Who is Afraid of Liquidity Risk?
Dynamic Portfolio Choice with Stochastic Illiquidity *

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Abstract

Recent empirical work documents large liquidity risk premiums in stock markets. We calculate the liquidity risk premiums demanded by large investors by solving a dynamic portfolio choice problem with stochastic price impact of trading, CRRA utility and a time-varying investment opportunity set. We find that, even with high trading-cost rates and substantial trading motives, the theoretically demanded liquidity risk premium is negligible, less than 3 basis points per year. Assuming forced selling during market downturn enlarges the liquidity risk premium to maximally 20 basis points per year, which is well below existing empirical estimates of the liquidity risk premium.

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Who is Afraid of Liquidity Risk?
Dynamic Portfolio Choice with Stochastic Illiquidity

Recent empirical work documents large liquidity risk premiums in stock markets. We calculate the liquidity risk premiums demanded by large long-term investors by solving a dynamic portfolio choice problem with stochastic price impact of trading, CRRA utility and a time-varying investment opportunity set. We find that, even with high trading-cost rates and substantial trading motives, the theoretically demanded liquidity risk premium is negligible, less than 3 basis points per year. Assuming forced selling during market downturn enlarges the liquidity risk premium to maximally 20 basis points per year, which is still well below existing empirical estimates of the liquidity risk premium.

*JEL classification:* G11; G12; G23

*Keywords:* Liquidity premium; Liquidity risk; Dynamic portfolio choice; Trading Costs; Price impact.
1 Introduction

Over the last 30 years, a growing literature has empirically analyzed the effect of illiquidity on asset prices. Recently, empirical work has focused in particular on the liquidity risk premium, which is a compensation for exposure to systematic liquidity shocks. Several articles document substantial liquidity risk premiums in realized returns (for example Pastor and Stambaugh (2003)), while other work finds that it is difficult to disentangle the liquidity risk premium from the direct effect of transaction costs on prices, sometimes called the liquidity level premium (Acharya and Pedersen (2005)). In addition, the liquidity risk factors are often correlated with other risk factors, such as market risk, volatility risk and the Fama-French (1993) size factor. This makes it nontrivial to empirically pin down the liquidity risk premium. Surprisingly, there is little theoretical work on the size of the liquidity risk premium. In this paper we therefore add to the debate on the liquidity risk premium by analyzing what size for the liquidity risk premium can be justified theoretically. We do this by calculating the liquidity risk premium demanded by large investors, in setting with dynamic portfolio choice, stochastic price impact of trading, CRRA utility and a time-varying investment opportunity set.

Our first key finding is that our benchmark setup generates a very small liquidity risk premium, which is well below most empirical estimates. This is even the case under quite extreme assumptions on the degree of liquidity risk and trading frequency. This result provides a benchmark for empirical work on the liquidity risk premium. In addition, as our setting follows as much as possible the standard portfolio choice framework, our work implies that nonstandard assumptions are necessary in theoretical models in order to have a chance at generating larger liquidity risk premiums.

Our second key finding is that in our setup the liquidity risk premium is always small relative to the liquidity level premium, which is the direct compensation for trading costs of a given asset (Amihud and Mendelson (1986)). Depending on the parameter settings, our model can generate a liquidity level premium of 1% to 2% per year, while the liquidity risk premium is at most 20 basis points. This provides some support for the empirical work that finds evidence for the existence of a substantial liquidity level premium.
Even though there is little theoretical work on the liquidity risk premium, several articles have developed theoretical models to understand the size of the liquidity level premium, including Constantinides (1986), Liu (2004), Lo, Mamaysky and Wang (2004), Jang, Koo, Liu and Loewenstein (2007). These articles study dynamic portfolio choice problems with transaction costs or other forms of illiquidity, but the degree of illiquidity is always constant and hence they cannot analyze the compensation demanded for liquidity risk. A few articles incorporate liquidity risk in theoretical asset pricing or portfolio choice problems (Acharya and Pedersen (2005), Lynch and Tan (2011), and Beber, Driessen and Tuijp (2012)). We compare to these works in more detail in the literature section.

We now explain our setup in more detail. Our approach is “partial equilibrium”. We model an investor solving a multi-period portfolio choice problem with stochastic illiquidity, and obtain liquidity level and risk premiums by calculating how much expected return the investor is willing to give up to remove illiquidity or illiquidity risk. In our setup, we aim to follow as much as possible the most common features of multi-period portfolio choice. We focus on a CRRA agent who solves a multi-period portfolio choice problem, maximizing expected utility of terminal wealth. There are two assets, a risk-free asset and a risky asset with lognormal returns, calibrated to match U.S. equity index data. We allow for predictability of asset returns by having a time-varying expected return that mean reverts over time, which we calibrate using the often-documented predictability of returns by the dividend-price ratio. As noted by Lynch and Tan (2011), incorporating predictability induces the agent to trade more, which in turn makes illiquidity more important.

There are various ways to model illiquidity, such as fixed transaction costs, proportional transaction costs, and periods where trading is not possible (see the literature section). We follow Garleanu and Pedersen (2013) and use transaction costs that are a quadratic function of transaction size. This is consistent with the idea that trading has price impact (Kyle (1985)). We choose this type of illiquidity as we want to focus on large investors, who are likely most important for the price formation in asset markets, and thus for the empirically observed liquidity premiums. For these large investors the price impact of trading is a key aspect of illiquidity. To incorporate liquidity risk we allow the price impact of trading to
change stochastically over time. This is consistent with empirical findings. For example, Amihud (2002) proposes the ILLIQ measure to estimate price impact and finds substantial time variation in this measure. In addition, this existing work has found that shocks to price impact are negatively correlated to market returns: price impact is higher in bad times. We incorporate such correlation in our setting as it likely amplifies liquidity risk premiums. Acharya and Pedersen (2005) focus on these liquidity covariances as the source of liquidity risk premiums.

We calibrate the parameters of the illiquidity process to match empirical estimates of price impact of large transactions (Bikker, Spierdijk and van der Sluis (2007)). We then solve the dynamic portfolio choice problem numerically by backward induction. We calculate the liquidity level premium as the expected return the investor is willing to give up to remove a constant level of price impact of trading, and the liquidity risk premium as the expected return the investor is willing to give up to remove the time-series variation of the price impact (but not the average level of the price impact). In our benchmark setting the agent has a 10-year horizon and trades annually. More frequent trading would lead to lower liquidity premiums as transaction sizes per trading round are smaller and hence total price impact is smaller.

In our benchmark parameter calibration, we find a rather small liquidity level premium of 17 basis points. The main reason for this small liquidity level effect is that investors endogenously choose to trade less in response to the presence of trading costs (as in Constantinides (1986)). Without trading costs investors rebalance their portfolio and trade to profit from the time-varying expected return. With trading costs, investors carefully trade off the benefits and costs of trading. The utility benefits of rebalancing and profiting from time-varying expected returns are rather small according to our calibrations, and hence even small trading costs strongly reduce the amounts traded. To see this quantitatively, we decompose the total premium into a part that directly compensates for average trading costs, which equals 4 basis points, and a part that captures the utility loss of deviating from the optimal weight in the risky asset (13 basis points). Lynch and Tan (2011) also study the effect of predictability on the liquidity level premium and find a somewhat larger
effect of 43 basis points, which is still below most empirical estimates of the liquidity level
premium.

Our key result is on the liquidity risk premium. In the benchmark setting, the liquidity
risk premium is below 1 basis point per year. This effect is due to the negative covariance
of the asset return and shocks to the price impact of trading. This effect is small for several
reasons. First, since the agent cares only very moderately about the level of trading costs,
variation in these trading costs does not affect the expected utility much either. Second,
even though the negative covariance between costs and returns implies that trading costs
are higher in bad states of the world, the agent can endogenously choose to trade less when
trading costs are currently higher than usual. This is a key difference between our approach
and the model of Acharya and Pedersen (2005), where agents always trade their entire
portfolio irrespective of the state of the world. By looking at a case with zero covariance
between price impact and asset returns, we also find that independent variation in the price
impact of trading has no meaningful effect on the agent’s utility.

We perform various robustness checks to validate this result. We vary risk aversion,
the covariance between costs and returns, and the level of price impact costs, and find that
all these aspects have only a very small effect on the liquidity risk premium. We then add
two nonstandard features to the setup in order to try to generate a larger liquidity risk
premium. First, we force the investor to completely build up his risky asset position at
time zero and fully sell off this position after some time. Even if we force the investor to
perform this building up and selling off every year, the liquidity risk premium is below 3
basis points, while the liquidity level premium is much higher at around 2% due to the much
higher trading amounts. The liquidity risk premium remains small in this case because the
“forced” buying and selling is fully anticipated in this setting. We therefore consider a
second case where we add “liquidity crisis” periods with funding liquidity shocks to the
model. In each period, if the market return is below (minus) one standard deviation, the
agent has to sell part of the risky asset. The size of the amount sold depends negatively on
the market return. This generates priced liquidity risk, as the amount traded depends on
the market return and thus on the state of the world. However, even in this rather extreme
setting, the maximum liquidity risk premium we obtain is 20 basis points per year, while the liquidity level premium is higher at 55 basis points. In addition, we show that this setting with forced selling implies return-turnover correlations that are much higher than observed in actual data. We also focus specifically on forced sales by mutual funds (caused by fund outflows), and again find that the correlation between these fund outflows and returns is much closer to zero in the data compared to our model with forced selling.

In sum, our results show that it is difficult to generate a large liquidity risk premium using standard preferences and dynamic portfolio choice. Nonstandard assumptions are necessary in order to generate a large liquidity risk premium.

The paper is organized as follows. Section 2 discusses related literature and contributions. Section 3 describes the dynamic portfolio choice problem with quadratic and time-varying trading costs. Section 4 solves the problem numerically. In Section 5, we calculate the implied liquidity level premiums and liquidity risk premiums under the benchmark setting. In Section 6 we introduce additional trading motives: a setting with a fixed frequency of rebuilding the portfolio and a setting with forced selling during market downturns. Section 7 compares the correlation between market returns and turnover implied by our model and that in market data, both for aggregate turnover and for outflows of mutual funds, followed by conclusions in Section 8.

2 Related Literature and Contributions

Several papers investigate the magnitudes of liquidity and liquidity risk premiums in financial markets, both theoretically and empirically.\(^1\),\(^2\) One major thread of the theoretical

\(^1\)Existing research uses different ways to model illiquidity: non-trading intervals (Diamond (1982), Ang, Papanikolaou and Westerfield (2014)), limits to trading quantities (Longstaff (2001)), trading only at deterministic times (Kahl, Liu, and Longstaff (2003), Koren and Szeidl (2003), Schwartz and Tebaldi (2006), and Longstaff (2009)). In this paper we focus on trading costs, a most common approach in both the liquidity pricing and portfolio choice literature (e.g. Constantinides (1986); Grossman and Laroque (1990); Vayanos (1998); Pastor and Stambaugh (2003); Lo, Mamaysky and Wang (2004); Acharya and Pedersen (2005)).

