Credit Market Frictions and the Linkage Between Micro and Macro Uncertainty*

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Abstract

This paper proposes a quantitative general equilibrium model with credit frictions to explain the observed comovement between micro uncertainty (dispersion of firm-level outcomes) and macro uncertainty (volatility of aggregate economic variables), and their countercyclicality. In a model with credit market frictions, an increase in firm cash flow dispersion leads to more firms receiving bad cash flows and claiming default on debt. Thus, credit frictions get more severe and the shock amplification associated with credit frictions get magnified. As a result, the economy becomes more volatile and macro uncertainty increases. The model quantitatively explains the comovement between micro and macro uncertainty, as well as their countercyclicality. Consistent with the model predictions, I find that in the data micro uncertainty, based on the dispersion of firm stock returns or sales growth, positively predicts future credit spreads.

JEL Codes: D8, E3, G12
Keywords: Macroeconomics, Uncertainty, Risks, Financial Frictions

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1 Introduction

Economic uncertainty has been demonstrated as an important driving force of the economy, recent studies find it can be used to explain business cycles, asset price fluctuations and financial crisis.\(^1\) Even though the nature and the measures of uncertainty may differ across studies, a common feature of uncertainty measures is that they appear to rise sharply in economic downturns and fall in booms.\(^2\) There are two types of uncertainty mostly used in the literature, namely micro and macro uncertainty. Micro uncertainty refers to the cross-sectional dispersion of firm-level outcomes, such as stock returns and sales growth, while macro uncertainty refers to the volatility of aggregate variables, such as the volatility of stock market indices. These two types of uncertainty are conceptually different, but there is a strong positive correlation between the two in the data\(^3\).

Previous literature has explored the impact of uncertainty on business cycles and asset prices. However, the comovement and interactions between micro and macro uncertainty has not yet been quantitatively investigated within a general equilibrium framework. To address this question, I build a quantitative general equilibrium model with credit frictions, based on Bernanke, Gertler, and Gilchrist (1999). The model explains the comovement between micro and macro uncertainty, as well as their countercyclicality as observed in the data.

In most frictionless real business cycle models, micro uncertainty does not matter for the aggregate economy, because the dispersion of firm-level outcomes cancels out at the aggregate level. However, in a macroeconomic model with credit frictions and default, micro uncertainty matters for the dynamics of the aggregate economy. In this model economy, there are entrepreneurs borrowing from creditors to finance their investment projects. Each entrepreneur receives cash flows by operating a firm. There is an idiosyncratic component associated with the cash flow, which has a time-varying dispersion. Through credit frictions, the dispersion of cash flows matters for aggregate economic variables. When cash flows received by entrepreneurs are more dispersed, more of them receive very low cash flows, which forces them to default on prenegotiated debt. Three outcomes are associated with more default. First, return dispersion increases, because more entrepreneurs experience zero gross equity returns due to default. Second, creditors demand higher credit spread. Third, more bankruptcy losses occur\(^4\), which reduce the entrepreneurs’ net worth, thus the leverage


\(^2\)See Bloom (2014) for the overview of the countercyclicality of uncertainty measures.

\(^3\)The micro and macro uncertainty in this paper indeed capture the risks at micro and macro level, respectively. The term uncertainty in this paper is different from Knightian uncertainty or ambiguity, where the objective probabilities or distributions are unknown, as in Epstein and Wang (1994), Ju and Miao (2012).

\(^4\)In the context of Bernanke et al. (1999), the bankruptcy losses are the monitoring costs paid by the
of the whole corporate sector increases. The worsening credit condition implies that the credit frictions become more severe. As we know from the literature\textsuperscript{5}, credit frictions amplify aggregate shocks which propagate throughout the economy. When default is likely to happen and the credit frictions become more severe, the shock amplification effect gets stronger, thus the economy is more sensitive to aggregate shocks. As a result, the economy becomes more volatile and macro uncertainty increases. Therefore micro and macro uncertainty are positively correlated.

However, the standard financial accelerator model, as of Bernanke et al. (1999), cannot quantitatively match the comovement between micro and macro uncertainty, and their counter-cyclicality. The reason is that even though the shock amplification effect of the financial frictions exists, however, the magnitude of the amplification effect is not strong enough. As a result, even though the aggregate economy becomes more volatile when micro-level dispersion is high, but the magnitude is not big enough.

In order to strengthen the shock amplification effect, the default behavior of the borrowers has to react more to aggregate shocks. In my model, the default behavior crucially depends on the asset price, because the borrowers’ debt repayment ability depends on the total value of assets. Therefore, if we allow asset price to be more responsive to aggregate shocks, the shock amplification effect of the credit market frictions will be magnified. In order to achieve this, I augment the model with endogenous growth and recursive preferences. The intuition follows the long-run risks literature. When agents prefer early resolution of uncertainty, together with more fluctuations in future cash flows, asset price will react more to aggregate shocks.

Endogenous growth creates more persistent variations in asset prices upon aggregate shocks, in comparison to the case of exogenous growth. This is because in standard macroeconomic models, marginal product of capital is typically assumed to be decreasing with respect to capital stock. An increase in productivity pushes up current investment and thus the economy accumulates more capital in the future. This leads to a dampening effect of future marginal product of capital. In my model economy, endogenous growth mitigates this general equilibrium effect, thus the impact of aggregate shocks on asset prices is more persistent. With more persistent shock propagation, asset price decrease even further upon bad shocks, which also leads to much lower value of asset held by entrepreneurs. The falling asset value deteriorates entrepreneurs’ debt repayment ability much more, which leads to much

\textsuperscript{5}Bernanke et al. (1999) discuss that in standard models of lending with asymmetric information, the external finance premium (credit spread) depends inversely on borrowers’ net worth. This inverse relationship enhances the swings in borrowing and thus in investment and production, which amplifies the fluctuations in the aggregate economy.
more default and deadweight loss. Thus output, consumption and investment also drop more. Therefore, thanks to the persistent shock propagation introduced by endogenous growth, the swings in asset prices and macroeconomic quantities are exacerbated, the economy is more sensitive to aggregate productivity shocks. As a result, when micro uncertainty is high, the economy is more sensitive to aggregate productivity shocks, thus it becomes more volatile and macro uncertainty increases. Hence the model delivers a strong comovement between micro and macro uncertainty.

The countercyclicality of micro and macro uncertainty follows deadweight loss due to default. When firm-level cash flow dispersion is high, more firms receive very low cash flows, more of them default. As a result, deadweight loss associated with default increases, output drops. Therefore, micro and macro uncertainty are countercyclical.

This model not only successfully replicates the dynamics of micro and macro uncertainty, it also provides rich empirical predictions. In the model, when micro uncertainty increases, more firms are likely to default and credit spread increases. Thus, micro uncertainty should have predictive power on future credit spreads. I use two different micro uncertainty to predict future credit spreads. I firstly construct a micro uncertainty measure as the idiosyncratic cross-section standard deviation (ICSV) of firm stock returns, which removes the common components to avoid the effects from business cycles. I regress credit spread, defined as a portfolio which is long in BAA bond and short in AAA bond, on ICSV. I find that one percentage increase in micro uncertainty leads to around 3 basis points increase in credit spread in the following month, which translates into more than 30 basis points annualized credit spread. This result is economically and statistically significant, since the average annual credit spread is around 80 basis points. Using the dispersion of firm sales growth as an alternative measure of micro uncertainty supports this finding.

The general equilibrium framework allows feedback effects between micro and macro uncertainty. When aggregate volatility increases, cash flows paid to an average entrepreneur become more volatile, this increases the likelihood of the default probability of an average firm, which in turn increases the cross-sectional dispersion. However, in this model economy where the time varying aggregate volatility is endogenously generated by micro level uncertainty, this channel plays a relatively smaller role. In the Appendix 6.1, I also discuss an economy with exogenously introduced aggregate stochastic volatility. Under this specification, when there is an increase in aggregate stochastic volatility, households’ precautionary saving effect dominates, which leads to more savings and investments. As a result, asset price increases, which strengthens the debt repayment ability of borrowers, thus micro uncertainty decrease, which is at odds with the data.

The model also implies that in bad states of the economy the loan default rate is high and
asset prices are low. As the economy recovers from the bad states, the expected future stock returns should be high. I test this prediction in the data by using nonperforming loans to total loans as a proxy for the loan default rate, and find that it significantly predicts future excess market returns, especially over long horizons. One percentage increase in loan default probability predicts 2.6% increase in the excess market returns in the following year. I also show that the model can quantitatively rationalize the credit spread and return predictability.

Related literature There is a large body of literature studying the impact of uncertainty shocks on business cycles and financial markets. Bloom (2009) documents that various measures of uncertainty are countercyclical. He argues that uncertainty shocks have strong real option effects. When uncertainty is high, investment opportunities deteriorates, firms freeze investment and hiring, they ‘wait and see’ until heightened uncertainty is resolved. Bloom et al. (2016) put this mechanism into a general equilibrium framework. Christiano, Motto, and Rostagno (2014) build a New Keynesian DSGE model with financial frictions, and perform a structural estimation. They find that uncertainty shocks, which propagate through financial frictions, account for large fluctuations in GDP and other macroeconomic variables. Gilchrist, Sim, and Zakrajšek (2014) build a model allowing uncertainty shocks to affect the economy through both of the two channels: the financial frictions and the real option effect. They find that uncertainty shocks affect the economy more via financial frictions. Other papers explore uncertainty shocks as a driving force for business cycles, e.g. Arellano, Bai, and Kehoe (2016), Bachmann and Bayer (2014). Basu and Bundick (2017), Bianchi, Ilut, and Schneider (2014). My paper complements this literature by explaining why micro and macro uncertainty comove, which is taken as exogenously by most of these papers.

There are a few papers studying the comovement between micro and macro uncertainty and their countercyclicality through information frictions. Kozeniauskas, Orlik, and Veldkamp (2016) study the common origin of uncertainty shocks in a partial equilibrium setup. They propose learning of aggregate disaster risks as the key mechanism driving different measures of uncertainty to comove. Benhabib, Liu, and Wang (2016) introduce endogenous information acquisition into a monopolistic competition model to explain the countercyclicality of micro and macro uncertainty. They also show that a two-way feedback exists between uncertainty and macroeconomic activities. The concept of uncertainty in this strand of literature is different than it in my paper. In their economy, the uncertainty is the Knightian uncertainty, where agents are not sure of the probability or distribution of future outcomes. In my paper, agents know the probability or distribution of future economic outcomes. My

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6Christiano et al. (2014) is also based on Bernanke et al. (1999) as in my paper. However, as I discussed extensively in Section 4.5, without endogenous growth and recursive preferences, Christiano et al. (2014) or Bernanke et al. (1999) cannot quantitatively match the comovement between micro and macro uncertainty.
definition of uncertainty captures the volatility, or the risks, of realized economic outcomes at micro and macro level. Additionally, previous studies are either in a partial equilibrium framework, or qualitative studies. My paper is a quantitative study in a general equilibrium framework.

Additionally, Gabaix (2011) argues that business cycle fluctuations can originate from idiosyncratic shocks at the firm level, because of the fat-tailed firm size distribution. This paper argues idiosyncratic risks matter to aggregate volatility through financial frictions, independent of firm size distribution. The research focus in this paper is about how idiosyncratic risks affect aggregate volatilities rather than aggregate quantities as in Gabaix (2011). More importantly, this paper quantitatively explains the comovement between uncertainty and credit spread.

Another strand of literature studies the countercyclicality of micro uncertainty. Bachmann and Moscarini (2012) argue that economic downturns lead to more dispersed prices, because the cost of experimenting new prices of firms are cheaper in recessions. Decker, D’Erasmo, and Moscoso Boedo (2016) argue that during economic down turns, firms optimally choose to access less markets to reduce costs. Therefore, firms’ risks are less diversified, which leads to more volatile firm-level outcomes. These papers only study the countercyclicality of micro uncertainty. My paper studies the comovement between micro and macro uncertainty and their countercyclicality.

This paper also relates to the literature studying the asset pricing implications of uncertainty, in particular the impact of uncertainty on the discount rates and firm growth options. Pástor and Veronesi (2006, 2009) show that high micro uncertainty increases the value of growth options relative to assets in place. On the other hand, high micro uncertainty will translate into systematic risk which pushes up the discount rates and thus depresses stock prices. Bansal, Kiku, Shaliastovich, and Yaron (2014) argue that high aggregate volatility pushes up discount rates which drive down asset prices. Ai and Kiku (2016) show that micro uncertainty increases the value of growth options, such that option-intensive firms, identified by idiosyncratic volatility, earn a lower premium. In my paper, upon an increase in micro uncertainty, investment opportunities deteriorate, so is Tobin’s $q$. The asset prices drop by so much that the increasing effect from the growth options is dominated, thus overall asset prices fall.

There are lots of papers which explore the empirical predictions of uncertainty (risks) on equity returns, such as Ang et al. (2006), Herskovic et al. (2016), Stambaugh et al. (2015), Garcia et al. (2014), Bollerslev et al. (2009). My work investigates whether micro uncertainty predicts credit spread.

This paper also connects to a large body of literature focusing on the role of financial
frictions on business cycles and asset prices. Such as Gertler and Kiyotaki (2010), Brunnermeier and Sannikov (2014), Carlstrom and Fuerst (1997), where financial frictions exacerbate adverse shocks to the economy. This paper also relates the literature focusing on the asset pricing implications of financial frictions, such as He and Krishnamurthy (2013), Gomes, Yaron, and Zhang (2003), where financial frictions increase risk premia.

The rest of the paper is structured as follows: In Section 2, I describe the uncertainty measures and discuss their link to credit frictions. In Section 3 I present the model setup. Section 4 presents the quantitative results, and discusses the mechanism of the model. Section 5 concludes.

2 Empirical Facts

In order to study the economic link between micro and macro uncertainty, I construct the corresponding measures of uncertainty and then present the empirical facts. Firstly, I show that the positive correlation between micro and macro uncertainty is statistically significant. Additionally, I show these two uncertainty measures comove with credit spread. Finally, I show that micro uncertainty is a robust predictor for future credit spread.

2.1 Data

Daily stock returns are from CRSP, my sample starts from January 1 of 1963 and ends in December 31 of 2016. Quarterly firm balance sheet data is obtained from Compustat, which starts from 1963:Q1 and ends in 2016:Q4. Monthly corporate bond yield is obtained from St. Louis Fed, Moody’s seasoned BAA and AAA corporate bond yield, from 1963 January until 2016 December. VIX index is from Chicago Board Option Exchange (CBOE). It is the implied volatility on S&P 500 stock market index of the next 30 days. The macro uncertainty measure of Jurado, Ludvigson, and Ng (2015) is downloaded from Sydney Ludvigson’s website. Annual industry level TFP is from NBER-CES Manufacturing Industry Database from 1958 until 2011. Earning to price ratio, long term yield on government bonds and net equity issuance are from Amit Goyal’s website (Welch and Goyal (2007)). All macroeconomic quantities, such as GDP, consumption and investment, are obtained from Bureau of Economic Analysis, from 1963Q1: until 2016:Q4.

