Index Investing, Market Risk Premium, and Capital Allocation∗

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February 17, 2019

Abstract

We study the equilibrium effects of the rise of indexing on market risk premium, capital allocation, and stock correlations. We show that these effects critically depend on the causes of the rise of indexing, the cost structure of information acquisition, and whether some investors can endogenously choose between active investors and indexers. For example, if the rise of indexing is due to an increase in the exogenous indexers, then the price informativeness in the index market decreases and the market risk premium increases, opposite to the finding of Bond and Garcia (2017). In addition, expected market capitalizations, variances of stock prices, stock price correlations, and the social welfare tend to decrease. In contrast, if the rise of indexing comes from a decrease in the participation cost in the index market, then the price informativeness in the index market goes up and the market risk premium goes down, while both the expected capital allocated to stocks and the social welfare increase. Our analysis provides testable implications that can help identify the driving force of the rise of indexing.

JEL Classification Codes: G11, G12, G14, D82.

Keywords: Indexing, Information Acquisition, Risk Premium, Capital Allocation, Correlations, Welfare.

∗We thank participants in the seminars at Fudan University, SUFE, SWUFE and Washington University in St. Louis for helpful comments.
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Abstract

We study the equilibrium effects of the rise of indexing on market risk premium, capital allocation, and stock correlations. We show that these effects critically depend on the causes of the rise of indexing, the cost structure of information acquisition, and whether some investors can endogenously choose between active investors and indexers. For example, if the rise of indexing is due to an increase in the exogenous indexers, then the price informativeness in the index market decreases and the market risk premium increases, opposite to the finding of Bond and Garcia (2017). In addition, expected market capitalizations, variances of stock prices, stock price correlations, and the social welfare tend to decrease. In contrast, if the rise of indexing comes from a decrease in the participation cost in the index market, then the price informativeness in the index market goes up and the market risk premium goes down, while both the expected capital allocated to stocks and the social welfare increase. Our analysis provides testable implications that can help identify the driving force of the rise of indexing.

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1. Introduction

Index investing has grown substantially during the past few decades. As a result of such growth, more investors enjoy lower investment fees and more index funds and ETFs save on stock picking costs. However, as such growth continues over the recent years, some major concerns arise in the industry, the media, and the academia. For example, will a significant rise of index investing hurt price discovery and thus reduce market efficiency? Will such rise reduce total capital investment, increase stock price correlation, and thus increase systemic risks? In this paper, we develop an equilibrium model to study how price discovery, capital allocation, and stock correlations changes after a rise of index investing. We show that qualitative and quantitative impact of index investing critically depends on what caused the growth of index investing and the cost structure of information acquisition. Our analysis highlights the importance of identifying the underlying driving force of the rise of index investing for the understanding of its impact and provides empirically testable implications that can help this identification.

In our model, in addition to a risk free asset, investors can also trade two stocks to maximize their expected utility at the end of period. For simplicity of analysis, we directly model the trading of two risky securities, the index and the non-index, whose combinations replicate the two stocks. Trading the risk free asset is free to all investors, but some investors must incur a participation cost (e.g., security learning cost) in order to trade the index and/or the non-index. We consider four types of investors: 1. active investors (“A investors”) who have zero cost for trading the index and the non-index, and thus always trade both risky assets; 2. discretionary investors (“D investors”) who must incur a participation cost to trade the index and a participation cost to trade the non-index. They endogenously choose to be savers who invest only in the risk free asset (“D0 investors”) or be indexers who only trade the index and the risk free asset (“DI investors”) or be active investors who trade all assets (“DA investors”); 3. exogenous indexers (“I investors”) who only trade the index and the risk free asset; and 4. liquidity traders (“L investors”) who must trade both risky assets to hedge their endowment risk. Only active traders and discretionary investors may acquire private information about the fundamental value of the risky assets by paying a cost.

In actual financial markets, most hedge funds can be viewed as “A investors” since they do not invest only in the index. Mutual funds, like “D investors”, can choose to switch to money market funds (“D0 investors”), or become passive funds (“DI investors”), or become active mutual funds (“DA investors”). Most index funds can be viewed as “I investors” who do not actively pick stocks or acquire private

\[1\] During 2016, actively-managed funds experienced $285 billion of outflows while passive funds attracted $429 billion of inflows. The proliferation of ETFs is now approaching 2,000 funds and nearly $3.0 trillion of asset under management.
information. “L investors” can be considered as individual traders or hedgers who trade assets for hedging needs instead of fundamental value of assets.

Our model allows us to study different possible causes of the rise of index investing. To make our main points clearer, we focus on three main possible causes in our analysis: 1. an increase in exogenous indexers; 2. a decrease in the participation cost in the index market; and 3. an increase in transparency of the non-index market. We show that the equilibrium effects of the rise of index investing on price discovery, capital allocation, and stock correlation critically depend on what causes the rise of indexing and how information acquisition by an investor in one market affects his cost of acquiring information in another market (“information acquisition externality” or “IAE” for short).

Consider first the case where the rise of index investment is because some discretionary investors become exogenous indexers for exogenous reasons, such as a significant increase in the participation cost in the non-index market for these investors or a belief that active investing cannot consistently outperform index investing after fees. In this case, because exogenous indexers who do not acquire private information crowd out discretionary indexers who acquire private information, the price informativeness of the index decreases and thus market risk premium increases. As a result, the total capital allocation to risky assets, price variances, and social welfare decrease.

In addition, stock correlations also decrease and can turn negative. Without indexing, the prices of the two stocks are uncorrelated because of the independence of their payoffs and the CARA preferences. When some investors are indexers, the correlation of Stocks 1 and 2 is driven by two opposing forces. First, when shocks are to the payoffs of the stocks, the stock prices tend to move in the same direction and thus exhibit a positive correlation. This is because with indexing, after a good shock to a stock’s payoff, indexers buy more of the stock which drives up the stock price. Because they are indexers, they also buy more of the other stock, and thus also drive up the price of the other stock. The case with a bad shock is similar. Second, when shocks are to the liquidity demands of the liquidity traders, the stock prices tend to move in the opposite directions because as the price of one stock goes down, the exposure of the indexers in the index goes down, and thus the indexers buy more of both stocks to rebalance the position, which drives the price of the other stock up. In short, with indexing, an informational shock tends to cause positive correlation, while a liquidity shock does the opposite. The net effect depends on the relative strength of the two forces.

If the rise of index investing is due to a decrease in the participation cost for trading the index, then price informativeness in the index market increases and thus market risk premium decreases. This is because as more and more investors switch from investing only in the risk-free asset to investing in the index. These new index investors optimally choose to acquire private information, thus the mass of information acquirers increases. As a result, the expected capital allocated to stocks, social
welfare, stock price variances, and stock correlations all increase. This is in sharp contrast with the first case where the rise of indexing is from an increase in the exogenous indexers.

Another possible reason for the rise of index investing is that it becomes less profitable to trade in the non-index market as a result of the improvement in the transparency in the non-index market. When the profitability in the non-index market decreases, more discretionary investors choose to be indexers. Both exogenous active traders and discretionary active traders optimally acquire less precise private information due to reduced profitable opportunities, yet the price informativeness in the non-index market *increases* because of improved non-index market transparency. When more discretionary active traders switch to index investment because of reduced profitability in the non-index market, the price informativeness in the index market increases, the market risk premium decreases, and the expected capital allocations increase if and only if the IAE is negative. This follows from similar intuitions as the previous case. When the IAE is negative, discretionary indexers acquire more precise private information about the index than that of discretionary active investors. As more discretionary investors choose to be indexers, the aggregate precision of private information about the index increases. In addition, the social welfare tends to increase because of the increased transparency of the non-index market.

Our paper highlights that it is important to understand the underlying mechanisms that drive the rise of index investing. These predictions can help researchers use empirically found relationship between the rise of index investment and price informativeness to identify what likely caused the increase in indexing and thus offer regulators some guidance on regulations adjustment if necessary.

The closest work to ours is Bond and Garcia (2017). Different from Bond and Garcia (2017), however, in our model, indexing is endogenous and some indexers can acquire private information about the assets they trade before and after they become indexers. In contrast to our above findings, Bond and Garcia (2017) find that as indexing increases, price informativeness improves, market risk premium declines, and social welfare decreases. The driving force behind the finding of Bond and Garcia (2017) on price informativeness and market risk premium is that all indexers in their model are essentially liquidity traders whose trades in a market only add noise and decrease the price informativeness. After they leave the non-index market, there is less noise trading in this market. As a result, the price informativeness in the non-index market increases, and thus the informed investors acquire less information in the non-index market. With the assumption of a negative IAE in their model, informed investors then acquire more information in the index market, which increases the price informativeness in the index market and lowers the market risk premium. In our model even in the case where we find the same qualitative results, the underlying driving force can be different from theirs. For example, consider the case where a rise of indexing is due to an increase in the participation cost in the non-index market and
the IAE is negative (as in Bond and Garcia (2017)). As reported above, because A and DA investors acquire information in the non-index market which increases their information acquisition cost in the index market, they acquire less precise information in the index market, opposite to the finding of Bond and Garcia (2017). However, as Bond and Garcia (2017), we also find that as indexing increases, the price informativeness in the index market increases and market risk premium decreases. As explained before, the driving force for obtaining these same qualitative results in this case as theirs is that the endogenous indexers (DI investors) obtain more precise information than both exogenous active traders and DA traders in the index market, and as the population of the endogenous indexers increases, the mass of investors who acquire more precise information increases, and thus the price informativeness in the index market increases and the market risk premium decreases.

