Factor Investing: Hierarchical Ensemble Learning*

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Abstract

We present a Bayesian hierarchical framework for both cross-sectional and time-series return prediction. Our approach builds on a market-timing predictive system that jointly allows for time-varying coefficients driven by fundamental characteristics. With a Bayesian formulation for ensemble learning, we examine the joint predictability as well as portfolio efficiency via predictive distribution. In the empirical analysis of asset-sector allocation, our hierarchical ensemble learning portfolio achieves 500% cumulative returns in the period 1998-2017, and outperforms most workhorse benchmarks as well as the passive investing index. Our Bayesian inference for model selection identifies useful macro predictors (long-term yield, inflation, and stock market variance) and asset characteristics (dividend yield, accrual, and gross profit). Using the selected model for predicting sector evolution, an equally weighted long-short portfolio on winners over losers achieves a 46% Sharpe ratio with a significant Jensen’s alpha. Finally, we explore an underexploited connection between classical Bayesian forecasting and modern ensemble learning.

Key Words: Hierarchical Model, Firm Characteristics, Market Timing, Portfolio Efficiency, Return Predictability, Risk Anomalies, Seemingly Unrelated Regressions

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1 Introduction

Bayesian methods are commonly used in portfolio efficiency studies because researcher can incorporate their prior beliefs and uncertainty of parameters is addressed by posterior distribution. In practice, estimation risk increases dramatically when we consider more than a couple of risky assets in the portfolio. With the rise interests of factor investing, risk anomalies, including risk factors and firm characteristics, asset allocation problems have gained tremendous attention. DeMiguel, Martin-Utrera, Nogales, and Uppal (2018) show the economic rationale of reducing transaction costs when considering a larger number of characteristics in asset allocation.

In this paper, we provide a Bayesian hierarchical model for both cross-sectional and time-series stock return prediction. First, we build a predictive system for the cross section of asset returns with macro market-timing predictors. Second, we systematically model the time-varying coefficients using fundamental characteristics for all assets. Third, the predictive system follows a hierarchical prior, such that the coefficient information for macro predictors and asset characteristics are shared across all assets. Finally, in the Bayesian portfolio analysis, our dynamic asset allocation procedure is based on re-estimating and then rebalancing the portfolios on a rolling window basis.

On one hand, searching for time-series stock return predictors has a long history in the field of empirical finance. Whereas the literature focuses on existence of return predictability for a single market index, Bayesian hierarchical model adds a convenient framework to study joint predictability for multiple assets. A predictive system across cross-sectional returns connects economic risk anomalies through the time-varying coefficients, as well as searching for the common predictors across assets. As a by-product, the Bayesian approach provides a predictive distribution, including the predictive returns and covariance matrix, for portfolio optimization.

On the other hand, long-short portfolio returns based on a firm characteristic can be treated as a proxy to an underlying risk factor. In the current literature, both fundamental asset characteristics and risk factors are the main drivers to explain returns in the cross section. The factor-based portfolio method applies asset pricing models to form expected returns and covariance matrix. In this paper, we use firm characteristics to perform a characteristics-based portfolio selection for the underlying risk factors. In particular, we follow the literature to project the time-varying coefficients on fundamental characteristics, such that the predictive returns are also driven by underlying risk
factors. We also compare the pros and cons of these two approaches for factor investing.

Our method marries Bayesian predictive distribution and hierarchical modeling. Notably, Bayesian predictive system is similar to modern ensemble learning: Using an average forecast from multiple models outperforms forecast from any model alone. The famous Bayesian additive regression trees (BART) of Chipman, George, and McCulloch (2010) is one early example of Bayesian ensemble learning and recently He, Yalov, and Hahn (2018) propose a modified version of BART with faster and better posterior estimation.

In this paper, we explore this unique connection to bridge the gap between Bayesian portfolio analysis and machine-learning forecasting. Our method provides a Bayesian interpretation for an ensemble learning approach. Bayesian hierarchical model allows us to assume a joint prior distribution on predictor coefficients across assets. Averaging multiple draws for the posterior predictor coefficients within a linear model is similar to taking the average of multiple ensemble models.

The primary goal of our model is to study the optimal portfolio choice for N risky assets, but not stock-bond (risk-free rate) allocation. We denote $r_{i,t+1}$ as the excess returns and $x_t$ as the vector of lag macro market-timing predictors. Our predictive regressions provide a system for cross-sectional return prediction.

$$r_{i,t+1} = \alpha_{i,t} + x_t^T\beta_{i,t} + \epsilon_{i,t+1}, \quad \forall i = 1, \cdots, N.$$ 

Market-timing predictive regressions are predictive models, but asset pricing factor models are explanatory models. We adopt fundamental characteristics-driven time-varying specifications for $\alpha_{i,t}$ and $\beta_{i,t}$, which are connected within a hierarchical structure for joint predictability. Using such lag predictors is convenient for one- or multi-step-ahead return prediction, whereas using risk factors of simultaneous returns is not. In particular, we follow the recent development of machine learning in asset pricing, such as Gu, Kelly, and Xiu (2018) and Feng, Polson, and Xu (2019), and use the combination of macro predictors and fundamental characteristics.