2.2 Uncertainty Measures

Micro uncertainty As in Bloom (2009), various measures of micro uncertainty strongly correlate with macro uncertainty. However, if there are common components driving firm-
level outcomes, e.g. business cycle conditions, and firms react differently to these common components, when these common components become more volatile, individual firm outcomes also become more dispersed. Therefore, these common components can lead to a positive correlation between micro and macro uncertainty. In order to mitigate of this effect, I remove the common components when constructing the micro uncertainty measure. It allows me to show the correlation between micro and macro uncertainty which are not driven by the common components.

I construct micro uncertainty measure based on stock returns using CRSP daily return data\textsuperscript{7} from 1963 until 2016. The micro uncertainty measure is constructed from the following two-step procedure. Firstly, I compute the idiosyncratic component of stock returns. It is constructed within every month $m$ by estimating a factor model using all available daily observations in that month. I take a linear structure model given by

$$R_{i,t} - R_f = \alpha_i + \beta_i F_t + \epsilon_{it}, \quad (1)$$

where $t$ denotes an observation made at day $t$ in a given month $m$, and $F_t$ is a set of factors considered in this regression, I specify $F_t$ as the Fama-French five-factor model (Fama and French (2015)). Alternatively, I also use the first ten principal components\textsuperscript{8} of the cross section of stock returns within month $m$, the results remain quantitatively similar. The idiosyncratic stock return is the residual $\epsilon_{it}$ from regression (1).

In the second step, I define micro uncertainty as the cross-section standard deviation of idiosyncratic stock returns (ICSV)

$$ICSV_t = \sqrt{\frac{1}{N_t} \sum_{i=1}^{N_t} (\epsilon_{i,t} - \bar{\epsilon}_t)^2},$$

where $\bar{\epsilon}_t$ is the mean of idiosyncratic returns of all stock at day $t$, and $N_t$ is the total number of stocks at day $t$. To construct monthly measure of ICSV, I average of the daily ICSV measure over $D_m$ days in a given month $m$,

$$ICSV_m = \frac{1}{D_m} \sum_{t=1}^{D_m} ICSV_t.$$

By observing equation (1), it becomes more clear how the common components can drive

\textsuperscript{7}I use the S&P 500 constitutes, the results remain quantitatively similar if I use all firms in CRSP if sufficiently amount of factors are controlled for.

\textsuperscript{8}As robustness, I also used the first fifteen principal components, the results remain quantitatively similar, all results are available upon request.
the correlation between micro and macro uncertainty. Suppose there are two firms, one firm has positive loadings, \( \beta_i' \), on all the factors, while the other firm has negative loadings. When the common factors, \( F_t \), become more volatile, the factors can reach more extreme realizations, then the returns of two firms become more dispersed because the common factors driving them moving to different directions.\(^9\) By removing the common components from the firm stock returns, the information contained in the residuals is only idiosyncratic.\(^{10}\)

Additionally, I use interquartile range (IQR) of firm year-on-year sales growth as another micro uncertainty measure. Every quarter, I compute the interquartile range of year-on-year sales growth of all firms in Compustat. The year-on-year sales growth is the current firm sales denominated by the sales four quarters ago. It is denoted as \( IQR(\Delta \text{Sales}) \). Following Bloom (2009), I only take firms with more than 150 quarters of data in Compustat quarterly accounts.

Finally, I also consider the cross-section standard deviation of TFP constructed from the NBER-CES Manufacturing Industry Database. It is denoted as \( CSV(\text{TFP}) \). The data is at annual frequency with industry identified at 4 digit SIC code level.

**Macro uncertainty** For macro uncertainty, I use three measures. The first one is the VIX index, which represents the market expectation of S&P 500 index return volatility of the next 30 days. It is widely used as a measure for macro uncertainty, as in Bloom (2009, 2014). The second one is the standard deviation of the daily S&P 500 index returns. The third one is the macro uncertainty measure from Jurado, Ludvigson, and Ng (2015), which captures the one-period ahead uncertainty.

Figure 1 shows the time series graph of uncertain measures and credit spread. The credit spread is the spread difference between BAA and AAA corporate bond yield. The shaded areas are NBER recessions. As we can notice that both micro and macro uncertainty rise sharply during recessions. The correlation between micro and macro uncertainty measures is also strong outside the recessions. Additionally, credit spread also fluctuates with micro and macro uncertainty measures. Due to data availability, the sample of VIX index only starts

\(^9\)One may argue the correlation between micro and macro uncertainty computed from stock returns may come from the fact that the volatility of the residual term \( \sigma(\varepsilon_{it}) \) may correlate with the volatility of the common factors \( \sigma(\beta_i'F_t) \). In the data, the idiosyncratic risk \( \sigma(\varepsilon_{it}) \), aggregate risk \( \sigma(\beta_i'F_t) \), and volatility of market return correlate with each other. As argued by Bartram et al. (2016), it is not clear why idiosyncratic risk \( \sigma(\varepsilon_{it}) \) correlates with aggregate risk \( \sigma(\beta_i'F_t) \). My paper can also be extended to explain the question raised by Bartram et al. (2016). Nevertheless, my paper focuses on explaining the comovement between micro and macro uncertainty. More importantly, the correlation between micro and macro uncertainty also holds for measures without using stock return data.

\(^{10}\)The way I construct the idiosyncratic component of returns is similar to Herskovic et al. (2016). Their common idiosyncratic volatility measure captures the time series variations of the average idiosyncratic volatility of all firms. However, in my study, ICSV measure captures the cross-section variations of the idiosyncratic return component.
from January of 1990.

Figure 1: **Uncertainty measures and credit spread**

This figure shows monthly time series plots of uncertainty measures, and credit spread. All measures are in percentage numbers. For macro uncertainty, I plot two measures in this figure, which are the VIX index, and the macro uncertainty measure $JLN$ from Jurado et al. (2015). The VIX index is a measure of the expected volatility of the next 30-day variance of S&P 500 index returns. For micro uncertainty, I use $ICSV^{FF}$, which is the cross-section standard deviation of residuals from the Fama French five-factor model, detailed construction is in Section 2.2. Credit spread ($Baa - Aaa$) is the spread between BAA and AAA corporate bond yield. Gray bars are NBER recessions. The VIX index is rescaled by multiplying with 0.01. $JLN$ is rescaled by multiplying with 2.

Table 1 shows selected micro and macro uncertainty measures are highly correlated: $ICSV^{FF}$, $IQR(\Delta Sales)$ and $CSV(TFP)$ strongly correlate with macro uncertainty measures, such VIX index, volatility of S&P 500 index return, and the JLN index from Jurado et al. (2015).\textsuperscript{11} All uncertainty measures are countercyclical, i.e. they all negatively correlated with GDP growth rate. We can also observe that credit spread strongly correlates with uncertainty measures.

\textsuperscript{11}Bloom (2009), Bloom et al. (2016) use more measures of micro and macro uncertainty based on different economics variable, e.g. industry output growth, GDP forecasts, etc, the countercyclical property of uncertainty remains robust.
Table 1: Correlations of uncertainty with other variables

This table reports the correlations between different uncertainty measures and credit spread, and GDP growth rate. Panel A reports the correlation between micro and macro uncertainty measures. Panel B reports the correlation between uncertainty measures, GDP growth rate and credit spread. For micro uncertainty, I use $ICSV^{FF}$, $IQR(\Delta Sales)$, and $CSV(TFP)$ as proxies. $ICSV^{FF}$ is the cross-section standard deviation of residuals from the Fama French five-factor model. $IQR(\Delta Sales)$ is the interquartile range of year-on-year sales growth of firms in Compustat. $CSV(TFP)$ is the cross-section standard deviation of TFP from NBER-CES Manufacturing Industry Database. For macro uncertainty, I use $JLN$, $VIX$ and $Vol(SPX)$ as proxies. $JLN$ is the macro uncertainty measure from Jurado et al. (2015). $VIX$ is the VIX index, which measures the market expectation of the volatility of S&P 500 index returns over the next 30 days. $Vol(SPX)$ is the volatility computed on the realized S&P 500 monthly index returns. $Baa - Aaa$ is the credit spread, defined as the difference between BAA and AAA corporate bond yield. All variables are at annual frequency. If a variable is available at higher frequency, I take the annual average. The sample for $VIX$ starts in 1990 and ends in 2016. For $ICSV^{FF}$, $IQR(\Delta Sales)$, $JLN$ index, $Vol(SPX)$, and $Baa - Aaa$, the sample starts in 1963 and ends in 2016. For $CSV(TFP)$ the sample starts in 1963 and ends in 2011.

Panel A: Correlations between uncertainty measures

<table>
<thead>
<tr>
<th></th>
<th>$JLN$</th>
<th>$VIX$</th>
<th>$Vol(SPX)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ICSV^{FF}$</td>
<td>0.38**</td>
<td>0.66***</td>
<td>0.72***</td>
</tr>
<tr>
<td>$IQR(\Delta Sales)$</td>
<td>0.60***</td>
<td>0.65***</td>
<td>0.32**</td>
</tr>
<tr>
<td>$CSV(TFP)$</td>
<td>0.57***</td>
<td>0.39**</td>
<td>0.35**</td>
</tr>
</tbody>
</table>

Panel B: Correlations with GDP growth rate and credit spread

<table>
<thead>
<tr>
<th></th>
<th>$\Delta GDP$</th>
<th>$Baa - Aaa$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$JLN$</td>
<td>-0.61***</td>
<td>0.79***</td>
</tr>
<tr>
<td>$VIX$</td>
<td>-0.48**</td>
<td>0.63***</td>
</tr>
<tr>
<td>$Vol(SPX)$</td>
<td>-0.37***</td>
<td>0.51***</td>
</tr>
<tr>
<td>$ICSV^{FF}$</td>
<td>-0.34***</td>
<td>0.33**</td>
</tr>
<tr>
<td>$IQR(\Delta Sales)$</td>
<td>-0.10</td>
<td>0.43***</td>
</tr>
<tr>
<td>$CSV(TFP)$</td>
<td>-0.39**</td>
<td>0.42***</td>
</tr>
</tbody>
</table>

*p < 0.10, ** p < 0.05, *** p < 0.01
2.3 Credit Spread and Micro Uncertainty

As I described in the previous section that credit spread strongly correlates with micro and macro uncertainty. The question naturally arises: does micro uncertainty predict future credit spreads? As shown in Table 2, I use ICSV measures constructed from Section 2.2 to predict future credit spreads. The cumulative credit spread between period $t$ and $t+h$ is defined as the holding period return of a portfolio, which is long in BAA bond and short in AAA bond, from period $t$ until period $t+h$. The predictive regressions are performed at monthly frequency, with horizon $h = 1, 2, 3, 6, 12$ months. In Panel A of Table 2, one percentage increase of ICSV leads to 2.6 basis points increase of credit spread in the following month, which translates into 31.2 basis points per annum. It is economically and statistically significant, since average credit spread is around 80 basis points per annum. In the lower panel, where I control for earning to price ratio, term spread, net equity issuance and inflation. As we can see that the predictive power of ICSV on credit spread is still statistically significant. Term spread is the difference between long term yield on government bonds and the T-bill. Net equity issuance is the ratio of 12-month moving sums of net issues by NYSE listed companies divided by the total market capitalization of NYSE.

The persistence of the regressor may lead to imprecise inference on the estimator, therefore I follow the procedure described in Welch and Goyal (2007) by imposing the null of no predictability to bootstrap the critical values. The result rejects the null hypothesis of no predictability at 5% confidence level for one period ahead predictive regression, which means that the slope coefficient in this predictive regression is statistically significant after controlling the persistence of the regressor. Additionally, in Appendix 6.2, I perform Bonferroni test proposed by Campbell and Yogo (2006), which takes into account the persistence of the predictor when calculating the finite-sample distribution of the test statistics. My results are still robust under this test.

In the Appendix 6.2, I also use the interquartile range of year-on-year firm sales growth to predict future credit spread, the results remain economically and statistically significant.

To summarize, micro and macro uncertainty measures are strongly correlated, and both are countercyclical. Additionally, micro uncertainty predicts future credit spread. These empirical results guide me to build up a quantitative general equilibrium model to understand the interaction between credit frictions, micro and macro uncertainty.

3 Model

In this section, I present a general equilibrium model with credit frictions, à la Bernanke, Gertler, and Gilchrist (1999) to reconcile the empirical findings. There are three types of
Table 2: Predictability of Credit Spread

This table reports the predictability of credit spread using ICSV measure. Sample period: 1964:M1 - 2016:M12 at monthly frequency. $ICSV^{FF}$ is the cross-section standard deviation of return residuals from Fama French five-factor model, as defined in Section 2.2. $CS_{t\rightarrow t+h} = \sum_{s=1}^{h} (Baa_{t+s} - Aaa_{t+s})$ is the cumulative credit spread. It is the holding period return of a portfolio, which is long in BAA bond and short in AAA bond, from period $t$ until period $t+h$. The credit spread is in percentage, at monthly frequency. $E/P$ is earning to price ratio, $Term\ Spread$ is the difference between long term yield on government bonds and the T-bill. $Net\ Equity\ Issuance$ is the ratio of 12-month moving sums of net issues by NYSE listed stocks divided by the total market capitalization of NYSE. Numbers in parentheses are standard errors estimated using Newey-West estimator allowing for 3 lags.

