Long-duration Bonds and Sovereign Defaults*

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Abstract

This paper extends the baseline framework used in recent quantitative studies of sovereign default by assuming that the governments can borrow using long-duration bond. This contrast with previous studies, which assume that the government can borrow using bonds that mature after one quarter. Once we assume that the government issues bonds with a duration similar to the average duration of sovereign bonds emerging economies, the model is able to generate a substantially higher and more volatile interest rate. This narrows the gap between the predictions of the model and the data, which indicates that the introduction of long-duration bonds may be a useful tool for future research about emerging economies. Our analysis is also relevant for the study of other credit markets.

JEL classification: F34, F41.

Keywords: Sovereign Default, Endogenous Borrowing Constraints, Bond Duration, Debt Dilution, Markov Perfect Equilibrium.

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1 Introduction

Business cycles in small emerging economies differ from those in developed economies. Emerging economies feature interest rates that are higher, more volatile and countercyclical (interest rates are usually acyclical in developed economies), and have higher output volatility, higher volatility of consumption relative to income, and more countercyclical net exports (see Aguiar and Gopinath (2007), Neumeyer and Perri (2005), and Uribe and Yue (2006)). Due to the high volatility and countercyclicality of the interest rate in emerging economies, a state-dependent interest rate scheme is a key ingredient in any model designed to explain the cyclical behavior of aggregate quantities and prices in these economies. In this respect, some studies assume an exogenous interest rate scheme.\textsuperscript{1} Other studies provide microfoundations for the interest rate scheme based on the risk of default.\textsuperscript{2} This is the approach taken by recent quantitative studies of sovereign default, which use modifications of the framework proposed by Eaton and Gersovitz (1981).\textsuperscript{3} The setup studied in the present paper belongs to this second class of models.

As in previous studies of sovereign default, we analyze a small open economy that receives a stochastic endowment stream of a single tradable good. The objective of the government is to maximize the expected utility of private agents. The government makes two decisions in every period. First, it decides whether to default on previously issued debt. The cost of defaulting is represented by an endowment loss that is incurred in the default period. Second, the government decides how much to borrow or save. The government can borrow (save) by issuing (buying) non-contingent bonds that are priced in a competitive market inhabited by a large number of identical risk-neutral lenders. Lenders have perfect information regarding the economy’s endowment.

The main difference between this paper and previous studies is the duration of bonds issued

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\textsuperscript{1}See, for instance, Aguiar and Gopinath (2007), Neumeyer and Perri (2005), Schmitt-Groh\’e and Uribe (2003), and Uribe and Yue (2006).


\textsuperscript{3}See Aguiar and Gopinath (2006), Arellano (2008), Arellano and Ramanarayanan (2006), Bai and Zhang (2006), Cuadra and Sapriza (2006a,b), D’Erasmo (2007), Eyigungor (2006), Hatchondo et al. (2006a,b, 2007b), Lizarazo (2005, 2006), Mendoza and Yue (2007), and Yue (2005). These models share the same blueprints with the models used in quantitative studies of household bankruptcy—see, for example, Athreya (2002), Athreya et al. (2007a,b), Chatterjee et al. (2005), Chatterjee et al. (2007), Li and Sarte (2006), Livshits et al. (2007), and Sánchez (2008).
by the government. Previous quantitative studies assume that sovereign bonds mature after one quarter. Instead, we allow the government to borrow using long-duration bonds. We assume that the government issues bonds that pay an infinite stream of coupons until a default is declared. The coupon payments promised over the life of a bond decrease at a constant rate. This assumption allows us to introduce debt instruments with a long duration in a simple and tractable way: the number of state variables is independent of the duration of debt.

We show that the predictions of the model change significantly once we assume that the government issues bonds with a duration that is close to the average duration in emerging economies—around four years, as documented by Cruces et al. (2002) and Cunningham et al. (2001). The mean annual spread—extra yield over the risk-free rate—increases from 0.1% in the model with one-quarter bonds to 2.5% in the model with four-year bonds. The spread volatility is also significantly higher when four-year bonds are assumed: it increases from 0.03% to 0.2%. The model with long-duration bonds is also able to replicate other salient features of emerging economies. Thus, our results narrow the gap between the predictions of the baseline model of sovereign default and the data, which indicates that introducing long-duration bonds may be a useful tool for future research about emerging economies. For instance, the modeling strategy undertaken in this paper would simplify significantly the study of the optimal maturity structure of sovereign debt (see, for example, Alfaro and Kanczuk (2007), Arellano and Ramanarayanan (2006), Bi (2006), and Broner et al. (2007)), and the study of the costs of debt dilution (see, for instance, Bi (2006), Bizer and DeMarzo (1992), Detragiache (1994), Hatchondo and Martinez (2007), Kletzer (1984), Sachs and Cohen (1982), and Tirole (2002)).

