector, the baseline calibration assumes fairly limited capital mobility by setting \( v = 1000 \). Sensitivity analysis explores less restricted mobility under \( v = 0.01 \) and 0.0001.

4. Government Consumption Effects

We analyze the effects of a government consumption shock and a public investment shock separately under three financing sources: domestic debt, external debt, and aid. A preserve-value, cumulative multiplier at \( k \) quarters after the shock is computed for both types of spending, as

\[
M^x(k) = \frac{\sum_{s=0}^{k} \left( \prod_{j=0}^{s} r_j^{-1} \right) \Delta x_s}{\sum_{s=0}^{k} \left( \prod_{j=0}^{s} r_j^{-1} \right) \Delta p_s^g g_s^j}, \quad x \in \{y, y^N, y^T, c, i, tb\}, \quad j \in \{C, I\}
\]

where \( \Delta \) denotes level changes from the steady state, \( r_0 \equiv 1 \), and \( r_t \equiv E_t \frac{R_t}{\pi_{t+1}} \) is the real interest rate. Tables 2 and 3 report the government consumption and public investment multipliers under the baseline calibration.

4.1. Different Financing Sources. Figure 1 compares the impulse responses to a government consumption increase under domestic debt (dotted-dashed lines), external debt (solid lines), and aid (dashed lines) financing. The size of the government consumption shock is 1 percent of steady-state output at time zero and decreases overtime according to the AR(1) process in (18). All the parameters across the three simulations are set to the baseline calibration, as listed in Table 1.
4.1.1. Domestic Debt Financing. When capital mobility is limited \((v = 1000)\), domestic financing produces the effects similar to those implied by typical neoclassical or New Keynesian models for a closed economy \(\text{(e.g., Baxter and King (1993), Forni et al. (2009), and Leeper et al. (2010))}\). Higher government consumption financed by domestic debt triggers a negative wealth effect among forward-looking savers as they anticipate higher future taxes to pay for the spending increase. The negative wealth effect drives up savers’ labor (hence aggregate labor due to the hand-to-mouth’s inelastic labor supply) and decreases their consumption.

In contrast, the hand-to-mouth households have the opposite consumption response for the first two quarters. Their positive consumption response is due to additional income brought by the government consumption increase. The short-run price rigidities prevent nontraded good prices from rising higher immediately than under flexible pricing, which exerts a stronger demand pressure on nontraded goods from the government consumption increase. The real wage rate in the nontraded good sector rises initially, despite more labor inputs, drawing labor to nontraded production. A higher wage rate thus raises hand-to-mouth households’ income and their consumption. Later, as higher government debt triggers tax rate increases and nominal rigidities dissipate, hand-to-mouth’s disposable income and consumption fall accordingly.

With domestic debt financing, the standard crowding-out story also prevails. As the government increases domestic borrowing, savers demand a higher interest rate to hold government debt. A higher interest rate crowds out private investment substantially. In later years, although investment declines become smaller, the investment level does not return to the steady-state ten years after the shock. In addition to the direct crowding-out effect by a government consumption increase, a persistent increase in the income tax rate also contributes to negative investment responses in the longer horizon.\(^{14}\)

\(^{14}\)The investment dynamics in the medium and long runs depend crucially on fiscal adjustment speeds. If \(\gamma\) in (19) is bigger, implying more aggressive adjustments, investment would fall more in earlier periods but return to the steady-state level sooner.
On the external side, the government’s relatively high demand for nontraded to traded goods pushes higher the relative price of nontraded goods, making the real exchange rate appreciate. The real appreciation, however, produces little movement in the current account. Although the real appreciation reduces the competitiveness of the traded good sector, weak domestic demand—as shown by the negative multipliers of consumption and investment (the top panel of Table 2)—also reduces demand for traded goods in the private sector, leaving the current account roughly unchanged.