### Panel A
$CS_{t\rightarrow t+h} = a + bICSV^{FF}_t + \varepsilon_{t+h}$

<table>
<thead>
<tr>
<th>$h$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>6</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ICSV^{FF}$</td>
<td>0.026***</td>
<td>0.052***</td>
<td>0.077***</td>
<td>0.136***</td>
<td>0.197***</td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td>(0.019)</td>
<td>(0.028)</td>
<td>(0.052)</td>
<td>(0.075)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.089</td>
<td>0.090</td>
<td>0.089</td>
<td>0.072</td>
<td>0.042</td>
</tr>
</tbody>
</table>

### Panel B
$CS_{t\rightarrow t+h} = a + bICSV^{FF}_t + cX_t + \varepsilon_{t+h}$

<table>
<thead>
<tr>
<th>$h$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>6</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ICSV^{FF}$</td>
<td>0.038***</td>
<td>0.078***</td>
<td>0.117***</td>
<td>0.218***</td>
<td>0.353***</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.017)</td>
<td>(0.024)</td>
<td>(0.045)</td>
<td>(0.066)</td>
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<tr>
<td>$E/P$</td>
<td>0.000***</td>
<td>0.001***</td>
<td>0.001***</td>
<td>0.002***</td>
<td>0.004***</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
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</tr>
<tr>
<td>$Term\ Spread$</td>
<td>0.007***</td>
<td>0.014***</td>
<td>0.020***</td>
<td>0.037***</td>
<td>0.050**</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.004)</td>
<td>(0.006)</td>
<td>(0.011)</td>
<td>(0.021)</td>
</tr>
<tr>
<td>$Net\ Equity\ Issuance$</td>
<td>-0.007***</td>
<td>-0.014***</td>
<td>-0.021***</td>
<td>-0.045***</td>
<td>-0.094***</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.003)</td>
<td>(0.005)</td>
<td>(0.010)</td>
<td>(0.020)</td>
</tr>
<tr>
<td>$Inflation$</td>
<td>0.003</td>
<td>0.009</td>
<td>0.019</td>
<td>0.065</td>
<td>0.245***</td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td>(0.019)</td>
<td>(0.026)</td>
<td>(0.047)</td>
<td>(0.084)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.366</td>
<td>0.384</td>
<td>0.398</td>
<td>0.417</td>
<td>0.434</td>
</tr>
</tbody>
</table>

Numbers in parenthesis are standard errors
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$
agents in the economy, households, creditors and entrepreneurs. Entrepreneurs operate firms
and borrow defaultable loans from creditors. There are two types of goods, consumption
and capital goods. They are produced by final goods producers and capital goods producers,
respectively. I start by describing the household’s problem, then I present the production
sectors, and the creditors and entrepreneurs’ problem.

3.1 Household

Time is discrete and infinite. There is a continuum of identical households in this economy.
The structure of the household follows the big family concept of Gertler and Kiyotaki (2010)
and Gertler and Karadi (2011). As it will become more clear latter, this allows borrowing and
lending under a representative household framework. Each household consists of two types of
family members: workers and entrepreneurs. Workers supply labor and return wages to the
household. Each entrepreneur operates a firm and transfer earnings back to the household.
Thus, the household effectively owns the firm that its entrepreneur operates. Within the
family, there is perfect consumption insurance, such that consumption decisions are made
altogether by the household.

The composition of the family members is always fixed at any time, with \( f \) fraction of
members being workers and \( 1 - f \) fraction being entrepreneurs. A family member can switch
between these two occupations. With probability \( \lambda \) the entrepreneurs continue operating
the firms. With probability \( 1 - \lambda \), entrepreneurs have to liquidate their net worth and
transfer the fund to the household, then they become workers. The same amount of workers
will randomly become entrepreneurs, keeping the composition of workers and entrepreneurs
fixed. The transfers paid to the households can be interpreted as the dividend. It will become
clear in Section 3.4 that a finite horizon for entrepreneurs will create borrowing incentives
for entrepreneurs. This is to rule out the case that entrepreneurs can accumulate enough net
worth to do all investment by self-financing, which eliminates borrowing in this economy.

The households are equipped with recursive preference as in Epstein and Zin (1989):

\[
U_t = \left\{ (1 - \beta)C_t^{1-\frac{1}{\psi}} + \beta(E_t[U_{t+1}^{1-\gamma}])^{\frac{1}{1-\gamma}} \right\}^{\frac{1}{1-\psi}},
\]

(2)

where \( \beta \) is the time discount rate, \( \gamma \) is the relative risk aversion, and \( \psi \) is the intertemporal
elasticity of substitution. Let \( C_t \) denote the households’ consumption.

Households can save through a risk free asset, \( B^f_t \), with gross return \( R^f_t \). The risk free
asset is traded among households themselves. Workers submit wages \( W_tL_t \) and entrepreneurs
transfer $\Pi_t$ amount of fund to household each period when liquidation happens.\footnote{One can also think there is a creditor living in each household, she takes the stochastic discount factor implied by the household as given. The creditor makes loan decisions separately. Because of no arbitrage condition of the bond pricing, the creditor does not transfer any profits back to the household which she belongs to.} To keep the problem simple, I assume households does not value leisure in their utility, thus the workers in the household supply labor inelastically.

The household chooses consumption, labor supply, corporate bond and risk free asset to maximize expected utility (2) subject to the following budget constraint,

$$C_t + B_{t+1} = R_t L_t + W_t L_t + \Pi_t. \quad (3)$$

Let $M_{t,t+1}$ denote the stochastic discount factor implied by the household’s optimization problem, $M_{t,t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left( \frac{U_{t+1}}{E_t \left[ U_{t+1} - \gamma \right]} \right)^{\frac{1}{\psi} - \gamma}$. The optimal choice of risk free asset reads,

$$E_t \left[ M_{t,t+1} R_{t+1} \right] = 1. \quad (4)$$

The bond valuation will be specified in Section 3.4.

### 3.2 Final Goods Producers

There is a continuum of islands, indexed by $j$, where $j \in [0,1]$. On each island, there is a representative firm. All firms on different islands are producing final consumption goods with an identical constant returns to scale Cobb-Douglas technology. Labor is perfectly mobile across firms and islands, while capital is island specific. Each firm $j$ produces final output $Y^j_t$ using the following production function

$$Y^j_t = \tilde{A}_t (\omega^j_t K^j_t)^{\alpha} (L^j_t)^{1-\alpha},$$

where $\alpha$ is the capital share, $\tilde{A}_t$ is the aggregate productivity shock, $K^j_t$ denotes the amount of capital used by firm, $L^j_t$ is labor input.

The idiosyncratic shock $\omega_t^j$ affects the efficiency units of capital on island $j$, where there is an entrepreneur operating the representative firm on that island. The idiosyncratic shock transforms capital $K_t^j$ into efficiency units $\omega_t^j K_t^j$. It will become clear in Section 3.4 that this idiosyncratic shock to efficiency units of capital is equivalent to a shock that affects the cash flows paid to the entrepreneurs. The shock $\omega_t^j$ is a random variable drawn from a log normal distribution, it is independent across time and across islands, it has mean of unity and standard deviation of $v_t$. I allow $v_t$ to be time varying, it is essentially the same as
the risk shock concept in Christiano, Motto, and Rostagno (2014). It controls the cross-section dispersion of idiosyncratic shock $\omega^j_t$. I assume the dispersion of idiosyncratic shock in period $t+1$, $v_t$, is observed at the end of period $t$, such that every agent in this economy already know how dispersed the idiosyncratic shock should be in period $t+1$ before making choices.\textsuperscript{13} I call the shock to standard deviation of idiosyncratic shock, $v_t$, the dispersion shock. The log of $v_t$ follows an AR(1) process. I use $F_t(\cdot)$ to denote the cumulative density function, and $f_t(\cdot)$ as the probability density function of the idiosyncratic shock at period $t$. The idiosyncratic shock can also be interpreted as the technology used by entrepreneurs for operating capital, it affects how efficient the entrepreneurs at running business\textsuperscript{14}.

Since labor is perfectly mobile, the wage rate is identical across all firms in this economy. Firms are maximizing their profits at time $t$ by choosing labor $L^j_t$,

$$\max_{L^j_t} Y^j_t - W_t L^j_t.$$ 

The optimality condition reads

$$\frac{\omega^j_t K^j_t}{L^j_t} = \left[ \frac{W_t}{A_t(1 - \alpha)} \right]^{1/\alpha},$$

which means that the efficiency capital to labor ratio is the same across all islands. Firms’ profits on island $j$ is then given by $Y^j_t - W_t L_t = \omega^j_t MPK_t K^j_t$, where $MPK_t \equiv \alpha \tilde{A}_t \left[ \frac{(1 - \alpha) A_t}{W_t} \right]^{1/\alpha-1}$ is the marginal product of effective capital.

**Endogenous growth** Additionally, following the argument of learning by doing as in Romer (1990), I assume the aggregate productivity is augmented by the aggregate stock of capital $K_t$,

$$\tilde{A}_t = A_t K_t^{1-\alpha},$$

where the log of $A_t$ follows an AR(1) process. There are two effects introduced by endogenous growth. The first one is that together with recursive preference, the model could deliver a sizable equity premium, also a low and smooth risk free rate, as in Bansal and Yaron (2004). The second effect is that it facilitates the amplification of aggregate productivity shocks through credit frictions, which helps the model to match the correlation between micro and macro uncertainty, it will be discussed more extensively in Section 4.5.

\textsuperscript{13}This is typical timing assumption in uncertainty shock literature, see Bloom (2009), Bloom et al. (2016), Christiano, Motto, and Rostagno (2014).

\textsuperscript{14}A similar setup can be seen in Nuño and Thomas (2017), there are many ways of introducing idiosyncratic shock in this type of models, e.g. Christiano, Motto, and Rostagno (2014), Carlstrom, Fuerst, and Paustian (2016).
3.3 Capital Goods Producers

At the end of period $t$, capital goods producers purchase $I_t$ amount of consumption goods and transform them into $\Lambda \left( \frac{I_t}{K_t} \right)$ amount of new capital goods. They also repair depreciated capital. Then they sell both the newly produced and repaired capital at price $q_t$. I assume all markets visited by capital goods producers are perfectly competitive, therefore price of capital $q_t$ is the same for all the agents in this economy. The production technology of the capital goods producers is constant returns to scale, which resembles the adjustment cost function as in Jermann (1998). The capital producers’ optimization problem is

$$\max_{I_t} q_t \Lambda \left( \frac{I_t}{K_t} \right) K_t - I_t,$$

where $I_t$ and $K_t$ are the aggregate investment and capital stock of the economy in period $t$. The optimality condition with respect to investment gives the marginal $q$, which is the price of capital in this economy,

$$q_t = \left[ \Lambda \left( \frac{I_t}{K_t} \right) \right]^{-1}.$$

By repairing depreciated capital and supplying newly produced capital, the evolution of capital is given by

$$K_{t+1} = (1 - \delta)K_t + \Lambda \left( \frac{I_t}{K_t} \right) K_t,$$

where $\delta$ is the capital depreciation rate. Note that the capital goods producers are the only agents cumulating capital.

3.4 Entrepreneurs and Creditors

At each island $j$, there is an entrepreneur indexed by $j$, who operates the representative firm on this island. At the end of each period $t$, an entrepreneur holds certain amount of net worth $N^j_t$. She decides the amount of capital that she wants to carry into period $t+1$, and purchases capital $K^j_{t+1}$ from the perfectly competitive capital goods market, at the market price $q_t$. She uses her net worth $N^j_t$ and loan $B^j_{t+1}$, which she borrowed externally, to finance the purchase. Therefore her budget constraint at the end of period $t$ is

$$q_t K^j_{t+1} = N^j_t + B^j_{t+1}. \quad (5)$$

$^{15}$The specification is $\Lambda \left( \frac{I_t}{K_t} \right) = \frac{a_1}{1-\xi} \left( \frac{I_t}{K_t} \right)^{1-1/\xi} + a_2$, $\xi$ is the elasticity parameter.
Note that the entrepreneurs do not borrow from their own household.

After purchasing capital, entrepreneur $j$ operates the firm on island $j$ to make production. At the end of period $t + 1$, she receives profits from this firm, $\omega_{t+1}^j MPK_{t+1} \cdot K_{t+1}^j$. Indeed, since there is only one entrepreneur on island $j$, the shock to this island is effectively also a shock to entrepreneur $j$. Following Bernanke et al. (1999) and Gertler and Kiyotaki (2010), I assume that after the production taken place, the undepreciated capital held by the entrepreneur, $\omega_{t+1}^j (1 - \delta) K_{t+1}^j$, must be liquidated in capital goods market, at competitive price $q_{t+1}$, and all new capital has to be purchased in the next period. Therefore the total amount of capital gain for entrepreneur $j$ in period $t + 1$ is $\omega_{t+1}^j [MPK_{t+1} + q_{t+1}(1 - \delta)]K_{t+1}^j$.

To simplify notation, I define

$$R_{t+1}^k = \frac{MPK_{t+1} + q_{t+1}(1 - \delta)}{q_t},$$

(6)

as the aggregate capital return on capital. Therefore the capital gain, or the cash flow, paid to entrepreneur $j$ by operating capital in period $t + 1$ is $\omega_{t+1}^j R_{t+1}^k q_{t+1} K_{t+1}^j$. It is clear now that the idiosyncratic shock to the efficiency units of capital, $\omega_{t+1}^j$, is equivalent to a shock to the capital gains, or the cash flows, paid to the entrepreneurs.

**The creditor and the debt contract** There is a representative risk neutral creditor in this economy, she takes the stochastic discount factor implied by the households as given. At the end of period $t$, every entrepreneur can enter a debt contract with the creditor in this economy. The debt contract specifies the amount of debt $B_{t+1}^j$ the entrepreneur borrows, and the negotiated loan rate $Z_{t+1}^j$. Note that the loan rate $Z_{t+1}^j$ is a pre-specified non-contingent rate for non-default loans. The realization of idiosyncratic shock is only known to the entrepreneurs. The creditor can only observe the realization of idiosyncratic shocks at the expense of monitoring costs, $\eta \omega_{t+1}^j R_{t+1}^k q_{t} K_{t+1}^j$. As discussed in Townsend (1979), it is optimal that the costly monitoring only happens if the borrower cannot honor the debt. When monitoring happens, the borrowers report their true states to the creditor.

After repaying the debt obligations in period $t + 1$, the net worth of entrepreneur $j$ becomes

$$N_{t+1}^j = \omega_{t+1}^j R_{t+1}^k q_{t} K_{t+1}^j - Z_{t+1}^j B_{t+1}^j,$$

(7)

where $N_{t+1}^j$ is the net worth in period $t + 1$ after debt repayment. When an entrepreneur experiences a sufficiently bad idiosyncratic shock, $\omega_{t+1}^j \leq \bar{\omega}_{t+1}^j$, such that the net worth is

16Since the portfolio held by the creditor is fully diversified, thus the bond return, which takes into account the recovery of default loans, should equal to the risk free rate implied by the household. Additionally, if these two pre-contingent rates of return do not equal, the creditor will only invest in one type of asset. In the end, it is equivalent to assume that the creditor takes the discount rate of the household as given.
not enough to repay the debt obligations, she declares default. The cut-off value of default for idiosyncratic shock $\bar{\omega}_{t+1}$ satisfies

$$\bar{\omega}_{t+1} q_t R^k_{t+1} K_{t+1}^j = Z_{t+1}^j B_{t+1}^j. \quad (8)$$

Note that the cutoff $\bar{\omega}_{t+1}$ is known in period $t + 1$, it is contingent on the aggregate capital return $R^k_{t+1}$.