\(^2\)Liquidity risk is defined in many different ways in existing work. For example, Huang (2003) defines it as randomly arriving liquidity shocks; in Vayanos (2004), it refers to the time variation of the need to liquidate; Ang, Papanikolaou and Westerfield (2014) focuses on the uncertainty of the length of non-trading interval. In this paper, we follow Acharya and Pedersen (2005) and define liquidity risk as the time variation of trading costs.
literature is the analysis of portfolio choice with trading costs. Most papers in this thread assume time constant trading-cost rates, which might be true for explicit costs (e.g. brokerage commissions) but is often not true for implicit trading costs (e.g. bid-ask spreads and price impact costs). As a starting point of this thread, Constantinides (1986) shows that for realistic proportional costs, the per-annum liquidity premium that must be offered to induce a constant relative risk aversion (CRRA) investor to hold the illiquid asset instead of an otherwise identical liquid asset is an order of magnitude smaller than the trading-cost rate itself. In subsequent work, Liu (2004) and Lo, Mamaysky and Wang (2004) use fixed trading costs. Realistic fixed trading costs can still hardly explain the large magnitude of the liquidity level premium. Longstaff (2001) limits the maximum amount of each transaction; and Garleanu (2008) models the illiquidity as the delay of trades.

The influence of trading costs largely relies on the trading frequency and the trading amounts. More trading leads to a larger liquidity level premium. The most popular way to achieve more trading is to add a time-varying investment opportunity set (return predictability or time-varying volatility). With this setting, many papers, such as Jang, Koo, Liu and Loewenstein (2007) and Lynch and Tan (2011), derive more trades and relatively larger liquidity level premiums.

Another choice is to create more trading motives with background risk. For example, Lynch and Tan (2011) include shocks to labor income in their model. Lo, Mamaysky and Wang (2004) and Garleanu (2008) both assume time-varying endowments in each period. Huang (2003) assumes that all investors face liquidity shocks and have to release their positions at some time point.

We add to this literature by letting trading-cost rates vary over time to study the magnitude of the liquidity risk premium, and we include forced selling during market downturns to further explore how this interacts with the time varying trading-cost rates and how it affects the liquidity risk premium.

Few theoretical studies include liquidity risk. The liquidity-adjusted CAPM proposed by Acharya and Pedersen (2005) assumes time varying trading-cost rates. It provides a unified framework for understanding the various channels through which liquidity risk may
affect asset prices. The primary limitation of liquidity-adjusted CAPM is that it is a one-period model. The trading frequency and trading amount are determined exogenously. In reality, both of them are determined endogenously by investors, and these decisions should affect liquidity level and liquidity risk premiums in equilibrium. To make the trading frequency and trading volume endogenous, a multi-period model is required. Beber, Driessen and Tuijp (2012) provide a multi-period extension of Acharya and Pedersen (2005), but continue to assume that investors do not trade at intermediate dates. In contrast, in our model the investor is allowed to rebalance and trade at intermediate dates.

To our best knowledge, Lynch and Tan (2011) and Garleanu and Pedersen (2013) are the only two dynamic portfolio choice papers assuming time-varying trading-cost rates while also having endogenously determined trading amounts and frequencies.

Lynch and Tan (2011) shows that permanent shocks on labor income and return predictability produce an additional trading motive and thus a first-order liquidity level premium. Their numerical results also show that time-varying trading-cost rates further inflate the premium since under their setting the trading-cost rate is high when the agent trades the most. Different from Lynch and Tan (2011), we study the portfolio choice problem of large institutional investors instead of individual investors and allow for price impact of trading. Institutional investors are more likely to be the marginal investors in financial markets. We thus use time-varying quadratic trading costs, instead of the percentage trading costs as Lynch and Tan (2011) do. In addition, institutional investors care more about funding liquidity shocks than labor income, therefore we assume exogenous funding liquidity shocks rather than labor income shocks. Finally we show that the forced selling of institutional investors during market downturns actually interacts with the time variation of trading costs and enlarges the liquidity risk premium.

Garleanu and Pedersen (2013) define a multi-period mean-variance portfolio choice problem, using additional assumptions on the objective function and return dynamics. Specifically, they assume that price changes (and not returns) are homoskedastic. They focus on the implications for portfolio choice, and do not calculate liquidity level or liquidity risk premiums. Different from their paper, we use a standard multi-period CRRA
utility framework, with standard dynamics of returns. In terms of portfolio choice, we do find similar implications as Garleanu and Pedersen (2013). Specifically, we confirm their conclusion that investors “aim in front of the target”: when chasing time-varying expected returns, investors balance trading costs, the utility benefits of these time-varying returns, and the extent to which these return opportunities are expected to disappear quickly over time or not. More generally, our paper provides useful implications to the trading cost management of long-term investors, showing how to balance trading costs, rebalancing and investment opportunities.

Our paper provides a benchmark to empirical work on liquidity level and liquidity risk premiums. A number of empirical papers (e.g. Amihud and Mendelson (1986), Amihud (2002), Pastor and Stambaugh (2003) and Acharya and Pedersen (2005)) find substantial differences in expected returns across portfolios sorted on liquidity measures, with a magnitude ranges from 4% to 7% per annum. Some recognize it as the premium for the level of illiquidity (Amihud and Mendelson (1986) and Amihud (2002)), while others understand it as the premium for liquidity risk (Pastor and Stambaugh (2003)) or both (Acharya and Pedersen (2005)).

3 Model

Our model follows as much as possible the standard approach to long-term dynamic portfolio choice. We thus solve a dynamic portfolio choice problem for a CRRA agent by maximizing his expected utility of terminal wealth. The model has two assets, a risk-free asset and a risky asset. We incorporate return predictability by allowing the expected return to vary over time. Our model deviates from the standard portfolio choice approach by including quadratic transaction costs (price impact costs) into the setting. Importantly, we allow the price impact of trading to change stochastically over time. In other words, we include liquidity risk. We now describe the setup in detail.

The investor has a finite investment horizon $T$ with initial wealth $W_0$. We assume that
the investor maximizes the expected CRRA utility of the terminal wealth, $W_T$,

$$E_0\left(\frac{W_T^{1-\gamma} - 1}{1 - \gamma}\right).$$  \hspace{1cm} (1)$$

The weight in the single risky asset at each time step, $\alpha_t$, $t = 1, 2, \ldots, T - 1$, serves as the control variable. The investor’s objective is to maximize the expected CRRA utility of the terminal wealth by choosing the dynamic investment strategy $(\alpha_1, \ldots, \alpha_{T-1})$,

$$\max_{\alpha_1, \alpha_2, \ldots, \alpha_{T-1}} E_0\left(\frac{W_T^{1-\gamma} - 1}{1 - \gamma}\right)$$

(2)

where the weight $\alpha_t$ can depend on the information set at time $t$.

We then turn to the asset menu. We assume the log risk-free rate $r_f$ is constant over time, and the log return of the risky asset is

$$r_{t+1} = \mu_t + \sigma_r u_{t+1}$$

(3)

where $\mu_t$ is the conditional mean of log return, and $\sigma_r$ is the volatility parameter. The return shock $u_{t+1}$ has a standard normal distribution. To generate trading motives that go beyond simple rebalancing towards the optimal weights, we incorporate time-varying expected returns. As in Merton (1971), Campbell and Viceira (1999), Barberis (2000), Wachter (2002), and Lynch and Tan (2011), we assume that $\mu_t$ depends on a persistent latent state variable $F_t$ which follows an AR(1) process with zero mean and standard normal shocks $v_{t+1}$,

$$F_{t+1} = \rho F_t + v_{t+1}$$

(4)

$$\mu_t = \mu_0 + a F_t.$$  \hspace{1cm} (5)

By allowing for persistence in expected returns (through $\rho$) this setup can capture the well-documented predictability of stock market returns by the price-dividend ratio (see, for example, Campbell and Shiller (1988), Fama and French (1988) and Cochrane (2008)), since
the price-dividend ratio has substantial persistence over time. We denote the correlation between return shocks $u_t$ and state variable shocks $v_t$ as $\text{Corr}(u_t, v_t) = \text{Corr}$. We will typically use negative values for this correlation, consistent with the stylized fact that shocks to expected returns correlate negatively with current returns. The loading parameter $a$ determines the magnitude of the time variation in $\mu_t$. Finally, $\mu_0$ is the long-run mean of the expected return.

The key innovation in our setup is to allow for stochastic trading costs. We use quadratic transaction costs as in Garleanu and Pedersen (2013). The expression for the dollar costs $TC_t$ of trading a dollar amount $V_t$ is given by

$$TC_t = \frac{1}{2} V_t^2 \sigma_r^2 \lambda_t$$

(6)

The trading costs $TC_t$ depend on $V_t^2$, rather than $V_t$ which is what constant proportional transaction costs would imply. Like Garleanu and Pedersen (2013), we normalize the price impact with the variance of returns $\sigma_r^2$. We multiply this by a stochastic trading cost parameter $\lambda_t$. This expression of quadratic transaction costs is consistent with the idea that trading has price impact (Kyle 1985). Under this setting, trading $V_t$ moves the price by

$$PI_t = V_t \sigma_r^2 \lambda_t$$

(7)

For a given trading amount $V_t$, the corresponding average proportional trading cost $c_t$ equals half the total price change, which can be written as

$$c_t = \frac{1}{2} PI_t = \frac{1}{2} V_t \sigma_r^2 \lambda_t$$

(8)

To incorporate liquidity risk, we allow price impact to change stochastically over time. Specifically, we let the log of $\lambda_t$ depend on the latent state variable $F_t$,

$$\ln \lambda_t = \ln \lambda_0 + bF_t,$$

(9)

where $b$ is the sensitivity of $\ln \lambda_t$ to $F_t$, and $\ln \lambda_0$ determines the long-run mean of $\ln \lambda_t$. 

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We use the same state variable for time variation in expected returns and trading costs because the numerical calculations increase exponentially in the number of state variables. In addition, with this setup we capture that illiquidity usually increases in bad times, when expected returns also increase (e.g. Pastor and Stambaugh (2003), and Acharya and Pedersen (2005)).

Given this setup we can work out how trading costs affect the long-term portfolio choice problem in more detail. The total trading amount at each time step is

\[ V_t = (\alpha_t - \alpha_{t-1})W_t \]  

(10)

where \( \alpha_{t-1} \) is the weight in risky asset before rebalancing. Substituting equation (10) into equation (6), the dollar trading costs are equal to

\[ TC_t(W_t, \alpha_t, \alpha_{t-1}, \lambda_t) = \frac{1}{2}((\alpha_t - \alpha_{t-1})W_t)^2 \sigma_r^2 \lambda_t \]  

(11)

We assume that all trading costs are paid from the risky asset. Then the level of wealth in next time step is

\[ W_{t+1} = (1 - \alpha_t)W_t exp(r_f) + (\alpha_t W_t - \frac{1}{2}((\alpha_t - \alpha_{t-1})W_t)^2 \sigma_r^2 \lambda_t) exp(r_{t+1}) \]  

(12)

and the weight of the risky asset before rebalancing in next time step is

\[ \alpha_{(t+1)-} = \frac{(\alpha_t W_t - \frac{1}{2}((\alpha_t - \alpha_{t-1})W_t)^2 \sigma_r^2 \lambda_t) exp(r_{t+1})}{W_{t+1}} \]  

(13)

The value function \( J \) at each time step \( t \) can be expressed as

\[ J(W_t, \alpha_{t-1}, F_t, t) = \max_{\alpha_t \ldots \alpha_{T-1}} E_t[\frac{W_t^{1-\gamma} - 1}{1 - \gamma}] \]  

(14)

and the Bellman equation for this dynamic portfolio choice problem is

\[ J(W_t, \alpha_{t-1}, F_t, t) = \max_{\alpha_t} E_t[J(W_{t+1}, \alpha_{(t+1)-}, F_{t+1}, t + 1)] \]  

(15)
The problem is solved using backward induction. We search numerically for the weight in risky asset $\alpha_t$ which maximizes the expected utility of the terminal wealth from the last period to the first.\(^3\) The optimal $\alpha_t$ is thus a function of wealth $W_t$, the pre-trading portfolio weight $\alpha_{t-}$, the state variables $F_t$, and time $t$.