As for welfare implications of indexing, Bond and Garcia (2017) find that the welfare of indexers decrease with indexing. In contrast, we show that the welfare of indexers may be improved with indexing. The difference in findings comes from the endogenizing of indexing and the consideration of the effect of the causes of indexing. With endogenous indexing, discretionary indexers can share the benefit of less competition in the non-index market as indexing increases, because otherwise they would choose to be active. Some causes of the rise of indexing such as an improvement in transparency of the non-index market tends to improve the welfare of some investors. Overall, our analysis complements Bond and Garcia (2017) by providing additional channels associated with a rise of index investing.

Our model extends the canonical Grossman and Stiglitz (1980) framework to a multi-asset setting to study the impact of indexing. As in Admati (1985), but different from Diamond and Verrecchia (1981), Ganguli and Yang (2009), and Bond and Garcia (2017), the liquidity trades are entirely exogenous in our model. However, this assumption is unlikely critical for our results because it is equivalent to a limiting case of endogenous liquidity trades as risk aversion approaches infinity. Van Nieuwerburgh and Veldkamp (2009) also study information acquisition in multi-asset markets. In their study, they use information acquisition capacity constraints to model the tension between acquiring information in different markets. In contrast, we explicitly model the cost of information acquisition that is a convex function of precisions. Qualitatively, the case with negative IAE in our model is similar to their setting in that acquiring more information in one market increases the cost of acquiring information in another market and thus tends to lower information acquisition in the other market. The case with positive IAE in our model captures another possibility where acquiring more information in one market helps lower the cost of information acquisition in another market. Qualitatively, this is similar to the case where the experience of acquiring information in one market helps information acquisition efficiency in another market. Baruch and Zhang (2017) also study the impact of indexing. They find that indexing does not affect the validity of the CAPM risk-return relation and
non-index portfolios suffer in terms of both Sharpe ratio and conditional payoff uncertainty. In contrast, we focus on how indexing affects price informativeness and market risk premium through its impact on information acquisition. Ganguli and Yang (2009) considers how multiple sources of information can lead to information acquisition complementarity and multiple equilibria. The information acquisition complementarity is qualitatively similar to a positive information acquisition externality in our model, but information acquisition complementarity endogenously arise in equilibrium, while information acquisition externality as we defined is determined by the exogenous information acquisition cost structure. Benchmarking to an index is qualitatively similar to indexing. Breugem and Buss (2017) show that benchmarking to an index reduces price informativeness and increases return volatility. In contrast to our model, a change in benchmarking is exogenous in Breugem and Buss (2017). Consequently, their results are also qualitatively different from ours in many cases.

The remainder of the paper proceeds as follows. In Section 2, we present the model. In Section 3 we derive the equilibrium and provide some comparative statics on information precision and price informativeness. In Section 4, we conduct numerical analysis to illustrate the impact of indexing that rises from different causes. We conclude in Section 5. All proofs are provided in the Appendix.

2. The Model

The Asset Market  We consider a one-period model where a continuum of investors can trade at time 0 one risk free and two stocks to maximize their expected utility at time 1. For \( i = 1, 2 \), Stock \( i \) has a final payoff of \( V_i \) with distribution \( N(\mu_i, \tau_i^{-1}) \) and correlation coefficient \( \rho \in (-1, 1) \) and a supply of 1 share. Trading the two stocks is equivalent to trading the market portfolio \( m \) (“index”) and the spread portfolio \( s \) (“non-index”), where the payoffs of the index \( m \) and the nonindex \( s \) are respectively \( V_m = \frac{V_1 + V_2}{2} \) and \( V_s = \frac{V_1 - V_2}{2} \). Then we have \( V_m \) and \( V_s \) are independently distributed as \( N(\mu_{vm}, \tau_{vm}^{-1}) \) and \( N(\mu_{vs}, \tau_{vs}^{-1}) \) respectively, where \( \mu_{vm} = \frac{\mu_1 + \mu_2}{2} \), \( \tau_{vm} = \frac{2}{1+\rho} \tau_i \), \( \mu_{vs} = \frac{\mu_1 - \mu_2}{2} \), and \( \tau_{vs} = \frac{2}{1-\rho} \tau_i \). In addition, the net supply of the index portfolio is 2 and that of the nonindex portfolio is zero. Because trading directly the two stocks is equivalent to trading the two portfolios and the payoffs of the index portfolio and the nonindex portfolio are independent, which greatly simplifies the analysis, in the subsequent analysis we will assume the investors trade directly the two portfolios instead and

\(^2\)The equal variance of \( V_1 \) and \( V_2 \) is assumed so that the index and the nonindex is independent which greatly simplifies the analysis. Without this assumption, one would need to solve for 10 coefficients instead of 5 and there would not seem to be closed form solutions. On the other hand, one can always vary the supply of a stock to change the variance of its payoff per share and solve for the equilibrium using the approach below. Therefore, the assumption of equal variance is for simplicity only.
solve the equilibrium prices of the portfolios. Then we back out the equilibrium prices for the stocks from the relationship between the payoffs of the stocks and the portfolios.

**Participation Costs and Types of Investors**

There is no participation cost in investing in the risk-free asset, but some investors incur participation costs in terms of utility loss (e.g., from time and attention consumption) before trading the index or the non-index portfolios.\(^3\)

A mass \(\lambda_A\) of investors are exogenous active investors (“A” investors) have zero cost of trading either the index or the non-index portfolio and thus always trade in both risky assets. They also acquire private information about both assets. These investors can represent funds that have relatively low cost in picking stocks. All other investors must incur a fixed participation cost \(k_m \geq 0\) before trading the index and incur a fixed participation cost \(k_s > 0\) before trading the non-index.

A mass \(\lambda_D\) of investors are discretionary traders (“D” investors) who can choose to participate in the index market by paying a fixed participation cost \(k_m \geq 0\) and to participate in the nonindex market by paying a fixed participation cost \(k_s > 0\). For simplicity, we assume \(k_s > k_m\) such that if it is optimal to trade in the nonindex portfolio, then it is also optimal to trade in the index portfolio. In equilibrium, an endogenously solved mass \(\lambda_{D0}\) of these discretionary traders choose to invest only in the risk free asset (“D0 investors”), a mass of \(\lambda_{DI}\) of these discretionary traders choose to invest only in the risk free asset and the index portfolio and thus become indexers (“DI” investors), and a mass of \(\lambda_{DA} = \lambda_D - \lambda_{DI} - \lambda_{D0}\) become active investors (“DA” investors) who trade all assets. “D” investors can represent funds who have relatively high cost of picking stocks and can switch to passive investing from active management.

A mass \(\lambda_I\) investors are exogenous indexers (“I” investors) who only invest in the index and the risk-free asset. These investors can represent most index funds that do not trade the non-index portfolio and do not acquire any private information. A change in the mass \(\lambda_I\) of exogenous indexers can be used to examine the impact of an exogenous change of indexing due to a large shock to some investors’ participation cost and information acquisition cost.

A mass \(\lambda_L\) of investors are liquidity traders (“L” investors) whose trading in both risky portfolios are exogenous. For \(j \in \{m, s\}\), each liquidity trader \(i\) has a random endowment \(e_{ji} = Z_j + u_{ji}\) shares of a nontraded asset \(j\) (e.g., two streams of labor income), with each share of the nontraded asset \(j\) paying the same amount at time 1 as \(V_j\), where \(Z_j\) and \(u_{ji}\) are independently distributed as \(N(0, \tau_{zj}^{-1})\) and \(N(0, \tau_{uj}^{-1})\) respectively, and \(\tau_{zj}\) and \(\tau_{uj}\) are all constants.\(^4\) \(Z_j\) (resp. \(u_{ji}\)) can be viewed as an

\(^3\)The case where the participation cost is monetary and is paid to the active investors yields the same qualitative results if the cost is exogenous (e.g., due to competition).

\(^4\)It is sufficient to assume that the payoff of the nontraded asset \(j\) is perfectly correlated with \(V_j\)
aggregate (resp. idiosyncratic) shock in the endowment. Liquidity traders have mean-variance preferences over the final wealth and infinite risk aversion. As a result, they must perfectly hedge the endowment risk by selling the same number of shares in markets m and s as their endowment of the nontraded assets at the market prices and have no incentive to acquire any information. As in Grossman and Stiglitz (1980), the presence of liquidity traders is necessary for the existence of an equilibrium in our model. The liquidity traders are similar to noise traders commonly assumed in the literature. The main difference is that we can measure liquidity traders’ utility by the expected terminal wealth which depends on market prices. Given this setup, we can shed some light on how indexing affects liquidity traders’ welfare. Because the total mass of investors is 1, we have $\lambda_A + \lambda_D + \lambda_I + \lambda_L = 1$.

Every investor has an initial endowment of 1 share of each of the two stocks. This implies that equivalently, every investor has an initial endowment of 2 shares of the index portfolio, but has no risk-free asset or the non-index portfolio. All non-liquidity-traders have constant absolute risk averse (CARA) preferences with a risk aversion coefficient of $\gamma > 0$.

In actual financial markets, hedge funds can be viewed as “A investors” who do not passively invest only in index funds. Mutual funds can be considered as “D investors” in our model. Some of them may choose to become active mutual funds (“DA investors”), some choose to be passive mutual funds (“DI investors”), and others choose to be money-market funds (“D0 investors”). Most index fund investors are considered as “I investors” who do not select stocks and do not care about acquiring private information about assets. Individual traders or hedgers who trade assets for hedging or rebalancing needs instead of fundamental value of assets can be viewed as “L investors.”