In Bayesian hierarchical modeling, each posterior model is drawn via different asset-specific predictability coefficients. Every predictor has a role in the predictive distribution for each asset by the posterior distribution. Therefore, the predictive distribution of asset returns summarizes both estimation uncertainty and model uncertainty (time-varying model specification). The key in-
puts to the mean-variance efficient portfolio of Markowitz (1952) is predictive expected returns and predictive covariance matrix. Our approach provides a predictive distribution of cross-sectional returns using lag macro predictors and fundamental characteristics.

We also perform a variety of empirical exercises that illustrate performance of our procedure in the U.S. equity market for portfolios in different sectors. Our hierarchical ensemble learning portfolio achieves 500% cumulative returns in the period 1998-2017, and outperforms all workhorse benchmarks, including the historical average, linear regression, the equally weighted portfolio, and the return of S&P 500 (SPY). Our predictive regressions consistently outperform the constant predictive regressions with the same market timing predictors.

From an investment perspective, we perform the Bayesian inference for model selection to identify useful macro predictors (long-term yield, inflation, and stock market variance) as well as asset characteristics (dividend yield, accrual, and gross profit). The small model, which only uses the useful predictors, has a close performance in the period 1998-2017. We use the small model to predict the top 3 and bottom 3 industries for a monthly sorting. Using the selected model for predicting sector evolution, an equally weighted long-short portfolio on winners over losers achieves a 46% Sharpe ratio with a significant Jensen’s alpha.

The rest of the paper is organized as follows. In section 1.1, we position our paper within the relevant literature in empirical asset pricing and Bayesian econometrics. Section 2 introduces our hierarchical model as well as its connection to ensemble learning. Section 3 shows the performance of our efficient portfolio, and the inference for portfolio characteristics and macro predictors. Section 4 concludes with directions for future research.

1.1 Related Literature

Our paper builds on several strands of Bayesian methods in asset pricing. Seminal references include Kandel and Stambaugh (1996) and Barberis (2000), who start the literature by considering estimation uncertainty in the optimal portfolio choice problem. They focus on the predictive distribution for returns and calculate the portfolio weights by the corresponding Bayesian estimates for the expected returns and covariance matrix. Both Avramov and Zhou (2010) and Jacquier and Polson (2011) provide excellent surveys for Bayesian methods of portfolio analysis and financial
econometrics. Polson and Tew (2000) introduce a Bayesian hierarchical structure and employ informative priors on the expected returns and covariance matrix. We follow their hierarchical structure on cross-sectional return predictability but also differs in estimating residual covariance matrix and allowing dynamic coefficients.

The body of literature on Bayesian portfolio analysis and asset pricing models is extensive and long-established. Pástor (2000) and Pástor and Stambaugh (2000) incorporate asset pricing model into the Bayesian portfolio problem, which enables risk-based or characteristics-based portfolio optimization. Avramov (2002) provides a Bayesian model averaging approach and concludes that an investor who ignores model uncertainty suffers considerable utility loses. They also find term and market premia are robust predictors. Avramov (2004) models excess returns on \( N \) investable assets with characteristics-driven time-varying alpha and time-varying risk premia. Avramov and Chordia (2006) extend this structure by allowing characteristics-driven beta. Our work benefits greatly from this body of literature, yet our modeling perspectives are different in a number of ways:

1. Our model is a predictive system instead of an asset pricing model. The difference is that a predictive system uses lag predictors, whereas an asset pricing model uses factors in the same period.

2. Our predictive system is naturally built for forecasting without assumptions on the predictor dynamics, whereas an asset pricing model requires the use of future beta and factors and assumptions on their dynamics (constant or time varying).

3. Our unconditional predictive regression contains both fundamental characteristics and macro predictors, whereas theirs is a predictive regression on fundamental characteristics.

4. We do not need to specify predictor dynamics on characteristics, whereas the literature adopts a vector autoregression to construct the predictive distribution.

The second related area is testing return predictability using macro market-timing predictors. The influential work of Welch and Goyal (2007) examine 14 predictor variables but find little forecasting power in univariate forecasting regressions. Cochrane (2008) applies a vector autoregression for returns and dividend growth to explore their joint stochastic dynamics and defend the re-
turn predictability. Rapach, Strauss, and Zhou (2010) find that using a combination of these macro predictors outperforms univariate forecasting regressions. Feng, He, and Polson (2018) provide a conditional linear return forecasting model within a neural network. Therefore, we build on this literature by adding time-varying coefficients on macro predictors.

Our paper is also closely related to recent literature on high dimensionality of cross-sectional asset pricing models. Feng, Giglio, and Xiu (2019) provide a high-dimensional inference method to tame the factor zoo for independent risk price. Gu, Kelly, and Xiu (2018) present a comprehensive empirical investigation of forecasting performance for multiple machine learning algorithms, whereas Han, He, Rapach, and Zhou (2018) give a forecast combination approach on the same characteristics library. Kozak, Nagel, and Santosh (2017) show a Bayesian shrinkage estimation for the stochastic discount factors. Kelly, Pruitt, and Su (2018) adopt a characteristics-driven time-varying coefficient model for principal component analysis. We have similar model specification with theirs, but apply a hierarchical shrinkage approach to evaluate both fundamental characteristics and macro predictors from an investment perspective.

