How does Price Informativeness affect Investment Sensitivity to Stock Price?

Itay Goldstein  Chong Huang  Qiguang Wang*

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Abstract

This paper studies a feedback model with both managerial learning and speculator learning. In equilibrium, stock price informativeness equals the product of speculator private information precision and supply shock precision; hence, market efficiency increases in either precision. These two different precisions, however, have different effects on investment-price sensitivity, due to the race between managerial learning and speculator learning. Investment-price sensitivity may decrease globally in speculator private signal precision, but first increase then decrease in supply shock precision. Also, investment becomes insensitive to price when supply shock is extremely volatile but remains significantly sensitive to price when speculators’ private signals become almost uninformative.

*Itay Goldstein, Wharton School of Business, University of Pennsylvania, itayg@wharton.upenn.edu; Chong Huang, Paul Merage School of Business, University of California, Irvine, chong.h@uci.edu; Qiguang Wang, School of Business, Hong Kong Baptist University, qiguangw@hkbu.edu.hk. For very helpful comments, we thank Philip Bond, David Hirshleifer, Liyan Yang, and participants at 2017 Finance Theory Group Summer School and the finance seminars at UC Irvine, National University of Singapore, and Hong Kong University.
1 Introduction

Financial economists have documented strong positive correlation between stock prices and corporate investments, which is also called investment sensitivity to price (or investment-price sensitivity).¹ While there is still no agreement on the reasons for such a correlation, many recent empirical studies, such as Chen, Goldstein, and Jiang (2007), Bakke and Whited (2010), Foucault and Fresard (2012), and Edmans, Jayaraman, and Schneemeier (2017), have provided supporting evidence for the managerial learning hypothesis. Under such a hypothesis, the stock price partially aggregates speculators’ private information about the firm’s economic fundamentals; hence, the manager will learn from the information gleaned from the price (price signal) and make the investment decision based on it. Consequently, the investment sensitivity to price should be increasing with the stock price informativeness, because the manager is able to learn more from a more informative price signal.

The managerial learning hypothesis in the literature, however, is developed in partial equilibrium settings where the stock price is assumed to be a linear increasing function of the firm’s economic fundamentals.² Although such an assumption seems plausible because the equilibrium pricing function is indeed linear in the firm’s value in rational expectations equilibrium models (Admati, 1985; Grossman and Stiglitz, 1980; Hellwig, 1980), it may not be true in a general equilibrium framework where the stock market feeds back to corporate investments through managerial learning. How, then, does the stock price informativeness affect the investment-price sensitivity when the stock price is endogenously determined? In addition, will different components of the price informativeness have the same effect on the investment-price sensitivity?

This paper addresses these questions. We study a model in which a firm’s manager makes investment decisions based on the firm’s stock price. Our model differs from a partial equilibrium setting in that the stock price is endogenously determined. Specifically, as in Goldstein, Ozdenoren, and Yuan (2013), there is a continuum of risk-neutral speculators who receive heterogeneous noisy private signals about the firm’s economic fundamentals; they then submit orders based on their private signals and the stock price. Hence, not only the manager but

¹Some early empirical evidence is documented in Barro (1990), Morck, Shleifer, and Vishny (1990), and Blanchard, Rhee, and Summers (1993). See also the survey by Bond, Edmans, and Goldstein (2012) for recent empirical findings.
²One exception is Goldstein, Ozdenoren, and Yuan (2013) who consider creditors’ capital providing problem. In their model, the speculators in the financial market can only submit market orders, and hence the speculators do not learn from the stock market. The speculator learning, however, turns out to be critical for all predictions in our model.
also the speculators are learning from the stock price. There are also random supply shocks that prevent the stock price from being perfectly revealing.

We show that in equilibrium, the price informativeness equals the product of the speculator private signal precision and the random supply shock precision. Hence, as either precision increases, the price signal becomes more informative. However, these two components of the price informativeness have different effects on the investment-price sensitivity. Specifically, under certain mild conditions, the investment-price sensitivity decreases globally in speculator private signal precision, but first increases then decreases in the supply shock precision. In addition, as the speculators’ private signals become almost uninformative, or the random supply shock is extremely volatile, the price signal is almost uninformative; surprisingly, while the investment is insensitive to the price in the latter case, the investment-price sensitivity remains significant in the former case.

The effect of the price informativeness on the investment-price sensitivity in our model arises from the race between the managerial learning and the speculator learning, which is missing in a partial equilibrium setting. Consider a marginal change of the random supply shock realization, which is observable to neither the manager nor the speculators. Such a change will affect both the corporate investment and the stock price through the realization of the price signal only, and therefore is equivalent to an exogenous change in the price signal. The investment-price sensitivity is then equal to the ratio of the investment-price signal sensitivity to the price-price signal sensitivity. The former is due to the managerial learning, and the later comes from the speculator learning. Unlike the linear pricing function in partial equilibrium settings, in our model, the equilibrium pricing function is nonlinear in the firm’s economic fundamentals, and thus the speculator learning is not a constant as the price informativeness changes. Therefore, the effect of the price informativeness on the investment-price sensitivity is determined by the comparison between its effects on the managerial learning and the speculator learning. Such a comparison is indeed the race between the managerial learning and the speculator learning.

We first discuss the global effect of the price informativeness on the investment-price sensitivity. Consider an increase in the effect of speculators’ private signal precision. In this case, the price signal becomes more precise, and so the managerial learning is stronger. However, the speculator learning is also stronger, since both their private signals and the price signal are more precise. In addition, the speculators rationally anticipate the changes in the manager’s learning and thus the induced changes in the optimal investment decision. Such an anticipation effect may dominate as their private signals become more precise. Hence, the speculator
learning grows faster than the managerial learning, and thus, the investment-price sensitivity decreases in the speculator private information precision globally.

The effect of an increase in the supply shock precision is relatively complicated. In such a case, both the manager and the marginal speculator rely more on the price signal. Hence, the ratio of the managerial learning to the speculator learning can either increase or decrease, depending on the relative precision of the price signal versus the speculators’ private signals. When the random supply shock precision is small, the managerial learning dominates, because the manager relies mostly on the price signal, whereas the marginal speculator updates her belief based mainly on her own private signal.

Therefore, when the random supply shock precision increases from a very low value, the investment-price sensitivity increases. As the random supply shock precision continues to increase to a certain level, both the manager and the speculators mainly learn from the price signal. In such a case, the anticipation effect becomes increasingly important for the marginal speculator learning if the signal realization is large. Thus, as the random supply shock precision increases at a high level, the speculator learning can catch up and ultimately dominates the managerial learning, resulting in a decreasing investment-price sensitivity.

The difference in effects on the investment-price sensitivity of the two components of the price informativeness is more salient when the price signal becomes almost uninformative. This occurs when either the speculators’ private signals become pure noises, or the random supply shock is extremely volatile. In such a case, the manager barely learns from the price signal, and hence, the managerial learning is trivial.

Then, the effects of the two components on the investment-price sensitivity is determined by their effects on the speculator learning. When the random supply shock is extremely volatile, the speculator’s learning remains significant, because they can still learn from their own private signals. Therefore, the ratio of the managerial learning to the speculator learning diminishes to zero as the random supply shock becomes extremely volatile, implying that the investment sensitivity to the stock price is trivial in this extreme case.

On the other hand, when the speculator private information precision approaches zero, the speculator learning also becomes trivial, because both their private signals and the price signal become pure noises. In addition, since the speculator private signal informativeness and the price signal informativeness diminishes to zero at the same order of speed, the managerial learning and the speculator learning become trivial at the same order of speed. As a result, the ratio of the managerial learning to the speculator learning converges to a positive number. Therefore, when the speculators’ private signals become almost pure noises, the stock price is
almost uninformative, but the investment-price sensitivity remains significant.

The asymptotic effect of speculator private information on the investment-price sensitivity is a unique feature in the model with the managerial learning. In a benchmark where the manager does not learn from the stock price for some exogenous reasons, the managerial learning is always zero and thus always dominates the marginal speculator learning in the investment-price sensitivity. Therefore, in the model without the managerial learning, the investment-price sensitivity is always zero.

Because different noises have heterogeneous effects on the investment-price sensitivity, we need to distinguish different noises when analyzing empirically the price informativeness effect on the investment-price sensitivity. Since it is usually difficult to identify different noises in the data, we propose to consider the financial market’s real effect, instead of the stock price’s real effect, when we study the relation between the financial market and the corporate decisions. Specifically, we propose to use the direct effect of the price informativeness on the firm’s real decisions as the measure. Such a measure is shown to have the following merits. First, the real effect of an increase in the precision of the speculators’ private signals is qualitatively same as that of an increase in the precision of the random supply shock. Hence, in empirical studies, we can directly consider the cross-sectional variation of the price informativeness, without needing to identify the reason for such variations.

Second, for a fixed realization of the price signal, the firm’s investment is globally monotonic in the price informativeness. For example, for a subsample with a sufficiently optimistic signal realization, when the stock price becomes more informative, the manager puts more weights on the price signal when updating her belief about the firm’s economic fundamentals. Since the realization of the price signal is sufficiently optimistic, the manager’s posterior about the firm’s economic fundamentals increases, which leads to an increase in the decision. On the other hand, the increase in the price signal precision will increase the manager’s posterior precision. This may have positive or negative effect on the manager’s decision. However, when the realization of the price signal is sufficiently optimistic, the first effect always dominates. Therefore, in this case, when the stock price becomes more informative, the firm’s real decision increases.

This paper contributes to a broader literature on the feedback effects of the financial market. The feedback effects have been analyzed in various contexts, such as firms’ investments (Ozdenoren and Yuan, 2008), mergers and acquisitions (Huang and Wang, 2017), creditors’ capital providing (Goldstein, Ozdenoren, and Yuan, 2013), and the government’s interven-
tions (Bond and Goldstein, 2015). Bond, Edmans, and Goldstein (2012) also survey recent theoretical and empirical studies of the feedback effects. We focus on the investment-price sensitivity in this paper, which is an important measure of the financial market’s feedback effect.

Our theory also contributes to the growing literature on how the composition of information available affects firms’ real decisions. Edmans, Heinle, and Huang (2016) show that when some information is soft, financial efficiency may have real costs, because a high-disclosure of soft information may distort the firm’s investment. In addition, Edmans, Jayaraman, and Schneemeier (2017) document that not only the total amount of information available but also the information source matters for the real effects of financial markets. Our paper complements these papers. Specifically, we show that different channels through which the stock price informativeness changes have different effects on the decision-price sensitivity. Therefore, if it is hard to identify the reason for the variation of the price informativeness, we may consider new measures of the feedback effect in empirical studies.

2 A General Model of Real Effects

We consider an economy that consists of a firm and a stock market. There are two dates, \( t = 0 \) and 1. At day 0, speculators trade in the stock market, and the market clearing determines the firm’s stock price. At day 1, the firm’s manager observes the stock price and then makes the corporate decisions. While the corporate decisions include various corporate activities, we refer to them as the firm’s investment in the model for the ease of exposition.