**Information Acquisition** For $t \in \{A, D\}$, $j \in \{m, s\}$, each investor $i$ of types A and D can observe independent private signals $Y_{tji}$ at time 0 about the payoff of the risky asset $j$, where

$$Y_{tji} = V_j + \varepsilon_{tji}, \quad j \in \{m, s\},$$

and all $\varepsilon_{tji}$ are independently distributed as $N(0, \tau_{tji}^{-1})$. The cost of acquiring private information with precisions $\tau_{tm}$ and $\tau_{ts}$ is $C_{t}(\tau_{tm}, \tau_{ts})$ for $t \in \{A, D\}$. We only consider symmetric equilibria where investors of the same type make the same trading and information acquisition decision. As a result, the precision choices are the same across investors of the same type, and thus we omit the $i$ index in the precision variables.

so that hedging motive is present. This is equivalent to assuming that liquidity traders are noise traders who have exogenous trading demand.

With finite risk aversion, the derivation is more complicated, but the qualitative results are the same.
Investors’ Problems  Let $P_m$ and $P_s$ be the time 0 equilibrium prices of the market portfolio and the non-index portfolio respectively, $I_t$ be time 0 information set of investor $i$ of type $t$, and $\Theta_{tji}$ be the number of shares of the $j$ portfolio bought by investor $i$ of type $t$ at time 0, for $j \in \{m, s\}$ and $t \in \{A, D, I, L\}$. At time 0, for $t \in \{A, D\}$ investor $i$ of type $t$ chooses $(\Theta_{tmi}, \Theta_{tsi})$ to solve

$$
\max E[-e^{-\gamma(W_{ti} - k_m 1_{t=DI}-(k_m+k_s)1_{t=DA})}|I_t] \tag{1}
$$

subject to the budget constraint

$$
\bar{W}_{ti} = 2V_m + \Theta_{tmi}1_{t\neq D0}(V_m - P_m) + \Theta_{tsi}1_{t=DA}(V_s - P_s) - C_t(\tau_{tm}, \tau_{ts}). \tag{2}
$$

At time 0, investor $i$ of type $I$—traders who only invest in the index chooses $\Theta_{imi}$ to solve

$$
\max E[-e^{-\gamma(W_{ti} - k_m)}|I_t] \tag{3}
$$

subject to the budget constraint

$$
\bar{W}_{ti} = 2V_m + \Theta_{imi}(V_m - P_m). \tag{4}
$$

For liquidity traders, $\Theta_{Lmi} = -(2 + Z_m + u_{mi})$ and $\Theta_{Lsi} = -(Z_s + u_{si})$.

Market-Clearing Condition  The time 0 equilibrium is $\{P_j, \Theta_{tji}, j \in \{m, s\}, t \in \{A, D, I, L\}\}$ such that $\Theta_{tji}$ solves the above problems for investor $i$ of type $t \in \{A, D, I, L\}$ and the following market clearing condition is satisfied:

$$
\sum_{t \in \{A, D, I, L\}} \int_{\Theta_{tji}} d\Theta_{tji} = 0, \quad j \in \{m, s\}. \tag{5}
$$

Choice of Indexing and Precision  Just before time 0, type D investors choose whether to only invest in the risk-free asset, or to become indexers, or to become active investors. In equilibrium, either all D investors strictly prefer investing only in the risk-free asset (i.e., $\lambda_{D0} = \lambda_D$) or all D investors strictly prefer indexing (i.e., $\lambda_{DI} = \lambda_D$) or all D investors strictly prefer to be active (i.e., $\lambda_{DA} = \lambda_D$) or each D investor is indifferent between indexers and active investors (i.e., $\lambda_{D0} = 0, \lambda_{DI} > 0, \lambda_{DA} > 0$) or each D investor is indifferent between the three choices (i.e., $\lambda_{D0} > 0, \lambda_{DI} > 0, \lambda_{DA} > 0$).
3. The Equilibrium

We first solve the equilibrium at time 0 given investors’ information and participation choice. We conjecture and later verify that

\[
P_m = a_m + b_m V_m - d_m Z_m, \quad P_s = a_s + b_s V_s - d_s Z_s,
\]

where \(a_m, b_m, d_m, b_s,\) and \(d_s\) are constants to be determined. For \(j \in \{m, s\},\) let

\[
\tau_j := \lambda_{A} \tau_{A_j} + (\lambda_{D} - \lambda_{DI} - \lambda_{D0}) \tau_{DI_j} + \lambda_{DI} \tau_{DI_j} 1_{j=m},
\]

and

\[
\rho_j = \frac{1}{\text{Var}[V_j|P_j]} = \tau_{vj} + \frac{\tau_{j}^2}{\gamma^2 \lambda_L^2} \tau_{zj}
\]

denote the total precision of private information and the price informativeness in market \(j.\) For given values of \(\tau_{vj}\) and \(\tau_{zj},\) price informativeness in market \(j\) increases with the total precision \(\tau_j\) of private information in market \(j.\)

We focus on linear symmetric equilibrium where investors of the same type choose the same trading strategy and the same information precision.

**Theorem 1** Given the signal precisions \(\tau_{tj},\) \(t \in \{A, DI, DA\},\) \(j \in \{m, s\},\) there is a unique linear symmetric equilibrium, and the equilibrium price coefficients are

\[
a_m = \frac{(1 - \lambda_L - \lambda_{D0}) \mu_{vm} \tau_{vm} - 2\gamma}{\tau_m + (1 - \lambda_L - \lambda_{D0}) \rho_m}, \quad (9)
\]

\[
b_m = 1 - \frac{(1 - \lambda_L - \lambda_{D0}) \mu_{vm} \tau_{vm}}{\tau_m}, \quad (10)
\]

\[
a_s = \frac{\lambda_{A} + \lambda_{DA}}{\tau_s} \mu_{vs} \tau_{vs}, \quad (11)
\]

\[
b_s = 1 - \frac{(\lambda_{A} + \lambda_{DA}) \mu_{vs} \tau_{vs}}{\tau_s}, \quad (12)
\]

\[
d_m = \frac{\gamma \lambda_{L} b_m}{\tau_m}, \quad d_s = \frac{\gamma \lambda_{L} b_s}{\tau_s}. \quad (13)
\]

Theorem 1 implies that the risk premium of the market portfolio \(m\) is equal to\(^6\)

\[
E[V_m - P_m] = \frac{2\gamma}{\tau_m + (1 - \lambda_L - \lambda_{D0}) \rho_m}. \quad (14)
\]

\(^6\)The risk premium of the non-index portfolio is zero because the aggregate supply of the non-index portfolio is zero and thus there is no aggregate risk.
In addition the expected price of the nonindex is

\[ E[P_s] = \mu_{vs}, \]  

(15)

because of the lack of aggregate risk in the non-index market.

For given values of \( \tau_{vm} \) and \( \tau_{zm} \), equation (14) implies that market risk premium and price informativeness \( \rho_m^* \) move in the opposition direction. This is because as price informativeness increases, the aggregate uncertainty in the market reduces and vise versa.

In terms of Stocks 1 and 2 prices, Equation (6) implies that we have

\[ P_1 = P_m + P_s, \quad P_2 = P_m - P_s, \]  

(16)

where \( P_1 \) and \( P_2 \) are the implied prices of Stock 1 and 2 respectively. As a result, we have the variance of \( P_1 \) and \( P_2 \) is

\[ \text{Var}(P_1) = \text{Var}(P_2) = \frac{b_m^2}{\tau_{vm}} + \frac{b_s^2}{\tau_{vs}} + \frac{d_m^2}{\tau_{zm}} + \frac{d_s^2}{\tau_{zs}}, \]  

(17)

and the correlation between Stocks 1 and 2 prices is

\[ \text{Corr}(P_1, P_2) = \frac{b_m^2/\tau_{vm} - b_s^2/\tau_{vs} + d_m^2/\tau_{zm} - d_s^2/\tau_{zs}}{b_m^2/\tau_{vm} + b_s^2/\tau_{vs} + d_m^2/\tau_{zm} + d_s^2/\tau_{zs}}. \]  

(18)

In addition, the expected market capitalizations of Stocks 1 and 2 before realizations of private signals are respectively

\[ EMC_1 = E[P_1] = \mu_{vm} + \mu_{vs} - \frac{2\gamma}{\tau_m + (1 - \lambda_L - \lambda_{D0})\rho_m}, \]  

(19)

and

\[ EMC_2 = E[P_2] = \mu_{vm} - \mu_{vs} - \frac{2\gamma}{\tau_m + (1 - \lambda_L - \lambda_{D0})\rho_m}. \]  

(20)

Equations (19) and (20) imply that the expected capitalization for both stocks increase with the price informativeness in the index market. Note that the price informativeness in the non-index market is irrelevant for the expected capitalization. This is because the total supply of the non-index portfolio is zero and thus the total capital allocated to the non-index portfolio is also zero.

Just before time 0, investors optimally choose the precisions of their private signals. We consider symmetric Nash equilibrium where investors of the same type choose the same precision and in equilibrium, each investor’s information precision is optimal given other investors’ choice.
Let $C_{tm}, C_{ts}, C_{tmm},$ and $C_{tss}$ denote respectively the first and the second derivative of $C_t$ with respect to $\tau_{tm}$ and $\tau_{ts}$, and $C_{tms}$ denote the cross derivative, for $t \in \{A, D\}$. To ensure the existence and the uniqueness of equilibrium, we make the following assumption:

**Assumption 1** $C_{tj} \geq 0$ for $j \in \{m, s\}$ with equality only at $\tau_{tm} = \tau_{ts} = 0$. $C_{tmm} > 0$, $C_{tss} > 0$, and $C_{tmm}C_{tss} - C_{tms}^2 \geq 0$ for $t \in \{A, D\}$.