4 Parameter Calibration and Numerical Solution

In this section, we further discuss how we solve this dynamic portfolio choice problem numerically with realistic parameter values calibrated to U.S. stock market data.

4.1 Parameter Values

We assume that the long-term expected annual return is 4% ($\mu_0 = 0.04$) and that the standard deviation of annual log return shocks is 10% ($\sigma_r = 0.10$).\(^4\) The risk free rate is 2% ($r_f = 0.02$), and the risk aversion level of our representative investor is set at 2.5 ($\gamma = 2.5$). Then, if there would be no trading costs and no time variation in expected returns ($\mu_t = \mu_0 = 0.04$), we obtain the usual myopic solution with a constant optimal weight $\alpha$ equal to

$$\frac{\mu_0 - r_f + \frac{\sigma_r^2}{2}}{\gamma \sigma_r^2} = 100\%$$

for all $t$. It means it is optimal for the representative investor to invest all his wealth in risky asset.\(^5\) Hence, even though our setting is partial equilibrium, with these parameters the investor will hold positions that are not very different from a representative investor. Of course, once we add trading costs and time-varying expected returns, the optimal weight will deviate from 100% over time. In the Appendix we also analyze cases with optimal myopic weights equal to 50% and 150% and find that this does not lead to a significant change in the magnitude of the liquidity risk premium. It is because the rebalancing trades

\(^3\)The numerical procedure is described in details in Appendix 9.2.

\(^4\)In unreported results we also solve a case with long-term expected return as large as 9%, and a standard deviation of returns shocks of 20%. We find similar values for the liquidity risk premium in this case.

\(^5\)Since we have only one risky asset and one investor in our economy, this assumption resembles the market clearing condition one would impose in a full equilibrium model. In the Appendix we also analyze the cases with optimal weights as 50% and 150%, and find it does not lead to a significant change in the magnitude of the liquidity risk premium. It is because the rebalancing trades are small even in those cases.
are small even in those cases.

We then turn to the trading costs. We use a high trading-cost rate with substantial time variation for our analysis. For the long-run mean of the price impact parameter $\lambda_c$, we take the estimates in Bikker, Spierdijk and van der Sluis (2007) who document that the price impact of a 1.5 million dollar trade is about 40 basis points. Using equation (7), this implies $\lambda_c = 26.88$. This value is also consistent with the numbers found in most papers of price impact. For example, Chan and Lakonishok (1997) find a price impact of about 54 bps, and Keim and Madhavan (1997) find a price impact about 30 bps to 65 bps. In addition, we allow the price impact to be 3 times larger in a robustness check.

Next we calibrate the stochastic time-series behavior of the price impact via the latent state variable $F_t$. To this end, we use the ILLIQ measure of Amihud (2002). ILLIQ, as $\lambda$ in our model, is a measure of price impact calculated as the absolute value of the daily return divided by the daily dollar trading volume. We calculate this measure for the market index from 1952 to 2010, aggregating the daily measure to an annual frequency. We calibrate the parameter $b$, which captures the conditional variation in $\lambda_t$, to match the conditional variation in the annual ILLIQ measure, which gives $b = 0.3149$. We set the annual time persistence of the state variable $F_t$ at 0.7 ($\rho = 0.7$), again calibrated using the monthly ILLIQ series. Under this setting, the 95% conditional confidence interval of $\lambda_t$ is $[0.29 \ast \lambda_c, 3.44 \ast \lambda_c]$, which means a 2 standard deviation positive (negative) shock on $\lambda_t$ makes it more than 3 times (less than one third) its long-term level. The time variation in price impact we assume here is therefore economically very substantial. For example, in 2008 the average ILLIQ value was about twice the level in 2006 and 2007, an increase well within the confidence interval.

Expected returns depend on the state variable $F_t$ via the parameter $a$. Assuming the dividend yield as the only predictor, we use dividend yield data from 1952 to 2010 for

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6Since we model $ln\lambda_t$ as a function of state variable $F_t$, instead of $\lambda_t$, to make sure the trading cost is positive. The long-run mean of the price impact parameter $\lambda_c$ does not equal to $\lambda_0$. 

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the calibration of this parameter $a$. The calibration using monthly data implies an annual standard deviation of shocks to expected returns of 0.92%. Hence we set the annual standard deviation of the shocks to expected returns at 1% ($a = 0.01$).

Finally, we set the initial wealth at 100 million dollars, ($W_0 = 1$), which is about the median size of U.S. hedge funds. Since hedge funds are more likely to be the marginal traders in the financial market and more likely to experience liquidity shocks than large mutual funds and pension funds do, we choose to use the median size of hedge funds in our benchmark setting. Considering there is only one risky asset in our economy, 100 million dollars holdings of one single asset is large enough to generate a significant price impact of trades. To further make sure we do not underestimate the liquidity risk premium, we also solve the problem with a higher level of wealth, which is equivalent to using a higher trading-cost rate $\lambda_t$ in our setting. We solve the portfolio choice problem for a horizon of 10 years, with an annual trading frequency. For the calculation of liquidity risk premium, we solve the problem for different values of the correlation between shocks on realized returns and price-impact rates, $Corr(v_t, u_t) = 0, -0.2, -0.4, -0.6$.

To sum up, in our assumptions we try to incorporate a substantial degree of illiquidity, in an attempt to try to generate substantial liquidity level and liquidity risk premiums. We assume high levels for the trading-cost rate, large time variation in it (from one third to 3 times the mean level), large trading motives (time-varying expected returns, and later a fixed frequency of rebuilding the portfolio and exogenous funding liquidity shocks), a single risky asset (no spreading of trades over stocks to reduce the price impact of trades), large institutional investors (high price impact of trades and high exposure to liquidity shocks), and an annual trading frequency (no spreading of trades within a year to reduce the price impact of trades).

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7First, we estimate the conditional standard deviation of the shocks to the dividend yield using an AR(1) model and the dividend yield data from 1952 to 2010. Then we estimate the coefficient of dividend yield in the regression of returns on lagged dividend yields using the data from 1952 to 2010 as well. The time variation parameter of expected returns, $a$, is estimated as the product of the conditional standard deviation and this regression coefficient.
4.2 Numerical Procedure

The dynamic portfolio choice problem is solved by backward recursion. Gaussian Quadrature is used to deliver the joint distribution of shocks on the state variable $F_t$ and return shocks $(v_t, u_t)$. Four points are used for each shock.

Before we present the liquidity premiums in the next section, we give an example of the portfolio behavior that our model generates. Figure 1 shows the weights in the risky asset both before and after rebalancing under the reference case with constant price impact but time-varying expected returns. Here we use a zero correlation between the expected return shocks and return shocks ($\text{Corr} = 0$), which implies there is no intertemporal hedging demand.

Figure 1 plots one simulation of the weights in the risky asset over 10 annual time steps, both the weights before rebalancing, $\alpha_{t_-}$, with black circles, and the weights after rebalancing and trading costs, $\alpha_{t_+}$, with red stars. The expression for the myopic optimal weight $\alpha_t^{\text{Myopic}}$ without trading costs is

$$\alpha_t^{\text{Myopic}} = \frac{\mu_t - r_f + \sigma^2 \gamma / 2}{\gamma \sigma^2}$$

which varies over time with the conditional expected return $\mu_t$. If the investor trades the entire way from $\alpha_{t_-}$ to $\alpha_t^{\text{Myopic}}$, he needs to pay a large amount of trading costs. On the other hand, if he does not trade at all and keeps the weight at $\alpha_{t_-}$, he loses too much utility by deviating from $\alpha_t^{\text{Myopic}}$. Therefore, it is optimal for the investor to trade partially towards the myopic optimal weight, $\alpha_t^{\text{Myopic}}$, as indicated by the pink crosses. The optimal amount to trade is decided by the trade-off between the marginal utility gain of getting closer to the aim and the marginal trading costs incurred. We will show later that both the loss of utility caused by deviation from $\alpha_t^{\text{Myopic}}$ and the actual trading costs incurred should be compensated in the form of a higher expected return (liquidity level premium). Besides, the investor resists to trade far away from the long-run average myopic demand $E(\alpha_t^{\text{Myopic}})$, the green dash line, since it will generate more trading costs in the future. These results
are consistent with the main findings in Garleanu and Pedersen (2013): when trading is
costly, the investor should trade partially towards the current aim, and also aim in front of
the target (consider the long-run optimal weight, $E(\alpha_t^{Myopic})$).

5 Liquidity Level Premium and Liquidity Risk Premium: Main Results

5.1 Defining and calculating the premiums

Trading costs make the investor deviate from the optimal solution in a frictionless market.
In a competitive market, investors should thus require a premium (higher expected return)
to compensate for the loss of utility caused by trading costs. Therefore, in this section,
we compute both the liquidity level premium, the premium compensating for the level of
trading costs, and the liquidity risk premium, the premium compensating for the time
variation of trading costs.

In this paper, the liquidity level premium is defined to be the increase in the long-term
expected return $\mu_0$ that the investor requires to be indifferent between having access to the
risky asset with rather than without trading costs. Similarly, the liquidity risk premium is
defined to be the increase in $\mu_0$ that the investor requires to be indifferent between having
access to the risky asset with rather than without time variation in trading costs (given a
mean level of trading costs). This approach is "partial equilibrium".

First, we discuss intuitively the sources of the liquidity level premium and the liquidity
risk premium. As we mentioned in previous section, investors should be compensated for
both the actual trading costs and the loss of utility caused by the deviation from the
optimal weight. The liquidity level premium measures such compensation in term of a
higher expected return. It is worth noting that the liquidity level premium thus depends
on both the trading-cost rate and the trading amount. The latter is a choice variable.

The liquidity risk premium captures the compensation for the utility loss caused by
time variation of trading-cost rates. The time variation of trading-cost rates has three
different effects on the utility of the investor: the variance of the cost rates ($Variance$
Effect), the covariance between trading costs and realized returns (Covariance Effect), and the additional freedom to choose the trading amount depending on the level of the cost rate in each period (Choice Effect).

1. Variance Effect: Since the investor is risk averse, he dislikes time variation of trading costs. A positive premium should be required as a compensation.

2. Covariance Effect: The investor dislikes to pay large amounts of trading costs during market downturns, hence dislikes a negative covariance between the trading costs and realized returns $Cov(c_t, r_t)$, and a positive premium should be required as a compensation. This is the main effect present in the liquidity CAPM of Acharya and Pedersen (2005).

3. Choice Effect: In a dynamic setting, investors respond actively to the time variation of cost rates. Investors trade more when cost rates are low and trade less when it is high. In this case, investors might actually benefit from the time variation of cost rates, and if this effect dominates a negative liquidity risk premium may be found. We show later that the liquidity risk premium is actually negative in some of our settings.

It is important to point out that in our setting the investor can control the liquidity risk to some extent. This is in contrast to Acharya and Pedersen (2005), where the investors sells all assets at the end of the period. To explain this in more detail, denote the total dollar holdings of the risky asset at time $t$ by $H_t$, then the realized trading costs $\hat{c}_t$ as a percentage of previous holdings $H_{t-1}$ are

$$\hat{c}_t = \frac{c_t V_t}{H_{t-1}} = \frac{1}{2} \frac{\sigma_r^2 \lambda_t V_t^2}{H_{t-1}}$$

In Acharya and Pedersen (2005) investors care about (co)variation of $c_t$. In our setup, the investor cares about (co)variation in $\hat{c}_t$, which is determined endogenously. By choosing trading amounts appropriately, the investor could choose to eliminate all risk in $\hat{c}_t$. Of course, this may not be optimal from an expected utility perspective. Below we calculate the optimal portfolio choice, and we will see that investors indeed trade less when the
trading-cost rate $\lambda_t$ is higher, so that the investor dampens the “realized” liquidity risk endogenously, leading to small \textit{Variance} and \textit{Covariance} effects.