With domestic debt financing, the impact output multiplier is 0.24 and falls to \(-0.71\) ten years after the initial shock (Table 2). Aside from the crowding out, the leak in the expansionary government consumption effects through negative traded output responses also contributes to the small multiplier for output. Compared to a fully closed economy, where the good account is also closed, the government consumption multiplier for output under domestic financing in our economy is smaller. When we assume that the government purchases only consist of nontraded goods (mimicking the close economy setting), the impact multiplier under domestic financing rises to 0.51, due to a much higher multiplier for the nontraded output.

4.1.2. External Debt Financing. Figure 1 shows that government consumption effects differ substantially between domestic debt (dotted-dashed lines) and external debt (solid lines) financing, particularly in consumption, investment, traded output, the real exchange rate, and the current account.

Large inflows of foreign exchange due to higher external borrowing appreciate the real exchange rate much more than with domestic financing. The more appreciated real exchange rate in turn reduces traded output by a larger magnitude in the first three years. The multiplier for traded output is \(-0.92\) by the end of the second year (compared to \(-0.16\) with domestic financing). Although perfectly competitive traded firms assumed in our model is an extreme assumption, the concentration of low-level technology manufacturing in LICs
suggests that firms are unlikely to have much pricing power in reality and hence can be quite susceptible to exchange rate fluctuations. Rodrik (2008) finds empirical support that the real exchange rate has an important impact on economic growth especially in developing countries, through the channel of traded good production as embedded in our model.

While traded output responses are quite negative with external debt financing, the output multiplier is bigger than with domestic debt financing. The impact multiplier is 0.43, compared to 0.24 with domestic financing. Moreover, the 10-year output multiplier remains positive at 0.17, versus −0.71 with domestic financing. The more expansionary effect of government consumption with external debt financing partly comes from less negative investment and more positive consumption responses. External financing expands the resource envelope for the domestic economy at least in the short run. Thus, it relieves the severe crowding-out effect of government consumption observed with domestic financing. Investment in the nontraded sector (not shown in the Figure) even turns positive for the first two years due to higher nontraded good demand. However, investment in the traded sector is more negative due to a more negative traded output response. The investment multiplier is about zero two years after the shock, compared to −0.66 with domestic financing.

In addition to a less negative investment response, the multiplier for consumption turns positive on impact with external debt financing. In particular, the hand-to-mouth’s consumption jumps up with external debt financing. The reduced crowding out effect further elevates the demand of nontraded goods, driving the real wage in the nontraded good sector and hence the overall wage rate rises higher with domestic financing. Given higher disposable income, the hand-to-mouth consume even more, which helps generate a positive multiplier for consumption.

In contrast to the little changes in the current account with domestic financing, the deficits rise substantially with external debt financing. The much appreciated real exchange rate, on the one hand, contracts traded output. On the other hand, it induces substitution of traded
goods in consumption and investment for nontraded goods, worsening the trade balance. The multiplier of trade balance is $-0.80$ at two years after the shock, compared to almost zero with domestic financing.

In the longer horizon, as the government spending effect wanes over time, the effects of fiscal adjustments become more dominant. Like domestic debt financing, external borrowing also triggers higher income tax rates, exerting a negative influence on investment and nontraded output. Traded output, instead, has a small positive response after year five. As the government provides the debt service payment to foreign creditors, it leads to a small real depreciation starting around year four, raising competitiveness of the traded good sector and its output. This positive response of traded output in later years mitigates the much negative traded output responses, producing a less negative, cumulative multiplier for traded output at $-0.68$ in 10 years, compared to $-0.92$ at two years after the shock.