The above assumption ensures the convexity of the cost function. The information acquisition externality (IAE) for type $t$ ($t = A, D$) investors can be measured by

$$\varphi_t \equiv -C_{tms} = -\frac{\partial^2 C_t(\tau_{tm}, \tau_{ts})}{\partial \tau_{tm}\partial \tau_{ts}}. \tag{21}$$

A negative $\varphi_t$ means that acquiring more information in one market increases the cost of information acquisition in another market. In this sense, there is a negative IAE across the two markets. The negative IAE can represent qualitatively the case where an investor has a fixed total information acquisition capacity, which implies that acquiring more information in one market lowers the capacity of acquiring information in another market, and as a result, information acquisition in the other market is reduced. A positive $\varphi_t$ means that information acquisition in one market reduces the cost of information acquisition in another market. In this sense, there is a positive IAE across the two markets. The positive IAE can represent qualitatively the case where the experience of acquiring information in one market helps make information acquisition in another market more efficient and thus lowers the cost of information acquisition in the other market.\(^7\)

Given the optimal choice of precisions of signals and trading strategies, D investors optimally choose whether to incur the participation cost $k$ to trade the non-index risky asset.

**Theorem 2** Under Assumption 1, there exists a unique linear symmetric equilibrium where the equilibrium information precisions solve the following five equations:

$$2\gamma C_{tj}(\tau_{tm}, \tau_{ts}) = \frac{1}{\tau_{tj} + \rho_j}, \quad t \in \{A, DA\}, j \in \{m, s\}, \tag{22}$$

\(^7\)A positive IAE is also qualitatively consistent with the situation where acquiring information in one market helps reveal information about another market, and thus paying a lower cost in the other market can give an investor the same precision information in total, for example, due to correlations among the different assets. The information acquisition complementarity as termed by the existing literature is qualitatively similar to a positive IAE in our model, but information acquisition complementarity endogenously arises in equilibrium, while IAE as we defined is determined by the exogenous information acquisition cost structure.
and
\[ 2\gamma C_{Dm}(\tau_{DIm}, 0) = \frac{1}{\tau_{DIm} + \rho_{m}}, \]
(23)

and the equilibrium masses \( \lambda_{D0} \) and \( \lambda_{DI} \) are such that either all \( D \) investors strictly prefer investing only in the risk-free asset (i.e., \( \lambda_{D0} = \lambda_D \)) or all \( D \) investors strictly prefer indexing (i.e., \( \lambda_{DI} = \lambda_D \)) or all \( D \) investors strictly prefer to be active (i.e., \( \lambda_{DA} = \lambda_D \)) or each \( D \) investor is indifferent between indexers and active investors (i.e., \( \lambda_{D0} = 0, \lambda_{DI} > 0, \lambda_{DA} > 0 \)) or each \( D \) investor is indifferent between the three choices (i.e., \( \lambda_{D0} > 0, \lambda_{DI} > 0, \lambda_{DA} > 0 \)).

Equation (22) is the first order conditions of the A and DA investors for the choice of precisions in the index and the non-index markets. Equation (23) is the first order condition of the DI investors for the choice of precision in the index market.

First, we examine how changes in the participation cost \( k \), in non-index market transparency \( \tau_{zs} \), and in the exogenous indexing \( \lambda_I \) affect endogenous fraction of indexers \( \eta^* \).

**Proposition 1**  
1. As the participation cost \( k \) increases, endogenous indexing \( \eta^* \) increases.
2. As the non-index market transparency \( \tau_{zs} \) increases, endogenous indexing \( \eta^* \) increases.
3. As the exogenous indexing \( \lambda_I \) increases, keeping \( \lambda_D + \lambda_I \) constant, endogenous indexing \( \eta^* \) decreases if \( \varphi_D = 0 \).

As expected, when it becomes more costly to trade in the non-index market, more discretionary investors optimally choose to be indexers. When the non-index market becomes more transparent, trading in the non-index becomes less profitable, and thus more discretionary traders choose to be indexers. When exogenous indexing increases, the equilibrium endogenous indexers decreases if there is no information acquisition externality.\(^8\)

One of the main questions we want to answer is how the rise of index investing affects price informativeness of both index and non-index markets. The following proposition implies that a rise in indexing due to either an increase of participation cost or an increase in exogenous indexing tends to decrease the price informativeness of the non-index. We will provide more detailed analysis and intuitions about this result in next section.

In addition, if there is no information acquisition externality, then the rise of endogenous indexing does not have any impact on the price informativeness of the

\(^8\)It can be shown that as the index market transparency \( \tau_{zm} \) increases, endogenous indexing \( \eta^* \) increases if and only if \( \varphi_D > 0 \).
index. In next section, we will show that how the rise of endogenous indexing affects the price informativeness of the index as well as the market risk premium critically depends on the sign of information acquisition externality $\varphi_D$ while the rise of indexing due to increases in the exogenous indexing always decreases the price informativeness of both the index and the non-index.

**Proposition 2** Suppose there is no information acquisition externality. Then

1. holding $\lambda_A$, $\lambda_D$, $\gamma$, $\lambda_L$, $\tau_{zm}$, and $\tau_{zs}$ constant, as the equilibrium mass of discretionary indexers increases, the price informativeness $\rho_m$ in the index market does not change, i.e., $\frac{\partial \rho_m}{\partial \eta^*} = 0$.

2. holding $\lambda_A$, $\lambda_D$, $\gamma$, $\lambda_L$, $\tau_{zm}$, and $\tau_{zs}$ constant, as the equilibrium mass of discretionary indexers increases, the price informativeness $\rho_s$ in the non-index market decreases.

One common concern on the rise of index investing is that indexers may free ride on active traders for information acquisition. The following proposition shows that while this is true in some cases, discretionary indexers may sometimes choose to acquire more precise information about the index than active investors and thus let others “free ride” on them.

**Proposition 3**

$$\text{Sign}(\tau_{DAm} - \tau_{DIm}) = \text{Sign}(\varphi_D),$$

(24)

where

$$\text{Sign}(x) = \begin{cases} 
1 & x > 0 \\
0 & x = 0 \\
-1 & x < 0.
\end{cases}$$

Proposition 3 shows that if the information acquisition externality is negative, then discretionary indexers acquire more precise information than discretionary active investors. In this case, discretionary active investors in some sense “free ride” on discretionary indexers in the index market.

The intuition is simple. When the information acquisition externality is negative (acquiring information in one market increases the cost of acquiring information in another market). Because discretionary active investors also acquire private information in the non-index market which increases their information acquisition cost in the index market, discretionary index investors’ equilibrium precision of private information about the index is therefore higher than that of discretionary active investors.

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*Note that exogenous indexers in our model are always getting a free ride on other traders. Active investors do pay to acquire private information even though the precision can be lower than that of discretionary indexers in the index market.*
4. Effects of the Rise of Index Investing

In this section we conduct numerical analysis of the equilibrium effects of the rise of indexing on price informativeness, market risk premium, and welfare. We focus on three possible causes of the rise of index investing:

1. Increases in exogenous indexers.
2. Increases in the participation cost $k_m$ for discretionary investors;
3. Increases in the market transparency $\tau_{zs}$ in the non-index market;

We will show that the relationship between indexing and market risk premium, welfare, and price informativeness depends critically on the causes of the rise of indexing. In the subsequent analysis, we use the following default parameter values: $\tau_{vm} = \tau_{vs} = \tau_{zm} = \tau_{zs} = 1$, $\gamma = 0.25$, $\mu_{vm} = 0.3$, $\mu_{vs} = 0.05$, $\lambda_A = 0.2$, $\lambda_D = 0.7$, $\lambda_f = 0$, $\lambda_L = 0.1$, $k_m = 0$, $k_s = 0.012$, and $C_t(\tau_{tm}, \tau_{ts}) = c_{tm}\tau_{tm}^2 + c_{ts}\tau_{ts}^2 + c_{tms}\tau_{tm}\tau_{ts}$ with $c_{tm} = c_{ts} = 0.01$, for $t = A, DI, DA$. To illustrate the impact of the sign of information acquisition externality, we plot three cases in all the figures below: positive IAE (i.e., $\varphi_D > 0$ with parameter value $c_{tms} = -0.005$); negative IAE (i.e., $\varphi_D < 0$ with parameter value $c_{tms} = 0.005$); and zero IAE (i.e., $\varphi_D = 0$).

Note that we do not attempt to calibrate our model to match empirical data. Instead, these default parameter values are chosen such that with minimum variations in some parameter values, we can illustrate all the key qualitative results.

For simplicity of exposition and to make our intuitions more clear, in this numerical section we use the same information cost parameter values for both A and D investors and thus the only difference between A and D investors lies in the participation cost. As a result, the optimal information precisions chosen by A investors and DA investors are always the same in both markets.\footnote{We also conduct similar analysis when A and D investors have different information cost parameter values. The main qualitative results are the same.}

4.1 Effects of Changes in Exogenous Indexers

In this subsection, as in Bond and Garcia (2017), we examine the impact of an exogenous increase in indexing. In practice, many passive index funds barely acquire private information about either index or non-index. Instead, they might spend great effort in retaining their existing clients and attracting more fund inflow. It is possible that some discretionary investors might simply switch to exogenous indexers and stop...
acquiring any private information. As this happens, the mass of exogenous indexers \( \lambda_I \) increases.