The rest of the article proceeds as follows. Section 2 introduces the model. Section 3 presents a recursive formulation of the model. Section 4 discusses the parameter values used to solve the model. Section 5 presents the results. Section 6 concludes.
2 The model

There is a single tradable good. The economy receives a stochastic endowment stream of this good, where

$$\log(y_t) = (1 - \rho) \mu + \rho \log(y_{t-1}) + \varepsilon_t,$$

with $|\rho| < 1$, and $\varepsilon_t \sim N(0, \sigma^2)$.

The objective of the government is to maximize the present expected discounted value of future utility flows of the representative agent in the economy, namely

$$E\left[\sum_{t=0}^{\infty} \beta^t u(c_t)\right],$$

where $\beta$ denotes the subjective discount factor and the utility function is assumed to display a constant coefficient of relative risk aversion, denoted by $\sigma$. That is,

$$u(c) = \frac{c^{(1-\sigma)} - 1}{1 - \sigma}.$$

The government makes two decisions in each period. First, it decides whether to default. A default implies that the government repudiates all current and future debt obligations implied by previously issued debt. Second, the government decides the amount of bonds that are going to be purchased or issued in the current period.

We assume that a bond issued in period $t$ promises an infinite stream of coupons, which decrease at a constant rate $\delta$. In particular, a bond issued in period $t$ promises to pay one unit of the good in period $t + 1$ and $(1 - \delta)^{s-1}$ units in period $t + s$ with $s \geq 2$. Note that the one-period-bond model is a particular case of our framework: it corresponds to $\delta = 1$.

Assuming a bond with the described coupon structure allows us to consider debt with duration longer than one period without increasing the dimensionality of the state space. If we would have assumed that the government could issue zero coupon bonds that mature $n$ periods ahead, it would have been necessary to keep track of the bonds issued from periods $t - 1$ to $t - n$.

Assuming that the coupon payments promised in a bond decrease over time allows us to calibrate the bond duration, which in our framework depends on $\delta$. We use the standard Macaulay definition of duration, according to which the duration of a bond is given by the sum of the
periods until each coupon payment divided by the bond price, when each period is weighted by
the discounted value of the corresponding coupon payment. That is,

\[ D \equiv \sum_{s=1}^{\infty} \frac{s C_s}{q (1+r^*)^s}, \]

where \( D \) denotes the duration of a bond that promises a coupon \( C_s \) in period \( s \) and has a price \( q \). The term \( r^* \) denotes the constant per-period yield delivered by the bond. According to this
definition and the coupon structure assumed in this paper, the duration of a bond is given by

\[ D = \frac{1 + r^*}{\delta + r^*}. \]

As in previous studies of sovereign default, we assume that there is a continuum of risk
neutral lenders who can borrow or lend at the risk-free rate \( r \). Lenders have perfect information
regarding the economy’s endowment.

Let \( b \) denote the amount of outstanding coupon claims at the beginning of the period. A
negative value of \( b \) implies that the government was a net issuer of bonds in the past. As
4 Each period, the borrowing game unfolds as follows. First, the government announces the size of
the current bond issuance. This will determine the coupon obligations at the beginning of the
following period, which are denoted by \( b' \). Then, each lender offers the government a price at
which he is willing to buy the bonds the government is issuing or to sell the bonds the government
wants to buy. The government then chooses one of the lenders with whom it will perform the
transaction. Finally, the transaction is performed and the current-period borrowing game ends.
This borrowing game is equivalent to the one assumed in recent quantitative studies of sovereign
default.5

4 We denote by \( B_t \) the nominal value of period-\( t \) debt, defined as the present value of the coupons issued in
previous periods, where future cash flows are discounted at the risk-free interest rate. Namely,

\[ B_t = \frac{b_t}{r + \delta}. \]