Overall, government consumption is more expansionary with external than with domestic debt financing. A few caveats, however, are worth noting. Our analysis assumes a constant risk premium regardless of the debt level. Also, our thought experiment has the spending shock occur when the economy is at the steady-state debt level. When an economy is in a high-debt state, a debt-financed spending increase can prompt foreign creditors to demand a much higher premium. The magnitude of fiscal adjustments then has to be bigger to service debt and to maintain debt sustainability. Under those circumstances, the negative effects of fiscal adjustments on output can offset further the expansionary effects of government consumption increases.\footnote{See Bi et al. (2013) for government spending effects when an economy is near its fiscal limits.}

4.1.3. \textit{Aid Financing}. Another common source of external financing in LICs is aid from international donors. Figure 1 shows that the effects with aid financing (dashed lines) are similar to those with external debt financing (dotted-dashed lines) especially in the short run. The main difference between the two external financing methods is in fiscal adjustments. Aid
financing does not require subsequent repayments. Thus, the tax rate remains at the steady-state level. As savers do not anticipate higher future taxes, their consumption does not change much. The multiplier for consumption is 0.52 on impact, mainly due to the hand-to-mouth positive consumption response as with external debt financing. In the longer horizon, investment with aid financing outperforms that with external debt financing because the income tax rate is lower. The multiplier for output is the highest with aid financing among the three methods; the cumulative output multiplier 20 years after the shock is 0.08 with debt financing and is 0.19 with aid financing.

Like external debt, aid financing leads to substantial appreciation in the real exchange rate, lowering traded output. Spending aid domestically often raises the well-known concern of Dutch disease. Empirically, Rajan and Subramanian (2011) find that manufacturing production is negatively affected by aid inflows, as captured by the implication here for an aid-financed spending increase. In our specification, Dutch disease is a general equilibrium phenomenon that the real exchange rate appreciates to reallocate production factors from the traded to nontraded good sector in order to cope with higher nontraded good demand. If the loss of traded output is persistent, it is likely that Dutch disease can do actual damage to the traded good sector. Berg et al. (2010) captures this by learning-by-doing externality in the TPF of the traded sector ($z^T$ in (13)). In that case, the loss in traded output induces declines in productivity, which can be persistent and generate a bigger fall in traded output.

4.2. Twin Deficit Hypothesis. Our results that financing sources can matter for government spending multipliers have implications on the twin-deficit hypothesis for developing countries. In developed economies with high capital mobility, many factors, such as trade price elasticity (Erceg et al. (2005)) and non-Ricardian saving behaviors (Kumhof and Laxton (2009)) are found to be important for the co-movements between the two deficits. In an environment with limited capital mobility, the extent to which fiscal deficits are financed by external borrowing is crucial for the hypothesis to hold. As we have shown, the capital
account does not move much along with the increase of government debt with domestic debt financing, but debt (hence fiscal deficits) and current account deficits both rise with external financing, consistent with the twin deficit hypothesis. We do not model exports and imports separately here. Implicitly, with external financing higher domestic demand results in higher imports and the appreciated real exchange rate reduces exports, driving up current account deficits.

Ilzetzki et al.’s (2013) estimate for a large sample of developing countries find insignificant responses of current account balance to a government spending increase. Our finding provides a potential explanation why this may be the case. Since countries vary in shares of capital mobility and financing sources for each spending increase, the twin deficit hypothesis needs not hold for developing countries on average.

5. Public Investment Effects

Figure 2 plots the impulse responses with the three financing methods under the baseline calibration for a public investment increase, equal to 1 percent of steady-state output at time zero. Table 3 presents the multiplier results based on (26).

5.1. Differences from Government Consumption Effects. The key differences in the effects of the two types of spending is that public investment accumulates to productive public capital used in production (10) and (13). Within each financing method, the multipliers for public investment are slightly smaller in the short run but substantially bigger in the longer horizon as public capital is built up over time. Upon observing an increase in public investment (productive spending), the negative wealth effect triggered by a government spending increase (as observed for a government consumption shock) is weakened because households also expect a higher stock of public capital, which would increase future production and income. Thus, consumption falls less than domestic financing or rises more with the two external financing methods. Because of this more positive (or less negative) consumption
response, the impact public investment multipliers for investment are more negative than the government consumption multiplier. In the longer horizon, as public capital increases, the productivity of private capital and labor increase, output of both sectors and hence consumption is higher relative to those for a government consumption shock. Despite a higher income tax rate, investment does not fall below the steady-state level for a public investment shock. In this case, the positive incentive to invest because of higher marginal product of private capital outweighs the disincentive from a higher tax rate on capital income.