Different from Bond and Garcia (2017), some investors (i.e., discretionary investors) in our model can endogenously choose to be indexers or to be active investors. After an increase in the exogenous indexers, some discretionary indexers may choose to switch to active investors. One natural question is then does the total mass of indexers, including both discretionary and exogenous indexers, increase? The left panel of Figure 1 shows that when the mass of the exogenous indexers \( \lambda_I \) increases, the equilibrium mass of discretionary indexers \( \lambda_{DI} \) decreases. This suggests the presence of a “crowding out” effect of exogenous indexing on endogenous indexing. In addition, the crowding out effect can over- or under-offset the increase in the mass of exogenous indexers \( \lambda_I \). This crowding-out effect arises because as some discretionary active investors become indexers, the competition in the non-index market decreases, and thus some discretionary indexers choose to switch back to be active investors. When there is a positive IAE, because of the extra information acquisition benefit, the crowding-out effect becomes stronger and can overcome the increase of the mass of exogenous indexers \( \lambda_I \). As a result, the total mass of indexers may decrease as the mass of exogenous indexers \( \lambda_I \) increases, as confirmed in the right panel of Figure 1. When there is a negative IAE, the total mass of indexers increases as the mass of the exogenous indexers increases. When almost all discretionary investors become exogenous indexers, there are not enough remaining discretionary investors to offset the increase, and therefore the total mass of indexers increases irrespective of the sign of IAE.

Although the total mass of indexers may decrease, Figure 1 shows that the mass of endogenous indexers \( \lambda_{DI} \) decreases irrespective of the sign of IAE. As a result, the mass of information acquiring investors in the index market decreases, and thus the price informativeness of the index decreases, which leads to a greater market premium, as shown in Figure 2. In contrast, the price informativeness of the non-index can be non-monotonic if there is positive IAE. As shown in Figure 3, the precisions of all informed traders in the non-index market increase. The non-monotonicity of the price informativeness of the non-index follows from the non-monotonicity of the total mass of traders who acquire information in the non-index market, as shown in the blue line (i.e., \( \varphi_D > 0 \)) in Figure 1. For example, if the total mass of indexers increases (e.g., the case when \( \varphi_D < 0 \) as in Figure 1) and thus the total mass of traders who acquire information in the non-index market decreases, then the price informativeness of the non-index decreases. In addition, as clearly illustrated in the left panel of Figure 3, when there is a large increase in the exogenous indexers, the price informativeness of the non-index decreases irrespective of the sign of IAE.

\(^{11}\)For example, some discretionary investors may experience a large shock to their participation cost and information acquisition cost so that it is optimal for them to become indexers and do not acquire any private information.
Opposite to our findings, Bond and Garcia (2017) find that market risk premium decreases and price informativeness in both markets increases when the rise of indexing is exogenous and the IAE is negative. This result is driven by their assumption that exogenous indexers cannot acquire information before becoming indexers. Because indexers in their model cannot acquire information and only trade for liquidity reasons in the non-index market before becoming indexers, when they leave the non-
Figure 3: The price informativeness and equilibrium precisions in the non-index market against the fraction of exogenous indexers $\lambda_I$. The default parameter values are: $\tau_{em} = \tau_{es} = \tau_{zm} = \tau_{zs} = 1$, $\gamma = 0.25$, $\mu_{em} = 0.3$, $\mu_{es} = 0.05$, $\rho = 0$, $k_m = 0$, $k_s = 0.01$, $\lambda_A = 0.2$, $\lambda_D = 0.7$, $\lambda_L = 0.1$, $c_{tm} = c_{ts} = 0.01$, $c_{tms} = -0.005$ ($\varphi_D > 0$) for the red curve, $c_{tms} = 0$ ($\varphi_D = 0$) for the green curve, and $c_{tms} = 0.005$ ($\varphi_D < 0$) for the blue curve, for $t = AI, DI, DA$.

index market, the price informativeness of the non-index increases. As a result, active investors reduce information acquisition about the non-index, and thus lowering the cost of information acquisition about the index. Consequently, active investors increase their information acquisition in the index market, which leads to greater price informativeness in the index market and a lower market risk premium.

We next turn our analysis to how index investing affects capital allocated to stocks, stock price variances, and stock price correlations. Figures 4 and 5 show that as the mass of exogenous indexers increases, both the expected market capitalizations of both stocks and the stock price variances as well as the correlation between the stocks decrease. As shown in Equations (19) and (20), the expected capitalization is increasing in the price informativeness in the index market, because investors are more willing to invest in the index when there is less adverse selection. Accordingly, with the decline in the price informativeness in the index market after the mass of exogenous indexers increases, the expected capitalizations decrease, as shown in Figure 2. As the capital provided to firms decrease, the real activity such as investment and production will be adversely affected. This shows that the rise of indexing can have real effect on the economy.

As Equations (9) and (17) imply, for sufficiently high price informativeness, as the price informativeness in the index (non-index) market increases, the variance of the index (non-index) price increases, because the prices become more sensitive to the changes in the payoff and in the liquidity demand. In addition, both Stock 1 and Stock 2 variances are equal to the sum of the variances of the index and the non-index prices. Because as the mass of the exogenous indexers increases, the price
informativeness in the index market decreases regardless of the sign of the IAE and the magnitude of the price informativeness in the index market is significantly greater than that in the non-index market, the variances of both stock prices decrease.

Without indexing, the prices of the two stocks are uncorrelated because of the independence of their payoffs and the CARA preferences. When some investors are indexers, the correlation of Stocks 1 and 2 is driven by two opposing forces. First, when shocks are to the payoffs of the stocks, the stock prices tend to move in the same direction and thus exhibit a positive correlation. This is because with indexing, after a good shock to a stock’s payoff, indexers buy more of the stock which drives up the stock price. Because they are indexers, they also buy more of the other stock, and thus also drive up the price of the other stock. The case with a bad shock is similar. Second, when shocks are to the liquidity demands of the liquidity traders, the stock prices tend to move in the opposite directions because as the price of one stock goes down, the exposure of the indexers in the index goes down, and thus the indexers buy more of both stocks to rebalance the position, which drives the price of the other stock up. In short, with indexing, an informational shock tends to cause positive correlation, while a liquidity shock does the opposite. The net effect depends on the relative strength of the two forces. As shown in Figure 5, with the mass of the exogenous indexers $\lambda_I$ increases, the price correlation monotonically decreases and turns negative when $\lambda_I$ is sufficiently high. This pattern appears because when $\lambda_I$ is small, a significant part of the shocks are informative of the payoffs, and thus the first force dominates and the correlation is positive; when $\lambda_I$ is large, information acquisition decreases and thus the shocks are mainly liquidity shocks, which leads to the dominance of the second force and thus the correlation turns negative.

![Figure 4](image)

Figure 4: The expected market capitalization of Stocks 1 and 2 against the fraction of exogenous indexers $\lambda_I$. The default parameter values are: $\tau_{vm} = \tau_{vs} = \tau_{zm} = \tau_{zs} = 1$, $\gamma = 0.25$, $\mu_{vm} = 0.3$, $\rho = 0$, $\mu_{vs} = 0.05$, $\rho = 0$, $k_m = 0$, $k_s = 0.01$, $\lambda_A = 0.2$, $\lambda_D = 0.6$, $\lambda_L = 0.1$, $c_{tm} = c_{ts} = 0.01$, $c_{tms} = -0.005$ for the blue line ($\varphi_D > 0$), $c_{tms} = 0$ for the green line ($\varphi_D = 0$), and $c_{tms} = 0.005$ for the red line ($\varphi_D < 0$), for $t = A, DI, DA$.

As for welfare, Figure 6 shows that if the rise of indexing is caused by the rise of the exogenous indexers, then the social welfare decreases regardless of the sign of the
4.2 Effects of Changes in Participation Cost

The participation cost $k_m$ for trading the index portfolio may change over time. For example, as more and more ETFs become available, the participation cost in the
Figure 7: The equilibrium mass of endogenous indexers $\lambda_{DI}$ against participation cost $k_m$. The default parameter values are: $\tau_{vm} = \tau_{zs} = \tau_{zm} = \tau_{zs} = 1$, $\gamma = 0.25$, $\mu_{vm} = 0.3$, $\mu_{vs} = 0.05$, $\rho = 0$, $k_s = 0.012$, $\lambda_A = 0.2$, $\lambda_D = 0.7$, $\lambda_I = 0$, $\lambda_L = 0.1$, $c_{tm} = c_{ts} = 0.01$, $c_{tms} = -0.005$ ($\varphi_D > 0$) for the red curve, $c_{tms} = 0$ ($\varphi_D = 0$) for the green curve, and $c_{tms} = 0.005$ ($\varphi_D < 0$) for the blue curve, for $t = A, DI, DA$.

index market decreases. Thus, it is possible that a lower participation cost in the index market contributes to the rise of indexing. Indeed, Figures 7 and 8 show that as the participation cost $k_m$ decreases, the equilibrium mass $\lambda_{DI}$ of indexers increases and the equilibrium mass $\lambda_{D0}$ of the investors who invest only in the risk-free asset decreases.