The value of debt to the creditor is given by:

$$B_{t+1}^j = E_t M_{t,t+1} \left\{ (1-\eta) R^k_{t+1} q_t K_{t+1}^j \int_0^{\bar{\omega}_{t+1}} \omega dF_t(\omega) + Z_{t+1}^j B_{t+1}^j [1 - F_t(\bar{\omega}_{t+1})] \right\}. \quad (9)$$

The non-arbitrage condition states that the creditor can lend the money to entrepreneurs, such that the discounted payoff of tomorrow, from making loans, should yield the same value as if the creditor hold $B_{t+1}^j$ amount of money today. The first term on the right hand side corresponds to expected revenues if entrepreneurs default with $\omega_{t+1}^j \leq \bar{\omega}_{t+1}$, which is the cash flow term specified in equation (3). The second term is the expected debt repayment if entrepreneurs honor the debt, with $\omega_{t+1}^j > \bar{\omega}_{t+1}$. The first term is net of monitoring costs. The monitoring cost is $\eta$ fraction of total cash flows paid to the default entrepreneurs.

**Entrepreneur’s optimization problem** At each period, $1-\lambda$ fraction of entrepreneurs are liquidated, and transfer their net worth to the household. These transfers deliver utility to the entrepreneurs, because this amount of money is finally consumed by the household which the entrepreneurs belong to. Since the household makes the consumption decision for all members in the same household, the entrepreneur should also value their net worth using the same stochastic discount factor as the household. Let $V_{t+1}^j$ denote the value function of entrepreneur $j$, then Bellman equation reads,

$$V_{t+1}^j(N_{t+1}^j) = \max_{K_{t+1}^j,\omega_{t+1}^j} E_t M_{t,t+1} \int_{\omega_{t+1}^j}^{\infty} \left\{ \lambda V_{t+1}^j(N_{t+1}^j) + (1-\lambda) N_{t+1}^j \right\} dF_t(\omega), \quad (10)$$

subject to flow budget constraint (5) and (7), additionally the creditor’s valuation of debt (9) has to be respected. Note that the integral starts from $\omega_{t+1}^j$, it means that the entrepreneur only value net worth if no default happens. If default happens, all the remaining value is collected by creditor. The entrepreneur no longer value the business and the remaining net worth is zero. The first term of the right hand side means, with probability $\lambda$ entrepreneur $j$ continues to operate the capital, therefore she receives the continuation value $V_{t+1}^j(N_{t+1}^j)$. 

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The second term represents that conditional on being liquidated with probability $1 - \lambda$, she transfers the remaining net worth, $N_{t+1}^j$, to the household.

By taking prices as given, the objective function (10), and constraints (5), (7) (9) are all linear, therefore the value function $V_t^j$ must be a linear function the state variable, net worth. I conjecture $V_t^j(N_t^j) = \mu_t^j N_t^j$, where $\mu_t^j$ is the marginal value of net worth for entrepreneur $j$.

I rewrite the optimization problem of the entrepreneur by plug in equation (5), (7) and (8). Then I normalize quantities by state variable $N_t^j$ and define leverage of entrepreneur $j$ as $\phi_t^j \equiv \frac{q_t K_{t+1}^j}{N_t^j}$. The entrepreneur $j$’s optimization problem can be written as

$$\mu_t^j = \max_{\phi_t^j, \omega_{t+1}} \left\{ E_t \left[ M_{t,t+1} \int_{\omega_{t+1}}^\infty [\mu_t^j + (1 - \lambda)] (\omega - \bar{\omega}_{t+1}^j) R_{t+1}^k \phi_t^j dF_t(\omega) \right] \right\}$$

s.t. $\phi_t^j - 1 = E_t M_{t,t+1} \left\{ R_{t+1}^k \phi_t^j \left[ \Gamma_t(\bar{\omega}_{t+1}^j) - \eta G_t(\bar{\omega}_{t+1}^j) \right] \right\}$,

where

$$\Gamma_t(\bar{\omega}_{t+1}^j) \equiv \int_0^{\omega_{t+1}^j} \omega dF_t(\omega) + \omega_{t+1}^j \int_{\omega_{t+1}^j}^\infty dF_t(\omega) = [1 - F_t(\bar{\omega}_{t+1}^j)] \omega_{t+1}^j + G_t(\bar{\omega}_{t+1}^j)$$

and

$$G_t(\bar{\omega}_{t+1}^j) \equiv \int_0^{\omega_{t+1}^j} \omega dF_t(\omega).$$

Here, $\Gamma_t(\bar{\omega}_{t+1}^j)$ represents the expected share of earnings received by creditors, and $1 - \Gamma_t(\bar{\omega}_{t+1}^j)$ is the expected share of earnings received by entrepreneurs.

Let the variables without subscript $j$ be the aggregate quantities, the aggregate evolution of entrepreneurs’ net worth is

$$N_{t+1} = \lambda \int N_{t+1}^j + (1 - \lambda[1 - F_t(\bar{\omega}_{t+1})]) \chi q_t K_{t+1}$$

or

$$N_{t+1} = \lambda(1 - \Gamma_t(\bar{\omega}_{t+1})) R_{t+1}^k q_t K_{t+1} + (1 - \lambda[1 - F_t(\bar{\omega}_{t+1})]) \chi q_t K_{t+1}.$$
3.5 Competitive Equilibrium

A competitive equilibrium is a set of quantities for households \( \{C_t, B^f_t, L_t\}_{t=0}^{\infty} \), quantities for entrepreneurs \( \{N^j_t, K^j_t, B^j_t\}_{t=0}^{\infty} \), quantities for creditors \( \{B^j_t\}_{t=0}^{\infty} \), and prices \( \{q_t, Z_t, R_k^t, R^f_t\}_{t=0}^{\infty} \), such that given prices, these quantities solve households, creditors and entrepreneurs’ optimization problems, firms maximize their profits, and markets clear. The market clearing conditions are

\[
K_t = \int_0^1 K^j_t dj \tag{15}
\]
\[
B_t = \int_0^1 B^j_t dj \tag{16}
\]
\[
L_t = \int_0^1 L^j_t dj \tag{17}
\]
\[
N_t = \int_0^1 N^j_t dj \tag{18}
\]
\[
\int_0^1 Y^j_t dj - D_t = C_t + I_t, \tag{19}
\]

together with equation (14), where \( D_t = \eta G_t(\bar{\omega}_{t+1})R_{t+1}^k q_t K_{t+1} \) is the monitoring costs, and \( j \in [0, 1] \). Equation (15) shows that the supply of capital from capital goods producer equals the demand of the entrepreneurs. Equation (16) says the bond demand by creditors equals the supply from entrepreneurs. I assume inelastic labor supply, therefore the aggregate equilibrium labor in equation (17) is always one. Equation (18) implies the net worth of all entrepreneurs sum up to aggregate net worth \( N_t \). In equation (19), the aggregate output less the monitoring cost equals the aggregate consumption and investment.

3.6 Equilibrium Asset Pricing

Since the conditions faced by all entrepreneurs are ex-ante identical, and the optimization problem characterized by equation (11) and (12) are independent of net worth, therefore all entrepreneurs will choose the same leverage ratio \( \phi^j_t = \phi_t \). Additionally, when creditor offers the debt contract, she does not know the idiosyncratic shock received by the entrepreneurs, therefore she offers the same loan rate \( Z^j_{t+1} = Z_{t+1} \) to all entrepreneurs, the cut-off for default is also the same for all entrepreneurs, \( \bar{\omega}^j_{t+1} = \bar{\omega}_{t+1} \) for any \( j \), as in equation (8). Indeed, the creditor holds a fully diversified portfolio of bonds by making loans to entrepreneurs with different idiosyncratic shocks. If the optimal \( \bar{\omega}^j_{t+1} \) and \( \phi^j_t \) is the same across entrepreneurs, then so is the marginal value of net worth \( \mu_t^j = \mu_t \) for any \( j \).

By dropping the subscript \( j \) and solve for the optimization problem of entrepreneurs
characterized by equation (11) and (12), the optimality conditions of entrepreneurs read

\[
E_t \left[ \tilde{M}_{t,t+1} \left\{ (1 - \Gamma_t(\bar{\omega}_{t+1})) R^k_{t+1} \phi_t \right\} \right] = 1 \quad (20)
\]

\[
E_t \left[ \tilde{M}_{t,t+1} \Gamma'_t(\bar{\omega}_{t+1}) \right] = E_t \left[ M_{t,t+1} (\Gamma'_t(\bar{\omega}_{t+1}) - \eta G'_t(\bar{\omega}_{t+1})) \right], \quad (21)
\]

where

\[
\tilde{M}_{t,t+1} = M_{t,t+1} \frac{\lambda \mu_{t+1} + 1 - \lambda}{\mu_t}, \quad (22)
\]

is the pricing kernel of entrepreneurs. It is the pricing kernel of the household, \(M_{t,t+1}\), augmented by the intertemporal change of entrepreneur’s marginal value of net worth. The nominator of the augmenting term, \(\lambda \mu_{t+1} + 1 - \lambda\), is the ex-post marginal value of net worth in period \(t + 1\). With probability \(\lambda\), entrepreneurs can receive continuation value. With probability \(1 - \lambda\), the net worth are liquidated and transferred to the household. The denominator \(\mu_t\) is the ex-ante marginal value of net worth in period \(t\).

Entrepreneurs in this economy are the marginal investors of equity, therefore their pricing kernel will price equity returns. The equity in this economy is the net worth held by the entrepreneurs, which is asset value less debt. Therefore the aggregate return on equity is indeed the return on net worth, defined as

\[
R^E_{t+1} = \frac{\int_0^1 N^j_{t+1} \omega^j_{t+1} R^k_{t+1} K^j_{t+1} - \int_0^1 N^j_{t+1} B^j_{t+1}}{\int_0^1 N^j_{t+1}} = \frac{(1 - \Gamma_t(\bar{\omega}_{t+1})) R^k_{t+1} \phi_t}{(1 - \Gamma_t(\bar{\omega}_{t+1})) R^k_{t+1} \phi_t}. \quad (23)
\]

Recall that \(1 - \Gamma_t(\bar{\omega}_{t+1})\) denotes the share of capital gains that goes to entrepreneurs, therefore the equity return in this economy is a levered return of the entrepreneurs’ claim on capital return. The leverage is defined as asset to equity ratio. I can rewrite equation (20) to obtain the pricing equation of aggregate equity returns,

\[
E_t \left[ \tilde{M}_{t,t+1} R^E_{t+1} \right] = 1. \quad (24)
\]

The equity return held by entrepreneur \(j\) is given by

\[
R^E_{t+1} \left( \frac{N^j_{t+1} \omega^j_{t+1} R^k_{t+1} K^j_{t+1} - Z^j_{t+1} B^j_{t+1}}{N^j_{t+1}} \right) = \frac{(\omega^j_{t+1} - \bar{\omega}_{t+1}) R^k_{t+1} \phi_t}{\eta}, \quad \text{if } \omega^j_{t+1} > \bar{\omega}_{t+1}
\]

\[
R^E_{t} = 0, \quad \text{if } \omega^j_{t+1} \leq \bar{\omega}_{t+1}.
\]

The above derivations take into account the expression for default threshold, as in equation (8). Note that the dispersion of equity returns note only depends on idiosyncratic shock, but also on aggregate conditions, such as aggregate capital return \(R^k_t\), default rate, etc.
For other asset classes in this economy, the risk free asset and debt, are priced by household’s stochastic discount factor, as in equation (4) and (9).

3.7 Uncertainty Measures in the Model

The model is solved and simulated at monthly frequency, it is not possible to compute the idiosyncratic cross-section volatility (ICSV) measure or the volatility of aggregate stock market returns based on daily variables. Therefore, in order to compare the data and the model, I construct the micro and macro uncertainty based on monthly data for both the model simulated data and the empirical data.

Micro uncertainty  In the model, I construct micro uncertainty as in Section 2.2, which is defined as the cross-section standard deviations of equity return residuals of factor regressions. By only look at the dispersion of residuals, this micro uncertainty measure mitigates the effect from aggregate fluctuations of common factors. Using model simulated monthly data, I firstly obtain residuals from 24-month non-overlapping rolling window regressions,

\[ R_{t}^{E,j} - R_{t}^{f} = \alpha^{j} + \beta^{j}(R_{t}^{E} - R_{t}^{f}) + \varepsilon_{t}^{j}, \]

where \( R_{t}^{E,j} - R_{t}^{f} \) is the individual firm excess equity return, and \( R_{t}^{E} - R_{t}^{f} \) is the market factor in the model.

After I obtain the residuals, for each month \( t \), I calculate the cross-section standard deviation of these residuals computed from rolling regressions, then I compute the micro uncertainty of the model, ICSV measure, as below,

\[ ICSV_{t} = \sqrt{\frac{1}{N_{t}} \sum_{i=1}^{N_{t}} (\varepsilon_{t}^{j} - \bar{\varepsilon}_{t})^2}. \]

For the empirical counterpart, I use the same procedure. Instead of regressing individual firm excess stock returns on only market factor, I regress individual firm excess returns on Fama-French five-factor model. Because there is only one common component in the model, but there are several well established common components in the data. The results remain similar if I use other specifications of the factors.

Macro uncertainty  Macro uncertainty is defined as the volatility of the aggregate equity returns \( \sigma(R^{E}) \), it is computed as the 24-month rolling window standard deviation of aggregate equity returns.
4 Quantitative Analysis

In this section I show the quantitative results of the model. Firstly the parameter choices are described. Secondly I show the performance of the model in terms of matching the moments of macroeconomic quantities and asset prices. Afterwards I show that the model can quantitatively explain the comovement between micro and macro uncertainty, and their countercyclicality observed in the data. Then the mechanism of credit frictions and endogenous growth in this model is discussed extensively. Finally I show that empirical predictions of the model are supported by the data.