As we can see from comparison of the three lines in Figure 2, various financing methods play similar roles in public investment effects as in government consumption effects. Between domestic and external debt financing, private investment is crowded out more with domestic financing. Also, the public investment multipliers for output are bigger with the two external financing methods than with domestic financing.

5.2. Investment Efficiency. Despite that public investment is reasonably productive—with an annual rate of return to public capital at 25 percent in the steady state, Table 3 shows that the public investment multipliers for output are below 1. The main reason for a small multiplier lies in the low investment efficiency assumed in the baseline, where one dollar of investment expenditure only delivers 40 cents of public capital ($\epsilon = 0.4$). This section explores the importance of investment efficiency on public investment effects.

The thought experiment assumes that the marginal efficiency of public investment is higher than the steady-state efficiency level of 0.4. To implement this, $\epsilon$ in (17) is replaced by $\epsilon_t$ such that

$$
\epsilon_t = \bar{\epsilon} \left( \frac{G_t^I - G_t^G}{G_t^I} \right) + \epsilon \left( \frac{G_t^I}{G_t^I} \right), \text{ if } G_t^I > G_t^G,
$$

where $\bar{\epsilon} = 0.8$ or $1$ is the marginal efficiency applied to the public investment expenditure exceeding the steady-state level.\textsuperscript{16} The experiment can be interpreted as that a new public investment expenditure can increase output more than the steady-state level, where the additional marginal efficiency is $\epsilon_t$.

\textsuperscript{16}An alternative experiment is to change the entire investment efficiency to $\epsilon = 0.8$ or $1$; i.e., investment efficiency is higher both in the steady state and additional public investment. This experiment, however, changes the steady state of the model so the results are not directly comparable to those under the baseline.
investment project (introduced by the shock) has higher efficiency, despite the historical low efficiency. For example, Chinese investment in Africa becomes increasingly diversified to include utilities, ports, roads, bridges, etc. Many of these projects are collaborated as joint ventures between Chinese companies and local governments and are likely to have higher investment efficiency than traditional infrastructure projects implemented by local governments alone.

Table 4 presents the cumulative multipliers under $\bar{\epsilon} = 0.8$ (the top two panels) and $\bar{\epsilon} = 1$ (the bottom two). Between the domestic and external debt financing, a higher marginal efficiency increases the multipliers for output especially in the longer horizon. In particular with external debt financing, the 10-year and 20-year multiplier increase to 1.11 and 1.63 from 0.65 and 0.86, respectively. Similarly with aid financing (not reported in the Table), the 10-year and 20-year multipliers further increase to 1.12 and 1.76. The 20-year output multiplier obtained with the two external financing methods are in line with the long-run output multiplier at 1.6 estimated by Ilzetzki et al. (2013) for the developing countries (without LICs included in the sample). In our model, if full efficiency is imposed ($\bar{\epsilon} = 1$) as commonly assumed for advanced economies, the 20-year cumulative output multipliers are 2.02, and 2.15 with external debt and aid financing.