5 Most of these studies describe a setup in which lenders offer to the government a price menu such that the
price associated with each borrowing level satisfies the lenders’ zero profit condition. After that, the government
chooses a point (borrowing and price) on the menu. It is easy to show that offering this price menu is an
equilibrium strategy of the game we describe.
The cost of a default is represented by an output loss \( \phi (y) \), which is incurred in the default period and may depend on the default-period output level \( y \). This assumption intends to capture the disruptions in economic activity caused by a default decision. It has been argued that a sovereign default increases the borrowing cost of domestic firms and, thus, it reduces output. Using micro-level data, Arteta and Hale (2006) find that sovereign debt crises are systematically accompanied by a large decline in foreign credit to domestic private firms. This may be the case because a sovereign default may indicate to investors a higher risk of expropriation or bad economic conditions, and therefore, it may reduce firms’ net worth and their ability to borrow (see Sandleris (2006) and the references therein). IMF (2002), Kumhof (2004), and Kumhof and Tanner (2005) discuss how financial crises that lead to severe recessions follow a sovereign default. Similarly, Kaminsky and Reinhart (1999) show that currency devaluations in developing countries tend to cause banking problems. Kobayashi (2006) presents a model in which a shock that disturbs the payments system causes a decrease in aggregate productivity. Mendoza and Yue (2007) study the link between sovereign-default risk and output.

As in recent studies of sovereign default, we focus on symmetric Markov Perfect Equilibria. That is, we assume that in each period, the government’s equilibrium default and borrowing decisions depend only on payoff relevant state variables, and that the government decides whether to pay back its current debt and its current-period borrowing level taking as given its future default decisions and borrowing levels. If the government could commit to future default and borrowing decisions, it would internalize the effect of its period-\( t \) decisions in every period before \( t \). This is not the case in Markov Perfect Equilibria.

3 Recursive formulation

Let \( V(b, y) \) denote the government’s value function at the beginning of a period, that is, before the default decision is made. Let \( \bar{V}(d, b, y) \) denote its value function after the default decision has been made. Let \( F \) denote the cumulative distribution function followed by next-period endowment (\( y' \)). For any price function \( q(b', y) \) at which the government can buy or sell bonds, the function \( V(b, y) \) satisfies the following functional equation:
\[
V(b, y) = \max_{d \in \{0, 1\}} \{d \tilde{V}(1, b, y) + (1 - d) \tilde{V}(0, b, y)\},
\]

where

\[
\tilde{V}(d, b, y) = \max_{b' \in \mathbb{R}} \left\{ u(c) + \beta \int V(b', y') F(dy'| y) \right\},
\]

\[
c = y - d\phi(y) + (1 - d)b - q(b', y)(b' - (1 - d)(1 - \delta)b) \geq 0.
\]

The term \(b' - (1 - d)(1 - \delta)b\) represents the current borrowing decision. If, for example, the government chooses \(b' = (1 - d)(1 - \delta)b\), it is neither issuing nor buying bonds. That is, the amount of coupons that will mature next period are solely determined by past debt issuances and the current default decision. If the government chooses \(b' < (1 - d)(1 - \delta)b\), it is issuing new bonds, and if \(b' > (1 - d)(1 - \delta)b\), the government is purchasing bonds.

The bond price that satisfies the lenders’ zero-profit condition is given by following functional equation:

\[
q^{ZP}(b', y) = \frac{1}{1 + r} \int [1 - h(b', y')] F(dy'| y) + \frac{1 - \delta}{1 + r} \int [1 - h(b', y')] q^{ZP}(g(h(b', y'), b', y'), y') F(dy'| y),
\]

where \(h(b, y)\) and \(g(d, b, y)\) denote the default and borrowing rules that lenders expect the government to follow in the future. The function \(g(d, b, y)\) determines the value of coupons that will mature next period. Note that the government computes its current default and borrowing decisions taking as given its future default and borrowing decisions, that is, \(h(b, y)\) and \(g(d, b, y)\).

In equilibrium, the optimal default and borrowing rules that solve problems (1) and (2) are equal to \(h(b, y)\) and \(g(d, b, y)\) at all possible states.

The first term in the right-hand side of equation (3) equals the expected value of the next-period coupon payment promised in a bond. The second term in the right-hand side of equation (3) equals the expected value of all other future coupon payments, which are summarized in the next-period bond price. Note that the second term is not present in the model with one-period bonds \((\delta = 1)\).