Figures 7 and 8 also suggest that, as participation cost $k_m$ decreases, the speed of the increase in $\lambda_{DI}$ and the speed of the decrease in $\lambda_{D0}$ first increases and then decreases. This is because the index market becomes more liquid as the number of investors who trade in index market increases until almost all the discretionary investors trade in the index market. In addition, the equilibrium mass of endogenous indexers is smaller and the equilibrium mass of discretionary investors who only invest in the risk-free asset is larger with positive IAE (i.e., $\varphi_D > 0$) than that with negative IAE (i.e., $\varphi_D < 0$) because the additional benefit of lowering information acquisition cost in the index market from participating in the non-index market make more discretionary investors to switch to active investors as shown in Figure 9. Different from the impact on $\lambda_{DI}$, as participation cost $k_m$ decreases, the mass of active discretionary investors decreases if there is positive information acquisition externality. The intuition is as follows. With positive information acquisition externality, more discretionary investors choose to be active for a given participation cost $k_m$. As the participation cost $k_m$ decreases, more discretionary investors invest in the index and acquire information, which reduces the benefit of acquiring more precise information for the active investors in the index market, and thus the benefit of lowering the information acquisition cost by trading in the non-index market decreases. As a result, the mass of discretionary active investors decreases.
Figure 8: The equilibrium mass of endogenous indexers $\lambda_{DI}$ against participation cost $k_m$. The default parameter values are: $\tau_{vm} = \tau_{vs} = \tau_{zm} = \tau_{zs} = 1$, $\gamma = 0.25$, $\mu_{vm} = 0.3$, $\mu_{vs} = 0.05$, $\rho = 0$, $k_s = 0.012$, $\lambda_A = 0.2$, $\lambda_D = 0.7$, $\lambda_I = 0$, $\lambda_L = 0.1$, $c_{tm} = c_{ts} = 0.01$, $c_{tms} = -0.005$ ($\varphi_D > 0$) for the red curve, $c_{tms} = 0$ ($\varphi_D = 0$) for the green curve, and $c_{tms} = 0.005$ ($\varphi_D < 0$) for the blue curve, for $t = A, DI, DA$.

Figure 9: The equilibrium mass of endogenous indexers $\lambda_{DI}$ against participation cost $k_m$. The default parameter values are: $\tau_{vm} = \tau_{vs} = \tau_{zm} = \tau_{zs} = 1$, $\gamma = 0.25$, $\mu_{vm} = 0.3$, $\mu_{vs} = 0.05$, $\rho = 0$, $k_s = 0.012$, $\lambda_A = 0.2$, $\lambda_D = 0.7$, $\lambda_I = 0$, $\lambda_L = 0.1$, $c_{tm} = c_{ts} = 0.01$, $c_{tms} = -0.005$ ($\varphi_D > 0$) for the red curve, $c_{tms} = 0$ ($\varphi_D = 0$) for the green curve, and $c_{tms} = 0.005$ ($\varphi_D < 0$) for the blue curve, for $t = A, DI, DA$. 
We now examine how the rise of index investing caused by decreases in the participation cost $k_m$ affects price informativeness of the index and market risk premium. Figure 10 shows that all informed participants in the index market (i.e., A, DA, and DI investors) decrease their information precisions as the participation cost $k_m$ decreases. This is because as more and more investors switch from investing only in the risk-free asset to investing in the index. These new index investors optimally choose to acquire private information, thus the mass of information acquirers increases and the marginal benefit of acquiring more precise information decreases. Because of this increase in the mass of information acquirers, even though each informed participant in the index market acquires less precise information, the price informativeness $\rho_m$ of the index still increases, as shown in the left panel of Figure 11. In addition, as implied by equation (14), market risk premium changes in the opposite direction to that of price informativeness $\rho_m^*$. Therefore, the market risk premium decreases with the rise of indexing caused by decreases in the participation cost $k_m$, as shown in the right panel of Figure 11. Compared to the case when the information acquisition externality is negative, the price informativeness is higher and the market risk premium is lower in the index market when the information acquisition externality is positive. This is because the positive externality lowers the information acquisition cost of the active investors who also invest in the non-index market.

One concern about the rapid growth of index investment is that indexers may free-ride on others for information acquisition, and thus reduce market information revelation. While this is always true for exogenous indexers (who do not acquire private information) and this is also true for discretionary indexers if the IAE is positive ($\varphi D > 0$), Proposition 3 shows that active investors may sometimes free ride on discretionary indexers for information acquisition if there is negative IAE.
Figure 11: The market risk premium (MRP) and price informativeness of the index against participation cost $k_m$. The default parameter values are: $\tau_{vm} = \tau_{vs} = \tau_{zm} = \tau_{zs} = 1$, $\gamma = 0.25$, $\mu_{vm} = 0.3$, $\mu_{vs} = 0.05$, $\rho = 0$, $k_s = 0.012$, $\lambda_A = 0.2$, $\lambda_D = 0.7$, $\lambda_I = 0$, $\lambda_L = 0.1$, $c_{tm} = c_{ts} = 0.01$, $c_{tms} = -0.005$ ($\varphi_D > 0$) for the red curve, $c_{tms} = 0$ ($\varphi_D = 0$) for the green curve, and $c_{tms} = 0.005$ ($\varphi_D < 0$) for the blue curve, for $t = A, DI, DA$.

($\varphi_D < 0$). Indeed, Figure 10 shows that when there is a positive IAE, discretionary indexers acquire less precise information than both active traders and discretionary active traders. But when there is a negative IAE, the opposite is true. In the case with a negative IAE, as indexing increases due to lower participation cost $k_m$, not only the information acquisition advantage of DI investors over A and DA investors increases, the fraction of investors with more precise information (i.e., the DI indexers) also increases, which further increases the free-riding extent of A and DA investors.

Now we turn to the impact of indexing on the price informativeness in the non-index market. Figure 12 shows that as indexing rises due to a decrease in the participation cost $k_m$ in the index market, active investors acquire less precise information and the price informativeness in the non-index market decreases if and only if the information acquisition externality is positive. With positive information acquisition externality, more discretionary investors choose to be active for a given participation cost $k_m$. As the participation cost $k_m$ decreases, more discretionary investors who invest in only the risk-free asset start to invest in the index and acquire information in the index market, which reduces the benefit of acquiring more precise information for the active investors in the index market. As a result, the benefit of the positive externality decreases and they acquire less precise information also in the non-index market. With negative information acquisition externality, the opposite is true.

Figure 13 shows that if the rise of indexing is caused by a decrease in the partici-

\[\text{Figure 11: The market risk premium (MRP) and price informativeness of the index against participation cost } k_m. \text{ The default parameter values are: } \tau_{vm} = \tau_{vs} = \tau_{zm} = \tau_{zs} = 1, \gamma = 0.25, \mu_{vm} = 0.3, \mu_{vs} = 0.05, \rho = 0, k_s = 0.012, \lambda_A = 0.2, \lambda_D = 0.7, \lambda_I = 0, \lambda_L = 0.1, c_{tm} = c_{ts} = 0.01, c_{tms} = -0.005 (\varphi_D > 0) \text{ for the red curve, } c_{tms} = 0 (\varphi_D = 0) \text{ for the green curve, and } c_{tms} = 0.005 (\varphi_D < 0) \text{ for the blue curve, for } t = A, DI, DA.\]

\[\text{Note that the precision with zero IAE (green line) is lower than that with negative IAE. This is because with negative IAE, there are less DA investors in the non-index market than with zero IAE, and thus the marginal benefit of acquiring more information is greater than with zero IAE, which makes it optimal to acquire more precise information than with zero IAE.}\]
Figure 12: The price informativeness and equilibrium precisions in Market s against participation cost $k_m$. The default parameter values are: $\tau_{vm} = \tau_{vs} = \tau_{zm} = \tau_{zs} = 1$, $\gamma = 0.25$, $\mu_{vm} = 0.3$, $\mu_{vs} = 0.05$, $\rho = 0$, $k_s = 0.012$, $\lambda_A = 0.2$, $\lambda_D = 0.7$, $\lambda_I = 0$, $\lambda_L = 0.1$, $c_{tm} = c_{ts} = 0.01$, $c_{tms} = -0.005$ ($\varphi_D > 0$) for the red curve, $c_{tms} = 0$ ($\varphi_D = 0$) for the green curve, and $c_{tms} = 0.005$ ($\varphi_D < 0$) for the blue curve, for $t = A, DI, DA$.

ipation cost in the index market, then on average the total capital invested in both stocks increase. This is because after a drop in the participation cost, some investors who previously invest only in the risk-free asset now starts to invest in stocks through the index.

Figures 14 and ?? show that if the rise of indexing is caused by a decrease in the participation cost in the index market, then both stock price variance and the correlations of the returns increase. In Figure ??, the correlation of stock payoffs is set to zero, therefore without indexing, the stock prices would be independent too given the CARA preferences. It is indexing that causes the stock return correlation because indexers effectively buy and sell the stocks in the index together and with the increase in the price informativeness in the index market, the effect of information motivated trades dominates that of liquidity motivated trades.

We next conduct an analysis of welfare. As shown in Figure 15, the social welfare in terms of the aggregate certainty equivalent wealth of investors increases as the participation cost $k_m$ decreases. As shown in Figure 16, the lower participation cost makes investors of Types D, I, and L better off, but it makes Type A investors (the exogenous active investors) worse off. The reduction in the welfare of the Type A investors comes from the increased competition in the index market and/or the non-index market as more investors start to invest in the index and/or non-index markets. However, this reduction in the welfare of Type A investors is dominated by the gain of other investors. In addition, the social welfare is the highest with positive information acquisition externality because of the lower information acquisition cost.