The powerful role of investment efficiency in raising the multiplier suggests that the key to take the advantage of public investment for promoting growth in LICs is to improve public investment efficiency. Our experiments involve a one-time public investment shock. While the effects are long-lasting, without continuous investment efforts to replenish depreciated capital, public capital eventually returns to the initial steady state level. To sustain the growth benefits of public investment, a permanent increase in the public investment level is also required such that the economy can move to a new steady-state with more public capital and output than in the initial steady state.

calibration. An economy with a higher public investment efficiency in the steady state implies a higher public capital to output ratio and hence lower marginal return to public capital (due to diminishing marginal return).
5.3. **Home Bias in Public Investment.** Another important factor in determining public investment multiplier, especially in the short run, is the degree of home bias in government purchases. The baseline assumes that both government consumption and public investment consist of 70 percent of nontraded goods ($\varphi^G = 0.7$). This assumption may seem reasonable for overall spending in higher income countries. In LICs many public investment projects, however, rely on imports of tools, equipments, engineers, and skilled workers to a large extent; thus, the degree of home bias in investment spending can be much lower than 0.7. We explore an alternative assumption that home bias in public investment is 0.4 and remains at 0.7 in government consumption.

To formalize this exercise, let $\varphi^{GC} = 0.7$ and $\varphi^{GI} = 0.4$ be the degrees of home bias in $g^C_t$ and $g^I_t$. Then, (16) is replaced with the following relative prices of government consumption and public investment to the CPI:

$$p^{GC}_t = \left[ \varphi^{GC} (p^N_t)^{(1-\chi)} + (1 - \varphi^{GC}) (s_t)^{(1-\chi)} \right]^{\frac{1}{1-\chi}},$$  \hspace{1cm} (28)$$

$$p^{GI}_t = \left[ \varphi^{GI} (p^N_t)^{(1-\chi)} + (1 - \varphi^{GI}) (s_t)^{(1-\chi)} \right]^{\frac{1}{1-\chi}}. \hspace{1cm} (29)$$

Also, the term $p^{G}_t (g^C_t + g^I_t)$ in (15) and (25) is replaced by $p^{GC}_t g^C_t + p^{GI}_t g^I_t. \hspace{1cm} (25)$

Figure 3 compares impulse responses for the variables that differ much between the baseline degree of home bias (solid lines) and the current specification with $\varphi^{GI} = 0.4$. The left (right) column assumes domestic (external) debt financing.\(^{18}\) Table 5 reports the public investment multipliers: The top two panels assume the baseline low investment efficiency of 0.4, and the bottom two assume the higher marginal investment efficiency with $\bar{\epsilon} = 0.8$.

Not surprisingly, under $\varphi^{GI} = 0.4$ traded output outperforms nontraded output, opposite to the pattern under the baseline degree of home bias. With domestic debt financing,\(^{17}\) in this new specification, we assume $p^{GC} = p^{GI} = 1$ in the steady state. In the original specification, $p^G = 1$ in the steady state, and the two specifications have the same steady state.\(^{18}\) The results with aid financing are skipped because of the similarity to external debt financing during the first five years.
higher government demand for traded goods leads to a real 
*depreciation*, generating a positive multiplier for traded output, reversing the real appreciation in the previous analysis.\textsuperscript{19} Consistent with Penati (1987), government purchases that consist of more traded goods can generate a real depreciation.

Also with domestic debt financing, consumption under \( \varphi^{GI} = 0.4 \) is more negative than the baseline. Price rigidities prevent the nontraded goods price from falling further, restricting the shifting of households’ demand toward nontraded goods. The declined production in the nontraded sector lowers its real wage and the overall wage rate, in contrast to the rising wage rate under the baseline degree of home bias. Falling labor income makes the hand-to-mouth’s consumption turn negative initially. As the nontraded output response turns negative, the impact multiplier for output also turns negative at \(-0.08\), compared to 0.20 under the baseline degree of home bias and investment efficiency. With external debt financing, the impact output multiplier remains positive but is also smaller than the one under the baseline degree of home bias; The impact output multiplier falls from 0.40 to 0.22.