**Investment Decisions** The firm’s value, denoted by \( \pi \), is determined by both the firm’s fundamental \( v \) and the investment level \( I \). The fundamental \( v \) is unknown to any agent in the model, and all agents have a common prior \( \mathcal{N}(v_0, \eta^{-1}) \) about it. The investment level \( I \) is chosen by the firm’s manager from the feasible set \([I_L, I_H]\). To manage the investment with the size \( I \), the manager incurs a private effort cost \( \Delta(I) \). We assume that \( \pi \) is strictly increasing in both \( v \) and \( I \), and \( \Delta \) is strictly increasing in \( I \). Hence, given her information set about \( v \),

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3 Whether the stock price in the secondary market affects the manager’s real decisions has also been discussed in empirical studies. For example, Luo (2005) finds that an acquirer is likely to learn from a negative market reaction after announcing an acquisition that the M&A deal is value-destroying, and so to cancel it.

4 While we interpret \( \Delta(I) \) as the manager’s effort costs in the model, it can be interpreted differently in many different applications. We may allow that \( \Delta \) decreases in the firm’s fundamental \( v \). That is, when the firm’s fundamentals are better, it is easier for the manager to manage a larger size of investments.
which is denoted by $\mathcal{F}$, the manager’s payoff maximization problem is
\[
\max_{I \in [I_L, I_H]} E[\pi(v, I) - \Delta(I)|\mathcal{F}] .
\] (1)

The manager’s strategy is then defined as a mapping $I : \mathcal{F} \rightarrow [I_L, I_H]$. In our model, for simplicity, we assume that the manager does not possess any private information, and hence, the manager’s information set $\mathcal{F}$ includes the stock price $P$ only. In Section 7, we relax this assumption by allowing the manager to have her own private signal. We show that all main results in our core model hold.

**Speculative Trading** There is a continuum of speculators in the market, who are uniformly distributed over $[0, 1]$ and indexed by $i$. Each speculator is risk neutral and trades to maximize his expected payoff. Before submitting the order, speculator $i$ receives a private signal $s_i = v + \epsilon_i$, where $\epsilon_i \sim \mathcal{N}(0, \gamma^{-1})$ is the noise term and is independent across speculators. Based on his private signal $s_i$ and any given stock price $P$, speculator $i$ submits a demand order $x_i \in [-1, 1]$. The trading restriction is imposed because the speculators are risk neutral in our model; otherwise, a speculator’s optimal asset holding may not be well defined. In practice, this can be justified by margin, capital, and liquidity constraints. Hence, speculator $i$’s trading strategy is a mapping from both his private signal and the stock price to the feasible stock holdings. Formally, speculator $i$’s trading strategy is defined as $d_i : \mathbb{R}^2 \rightarrow [-1, 1]$. Each speculator $i$ then solves the following maximization problem:
\[
\max_{d \in [-1, 1]} E[(\pi(v, I) - P) \cdot d|s_i, P] \text{ for all } s_i \text{ and } P .
\] (2)

**Random Supply Shock** Aggregating the speculators’ individual orders leads to the total demand scheme by speculators$^5$ $D(v, P) = \int_0^1 d(s, P) d\Phi(\sqrt{\gamma}(s - v))$, where $\Phi(\cdot)$ is the cumulative distribution function (cdf) of a standard normal distribution and $\Phi(\sqrt{\gamma}(s - v))$ represents the cross-sectional distribution of private signals $s_i$ conditional on the realization of $v$. There is an exogenous stochastic supply of the firm’s stock, denoted by $S(\xi)$. We assume that
\[
S(\xi) = 1 - 2\Phi(\xi)^6
\] (3)

$^5$We assume that the Law of Large Numbers applies to the continuum of speculators so that conditional on $v$ the cross-sectional distribution of signal realizations ex post is the same as the ex-ante distribution of speculators’ signals.

$^6$Alternatively, we can assume that the supply increases in the price $P$: $S(\xi, P) = 1 - 2\Phi(\xi + h(P))$, with
where \( \xi \sim N(0, \beta^{-1}) \) represents liquidity shocks. The variance of \( \xi, \beta^{-1} \), measures the volatility of the random supply shock. The specific structure of the exogenous random supply \( S(\xi) = 1 - 2\Phi(\xi) \) is inelastic and contains a noise term to prevent the price from fully revealing the true fundamental. It follows Albagli, Hellwig, and Tsyvinski (2013) for tractability, and it is similar to the exogenous random supply functions employed by Hellwig, Mukherji, and Tsyvinski (2006) and Goldstein, Ozdenoren, and Yuan (2013).

**Equilibrium** In order to clear the market, the stock price \( P \) is set such that \( D(v, P) = S(\xi) \). We are then interested in a perfect Bayesian equilibrium.

**Definition 1 (Equilibrium)** An investment strategy by the manager \( I^{*}(P) \): \( \mathbb{R} \to [I_{L}, I_{H}] \), a symmetric trading strategy \( d^{*}(s, P) : \mathbb{R}^{2} \to [-1, 1] \), and a price function \( P(v, \xi) \) constitute a perfect Bayesian equilibrium if

1. for the manager, \( I^{*}(P) \in \text{argmax}_{I \in [I_{L}, I_{H}]} \mathbb{E}[\pi(v, I) - \Delta(I)|P] \) for all \( P \);
2. for any speculator \( i \in [0, 1] \), \( d(s_{i}, P) \in \text{argmax}_{d \in [-1, 1]} \mathbb{E}[(\pi(v, I^{*}) - P) \cdot d|s_{i}, P] \) for all \( s_{i} \) and \( P \);
3. market clears: \( D(v, P) = S(\xi) \); and
4. \( \mathbb{E}[:|P] \) and \( \mathbb{E}[:|s_{i}, P] \) are calculated with respect to the posterior probability measures using Bayes’ rule.

**3 Equilibrium Characterization**

In this section, we provide conditions for the existence of an equilibrium of the model. We then characterize an equilibrium, in which the manager and the speculators learn from the stock price, which partially aggregates speculators’ private signals.

**3.1 Optimal Investment**

Because speculators’ private signals about \( v \), as well as the random supply shock \( \xi \), are normally distributed, we conjecture that the posterior of \( v \) conditional on the equilibrium stock price \( P \) is also normally distributed; that is, \( v|P \sim N(\mu_{v|P}, \sigma_{v|P}^{2}) \). Since the manager only

\[ h'(P) > 0. \] As evident below, this leads to an different market clearing condition. However we show in the Online Appendix that this does not change our results.
observes the stock price $P$, $\mu_{v|p}$ and $\sigma_{v|p}$ fully characterize her posterior about $v$. Hence, for any given investment $I$, the manager’s objective function can be rewritten as

$$\mathbb{E}[\pi(v, I) - \Delta(I)|P] \equiv \Pi(I, \mu_{v|p}, \sigma_{v|p}) - \Delta(I),$$  \hspace{1cm} (4)

and the maximization problem becomes

$$\max_{I \in [I_L, I_H]} \Pi(I, \mu_{v|p}, \sigma_{v|p}) - \Delta(I).$$  \hspace{1cm} (5)

We make the following standard assumptions about $\Pi(I, \mu, \sigma)$ and $\Delta(I)$. These assumptions constitute a set of sufficient condition such that for any given $\mu_{v|p}$ and $\sigma_{v|p}$, the manager has a unique optimal investment.

**Assumption 1** The functions $\Pi(I, \mu, \sigma)$ and $\Delta(I)$ satisfy the following assumptions.

1. $\Pi(I, \mu, \sigma)$ is at least twice-differentiable in $I$, $\mu$, and $\sigma$. It is strictly increasing in both $I$ and $\mu$, and it is strictly concave in $I$. In addition, the expected marginal ROI (return on investment) is also increasing in $\mu$, i.e., $\frac{\partial^2 \Pi(I, \mu, \sigma)}{\partial I \partial \mu} > 0$.

2. $\Delta(I)$ is at least twice-differentiable, strictly increasing, and strictly convex in $I$.

3. For any $\mu$ and $\sigma$, there is $I^* \in (I_L, I_H)$ such that $\Pi(I, \mu, \sigma) - \Delta(I)$ is maximized at $I^*$.

Because $\Pi(I, \mu_{v|p}, \sigma_{v|p})$ is strictly concave in $I$, and $\Delta(I)$ is strictly convex, for any given $\mu_{v|p}$ and $\sigma_{v|p}$, the manager’s optimal investment level $I^*$ is unique and is characterized by the first-order condition:

$$\frac{\partial \Pi(I, \mu_{v|p}, \sigma_{v|p})}{\partial I} - \frac{\partial \Delta(I)}{\partial I} = 0.$$  \hspace{1cm} (6)

In addition, Assumption 1 requires that the expected marginal ROI (return on investment) increases in the posterior mean $\mu$ of the firm’s fundamental $v$. This assumption is not only economically intuitive but also guarantees the existence of the equilibrium.

We next characterize the manager’s learning incentive. The equilibrium we characterize in the following sections makes sense only if learning increases the manager’s ex-ante expected payoff. Lemma 1 below shows that the manager to learn from the stock price.

**Lemma 1** If $v|P \sim \mathcal{N}(\mu_{v|p}, \sigma^2_{v|p})$ and, in addition to Assumption 1, $\Pi(I, \mu, \sigma)$ is convex in $\mu$ and is decreasing in $\sigma$, then the manager’s ex-ante payoff is strictly decreasing in $\sigma^2_{v|p}$. In particular, the
manager gets a strictly higher ex-ante payoff by learning from the stock price $P$ than by refraining from learning.

Intuitively, the conditional standard deviation $\sigma_{v|P}$ is exogenous and has three effects on the manager’s ex-ante payoff. It first impacts the manager’s conditional estimation about the firm’s economic fundamentals and thus changes the conditional mean $\mu_{v|P}$. Also, since $\Pi(I, \mu, \sigma)$ decreases in $\sigma$, a decrease in $\sigma_{v|P}$ will increase the manager’s conditional payoff for any realization and so will increase the manager’s ex-ante payoff. Finally, changes in $\sigma_{v|P}$ also changes $I^*$, but it follows from the Envelop Theorem that this effect does not affect the manager’s ex-ante payoff. We emphasize that the assumptions stated in Lemma 1 are sufficient but not necessary. For example, in Huang and Wang (2017), $\Pi(I, \mu, \sigma)$ may not be decreasing in $\sigma$, but the manager still has strict incentives to learn from the stock price.

### 3.2 Trading in the Stock Market

Because the speculators in our model are risk neutral, in an equilibrium, speculator $i$ follows a cutoff trading strategy characterized by $g(P)$ as below:

$$d(s_i, P) = \begin{cases} 1, & \text{if } s_i > g(P) \\ \in [-1, 1], & \text{if } s_i = g(P) \\ -1, & \text{if } s_i < g(P) \end{cases}$$

(7)

That is, given the stock price $P$, if speculator $i$’s private signal is greater than a price-dependent cutoff, she will buy one share of the firm’s stocks; if her private signal lands below such a cutoff, she will sell one share of the firm’s stocks.