Finally, our paper is related to the area of Bayesian model comparison and hypothesis testing. Connolly (1991) applies Bayesian posterior-odds analysis to show the instability of “the weekend effect”. Pástor and Stambaugh (2000) study the Bayesian factor model comparison from an investment perspective, while Lopes and West (2004) explore reversible jump MCMC methods for latent factor models. Li, Liu, and Yu (2015) develop a Bayesian Chi-squared test to overcome the disadvantages of using a Bayes factor. Lopes and Polson (2018) re-examine the Bayes factor from the perspective of an a priori assessment of the test statistic distribution. We apply Chi-squared test to perform a model selection for fundamental characteristics and macro predictors.

2 Methodology

First, our predictive model is introduced. Section 2.2 shows the reformulation of the predictive system as a seemingly unrelated regression model. Section 2.3 discusses a hierarchical prior on the regression coefficients. The Markov chain Monte Carlo scheme is presented in section 2.4. Finally, section 2.5 shows the use of predictive distribution for portfolio optimization.
2.1 Predictive Modeling

Suppose a Bayesian investor observes historical return $R_t$ of $N$ assets, a length $P$ vector of asset characteristics $z_t$ for each stock and macro market-timing predictors $x_t$ which is a length $Q$ vector. The investor updates the mean-variance efficient portfolio at time period $t + 1$ based on a joint predictive distribution $f(R_{t+1} \mid D_t)$ of these $N$ assets for every period.

$$R^p_{t+1} = W_t^\top R_{t+1},$$

where $R_{t+1} = (r_{1,t+1}, \cdots, r_{N,t+1})^\top$, $W_t$ is the portfolio weight estimated at the end of period $t$ and $R^p_{t+1}$ is return of the portfolio. We denote all historical data observed up to period $t$, including asset returns, characteristics, and macro predictors as $D_t$.

We follow the Bayesian predictive regression model in Kandel and Stambaugh (1996), but adopt a conditional predictive formulation with time-varying coefficients. For each asset $i$ and time period $t$, return of asset $i$ at time period $t + 1$ is modeled as

$$r_{i,t+1} = \alpha_{i,t} + x_t^\top \beta_{i,t} + \epsilon_{i,t+1},$$

where $x_t$ is the vector for $Q$ macro predictors. Residual vector $(\epsilon_{1,t+1}, \cdots, \epsilon_{N,t+1})^\top$ contains shocks to all asset returns and are assumed to follow a multivariate normal distribution $N(0, \Sigma)$ with full covariance matrix. Coefficients $\alpha_{i,t}$ and $\beta_{i,t}$ are assumed to be time-varying, driven by the asset characteristics as follows

$$\alpha_{i,t} = \eta_i^a + z_{i,t}^\top \theta_i^a,$$
$$\beta_{i,t} = \eta_i^b + \theta_i^b z_{i,t},$$

where $\theta_i^b$ is a matrix coefficient of size $Q \times P$ and $z_{i,t}$ is the vector for $P$ portfolio characteristics. Avramov (2004) gives similar time-varying coefficients setup, but assumes a common factor structure for all assets and factor loadings and alphas are driven by lagged stock characteristics. If we plug the time-varying coefficients into equation (2), we obtain an unconditional predictive regres-
sion on \( z_t, x_t \), and their interactions \( z_t \otimes x_t \):

\[
 r_{i,t+1} = \eta_i^a + z_{i,t}^T \theta_i^a + x_{i,t}^T \eta_i^b + (x_t \otimes z_{i,t})^T \theta_i^b + \epsilon_{i,t+1}. \tag{5}
\]

Estimating covariance matrix of many assets is a hard problem. In previous literature, people either assume a low dimensional factor structure and independent residual term or build predictive regression for each individual asset independently. If we consider \( N \) investable assets jointly, we have a multiple-response model involved with group selection. For a Bayesian investor, a natural way for predicting multiple risk assets is to adopt a hierarchical structure for the predictor coefficients to study their joint predictability. In the next subsection, we follow the factor hierarchical approach of Polson and Tew (2000) and present our hierarchical setup with the matrix formulation of seemingly unrelated regressions.

### 2.2 Seemingly Unrelated Regressions

Before discussing about our main model, we simplify notations of our main equation (5) as

\[
 r_{i,t+1} = f_{i,t}^T b_i + \epsilon_{i,t}, \tag{6}
\]

where \( f_{i,t} = [1, z_{i,t}, x_t, (x_t \otimes z_{i,t})] \), \( \otimes \) denotes Kronecker product. \( b_i \) is a vector of all coefficients \( b_i = [\eta_i^a, \theta_i^a, \eta_i^b, \theta_i^b] \). For each asset \( i \), stack equations of different time period \( t \) as

\[
 r_i = f_i^T b_i + \epsilon_i, \tag{7}
\]

where \( r_i = (r_{i,2}, \ldots, r_{i,T+1})^T \), \( \epsilon_i = (\epsilon_{i,1}, \ldots, \epsilon_{i,T})^T \), and \( f_i \) is a matrix with \( T \) rows. The predictive coefficient \( b_i \) is the predictor coefficient that can be learned jointly from all assets. Here, \( b_i \) implies which macro market-timing signals are useful, as well as which characteristics drive the time-varying coefficients. If we work on the unconditional expression in equation (5), the predictive system for all \( N \) assets can be reformulated into a seemingly unrelated regressions (SUR) setup.
The SUR setup is organized by assets. Stacking all equations asset by asset, we have

\[ R = FB + E, \]  

(8)

where

\[
R = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_N \end{bmatrix}, \quad F = \begin{bmatrix} f_1 & 0 & 0 & 0 \\ 0 & f_2 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & f_N \end{bmatrix}, \quad B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{bmatrix}, \quad E = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_N \end{bmatrix}.
\]

Here, \( R \) is a \( NT \times 1 \) vector of stacked vector of firm returns, \( F \) is a \( NT \times NK \) block diagonal matrix, \( B \) is an \( NK \times 1 \) vector, and \( E \) is \( NT \times 1 \) stacked vector of residuals.