**Definition 1** A recursive competitive equilibrium is characterized by

1. a set of value functions \(\tilde{V}(d, b, y)\) and \(V(b, y)\),
2. a default decision rule \( h(b, y) \) and a borrowing rule \( g(d, b, y) \),

3. and a bond price function \( q^{ZP}(b', y) \),

such that:

(a) given \( h(b, y) \) and \( g(d, b, y) \), \( V(b, y) \) and \( \tilde{V}(d, b, y) \) satisfy functional equations (1) and (2), respectively, when the government can trade bonds at \( q^{ZP}(b', y) \);

(b) given \( h(b, y) \) and \( g(d, b, y) \), the bond price function \( q^{ZP}(b', y) \) offered to the government satisfies the lenders’ zero-profit condition implicit in equation (3);

(c) the default rule \( h(b, y) \) and borrowing rule \( g(d, b, y) \) solve the dynamic programming problem defined by equations (1) and (2), when the government can trade bonds at \( q^{ZP}(b', y) \).

As discussed by Krusell and Smith (2003), typically, there is a problem of multiplicity of Markov-perfect equilibria in infinite-horizon economies. In order to avoid this problem, we analyze the equilibrium that arises as the limit of the finite-horizon economy.

4 Parameterization

The model is solved numerically using value function iteration and spline interpolation.\(^6\) Table 1 presents the parameterization used in this paper.

We assume a coefficient of relative risk aversion of 2, which is within the region of accepted values in studies of real business cycles. A period in the model refers to a quarter. The risk-free interest rate is set equal to 1%. As in Hatchondo et al. (2006b), the parameter values that govern the endowment process are chosen so as to mimic the behavior of GDP in Argentina from the fourth quarter of 1993 to the third quarter of 2001. The parameterization of the output process

\(^6\)The algorithm finds two value functions, \( \tilde{V}(1, b, y) \) and \( \tilde{V}(0, b, y) \). Convergence in the equilibrium price function \( q(b', y) \) in equation (3) is also assured.
is similar to the parameterization used in other studies that consider a longer sample period (see Aguiar and Gopinath (2006)).

As in previous studies (see, for example, Aguiar and Gopinath (2006)), we assume that \( \phi(y) = \lambda y \). The value of \( \lambda \) is taken from Hatchondo et al. (2007b). They find that in a baseline model of sovereign default, a value of \( \lambda \) equal to 8.3% delivers a default cost that is equivalent to the cost implied by the process of output loss assumed in Aguiar and Gopinath (2006) (a loss of 2% per period for an average of 10 periods).

The discount factor is relatively low but higher than what is assumed in previous studies that assume a cost of default similar to the one assumed in this paper (for instance, Aguiar and Gopinath (2006) assume \( \beta = 0.8 \)). Low discount factors may be a result of political polarization in emerging economies (see Amador (2003) and Cuadra and Sapriza (2006a)).

We present results for two values of \( \delta \). When \( \delta = 1 \), bonds have a duration of one period, which is the case analyzed in previous studies. When \( \delta = 0.053125 \), bonds have a duration of four years if future coupon payments are discounted at the risk-free rate \( r \) (the duration would be 3.66 years if future coupon payments where discounted using the average yield implicit in the equilibrium bond prices we find in Section 5). Cruces et al. (2002) report that the average duration of Argentinean bonds included in the EMBI index was 4.13 years in 2000. This number is not significantly different from what is observed in other emerging economies. Cruces et al.
(2002) find an average duration of 4.77 with a standard deviation of 1.52, based on a sample of 27 emerging economies.

5 Results

This section focuses on the ability of the model to replicate some stylized facts about the macroeconomic behavior of emerging economies. We simulate the model for a number of periods that allows us to extract 500 samples of 32 consecutive periods before a default. We focus on samples of 32 periods because we want to compare the artificial data generated by the model with Argentine data from the fourth quarter of 1993 to the third quarter of 2001. This is the same period that is considered in Hatchondo et al. (2006b). The macroeconomic behavior observed over that period displays the same qualitative features that can be observed using a longer sample period or using data from other emerging markets (see, for example, Aguiar and Gopinath (2007), Neumeyer and Perri (2005), and Uribe and Yue (2006)). The only exception is that the volatility of consumption is slightly lower than the volatility of income, while emerging market economies tend to display a higher volatility of consumption relative to income. Finally, in order to facilitate the comparison with the data, we only consider sample paths where the last default was declared at least two periods before the beginning of each sample.