The cutoff trading strategy follows from the market microstructure and the firm’s equilibrium strategy. Given a price $P$, when $s_i$ is high, speculator $i$ is optimistic about the firm’s fundamental, and she would like to hold the largest possible long position. In contrast, when $s_i$ is low, speculator $i$ would like to hold a short position of the firm’s share. Hence, there must be a speculator with the private signal $s_i = g(P)$, who is indifferent between trading and not trading. We call such a speculator the “marginal” speculator.

The threshold point $g(P)$ is price-contingent, because an increase in the stock price $P$ has three effects on the marginal investor’s decisions. First, an increase in $P$ increases the cost of holding the firm’s stock, which will reduce a speculator’s incentives to hold a long position. This is commonly referred to as a “substitution effect”. Second, as the price aggregates
dispersed information, a higher stock price suggests that the firm has better fundamentals, which imply a higher value of the firm’s stock. Hence, the price also has the “information effect”. And finally, an increase in the stock price induces a higher investment level, which will also increase the firm’s value. We call it the “feedback effect”.

In typical REE models without the feedback effect, the substitution effect always dominate the information effect, the speculators’ trading decisions are strategic substitutes to each other, and a unique equilibrium exits. In contrast, with feedback effect, strategic complementarities can arise, and price multiplicity may emerge, as modeled by Ozdenoren and Yuan (2008). In our model, however, Assumption 1 guarantees that the feedback effect does not dominate; as a result, we rule out price multiplicity. This largely simplifies our equilibrium analysis.

Given the demand function in (7), we can calculate the aggregate demand from informed speculators:

\[ D(v, P) = \int_{g(P) - v}^{\infty} d \Phi(\sqrt{\gamma} \epsilon) - \int_{-\infty}^{g(P) - v} d \Phi(\sqrt{\gamma} \epsilon) = 1 - 2 \Phi(\sqrt{\gamma}(g(P) - v)). \] (8)

It then follows from the market clearing condition \( D(v, P) = S(\xi) \) that

\[ g(P) = v + \frac{\xi}{\sqrt{\gamma}}. \] (9)

We define \( z \equiv g(P) = v + (1/\sqrt{\gamma}) \xi \). Then, \( z \) is the public signal that is gleaned from the price; we then call it the “price signal.” Since \( z \) is a sufficient statistic of the stock price (verified in Lemma 2 below), conditioning on the stock price \( P \) is equivalent to conditioning on the price signal \( z \). As a result, we use \( E[\cdot | z] \) in place of \( E[\cdot | P] \) wherever it facilitates the discussion.

For any price \( P \), with the optimal investment function \( I^* \) given by equation (6), we can now write speculator \( i \)'s expected payoff from buying one share of the firm’s stocks given his information set:

\[ E[\pi(v, I^*) - P | s_i, P] = \Pi(I^*, \mu_v|s_i, P, \sigma_v|s_i, P) - P. \] (10)

Then, \( d(s_i, P) = 1 \) if and only if equation (10) is positive. From the proposed cutoff trading strategy in equation (7), the marginal speculator with the private signal \( g(P) \) is indifferent between buying or selling the stock. Hence, substitute \( s_i = g(P) \) into equation (10), we have
speculator $i$’s payoff equal to 0. Formally,

$$E[\pi(v, I^*) - P|s_i = g(P), P] = 0. \quad (11)$$

Or, using the sufficient statistic $z$, we have

$$P = E[\pi(v, I^*)|s_i = z, z] = \Pi(I^*, \mu_{v|s_i=z,z}, \sigma_{v|s_i=z,z}). \quad (12)$$

Therefore, the stock price $P$ in equilibrium is determined by the marginal speculator’s expectation of the firm’s value, conditional on his own private signal $s_i = z$ and public signal $z$.

### 3.3 An Equilibrium with Managerial Learning

It follows from Lemma 1 and the definition of the price signal that the manager makes the optimal investment decision based on both the prior belief and the price signal $z$. Hence, the manager’s optimal investment $I^*$ characterized in equation (6), the speculators’ investment strategies characterized in equation (7), and the stock price $P$ determined in equation (12) constitute an equilibrium, provided that our conjectures that $v|P$ has the distribution $\mathcal{N}(\mu_{v|P}, \sigma^2_{v|P})$ and that $z$ is a sufficient statistic of $P$ are correct given all agents’ strategies.

Given the speculators’ investment strategies, when the stock market is cleared, equation (9) shows that conditional on $P$ (and thus $g(P)$), $v$ is distributed normally with mean $(\eta v_0 + \gamma \beta g(P)) / (\eta + \gamma \beta)$ and precision $\eta + \gamma \beta$.

For the above information structure and the inference to hold, we need to show that $z = g(P)$ is monotonic in $P$ in equilibrium (hence $P$ must be monotonic in $z$). This has to hold because the cutoff trading strategy stipulates that $g(P)$ must be a function of $P$ and the indifference condition (12) represents $P$ as a function of $g(P)$. Therefore, we show in Lemma 2 that the indifference condition (11) characterizes a one-to-one mapping between $P$ and $g(P)$.

**Lemma 2** Under Assumption 1, given the speculators’ strategies characterized in equation (11), the price signal $z = g(P)$ is strictly increasing in $P$.

It then follows from Lemma 1 and Lemma 2 that our model has an equilibrium in which the manager learns from the stock price. The equilibrium is characterized in Proposition 1 below.
Proposition 1 Under Assumption 1, the model has a monotone equilibrium, in which

1. the optimal investment $I^*$ is the unique solution to the equation (6);

2. speculators employ a symmetric cutoff trading strategy, described in equation (7), with the threshold $g(P)$ uniquely determined by equation (11); and

3. the market price $P$ is uniquely determined by equation (12).

Importantly, in the equilibrium, the manager and the marginal speculator observe the same signal realization $z$, because the realization of the marginal speculator’s private signal is the same as that of the price signal. However, the manager and the marginal speculator have different posteriors about the firm’s economic fundamentals. In particular, while the signal realization $z$ affects the manager’s posterior through the price signal only, it affects the marginal speculator’s posterior also through his own private signal. Therefore, there is an information wedge between the manager and the marginal speculator: the manager and the marginal speculator learn differently from the same signal realization. It will become clear in Section 4 that such an information wedge plays a critical role in the analysis of the relation between the manager’s investment decision and the stock price.

4 Investment-price Sensitivity

In this section, we analyze the equilibrium properties of the relation between the manager’s investment decision and the stock price (investment-price sensitivity). The investment-price sensitivity is widely used in empirical studies as a measure of feedback effect between the stock price and the firm’s real decisions (for example, Chen, Goldstein, and Jiang (2007)). Therefore, the equilibrium properties derived in this section have important empirical implications.

Because both the price and the investment are endogenous in our model, we consider exogenous shocks to the stock market in order to study the investment-price sensitivity. Suppose that there is a negative shock to the noise supply, which will push up the stock price. As a result, the price signal would also increase, and the manager will increase the investment accordingly. Moreover, the increase in the investment implies an increase in the firm’s value, leading to a higher aggregate demand for the firm’s stock from the speculators and hence a further increase in the stock price. The stock price and the investment keep increasing in such a feedback loop, until a new equilibrium is reached. Therefore, we measure the investment-
price sensitivity as the ratio of the accumulated change in the investment to that in the stock price, as the market adjust to the new equilibrium.

Formally, we consider a marginal increase in \( \xi \). Then, the investment-price sensitivity is measured by

\[
\frac{dI^*}{dP} = \frac{\partial I^*/\partial \xi}{\partial P/\partial \xi}.
\]

Furthermore, it follows from equation (6) and equation (12) that in equilibrium, the random supply shock \( \xi \) affects both the investment \( I^* \) and the stock price \( P \) through the signal realization \( z \) only. So, we calculate the equilibrium investment-price sensitivity as

\[
\frac{dI^*}{dP} = \frac{\partial I^*/\partial z}{\partial P/\partial z} = \frac{\partial I^*/\partial z}{\partial P/\partial z}.
\] (13)

4.1 Almost Uninformative Stock Price

We first analyze the question whether the investment-price sensitivity is still significant when the stock price becomes almost uninformative. Because \( z \) is the price signal, a reasonable measure of the informativeness of the stock price is the precision of \( z \). Since \( z \) is defined as \( v + \xi / \sqrt{\gamma} \), it is normally distributed with precision \( \gamma \beta \), which is also the informativeness of the price signal. Then, as either \( \gamma \to 0 \) or \( \beta \to 0 \), the stock price becomes almost uninformative; put differently, when either the speculators’ private signals are almost uninformative, or the stock market is extremely noisy (i.e., the noise supply shock has an extremely high variance), the stock price becomes almost uninformative. Then, our question can be reformulated as whether \( dI^*/dP \) converges to 0, when either \( \gamma \to 0 \) or \( \beta \to 0 \).

As shown in Proposition 1, the stock price affects the firm’s investment through managerial learning in equilibrium. Then, it seems intuitive to argue that as the stock price becomes almost uninformative, the manager can barely learn the firm’s fundamentals from the stock price, and hence, the investment-price sensitivity should become insignificant. Because this argument does not depend on how the stock price becomes almost uninformative, as either \( \gamma \to 0 \) or \( \beta \to 0 \), the investment-price sensitivity should be nearly zero.

Proposition 2, however, shows that this intuitive argument is incomplete. In particular, the asymptotic behavior of the investment-price sensitivity depends on how the price becomes almost uninformative. Surprisingly, when \( \gamma \to 0 \), \( dI^*/dP \) is strictly positive. This suggests that if it is the almost uninformative speculators’ private signals that lead to almost uninformative stock price, the investment-price sensitivity will remain significant.
Proposition 2 In the equilibrium characterized in Proposition 1,

\[ \frac{dI^*}{dP} \rightarrow \begin{cases} 
  c, & \text{as } \gamma \to 0 \\
  0, & \text{as } \beta \to 0 
\end{cases} \]

where \( c > 0 \) is a constant.

We leave the proof of Proposition 2 in the Appendix. Here, we elaborate the intuition of such a surprising result. Specifically, it follows from equation (13) that the investment-price sensitivity can be factorized into two components. The first is \( \partial I^*/\partial z \), the numerator of the last fraction in equation (13), which is the effect of the signal realization on the manager’s investment decision. We therefore call it *managerial learning*. From the manager’s first-order condition (equation (6)), a change of the price signal affects the optimal investment decision by affecting the manager’s posterior estimation of \( v \), which, by Bayes rule, can be written as

\[ \mu_{v|z} = \frac{\eta v_0 + \gamma \beta z}{\eta + \gamma \beta}. \] (14)

Hence, a change in \( z \) will change the manager’s posterior mean \( \mu_{v|z} \) by \( \frac{\gamma \beta}{\eta + \gamma \beta} \). Therefore, as either \( \gamma \to 0 \) or \( \beta \to 0 \), the effect of \( z \) on \( \mu_{v|z} \) and thus the effect of \( z \) on the optimal investment \( I^* \) are close to 0. That is, when either the speculators’ private signals become almost uninformative or the random supply shock is extremely volatile, the price signal \( z \) becomes a white noise. Consequently, the manager barely updates her estimation of \( v \) based on a change in \( z \). Therefore, as the stock price becomes almost uninformative, the managerial learning effect becomes trivial.