We differ from Polson and Tew (2000) in that the covariance matrix of the stacked residual \( E \) is assumed to be \( \Omega = \Sigma \otimes I_N \), where \( \Sigma \) is the covariance matrix for cross-sectional residuals and \( I_N \) is a \( N \times N \) identity matrix. This part of our model differs from the standard SUR setup, we assume cross section covariance but no time series covariance of residuals. We update \( \Omega \) by updating \( \Sigma \) in the Gibbs sampling discussed in section 2.4. We simply assume an inverse-Wishart prior on \( \Sigma \),

\[ \Sigma \sim IW(\nu_\Sigma, V_\Sigma). \]  

(9)

The next section shows details of hierarchical prior on regression coefficients.

### 2.3 Hierarchical Prior

The current empirical literature mainly focuses on the time-series predictability evidence for index returns, such as S&P500. Similar to the field of cross-sectional asset pricing, researchers are also interested in the common predictors across assets, such as portfolios of different sectors. To study the average predictability across assets, we assume \( b_i \) is independent and identical draw
from the following mutivariate hierarchical prior

\[ b_i \sim N(\bar{b}, \Delta_b) \quad \forall i = 1, \cdots, N, \]  

\[ \bar{b} \sim N(0, \Delta_{\bar{b}}), \]  

\[ \Delta_b \sim IW(\nu_b, V_b), \]  

where \( \bar{b} \) is a mean vector and \( V_b \) is a diagonal covariance matrix. The hierarchical structure does help for shrinking the coefficients across all assets. However, simply testing \( H_0 : \bar{b} = 0 \) does not reveal the joint predictability implication. The ideal test should be about the covariance matrix parameter \( \Delta_b \), such that there is no cross-sectional difference for \( b_i \).

The likelihood function is multivariate normal:

\[
l(E \mid B, \Omega) \propto |\Omega|^{-\frac{1}{2}} \exp \left\{ -\frac{1}{2}(R - FB)^T \Omega^{-1} (R - FB) \right\}.\]  

The joint posterior can be expressed as

\[
p(B, \Omega, \bar{b}, \Delta_b \mid R, F) \propto l(E \mid B, \Omega)p(\Omega)p(B \mid \bar{b}, \Delta_b)p(\bar{b})p(\Delta_b).\]  

The next section describes Gibbs sampler of the model above.

### 2.4 Markov chain Monte Carlo Scheme

For the univariate predictive regression, deriving its predictive distribution and obtaining the conditional mean and covariance matrix is common. However, it is hard to get a closed form predictive distribution of our hierarchical prior model. Therefore, we discuss our Markov chain Monte Carlo scheme in this subsection. Most steps of the Gibbs sampler is similar to Polson and Tew (2000), but the sampling scheme of residual covariance \( \Omega \) is different. We assume \( \Omega = \Sigma \otimes I_N \) and draw \( \Sigma \) from inverse-Wishart distribution.

1. Update \( B \) through a multivariate normal distribution:

\[
B \mid \bar{b}, \Delta_b, \Omega, R, F \sim N\left(b^*, (F^T \Omega^{-1} F + I_N \otimes \Delta_b^{-1})^{-1}\right),
\]
where \( b^* = (F^\top \Omega^{-1} F + I_N \otimes \Delta_b^{-1})^{-1} \times (F^\top \Omega^{-1} R + (I_N \otimes \Delta_b^{-1})(t_N \otimes \tilde{b})) \), and \( t_N \) denotes an \( N \times 1 \) vector of ones.

(2) Update \( \tilde{b} \) through a multivariate normal distribution:

\[
\tilde{b} \mid B, \Delta_b \sim N \left( \frac{1}{N}(t_N \otimes I_K)^\top B, \frac{1}{N} \Delta_b \right),
\]  

where \( t_N \) denotes an \( N \times 1 \) vector of ones and \( I_K \) is \( K \times K \) dimensional identity matrix.

(3) Update \( \Delta_b \) through an inverse-Wishart distribution:

\[
\Delta_b \mid B, \tilde{b} \sim IW \left( \nu_b + N, ((D - \tilde{b} \otimes 1_N^\top)(D - \tilde{b} \otimes 1_N^\top)^\top + V_b) \right)^{-1}.
\]  

(4) Update \( \Sigma \) through an inverse-Wishart distribution:

\[
\Sigma \mid B, R, F \sim IW(\nu_\Sigma + T, V_\Sigma + \hat{E}^\top \hat{E}),
\]  

where \( \hat{E} = [\hat{e}_1, \cdots, \hat{e}_N] \) is a \( T \times N \) matrix of residuals, \( \hat{e}_i = r_i - f_i^\top b_i \). Note that \( \Omega = \Sigma \otimes I_N \).