We now describe in detail how we calculate the premiums in our benchmark setup. We solve four different cases of the portfolio choice problem:

\textit{Case 1:} Constant expected returns and \textit{no} trading costs: $a = 0, b = 0, \lambda_c = 0$.

\textit{Case 2:} Time-varying expected returns and \textit{no} trading costs: $a = 0.01, b = 0, \lambda_c = 0$.

\textit{Case 3:} Time-varying expected returns and \textit{constant} trading costs: $a = 0.01, b = 0, \lambda_c = 26.88$.

\textit{Case 4:} Time-varying expected returns and \textit{time-varying} trading costs: $a = 0.01, b = 0.315$.

We calculate the expected utility for each case. The initial position in risky asset is assumed to be at 100%, the long-term optimal weight in the risky asset in \textit{Case 1}. The case with constant expected return and no trading costs, \textit{Case 1}, is used as the reference case for the calculation of both the liquidity level premium and liquidity risk premium. Specifically, for each of the \textit{Cases 2, 3, and 4}, we find the corresponding level of expected return in the reference case (\textit{Case 1}) which makes the investor indifferent between holding the risky asset in the specific case and the reference case. This implies that the investor has the same expected utility in both cases. Then we compare the corresponding levels of expected returns across different cases. The difference of corresponding expected returns for \textit{Case 2} versus \textit{Case 3} is recorded as the liquidity level premium, and the difference of the corresponding expected returns for \textit{Case 3} versus \textit{Case 4} is recorded as the liquidity risk premium.

[Insert Table 2 about here]

\textbf{5.2 Benchmark Results}

Table 2 reports the liquidity level premium and liquidity risk premium under the benchmark setting, for different values of the correlation between the return shocks and shocks on
trading-cost rate (shocks on state variable $F_t$), $Corr = 0, -0.2, -0.4, -0.6$. The first key finding is that the magnitudes of the liquidity risk premium are extremely small, ranging from -0.62 to -0.10 basis points, significantly smaller than the liquidity level premium, which ranges from 16.4 to 15.7 basis points. Perhaps surprisingly, the liquidity risk premium is actually negative, though of negligible size. We return to this issue below.

Table 2 also shows the effect of correlation between liquidity and returns (the Covariance Effect). We calculate this as the difference of the corresponding expected returns with nonzero correlation between trading-cost rates and returns, $Cov(\lambda_t, r_t) \neq 0$, and the case with zero correlation, $Cov(\lambda_t, r_t) = 0$. In line with the Acharya and Pedersen (2005) model, the premium for $Cov(\lambda_t, r_t)$ indeed increases as the correlation between returns and cost rates becomes more negative, but the effect is always smaller than 1 basis points and accounts for 3.3% of the total liquidity premium at most. As discussed above, investors endogenously choose to trade less when trading costs are higher, which dampens the effects of liquidity risk.

The small liquidity risk premiums found in our analysis conflict with the empirical literature on the liquidity risk premium (e.g. Pastor and Stambaugh (2003) and Acharya and Pedersen (2005)), where the covariance between trading-cost rates and returns, $Cov(c_t, r_t)$, generates a large liquidity risk premium. Specifically, the benchmark estimates of Acharya and Pedersen (2005) imply an annual liquidity risk premium of 15 basis points for the most liquid stock portfolio, increasing to 125 basis points for the least liquid portfolio.\footnote{To calculate these numbers, we use specification 1 in Table 4A of Acharya and Pedersen (2005) and multiply the price of risk with the three liquidity betas in Table 1.} Pastor and Stambaugh (2003) add a market liquidity factor to the Fama-French three-factor model. Sorting on liquidity betas, they find that the annual excess return spread of stocks with high versus low liquidity betas is equal to 7.5%.

As mentioned above, we find negative liquidity risk premiums in Table 2. This implies that the Choice Effect dominates the Variance Effect and Covariance Effect: the investor actually prefers to have time variation in trading-cost rates since he/she can react according to the realized cost rate and thus variation in cost rates increases the expected utility. To better understand how the Choice Effect generates a negative liquidity risk premium, we
plot the expected utility of the terminal wealth as a function of the trading-cost parameter \( \lambda_c \) under the optimal strategy in Figure 2 (the solid curve), for the reference case with a time constant trading-cost rate and zero correlation between return shocks and shocks on \( F_t \). Obviously, the expected utility decreases with the increase of trading-cost parameter \( \lambda_c \). However, when \( \lambda_c \) becomes larger, investors choose to trade less. This diminishes the effect of higher values of \( \lambda_c \) on expected utility. Therefore, the expected utility is strictly convex in \( \lambda_c \). Then if \( \lambda_c \) is stochastic instead of deterministic, the expect utility of the investor will always be higher than the expect utility at the mean level of \( \lambda_c \). Hence, when we introduce time variation into the trading-cost parameter \( \lambda \), the \textit{Choice Effect} makes the liquidity risk premium negative. Economically, this effect is rather small however.

We now turn to the liquidity level premium. In Table 2 we see that the liquidity level premium, ranging from 15.7 to 16.5 basis points, is much smaller than the price impact costs of an average trade, which equal 65 basis points in the benchmark setting.\(^9\) This result is consistent with Constantinides (1986), who shows that the liquidity premium is an order of magnitude smaller than the trading-cost rate itself. The reason is that investors trade less if the trading-cost rate is high. Under the benchmark setting, the main trading motive is to trade towards the time-varying myopic aim introduced by the time-varying expected return \( \mu_t \). The investor also wants to rebalance the portfolio over time, but these effects are relatively small if the portfolio weights are close to 100%.\(^10\) Table 3 decomposes the liquidity level premium into a component that compensates for the direct trading costs and a component that measures the utility loss of deviating from the (frictionless) optimal weight. We see that only a small fraction of the liquidity level premium compensates for actual trading costs, about 4.18 basis points.\(^11\) The remaining part of the liquidity level premium, which is necessarily compensation for the utility loss of deviating from the myopic optimal weight, is larger at 12.7 basis points. Hence the investor chooses to trade relatively little

\(^9\)The average annual trading amount for the benchmark setting is 4.9 million dollars, which corresponds to price impact costs of 65.3 basis points: \( 1/2 \times 4.9 \times 0.01 \times 26.88 \) according to equation (7).

\(^10\)In the Appendix we solve the model for cases where the myopic optimal weight equals 50% and 150%, respectively, and find similar results for the liquidity (risk) premiums. It is because the rebalancing trades are small even in those cases.

\(^11\)We calculate the realized trading costs as a percentage of total wealth for each time step of each simulation, and use the average value across all 10,000 simulations and all 10 steps each to measure the component in the liquidity level premium that captures the actual trading costs.
and deviate substantially from the optimal portfolio weight. This result is in accordance with Garleanu and Pedersen (2013) who analytically show that investors balance between the trading costs and having suboptimal portfolios.

[Insert Table 3 about here]

To further investigate how the level of trading-cost rate affects the liquidity risk premium as well as the liquidity level premium, we solve the dynamic problem for different values of the trading-cost parameter, from $3.36(=1/8 \times 26.88)$ to $107.52(=4 \times 26.88)$. The correlation between return shocks and shocks on $F_t$, $\text{Corr}(u_t,v_t)$ is set to -0.3 in all cases. Table 3 shows that the liquidity risk premium changes only slightly from -0.49 basis points to -0.30 basis points when the cost rate becomes 4 times larger. Considering that such a cost rate corresponds to a 1.6% price impact for a 1.5 million $ trade, the liquidity risk premium of -0.30 basis points is indeed negligible. It is worth noting that in our model an increase in trading-cost parameter $\lambda_c$ is equivalent to an increase in wealth level. It means the liquidity premiums calculated for the setting with initial level of wealth as 100 million dollars and $\lambda_c=4 \times 26.88$ are the same compared to a setting with initial level of wealth of 400 million dollars and $\lambda_c = 26.88$. Therefore, the results shown in Table 3 also indicate that the liquidity risk premium is small for higher levels of wealth.

[Insert Figure 3 about here]

Consistent with Constantinides (1986) and Garleanu and Pedersen (2013), we find that investors trade less when the trading-cost rate is higher. Table 3 and Figure 3 both show that the average dollar trading amount per year decreases from 4.9 million dollars to 2.0 million dollars when the cost rate becomes 4 times the benchmark level, and it increases to 14.8 million dollars when the cost rate becomes 1/8 of the benchmark level. In addition, Figure 3 shows that the optimal trading amount is decreasing and convex in the cost rate. Hence, the relative importance of direct trading costs for the liquidity level premium decreases monotonically when the trading-cost rate increases, from 64% for $\lambda_c = 1/8 \times 26.88$ to 14% for $\lambda_c = 4 \times 26.88$, and the utility loss of suboptimal portfolio holdings thus becomes more important.
Figure 4 shows that the equilibrium liquidity premium is increasing and concave in the trading-cost rate. This is different from, for example, Acharya and Pedersen (2005), where both the trading amount and the trading frequency are exogenous, and hence total trading costs and the liquidity premium increase linearly with the cost rate.

6 Introducing Additional Trading Motives

Until now, we assumed that the main trading motive is chasing the time-varying expected returns and found small liquidity risk premiums. In this section, we introduce some non-standard trading motives in an attempt to generate larger liquidity risk premiums. First, we let investor build up and down their portfolio over the investment horizon. Second, we impose funding liquidity shocks on our investor, which force her to sell part of the risky asset portfolio.

6.1 Building Up and Scaling Down the Portfolio

One main thread of the liquidity literature (e.g. Amihud and Mendelson 1986 and Acharya and Pedersen 2005) effectively assumes that investors buy their entire portfolio at the beginning of a period, and completely sell it after some periods. Following this idea, we now solve the dynamic portfolio choice problem under the assumption that investors have to build up their portfolio at time 1 and sell it completely after $\tau$ years. The goal is to see whether this assumption helps to generate a larger liquidity risk premium compared with the benchmark results in Section 5.

We solve the problems for different values of $\tau$: every year ($\tau = 1$), every 2 years, 5 years and 10 years. Obviously, $\tau = 1$ is an extreme assumption, since the investor is forced to rebuild the portfolio every year. To illustrate our setup, Figure 5 plots the trajectory of the optimal weights invested in the risky asset when $\tau$ is equal to 10 years. The figure
shows that it is optimal for investors to gradually build up and down the portfolio to reduce the price impact of trades, consistent with Garleanu and Pedersen (2013).

Using the same partial equilibrium approach as in Section 5, we calculate the liquidity level premiums and liquidity risk premiums for different values of $\tau$. Table 4 shows that liquidity risk premiums are still very small, from 0.85 bps to 2.66 basis points. These are particularly small because the liquidity level premium increases substantially due to the high trading amounts, with values between 94 bps to 210 bps. In sum, adding a fixed frequency of building up and selling the portfolio does not help to generate a large liquidity risk premium.

In addition, similar to the increase of the trading-cost rate, as forced selling and rebuilding of the portfolio becomes more frequent, it is optimal for investor to invest less into the risky asset and thus trade less and pay less trading costs. As Table 4 shows, the direct trading cost component of the liquidity level premium and the average trading amount per year both decrease as the rebuilding of the portfolio becomes more frequent.