The second component is the denominator in equation (13), \( \partial P/\partial z \), which is the key to understanding Proposition 2. This component shows how a signal realization change will affect the equilibrium price. Recall that in the equilibrium, the relation between the stock price and the price signal is characterized by equation (12), which shows that the marginal speculator’s estimation of the firm’s value equals the stock price. So, we call this second component *marginal speculator learning*.

In order to see the relation between the stock price and the price signal, we consider a marginal increase in the signal realization. This increases the firm’s value through the managerial learning channel and should lead to a higher price. However, this effect is trivial since the manager barely updates her posterior or changes the optimal investment when the
price signal is extremely noisy. Then, its impact on the price, if any, should be through the marginal speculator’s belief updating. By Bayes rule, the marginal speculator’s posterior estimation of $v$ is

$$
\mu_{v|s_i=z,x} = \frac{\eta v_0 + \gamma z + \gamma \beta z}{\eta + \gamma + \gamma \beta}.
$$

First, consider the case that the speculators’ private signals are almost uninformative; that is, $\gamma \to 0$ while $\beta$ is fixed. In such a case, an increase in the signal realization $z$ will not change the marginal speculator’s posterior estimation about the firm’s fundamentals $v$ much. That is, when $\gamma$ is very small, $\mu_{v|s_i=z,x}$ barely changes when $z$ has a marginal change. Because the equilibrium stock price equals the marginal speculator’s estimation of the firm value, almost no change of the marginal speculator’s posterior implies almost no change of the equilibrium stock price. As a result, the marginal speculator learning is also trivial when $\gamma$ is very small. Furthermore, the managerial learning and the marginal speculator learning becomes trivial at the same speed of the speculators’ private signals becoming uninformative, resulting in a positive constant investment-price sensitivity at the limit.

In the second case, the random supply shock is extremely volatile; that is, $\beta \to 0$ while $\gamma$ is fixed. As the price becomes uninformative, the marginal speculator assigns almost zero weight to the price signal. Consequently, his posterior mean is entirely determined by his private signal to which he assigns a constant weight $\gamma$. Therefore, when the signal realization has a marginal increase, the marginal speculator’s posterior has a non-trivial change. Therefore, even when the random supply shock becomes extremely volatile, the marginal speculator learning remains significant. As a result, the managerial learning dominates the investment-price sensitivity, and consequently, when the random supply shock is extremely volatile, the investment-price sensitivity converges to zero.

Yang (2018) also discusses the asymptotic behavior of the investment-price sensitivity, as the liquidity trading becomes extremely volatile. However, his paper features the firm’s optimal information disclosure, and as the liquidity trading becomes extremely volatile, the firm optimally disclose very precise information. Then, the asset price becomes extremely informative, leading to a significant investment-price sensitivity. In our setting, however, the firm does not disclose any information, and therefore, when the liquidity trading becomes extremely volatile, the stock price becomes almost uninformative. Therefore, the managerial learning is trivial, while the marginal speculator learning remains significant, and so the investment-price sensitivity becomes trivial.

We further show that the differing asymptotic behavior above is a special feature of the
model with managerial learning. To see this, we consider a benchmark in which all agents commonly know that the manager does not learn from the price signal. Such a no-learning assumption may be justified by the cost of extracting information from the stock price or the manager’s bounded rationality, such as limited attention.

It then follows from equation (6) that, in such a benchmark, the manager’s optimal investment decision is determined solely by the prior belief characterized by \( v_0 \) and \( \eta \). Therefore, the managerial learning effect \( \partial I^*/\partial z = 0 \). This leads to Proposition 3 below, which shows that in a model without the managerial learning, as the speculators’ private signals become almost uninformative, the investment-price sensitivity is insignificant.

**Proposition 3** If it is commonly known that the manager does not learn from the price signal,

\[
\lim_{\gamma \to 0} \frac{dI^*}{dP} = \lim_{\beta \to 0} \frac{dI^*}{dP} = 0. \tag{16}
\]

### 4.2 Monotonicity of Investment-price Sensitivity

In this section, we discuss how the stock price informativeness affects the investment-price sensitivity globally. We show that the investment-price sensitivity can either increase or decrease in the stock price informativeness, and the monotonicities of the investment-price sensitivity in \( \gamma \) and in \( \beta \) are also different.

It also seems intuitive to argue that as the stock price becomes more informative, the manager puts more weights on the price signal when updating her belief about the firm’s fundamentals \( v \). Therefore, the same change in the price will lead to a bigger change in the manager’s posterior and thus a greater change in the optimal investment, implying that the investment-price sensitivity is increasing in the informativeness of the stock price.

This argument, however, leaves out two things. First, the price informativeness can directly impact how the optimal investment varies with the manager’s belief. Specifically, it is possible that the optimal investment becomes less sensitive to the manager’s belief as the price informativeness increases. Formally, the managerial learning effect \( \partial I^*/\partial z \) can be factorized as

\[
\frac{\partial I^*}{\partial z} = \frac{\partial I^*(\mu_{v|z}, \sigma_{v|z})}{\partial \mu_{v|z}} \times \frac{\partial \mu_{v|z}}{\partial z} \quad . \tag{17}
\]

Here, the first term in equation (17), \( \frac{\partial I^*(\mu_{v|z}, \sigma_{v|z})}{\partial \mu_{v|z}} \), determines the sensitivity of the investment to the manager’s posterior estimation, and so we call it the *feedback multiplier*. The second
term, \( \frac{\partial \mu_{v|z}}{\partial z} = \frac{\gamma \beta}{\gamma \beta + \eta} \), measures the manager’s belief updating due to a change in \( z \).

Importantly, while the manager’s belief updating strictly increases in the price informativeness, the feedback multiplier may decrease in the price informativeness. Consider, for example, that the feedback multiplier itself increases in the posterior mean. Then, when \( z < v_0 \), an increase in \( \gamma \) or \( \beta \) will lead to a decrease in the manager’s posterior \( \mu_{v|z} \), because the manager puts heavier weights on the price signal, which is smaller than the prior mean of the economic fundamentals. Since the feedback multiplier increases in the posterior mean, the feedback multiplier will be smaller when either \( \gamma \) or \( \beta \) is larger.

When the effect of the price informativeness on the feedback multiplier dominates its effect on the belief updating, the total managerial learning effect may decrease in the price informativeness. Lemma 3 provides sufficient conditions for such a decreasing relation.

**Lemma 3** If the ratios \( \frac{\partial^2 I^*}{\partial \mu \partial \sigma^2} \) and \( \frac{\partial I^*}{\partial \mu} \) are bounded, then the managerial learning effect \( \frac{\partial I^*}{\partial z} \) may decrease in \( \gamma \) and \( \beta \).

Second, while the manager puts more weights on the price signal in her belief updating, so does the marginal trader. This implies that the price will vary with the price signal. In equilibrium, the price aggregates the dispersed information and also clears the market. To draw further insight, we decompose the marginal speculator learning. From the marginal speculator’s indifference condition (12), we have

\[
\frac{\partial P}{\partial z} = \frac{\partial \Pi}{\partial I^*} \left( I^*, \mu_{v|s_i=z,z}, \sigma_{v|s_i=z,z} \right) \frac{\partial I^*}{\partial \mu_{v|z}} \frac{\partial \mu_{v|z}}{\partial z} + \frac{\partial \Pi}{\partial \mu_{v|s_i=z,z}} \frac{\partial \mu_{v|s_i=z,z}}{\partial z}
\]

The first part is an anticipation effect. When the signal realization \( z \) changes, the marginal speculator anticipates that the manager is also updating her belief and changing the optimal investment. Interestingly, the effect of a marginal change in the signal realization on the marginal speculator’s estimation of the firm’s value differs from that on the manager’s estimation, because the manager and the marginal speculator estimate the firm’s value based on different information sets. This is reflected in the term \( \frac{\partial \Pi}{\partial I^*} \left( I^*, \mu_{v|s_i=z,z}, \sigma_{v|s_i=z,z} \right) \frac{\partial \mu_{v|z}}{\partial z} \), which is called the anticipation multiplier. By Assumption 1, such a multiplier increases in the posterior mean \( \mu_{v|s_i=z,z} \). Therefore, when the marginal speculator is optimistic about the firm’s

\[7\text{That is, } \frac{\partial^2 I^*}{\partial \mu \partial \sigma^2} > 0.\]
fundamentals, the anticipation effect will be amplified and adds greatly to the total marginal speculator learning effect. In contrast, when \( z \) is negative and the marginal speculator is pessimistic, the anticipation effect will be dampened.

The second component is the marginal speculator’s belief updating effect. Note that because the marginal speculator learns from both the price signal and his own private signal, we have

\[
\frac{d\mu_{v|s_i=z,z}}{dz} = \frac{\partial \mu}{\partial z} + \frac{\partial \mu}{\partial s_i} \Bigg|_{z_i=z} = \frac{\gamma + \gamma \beta}{\eta + \gamma + \gamma \beta}.
\]

In equilibrium, the marginal speculator’s private signal and the price signal have the same realization, and so as the signal realization changes marginally, the marginal speculator updates based on both his private signal and the price signal.

Under Assumption 1, the anticipation effect and the belief updating effect both contribute positively to the marginal speculator learning from the signal realization \( z \). In contrast, the managerial learning effect only derives from her belief updating. Therefore, the marginal speculator learning should be larger than the managerial learning, especially when \( z \) is large and the marginal speculator is optimistic.

If this is the case, then an increase in the speculators’ private signal precision, \( \gamma \), can decrease the investment-price sensitivity. As \( \gamma \) increases, the price becomes more informative. Hence, both the manager and the marginal speculator rely more on the price signal when updating their beliefs. However, the increase in \( \gamma \) also reinforces the marginal speculator’s belief updating effect because he also learns from his own private signal, which now also becomes a more informative signal. In addition, when \( z \) is large (relative to \( v_0 \)), the increase in \( \gamma \) implies a higher \( \mu_{s_i=z,z} \) and hence a higher marginal ROI, leading to an even stronger anticipation effect. Therefore, the effect of the increase in \( \gamma \) on the marginal speculator learning effect dominates that on the managerial learning, leading to a decreasing investment-price sensitivity. If the marginal speculator learning is greater than the managerial learning effect (i.e., \( \partial P/\partial z > \partial I^*/\partial z \)) when \( \gamma \) is close to zero,\(^8\) then it is possible that the investment-price sensitivity globally decreases in \( \gamma \).

Now consider an increase in the liquidity trading precision \( \beta \). We start with a small \( \beta \).