Then we finish updating \( \Omega \).

2.5 Ensemble Forecast and Portfolio Optimization

The idea of ensemble learning is to use an average of multiple forecasts to reduce the variance of prediction. For an unconditional linear predictive regression in equation (5), an ensemble learning approach is equivalent to plug in the average of the predictor coefficients \( b_i \). Since we plug in posterior mean of parameters when predict return of the next time period, Bayesian estimate of \( b_i \) is the average from the posterior draws. Therefore, the Bayesian forecast is the ensemble forecast:

\[
\widehat{r}_{i,t+1}^{(j)} = f_{i,t} b_{i}^{(j)}
\]  

\[
\frac{1}{J} \sum_{j=1}^{J} \widehat{r}_{i,t+1}^{(j)} = f_{i,t} \frac{1}{J} \sum_{j=1}^{J} b_{i}^{(j)}.
\]
Given the Bayesian predictive distribution \( f(R_{t+1} \mid D_t) \), we can easily obtain the conditional expectation and covariance matrix for \( R_{t+1} \), as well as the optimal portfolio weights.

The hierarchical prior \( N(\overline{b}, \Delta_h) \) incorporates a second part for the ensemble learning idea. First, return predictability has to be shared across all assets. The hierarchical prior represents cross-sectional predictability with the same prior mean. Second, we focus on average signal for a predictor. We can simply perform a posterior inference on \( \overline{b} \) for whether macro predictor is useful in the conditional formulation. We can also infer which characteristics drive time-varying predictor coefficients. Finally, Bayesian shrinkage prior acts like a regularization penalty on the average signal and helps out-of-sample forecasting.

We follow Bayesian portfolio analysis by simply plug in conditional expectation \( \mathbb{E}(R_{t+1} \mid D_t) \) and covariance matrix \( \text{Cov}(R_{t+1} \mid D_t) \) for optimal portfolio weight calculation. Specifically, the portfolio is built to maximize the mean-variance utility function:

\[
U(W) = \exp \left\{ \mathbb{E}(R_{p,t+1}) - \frac{\gamma}{2} \text{Var}(R_{p,t+1}) \right\},
\]

where \( R_{p,t+1} = W^\top R_{t+1} \) is the future portfolio return and \( \gamma \) is the coefficient for risk aversion. We simply restrict the short selling and require \( \sum W_i = 1 \) and \( W_i \geq 0 \).

We observe macro predictors and fundamental characteristics in the current period. Given that time-varying predictor coefficients are driven by fundamental characteristics, our framework is a convenient way to provide or update the one-step- or multi-step-ahead optimal portfolio weights. We illustrate this convenient usefulness in the section of empirical analysis.

3 Empirical Study

First, the background of data is introduced. Implementation details and comparison metrics are listed in section 3.2. Section 3.3 provides an out-of-sample comparison for our optimal portfolio performance and other workhorse benchmarks. In section 3.4, we perform a Bayesian inference on the predictor selection and show a dimension-reduced model.
3.1 Data

We provide an asset allocation analysis on the cross section of the U.S. equity market for portfolios in different sectors. We follow Fama-French annual industry classification\(^1\) and download their industry portfolio returns. Given that we only consider risky asset allocation, all returns used are excess returns by subtracting the risk free rate.

To compare portfolio optimization methods for different numbers of assets, three different versions of classifications are evaluated: 10, 30, and 49 industries. We attempt to explore the asset sector allocation at different levels of classifications. We have also added the case for size and book-to-market 25 sorted portfolios as a benchmark comparison.

The data sample begins from January 1978 and ends on January 2018. We provide a rolling-window forecast for the recent 20 years. We also offer portfolio performance measures for the first and second halves of the sample. Our dynamic asset allocation procedure is based on re-estimating and then rebalancing portfolios on a rolling window basis. Different rolling window widths are considered, including 120-month, 180-month, and 240-month windows. Each time the model is trained with data within the rolling window and predict return of the next month, then the window moves forward.

A combination of fundamental characteristics and macro market-timing predictors is considered, which explore both time-series and cross-sectional return prediction. Welch and Goyal (2007) study time-series return prediction of S&P 500 using market-timing predictors, and we pick five macro predictors from them, including the treasury-bill rate, inflation, long-term yield, stock market variance of S&P 500, and lag excess market return.

The 10 portfolio characteristics include workhorse risk anomalies for the cross-sectional return prediction. The chosen firm characteristics include all main categories, such as the book-to-market ratio, earning-price ratio, investment growth, return on equity, inventory, accrual, dividend yield, gross profit, capitalization ratio, and asset turnover. When calculating portfolio characteristics, we use cross-sectional average for firm characteristics.

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\(^1\)They form industry portfolios at the end of June of year \(t\) with the Compustat SIC codes for the fiscal year ending in the calendar year \(t-1\). We follow the same annual portfolio construction to calculate the fundamental characteristics.
3.2 Implementation Procedures and Comparison Metrics

To gauge the performance of our new model, we compare our approach with alternative methods using annualized Sharpe ratio, t-statistics for Jensen’s alpha, mean-variance utility specified in equation 21, as well as portfolio turnover ratio. Turnover ratio is evaluated as below:

$$\text{TO} = \frac{1}{T} \sum_{t=1}^{T} \sum_{j=1}^{N} \left| w_{j,t+1} - w_{j,t+} \right|,$$

where $w_{j,t+}$ is the portfolio weight of security j before rebalancing at time period $t + 1$ and $w_{j,t+1}$ is the portfolio weight after rebalancing. Intuitively the turnover ratio can be interpreted as the average fraction of the portfolio value that has been bought or sold for the monthly rebalancing.