4.1 Calibration

Table 3: Benchmark Calibration - Monthly

This table presents the parameters used in the Benchmark model at monthly frequency.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Values</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount rate</td>
<td>$\beta$</td>
<td>0.999</td>
<td>RBC</td>
</tr>
<tr>
<td>Relative risk aversion</td>
<td>$\gamma$</td>
<td>6</td>
<td>LRR</td>
</tr>
<tr>
<td>IES</td>
<td>$\psi$</td>
<td>1.4</td>
<td>LRR</td>
</tr>
<tr>
<td>Capital share in production</td>
<td>$\alpha$</td>
<td>0.33</td>
<td>RBC</td>
</tr>
<tr>
<td>Depreciation rate of capital</td>
<td>$\delta$</td>
<td>0.07/12</td>
<td>RBC</td>
</tr>
<tr>
<td>Investment adjustment cost</td>
<td>$\xi$</td>
<td>1.6</td>
<td>std($I$)</td>
</tr>
<tr>
<td>Entrepreneur survival rate</td>
<td>$\lambda$</td>
<td>0.99</td>
<td>Gertler and Kiyotaki (2010)</td>
</tr>
<tr>
<td>Monitoring costs</td>
<td>$\eta$</td>
<td>0.38</td>
<td>BGG (1999)</td>
</tr>
<tr>
<td>Equity injection to new entrepreneurs</td>
<td>$\chi$</td>
<td>0.195</td>
<td>BGG (1999)</td>
</tr>
<tr>
<td>Average volatility of idiosyncratic shock</td>
<td>$\bar{v}$</td>
<td>0.22</td>
<td>Leverage</td>
</tr>
<tr>
<td>Persistence of TFP</td>
<td>$\rho^A$</td>
<td>0.998</td>
<td>$\rho(Y)$</td>
</tr>
<tr>
<td>Std of TFP shock</td>
<td>$\sigma^A$</td>
<td>0.007</td>
<td>std($Y$)</td>
</tr>
<tr>
<td>Persistence of dispersion</td>
<td>$\rho^v$</td>
<td>0.988</td>
<td>Christiano et al. (2014)</td>
</tr>
<tr>
<td>Std of dispersion shock</td>
<td>$\sigma^v$</td>
<td>0.021</td>
<td>std(Credit Spread)</td>
</tr>
</tbody>
</table>

The model is calibrated at the monthly frequency. The choice of parameters and the corresponding moments are shown in Table 3. There are three blocks of parameters. The first block contains the household’s preference parameters. The relative risk aversion is set to $\gamma = 6$, and the intertemporal elasticity of substitution $\psi$ is set to 1.4, in line with Bansal and Yaron (2004). The second block contains the parameters related to production and entrepreneurs. Capital share $\alpha$ is set to 0.33, and depreciation rate is 7% per annum. These parameters are in line with the RBC literature. The capital adjustment cost parameter $\xi$ is
set to 1.6, in order to match the volatility of investment growth rate. The probability of an entrepreneur to survive from liquidation is set to $\lambda = 0.99$, which implies an average 10-year corporate duration as in Gertler and Kiyotaki (2010). The credit frictions related parameters are set to match (i) steady state investment to output ratio is at 0.18, as in the data, (ii) credit spread, the difference between average $Z$ and $R^f$ is at 0.6%, and (iii) a leverage ratio of $\phi = 2$. This leads to a standard deviation of the idiosyncratic productivity shock $\bar{v} = 0.22$, a monitoring cost parameter of $\eta = 0.38$, and a value for the parameter controls the initial net worth of new entering entrepreneurs $\chi = 0.195$.

The last block contains shock parameters. Idiosyncratic shocks to islands are following Christiano et al. (2014), $\log \omega \sim N(\frac{-\nu^2}{2}, \nu_t)$. The evolution of TFP shocks and the shocks to dispersion $v_t$ are given by

$$
\log(A_t) = \rho^A \log(A_{t-1}) + \sigma^A \varepsilon^A_t, \quad \varepsilon^A_t \sim N(0, 1)
$$

$$
\log(v_t) - \log(\bar{v}) = \rho^v (\log(v_{t-1}) - \log(\bar{v})) + \sigma^v \varepsilon^v_t, \quad \varepsilon^v_t \sim N(0, 1).
$$

The two shocks $\varepsilon^A_t$ and $\varepsilon^v_t$ are independent from each other.\textsuperscript{18} For TFP shocks, $\rho^A$ and $\sigma^A$ are set to match the autocorrelation and volatility of output, respectively. For the parameters governing idiosyncratic shocks, $\rho^v = 0.988$ is roughly in line with Christiano et al. (2014), and $\sigma^v$ is set to jointly match the volatility of credit spread and default probability.

The model is solved using a third-order perturbation method.

\subsection*{4.2 Simulation}

In this section, I compare model simulated moments to their data counter part. I simulate the model at monthly frequency and report the moments of annualized variables. I firstly simulate the model for 3,600 months and drop the first 600 months to avoid the dependence of initial conditions. Hence 3,000 months are left, which corresponds to 250 years. Then I simulate a panel of 5,000 firms. Table 4 reports moments of the simulated data versus the corresponding moments computed from empirical data. I call the shocks to the standard deviation of the idiosyncratic shocks $\varepsilon^v_t$ as dispersion shocks.

\textbf{Aggregate quantities} The first block of Table 4 reports the moments of macroeconomic quantities, such as volatilities and persistence of output, consumption and investment

\textsuperscript{18}One may argue that if the volatility of TFP is time-varying and correlates with the dispersion of idiosyncratic shocks, then macro and micro uncertainty are correlated by the correlation structure of the shocks alone. However, in the data, the correlations between volatility of aggregate TFP and various dispersion measures are statistically insignificant.
Table 4: Model Simulations

This table compares the moments of the model simulated data and the moments from empirical data (1963-2016). All variables are annualized. The first block reports basic statistics of macroeconomic quantities. The moments in the second block are of asset prices and default rate. The last block reports the correlation between micro and macro uncertainty, and the cyclical properties of uncertainty measures. The micro uncertainty measure is ICSV, it is the cross-section standard deviations of equity return residuals of factor regressions. In the model simulated data, the factor used is the market factor. In the empirical data it is the Fama French five-factor model. The macro uncertainty is the volatility of aggregate equity returns. Column "Benchmark" reports the moments under calibration specified in Table 3, with both TFP and dispersion shocks. The last two columns show the moments of models with only TFP shocks, dispersion $v$ is constant.

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Benchmark</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(\Delta y)$</td>
<td>2.51</td>
<td>2.17</td>
</tr>
<tr>
<td>$\sigma(\Delta c)$</td>
<td>2.27</td>
<td>2.26</td>
</tr>
<tr>
<td>$\sigma(\Delta i)$</td>
<td>6.53</td>
<td>4.31</td>
</tr>
<tr>
<td>$AC_1(\Delta y)$</td>
<td>0.33</td>
<td>0.31</td>
</tr>
<tr>
<td>$corr(\Delta y, \Delta c)$</td>
<td>0.86</td>
<td>0.96</td>
</tr>
<tr>
<td>$corr(\Delta y, \Delta i)$</td>
<td>0.83</td>
<td>0.59</td>
</tr>
<tr>
<td>$corr(\Delta c, \Delta i)$</td>
<td>0.66</td>
<td>0.37</td>
</tr>
<tr>
<td>$E[R^E - R^f] %$</td>
<td>6.51</td>
<td>8.76</td>
</tr>
<tr>
<td>$\sigma(R^E - R^f) %$</td>
<td>16.66</td>
<td>7.58</td>
</tr>
<tr>
<td>$E[R^f] %$</td>
<td>1.17</td>
<td>1.93</td>
</tr>
<tr>
<td>$\sigma(R^f) %$</td>
<td>0.89</td>
<td>0.89</td>
</tr>
<tr>
<td>$E[Z - R^f] %$</td>
<td>0.96</td>
<td>0.45</td>
</tr>
<tr>
<td>$\sigma(Z - R^f) %$</td>
<td>0.44</td>
<td>0.52</td>
</tr>
<tr>
<td>$E[F] %$</td>
<td>2.22</td>
<td>1.05</td>
</tr>
<tr>
<td>$\sigma(F) %$</td>
<td>1.35</td>
<td>1.45</td>
</tr>
<tr>
<td>$corr(\sigma(R^E), ICSV)$</td>
<td>0.57</td>
<td>0.57</td>
</tr>
<tr>
<td>$corr(\Delta y, \sigma(R^E))$</td>
<td>-0.36</td>
<td>-0.37</td>
</tr>
<tr>
<td>$corr(\Delta y, ICSV)$</td>
<td>-0.33</td>
<td>-0.22</td>
</tr>
<tr>
<td>$corr(Z - R^f, \sigma(R^E))$</td>
<td>0.50</td>
<td>0.73</td>
</tr>
</tbody>
</table>
growth rates. The data simulated by the benchmark model is broadly consistent with the moments computed from the data.

**Asset prices** The moments of asset prices are listed in the second block of table 4. The equity premium generated by the model is sizable 8.76%, it is roughly in line with the data. This is due to the fact that entrepreneurs are the marginal investors of equity, their pricing kernel prices equity, as in equation (24). Two effects leads to this sizable equity premium. Firstly the entrepreneurs’ pricing kernel is more volatile. We can observe that the entrepreneurs’ pricing kernel, as in equation (22), depends on the marginal value of net worth, which is very volatile in this economy. The reason is that the valuation of net worth depends on the return on capital, as in equation (11), which is very volatile. Secondly, since the model is equipped with endogenous growth and preference for early resolution of uncertainty, the long-run risk channel helps to deliver a high equity premium, as discussed in Bansal and Yaron (2004). The volatility of equity premium is relatively low in comparison to the data. This is extensively discussed in the production based asset pricing literature that this type of models naturally generate very low volatilities in returns, see Ai, Croce, and Li (2013), Croce (2014), Kung and Schmid (2015). However, I can potentially increase the shock to dispersion of the idiosyncratic shock so that investment becomes more volatile. Then the marginal $q$ also becomes more volatile, thus the volatility of equity return goes up, but this would lead the model no longer matching the moments of macroeconomic quantities anymore. For example, investment and consumption become too volatile, correlation between investment and consumption becomes negative.19

Credit spread moments are also well matched, both the first and second moments. Default probability in the model is 1.05% in comparison to 2.2% in the data. The volatility of default probability is also in line with the data. In the data, I use non-performing loans to total value of loans as the proxy for loan default rate. It is downloaded from St. Louis Fed. Non-performing loans are defined as loans that bank managers classify as 90-days or more past due or nonaccrual in the call report.

**Dynamics of uncertainty** The last block of Table 4 shows the correlation between micro and macro uncertainty measures, and their cyclical properties.

The micro uncertainty measure $ICSV$ reported in the “Data” column of Table 4 is computed following exactly the same procedure. The only difference is that in the “Data” column, the factors used are the Fama-French five-factors, instead of only the market factor.

---

19Nezafat and Slavík (2014) introduce financial shocks into Kiyotaki and Moore (2012) framework, where financial shocks directly affect investors’ investment opportunities, their model can obtain a much higher volatility of the equity premium without any capital adjustment cost.
The correlation between micro and macro uncertainty, \( \text{corr}(\sigma(\text{RE}), ICSV) \), is of 57%, which is consistent with the data. Additionally, the model successfully generates the counter-cyclicality of micro and macro uncertainty. Macro uncertainty \( \sigma(\text{RE}) \) has a correlation with output growth of -36%, it matches the data very well. Meanwhile micro uncertainty has a correlation with output growth of -22%, roughly in line with the data. The correlation between credit spread and macro uncertainty \( \text{corr}(Z - R^f, \sigma(\text{RE})) \) is a bit higher than in the data.

4.2.1 Credit Spread Predictability

In this section I show that this model can replicate the predictability of credit spread as documented in Section 2.3. That is micro uncertainty measure based on returns, ICSV, positively predicts future credit spreads. The model is simulated at monthly frequency. Therefore in order to make comparison, I compute ICSV as the cross-section standard deviations of the residuals, from 24-month non-overlapping rolling regressions of the Fama French five-factor model in the data.

Table 5: Credit Spread Predictability

This table compares the results between credit spread predictive regressions performed on empirical data (Panel A) and the model simulated data (Panel B). The regressions are performed at monthly frequency. \( CS_{t \rightarrow t+h} \) denotes the cumulative credit spread from period \( t \) to period \( t+h \), in percentage numbers. The micro uncertainty measure \( ICSV_t \), is the cross-section standard deviations of equity return residuals from factor regressions. In the model simulated data, the factor used is the market factor. In the empirical data it is the Fama French five-factor model. Numbers in parentheses are standard errors estimated using the Newey-West estimator.

\[
CS_{t \rightarrow t+h} = a + bICSV_t + \varepsilon_{t+h}
\]

<table>
<thead>
<tr>
<th>( h )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>6</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>ICSV</td>
<td>0.004</td>
<td>0.007</td>
<td>0.011</td>
<td>0.022</td>
<td>0.032</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.003)</td>
<td>(0.005)</td>
<td>(0.009)</td>
<td>(0.014)</td>
</tr>
</tbody>
</table>

Panel A: Data

<table>
<thead>
<tr>
<th>( h )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>6</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>ICSV</td>
<td>0.008</td>
<td>0.015</td>
<td>0.021</td>
<td>0.040</td>
<td>0.068</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.002)</td>
<td>(0.004)</td>
</tr>
</tbody>
</table>

Panel B: Model

Table 5 compares the results of credit spread predictive regressions performed on empirical
data (Panel A) and the model simulated data (Panel B). The model is doing a good job at replicating the predictability of credit spreads using ICSV measure. The slope coefficient increases with horizon, as in the data. The predictability comes from the fact that when entrepreneurs’ cash flows are more dispersed, return dispersion is higher and default is more likely to happen, therefore future credit spread increases.

4.3 Impulse Responses

In order to understand the shock propagation in this economy, I show impulse responses in Figure 2. In period one, I introduce two independent contractionary shocks. One is a negative shock to TFP, see the red dotted line. The other one is a positive dispersion shock, see the black solid line. Both of the shock magnitudes are of one standard deviation of the corresponding processes. The left panel shows the dynamics of macroeconomic variables, while the right panel shows the dynamics of variables related to asset prices and default.

Firstly, I briefly discuss the impact of TFP shocks as shown in Figure 2. Focusing on the left panel, upon a negative TFP shock, investment, output and consumption growth drop immediately. The impact of the negative TFP shock on net worth is stronger because more default is happening, thus leverage $\phi$ increases. Focusing on the right panel. Asset price $q$ decreases when productivity is low. Aggregate return on equity $r^E$ and capital return $r^K$ follows the dynamics of the asset price $q$. As in equation (8), the default threshold $\omega$ falls if return on capital and asset price fall. When default threshold $\omega$ increases, it pushes up default rate $F$. Creditors charge higher loan rate if more entrepreneurs are likely to default, therefore credit spread $z - r^f$ increases, as shown in the down right panel. Considering the credit spread $z - r^f$ demanded by creditor increases, the default threshold would increase even further.