[Insert Table 4 about here]

6.2 Funding Liquidity Shocks

We now include funding liquidity shocks into our setting. During periods of crisis (e.g., the 1987 market crash, the 1997 Asian crisis, the Russian debt crisis of 1998, the hedge-funds meltdown of 2007, and the 2008 financial crisis), market liquidity goes down, trading-cost rates go up, and at the same time, institutional investors are often forced to sell a substantial amount of their positions in order to satisfy regulatory restrictions or deal with fund outflows.\textsuperscript{12} To investigate how large a liquidity risk premium can be generated by forced trading during market downturns, we add exogenous funding liquidity shocks into our model in a reduced-form way. We let the occurrence of these funding shocks depend

\textsuperscript{12}In the real world, investors are often forced to release part of their positions when the market goes down. For example, the mutual fund literature has a long history of documenting the flow-performance sensitivity (e.g. Warther (1995), Sirri and Tufano (1998), Froot, O’connell and Seasholes (2001), Huang, Wei and Yan (2007)). They all show there are more fund outflows during market downturns. Brunnermeier and Pedersen (2009) claim that investors’ capital and margin requirements are binding when the market deteriorates, which forces them to reduce their holdings.
on the market return, to capture the idea that funding liquidity shocks likely occur in bad times.

Concretely, we assume that, if the realized risky asset return \( r_{t+1} \) is more than one standard deviation below the conditional mean, the investor is forced to release a proportion (or all) of her positions in the risky asset. The proportion increases linearly in the size of the return. If the return is more than 3 times the standard deviation below the mean, the investor has to sell the entire position. In formulas, the effect of liquidity shocks on the portfolio weight in the risky asset before rebalancing, which we denote \( \Delta \alpha_{(t+1)-} \), is as follows:

- If \( \mu_t - 3\sigma_r \leq r_{t+1} \leq \mu_t - \sigma_r \), the forced selling implies a change in the portfolio weight for the risky asset of \( \Delta \alpha_{(t+1)-} = \alpha_{(t+1)-} + \frac{r_{t+1} - (\mu_t - \sigma_r)}{2\sigma_r} \), with \( \Delta \alpha_{(t+1)-} = 0 \), if \( r_{t+1} \geq \mu_t - \sigma_r \).

- If \( r_{t+1} \leq \mu_t - 3\sigma_r \), the investor is forced to sell the entire position in the risky asset: \( \Delta \alpha_{(t+1)-} = -\alpha_{(t+1)-} \).

Similarly to the benchmark setting, we solve the portfolio choice problem to calculate the liquidity level premium and liquidity risk premium.

By introducing forced selling we influence the realized trading costs \( \hat{c}_t = c_t V_t H_{t-1} \). As discussed in Section 5, in the benchmark setting the investor dampens the variation in these realized trading costs by trading less if the cost-rate \( c_t \) is higher. This lowers the liquidity risk premium. Now, we effectively force the investor to trade more in bad times, when returns are low and cost-rates are high. Hence, we increase the variation in realized trading costs, and this could potentially lead to higher liquidity risk premiums.

We solve the dynamic portfolio choice problem with funding liquidity shocks for three values of the correlation between return shocks and shocks on trading-cost rates \( \text{Corr}(u_t, v_t) \), 0, -0.3, -0.6. Table 5 shows liquidity level premiums and liquidity risk premiums in this setting. We find that the total liquidity risk premium under this setting is significantly larger than before, 11.5 basis points for the case with \( \text{Corr}(u_t, v_t) = -0.3 \), and 20.8 basis points for the case with \( \text{Corr}(u_t, v_t) = -0.6 \). It accounts for a nonnegligible fraction of the
total liquidity premium, 18% and 28% respectively.

[Insert Table 5 about here]

Table 5 also shows that the liquidity risk premium is very small if $\text{Corr}(u_t, v_t) = 0$, just above 2 basis points. In this case, investors are forced to sell when market returns are low, but trading costs are not higher than usual. Hence, liquidity risk is not very important to the investor, and only the liquidity level premium increases relative to the benchmark case (Table 2) because the investor simply has to trade more. Hence, to generate a nonnegligible liquidity risk premium it is crucial that funding liquidity shocks correlate with market liquidity shocks.

Finally, we solve the problem with funding liquidity shocks for different levels of the trading-cost rates $\lambda$, from $3.36 (= 1/8 \times 26.88)$ to $107.52 (= 4 \times 26.88)$. The correlation between return shocks and shocks to the trading-cost rate is set to -0.3 for all cases. We see from Table 6 that both the liquidity level premium and liquidity risk premium increase with the trading-cost rate. As the cost rate becomes 4 times as large as before, the liquidity risk premium increases from 11.53 bps to 21.54 bps, but the relative importance decreases from 18% of total liquidity premium to 12%.

[Insert Table 6 about here]

To sum up, in this section we find that we can only generate a nonnegligible liquidity risk premium if investors are forced to trade substantially during a market downturn, and the trading-cost rate goes up at the same time. Even in this case the liquidity risk premium is still quite modest in economic terms. An obvious question is how extreme our assumptions on funding liquidity shocks and forced trading are. We address this in the next section.

7 How Extreme are the Funding Liquidity Shocks?

We have shown that our benchmark setup generates very small liquidity risk premiums. As shown in Section 6, one nonstandard assumption which has a chance at generating larger liquidity risk premiums is when we allow for funding liquidity shocks, which lead to
forced selling during market downturns. In this section we assess how realistic this setting with forced selling is. We do this in two ways. First, we note that this forced selling introduces a large correlation between the portfolio turnover and the realized return. We therefore compare the return-turnover correlation as observed in the U.S. stock market to the correlation implied by our model (Section 7.1 and 7.2). Second, we focus on a set of investors who often encounter forced selling via fund outflows: mutual funds. We study the correlation between returns and aggregate fund outflows in Section 7.3.

We will see that the correlation between aggregate turnover and returns and the correlation between fund outflows and returns are both much smaller in the data than in our model with funding liquidity shocks. This shows that our model with forced selling during market downturns is quite extreme.

\section{Aggregate Turnover and Market Returns}

Since both market turnover and market liquidity of U.S. stock market are highly persistent over time, we first do an AR(1) regression for the log values of both turnover and ILLIQ to capture the innovations. We use annual data from 1966 to 2010.

\begin{equation}
\ln Trn_{t+1} = \alpha_{\text{trn}} + \rho_{\text{trn}} \ln Trn_{t} + \varepsilon_{\text{trn}}^{t+1}
\end{equation}

\begin{equation}
\ln ILLIQ_{t+1} = \alpha_{\text{ILLIQ}} + \rho_{\text{ILLIQ}} \ln ILLIQ_{t} + \varepsilon_{\text{ILLIQ}}^{t+1}
\end{equation}

We find that both the market turnover and ILLIQ are quite persistent at the annual frequency. The time persistence of market turnover, $\rho_{\text{trn}}$ in equation (19), is 0.9987. The time persistence of ILLIQ, $\rho_{\text{ILLIQ}}$ in equation (20), is 0.9735.

Table 7 reports the correlations between the annual market excess returns, the innovations in market liquidity and the innovations in market turnover from 1966 to 2010, and the covariance between the annual market excess returns and the innovations in market turnover. In accordance with higher trading during market downturns, Panel C of Table 7 reports a negative correlation (-0.170) between market excess returns and market turnovers.
when market excess returns $R_M - r_f$ are negative, but it is not significant. The correlation between market excess returns and market turnover is positive for the entire sample (0.203). In addition, Panel B of Table 7 reports a significant negative correlation (-0.584) between market excess returns and the innovations in ILLIQ, showing that the market is more liquid during bull markets than bear markets, and a negative correlation (-0.281) between the innovations in ILLIQ and the innovations in market turnover. This indicates that investors on average trade more when the market is relatively more liquid, in line with the predictions of our benchmark model.

Looking at the level of market turnover, the data give an average annual market turnover of about 66%, which is substantially larger than the 5% - 10% in the simulations of our model. Therefore our model is not able to capture the high turnover in the stock market. Previous literature suggests excess turnover could be caused by noise trades of investors, high-frequency traders and the large variation of investment sentiment, and those are not included in our setting.

### 7.2 Aggregate Turnover: Comparison with Simulated Results

For each setting of our model, we simulate 10,000 trajectories of the annual stock return and annual turnover. Each trajectory has 10 annual steps. Then for each trajectory of turnover, we do the AR(1) regression, equation (19), to calculate the innovation in the log turnover. The correlation and covariance between excess returns and innovations in turnover are calculated across all 10,000 simulations with 10 steps each.

Table 8 reports the correlation between the annual returns and annual turnover for the simulations of our model, and Table 9 reports the covariance.

Now we compare the simulated results with the market data. In general, the turnover-return covariance in our simulated data is qualitatively comparable with the market data for our benchmark setting (Section 5). Turning to the correlation between the excess returns and innovations in turnover, $\text{Corr}(R_m - r_f, \Delta \ln Trn)$, Panel A in Table 8 reports that for the benchmark setting with $\text{Corr}(u_t, v_t)$ between -0.4 and -0.6, the $\text{Corr}(R_m - r_f, \Delta \ln Trn)$ for the entire sample ranges from -0.010 to 0.023, which is smaller than the 0.203 in market
data. This mostly positive correlation in our model is mainly caused by the fact that the investor trades more when the market is liquid, but this effect generates only slightly positive correlations. When we condition on positive market returns, the model-implied value for $\text{Corr}(R_m - r_f, \Delta \ln Trn)$ ranges from -0.014 to 0.049, slightly higher than the -0.056 in the data. When we condition on negative market returns, $\text{Corr}(R_m - r_f, \Delta \ln Trn)$ ranges from -0.087 to -0.110, of similar order but slightly smaller than the empirical estimate (-0.170).

The magnitude of the covariance between the excess returns and innovations in turnover, $\text{Cov}(R_m - r_f, \Delta \ln Trn)$, in our simulated data, Panel A in Table 9, is comparable to that in the market data, ranging from $-51.0$ to $16.2 \times 10^{-4}$.

Now we turn to the model with funding liquidity shocks. Panel B in Table 8 reports that in this case the return-turnover correlations implied by the model become too negative. With $\text{Corr}(u_t, v_t) = -0.6, \text{Corr}(R_m - r_f, \Delta \ln Trn)$ shoots up to -0.22 for the entire sample, and -0.673 when we condition on negative market returns. This implies that the setup with funding liquidity shocks implies quite extreme trading behavior of our investor compared to what we see on average in the data. The fact that such a strong assumption of liquidity risk still only generates a 20 basis-point liquidity risk premium strengthens our conclusion that liquidity risk premiums in standard portfolio choice models are quite small. Consistently, the covariances reported in Panel B of Table 9 are substantially higher than that in the data in absolute terms.

The simulation analysis also provides a more general understanding of the trading behavior implied by our model. There are mainly four effects affecting return-turnover correlations and covariances.

1. Time-varying expected returns: Investors trades more when the realized return is high in absolute terms, since this corresponds to a substantial change in the time-varying expected return if $\text{Corr}(u_t, v_t)$ is nonzero. We see this from the column with $\text{Corr}(u_t, v_t) = -0.6$ in the Panel A of Table 8 and Table 9 for the benchmark setting where time-varying expected returns play the most crucial role. Both $\text{Corr}(R_m - r_f, \Delta \ln Trn)$ and $\text{Cov}(R_m - r_f, \Delta \ln Trn)$ are positive when conditioning on $R_m > r_f$ and negative when $R_m < r_f$.

2. Time-varying price impact of trading: Investor trades more when the market is more
liquid (high realized return comes with high market liquidity when $Corr(u_t, v_t)$ is negative in our model). See row ‘Entire sample’ in both Panel A and B of Table 8 and Table 9. Both $Corr(R_m - r_f, Δ \ln Trn)$ and $Cov(R_m - r_f, Δ \ln Trn)$ increase as $Corr(u_t, v_t)$ becomes more negative from 0 to -0.6.