\(^8\)To see this, note that for any ratio \( f(x)/g(x) \) where \( f(x) > 0, g(x) > 0 \) for all \( x > 0 \), then

\[
\left( \frac{f}{g} \right)' = \frac{f'g - fg'}{g^2} < 0 \quad \forall x > 0,
\]

if \( f' < g' \) for all \( x > 0 \) and \( \lim_{x \to 0} [f(x) - g(x)] < 0 \).
In this case, although the price signal is very noise, both the manager’s and the marginal speculator’s belief updating from the signal realization $z$ will increase. Since the manager does not have private signals, his belief updating mainly relies on the price signal. On the other hand, the marginal speculator possesses his own private signal and mainly learns from it when the price signal is very noisy. Therefore, the increase in the manager’s learning can be much more pronounced. In contrast, as $\beta$ is large, such difference in belief updating becomes smaller. Meanwhile, the marginal speculator’s anticipation effect becomes more important, especially when the signal realization $z$ is sufficiently optimistic (much larger than $v_0$). This is because an increase in $\beta$ always leads to an increase in the posterior belief $\mu_{v|s_i=r,z}$, which in turn implies an increase in the marginal ROI. As the anticipation effect contributes more and more to the overall learning effect, the marginal speculator’s incremental learning effect catches up and ultimately dominate the incremental managerial learning effect when $\beta$ is large. In sum, we conjecture that it is possible for the investment-price sensitivity to first increase and then decrease in $\beta$.

Consistent with our argument, we show numerically that the investment-price sensitivity is not necessarily increasing in the stock price informativeness. And the above possible scenarios can arise and render the investment-price sensitivity being locally or globally decreasing in the price informativeness.

To illustrate this, we assume that $\pi(v,I) = e^v I$ and $\Delta(I) = 0.5I^2$. We solve the equilibrium analytically in Section 6 and plot the investment-price sensitivity as a function of $\gamma$ and $\beta$ below. This plot shows that there are only two cases in which the investment-price sensitivity is strictly increasing in the stock price informativeness. The first is when $\gamma$ is sufficiently small. In such a case, the investment-price sensitivity is strictly increasing in $\beta$. That is, when the quality of the speculators’ private signals is sufficiently low, the investment-price sensitivity is stronger as the random supply shock becomes less volatile. The other case is when $\beta$ is sufficiently small; then, the investment-price sensitivity is strictly increasing $\gamma$. Hence, when the random supply shock is extremely volatile, the investment-price sensitivity is strictly increasing in the precision of the speculators’ private signals.

In other cases, however, the investment-price sensitivity is not strictly increasing in either $\gamma$ or $\beta$. For example, when $\beta$ is fixed at a relatively high value, the investment-price sensitivity is strictly decreasing in $\gamma$. Also, when $\gamma$ is fixed at a relatively high value, as $\beta$ increases, the investment-price sensitivity increases first and then decreases. Both these scenarios are consistent with our early conjectures.

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9This result is also observed when we allow the manager to possess his own private information.
Figure 1: Monotonicity of Investment-Price Sensitivity in Price Informativeness

\[ \pi(v, I) = e^{vI}, \text{ and } \Delta(I) = \frac{1}{2}I^2. \] Parameter values are set as follows: \( v_0 = 0, \eta = 1, \) and \( z = 2. \)

5 Direct Effect of Price Informativeness on Real Decision

As discussed in Section 4, the investment-price sensitivity captures not only the managerial learning effect (which is also the real effect of the financial market) but also the stock price’s information aggregation and market clearing effect. Therefore, to empirically test the managerial learning effect, or the real effect of the financial market, we may need a pure measure of the real effect. In this section, we discuss an alternative measure to gauge the effect of managerial learning — the direct effect of the stock price informativeness on the investment.

Proposition 4 below shows the equilibrium properties of the direct effect of the stock price informativeness on the investment.

**Proposition 4** Assume that for any \( I, \mu, \) and \( \sigma, \) the ratio \( \frac{\partial^2 \Pi(I, \mu, \sigma^2)}{\partial I \partial \mu} \) is bounded. Then, there exists \( \bar{z} \) and \( z \) (with \( \bar{z} > v_0 > z \)) such that

\[
\frac{\partial I^*}{\partial \gamma \beta} \begin{cases} 
> 0, & \text{when } z > \bar{z}; \\
< 0, & \text{when } z < \bar{z}.
\end{cases}
\]  

(19)

We note several important features of the direct effect of the stock price informativeness on
the investment presented in Proposition 4. First, the effect of the speculators’ private signals
(\(\gamma\)) is qualitatively consistent with the effect of the precision of the random supply shock
(\(\beta\)). Therefore, when considering the effect of the stock price informativeness, we no longer
need to distinguish the reason for the cross-sectional variations of the price informativeness
(whether they arise from the different precisions of the speculators’ private signals or from
the different volatility of the random supply shocks), which has been shown, in Section 4, to
be critical when we employ the investment-price sensitivity to measure the financial market’s
real effect.

Furthermore, equation (19) shows that the firm’s investment is monotonic in the stock price
informativeness. Specifically, when the realized price signal is sufficiently positive (greater
than a threshold \(\bar{z}\)), an increase in the stock price informativeness increases the firm’s in-
vestment; but when the realized price signal is sufficiently negative (smaller than another
threshold \(z\)), an increase in the stock price informativeness decreases the firm’s investment.

Indeed, an increase in the stock price informativeness has two effects on the firm’s in-
vestment. Let’s consider the case that \(z > \bar{z}\). On the one hand, an increase in the price
informativeness will make the manager puts more weights on the price signal when updat-
ing her belief about the firm’s fundamentals, resulting in a higher posterior mean when \(z\) is
greater than \(v_0\). On the other hand, an increase in the price informativeness always increases
the manager’s posterior precision. While higher posterior mean always increases the optimal
investment, the effect of posterior precision on the investment level is uncertain unless further
assumption is added. However, with the minimum condition imposed in Proposition 4, the
effect of posterior mean always dominates the effect of posterior precision when \(z\) is large and
therefore the investment increases in the price informativeness. When \(z < \bar{z} < v_0\), an increase
in \(\gamma \beta\) still increases the posterior precision but its impact on the posterior mean is reversed.
And when \(z < \bar{z}\), the negative effect on the investment resulted from reduced posterior mean
starts to dominates, and the investment starts to decrease in \(\gamma \beta\).

6 An Application

In this section, we assume specific functional form for the manager’s investment problem and
show that the equilibrium properties of the investment-price sensitivity and those of the direct
effect of the price informativeness on the investment hold in this application. Since we are
able to solve the model analytically, tighter boundaries and sharper prediction can be derived.
We specify the firm’s value and the manager’s private cost as follows.

\[
\pi(v, I) = e^v I \quad \text{and} \quad \Delta(I) = \frac{1}{2} I^2. \tag{20}
\]

Given these payoff functions, the optimal investment is

\[
I^* = E[e^v | z] = \exp \left( \frac{\eta v_0 + \gamma \beta z + 1/2}{\eta + \gamma \beta} \right).
\]

Using the indifference condition in (12), the equilibrium price is

\[
P = E[e^v | s_i = z, z] I^* = \exp \left( \frac{\eta v_0 + \gamma \beta z + 1/2}{\eta + \gamma \beta} + \frac{\eta v_0 + (\gamma \beta + \gamma) z + 1/2}{\eta + \gamma \beta + \gamma} \right),
\]

and the investment-price sensitivity is then

\[
\frac{\partial I^*}{\partial P} = \exp \left( -\frac{\eta v_0 + (\gamma \beta + \gamma) z + 1/2}{\eta + \gamma \beta + \gamma} \right) \frac{\beta(\eta + \gamma + \gamma \beta)}{\eta + 2\beta^2 \gamma + 2\beta(\gamma + \eta)}. \tag{21}
\]

Recall that \( z = v + \xi / \sqrt{\gamma} \) is the price signal. Hence, the precision of \( z \) is \( \gamma \beta \), which is the product of the precision of the speculators’ private signals (\( \gamma \)) and the precision of the random supply shock (\( \beta \)). The stock price becomes almost uninformative, when either \( \gamma \to 0 \) or \( \beta \to 0 \). However, the investment-price sensitivity is rather different in these two asymptotic cases.

First, consider the case that \( \gamma \to 0 \). It follows from equation (21) that the investment-price sensitivity is bounded away from 0. Formally,

\[
\frac{\partial I^*}{\partial P} \to \exp \left( -v_0 - \frac{1}{2\eta} \right) \frac{\beta}{1 + 2\beta} > 0.
\]

However, when \( \beta \to 0, \partial I^*/\partial P \to 0 \).

We next turn to monotonicities of the investment-price sensitivity in the price informativeness. The analytic results confirm our conjecture and the numerical results presented in Section 4.2 and is summarized in Lemma 4 below.

**Lemma 4** Given the investment example in equation (20), the investment-price sensitivity has the
following properties:

1. When \( z < v_0 + 1/(2\eta) \), \( \partial I^*/\partial P \) increases in both \( \gamma \) and \( \beta \);

2. When \( z > v_0 + 1/(2\eta) \),
   - \( \partial I^*/\partial P \) decreases in \( \gamma \) when \( \beta > \bar{\beta} \);
   - \( \partial I^*/\partial P \) always first increases and eventually decreases in \( \beta \);

3. \( \partial I^*/\partial P \) strictly decreases in \( z \).

where \( \bar{\beta} \) is a constant defined in the appendix.

These results confirm the intuition in Section 4.2. The general pattern is that when \( z \) is large relative to \( v_0 \), increases in either \( \gamma \) or \( \beta \) increases the marginal speculator posterior about the productivity \( v \). As she becomes more optimistic, her estimate of the marginal ROI at the investment level \( I^* \) also increases, which further amplifies the anticipation effect.

It is also easy to verify that \( \frac{\partial I^*/\partial \sigma^2 \partial I^*/\partial \mu}{\partial I^*/\partial \gamma \partial I^*/\partial \beta} = \frac{1}{2} \). Hence, \( \frac{\partial I^*/\partial \sigma^2 \partial I^*/\partial \mu}{\partial I^*/\partial \gamma \partial I^*/\partial \beta} \) is bounded. Moreover, since the ratio is a constant, \( z = z = v_0 + (2\eta)^{-1} \) (tighter bounds than characterized in Proposition 4). Thus, Proposition 4 holds:

\[
\frac{\partial I^*}{\partial \gamma \beta} \begin{cases} > 0, & \text{when } z > v_0 + (2\eta)^{-1}; \\ < 0, & \text{when } z < v_0 + (2\eta)^{-1}. \end{cases} \tag{22}
\]

Finally, we verify that Lemma 1 holds. That is, the manager benefits from the price signal.\(^10\) The ex-ante expected payoff of the manager if she learns is

\[
E_z \left[ \frac{1}{2} I^{*2} \right] = E_z \left[ \frac{1}{2} \exp \left( 2\eta v_0 + \gamma \beta z + 1/2 \right) \right] \\
= \frac{1}{2} \exp \left( 2v_0 + 2\eta - \frac{1}{\eta + \gamma \beta} \right)
\]

Given the above expression, the ex-ante payoff is strictly increasing in \( \tau_z = \gamma \beta \). Thus, the manager has incentives to learn from the price signal.

\(^{10}\) We find that in the literature on the feedback effect between the financial market and the corporate decisions, very few works seriously check this property.
7 Manager’s Private Signal

In this section, we allow the manager to possess her own private signal. We then solve the equilibrium and show that main results from the base model still hold.

Denote \( y = v + \epsilon \) the private signal that the manager has, where \( \epsilon \sim \mathcal{N}(0, \theta^{-1}) \). We first characterize the equilibrium. The signal that can be inferred from the price is again assumed to be \( z = g(P) = v + \xi / \sqrt{\eta} \), which will be confirmed later.