The impact of dispersion shocks works through the debt contract in this model. The return on levered equity resembles the payoff of an investor buying a European call option. The payoff structure faced by the creditor mimics an investor writing a European put option. When dispersion of idiosyncratic shock increases, the riskiness of assets goes up, which benefits the equity holders. The reason is that equity holders just need to pay back a pre-specified amount of debt obligation, the excess amount of profits are retained. The creditors would like to cut the credit supply and raise the loan rate when uncertainty is high, because more entrepreneurs are likely to default on loans when cash flow dispersion is high.

In this model, higher dispersion benefits the equity holders in expense of creditors, but the impact on creditor dominates. Therefore creditors cut credit supply and investment falls, as shown in the left panel of Figure 2. Consumption increases upon a positive innovation in dispersion. This is because, in this model capital is pre-determined and labor is inelastic.
This figure plots the impulse responses with respect to a one-standard-deviation negative shock to aggregate productivity or a one-standard-deviation positive shock to dispersion. The y axis is the percentage deviations from the steady state. The black solid lines are with respect to dispersion shock, the red dashed lines are with respect to TFP shock. One period is one month. $\phi$ is the leverage of entrepreneurs, $q$ is the asset price, $r^E$ is the equity return, $\overline{\omega}$ is the default cutoff, $F$ is the default probability and $z - r^f$ is the credit spread. All parameters are calibrated as in Table 3.
thus output growth is not much affected by the dispersion shock.\(^{20}\) Investment drops but output stays stable. According to final goods market clearing condition, this implies that consumption has to increase in order to absorb the drop in investment upon dispersion shock. The negative comovement between consumption and investment introduced by uncertainty shock is discussed in Basu and Bundick (2017).

As shown in the right panel of Figure 2, variations in the dispersion of idiosyncratic cash flows introduce much stronger fluctuations in the loan default rate $F$ and the credit spread $z - r^f$ than TFP shock, since it directly controls the fraction of entrepreneurs receive very bad cash flows. As in the down right panel in Figure 2, the reaction of credit spread to a one-standard-deviation positive dispersion shock is roughly five times stronger than it to a one-standard-deviation negative TFP shock.

### 4.4 Micro Uncertainty and Amplification of Financial Friction

The key mechanism of this model is that when the idiosyncratic shocks to entrepreneurs are more dispersed, default and costly monitoring are more likely to happen. Thus, more default losses occur, net worth shrinks and leverage increases. The credit friction is more severe and the economy is more sensitive to aggregate shocks. As a result, the amplification effect is stronger, and the economy becomes more volatile.

In order to highlight this mechanism, Figure 3 shows the impulse responses of the economy upon only TFP shocks, but under different levels of idiosyncratic cash flow dispersion. For high dispersion specification, the standard deviation of the idiosyncratic shock is fixed at $\bar{\nu} = 20.7\%$, while for low dispersion specification, I set $\bar{\nu} = 24.5\%$. The black solid (red dashed) line shows the impulse responses upon a one-standard-deviation TFP shock under high (low) dispersion. The shock size and persistence of the TFP shock for high and low dispersion specifications are exactly the same, as shown in the upper left panel of Figure 3. Note that under high dispersion, the economy responds more strongly to aggregate TFP shocks. Investment, asset price and equity return drop more upon the negative TFP shock. Leverage, default threshold, default probability and credit spread increase more in response to the negative TFP shock.

Below I discuss the mechanism why economy reacts more under higher micro-level dispersion. When the cash flows paid to the entrepreneurs are more dispersed, more of them are likely to receive very low cash flows and are not able to pay back the debt. Thus more default happens, monitoring costs associated with bankruptcy increase and net worth shrinks. Hence, the leverage of entrepreneurs increases and the credit frictions get more severe. The economy

\(^{20}\)Output is in fact affected by dispersion shock, but the magnitude is one order smaller than that of the TFP shock.
is more sensitive to aggregate productivity shocks. As a result, the economy becomes more volatile and macro uncertainty increases. In short, by moving from low to high dispersion states, the shock amplification effect from the credit frictions, or the financial accelerator as in Bernanke et al. (1999), gets stronger. As the dispersion of cash flow is varying over time, this will also lead to a time varying volatility of the aggregate economy. Thus micro and macro uncertainty comove with each other.

Figure 3: Impulse responses of TFP shock under different dispersion

This figure plots the impulse responses with respect to a one-standard-deviation negative shock to aggregate productivity. The y axis denotes the percentage deviations from the steady state. The black solid lines are the impulse responses under TFP shock, in which the standard deviation of idiosyncratic shock fixed at $\bar{v} = 24.5\%$. The red dashed lines are the impulse responses with respect to TFP shock, in which $\bar{v} = 20.7\%$. The size and process of the TFP shock are exactly the same for both high and low dispersion cases. One period is one month. $\phi$ is the leverage of entrepreneurs, $\omega$ is the default cutoff, $q$ is the asset price, $r^E$ is the equity return, $F$ is the default probability and $z - r^f$ is the credit spread.

Through out this paper, I argue that the variations in micro uncertainty can drive aggre-
gate volatility to be time-varying. However, it is also possible the direction goes the other way around, meaning that aggregate volatility may drive micro level dispersion to be time-varying. Appendix 6.1 shows that if I allow stochastic aggregate volatility and shut down exogenous shocks to micro level dispersion, this would imply a negative correlation between macro uncertainty and credit spread, which is at odds with the data. The results and the mechanism are presented and discussed in the Appendix 6.1 as well.

4.5 Endogenous Growth and Credit Frictions

This section discusses why endogenous growth is crucial to replicate the comovement between micro and macro uncertainty, while keeping the dynamics of macroeconomic variables in line with the data.

For models with credit frictions as in Bernanke, Gertler, and Gilchrist (1999), in order to generate enough comovement between micro and macro uncertainty, the model should fulfill the following two conditions. First, the magnitude of shock amplification due to credit frictions has to differ between high and low idiosyncratic cash flow dispersion states. Second, shock amplification due to credit frictions should be quantitatively large. The first condition ensures that when the economy moves from low to high dispersion states, the economy becomes more sensitive to aggregate shocks, hence it becomes more volatile. The second condition is a complement to the first one. If the shock amplification effect from the credit frictions is too small, even though the economy exhibits different sensitivities to aggregate shocks under different dispersion states, the volatility of the economy would not change much quantitatively. Therefore the shock amplification effect from credit frictions has to be sufficiently large.

Endogenous growth reinforces the shock amplification effect of credit frictions to be much stronger, which strengthens the second condition. Below I firstly show quantitatively that models without endogenous growth cannot jointly match the comovement between micro and macro uncertainty, and the moments of macroeconomic quantities. Then I discuss the importance of endogenous growth.

4.5.1 Models Without Endogenous Growth

In order to show the effect of endogenous growth quantitatively, I simulate models without endogenous growth. I call them short-run risks models, which are calibrated using the same procedure as the benchmark model, except that the TFP process is purely exogenous and not augmented by aggregate capital. The results are shown in Table 6. Following the convention of the business cycle literature for short-run risks models, instead of reporting
growth rates of macroeconomic quantities as in Table 4, the macro moments reported in Table 6 are computed based on growth rates of the annualized simulated data. The column “Data” reports the moments computed from empirical data. Column “Benchmark” and “Endo+CRRA” report the moments from the models with endogenous growth, in which the aggregate TFP, $\bar{A}_t$, is augmented by aggregate capital stock, $\bar{A}_t = A_t K_t^{1-\alpha}$. The difference is that column “Endo+CRRA” reports the results from a model with endogenous growth, but with constant relative risk aversion (CRRA) preferences, instead of recursive preferences. Column “BGG” is a model without endogenous growth, equipped with CRRA preferences. Its TFP processes are purely exogenous, following AR(1) process. “BGG” is calibrated to match the macroeconomic dynamics, such as the volatility of output, consumption and investment, and the correlation among them. However, the correlation between micro and macro uncertainty is 26%, which is half as it is in the data. The countercyclicality of micro and macro uncertainty is also much weaker than in the data.

4.5.2 The Interaction Between Credit Frictions and Endogenous Growth

In this section, I show that by augmenting the model with endogenous growth, the dampening effect is mitigated.

The role of credit frictions The role of financial frictions in lots of macroeconomic models is to provide a channel such that transitory shocks have a persistent impact on aggregate quantities and prices. For the case of costly state verification as in this paper, the key variable is the asset price which determines the resources available to entrepreneurs to pay back the debt obligations and also the amount of money collected by creditors when default happens. Therefore when the asset price falls, less resources are available for entrepreneurs to pay back their debt obligations and more default happens. In the case of default, creditors receive a fraction of the remaining asset of the borrower instead of the debt payments, but the collected asset is of less value because of low asset price. As a result, the creditors are also worse off, they cut their credit supply and raise the loan rate to ensure that only entrepreneurs with sufficiently high asset value can participate the debt borrowing. Thus, investment drops due to less credit supply, which causes the asset price (marginal $q$) to drop even further. The falling asset price and the default reinforce each other. Hence, in order to strengthen the shock amplification, which works through the fluctuations in asset prices, I augment

\textsuperscript{21}For the case of exogenous or endogenous leverage constraint in Kiyotaki and Moore (1997) or Gertler and Kiyotaki (2010), the asset price determines the amount of collateral that can be pledged, therefore if the asset price shrinks upon an adverse shock, less collateral will be available for borrower, then debt financing drops, investment drops, which causes asset price to drop even further, then the feed back effect continues.
Table 6: Model Simulations

This table compares the simulated moments of the model with and without endogenous growth. The statistics of macro quantities in the data are computed from annualized variables. The micro uncertainty measure is ICSV, it is the cross-section standard deviations of equity return residuals of factor regressions. In the model simulated data, the factor used is the market factor. In the empirical data it is the Fama-French five-factor model. The macro uncertainty is the volatility of aggregate equity returns, computed within each year. Column “Data” is the statistics of data, column “Benchmark” column is of moments computed from benchmark model. Column “BGG” is the model without endogenous growth. Column “BGG” is calibrated to match the moments of macroeconomic quantities as in Bernanke et al. (1999).

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Benchmark</th>
<th>Endo+CRRA</th>
<th>BGG</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(\Delta y)$</td>
<td>2.05</td>
<td>2.17</td>
<td>2.08</td>
<td>2.41</td>
</tr>
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<td>2.26</td>
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<td>2.36</td>
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<td>3.72</td>
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<tr>
<td>$AC_1(\Delta y)$</td>
<td>0.55</td>
<td>0.31</td>
<td>0.30</td>
<td>0.01</td>
</tr>
<tr>
<td>$corr(\Delta y, \Delta c)$</td>
<td>0.87</td>
<td>0.96</td>
<td>0.99</td>
<td>1.00</td>
</tr>
<tr>
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<td>0.59</td>
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</tr>
<tr>
<td>$corr(\Delta c, \Delta i)$</td>
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<td>0.37</td>
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</tr>
<tr>
<td>$E[R^E - R^f] %$</td>
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<td>8.76</td>
<td>0.71</td>
<td>4.30</td>
</tr>
<tr>
<td>$\sigma(R^E - R^f %$</td>
<td>16.66</td>
<td>7.58</td>
<td>1.00</td>
<td>4.18</td>
</tr>
<tr>
<td>$E[R^f %$</td>
<td>1.17</td>
<td>1.93</td>
<td>6.31</td>
<td>0.16</td>
</tr>
<tr>
<td>$\sigma(R^f %$</td>
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<td>0.89</td>
<td>1.40</td>
<td>1.48</td>
</tr>
<tr>
<td>$E[Z - R^f %$</td>
<td>0.96</td>
<td>0.45</td>
<td>0.04</td>
<td>0.29</td>
</tr>
<tr>
<td>$\sigma(Z - R^f %$</td>
<td>0.44</td>
<td>0.52</td>
<td>0.07</td>
<td>0.29</td>
</tr>
<tr>
<td>$E[F] %$</td>
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<td>0.69</td>
</tr>
<tr>
<td>$\sigma(F) %$</td>
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<td>1.45</td>
<td>0.19</td>
<td>0.77</td>
</tr>
<tr>
<td>$corr(\sigma(R^E), ICSV)$</td>
<td>0.57</td>
<td>0.57</td>
<td>0.35</td>
<td>0.26</td>
</tr>
<tr>
<td>$corr(\Delta y, \sigma(R^E))$</td>
<td>-0.36</td>
<td>-0.37</td>
<td>-0.20</td>
<td>-0.17</td>
</tr>
<tr>
<td>$corr(\Delta y, ICSV)$</td>
<td>-0.33</td>
<td>-0.22</td>
<td>-0.07</td>
<td>-0.09</td>
</tr>
<tr>
<td>$corr(Z - R^f, \sigma(R^E))$</td>
<td>0.50</td>
<td>0.73</td>
<td>0.56</td>
<td>0.50</td>
</tr>
</tbody>
</table>
the model endogenous growth, which allows the asset price to response more strongly to aggregate shocks.

**The role of endogenous growth** Endogenous growth offers an additional channel, which makes the aggregate productivity shock to have a stronger and more persistent impact on asset prices. Below I illustrate the channel analytically. I plug in the definition of capital return equation (6) into (20), to obtain the valuation of the asset price,

\[ q_t = E_t \left[ \tilde{M}_{t,t+1} \left\{ (1 - \Gamma_t(\tilde{\omega}_{t+1})) \left( MPK_{t+1} + q_{t+1}(1 - \delta)\phi_t \right) \right\} \right]. \]

Iterating forward, \( q_t \) can be expressed as the present value of the infinite sum of all future payoffs,

\[ q_t = \sum_{s=1}^{\infty} (1 - \delta)^s E_t \left[ \Omega_{t,t+s} \alpha A_{t+s} K_{t+s}^{\alpha-1} \right], \quad (25) \]

where \( \Omega_{t,t+s} = \prod_{\tau=1}^{s} \tilde{M}_{t+\tau-1,t+\tau}(1 - \Gamma_{t+\tau-1})\phi_{t+\tau-1} \) is the effective discount rate of the price of capital, which takes into account the fluctuations in leverage \( \phi_t \) and the share that goes to the entrepreneurs \( 1 - \Gamma_t \). Equation (25) states that the price of capital \( q_t \), is the net present value of marginal productivity of capital in all future periods.