When the marginal investment efficiency is higher (\( \bar{\epsilon} = 0.8 \)), the bottom two panels of Table 5 show similar comparison results between the two degrees of home bias. In summary, a lower degree of home bias in government spending introduces more leak in the short-run expansionary effects. As most demand of higher government purchases is met by traded goods (or imports), and the Keynesian effects are weakened to stimulate domestic production. In the longer horizons, as productive public capital is built up, the positive output effect of public investment is mainly supported by the higher productivity of private production factors. The difference in cumulative multipliers for output thus narrows for various degrees of home bias in government purchases. For example, the 10-year public investment multiplier

\textsuperscript{19}Empirically, whether a government spending increase results in a real depreciation or appreciation is inconclusive. Many empirical studies using data of developed countries find real depreciation (e.g., Kim and Roubini (2008) and Monacelli and Perotti (2010)). Ilzetzki et al. (2013) find a brief but significant real appreciation for the developing country sample.
for output under $\varphi^{GI} = 0.4$ is 0.59 with domestic financing, versus 0.65 under the baseline degree of home bias.

6. Sensitivity Analysis on Capital Mobility

So far, we have shown that government spending effects can be quite different with various domestic/external financing sources. Throughout the simulations, international capital mobility in the private sector is fairly limited. In this section, we use a government consumption increase to show that the importance of accounting for domestic/external financing sources in government spending effects largely vanishes when capital mobility is high.

Figure 4 compares impulse responses for a government consumption shock (of the same size in Figure 1) with external debt financing under various capital account mobility: $v = 1000$ (baseline), $v = 0.01$, and $v = 0.0001$. With higher capital mobility, households counteract the government’s external borrowing, and the economy as a whole borrows less externally. An initially much appreciated real exchange rate from rising external government debt discourages households from borrowing externally because the marginal benefit of additional borrowing in units of local goods decreases. In equilibrium, a smaller total external debt generates a less appreciated real exchange rate. Meanwhile, as households retire external debt, fewer resources are available for the domestic economy, producing similar government consumption effects with domestic debt financing, as shown in Figure 5 for government consumption effects under $v = 0.0001$.

7. Conclusion

We construct a DSGE model to study the government consumption and public investment effects in LICs. The framework takes into account LIC specific features, important for
assessing government spending effects. These features include a large share of hand-to-mouth households, fairly limited international capital mobility in the private sector, allowing for external debt and aid financing, and low public investment efficiency.

With limited capital mobility, domestic/external financing sources for government spending matter for either type of the spending effects. External financing increases the resources available for the economy at least in the short run, mitigating the crowding-out effects of government spending. A large inflow of foreign exchange with external financing, however, appreciates the real exchange rate and reduces traded output. The cumulative multiplier for output in the longer horizon with aid financing is generally higher than with external debt financing because persistent fiscal adjustments (such as higher income tax rates) triggered by government debt have a negative effect on output. The general impression that capital scarcity in LICs renders productive public investment a large positive growth effect may not be readily delivered in LICs. While the return to public capital can be high, the output multiplier can still be much below 1 if the public investment efficiency remains low.

The model is currently calibrated to an average LIC. When a sufficient length of quarterly data becomes available for some LIC country, the model should be fitted to data to evaluate the quantitative importance of LIC features added in this model. Moreover, as we have shown that some parameters, such as public investment efficiency and degree of home bias in government purchases, is crucial in determining the size of public investment multipliers, micro evidence or efforts in searching for country-specific estimates, such as investment efficiency estimate pursued by Hurlin and Arestoff (2010) for Mexico and Colombia, would be important for applying the model to evaluate country-specific spending effects.

Acknowledgment: We thank Andrew Berg, Eric Leeper, David Savitski, and Todd Walker for helpful comments and Pranav Gupta for great research assistance. This paper is part of a research project supported by the U.K.’s Department for International Development. The views expressed herein are those of the authors and should not be attributed to the DFID, the IMF, its Executive Board, or its management.
This appendix describes the equilibrium conditions for optimization problems of the households and firms. When solving the savers’ utility maximization problem, let $\lambda^a_t$ be the Lagrangian multiplier for the budget constraint, and $Q^N_t$ and $Q^T_t$ be the Lagrangian multipliers for the law of motion of capita in each sector. Then, Tobin’s $Q$ for $k^N_t$ and $k^T_t$ are $q^N_t = \frac{Q^N_t}{\lambda^a_t}$ and $q^T_t = \frac{Q^T_t}{\lambda^a_t}$.