The manager’s optimization problem now becomes

\[
\max_I \Pi(I, \mu_{v|y,z}, \sigma_{v|y,z}) - \Delta(I), \quad (23)
\]

where \( \mu_{v|y,z} = \frac{\eta v_0 + \theta v + \gamma \beta z}{\eta + \theta + \gamma \beta} \) and \( \sigma_{v|y,z} = (\eta + \theta + \gamma \beta)^{-1} \) jointly characterize the manager’s posterior belief. By Assumption 1, the optimal investment, denoted by \( I^m(\mu_{v|y,z}, \sigma_{v|y,z}) \), is determined through the first-order condition:

\[
\frac{\partial \Pi(I, \mu_{v|y,z}, \sigma_{v|y,z})}{\partial I} = \frac{\partial \Delta(I)}{\partial I}. \quad (24)
\]

From the manager’s perspective, conditioning on his own private signal and the price signal is equivalent to updating his belief first with the private signal and then learning from the price based on newly updated the prior belief \((\mu_{v|y}, \sigma_{v|y})\). Thus, the managerial learning incentive can be characterized in the same way as in Lemma 1.

Speculators use the same cutoff trading strategies as in our core model. Then speculator \( i \)’s expected payoff from buying one unit of the shares given his information set is:

\[
E[\pi(v, I^m) - P|s_i, z] = E[E[\pi(v, I^m) - P|y, s_i, z]|s_i, z] = E[\Pi(I^m, \mu_{v|y,s_i,z}, \sigma_{v|y,s_i,z})|s_i, z] - P
= \int_{-\infty}^{+\infty} \Pi(I^m, \mu_{v|y,s_i,z}, \sigma_{v|y,s_i,z})f(y|s_i, z)dy - P, \quad (25)
\]

where \( f(y|s_i, z) \) is the pdf of the distribution of \( y \) conditional on \( s_i \) and \( z \). Speculator \( i \) essentially uses his own information to predict the manager’s private signal, estimates the expected payoff for each given realization of \( y \), and then averages all the expected payoffs across the
support of $y$ conditional on $s_i$ and $z$. $\mu_{v|y,s_i,z}$ and $\sigma_{v|y,s_i,z}$ characterizes the speculator’s posterior for a given realization of $y$.

Note that $\Pi(I^m, \mu_{v|y,s_i,z}, \sigma_{v|y,s_i,z})$ increases in $s_i$ (since it is increasing in $\mu_{v|y,s_i,z}$). Moreover, an increase in $s_i$ shifts the conditional distribution $f(y|s_i, z)$ to the right. By the first-order stochastic dominance, equation (25) increases in $s_i$. In other words, the speculator’s expected payoff increase with their private signal $s_i$.

The indifference condition is now

$$\int_{-\infty}^{+\infty} \Pi(I^m, \mu_{v|y,s_i=z}, \sigma_{v|y,s_i=z}) f(y|s_i=z) dy = P = 0. \quad (26)$$

Using the same argument as before, the right-hand side of equation (26) strictly increases in $g$. Therefore, Lemma 2 holds. For any $g \in \mathbb{R}^+$, the market price $P$ exists and is uniquely determined by (26) by plugging in $g(P) = z = v + \xi / \sqrt{\gamma}$, which is derived from the same market clearing condition as in the base model:

$$P = \int_{-\infty}^{+\infty} \Pi(I^m, \mu_{v|y,s_i=z}, \sigma_{v|y,s_i=z}) f(y|s_i=z) dy \quad (27)$$

We summarize the results and characterize the equilibrium in Proposition 5

**Proposition 5** Under Assumption 1, the model that allows the manager to possess private signals has a monotone equilibrium, in which

1. the optimal investment $I^*$ is the unique solution to the equation (24);

2. speculators employ a symmetric cutoff trading strategy, described in equation (7), with the threshold $g(P)$ uniquely determined by equation (26); and

3. the market price $P$ is uniquely determined by equation (27).

Our next set of results show that Proposition 2 and Proposition 4 hold.

**Proposition 6** In the equilibrium characterized by Proposition 5,

$$\frac{\partial I^*}{\partial P} \rightarrow \begin{cases} c, & \text{as } \gamma \to 0 \\ 0, & \text{as } \beta \to 0 \end{cases},$$

where $c > 0$ is a constant.
This result once again demonstrates that the information wedge has far-reaching implications on the equilibrium relation between the manager’s action and the stock price. And its effect is robust to alternative information structures for the manager, even when the manager departs from rational Bayesian updating. For example, we can alter the interpretation of $y$ and assume that its subjective precision conditional on $v$ is $\theta$ but its objective precision is only $\theta_1 < \theta$. This scenario essentially models the managerial overconfidence, and the equilibrium still exists and the investment-price relation still depends differentially on $\gamma$ and $\beta$.

As pointed earlier, the manager’s inference can be considered as learning from the price with the prior $(\mu_{v|y}, \sigma_{v|y})$. With this treatment, the direct effect of the price informativeness $\gamma \beta$ on the investment decision $I^*$ can be characterized in the similar fashion.

**Proposition 7** If the ratio $\frac{\partial^2 \Pi(I, \mu, \sigma)}{\partial I \partial \mu}$ is bounded. Then, for the investment $I^*$ determined by (24), there exists $z$ and $\bar{z}$ (with $\bar{z} > \mu_{v|y} > z$)

$$\frac{\partial I^*}{\partial \gamma \beta} \begin{cases} > 0, & \text{when } z > \bar{z} \\ < 0, & \text{when } z < \bar{z} \end{cases}$$

(28)

**8 Conclusion**

This paper contributes to the fast growing literature on the feedback between the financial market and corporate decisions. We develop a general theory, in which both the manager and the speculators learn from the stock price, which partially aggregates the dispersed information in the financial market. We uncover several surprising equilibrium properties of the decision-price sensitivity. First, when the stock price becomes almost uninformative due to the almost uninformative private signals of the speculators, the manager barely learns from the stock price, but the decision-price sensitivity remains significant. In contrast, if it is the extremely volatile random supply shock that leads to the almost uninformative stock price, the decision-price sensitivity is insignificant. Second, the decision-price sensitivity is not monotonic in the price informativeness, and its monotonicity in the precision of the speculators’ private signals differs qualitatively from that in the precision of the random supply shock.

These surprising equilibrium properties are due to the difference in the manager’s learning and the marginal speculator’s learning, even though the manager and the marginal speculator observe the same signal realization. Such a difference, which we call the information wedge, determines the asymptotic behavior of the decision-price sensitivity. In particular, when the
speculators’ private signals become almost uninformative, the information wedge diminishes, and thus the managerial learning effect and the marginal speculator’s learning effect cancel each other in determining the decision-price sensitivity. But when the random supply shock becomes extremely volatile, the information wedge becomes infinitely large, and so the managerial learning effect dominates. Then, since the managerial learning effect becomes trivial in this case, the decision-price sensitivity is insignificant.

In addition, because the stock price is determined by the marginal speculator’s indifference condition, it is not equivalent to the price signal. Therefore, the decision-price sensitivity is not monotonic in the price informativeness. Since a change in the precision of the speculators’ private signals and a change in the volatility of the random supply shock have different effects on the price formation process, they have asymmetric effects on the decision-price sensitivity.

We then propose the direct effect of the price informativeness on the corporate decision as a new measure of the financial market’s real effect. With such a new measure, empirical studies can consider the price informativeness effect without identifying the reason for the variation of the price informativeness. In addition, we provides conditions by which the sign of the direct effect of the price informativeness on the corporate decision can be characterized.

Our paper has both theoretical and applied contributions. From the theoretical perspective, we provides a rather general model about the feedback effect between the financial market and corporate decisions. Our analysis does not rely on an explicit equilibrium pricing function; instead, we analyze the equilibrium properties based only on the implicit equilibrium characterization. Our conclusions also show the importance of a formal model of the trading in the financial market, because simply assuming the price equal to the firm value based on the price signal or treating the price as an exogenous public signal will provide biased results and incomplete economic intuitions. From the empirical perspective, we argue that the decision-price sensitivity may not be a pure measure of the feedback effect, because the testing results will then capture the marginal speculator’s learning effect. Also, employing the decision-price sensitivity to measure the feedback effect, we need to identify the reason for the variation of the price informativeness. Therefore, we propose a new measure of the feedback effect — the direct effect of the price informativeness on the corporate decisions.

One interesting extension is when the speculators are risk-averse. Unfortunately, the methods developed in this paper do not apply to such a model; and it seems that one needs to solve the explicit pricing function for the equilibrium analysis. Therefore, we leave it for the future studies.
Omitted Proofs

In this section, we present the proofs.

Proof of Lemma 1. Given that the \( v \) is normal conditional on \( P \), we will use a sufficient statistic \( z \) for \( P \) to simplify the proof below. The existence and uniqueness of \( z \) is proved later on. We can calculate the manager’s ex-ante expected payoff as follows by treating the posterior mean \( \mu_{v|P} \sim N'(\bar{v}, \sigma) \) as the random variable over which the expectation is taken:

\[
\mathbb{E} \left[ \Pi(I^*, \mu_{v|P}, \sigma_{v|P}) - \Delta(I^*) \right] = \int_{-\infty}^{+\infty} (\Pi(I^*, \mu, \sigma_{v|P}) - \Delta(I^*)) \phi(x, v_0, \sigma_{\mu}) \, d\mu,
\]

where \( \sigma^2_{\mu} = \sigma^2_v - \sigma^2_{v|P} \) by the law of total variance. Take derivative of the above equation w.r.t. \( \sigma_{v|P} \) and use the envelope theorem, we have

\[
\frac{\partial}{\partial \sigma_{v|P}} \int_{-\infty}^{+\infty} (\Pi(I^*, \mu, \sigma_{v|P}) - \Delta(I^*)) \phi(x, v_0, \sigma_{\mu}) \, d\mu = \int_{-\infty}^{+\infty} \frac{\partial \Pi(I^*, \mu, \sigma_{v|P})}{\partial \sigma_{v|P}} \phi(x, v_0, \sigma_{\mu}) \, d\mu + \int_{-\infty}^{+\infty} (\Pi(I^*, \mu, \sigma_{v|P}) - \Delta(I^*)) \frac{\partial \phi(x, v_0, \sigma_{\mu})}{\partial \sigma_{\mu}} \frac{\partial \sigma_{\mu}}{\partial \sigma_{v|P}} \, d\mu \tag{A.1}
\]

Since \( \Pi(I, \mu, \sigma) \) is decreasing in \( \sigma \), the first part in the equation (A.1) is negative. Moreover, note that \( \sigma_{\mu} \) is decreasing in \( \sigma_{v|P} \). Combined with the convexity of \( \Pi(I, \mu, \sigma) \) in \( \sigma \) and applying the second order stochastic dominance \( \frac{\partial^2 \phi(x, v_0, \sigma_{\mu})}{\partial \sigma_{\mu}^2} \) is effectively a mean-preserving spread of \( \mu \) and \( (\Pi(I^*, \mu, \sigma_{v|P}) - \Delta(I^*)) \) is a convex function of \( \mu \), the second part is also negative. ■