Table 2 reports moments in the data and in our simulations.\(^7\) The trade balance \((TB)\) is expressed as a fraction of output \((Y)\). The interest rate spread \((R_s)\) represents the margin of extra yield over the risk-free rate and is expressed in annual terms.\(^8\) The logarithm of income and consumption are denoted by \(y\) and \(c\), respectively. The standard deviation of \(x\) is denoted by \(\sigma(x)\) and is reported in percentage terms. The correlation between \(x\) and \(z\) is denoted by

\(^7\)The data for output, consumption, and trade balance was obtained from the Argentinean Finance Ministry. The spread before the first quarter of 1998 is taken from Neumeyer and Perri (2005), and from the EMBI Global after that.

\(^8\)Let \(r^*\) denote the per-period constant yield implied by a price \(q(b', y)\), namely

\[
r^* = \frac{1}{q(b', y)} - \delta.
\]

The annualized spread is given by \(R_s = \left(\frac{1 + r^*}{1 + r}\right)^4 - 1\).
\( \rho(x, z) \). The moments were computed using the detrended series, where the trends are found using the Hodrick-Prescott filter with a smoothing parameter of 1,600. The moments reported in the table are chosen so as to evaluate the ability of the model to replicate the distinctive business cycle properties of emerging economies. Relative to developed economies, emerging economies feature higher, more volatile and countercyclical interest rates; higher volatility of consumption relative to income; and more countercyclical net exports.

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>One quarter</th>
<th>Four years</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma(y) )</td>
<td>3.17</td>
<td>3.15</td>
<td>3.07</td>
</tr>
<tr>
<td>( \sigma(c) )</td>
<td>2.98</td>
<td>3.24</td>
<td>3.14</td>
</tr>
<tr>
<td>( \sigma(TB/Y) )</td>
<td>1.35</td>
<td>0.17</td>
<td>0.12</td>
</tr>
<tr>
<td>( \sigma(R_s) )</td>
<td><strong>2.51</strong></td>
<td><strong>0.03</strong></td>
<td><strong>0.23</strong></td>
</tr>
<tr>
<td>( \rho(c, y) )</td>
<td>0.97</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>( \rho(TB/Y, y) )</td>
<td>-0.69</td>
<td>-0.46</td>
<td>-0.56</td>
</tr>
<tr>
<td>( \rho(R_s, y) )</td>
<td>-0.65</td>
<td>-0.94</td>
<td>-0.86</td>
</tr>
<tr>
<td>( \rho(R_s, TB/Y) )</td>
<td>0.56</td>
<td>0.73</td>
<td>0.81</td>
</tr>
<tr>
<td>( E(R_s) )</td>
<td><strong>7.44</strong></td>
<td><strong>0.11</strong></td>
<td><strong>2.54</strong></td>
</tr>
</tbody>
</table>

Table 2: Business cycle statistics. The second column is computed using data from Argentina from 1993 to 2001. The third column reports the mean value of each moment over the first 500 samples of 32 periods before a default in an economy where the government issues bonds with a duration of one quarter. The fourth column reports the mean value of each moment over the first 500 samples of 32 periods before a default in an economy where the government issues bonds with a duration of four years.

Table 2 shows that the mean annual spread increases from 0.1% in the model with one-quarter bonds to 2.5% in the model with four-year bonds. The spread volatility is also significantly higher in the model with four-year bonds; it increases from 0.03% to 0.2%. The table also shows that introducing four-year bonds does not damage the ability of the model to replicate other features of the data.\(^9\) The difference in the behavior of the spread is also apparent in Figure 1, which

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\(^9\)As in the benchmark case with one-quarter bonds, debt levels obtained with four-year bonds are low: the nominal debt level as a fraction of quarterly output increases from 7.5% in the model with one-quarter bonds to 8.4% in the model with four-year bonds. As it is illustrated in Figure 2, equilibrium debt levels generated by the model are directly linked to the cost of defaulting, which in our model represents 8.3% of quarterly output.
shows the combinations of income realizations and spread levels chosen in equilibrium in the simulations of the model with one-quarter and four-year bonds.

Figure 1: Endowment realizations and equilibrium spread levels in the economies with one-quarter and four-year bonds. The plot shows 500 samples of 32 periods before a default episode.