In the case of exogenous growth, in which the aggregate productivity is not augmented by capital, \( \tilde{A}_t = A_t \), thus the valuation of the asset price is

\[ q_t = \sum_{s=1}^{\infty} (1 - \delta)^s E_t \left[ \Omega_{t,t+s} \alpha A_{t+s} K_{t+s}^{\alpha-1} \right], \quad (26) \]

where \( \log(A_t) \) follows AR(1) process. Note that as long as the marginal product of capital is decreasing with respect to capital, as shown in equation (26), the impact of TFP shock will be dampened over time. An increase in TFP directly pushes up marginal productivity of capital by increasing aggregate productivity. I call this the direct effect. The second effect is the general equilibrium effect. When productivity is high, firms invest more, thus future capital stock increases. Due to the fact that the marginal productivity of capital is decreasing with respect to capital, an increase in the future capital stock mitigate the direct effect in the future periods by reducing the marginal productivity of capital in future periods. The dampening effect from general equilibrium exists as long as the TFP process is exogenous, e.g. the productivity with long-run risks specification as in Croce (2014), or a stationary AR(1) process as in the classic RBC literature.

The dampening effect does not exist in the case of endogenous growth. If I incorporate
the specification of endogenous growth, $\bar{A}_t = A_t K_t^{1-\alpha}$, into equation (25), the valuation of the asset price is

$$q_t = \sum_{s=1}^{\infty} (1 - \delta)^s E_t \left[ \Omega_{t,t+s} \alpha A_{t+s} \right].$$

As shown in equation (27), it implies that $MPK_t = A_t$. Therefore an increase in future capital stock does not lead to a lower future marginal productivity of capital. Hence there is no general equilibrium effect which dampens the direct effect. As a result, the asset price $q_t$ responds more persistently to TFP shocks, and the economy is more sensitive to TFP shocks.

**The role of intertemporal elasticity of substitution** As we know from the long-run risks literature, when investors show early resolution of uncertainty, they care fluctuations in future cash flows. Therefore, these future cash flow fluctuations will be priced by the investors, which make asset price to be more responsive to aggregate productivity shocks.

The condition for investors to have early resolution of uncertainty, IES has to be higher than the inverse of relative risk aversion, $\psi > 1/\gamma$. Table 6 column “Endo+CRRA” shows the results of a model in which the household is equipped with constant relative risk aversion (CRRA) preferences. CRRA preferences lead the households to be indifferent between early and late resolution of uncertainty. I achieve this by setting relative risk aversion $\gamma = 1/\psi = 2$.

The model generates a small positive correlation between micro and macro uncertainty, at the level of 0.35 in comparison to 0.57 in the data. This is because the household is indifferent between early and late resolution of uncertainty. Even though there is a lot of fluctuations in the future cash flows due to endogenous growth, these fluctuations are not priced by the households. Thus, the asset price $q$ does not fluctuate much with future cash flows, and the shock amplification effect from the credit frictions is weak.

Additionally, without early resolution of uncertainty, the equity premium is small, only 0.7% and the risk free rate is too high at 6%.

### 4.6 Discussion of the Debt Contract

In this section, I discuss why the debt contract adopted in this model is important to produce a countercyclical default rate and credit spread, as well as a positive correlation between micro uncertainty and the credit spread.

There are two opposing effects introduced by high cash flow dispersion. The first effect potentially strengthens entrepreneurs’ debt repayment ability. The equity holders’ payoff structure resembles that of a European call option. When cash flow dispersion is high, the riskiness of the firm’s assets increases, which benefits the equity holders at the expense of
the creditors. Essentially the equity holders enjoy the upside risk (from right tail of the cash flow) that comes with high dispersion in cash flow. If the resources can be used to pay back debt obligation is the equity value, upon an increase in cash flow dispersion, the expected equity value increases, which strengthens ability of entrepreneurs for repaying debt. Hence less default happens and the credit spread decreases.

By introducing entrepreneurs into this type of model, the resources can be used to pay back debt obligation is indeed the asset value. Therefore an increases in cash flow dispersion may push up equity value, but the asset value may decrease. When the over all effects lead asset prices to fall, the asset value and debt repayment ability decreases.

The first effect on equity value I described above is dominated by the second effect, which I describe below. When cash flow dispersion increases, creditors see more entrepreneurs are likely to default because more of them are receiving very low cash flows. Hence, creditors cut credit supply and increase loan rate, thus credit spread increases and investment drops. The marginal $q$ and asset value decrease, which reduces the ability of entrepreneurs to repay their debt. This pushes up the default threshold and leads to even more default. Additionally, the falling asset prices decrease the value collected by creditors upon default. Both entrepreneurs and creditors are worse off due to the falling asset prices, investment and asset prices drop even further. Therefore the overall effect of an increase in cash flow dispersion leads to drop in investment and asset prices, and an increase in credit spread. The positive correlation between micro uncertainty and credit spread is due to the fact that the second effect dominates, that is high cash flow dispersion leas to low asset prices and high default threshold.

The effect of TFP shocks under this type of contract is extensively discussed in existing literature, as in Bernanke, Gertler, and Gilchrist (1999), Carlstrom, Fuerst, and Paustian (2016), etc. A negative TFP shock drives down entrepreneurs’ net worth, pushes up leverage and default rates, thus credit spread increases.

### 4.7 Empirical Predictions

The model implies that when cash flow dispersion is high, more firms are likely to default on their loans, credit supply and investment drops, and the price of capital falls. Like most production based asset pricing models, the price of equity moves one-to-one with the price of capital. Therefore, asset prices fall with an increasing default probability, it implies that future asset prices recover and the expected future returns increase. As a result, default rate should positively predict future stock returns.\footnote{There is mixed empirical evidence regarding whether idiosyncratic risk measures can predict future market returns or not, see e.g. Goyal and Santa-Clara (2003), Bali et al. (2005), Garcia et al. (2014).} Guided by the model, I test whether realized
loan default rate can predict future market excess returns or not.

I use nonperforming loans to total loans ratio as the proxy for the loan default rate, which is downloaded from the St. Louis Fed. Nonperforming loans are the loans that banks classify as 90 days or more past due or non-accrual in the call report. I regress market excess returns on lagged loan default rate, at various horizons. Table 7 presents the results. Panel A shows the results from the empirical data, while Panel B shows the results from the model simulated data.

In Panel A, as we can see that the loan default rate positively predicts future excess returns, it is more significant at longer horizons, e.g. 4 quarters or even longer. The model successfully captures the magnitude and the increasing pattern of the slope coefficients.

Table 7: Return Predictability

This table compares the results between return predictive regressions performed on the data (Panel A) and the model (Panel B). The regressions are performed at quarterly frequency. $R_{t \rightarrow t+h}^{ex}$ denotes the cumulative excess market return from period $t$ to $t+h$, in percentage numbers. The regressor is loan default rate, $F$, measured in percentage numbers. In Panel A, the loan default rate is measured by nonperforming total loans to total loans, downloaded from St. Louis Fed, from 1990:Q1 until 2016:Q4. In Panel B, the regressor is measured using the fraction of default loans in the model. Numbers in parentheses are standard errors estimated using Newey-West estimator.

\[
R_{t \rightarrow t+h}^{ex} = a + bF_t + \varepsilon_{t+h}
\]

<table>
<thead>
<tr>
<th>$h$</th>
<th>1</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F$</td>
<td>0.504</td>
<td>2.619**</td>
<td>5.903***</td>
<td>9.561***</td>
<td>13.455***</td>
</tr>
<tr>
<td></td>
<td>(0.524)</td>
<td>(1.115)</td>
<td>(2.012)</td>
<td>(2.345)</td>
<td>(2.194)</td>
</tr>
</tbody>
</table>

Panel B: Model

<table>
<thead>
<tr>
<th>$h$</th>
<th>1</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F$</td>
<td>1.256***</td>
<td>3.965***</td>
<td>5.973***</td>
<td>6.946***</td>
<td>8.016***</td>
</tr>
<tr>
<td></td>
<td>(0.086)</td>
<td>(0.260)</td>
<td>(0.484)</td>
<td>(0.776)</td>
<td>(1.090)</td>
</tr>
</tbody>
</table>

Numbers in parathenises are standard errors
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Robustness I also perform the same predictive regressions, but additionally control for other popular predictors. The results remain robust. Additionally, following Campbell and

However, the micro uncertainty measure considered in this paper, ICSV, has an insignificant correlation with market return and cannot predictive future market returns by construction. The reason is that the regression residuals from Fama French five-factor model are orthogonal to the market return.
Yogo (2006), I perform the Benferroni test, which takes into account the persistence of the predictor when calculating the finite sample properties of the estimates. The predictability remains after taking the persistence into account. The results for robustness checks are reported in Appendix 6.2.

5 Conclusion

This paper presents a general equilibrium model to quantitatively explain the strong positive correlation between micro and macro uncertainty, and their countercyclicality. The model builds on a classical financial accelerator model with a time-varying dispersion of idiosyncratic shocks, which affects the cash flows paid to the firms. The firms are the borrowers in this economy. When the dispersion of cash flows received by the borrowers is high, default is more likely to happen and equity return dispersion increases. In the meanwhile more deadweight loss occur, output drops. More default and deadweight loss make the amplification effect of credit frictions stronger. Hence the economy becomes more sensitive to aggregate shocks and more volatile, leading to an increase in macro uncertainty. Therefore, linked by credit market frictions, micro and macro uncertainty commove together and become countercyclical. More importantly, augmenting the model with endogenous growth and recursive preferences are crucial to quantitatively explain the correlation between micro and macro uncertainty. These two components magnify the shock amplification due to credit frictions, by allowing the aggregate productivity shocks to have a long-run impact on the marginal productivity of capital.

The model has rich empirical predictions on the credit spread and return predictability. Consistent with the model, I document that in the data micro uncertainty, based on the dispersion of equity returns or sales, can predict future credit spreads. Additionally, the loan default rate, measured by nonperforming loans to total loans, predicts future excess market returns. The model can quantitatively rationalize the predictability found in the data.
References


6 Appendix

6.1 Stochastic Aggregate Volatility and Uncertainty Comovement

The main argument of the paper is that the time-varying aggregate volatility comes from the interaction between time-varying micro level dispersion and credit frictions. This section shows that if I allow stochastic volatility in the aggregate production process and shut down the time-varying dispersion of cash flow shocks to entrepreneurs, the correlation between micro and macro uncertainty is quantitatively insignificant. Additionally, I also show that the stochastic aggregate volatility produces negative correlation between aggregate volatility and credit spread, which is counterfactual.

In order to introduce stochastic volatility in the aggregate productivity process, I specify the productivity process as the following,

\[
\log(A_t) = \rho_A \log(A_{t-1}) + \sigma_A^{t-1} \varepsilon_t^A \\
\log(\sigma_t^A) - \log(\bar{\sigma}^A) = \rho^\sigma (\log(\sigma_{t-1}^A) - \log(\bar{\sigma}^A)) + \sigma^\sigma \varepsilon_t^\sigma, \quad \varepsilon_t^\sigma \sim \text{N}(0, 1),
\]

where the \(\sigma_t^A\) is the stochastic volatility of productivity. Following the timing convention of Bloom (2009), the volatility of productivity process is known to all agents before production takes place.

I set \(\rho^\sigma = 0.988\) and \(\sigma^\sigma = 0.05\) in order to match the volatility of the macro quantities. Table 8 reports the moments of the model with stochastic volatility in production process, in comparison with the benchmark model and the data. Column “Stoch. Vol.” reports the moments from the model with stochastic volatility in productivity and no exogenous dispersion in the cash flows paid to the entrepreneurs. As we can see that the correlation between micro and macro uncertainty defined on equity returns, \(\text{corr}(\sigma(R^E), ICSV)\) is even negative, at -0.31. It is far away from the data counter part.

**Impulse responses** In order to further explore the mechanism why stochastic aggregate volatility of productivity generates negligible comovement of uncertainty, I plot the impulse responses in Figure 4. As shown in the left panel of Figure 4, upon an increase in the volatility of aggregate productivity, households increase saving due to precautionary saving effect, thus investment increases and consumption falls. The increasing investment leads to an increase in asset price \(q\), as shown in the upper right panel.\(^{23}\) The increasing asset price pushes up the asset value held by borrowers, which in turn strengthens their debt repayment ability.

\(^{23}\)In most production based economy like this paper, the Tobin’s marginal \(q\) is indeed the asset price, therefore investment and asset price moves one-to-one.
Table 8: Model Simulation

This table compares the simulated moments of the benchmark model and the model with stochastic volatility in productivity. All macroeconomic quantities are computed from annualized variables. The first block reports basic statistics of macroeconomic quantities. The moments in the second block are of asset prices and default rate. The last block reports the correlation between micro and macro uncertainty, and the cyclical properties of uncertainty measures. The micro uncertainty measure is ICSV, it is the cross-section standard deviations of equity return residuals of factor regressions. In the model simulated data, the factor used is the market factor. In the empirical data it is the Fama French five-factor model. The macro uncertainty is the volatility of aggregate equity returns. Column "Benchmark" reports the moments under calibration specified in Table 3, with both TFP and dispersion shocks. The column “Stoch Vol” reports the moments simulated from the model with stochastic volatility in productivity process, as specified in equation (28), but with constant idiosyncratic shock to firms.