Savers’ first order condition (FOC) for $c^a_t$:

$$\lambda^a_t = (c^a_t)^{-\sigma} \quad (A.1)$$

Savers’ FOC for $b^a_t$:

$$\lambda^a_t = \beta E_t \left( \frac{\lambda^a_t R_t}{\pi_{t+1}} \right) \quad (A.2)$$

Savers’ FOC for $b^{hs,a}_t$:

$$s_t \lambda^a_t \left[ 1 - v \left( b^{hs,a}_t - 1 \right) \right] = \beta E_t \left( \frac{\lambda^a_{t+1} s_{t+1} R^*}{\pi^*} \right) \quad (A.3)$$

Savers’ FOC for $m^a_t$:

$$(m^a_t)^{-\xi} = \lambda^a_t - \beta \frac{\lambda^a_{t+1}}{\pi_{t+1}} \quad (A.4)$$

Combining (A.3) with (A.4) yields

$$(m^a_t)^{-\xi} = \lambda^a_t \left( \frac{R_t - 1}{R_t} \right) \quad (A.5)$$

In its log-linearized form (denoted variables in percentage deviation from steady state by $\hat{}$), the demand function for the real money balance is

$$\hat{m}^a_t = \frac{\sigma}{\xi} c^a_t - \frac{1}{\xi (R - 1)} \hat{R}_t, \quad \xi, \sigma > 0, R > 1, \quad (A.6)$$

where $-\frac{1}{\xi (R - 1)}$ is the interest elasticity for the demand of the real money balance.

Savers’ FOC for $l^a_t$:

$$(l^a_t)^\psi = \lambda^a_t w_t (1 - \tau_t) \quad (A.7)$$

Savers’ FOC for $k^N_t$:

$$q^N_t = \beta_t E_t \frac{\lambda^a_t}{\lambda^a_t} \left[ (1 - \tau_{t+1}) r^N_{t+1} - \frac{\kappa}{2} \left( \frac{i^N_{t+1}}{k^N_{t+1}} - \delta \right)^2 + \kappa \left( \frac{i^N_{t+1}}{k^N_{t+1}} - \delta \right) \left( \frac{i^N_{t+1}}{k^N_{t+1}} \right) + q^N_{t+1} (1 - \delta) \right] \quad (A.8)$$
Savers’ FOC for $k_t^T$:

$$q_t^T = \beta_t E_t \frac{\lambda^{T+1}_t}{\lambda_t} \left[ (1 - \tau_t) r_{t+1}^T - \frac{\kappa}{2} \left( \frac{i_{t+1}^T}{k_t^T} - \delta \right)^2 + \kappa \left( \frac{i_{t+1}^T}{k_t^T} - \delta \right) \left( \frac{i_{t+1}^T}{k_t^T} \right) + q_{t+1}^T (1 - \delta) \right]$$  \hspace{1cm} (A.9)

Savers’ FOC for $i_t^N$

$$q_t^N = 1 + \kappa \left( \frac{i_t^N}{k_{t-1}^N} - \delta \right)$$  \hspace{1cm} (A.10)

Savers’ FOC for $i_t^T$

$$q_t^T = 1 + \kappa \left( \frac{i_t^T}{k_{t-1}^T} - \delta \right)$$  \hspace{1cm} (A.11)

Labor supplied to the traded good sector:

$$l_t^N = \varphi^l \left( \frac{w_t^N}{w_t} \right)^{\chi^l} l_t$$  \hspace{1cm} (A.12)