3. Forced sales during crisis: The investor is forced to sell when the realized return is low. See Panel B of Table 8 and Table 9 for the setting with funding liquidity shocks. Both $Corr(R_m - r_f, Δ \ln Trn)$ and $Cov(R_m - r_f, Δ \ln Trn)$ are negative for the entire sample and the negative sample. It is the most dominant effect in this setting.

4. Wealth effect: A higher wealth level means larger price impact, and higher realized returns lead to higher wealth level. See column $Corr(u_t, v_t) = 0$ in the Panel A of Table 8 and Table 9 for the benchmark setting. Both $Corr(R_m - r_f, Δ \ln Trn)$ and $Cov(R_m - r_f, Δ \ln Trn)$ are negative when $Corr(u_t, v_t) = 0$ for the entire sample under the benchmark setting.

### 7.3 Aggregate Mutual Fund Flows and Market Returns

One may argue that the equilibrium liquidity risk premium is mostly determined by large institutional investors, who face larger forced selling than retail investors. In this subsection, we therefore compare the assumption of forced selling in our model with the actual fund flows of mutual funds. We use the data of all mutual funds from 1966 to 2016 in the CRSP Mutual Fund database. We only keep equity funds with more than 60% invested in US common stocks in our sample. It leaves us 37,219 funds in total. Then we study the correlation between the aggregate fund flows of all funds in our sample and the market excess returns\(^\text{13}\).

Following prior literature (e.g., Chen, Hong, Huang and Kubik (2004), Alexander, Cicci and Gibson (2007), Coval and Stafford (2007)), we estimate fund flows using the CRSP series of monthly TNA and returns. The net flow of funds to mutual fund $i$ during month $t$ is defined as \(^\text{13}\)This is different from the flow-performance sensitivity documented in the mutual fund literature, which is at the individual fund level instead of the aggregate level.
\[ FLOW_{i,t} = TNA_{i,t} - TNA_{i,t-1} \times (1 + R_{i,t}) \] (21)

Where \( TNA_{i,t} \) is the total net assets (TNA) for fund \( i \) at the end of month \( t \), and \( R_{i,t} \) is the monthly return for fund \( i \) over month \( t \).\(^{14}\) For the years 1991 to 2016, we aggregate the flows of all funds every year (denoted as \( FLOW_y \)), and aggregate the TNAs of all funds at the end of each year (denoted as \( TNA_y \)). For the years 1966 to 1990, where only annually or quarterly data of TNAs are available, we calculate the fund flows as the change of fund TNA every year adjusted by their cumulative returns in this year.

Then we calculate the percentage flow as the aggregate flow as a percentage of aggregate TNA at the end of last year as

\[ flow_y = \frac{FLOW_y}{TNA_{y-1}} \] (22)

The summary statistics of \( flow_y \) and the market excess returns \( R_m - r_f \) are reported in Table 10, Panel A. The correlations and covariances of \( flow_y \) and \( R_m - r_f \) are reported in Panel B. Table 10 shows the equity funds in our sample have an average inflow of 4.4% per year from 1966 to 2016. The correlation between fund flows and market returns in the same year is small, only about 0.107, and it increases to 0.216 when the sample is limited to the years with negative market returns \((R_m - r_f < 0)\). Because the number of observations are limited in the annual analysis, none of those correlations are statistically significant.

We then compare these correlations to what is implied by our model with funding liquidity shocks (Section 6.2). Table 11 reports the correlation between the annual returns and the fraction of holdings that our investor is forced to sell in our setting with liquidity shocks based on 10,000 simulations. The average size of the forced sales in our setting is -4.2%. Since we only have forced selling in our setting, the maximum value of this "fund flow" is 0. Panel B shows the correlation between forced sales and annual returns is high, 0.603 for the entire sample and 0.897 for the sample with negative returns. The covariance

\(^{14}\)To account for funds newly established and funds dropping out of the market, for each fund, we set the flow of the first observation as the TNA reported in that month, and we add one more observation after the last observation of each fund and set the flow equal to -(TNA of the previous month).
is also substantially larger than in the data. Therefore our setting with forced selling seems much more extreme than the actual forced selling faced by mutual funds.

We also report the results for monthly fund flows and monthly market returns from January 1991 to December 2016 in Table 12. The correlation of monthly flows and monthly market excess returns is 0.217 for the entire sample and 0.313 for the months with negative market returns \((R_m - r_f < 0)\), which is still substantially smaller than the value of 0.897 implied by our model. The correlation between fund flows and the 1-month lagged market returns is small and not significantly different from zero (not reported).

8 Conclusions

In this paper we solve a dynamic portfolio choice problem with stochastic illiquidity, CRRA utility and a time-varying expected return. Our goal is to generate theoretical predictions for the liquidity risk premium that large investors demand. Our main finding is that this liquidity risk premium is negligible, less than 1 bp per year, in our benchmark setting with time-varying expected returns. This is much smaller than what is typically found empirically, see the seminal contributions of Pastor and Stambaugh (2003) and Acharya and Pedersen (2005).

Only when we include large funding liquidity shocks leading to forced sales in market downturns, do we find a nonnegligible liquidity risk premium of about 20 basis points per year. We show that it is crucial that trading costs increase when funding liquidity shocks occur to generate this level for the liquidity risk premium. In other words, funding liquidity and market liquidity need to be strongly correlated. However, we show that our extension with funding liquidity shocks implies return-turnover correlations that are much higher than in the data.

Understanding the gap between our theoretical predictions and empirical estimates is clearly interesting for future research. On the theoretical side, potential directions are to allow for non-standard preferences and behavioral trading motives. On the empirical side, isolating liquidity risk from other risk factors, such as volatility risk, could be interesting.
9 Appendix

Figure 1: Weights in risky asset under benchmark setting (1 simulation)

This figure plots one simulation of the weights in the risky asset from time 1 to 10 (10 years), for the benchmark setting with a time constant trading-cost rate and zero correlation between returns and costs. The initial weight is set as $\alpha_0 = 100\%$. In each time step $t$ (1 to 10), we plot both the weight before rebalancing $\alpha_t^-$, the black circle, and the weight after rebalancing and paying trading costs $\alpha_t^+$, the red star. The pink cross denotes the myopic optimal weight in each time step, $\alpha_t^\text{Myopic}$, and the green dash line is the long-run optimal weight, $\alpha_t^\text{LongRun}$ which equals 100\%.
This figure plots the expected utility of the terminal wealth as a function of the trading-cost parameter $\lambda_c$ under the optimal strategy, $\max_{\alpha_0, \ldots, \alpha_{T-1}} E_0[U(W_T)]$, for the benchmark setting with a time constant trading-cost rate.
Figure 3: Average trading amount as a function of the trading-cost rate $\lambda_c$

This figure plots the average annual trading amount as a function of the trading-cost parameter $\lambda_c$ under the optimal strategy, for the benchmark setting with a time-varying trading-cost rate. The correlation between returns and costs is -30%: $\text{Corr}(u_t, v_t) = -0.3$. Initial wealth is 100 million.
Figure 4: Total liquidity premium as a function of the trading-cost rate $\lambda_c$

This figure plots the total annualized liquidity premium as a function of the trading-costs parameter $\lambda_c$ under the optimal strategy, for the benchmark setting with a time-varying trading-cost rate. The correlation between returns and costs is -30\%: $\text{Corr}(u_t, v_t) = -0.3$. 

36
This figure plots the average trajectory of the optimal weight in the risky asset for the case of building up the portfolio from zero at time 1 and releasing all the positions before the end of year 10 (time 11). The correlation between returns and costs is -30\%: \textit{Corr}(u_t, v_t) = 0.3. The weight is averaged across 10,000 simulations.
Table 1: Summary of Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Explanations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_0$</td>
<td>0.04</td>
<td>Long-run mean of the expected return</td>
</tr>
<tr>
<td>$r_f$</td>
<td>0.02</td>
<td>Risk-free rate</td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>0.10</td>
<td>Return volatility</td>
</tr>
<tr>
<td>$\lambda_c$</td>
<td>26.88</td>
<td>The long-run mean of the price impact parameter $\lambda_t$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.7</td>
<td>AR(1)-coefficient of the state variable $F_t$</td>
</tr>
<tr>
<td>$a$</td>
<td>0.01</td>
<td>Loading of the time-varying expected return on $F_t$</td>
</tr>
<tr>
<td>$b$</td>
<td>0.3149</td>
<td>Loading of the price impact parameter $\lambda_t$ on $F_t$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0, −0.3, −0.6</td>
<td>Correlation between shocks to $F_t$ and $r_t$;</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>2.5</td>
<td>Risk aversion level of the investor</td>
</tr>
<tr>
<td>$W_0$</td>
<td>100</td>
<td>Initial wealth (100 million U.S. dollar)</td>
</tr>
<tr>
<td>$T$</td>
<td>11</td>
<td>Horizon (10 years of rebalancing, with annual steps)</td>
</tr>
</tbody>
</table>
Table 2: Liquidity risk premium for the benchmark setting

This table reports the liquidity level premium and liquidity risk premium under the benchmark setting, for different values of the correlation between the returns and trading costs, $Corr(u_t, v_t)$, 0, -0.2, -0.4, -0.6. The liquidity level premium is defined as the increase in the long-term mean of the expected return, $\mu_0$, on the risky asset that the investor requires to be indifferent between having access to the risky asset with rather than without trading costs. The liquidity risk premium is the increase in $\mu_0$ that the investor requires to be indifferent between having access to the risky asset with rather than without time variation in trading costs. The premium for the covariance effect is calculated as the increase in the liquidity risk premium when lowering $Corr(u_t, v_t)$ from 0 to -0.2, -0.4 and -0.6, respectively. All premiums are reported in basis points (bps).

<table>
<thead>
<tr>
<th>$Corr(u_t, v_t)$</th>
<th>0</th>
<th>-0.2</th>
<th>-0.4</th>
<th>-0.6</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Liquidity Level Premium (Total)</strong></td>
<td>16.43</td>
<td>15.86</td>
<td>16.48</td>
<td>15.68</td>
</tr>
<tr>
<td><strong>Liquidity Risk Premium (Total)</strong></td>
<td>-0.623</td>
<td>-0.582</td>
<td>-0.353</td>
<td>-0.104</td>
</tr>
<tr>
<td><strong>Total Liquidity Premium</strong></td>
<td>15.80</td>
<td>15.27</td>
<td>16.12</td>
<td>15.58</td>
</tr>
<tr>
<td><em>liquidity risk premium covariance effect</em></td>
<td>0.000</td>
<td>0.042</td>
<td>0.270</td>
<td>0.519</td>
</tr>
</tbody>
</table>
This table reports the liquidity level premium (decomposed into a direct trading cost component and a utility loss component), the liquidity risk premium and the average annual trading amount across different trading-cost rates $\lambda_c$, from $3.36(1/8*26.88)$ to $107.52(4*26.88)$. The direct trading cost component is the premium compensating for actual trading costs paid, and the utility loss component is the premium compensating for the utility loss caused by deviating from the optimal weight in a frictionless market. Both the compensation for trading costs and the average trading amount per year are calculated through 10,000 simulations with 10 steps each. All premiums are reported in basis points (bps), and the correlation between the returns and trading costs equals -30%: $corr(u_t,v_t) = -0.3$.