Three empirical exercises are designed to compare different methods. Each exercise has different data input but the dynamic portfolio optimization and rebalancing designs are the same for all exercises. Here are the step-by-step details.

- We estimate the model from a rolling window of historical data. For a good convergence condition, we run 2,000 MCMC samples and burn the first 1,000 draws.

- To predict asset return of time period $t + 1$, we plug the latest observation for $x_t$ and $z_t$ into the ensemble learning model with parameters set at posterior mean.

- The predictive (conditional) covariance matrix is a byproduct estimated from the Bayesian model, which is a huge plus for the Bayesian predictive distribution approach.

- To maximize the mean-variance utility, we calculate the long-only (non-negative) optimal portfolio weights. We follow Avramov (2002), set risk aversion parameter $\lambda = 1.68$.

- Suppose rolling window width is $m$. For every new time period $t + 1$, estimate model parameters with data from $t - m + 1$ to time $t$. Then we predict asset return at time $t + 1$ (1-month ahead, we also predict 3-month or 6-month ahead return in empirical exercises) and rebalance our long-only portfolio weight accordingly. we repeat the rolling window estimation and recalculate the optimal long-only portfolio weights.
• However, for the period-by-period portfolio rebalancing, we only allow a maximum 20% of change for each individual asset position.

3.3 Portfolio Performance

First, we perform our portfolio optimization approach on different levels of classifications for industry returns, such as 10, 30, and 49 industries. A finer industry classification helps us obtain a more accurate estimate for \( z_t \), the industry-average fundamental characteristics. However, a larger cross section also induces higher estimation errors which might cause the mean-variance optimal diversification to underperform a naive diversification, the equally weighted portfolio. The goal of this exercise is to evaluate this trade-off.

The results are presented in Table 1 and Figure 1. We use a fixed 120-month rolling window to evaluate the portfolio performances in different sample periods. First of all, our hierarchical ensemble learning (HEL) portfolio has the highest Sharpe ratios for different industry classifications and sample periods. Second, a dynamic monthly updated portfolio by HEL has accumulated the highest cumulative returns, especially in the second period 2008 - 2017, and achieved almost 500% returns in twenty years.

On one hand, the regression model, which has constant coefficients on macro predictors \( x_t \), does not work well when \( N \) is larger. On the other hand, our characteristics-driven time-varying coefficient model works better when \( N \) is large, because the model learns a better hierarchical structure from a larger cross section. Finally, for the whole sample 1998 - 2017, all HEL portfolios have significant Jensen’s alpha over the market portfolio, as well as the highest mean-variance utilities over other competing methods.

Second, we perform our hierarchical ensemble stock return prediction on different horizons, namely, the 1-, 3-, and 6-month horizons. The presidential address in Cochrane (2011) shows stronger return predictability on a long-run regression for longer-horizon returns, especially for those predictors with serial correlation. We repeat the predictability evaluation using future 3-month \( \sum_{s=t+1}^{t+3} r_s \) as well as 6-month \( \sum_{s=t+1}^{t+6} r_s \) horizons. For ease of comparison, we still update the dynamic portfolio on a monthly basis, though those long-run forecasting models do optimize on a monthly basis. We are interested in the trade-off between this long-run forecasting
“mis-specification” and the higher signal-to-noise ratio.

The results are in Table 2 and Figure 2. We also use a fixed 120-month rolling window to evaluate the portfolio performances in different sample periods, but the returns on left-hand-side are future 1-, 3-, and 6-month cumulative returns. The first finding is, by higher cumulative returns and Sharpe ratios for the dynamic portfolios, we can see the 1-month prediction model works better than the other two. The reason might be, the monthly portfolio rebalancing is not consistent with the long-run return prediction. By the decreasing trend from 1-, 3-, to 6-month portfolio performances, the prediction consistency does matter in the Bayesian portfolio analysis.

3.4 Predictor Evaluation

A second illustration of our method is related to the Bayesian inference and predictor selection. The goal is to evaluate the effect of each macro predictor \( x_i \) and fundamental characteristics \( z_i \). We want to understand whether the macro predictor is useful for predicting the cross section of returns. If predictability is possible, we want to understand whether the predictor coefficients are time varying and driven by some fundamental characteristics. Given the interaction between macro predictors and characteristics, the unconditional model is relatively high dimensional. If we confirm a small number of useful macro predictors and fundamental characteristics, we find an interpretable characteristics-based predictive model.