Proof of Lemma 2. Given equation 12, we only need to show that the its RHS is strictly increasing in \( z \) (or \( g(P) \)).

\[
\frac{\partial \Pi(I^*, \mu_{v|\sigma_0=s_z}, \sigma_{v|z})}{\partial z} = \frac{\partial \Pi}{\partial I^*} \frac{\partial I^*}{\partial \mu_{v|z}} \frac{\partial \mu_{v|z}}{\partial z} + \frac{\partial \Pi}{\partial \mu_{v|z}} \frac{\partial \mu_{v|z}}{\partial z},
\]

where

\[
\frac{\partial I^*(\mu_{v|z}, \sigma_{v|z})}{\partial \mu_{v|z}} = -\frac{\partial^2 \Pi}{\partial \mu_{\sigma}^2} > 0,
\]

\[
\frac{\partial \Pi}{\partial I^*} > 0, \quad \frac{\partial \Pi}{\partial \mu_{v|z}} > 0, \quad \frac{\partial \mu_{v|z}}{\partial z} = \frac{\gamma \beta}{\eta + \gamma \beta} > 0, \quad \text{and} \quad \frac{\partial \mu_{v|z}}{\partial z} = \frac{\eta \beta + \gamma}{\eta + \gamma \beta + \gamma} > 0. \tag{A.2}
\]

Proof of Proposition 1. To prove the existence of the equilibrium, we simply show that the proposed
optimal investment strategy in (6), the cutoff trading strategy in (7) and (11), and the market price in (12) indeed consist of an equilibrium and satisfy Definition 1. Note that we do not establish the uniqueness of the equilibrium. There could exist other equilibria under which the speculators employ complicated non-linear trading strategies. We do, however, show that the equilibrium is unique when the speculators follow cutoff trading strategies.

We start with the proposed pricing function in (12). With this price, we next demonstrate that the characterized optimal investment, optimal trading strategy exist, are unique, and satisfy the manager’s as well as the speculators’ maximization problems. Last, with the trading strategies, the market clears and yield the pricing function as we initially proposed, thus completing the proof.

First, under Assumption 1, the manager’s objective function is single peaked and its global maximum can be characterized by its FOC as in (6) for any given information set.

We used the proposed pricing function to derive the optimal investment $I^\ast$. Next, we confirm that, with $I^\ast$, investors follow the cutoff strategies characterized by $g(P)$, which leads to the pricing function as we initially proposed. Investor $i$’s expected payoff from buying one unite of the share given its information set is

$$E[\pi(v, I^\ast)|s_i, P] = \Pi(I^\ast, \mu_{v|s_i, P}, \sigma_{v|s_i, P}) - P.$$  

Assumption 1 guarantees that this function is increasing in $s_i$ (since $\mu_{v|s_i, P}$ is strictly increasing in $s_i$). In other words, investor $i$’s profit of holding one share of the stock is increasing in his signal $s_i$. It thus supports the cutoff strategies proposed in (7).

However, we have not so far established the existence of $g(P)$ for a given $P$. $g(P)$ is determined through the indifference condition in (11)

$$\Pi(I^\ast, \mu_{v|s_i=g(P), P}, \sigma_{v|s_i, P}) = P.$$  

The LHS of the above equation is strictly increasing in $g(P)$. If $\Pi(I^\ast, \mu, \sigma_{v|s_i, P})$ is unbounded in $\mu$, then the solution for $g(P)$ for a given $P$ exists and is unique. If $\Pi(I^\ast, \mu, \sigma_{v|s_i, P})$ is bounded, then $P$ is also bounded by the same boundaries as evident from (12). Then by the monotonicity, the solution of $g(P)$ still exists and is unique.

The last step to complete the proof is to derive the pricing function as in (12), which is followed
directly from (8), (9), and (11).

\[ \text{Proof of Proposition 2.} \text{ Using (11), } P = \Pi (I^*, \mu_{v|s_i=z, z}, \sigma_{v|s_i=z, z}) \text{, we have} \]

\[ \frac{\partial P}{\partial z} = \frac{\partial \Pi (I^*, \mu_{v|s_i=z, z}, \sigma_{v|s_i=z, z})}{\partial I^*} \frac{\partial I^* (\mu_{v|z}, \sigma_{v|z})}{\partial z} + \frac{\partial \Pi (I^*, \mu_{v|s_i=z, z}, \sigma_{v|s_i=z, z})}{\partial \mu_{v|s_i=z, z}} \frac{\partial \mu_{v|s_i=z, z}}{\partial z}. \]  

(A.2)

Rearrange,

\[ \frac{\partial I^*}{\partial P} = \left[ \frac{\partial \Pi (I^*, \mu_{v|s_i=z, z}, \sigma_{v|s_i=z, z})}{\partial I^*} + \frac{\partial \Pi (I^*, \mu_{v|s_i=z, z}, \sigma_{v|s_i=z, z})}{\partial \mu_{v|s_i=z, z}} \frac{\partial \mu_{v|s_i=z, z}}{\partial I^*} \right]^{-1}. \]  

(A.3)

Recall that \( z = v + 1/\sqrt{\gamma} \epsilon \) is the price signal. The precision of \( z \) is \( \gamma \beta \), which is the product of the precision of the speculators’ private signal \( \gamma \) and the precision of the supply shock \( \beta \). Prices become pure noises when either \( \gamma \to 0 \) or \( \beta \to 0 \). However, feedback effects are rather different under these limiting cases.

First, consider the asymptotic when \( \gamma \to 0 \). \( \Pi \) is a smooth function, therefore

\[ \frac{\partial \Pi (I^*, \mu_{v|s_i=z, z}, \sigma_{v|s_i=z, z})}{\partial I^*} \to \frac{\partial \Pi (I^* (v_0, 1/\sqrt{\eta}), v_0, 1/\sqrt{\eta})}{\partial I^*} = \frac{\partial \Delta (I^* (v_0, 1/\sqrt{\eta}), v_0, 1/\sqrt{\eta})}{\partial I^*} > 0, \]

where the equality follows from the manager’s optimization problem and Assumption 1, and

\[ \frac{\partial \Pi (I^*, \mu_{v|s_i=z, z}, \sigma_{v|s_i=z, z})}{\partial \mu_{v|s_i=z, z}} \to \frac{\partial \Pi (I^*, \mu, \sigma)}{\partial \mu} \bigg|_{I^*=I^* (v_0, 1/\sqrt{\eta}), \mu=v_0, \sigma=1/\sqrt{\eta}} > 0. \]

Moreover, \( \frac{\partial \mu_{v|s_i=z, z}}{\partial I^*} = \frac{\partial \mu_{v|s_i=z, z}}{\partial I^* (\mu_{v|s_i=z, z}, \sigma_{v|s_i=z, z})/\partial z} \), where

\[ \frac{\partial \mu_{v|s_i=z, z}}{\partial z} = \frac{\gamma + \gamma \beta}{\eta + \gamma + \gamma \beta} \to 0, \]

and

\[ \frac{\partial I^* (\mu_{v|z}, \sigma_{v|z})}{\partial z} = \frac{\partial I^* (\mu_{v|z}, \sigma_{v|z})}{\partial \mu_{v|z}} \frac{\partial \mu_{v|z}}{\partial z} = \frac{\partial I^* (\mu_{v|z}, \sigma_{v|z})}{\partial \mu_{v|z}} \frac{\gamma \beta}{\eta + \gamma} \to 0. \]
Thus,

\[
\frac{\partial \mu_{v|s_i=z,z}}{\partial I^*} = \frac{\gamma + \gamma \beta}{\eta + \gamma + \gamma \beta} \frac{\partial I^*(\mu_{v|s_i},\sigma_{v|s_i})/\partial \mu_{v|p}}{\partial I^*(v_0,1/\sqrt{\eta})/\partial v_0} > 0. \quad (A.4)
\]

It then follows, using (A.2),

\[
\lim_{\gamma \to 0} \frac{\partial I^*}{\partial P} = \left[ \frac{\partial \Delta(I^*(v_0,1/\sqrt{\eta}),v_0,1/\sqrt{\eta})}{\partial I^*} + \frac{1 + \beta}{\beta} \frac{\partial \Pi(I^*(v_0,1/\sqrt{\eta}),v_0,1/\sqrt{\eta})}{\partial v_0} \right]^{-1} > 0. \quad (A.5)
\]

When \( \beta \to 0 \), the only difference is \( \partial \mu_{v|s_i=z,z}/\partial z \) in (A.4), which now approaches +\( \infty \) as \( \beta \) approaches 0. Consequently,

\[
\lim_{\beta \to 0} \frac{\partial I^*}{\partial P} = 0.
\]

\[\blacksquare\]

**Proof of Proposition 3.** If the manager does not learn from the stock market, the optimal investment will be determined by (6) with \( \mu = v_0 \) and \( \sigma = 1/\sqrt{\eta} \). Hence, \( \partial I^*/\partial \gamma \beta = 0 \) for all \( \gamma \) and \( \beta \). The price will be a sideshow and is given by

\[
P = E[\pi(v, I^*(\mu_v, \sigma_v))] = \Pi(I^*(\mu_v, \sigma_v), \mu_{v|s_i=z,z}, \sigma_{v|s_i=z,z})
\]

Therefore, \( \partial z/\partial P \) is well defined and it is positive for all \( \gamma \) and \( \beta \). Taken together, the investment-price sensitivity \( \frac{\partial I^*}{\partial P} = \frac{\partial I^*}{\partial z} \) is 0 \( \forall \gamma > 0, \beta > 0 \). \[\blacksquare\]

**Proof of Proposition 4.** For convenient expositions, we express the optimal investment as a function of posterior mean and posterior variances.

\[
\frac{\partial I^*(\mu_{v|z}, \sigma_{v|z}^2)}{\partial \gamma \beta} = \frac{\partial I^*(\mu_{v|z}, \sigma_{v|z}^2)}{\partial \mu_{v|z}} \frac{\mu_{v|z}}{\partial \gamma \beta} + \frac{\partial I^*(\mu_{v|z}, \sigma_{v|z}^2)}{\partial \sigma_{v|z}^2} \frac{\sigma_{v|z}^2}{\partial \gamma \beta} = \frac{\partial I^*(\mu_{v|z}, \sigma_{v|z}^2)}{\partial \mu_{v|z}} \left[ \frac{\eta(z-v_0)}{((\eta + \gamma \beta)^2) - 1} \right].
\]

Given the above expression, the sign is determined by \( \eta(z-v_0) - \frac{\partial I^*/\partial \sigma_{v|z}^2}{\partial I^*/\partial \gamma}. \) With the assumption in
Proposition 4, let \( l \) and \( \bar{l} \) be the lower bound and upper bound of \( \frac{\partial l}{\partial \beta} = \frac{\partial \bar{l}}{\partial \beta} \). It then follows that \( \frac{\partial l}{\partial \beta} > 0 \) (\( < 0 \)) if \( z > \bar{z} = \bar{l}/\eta + v_0 \) (\( z < \bar{z} = l/\eta + v_0 \)).