To summarize, if the rise of indexing is due to decreased cost of participating in the index market, then the price informativeness of the index tends to increase and
Figure 13: The expected market capitalization of Stocks 1 and 2 against participation cost $k_m$. The default parameter values are: $\tau_{vm} = \tau_{zs} = \tau_{zm} = \tau_{vs} = 1$, $\gamma = 0.25$, $\mu_{vm} = 0.3$, $\mu_{vs} = 0.05$, $\rho = 0$, $k_s = 0.012$, $\lambda_A = 0.2$, $\lambda_D = 0.6$, $\lambda_I = 0.1$, $\lambda_L = 0.1$, $c_{tm} = c_{ts} = 0.01$, $c_{tms} = -0.005$ for the blue line ($\varphi_D > 0$), $c_{tms} = 0$ for the green line ($\varphi_D = 0$), and $c_{tms} = 0.005$ for the red line ($\varphi_D < 0$), for $t = A, DI, DA$.

Figure 14: The variance and correlation of stock prices against participation cost $k_m$. The default parameter values are: $\tau_{vm} = \tau_{vs} = \tau_{zm} = \tau_{zs} = 1$, $\gamma = 0.25$, $\mu_{vm} = 0.3$, $\mu_{vs} = 0.05$, $\rho = 0$, $k_s = 0.012$, $\lambda_A = 0.2$, $\lambda_D = 0.6$, $\lambda_I = 0.1$, $\lambda_L = 0.1$, $c_{tm} = c_{ts} = 0.01$, $c_{tms} = -0.005$ for the blue line ($\varphi_D > 0$), $c_{tms} = 0$ for the green line ($\varphi_D = 0$), and $c_{tms} = 0.005$ for the red line ($\varphi_D < 0$), for $t = A, DI, DA$.

thus market risk premium decreases, but the price informativeness in the non-index market decreases. In addition, total capital allocated to the stocks increase, and so do the variances and correlation of stock prices. Moreover, the social welfare tends to be increased.

4.3 Effects of Changes in Transparency of the Non-Index Market

We now look at another possible reason for the rise of index investment. Holding everything else constant, if the non-index market becomes more transparent and thus
trading in the non-index market becomes less profitable for informed investors, then we should expect that more discretionary traders optimally choose to be indexers. A
measure of the transparency of the non-index market in our model is the precision of the liquidity trades $\tau_{zs}$ in the non-index market. As $\tau_{zs}$ increases, the non-index market becomes more transparent in the sense that price in the non-index market becomes more informative, as can be seen from Equation (8).

Figure 17: The equilibrium mass of endogenous indexers $\eta^*$ against non-index market transparency $\tau_{zs}$. The default parameter values are: $\tau_{vm} = \tau_{vs} = \tau_{zm} = \tau_{zs} = 1$, $\gamma = 0.25$, $\mu_{vm} = 0.3$, $\mu_{vs} = 0.05$, $\rho = 0$, $k_m = 0$, $k_s = 0.01$, $\lambda_A = 0.2$, $\lambda_D = 0.7$, $\lambda_I = 0$, $\lambda_L = 0.1$, $c_{tm} = c_{ts} = 0.01$, $c_{tms} = -0.005$ ($\varphi_D > 0$) for the red curve, $c_{tms} = 0$ ($\varphi_D = 0$) for the green curve, and $c_{tms} = 0.005$ ($\varphi_D < 0$) for the blue curve, for $t = A, DI, DA$.

Consistent with our intuition, as illustrated in Figure 17, the equilibrium mass of indexers indeed increases regardless of the sign of the IAE as the non-index market transparency $\tau_{zs}$ improves, because the profitability from trading in the non-index market decreases.

In contrast to the case when the rise of index investment is due to a lower participation cost $k_m$ in the index market, the left panel of Figure 18 shows that when the rise of index investment is due to the increased transparency of the non-index market, price informativeness in the non-index market increases regardless of the sign of IAE. This is because the increase in transparency $\tau_{zs}$ dominates the effect of information acquisition externality.

Also different from the case when the rise of index investing is caused by decreases in the participation cost in the index market, the right panel of Figure 18 illustrates that how market risk premium and price informativeness in the index market ($\rho^*_m$) change with the rise of indexing critically depends on the sign of IAE (i.e., the sign of $\varphi_D$). More specifically, if there is positive information acquisition externality (i.e., $\varphi_D > 0$), then $\rho^*_m$ decreases. The opposite is true if there is a negative information acquisition externality (i.e., $\varphi_D < 0$). The intuition is as follows. When there is positive information acquisition externality (i.e., $\varphi_D > 0$), because both A investors and DA investors also acquire information in the non-index market, they have a
Figure 18: The price informativeness against the non-index market transparency $\tau_{zs}$. The default parameter values are: $\tau_{vm} = \tau_{vs} = \tau_{zm} = \tau_{zs} = 1$, $\gamma = 0.25$, $\mu_{vm} = 0.3$, $\mu_{vs} = 0.05$, $\rho = 0$, $k_m = 0$, $k_s = 0.01$, $\lambda_A = 0.2$, $\lambda_D = 0.7$, $\lambda_I = 0$, $\lambda_L = 0.1$, $c_{tm} = c_{ts} = 0.01$, $c_{tms} = -0.005$ ($\varphi_D > 0$) for the red curve, $c_{tms} = 0$ ($\varphi_D = 0$) for the green curve, and $c_{tms} = 0.005$ ($\varphi_D < 0$) for the blue curve, for $t = A, DI, DA$.

lower cost of information acquisition in the index market, and thus obtain more precise information about the index than DI investors, as shown in the blue lines ($\varphi_D > 0$) in Figure 10. As the non-index market becomes more transparent, more and more discretionary active investors who acquire more precise information in the index market switch to discretionary indexers who acquire less precise information. As a result, the price informativeness of the index decreases, and the market risk premium increases.

On the other hand, as the transparency of the non-index market increases, both exogenous active traders and discretionary active traders optimally acquire less precise information in the non-index market. If the information acquisition externality is negative, the reduction in the information acquisition in the non-index market lowers the cost of acquiring information and increases the benefit of trading in the index market. Therefore, both active traders and discretionary traders acquire more precise information about the index which leads to a higher price informativeness of the index and a lower market risk premium.

As for the impact of indexing on capital allocated to stocks, Figure 19 shows that if the rise of indexing is caused by a decrease in the profitability in the non-index market, then on average the total capital invested in both stocks increase if and only if the information acquisition externality is negative. This is because (1) both Stocks 1 and 2 are equivalent to long the index plus a position in the nonindex; (2) as price informativeness increases, investors are more willing to invest in the index and thus expected market cap of the index market increases; and (3) the total supply of the non-index is zero and thus the expected price of the nonindex is $\mu_{vs}$ as shown in (.), which is independent of the price informativeness of the index.
Figure 19: The expected market capitalization of Stocks 1 and 2 against participation cost $k_m$. The default parameter values are: $\tau_{vm} = \tau_{vs} = \tau_{zm} = \tau_{zs} = 1$, $\gamma = 0.25$, $\mu_{vm} = 0.3$, $\mu_{vs} = 0.05$, $\rho = 0$, $k_m = 0$, $k_s = 0.01$, $\lambda_A = 0.2$, $\lambda_D = 0.6$, $\lambda_I = 0.1$, $\lambda_L = 0.1$, $c_{tm} = c_{ts} = 0.01$, $c_{tms} = -0.005$ for the blue line ($\varphi_D > 0$), $c_{tms} = 0$ for the green line ($\varphi_D = 0$), and $c_{tms} = 0.005$ for the red line ($\varphi_D < 0$), for $t = A, DI, DA$.

Figure 20 shows that both stock price variance and correlation can be non-monotonic in the profitability of the non-index market. Because as shown above, the price informativeness of the non-index always increases irrespective of the sign of the information acquisition externality, but the price informativeness of the index decreases with the transparency $\tau_{zs}$ if and only if the externality if positive, this results in the pattern shown in Figure 20: with positive externality, the variance first decreases and then increases with the transparency if the externality is positive, otherwise it increases with the transparency.

On the other hand, the correlation of Stocks 1 and 2 is monotonically increasing in the ratio of the index variance to the non-index variance. If the externality is not negative, then the ratio of the price informativeness of the index to the non-index decreases as shown above. This implies that the ratio of the index variance to the non-index variance decreases, and thus the correlation decreases with the non-index market transparency. If the externality is negative, then the price informativeness in the index market and that in the non-index market change in the same direction. Therefore, if the price informativeness of the index increases faster than the price informativeness of the non-index, then the ratio of the price informativeness may first increase and then decrease, which causes the correlation first increases and then decreases with the transparency in the non-index market, as shown in red line of the right subfigure in Figure 20.

As for welfare, Figure 21 shows that if the rise of indexing is caused by the improvement in the transparency in the non-index market, then the social welfare can increase regardless of the sign of the information acquisition externality. This is because as the transparency in the non-index market improves, while some investors like the active investors may be worse-off, the uninformed investors such as the liquidity traders may be better-off due to less information asymmetry.
5. Conclusion

We study the equilibrium effects of the rise of indexing on price discovery, capital allocation, and stock correlations. We show that these effects critically depend on the causes of the rise of indexing, the cost structure of information acquisition, and whether some investors can endogenously choose between active investors and indexers. For example, if the rise of indexing is due to an increase in the exogenous indexers, as in Bond and Garcia (2017), then the price informativeness in the index market decreases and the market risk premium increases, opposite to the finding of Bond and Garcia (2017). In addition, expected market capitalizations, variances of...
stock prices, stock price correlations, and the social welfare tend to decrease. In contrast, if the rise of comes from a decrease in the participation cost in the index market, then the price informativeness in the index market goes up and the market risk premium goes down. In addition, the social welfare increases. Moreover, we show that because discretionary indexers can acquire information before and after they become indexers. As a result, active investors may sometimes free ride on discretionary indexers for information acquisition.