In order to understand why the introduction of long-duration bonds induce higher spreads, first, one should note that with long-duration bonds, the spread paid by the government not only depends on the default probability in the next-period, but it also depends on the default probability in every other future period. This is apparent in the equilibrium bond price presented in equation (3). With long duration bonds, the equilibrium bond price depends on both the next-period default decision $h(b', y')$ and the next-period equilibrium bond price $q^{ZP}(g(h(b'y'), b', y'), y')$, which captures the expected default probability in every future period. In contrast, with one-period bonds ($\delta = 1$), the equilibrium bond price only depends on the next-period default decision $h(b', y')$ and is independent of the default probability in every other

There are costs of defaulting that we do not consider (see Hatchondo et al. (2007a) for a discussion of defaulting costs). Besides, the model assumes that the government cannot save and borrow at the same time, that all debt is held by foreigners, and that the recovery rate on debt in default is zero. These simplifying assumptions restrict the ability of the model to account for higher debt levels. Even though debt levels generated by the model are low, they are not far away from levels in previous studies, and the stripped-down version of the baseline model studied in this paper has the advantage that it enables us to highlight the role played by the bond duration in a simple way.
future period.

Figure 2 describes how the relationship between bond prices and future default probabilities restricts the current borrowing set available to the government. The graph presents the menu of combinations of next-period debt levels and annual spread yields from which the government can choose. We plot the spread instead of the bond price and the nominal next-period debt level \(\frac{b'}{b+1}\) instead of \(b'\) in order to facilitate the comparison across different bond durations.

![Diagram](image)

Figure 2: Menu of combinations of spread yields and next-period debt levels \(\frac{b'}{b+1}\) available to the government. The left (right) panel is obtained assuming bonds with a duration of one quarter (four years). The solid dots illustrate the optimal decision of a government that inherits a debt level equal to the average debt observed in our simulations for each duration. The low (high) value of \(y\) correspond to an endowment realization that is two standard deviations below (above) the unconditional mean.

The left panel of Figure 2 shows that with one-quarter bonds, the government can choose next-period debt levels that imply a spread arbitrarily close to zero. That is, the government can choose a next-period debt level that will not be paid back solely when the economy is hit in the next period with extraordinarily low endowment realizations.

In contrast, the right panel of Figure 2 shows that with four-year bonds, even if the government chooses a debt level—for the next period—that is close to zero, the discount in the bond price would reflect a default premium substantially higher than zero. It is still true that when the next-period debt is close to zero, the probability of a default in the next period is negligible. However, equilibrium default probabilities in other future periods are significantly away from zero. That is, lenders anticipate that there is a positive probability that the government will
not honor all future coupon payments, and therefore, they charge a higher spread in the current period. Even though the government can commit to choose a debt level that implies a nearly zero default probability in the following period, the government’s future borrowing and default decisions imply significant default probabilities in other future periods. This finding resembles the result in environments where the borrower does not have within-period commitment to exclusive borrowing contracts (see Bizer and DeMarzo (1992) and Hatchondo and Martinez (2007)).

Next, we discuss the Euler Equation that determines the optimal borrowing decision, which will help explain the optimal choices described in Figure 2. In order to simplify the notation, we do not write consumption, default, and future borrowing as functions of the state variables, and we denote as \( \tilde{b} = (1 - d)(1 - \delta)b \) the interim value of next-period coupon obligations. We use \( x' \) to denote the optimal value of the variable \( x \) in the following period, and \( f_i(x_1, ..., x_n) \) to denote the first-order derivative of the function \( f \) with respect to the argument \( x_i \).

\[
u_1(c) q^{ZP}(b', y) = \beta \int u_1(c') (1 - d')[1 + q^{ZP}(b'', y')(1 - \delta)] f'(dy' | y) - u_1(c) q^{ZP}(b', y)(b' - \tilde{b}). \tag{4}
\]

The left-hand side of equation (4) represents the marginal benefit of borrowing. By issuing one extra bond today, the government obtains \( q^{ZP}(b', y) \) extra units of current consumption. The right-hand side of equation (4) represents the marginal cost of borrowing. The first term in the right-hand side shows that by borrowing more, the government increases its debt obligations, which in turn reduces future consumption. The second term in the right-hand side shows that by announcing a higher issuance volume today, the government decreases the issuance price of every bond it issues (rather than merely the price of the last unit issued), which in turn decreases current consumption. We shall refer to the second term in the right-hand side of equation (4) as the current cost of borrowing.