**Proof of Proposition 6.** Differentiating equation (26) w.r.t. \( z \), we get

\[
\frac{\partial P}{\partial z} = \int_{-\infty}^{+\infty} \Pi(I^m, \mu_{v|y,z_1 = z, z}, \sigma_{v|y,z_1 = z}) \frac{\partial f(y|z_i = z, z)}{\partial z} dy \\
+ \int_{-\infty}^{+\infty} \frac{\partial \Pi(I^m, \mu_{v|y,z_1 = z, z}, \sigma_{v|y,z_1 = z})}{\partial I^m} \frac{\partial I^m}{\partial \mu_{v|y,z}} f(y|z_i = z, z) dy \\
+ \int_{-\infty}^{+\infty} \frac{\partial \Pi(I^m, \mu_{v|y,z_1 = z, z}, \sigma_{v|y,z_1 = z})}{\partial \sigma_{v|y,z}} \frac{\partial \sigma_{v|y,z}}{\partial \mu_{v|y,z}} f(y|z_i = z, z) dy 
\]

(A.6)

Simplify each component,

\[
A = \int_{-\infty}^{+\infty} \Pi(I^m, \mu_{v|y,z_1 = z, z}, \sigma_{v|y,z_1 = z}) \sigma_{v|y,z_1 = z}^{-2} (y - \mu_{y|z_1 = z}) \frac{\gamma + \gamma \beta}{\eta + \gamma \beta} f(y|z_i = z, z) dy 
\]

(A.7)

\[
B = \int_{-\infty}^{+\infty} \frac{\partial \Pi(I^m, \mu_{v|y,z_1 = z, z}, \sigma_{v|y,z_1 = z})}{\partial I^m} \frac{\partial I^m}{\partial \mu_{v|y,z}} \frac{\gamma \beta}{\eta + \gamma \beta} f(y|z_i = z, z) dy 
\]

(A.8)

\[
C = \int_{-\infty}^{+\infty} \frac{\partial \Pi(I^m, \mu_{v|y,z_1 = z, z}, \sigma_{v|y,z_1 = z})}{\partial \sigma_{v|y,z}} \frac{\partial \sigma_{v|y,z}}{\partial \mu_{v|y,z}} \frac{\gamma \beta + \gamma}{\eta + \gamma + \gamma \beta} f(y|z_i = z, z) dy 
\]

(A.9)

Further notice that

\[
\frac{\partial I^m(\mu_{v|y,z}, \sigma_{v|y,z})}{\partial z} = \frac{\partial I^m(\mu_{v|y,z}, \sigma_{v|y,z})}{\partial \mu_{v|y,z}} \frac{\gamma \beta}{\eta + \gamma + \gamma \beta} 
\]

(A.10)

Therefore,

\[
\frac{\partial P}{\partial I^m} = \frac{\partial P}{\partial z} \frac{\partial z}{\partial I^m} = \left[ \frac{\partial I^m(\mu_{v|y,z}, \sigma_{v|y,z})}{\partial \mu_{v|y,z}} \right]^{-1} \times \\
\int_{-\infty}^{+\infty} \left[ A_1 \frac{1 + \beta}{\beta} \frac{\eta + \gamma + \gamma \beta}{\eta + \gamma + \gamma \beta} + B_1 B_2 + C_1 \frac{1 + \beta}{\beta} \frac{\eta + \gamma + \gamma \beta}{\eta + \gamma + \gamma \beta} \right] f(y|z_i = z, z) dy 
\]

(A.11)
where equation (A.12) follows from the first-order stochastic dominance (A.11), it follows that the investment-price sensitivity
\[ \frac{\partial I^m}{\partial w|y,\sigma^2|y,z} = \frac{\partial I^m(w|y,\sigma^2|y)}{\partial w} > 0 \]

Then, as \( \gamma \to 0 \)

\[ \lim_{\gamma \to 0} \int_{-\infty}^{+\infty} \frac{1 + \beta \eta + \theta + \gamma \beta}{\beta} \frac{\partial \prod(y,\mu_y|y)}{\partial \mu_y} \prod(y,\mu_y|y) \phi(y,\mu_y,\sigma_y)dy > 0 \]

\[ \lim_{\gamma \to 0} \int_{-\infty}^{+\infty} B_1 B_2 f(y|s_i = z,\xi)dy \]

\[ \lim_{\gamma \to 0} \int_{-\infty}^{+\infty} C_1 \frac{1 + \beta \eta + \theta + \gamma \beta}{\beta} \frac{\partial \prod(y,\mu_y|y)}{\partial \mu_y} \prod(y,\mu_y|y) \phi(y,\mu_y,\sigma_y)dy > 0, \] (A.12)

(A.13)

where equation (A.12) follows from the first-order stochastic dominance (\( \prod(I^m,\mu_y|y,\sigma^2|y) \)) is an increasing function of \( y \) by Assumption 1 and is independent of \( \mu_y \). Plug the above results into equation (A.11), it follows that the investment-price sensitivity \( \partial I^m/\partial P \) approaches a positive constant as \( \gamma \to 0 \).

Now, consider the case that \( \beta \to 0 \).

\[ \lim_{\beta \to 0} \frac{\partial I^m}{\partial w|y,\sigma^2|y,z} = \frac{\partial I^m(w|y,\sigma^2|y)}{\partial w} > 0 \]

\[ \lim_{\beta \to 0} \int_{-\infty}^{+\infty} A_1 \frac{1 + \beta \eta + \theta + \gamma \beta}{\beta} \frac{\partial \prod(I^m|y,\mu_y|y,\sigma^2|y)}{\partial \mu_y} \prod(I^m|y,\mu_y|y,\sigma^2|y) \phi(y,\mu_y,\sigma_y)dy = +\infty \]

\[ \lim_{\beta \to 0} \int_{-\infty}^{+\infty} B_1 B_2 f(y|s_i = z,\xi)dy \]

\[ \lim_{\beta \to 0} \int_{-\infty}^{+\infty} C_1 \frac{1 + \beta \eta + \theta + \gamma \beta}{\beta} \frac{\partial \prod(I^m|y,\mu_y|y,\sigma^2|y)}{\partial \mu_y} \prod(I^m|y,\mu_y|y,\sigma^2|y) \phi(y,\mu_y,\sigma_y)dy = \Delta \] (A.14)

\[ \lim_{\beta \to 0} \int_{-\infty}^{+\infty} C_1 \frac{1 + \beta \eta + \theta + \gamma \beta}{\beta} \frac{\partial \prod(I^m|y,\mu_y|y,\sigma^2|y)}{\partial \mu_y} \prod(I^m|y,\mu_y|y,\sigma^2|y) \phi(y,\mu_y,\sigma_y)dy = +\infty, \]
where $\Delta$ is a finite constant. The above results indicate that real effects measured by investment-price sensitivity converges to 0 when $\beta \to 0$. ■

**Proof of Lemma 4.** Differentiate (21) w.r.t. $\gamma$ and simplify. Its sign is determined by the following linear function in $\gamma$: $a\gamma + b$, where

$$a = 2\eta + 2\beta^2(2\eta v_0 - 2\eta z + 1) + 2\beta(\eta + 2\eta v_0 - 2\eta z + 1) \quad (A.15)$$

$$b = \eta + 2\beta \eta (2\eta v_0 - 2\eta z + 1) + 2\eta^2(v_0 - z + 1) \quad (A.16)$$

When $z < v_0 + (2\eta)^{-1}$, then $a > 0$ and $b > 0$ for any $\beta > 0$. Then the investment-price sensitivity strictly increases in $\gamma$.

When $z > v_0 + 1/(2\eta)$, $a$ becomes a quadratic function in $\beta$. This quadratic function is positive when $\beta$ approaches zero and is concave in $\beta$. It has exactly one positive root, denoted by $\beta_0$. On the other hand, $b$ is downward slopping linear function in $\beta$. Let $\beta_1$ be such that $b < 0$ for $\beta > \beta_1$. Then, for any $\beta > \bar{\beta} \equiv \max\{\beta_0, \beta_1\}$, $a < 0$ and $b < 0$; thus, investment-price sensitivity decreases in $\gamma$.

Differentiate (21) w.r.t. $\beta$ and simplify. Its sign is determined by the following cubic function in $\beta$: $a\beta^3 + b\beta^2 + c\beta + d$, where

$$a = 2\gamma^2(2\eta v_0 - 2\eta z + 1) \quad (A.17)$$

$$b = 2\gamma^3(2\eta v_0 - 2\eta z + 1) + \eta(2\eta v_0 - 2\eta z + 1) \quad (A.18)$$

$$c = \gamma^2\gamma(6\gamma + 2\eta(v_0 - z + 3) + 1) \quad (A.19)$$

$$d = 2\eta(\gamma + \eta)^2 \quad (A.20)$$

When $z < v_0 + (2\eta)^{-1}$, $a > 0$, $b > 0$, and $c > 0$. So the investment-price sensitivity is strictly increasing in $\beta$.

When $z > v_0 + 1/(2\eta)$, $a < 0$ and $d > 0$. So the cubic function is positive when $\beta$ is small and is negative when $\beta$ is large.

Differentiate (21) w.r.t. $z$ and simplify:

$$-\frac{\beta(\beta + 1)\gamma}{2\beta^2\gamma + 2\beta(\gamma + \eta) + \eta} \exp\left(-\frac{2\eta v_0 + 2(\beta + 1)\gamma z + 1}{2(\beta \gamma + \gamma + \eta)}\right) < 0$$

■
Proof of Lemma 3. Differentiate (17) w.r.t. $\gamma\beta$ and suppress the subscripts of $\mu_{v|z}$ and $\sigma_{v|z'}^2$.

$$\frac{\partial^2 I^*}{\partial \mu \partial \gamma \beta} = \left( \frac{\partial^2 I^*}{\partial \mu^2} \frac{(z - v_0)\eta}{(\eta + \gamma \beta)^2} - \frac{\partial^2 I^*}{\partial \mu \partial \sigma^2} \frac{1}{(\eta + \gamma \beta)^2} \right) \frac{\gamma \beta}{\eta + \gamma \beta} + \frac{\partial I^*}{\partial \mu} \frac{\eta}{(\eta + \gamma \beta)^2} \tag{A.21}$$

Suppose that $I^*$ is convex in $\mu$, i.e., $\partial^2 I^*/\partial^2 \mu > 0$. (A.21) is negative if

$$z - v_0 < \frac{\partial^2 I^*/\partial \mu \partial \sigma^2}{\partial^2 I^*/\partial \mu^2} \frac{1}{\eta} - \frac{\partial I^*/\partial \mu}{\partial^2 I^*/\partial \mu^2} \frac{\eta + \gamma \beta}{\gamma \beta}$$

Since both $\frac{\partial^2 I^*/\partial \mu \partial \sigma^2}{\partial^2 I^*/\partial \mu^2}$ and $\frac{\partial I^*/\partial \mu}{\partial^2 I^*/\partial \mu^2}$ are bounded, and $\frac{\eta + \gamma \beta}{\gamma \beta}$ is also bounded for $\gamma \beta \in [c, +\infty]$ for any $c > 0$. Hence, (A.21) can be satisfied for $\gamma \beta \in (c, +\infty)$. The proof is similar when $I^*$ is concave in $\mu$. ■
References


36