<table>
<thead>
<tr>
<th>Mean value of trading-cost parameter $\lambda_c$</th>
<th>3.36</th>
<th>6.72</th>
<th>13.44</th>
<th>26.88</th>
<th>53.76</th>
<th>107.52</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Liquidity Level Premium</strong> (direct trading cost component)</td>
<td>4.83</td>
<td>5.30</td>
<td>5.35</td>
<td>4.18</td>
<td>3.45</td>
<td>2.88</td>
</tr>
<tr>
<td><strong>Liquidity Level Premium</strong> (utility loss component)</td>
<td>2.88</td>
<td>5.45</td>
<td>8.56</td>
<td>12.65</td>
<td>15.69</td>
<td>18.05</td>
</tr>
<tr>
<td><strong>Liquidity Risk Premium (Total)</strong></td>
<td>-0.167</td>
<td>-0.316</td>
<td>-0.393</td>
<td>-0.494</td>
<td>-0.370</td>
<td>-0.297</td>
</tr>
<tr>
<td><strong>Total Liquidity Premium</strong></td>
<td>7.54</td>
<td>10.44</td>
<td>13.51</td>
<td>16.34</td>
<td>18.77</td>
<td>20.63</td>
</tr>
<tr>
<td><strong>avg. trading amount (million$)</strong></td>
<td>14.8</td>
<td>10.8</td>
<td>7.7</td>
<td>4.9</td>
<td>3.1</td>
<td>2.0</td>
</tr>
</tbody>
</table>
Table 4: Liquidity risk premium with building up and scaling down the portfolio

This table reports the liquidity level premium (decomposed into the direct trading cost component and the utility loss component), the liquidity risk premium and the average trading amount per year for different frequencies of rebuilding the portfolio (every 1, 2, 5 and 10 years, respectively), and for 3 values of the correlation between returns and trading costs ($Corr(u_t, v_t) = 0, -0.3, -0.6$) for each frequency. Both the direct trading cost component and the average trading amount per year are calculated through 10,000 simulations with 10 steps each. All premiums are reported in basis points (bps).

<table>
<thead>
<tr>
<th>in bps</th>
<th>$Corr(u_t, v_t)$</th>
<th>Frequency of rebuilding (per $\tau$ years)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td><strong>Liquidity Level Premium</strong></td>
<td>0</td>
<td>8.37</td>
</tr>
<tr>
<td>(direct trading cost component)</td>
<td>-0.3</td>
<td>7.98</td>
</tr>
<tr>
<td></td>
<td>-0.6</td>
<td>7.85</td>
</tr>
<tr>
<td><strong>Liquidity Level Premium</strong></td>
<td>0</td>
<td>184.35</td>
</tr>
<tr>
<td>(utility loss component)</td>
<td>-0.3</td>
<td>195.38</td>
</tr>
<tr>
<td></td>
<td>-0.6</td>
<td>202.44</td>
</tr>
<tr>
<td><strong>Liquidity Risk Premium (Total)</strong></td>
<td>0</td>
<td>1.509</td>
</tr>
<tr>
<td></td>
<td>-0.3</td>
<td>1.645</td>
</tr>
<tr>
<td></td>
<td>-0.6</td>
<td>1.859</td>
</tr>
<tr>
<td><strong>Total Liquidity Premium</strong></td>
<td>0</td>
<td>194.23</td>
</tr>
<tr>
<td></td>
<td>-0.3</td>
<td>205.01</td>
</tr>
<tr>
<td></td>
<td>-0.6</td>
<td>212.15</td>
</tr>
<tr>
<td><strong>avg. trading amount (million $s)</strong></td>
<td>0</td>
<td>9.49</td>
</tr>
<tr>
<td></td>
<td>-0.3</td>
<td>9.26</td>
</tr>
<tr>
<td></td>
<td>-0.6</td>
<td>9.18</td>
</tr>
</tbody>
</table>
Table 5: Liquidity risk premium with funding liquidity shocks (forced selling)

This table reports liquidity level premiums and liquidity risk premiums for the setting with funding liquidity shocks (forced selling). We report it for 3 values of the correlation between returns and trading costs ($\text{Corr}(u_t, v_t) = 0, -0.3, -0.6$) separately. We report the liquidity level premium (direct trading cost component), liquidity level premium (utility loss component) and the total liquidity risk premium. All premiums are reported in basis points (bps).

<table>
<thead>
<tr>
<th>$\text{Corr}(u_t, v_t)$</th>
<th>0</th>
<th>-0.3</th>
<th>-0.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquidity Level Premium (direct trading cost component)</td>
<td>29.92</td>
<td>31.94</td>
<td>33.19</td>
</tr>
<tr>
<td>Liquidity Level Premium (utility loss component)</td>
<td>19.47</td>
<td>20.72</td>
<td>21.41</td>
</tr>
<tr>
<td>Liquidity Risk Premium (Total)</td>
<td>2.22</td>
<td>11.53</td>
<td>20.83</td>
</tr>
<tr>
<td><strong>Total Liquidity Premium</strong></td>
<td>51.61</td>
<td>64.19</td>
<td>75.44</td>
</tr>
</tbody>
</table>
Table 6: Liquidity risk premium with funding liquidity shocks for different levels of trading-cost rates

This table reports liquidity level premiums, liquidity risk premiums and average trading amount per year under the setting with funding liquidity shocks for different levels of the mean trading-cost rate $\lambda_c$, from 3.36 ($1/8\times 26.88$) to 107.52 ($4\times 26.88$). The correlation between returns and trading costs equals -30%: $\text{Corr}(u_t, v_t) = -0.3$. We report the liquidity level premiums (direct trading cost component), liquidity level premium (utility loss component) and the total liquidity risk premiums. Both the direct trading cost component and the average trading amount per year are calculated through 10,000 simulations with 10 steps each. All premiums are reported in basis points (bps).

<table>
<thead>
<tr>
<th>Average value of trading-cost parameter $\lambda_c$</th>
<th>3.36</th>
<th>6.72</th>
<th>13.44</th>
<th>26.88</th>
<th>53.76</th>
<th>107.52</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquidity Level Premium (direct trading cost component)</td>
<td>10.93</td>
<td>15.48</td>
<td>21.79</td>
<td>31.94</td>
<td>49.51</td>
<td>86.53</td>
</tr>
<tr>
<td>Liquidity Level Premium (utility loss component)</td>
<td>2.17</td>
<td>5.91</td>
<td>11.96</td>
<td>20.72</td>
<td>34.58</td>
<td>66.56</td>
</tr>
<tr>
<td>Liquidity Risk Premium (Total)</td>
<td>4.88</td>
<td>7.52</td>
<td>9.92</td>
<td>11.53</td>
<td>13.71</td>
<td>21.54</td>
</tr>
<tr>
<td>Total Liquidity Premium</td>
<td>17.98</td>
<td>28.91</td>
<td>43.67</td>
<td>64.19</td>
<td>97.81</td>
<td>174.62</td>
</tr>
<tr>
<td>avg. trading amount (million $$s)</td>
<td>20.69</td>
<td>16.04</td>
<td>12.22</td>
<td>9.58</td>
<td>8.03</td>
<td>7.68</td>
</tr>
</tbody>
</table>
Table 7: Correlation and Covariance between Market Returns, Innovations in ILLIQ and Turnover (annually)

This table reports correlations between annual market excess returns, innovations in market liquidity ($\Delta \ln ILLIQ$) and innovations in market turnover ($\Delta \ln Trn$) from 1966 to 2010, and the covariance between the annual market excess returns and the innovations in market turnover. Panel A has summary statistics, Panel B provides the correlations, and Panel C tabulates the correlations and covariance of the annual market excess returns and the innovations in market turnover for the entire sample, the subsample with positive returns only ($R_m - r_f > 0$), and subsample with negative returns only ($R_m - r_f < 0$) separately.

### Panel A: Summary Statistics

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std.Dev</th>
<th># obs</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_m - r_f$</td>
<td>0.056</td>
<td>0.185</td>
<td>45</td>
<td>-0.399</td>
<td>0.321</td>
</tr>
<tr>
<td>$\Delta \ln Trn$</td>
<td>0.000</td>
<td>0.139</td>
<td>44</td>
<td>-0.318</td>
<td>0.252</td>
</tr>
<tr>
<td>$\Delta \ln ILLIQ$</td>
<td>0.000</td>
<td>0.244</td>
<td>44</td>
<td>-0.431</td>
<td>0.760</td>
</tr>
</tbody>
</table>

### Panel B: Correlations for Entire Sample

<table>
<thead>
<tr>
<th></th>
<th>$R_m - r_f$</th>
<th>$\Delta \ln ILLIQ$</th>
<th>$\Delta \ln Trn$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_m - r_f$</td>
<td>1</td>
<td>-0.584***</td>
<td>0.203</td>
</tr>
<tr>
<td>$\Delta \ln ILLIQ$</td>
<td>1</td>
<td>-0.281*</td>
<td></td>
</tr>
<tr>
<td>$\Delta \ln Trn$</td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Panel C: Correlation and Covariance of Returns and Turnovers

<table>
<thead>
<tr>
<th></th>
<th>$Corr(R_m - r_f, \Delta \ln Trn)$</th>
<th>$Cov(R_m - r_f, \Delta \ln Trn) \times 10^{-4}$</th>
<th># obs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entire sample</td>
<td>.203</td>
<td>51.9</td>
<td>44</td>
</tr>
<tr>
<td>$R_m - r_f &gt; 0$</td>
<td>-.056</td>
<td>-.66</td>
<td>30</td>
</tr>
<tr>
<td>$R_m - r_f &lt; 0$</td>
<td>-.170</td>
<td>-.259</td>
<td>14</td>
</tr>
</tbody>
</table>
Table 8: Correlation of the Returns and Innovations in Turnovers (simulation results)

This table reports the correlation between the annual returns and the innovations in turnover ($\Delta \ln Trn$) for different settings of our model: Panel A for the benchmark setting with time-varying trading-cost rates, and Panel B for the setting with funding liquidity shocks (forced selling) and time-varying trading-cost rates. For each setting, we report the correlation values for cases with different values of the correlation between returns and trading costs, $Corr(u_t, v_t)$. We also report them for the entire sample, the subsample with positive returns only ($R_m - r_f > 0$), and the subsample with negative returns only ($R_m - r_f < 0$). We do 10,000 simulations for each case within each setting.

### Panel A: Benchmark setting with time-varying trading-cost rates

<table>
<thead>
<tr>
<th>$Corr(R_m - r_f, \Delta \ln Trn)$</th>
<th>$Corr(u_t, v_t)$</th>
<th>0</th>
<th>-0.2</th>
<th>-0.4</th>
<th>-0.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entire sample</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_m - r_f &gt; 0$</td>
<td></td>
<td>-0.100***</td>
<td>-0.058***</td>
<td>-0.010***</td>
<td>0.023***</td>
</tr>
<tr>
<td>$R_m - r_f &lt; 0$</td>
<td></td>
<td>-0.074***</td>
<td>-0.031***</td>
<td>-0.014***</td>
<td>0.049***</td>
</tr>
</tbody>
</table>

### Panel B: Setting with funding liquidity shocks and time-varying trading-cost rates

<table>
<thead>
<tr>
<th>$Corr(R_m - r_f, \Delta \ln Trn)$</th>
<th>$Corr(u_t, v_t)$</th>
<th>0</th>
<th>-0.3</th>
<th>-0.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entire sample</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_m - r_f &gt; 0$</td>
<td></td>
<td>-0.328***</td>
<td>-0.303***</td>
<td>-0.220***</td>
</tr>
<tr>
<td>$R_m - r_f &lt; 0$</td>
<td></td>
<td>-0.087***</td>
<td>-0.047***</td>
<td>0.091***</td>
</tr>
</tbody>
</table>
Table 9: Covariance of the Returns and Innovations in Turnover (simulation results)

This table reports the covariance between the annual returns and the innovations in turnover ($\Delta \ln Trn$) for different settings of our model: Panel A for the benchmark setting with time-varying trading-cost rates, and Panel B for the setting with funding liquidity shocks (forced selling) and time-varying trading-cost rates. For each setting, we report the covariance values for cases with different values of the correlation between returns and trading costs, $\text{Corr}(u_t, v_t)$. We also report them for the entire sample, the subsample with positive returns only ($R_m - r_f > 0$), and the subsample with negative returns only ($R_m - r_f < 0$) separately. We do 10,000 simulations for each case within each setting.