The unconditional predictive model in equation 5 is built for forecasting and is estimated as hierarchical seemingly unrelated regressions. However, for interpretation, we should use the conditional predictive model as specified in section 2.1:

\[
\begin{align*}
  r_{i,t+1} &= \alpha_{i,t} + x_i^T \beta_{i,t} + \epsilon_{i,t+1} \\
  \alpha_{i,t} &= \eta_{i}^a + z_{i,t} \theta_{i}^a \\
  \beta_{i,t} &= \eta_{i}^b + \theta_{i}^b z_{i,t}.
\end{align*}
\]

To evaluate the predictability of one macro predictor, we need to examine if the corresponding \( \beta_{i,t} \) is zero. Hence, we need to check if the corresponding \( \eta_{i}^b \) and \( \theta_{i}^b \) are jointly zero. In the unconditional formulation, we need to test whether 11 parameters (10 interactions plus one intercept) are
jointly zero for the macro predictor and all its interaction with characteristics.

To evaluate the usefulness of one fundamental characteristic, we need to examine if the corresponding $\theta^e_i$ and $\theta^b_i$ are jointly zero. In the unconditional formulation, we need to test whether six (five interactions plus one intercept) parameters are jointly zero for the characteristic and all its interaction with macro predictors. The hierarchical structure does help shrink the coefficients across assets, but it does not help explain the usefulness of the predictor.

For Bayesian estimates, the ensemble learning estimates over thousands of posterior samples, it is convenient to calculate the posterior distribution for the vector parameter and the covariance matrix. To test the predictors individually, one approach is a fast calculation for the Chi-square test statistic over the posterior sample. Given that we also have a rolling-window re-estimation, we obtain period-by-period posterior Chi-square statistics, which is a convenient way to evaluate the existence and strength of the non-stationary predictor. This rolling-window estimation makes the simple Chi-square statistic evaluation preferable to the Bayes factor approach, which requires intensive computation for rolling window predictions.

We plot the histograms of posterior Chi-square statistics of the first exercise in section 3.3 for the period 1998 - 2017 in Figure 3. The 90% cutoffs are 17.3 for macro predictors and 10.6 for characteristics. We find significant macro predictors (long-term yield, inflation, and stock market variance) as well as asset characteristics (dividend yield, accrual, and gross profit). If we re-evaluate the portfolio on this selected model, we also find a light performance over the full model: similar Sharpe ratios and utilities, significant Jensen’s alpha, but lower turnover ratios. The results presented in Table 3 and Figure 4, which can be compared with Table 1 and Figure 1.

The last exercise is to plot the cumulative returns of the first three winning industries and first three losing industries predicted by the model on a monthly rebalancing. The industry winners and losers are sorted on the predictive returns of our model. In Figure 5, we find a strikingly high cumulative returns for the winner portfolio, over 1,000%, where the other five sorted portfolios keep a consistent order predicted by our method. We consider a long-short strategy: long the equally weighted portfolios for the first three winners and short the equally weighted portfolios for the first three losers. Such HEL predicted long-short portfolios have significant Jensen’s alphas for

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2Earnings-to-price is significant in this figure, but not as robust as accrual in other figures. We keep accrual for the variable selection. The prediction performance does not change if we replace accrual with earnings-to-price.
the whole sample period 1998 - 2017, with respect to 30 and 49 industry portfolios.

4 Conclusion

In this paper, we propose a hierarchical ensemble learning framework for Bayesian portfolio analysis. In particular, we provide a market-timing and characteristics-based predictive model for both predicting stock returns and the optimal asset allocation.

The Bayesian hierarchical structure works with the cross-sectional prediction model because it helps shrink the coefficients across all assets. However, adding a prior or regularization on the mean of the predictor coefficient does not reveal the joint predictability implication. The ideal procedure should be adding a prior or regularization on the diagonal of the covariance matrix, such that no cross-sectional difference exists for the useless predictor coefficients. One possible direction of future research is the specification on this hierarchical prior for the field of asset pricing.

Taken together, our results provide promise for the continuing progress of Bayesian methods in empirical asset pricing. The traditional factor-based portfolio optimization applies the asset pricing model to form the expected returns and covariance matrix, but asset pricing factor models are explanatory models of simultaneous data. Applying asset pricing models for time-series return prediction requires additional specification on the dynamics for risk factors. In this paper, we use firm characteristics to perform a characteristics-based portfolio selection for the underlying risk factors. These market-timing predictive regressions are predictive models with the exposures of underlying risk factors. Finally, our empirical results confirm that the literature that combines market-timing predictors and fundamental characteristics is also useful in the asset allocation problem.

Lastly, the recent trend of machine learning methods in finance is connected to classical Bayesian modeling in several ways. Most machine learning methods are originally built for model prediction in a high signal-to-noise environment. However, the area of time-series return prediction has a low signal-to-noise ratio and is non-stationary, and thus demands a Bayesian interpretation. Bayesian priors help stabilize the prediction model, and the possibility of Bayesian inference complements the current empirical literature of machine learning. Our hierarchical ensemble learning basically connects to a machine learning concept within the Bayesian formulation. Interpreting other machine learning methods require future research.
References


### Table 1: Performance Statistics for Different Industries

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<td>48% 1.17</td>
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This table provides the performance statistics of the dynamic monthly updated portfolios using a 120-month rolling window. We summarize returns of multiple methods for 1-month-ahead return prediction, including hierarchical ensemble learning (HEL), regression (OLS), moving average (MA), and the equally weighted portfolios (EW). The summary statistics include the annualized Sharpe ratio (SR), the t-statistics for Jensen’s alpha, the monthly turnover percentage (TO), and the mean-variance utility. Results of three sample periods are reported.
Table 2: Performance Statistics for Different Step-Ahead Forecasts