The current cost of borrowing helps in understanding why the government does not want to accept a high spread in the model with one-period bonds. The solid dots in Figure 2 illustrate the government’s optimal borrowing decisions. It can be seen that with one-period bonds, the government tends to choose borrowing levels that command a very low default premium. The reason is that the spread charged by lenders becomes very responsive to the chosen debt level past
some borrowing amount, which raises the current cost of borrowing to levels that discourage the government from choosing spread levels significantly away from zero (see Aguiar and Gopinath (2006) and Hatchondo et al. (2007b) for a more thorough discussion of this mechanism). With long-duration bonds, the government also stays away from debt levels for which the current cost of borrowing is high but that occurs at spread levels that are substantially above zero, and thus, the optimal current borrowing decision commands a higher default premium than in the economy with one-period bonds.

We should mention that Arellano (2008) succeeds in presenting a quantitative model with one-quarter bonds that generates a significant mean spread. This is achieved by assuming a cost of defaulting that is more sensitive to current income than the one assumed in this paper. Our findings suggest that introducing long-duration bonds could improve the ability of the model to replicate the observed spread behavior when the default cost resembles the one used in Arellano (2008). In her model, when economic conditions are good, the spread generated by the model is close to zero, which is not observed in the data—see Figure 5 in Arellano (2008). This could be corrected by introducing long-duration bonds. The reason is that even when economic conditions are good and the probability of a default in the following period is small, the spread may be significantly away from zero because lenders anticipate a higher default probability in other future periods.

The current cost of borrowing also shows that by introducing long-duration bonds, the model incorporates the incentives to overborrow discussed in the literature about debt dilution. An increase in the current borrowing level raises the default probability of all outstanding debt. In the model with long-duration bonds, the government only internalizes as a cost the increase in the default probability of the bonds issued today. It does not internalize as a cost the increase in the default probability of bonds issued in past periods. In contrast, in the model with one-period bonds, the government does not have the option to dilute previously issued debt. The reason is that with one-period bonds, when the government decides the current borrowing level, the outstanding debt level is zero, either because the government has honored its debt obligations

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at the beginning of the period or because it has defaulted.

Equation (4) displays an additional mechanism that helps the model with long-duration bonds generate a higher spread. The introduction of long-duration bonds reduces the fraction of debt that needs to be rolled over in a given period in order to keep the debt level constant (this fraction is equal to $\delta$). This tends to reduce the current cost of borrowing by decreasing the current issuance needs, captured by the term $b' - \tilde{b}$ in equation (4). With one-quarter bonds, the government has to pay back its entire debt stock in every period it does not default, which leads to relatively high issuance volumes. In contrast, with four-year bonds, the government only rolls over a small fraction of its debt, and therefore, the decrease in the current issuance price is weighted by a smaller number (issuance volume).

Finally, we discuss why introducing long-duration bonds allows the model to generate a higher spread volatility. Figure 2 shows that in the model with four-year bonds, the menu of interest rates available to the government is more sensitive to the current endowment realization than in the model with one-period bonds. As explained above, the reason for this is that with long-duration bonds, lenders anticipate the behavior of the government in every future period, not just in the next period. The right panel of Figure 2 shows that even when the government chooses debt levels that are close to zero and the next-period default probability is almost zero independently of the current income, lenders still charge a different spread depending on current income. In part, this is the case because lower income levels today imply lower income levels in future periods, and thus, the lender anticipates that on the equilibrium path the government will tend to choose borrowing levels that will increase the likelihood of a future default. The difference between the menu of spread and debt levels available to the government with one-quarter and four-year bonds leads the government to accept spread levels that are more sensitive to income when four-year bonds are assumed (see the dots in Figure 2), and thus, it leads to a higher spread volatility.
6 Conclusions

We have presented an extended version of the baseline model of sovereign default, which allows us to increase the assumed duration of sovereign bonds. We have shown that when the model is parameterized to display a bond duration similar to the one observed in the data, the mean and the volatility of the interest rate are substantially larger than when one-quarter bonds are assumed. Replicating the behavior of interest rates in emerging economies is an important goal in studies that analyze the macroeconomic behavior of these economies. Our results narrow the gap between the predictions of the baseline model and the data significantly, indicating that the extended model is a useful tool for future research.
References