Labor supplied to the nontraded good sector:

$$l_t^T = \varphi^l \left( \frac{w_t^T}{w_t} \right)^{\chi^l} l_t$$  \hspace{1cm} (A.13)

The aggregate real wage rate:

$$w_t = \left[ \varphi^l \left( w_t^N \right)^{1+\chi^l} + (1 - \varphi^l) \left( w_t^T \right)^{1+\chi^l} \right]^{\frac{1}{1+\chi^l}}$$  \hspace{1cm} (A.14)

Nontraded good firms’ FOC for $p_t^N$:

$$\Pi_t^N = \beta E_t \left( \frac{\lambda^{T+1}_t}{\lambda_t} \Pi_{t+1}^N \frac{y_{t+1}^N p_{t+1}^N}{y_t^N p_t^N} \right) + \frac{\theta}{\alpha^N \zeta (1 - \nu)} \frac{w_t^N l_t^N}{p_t^N y_t^N} + \frac{1 - \theta}{\zeta}$$  \hspace{1cm} (A.15)

where $\Pi_t^N = \frac{\pi^N_t}{\pi_{t-1}^N} \left( \frac{\pi^N_t}{\pi_{t-1}^N} - 1 \right)$.

Nontraded good firms’ FOC for $l_t^N$:

$$w_t^N l_t^N = (1 - \nu) p_t^N \alpha^N y_t^N$$  \hspace{1cm} (A.16)

Nontraded good firms’ FOC for $k_t^N$: is

$$r_t^N k_{t-1}^N = (1 - \nu) p_t^N (1 - \alpha^N) y_t^N$$  \hspace{1cm} (A.17)

The above two equations can be combined as

$$(1 - \alpha^N) w_t^N l_t^N = \alpha^N r_t^N k_{t-1}^N$$  \hspace{1cm} (A.18)
Traded good firms’ FOC for $l^T_t$:

$$w^T_t l^T_t = (1 - \iota) s_t \alpha^T y^T_t$$  \hspace{1cm} (A.19)

Traded good firms’ FOC for $k^T_t$:

$$r^T_t k^T_{t-1} = (1 - \iota) s_t (1 - \alpha^T) y^T_t$$  \hspace{1cm} (A.20)
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Table 1. Baseline Calibration and Some Steady-State Values
### Table 2. Government consumption multipliers: baseline calibration.
See (26) for the multiplier calculation.

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### Table 3. Public investment multipliers: baseline calibration. See (26) for the multiplier calculation.

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### Table 4. Public investment multipliers: higher investment efficiency. See (26) for the multiplier calculation.

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### Table 5. Public investment multipliers: less home bias in $G_t^I$ ($\varphi^{GI} = 0.4$). See (26) for the multiplier calculation.

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<td>0.58</td>
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Figure 1. Impulse responses to a government consumption increase: baseline calibration. The x-axis is in years. The y-axis is in percent deviation from the steady state unless noted in parentheses.
Figure 2. Impulse responses to a public investment: baseline calibration. The x-axis is in years. The y-axis is in percent deviation from the steady state unless noted in parentheses.
Figure 3. Public investment effects with different degree of home bias. The baseline assumes $\varphi^G = 0.7$. The less-home-bias scenario assumes $\varphi^{GC} = 0.7$ and $\varphi^{GI} = 0.4$. The x-axis is in years. The y-axis is in percent deviation from the steady state unless noted in parentheses.
Figure 4. Government consumption effects under different capital mobility. The x-axis is in years. The y-axis is in percent deviation from the steady state. The size of the government consumption shock is the same as Figure below.

Figure 5. Government consumption effects with nearly open capital account. The x-axis is in years. The y-axis is in percent deviation from the steady state.
References


Bi, H., Shen, W., Yang, S.-C. S., 2013. Fiscal limits, external debt, and fiscal policy in developing countries. Manuscript, International Monetary Fund.


International Monetary Fund, 2011. Revenue Mobilization in Developing Countries. Washington, D.C.