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<tr>
<td>SPY</td>
<td>42%</td>
<td>1.68</td>
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1-month

| HEL    | 58% | 1.85 | 63% | 0.62% | 58% | 2.17 | 64% | 0.60% | 58% | -0.32 | 62% | 0.64% |
| MA     | 31% | 0.01 | 23% | 0.15% | 35% | 0.93 | 26% | 0.22% | 27% | -0.74 | 20% | 0.09% |
| OLS    | 31% | 0.17 | 47% | 0.13% | 41% | 1.12 | 40% | 0.36% | 18% | -1.47 | 55% | -0.10% |
| EW     | 54% | 1.72 | 4% | 0.54% | 44% | 1.85 | 4% | 0.38% | 63% | -0.34 | 3% | 0.71% |
| SPY    | 42% | 1.68 | 0% | 0.36% | 36% | 1.11 | 0% | 0.29% | 48% | 1.17 | 0% | 0.44% |

3-month

| HEL    | 58% | 1.88 | 57% | 0.64% | 59% | 2.35 | 59% | 0.63% | 57% | -0.40 | 55% | 0.64% |
| MA     | 31% | 0.01 | 23% | 0.15% | 35% | 0.93 | 26% | 0.22% | 27% | -0.74 | 20% | 0.09% |
| OLS    | 25% | -0.24 | 27% | -0.06% | 24% | 0.45 | 29% | -0.18% | 26% | -0.88 | 26% | 0.05% |
| EW     | 54% | 1.72 | 4% | 0.54% | 44% | 1.85 | 4% | 0.38% | 63% | -0.34 | 3% | 0.71% |
| SPY    | 42% | 1.68 | 0% | 0.36% | 36% | 1.11 | 0% | 0.29% | 48% | 1.17 | 0% | 0.44% |

6-month

This table provides the performance statistics of the dynamic monthly updated portfolios by a 120-month rolling window. The three panels correspond to 1-, 3-, and 6-month-ahead return prediction. We summarize returns of multiple methods for 49 industry portfolios, including hierarchical ensemble learning (HEL), regression (OLS), moving average (MA), and the equally weighted portfolios (EW). The summary statistics include the annualized Sharpe ratio (SR), the t-statistics for Jensen’s alpha, the monthly turnover percentage (TO), and the mean-variance utility. Results of three sample periods are reported.
Table 3: Performance Statistics for Different Industries: Selected Model

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This table provides the performance statistics of the dynamic monthly updated portfolios by a 120-month rolling window, for the selected model (identified in Figure 3). We summarize returns of multiple methods for 1-month-ahead return prediction, including hierarchical ensemble learning (HEL), regression (OLS), moving average (MA), the equally weighted portfolios (EW), and the long-short industry portfolio (LS). The summary statistics include the annualized Sharpe ratio (SR), the t-statistics for Jensen’s alpha, the monthly turnover percentage (TO), and the mean-variance utility. Results of three sample periods are reported.
Figure 1: Cumulative Performance for Different Industries

These figures provide the cumulative performance of the dynamic monthly updated portfolios by a 120-month rolling window. The three panels correspond to 10, 30, and 49 industry portfolios. We plot the cumulative returns of multiple methods for 1-month-ahead return prediction, including hierarchical ensemble learning (HEL), regression (OLS), moving average (MA), and the equally weighted portfolios (EW). We also add S&P 500 (SPY) as a benchmark for the passive investment.
Figure 2: Cumulative Performance for Different Step-Ahead Forecasts

These figures provide the cumulative performance of the dynamic monthly updated portfolios by a 120-month rolling window. The three panels correspond to 1-, 3-, and 6-month-ahead return prediction. We plot the cumulative returns of multiple methods for 49 industry portfolios, including hierarchical ensemble learning (HEL), regression (OLS), moving average (MA), and the equally weighted portfolios (EW). We also add S&P 500 (SPY) as a benchmark for the passive investment.
These figures provide the histogram of 240 $\chi^2$ statistic estimates for the predictor testing during the period 1998 - 2017. The $\chi^2$ test is a standardized Bayesian estimate for the coefficient vector of the predictor and its interactions. The first column corresponds to five macro predictors, where the 90% cutoff is 17.3. The last two columns correspond to ten fundamental characteristics, where the 90% cutoff is 10.6. We find Long-term Yield, Inflation, Stock Market Variance, Dividend Yield, Accrual, and Gross Profit significant.
These figures provide the cumulative performance of the dynamic monthly updated portfolios by a 120-month rolling window, for the selected model (identified in Figure 3). The three panels correspond to 10, 30, and 49 industry portfolios. We plot the cumulative returns of multiple methods for 1-month-ahead return prediction, including hierarchical ensemble learning (HEL), regression (OLS), moving average (MA), and the equally weighted portfolios (EW). We also add S&P 500 (SPY) as a benchmark for the passive investment.
These figures provide the cumulative performance of the dynamic monthly sorted portfolios by a 120-month rolling window, for the selected model (identified in Figure 3). The three panels correspond to 10, 30, and 49 industry portfolios. We plot the cumulative returns of the first three winning industries and first three losing industries predicted by the model on a monthly rebalancing. The industry winners and losers are sorted on the predictive returns of our model.