<table>
<thead>
<tr>
<th>Data</th>
<th>Benchmark</th>
<th>Stoch Vol</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(\Delta y)$</td>
<td>2.51</td>
<td>2.17</td>
</tr>
<tr>
<td>$\sigma(\Delta c)$</td>
<td>2.27</td>
<td>2.26</td>
</tr>
<tr>
<td>$\sigma(\Delta i)$</td>
<td>6.53</td>
<td>4.31</td>
</tr>
<tr>
<td>$AC_1(\Delta y)$</td>
<td>0.33</td>
<td>0.31</td>
</tr>
<tr>
<td>$corr(\Delta y, \Delta c)$</td>
<td>0.86</td>
<td>0.96</td>
</tr>
<tr>
<td>$corr(\Delta y, \Delta i)$</td>
<td>0.83</td>
<td>0.59</td>
</tr>
<tr>
<td>$corr(\Delta c, \Delta i)$</td>
<td>0.66</td>
<td>0.37</td>
</tr>
<tr>
<td>$E[R^E - R^f] %$</td>
<td>6.51</td>
<td>8.76</td>
</tr>
<tr>
<td>$\sigma(R^E - R^f) %$</td>
<td>16.66</td>
<td>7.58</td>
</tr>
<tr>
<td>$E[R^f] %$</td>
<td>1.17</td>
<td>1.93</td>
</tr>
<tr>
<td>$\sigma(R^f) %$</td>
<td>0.89</td>
<td>0.89</td>
</tr>
<tr>
<td>$E[Z - R^f] %$</td>
<td>0.96</td>
<td>0.45</td>
</tr>
<tr>
<td>$\sigma(Z - R^f) %$</td>
<td>0.44</td>
<td>0.52</td>
</tr>
<tr>
<td>$E[F] %$</td>
<td>2.22</td>
<td>1.05</td>
</tr>
<tr>
<td>$\sigma(F) %$</td>
<td>1.35</td>
<td>1.45</td>
</tr>
<tr>
<td>$corr(\sigma(R^E), ICSV)$</td>
<td>0.57</td>
<td>0.57</td>
</tr>
<tr>
<td>$corr(\Delta y, \sigma(R^E))$</td>
<td>-0.36</td>
<td>-0.37</td>
</tr>
<tr>
<td>$corr(\Delta y, ICSV)$</td>
<td>-0.33</td>
<td>-0.22</td>
</tr>
<tr>
<td>$corr(Z - R^f, \sigma(R^E))$</td>
<td>0.50</td>
<td>0.73</td>
</tr>
</tbody>
</table>
Figure 4: Impulse Responses

This figure plots the impulse responses with respect to a one-standard-deviation positive shock to the volatility of aggregate productivity. The y axis is the percentage deviations from the steady state. One period is one month. $\phi$ is the leverage of entrepreneurs, $q$ is the asset price, $r^E$ is the equity return, $\omega$ is the default cutoff, $F$ is the default probability and $z - r^f$ is the credit spread.
Thus more of the borrowers are able to honor their debt, less default occurs, credit spread decreases, as shown in the right panel of Figure 4. This will alleviate the credit frictions, weaken the shock amplification effect of the frictions. Thus, an exogenous increase in the volatility of the productivity process drives up overall volatility, but it also weakens shock amplification of the financial frictions, thus these two effects offset each other in term of the volatility of the economy. As one can see that given the same amount of volatility in term of macroeconomic quantities, the volatility of equity $\sigma(R^E - R^f)$ in column “Stoch. Vol.” is smaller than it in the benchmark column.

**Dynamic of uncertainty**  In absence of exogenous variations in the dispersion of firm cash flows, the heterogeneity coming from micro level is fixed, therefore the time-varying dispersion pattern at the micro level is only driven by aggregate fluctuations. In this model, the micro level heterogeneity comes from the aggregate variable is the default threshold, or the default probability. Because default firms have zero gross equity returns, while non-default firms have positive gross equity returns. When more firms default, micro level heterogeneity of equity returns increases. As we discussed earlier that upon an increase in the volatility of productivity, investment and asset price increase, debt repayment ability of borrowers get strengthened, less default is going to happen. Hence, the return dispersion decreases, which implies that micro uncertainty falls when aggregate productivity is more volatile. The mechanism leads to a negative correlation between micro and macro uncertainty.

### 6.2 Robustness of Predictive Regressions

**Credit Spread predictability using other measures** Table 9 shows that micro uncertainty measures, computed as the interquartile range of firm sales growth, can also significantly predict future credit spreads. I regress the cumulative yield difference between BAA and AAA corporate bond index on the interquartile range of firm sales growth. The predictive regression is performed at quarterly frequency. The results remain significant after controlling for earning to price ratio, term spread, net equity issuance, inflation and GDP growth rate.

**Return predictability** Table 10 reports return predictability using loan default rates, with popular control variables, including net equity issuance, term spread, inflation, credit spread, consumption to wealth ratio. All these control variables are downloaded from Amit Goyal’s website. Dividend yield is not included because it strongly correlates with default probability, at 53%, which causes multicollinearity problem.
Robustness of predictability The predictors used in this study, ICSV and loan default rate, both have high persistence, therefore in order to take into account the persistence when doing inference on the estimates, I use the Bonferroni test proposed by Campbell and Yogo (2006), which takes into account the persistence when calculating the finite-sample distribution of the estimates. The persistence of \( ICSV^FF \) and default probability are 0.866 and 0.986, respectively. Table 11 shows that these two variables are robust predictors after taking into account of the persistence, both are significant at 95% confidence interval.
Table 9: **Credit Spread Predictability - Sales**

This table reports predictability of credit spread using dispersion of sales growth. The dispersion of sales growth is measured as interquartile range (IQR) of year-on-year sales growth. The sample starts in 1962:Q2 and ends in 2016:Q4. $CS_{t \rightarrow t+h} = \sum_{s=1}^{h} (Baa_{t+s} - Aaa_{t+s})$ is the cumulative credit spread. It is the holding period return of a portfolio, which is long in BAA bond and short in AAA bond, from period $t$ until period $t+h$. The credit spread is in percentage, at quarterly frequency. *IQR Sales Growth* is the interquartile of firm sales growth. *E/P* is earning to price ratio, *Term Spread* is the difference between long term yield on government bonds and the T-bill. *Net Equity Issuance* is the ratio of 12-month moving sums of net issues by NYSE listed stocks divided by the total market capitalization of NYSE. Numbers in parentheses are standard errors estimated using Newey-West estimator allowing for 3 lags.

### Panel A

\[
CS_{t \rightarrow t+h} = a + b IQR(\Delta Sales)_t + \varepsilon_{t+h}
\]

<table>
<thead>
<tr>
<th>$h$</th>
<th>1</th>
<th>4</th>
<th>8</th>
<th>12</th>
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<tbody>
<tr>
<td>IQR(\Delta Sales)$_t$</td>
<td>0.046***</td>
<td>0.236***</td>
<td>0.472***</td>
<td>0.631***</td>
<td>0.774***</td>
</tr>
<tr>
<td></td>
<td>(0.016)</td>
<td>(0.063)</td>
<td>(0.091)</td>
<td>(0.124)</td>
<td>(0.162)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.095</td>
<td>0.185</td>
<td>0.215</td>
<td>0.199</td>
<td>0.180</td>
</tr>
</tbody>
</table>

### Panel B

\[
CS_{t \rightarrow t+h} = a + b IQR(\Delta Sales)_t + cX_t + \varepsilon_{t+h}
\]

<table>
<thead>
<tr>
<th>$h$</th>
<th>1</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>IQR(\Delta Sales)$_t$</td>
<td>0.033***</td>
<td>0.164***</td>
<td>0.296***</td>
<td>0.376***</td>
<td>0.470***</td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td>(0.035)</td>
<td>(0.058)</td>
<td>(0.102)</td>
<td>(0.139)</td>
</tr>
<tr>
<td>E/P</td>
<td>0.004***</td>
<td>0.011***</td>
<td>0.019***</td>
<td>0.025***</td>
<td>0.034***</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.003)</td>
<td>(0.005)</td>
<td>(0.006)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>Term Spread</td>
<td>0.084***</td>
<td>0.245***</td>
<td>0.155</td>
<td>0.056</td>
<td>0.096</td>
</tr>
<tr>
<td></td>
<td>(0.024)</td>
<td>(0.082)</td>
<td>(0.168)</td>
<td>(0.244)</td>
<td>(0.289)</td>
</tr>
<tr>
<td>Net Equity Issuance</td>
<td>-0.056***</td>
<td>-0.274***</td>
<td>-0.465***</td>
<td>-0.599***</td>
<td>-0.642***</td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
<td>(0.065)</td>
<td>(0.110)</td>
<td>(0.150)</td>
<td>(0.201)</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.096**</td>
<td>0.626***</td>
<td>1.336***</td>
<td>2.042***</td>
<td>2.496***</td>
</tr>
<tr>
<td></td>
<td>(0.047)</td>
<td>(0.164)</td>
<td>(0.288)</td>
<td>(0.487)</td>
<td>(0.627)</td>
</tr>
<tr>
<td>GDP Growth</td>
<td>-0.092**</td>
<td>-0.401***</td>
<td>-0.568***</td>
<td>-0.496</td>
<td>-0.303</td>
</tr>
<tr>
<td></td>
<td>(0.038)</td>
<td>(0.122)</td>
<td>(0.191)</td>
<td>(0.306)</td>
<td>(0.407)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.526</td>
<td>0.615</td>
<td>0.616</td>
<td>0.588</td>
<td>0.542</td>
</tr>
</tbody>
</table>

Numbers in parenthesis are standard errors
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$
Table 10: Return Predictability

This table reports the return predictability of default probability after controlling for net equity issuance, term spread, inflation, credit spread, consumption to wealth ratio\(^{24}\). The regressions are performed at quarterly frequency. \(R_{t \rightarrow t+h}^{ex}\) denotes the cumulative market return in excess of risk free rate, from period \(t\) to \(t+h\), in percentage numbers. The key regressor is the loan default rate, \(F\), measured in percentage numbers. Net Equity Issuance is the ratio of 12-month moving sums of net issues by NYSE listed stocks divided by the total market capitalization of NYSE. Term Spread is the difference between long term yield on government bonds and the T-bill. Inflation is the growth rate of CPI. Credit Spread is the difference between yield of BAA and AAA corporate bond. CAY is the consumption to wealth ratio downloaded from Sydney Ludvigson’s website. Numbers in parentheses are standard errors estimated using Newey-West estimator allowing for 3 lags. Numbers in parentheses are standard errors estimated using Newey-West estimator.

\[
R_{t \rightarrow t+h}^{ex} = a + bF_t + cX_t + \varepsilon_{t+h}
\]

<table>
<thead>
<tr>
<th>(h)</th>
<th>1</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>(F)</td>
<td>1.012</td>
<td>3.190**</td>
<td>5.144***</td>
<td>8.264***</td>
<td>11.850***</td>
</tr>
<tr>
<td></td>
<td>(0.797)</td>
<td>(1.526)</td>
<td>(1.759)</td>
<td>(1.982)</td>
<td>(2.019)</td>
</tr>
<tr>
<td>(Net\ \text{Equity Issuance})</td>
<td>0.630</td>
<td>2.939</td>
<td>-0.416</td>
<td>-3.855*</td>
<td>-2.668</td>
</tr>
<tr>
<td></td>
<td>(0.759)</td>
<td>(1.841)</td>
<td>(1.748)</td>
<td>(1.974)</td>
<td>(1.819)</td>
</tr>
<tr>
<td>(Term\ \text{Spread})</td>
<td>-0.876</td>
<td>-2.092</td>
<td>4.715*</td>
<td>9.724***</td>
<td>8.503***</td>
</tr>
<tr>
<td></td>
<td>(1.004)</td>
<td>(2.336)</td>
<td>(2.614)</td>
<td>(2.377)</td>
<td>(2.616)</td>
</tr>
<tr>
<td>(Inflation)</td>
<td>0.018</td>
<td>-4.984</td>
<td>-8.173*</td>
<td>-7.331**</td>
<td>-5.696</td>
</tr>
<tr>
<td></td>
<td>(1.730)</td>
<td>(3.073)</td>
<td>(4.343)</td>
<td>(3.582)</td>
<td>(3.824)</td>
</tr>
<tr>
<td>(Credit\ \text{Spread})</td>
<td>-0.082</td>
<td>6.653</td>
<td>-6.676</td>
<td>-20.445**</td>
<td>-11.971</td>
</tr>
<tr>
<td></td>
<td>(3.877)</td>
<td>(7.505)</td>
<td>(9.796)</td>
<td>(9.743)</td>
<td>(10.209)</td>
</tr>
<tr>
<td>(CAY)</td>
<td>0.250</td>
<td>1.736*</td>
<td>7.385***</td>
<td>12.408***</td>
<td>13.219***</td>
</tr>
<tr>
<td></td>
<td>(0.478)</td>
<td>(1.029)</td>
<td>(1.663)</td>
<td>(1.981)</td>
<td>(2.604)</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.045</td>
<td>0.231</td>
<td>0.407</td>
<td>0.616</td>
<td>0.651</td>
</tr>
</tbody>
</table>

Numbers in parenthesis are standard errors
* p < 0.10, ** p < 0.05, *** p < 0.01
Table 11: **Bonferroni Test of Predictability**

This table reports the Bonferroni test of predictability, using the procedure described in Campbell and Yogo (2006). $CS_{t \rightarrow t+h}$ denotes the cumulative credit spread from period $t$ to $t+h$, in percentage, at monthly frequency. $R_{t \rightarrow t+h}^{ex}$ denotes the cumulative excess market return from period $t$ to $t+h$, in percentage, at quarterly frequency. $ICSV_{t}^{FF}$ is the cross-section standard deviation of return residuals from Fama French five-factor model, as defined in Section 2.2. $F_{t}$ is the default loan to total loan ratio. The table reports the estimate of slope coefficient in the predictive regression, $b$, and 95% confidence interval based on the Bonferroni test.

**Panel A: Credit Spread Predictability - Monthly**

$$CS_{t \rightarrow t+h} = a + bICSV_{t}^{FF} + \varepsilon_{t+h}$$

$$ICSV_{t}^{FF} = \mu + \rho ICSV_{t-1}^{FF} + u_{t}$$

<table>
<thead>
<tr>
<th>$h$</th>
<th>$b$</th>
<th>95% Conf. Interval</th>
<th>$corr(\varepsilon,u)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.026</td>
<td>[0.024,0.036]</td>
<td>-0.075</td>
</tr>
<tr>
<td>2</td>
<td>0.052</td>
<td>[0.045,0.071]</td>
<td>-0.059</td>
</tr>
<tr>
<td>3</td>
<td>0.077</td>
<td>[0.604,0.102]</td>
<td>-0.043</td>
</tr>
<tr>
<td>6</td>
<td>0.136</td>
<td>[0.104,0.180]</td>
<td>-0.023</td>
</tr>
<tr>
<td>12</td>
<td>0.197</td>
<td>[0.130,0.277]</td>
<td>-0.013</td>
</tr>
</tbody>
</table>

**Panel B: Return Predictability - Quarterly**

$$R_{t \rightarrow t+h}^{ex} = a + bF_{t} + \varepsilon_{t+h}$$

$$F_{t} = \mu + \rho F_{t-1} + u_{t}$$

<table>
<thead>
<tr>
<th>$h$</th>
<th>$b$</th>
<th>95% Conf. Interval</th>
<th>$corr(\varepsilon,u)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.504</td>
<td>[-0.164,1.843]</td>
<td>-0.076</td>
</tr>
<tr>
<td>4</td>
<td>2.619</td>
<td>[2.081,5.891]</td>
<td>-0.151</td>
</tr>
<tr>
<td>8</td>
<td>5.903</td>
<td>[4.812,10.516]</td>
<td>-0.147</td>
</tr>
<tr>
<td>12</td>
<td>9.561</td>
<td>[8.590,15.276]</td>
<td>-0.164</td>
</tr>
<tr>
<td>16</td>
<td>13.455</td>
<td>[12.129,19.508]</td>
<td>-0.157</td>
</tr>
</tbody>
</table>