**Panel A: Benchmark setting with time-varying trading-cost rates**

<table>
<thead>
<tr>
<th></th>
<th>$\text{Cov}(R_m - r_f, \Delta \ln Trn)$</th>
<th>$\text{Corr}(u_t, v_t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$^{(\times 10^{-4})}$</td>
<td>0  -0.2  -0.4  -0.6</td>
</tr>
<tr>
<td>Entire sample</td>
<td>-93.5   -61.0  -10.9  16.2</td>
<td></td>
</tr>
<tr>
<td>$R_m - r_f &gt; 0$</td>
<td>-33.2   -15.9  -7.7   25.7</td>
<td></td>
</tr>
<tr>
<td>$R_m - r_f &lt; 0$</td>
<td>-32.1   -18.5  -44.1 -51.6</td>
<td></td>
</tr>
</tbody>
</table>

**Panel B: Setting with liquidity shocks and time-varying trading-cost rates**

<table>
<thead>
<tr>
<th></th>
<th>$\text{Cov}(R_m - r_f, \Delta \ln Trn)$</th>
<th>$\text{Corr}(u_t, v_t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$^{(\times 10^{-4})}$</td>
<td>0  -0.3  -0.6</td>
</tr>
<tr>
<td>Entire sample</td>
<td>-351.6  -327.3  -235.9</td>
<td></td>
</tr>
<tr>
<td>$R_m - r_f &gt; 0$</td>
<td>-36.4   -19.9   38.4</td>
<td></td>
</tr>
<tr>
<td>$R_m - r_f &lt; 0$</td>
<td>-393.2  -384.3  -391.9</td>
<td></td>
</tr>
</tbody>
</table>
Table 10: Correlation between Market Returns and Aggregate Fund Flows (Annual)

This table reports the correlations and covariances between market excess returns and aggregate mutual fund flows at an annual frequency from 1966 to 2016. Panel A shows the summary statistics, Panel B tabulates the correlations and covariances for the entire sample, the subsample with positive returns only ($R_m - r_f > 0$), and subsample with negative returns only ($R_m - r_f < 0$) separately.

**Panel A: Summary Statistics**

<table>
<thead>
<tr>
<th></th>
<th># Obs</th>
<th>Mean</th>
<th>Std.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>$flow_y$</td>
<td>55</td>
<td>0.044</td>
<td>0.090</td>
<td>-0.143</td>
<td>0.311</td>
</tr>
<tr>
<td>$R_m - r_f$</td>
<td>56</td>
<td>0.068</td>
<td>0.176</td>
<td>-0.383</td>
<td>0.352</td>
</tr>
</tbody>
</table>

**Panel B: Correlation and Covariance of Returns and Concurrent Flows**

<table>
<thead>
<tr>
<th></th>
<th>$Corr(R_m - r_f, flow_y)$</th>
<th>$Cov(R_m - r_f, flow_y) \times 10^{-4}$</th>
<th># obs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entire sample</td>
<td>.107</td>
<td>16.9</td>
<td>55</td>
</tr>
<tr>
<td>$R_m - r_f &gt; 0$</td>
<td>.028</td>
<td>2.6</td>
<td>39</td>
</tr>
<tr>
<td>$R_m - r_f &lt; 0$</td>
<td>.216</td>
<td>13.8</td>
<td>16</td>
</tr>
</tbody>
</table>

Table 11: Model-implied Correlation of Returns and Forced Sales

This table reports the correlation between annual returns and the fraction of holdings our investor is forced to sell, which we denote $Outflow_y$, in our setting with liquidity shocks (Section 6.2). Panel A shows the summary statistics, Panel B reports the correlation. We report this correlation for the entire sample and the subsample with negative returns only ($R_m - r_f < 0$) separately. We do 10,000 simulations of our theoretical model to calculate these numbers.

**Panel A: Summary Statistics**

<table>
<thead>
<tr>
<th></th>
<th># Obs</th>
<th>Mean</th>
<th>Std.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Outflow_y$</td>
<td>10,000</td>
<td>-0.042</td>
<td>0.129</td>
<td>-1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>$R_m - r_f$</td>
<td>10,000</td>
<td>0.020</td>
<td>0.101</td>
<td>-0.416</td>
<td>0.464</td>
</tr>
</tbody>
</table>

**Panel B: Correlation and Covariance of Returns and Concurrent Flows**

<table>
<thead>
<tr>
<th></th>
<th>$Corr(R_m - r_f, Outflow_y)$</th>
<th>$Cov(R_m - r_f, Outflow_y) \times 10^{-4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entire sample</td>
<td>.603***</td>
<td>79.7</td>
</tr>
<tr>
<td>$R_m - r_f &lt; 0$</td>
<td>.897***</td>
<td>95.6</td>
</tr>
</tbody>
</table>
Table 12: Correlation between Market Returns and Aggregate Fund Flows (monthly)

This table reports the correlations and covariances between market excess returns and aggregate mutual fund flows at an monthly frequency from 1991 January to 2016 December. Panel A shows the summary statistics, Panel B tabulates the correlations and covariances for the entire sample, the subsample with positive returns only ($R_m - r_f > 0$), and subsample with negative returns only ($R_m - r_f < 0$) separately.

**Panel A: Summary Statistics**

<table>
<thead>
<tr>
<th></th>
<th># Obs</th>
<th>Mean</th>
<th>Std.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>flow_m</td>
<td>312</td>
<td>0.006</td>
<td>0.008</td>
<td>-0.015</td>
<td>0.068</td>
</tr>
<tr>
<td>$R_m - r_f$</td>
<td>312</td>
<td>0.007</td>
<td>0.042</td>
<td>-0.172</td>
<td>0.114</td>
</tr>
</tbody>
</table>

**Panel B: Correlation and Covariance of Returns and Concurrent Flows**

<table>
<thead>
<tr>
<th></th>
<th>Corr($R_m - r_f, flow_m$)</th>
<th>Cov($R_m - r_f, flow_m$) ($* 10^{-4}$)</th>
<th># obs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entire sample</td>
<td>.217</td>
<td>.07</td>
<td>311</td>
</tr>
<tr>
<td>$R_m - r_f &gt; 0$</td>
<td>-.092</td>
<td>-.16</td>
<td>196</td>
</tr>
<tr>
<td>$R_m - r_f &lt; 0$</td>
<td>.313**</td>
<td>.62</td>
<td>115</td>
</tr>
</tbody>
</table>
9.1 Appendix: Robustness Check for the Effect of Rebalancing on the Liquidity Risk Premium

Table A1 and A2 show that the liquidity risk premium is still negligible even if the long-run optimal weight is 50% ($\gamma = 5$) and 150% ($\gamma = 5/3$). In this case, the investor needs to rebalance more compared to the benchmark case (where the long-run optimal weight is 100% in absence of hedging demands and transaction costs). Hence, the need to rebalance does not affect our conclusions.
Table A1: Liquidity risk premium for the benchmark setting with optimal weight as 50%

This table reports the liquidity level premiums and liquidity risk premiums under the benchmark setting with an optimal frictionless myopic weight of 50% on the risky asset ($\gamma = 5$), for different values of the correlation between returns and trading costs, $\text{Corr}(u_t, v_t)$, 0, -0.2, -0.4, -0.6. All premiums are reported in basis points (bps).

<table>
<thead>
<tr>
<th>$\text{Corr}(u_t, v_t)$</th>
<th>0</th>
<th>-0.2</th>
<th>-0.4</th>
<th>-0.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquidity Level Premium (Total)</td>
<td>17.14</td>
<td>17.76</td>
<td>19.26</td>
<td>18.75</td>
</tr>
<tr>
<td>Liquidity Risk Premium (Total)</td>
<td>-0.682</td>
<td>-0.635</td>
<td>-0.398</td>
<td>0.069</td>
</tr>
<tr>
<td><strong>Total Liquidity Premium</strong></td>
<td>16.46</td>
<td>17.12</td>
<td>18.86</td>
<td>18.82</td>
</tr>
<tr>
<td><strong>liquidity risk premium covariance effect</strong></td>
<td>0.000</td>
<td>0.047</td>
<td>0.283</td>
<td>0.750</td>
</tr>
</tbody>
</table>
Table A2: Liquidity risk premium for the benchmark setting with optimal weight as 150%

This table reports the liquidity level premiums and liquidity risk premiums under the benchmark setting with an optimal frictionless myopic weight of 150% on the risky asset ($\gamma = 5/3$), for different values of the correlation between returns and trading costs, $\text{Corr}(u_t, v_t)$, 0, -0.2, -0.4, -0.6. All premiums are reported in basis points (bps).

<table>
<thead>
<tr>
<th>$\text{Corr}(u_t, v_t)$</th>
<th>0</th>
<th>-0.2</th>
<th>-0.4</th>
<th>-0.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquidity Level Premium (Total)</td>
<td>15.12</td>
<td>13.70</td>
<td>12.98</td>
<td>11.31</td>
</tr>
<tr>
<td>Liquidity Risk Premium (Total)</td>
<td>-0.569</td>
<td>-0.473</td>
<td>-0.354</td>
<td>-0.077</td>
</tr>
<tr>
<td>liquidity risk premium for covariance effect</td>
<td>0.000</td>
<td>0.096</td>
<td>0.215</td>
<td>0.492</td>
</tr>
</tbody>
</table>
9.2 Numerical Procedure in Detail

The model is solved using backward induction. In the last period, the value function corresponds to the CRRA utility of the final wealth. We can use this value function to compute the decision variable, the optimal weight in the risky asset $\alpha_{T-1}$, for the previous period, and given these, obtain the corresponding value function. This procedure is then iterated backwards.

We optimize over the space of the decision variable $\alpha_t$ using standard grid search. We use 50 grids for $\alpha_t$ in the benchmark case with a lower bound of 50% and an upper bound of 150%. The upper and lower bounds for the decision variable are chosen to be nonbinding in all periods. To increase the accuracy of the grid search, we first obtain the distribution of $\alpha_t$ across all time periods under equally spaced grids. Then we choose the grid points optimally using the inverse density of this distribution, making the density the same across all grids.

The state-space is also discretized. To reduce the overall computational burden, we search for each state variable a relatively small number of grids that still guarantees the accuracy of our solution, by solving the model with different number of grids for each variable. In the paper, we approximate the density function for returns in the risky asset and innovations in driving factor $F_t$ using gaussian quadrature methods, with 4 nodes each. We use 20 grids for the weight in risky asset before rebalancing $\alpha_{t-1}$, 10 grids for the driving factor $F_t$, and 16 grids for the wealth level $W_t$. All grid points are chosen optimally using the inverse density of the distributions of their corresponding state variables, which are firstly estimated under equally spaced grids. In order to evaluate the value function corresponding to the wealth level that do not lie in the chosen grid, we used a cubic spline interpolation in the log wealth level. This interpolation has the advantage of being continuously differentiable and having a nonzero third derivative, thus preserving the prudence feature of the utility function. Since the lower bound on wealth level is strictly positive, the value function at each grid point is also bounded below. This fact makes the spline interpolation work quite well given a sufficiently fine discretization of the state-space.
References


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  paper, London School of Economics.