Our analysis highlights that the rise of indexing might have opposite effects if the mechanisms that drive the rise of indexing are different. These predictions can help researchers use empirically found relationship between the rise of indexing and price informativeness to identify what likely caused the increase in indexing and thus offer regulators some guidance on regulations adjustment if necessary.
References


Appendix A

In this Appendix, we provide proofs of analytical results.

Proof of Theorem 1:

For \( t = A, DA, DI, I \) and \( j = m, s \), the optimal number of shares of security \( j \) bought by investor \( i \) of type \( t \) is

\[
\Theta_{tji} = \frac{E[V_j|I_i] - P_j}{\gamma \text{Var}[V_j|I_i]} - s_j,
\]

where \( s_m = 1 \) and \( s_s = 0 \). The information set of investor \( i \) of type \( t \) is \( I_i = (Y_{tmi}, Y_{tsi}, P_m, P_s) \), where the precision of \( Y_{tji} \) \((j = m, s)\) is zero for \( t = I, L \), and the precision of \( Y_{tsi} \) is zero \( t = DI \). Because \( Y_{tmi} \) and \( Y_{tsi} \) are independent, and \( V_m \) and \( V_s \) are independent, the conditional expectation of \( V_j \) only depends on \((Y_{tji}, P_j)\). Direct computation yields that for \( t = A, D, I \) and \( j = m, s \),

\[
E[V_j|I_i] = \frac{d_j^2(Y_{tji}\tau_{tji} + \mu_{v_j}\tau_{v_j}) + b_j(-a_j + P_j)\tau_{z_j}}{d_j^2(\tau_{tji} + \tau_{v_j}) + b_j^2\tau_{z_j}},
\]

and

\[
\text{Var}[V_j|I_i] = \left(\tau_{tji} + \tau_{v_j} + \frac{b_j^2}{d_j^2}\tau_{z_j}\right)^{-1},
\]

where \( a_s = 0 \) and \( \mu_{vs} = 0 \). Using the market clearing conditions

\[
\sum_{t \in \{A, DI, DA, I\}} \int \Theta_{tji}di - \lambda_LZ_j = 0,
\]

where the integration is over all investors of the same type, Setting the coefficients of \( V_j \) and \( Z_j \) and the constant term to be zero, we get the results in Theorem 1.

Proof of Theorem 2:

Define \( r_j = \frac{\tau}{\lambda\gamma} \) for \( j = m, s \). First, it is straightforward to show that type A and type DA investors choose the precisions \((\tau_{tmi}, \tau_{tsi})\) to maximize

\[
-\gamma C_t(\tau_{tmi}, \tau_{tsi}) + \frac{1}{2} \log(\tau_{tmi} + \tau_{vm} + r_m^2\tau_{zm}) + \frac{1}{2} \log(\tau_{tsi} + \tau_{vs} + r_s^2\tau_{zs}), \quad t \in \{A, DA\},
\]

\[
(A-5)
\]
while type DI investors choose the precision $\tau_{tmi}$ to maximize

$$\quad -\gamma C_t(\tau_{tmi}, 0) + \frac{1}{2} \log(\tau_{tmi} + \tau_{vm} + r_m^2 \tau_{zm}), \quad t = DI. \quad (A-6)$$

Under Assumption 1, it can be easily verified that the objective functions are all globally strictly concave in the choice precision variables, and therefore given $r_m$ and $r_s$, there are unique solutions. Since investors of the same type choose the same precisions, we omit the index $i$. Define the optimal precision functions as

$$\quad \tau^*_t = f_t^j(r_m, r_s), \quad j \in \{m, s\}, \quad t \in \{A, DA, DI\}, \quad (A-7)$$

with $f_s^{DI}(r_m, r_s) = 0$. Taking derivatives with respect to $r_m$ in the first order conditions (22)-(23), Assumption 1 then implies that for $t = A, DA$,

$$\quad \frac{\partial f^t_m(r_m, r_s)}{\partial r_m} < 0, \quad \text{Sign} \left( \frac{\partial f^t_m(r_m, r_s)}{\partial r_s} \right) = -\text{Sign} (\varphi_D) \quad (A-8)$$

and

$$\quad \frac{\partial f^{DI}_m(r_m, r_s)}{\partial r_m} < 0, \quad \text{Sign} \left( \frac{\partial f^{DI}_m(r_m, r_s)}{\partial r_s} \right) = 0 \quad (A-9)$$

By a similar argument, we have for $t = A, DA$,

$$\quad \frac{\partial f^t_s(r_m, r_s)}{\partial r_s} < 0, \quad \text{Sign} \left( \frac{\partial f^t_s(r_m, r_s)}{\partial r_m} \right) = -\text{Sign} (\varphi_D) \quad . \quad (A-10)$$

Note that in equilibrium

$$\quad r_m = \frac{\lambda_A \tau_{Am} + (\lambda_D - \eta^*) \tau_{DAm} + \eta^* \tau_{DIM}}{\gamma \lambda_L} \quad (A-11)$$

and

$$\quad r_s = \frac{\lambda_A \tau_{As} + (\lambda_D - \eta^*) \tau_{DAs}}{\gamma \lambda_L} \quad . \quad (A-12)$$

Therefore, we must show that there is a unique solution $(r^*_m, r^*_s)$ to the equations

$$\quad f_m(r_m, r_s) \equiv \lambda_A f^A_m(r_m, r_s) + (\lambda_D - \eta^*) f^{DA}_m(r_m, r_s) + \eta^* f^{DI}_m(r_m, r_s) - \gamma \lambda L r_m = 0 \quad (A-13)$$

and

$$\quad f_s(r_m, r_s) \equiv \lambda_A f^A_s(r_m, r_s) + (\lambda_D - \eta^*) f^{DA}_s(r_m, r_s) - \gamma \lambda L r_s = 0 \quad . \quad (A-14)$$
It is clear that for any given \( r_s \), \( f_m(0, r_s) > 0 \) and \( f_m(\infty, r_s) < 0 \) because as optimal precisions \( f_{im}^A(0, r_s) \geq 0 \), \( f_{im}^{DA}(0, r_s) \geq 0 \), \( f_{im}^{Dl}(0, r_s) > 0 \), and \( f_m^t(\infty, r_s) = 0 \) for \( t \in \{A, DA, DI\} \) as implied by the first order conditions. In addition, we have \( \frac{\partial f_m(r_m, r_s)}{\partial r_m} > 0 \) by (A-8) for any given \( r_s \). Therefore, for any given \( r_s \), there is a unique positive solution \( r_m = g(r_s) \) such that equation (A-13) holds. In addition, by implicit function theorem,

\[
g'(r_s) = -\frac{\partial f_m(r_m, r_s)}{\partial r_m}.
\]  

(A-15)

Plugging \( r_m = g(r_s) \) into the second equation (A-14), we have \( f_s(g(r_s), r_s) = 0 \). We have \( f_s(g(0), 0) > 0 \) because the precisions \( f_{is}^A(r_m, r_s) \) and \( f_{is}^{DA}(r_m, r_s) \) are all positive for any \( r_m \) and \( f_s(g(\infty), \infty) < 0 \) because \( f_{is}^A(r_m, \infty) = 0 \) and \( f_{is}^{DA}(r_m, \infty) = 0 \) for any \( r_m \). In addition, using the first order conditions and Assumption 1, through straightforward but tedious calculation of Jacobian matrix \( J \) of \( (f_m, f_s) \), one can show that

\[
\frac{df_s(g(r_s), r_s)}{dr_s} = \frac{\partial f_s(r_m, r_s)}{\partial r_m} g'(r_s) + \frac{\partial f_s(r_m, r_s)}{\partial r_s} = \frac{|J|}{\frac{\partial f_m(r_m, r_s)}{\partial r_m}} < 0
\]

(A-16)

for all \( r_s > 0 \), because \(|J|\) can be verified to be strictly positive. By continuity and monotonicity, there must exist a unique solution \( r_s^* > 0 \) to \( f_s(g(r_s), r_s) = 0 \) which implies that there exist unique \( r_m^* = g(r_s^*) > 0 \) and \( r_s^* > 0 \) that solve \( f_m(r_m, r_s) = 0 \) and \( f_s(r_m, r_s) = 0 \). Therefore, there exists a unique equilibrium for a given \( \eta^* \). To show the existence and uniqueness of \( \eta^* \), note that when the participation cost \( k = 0 \), it is always better to invest in both of the risky assets because of the diversification effect, so the fraction of DI among discretionary investors is zero, while when \( k = \infty \), it is always better to always invest only in the market portfolio, and so the fraction of DI is one. The negative of the log of the ratio of the utility of DA investors to that of DI investors, which has the same sign of the difference in utilities, is equal to

\[
h(\eta) = -\gamma(C_D(\tau_{DAm}, \tau_{DIM}) + k) - \frac{1}{2} \log \left( \frac{\tau_{em} + \tau_{DIM} + \tau_{m}^2 \tau_{zm}}{\tau_{em} + \tau_{DAm} + \tau_{m}^2 \tau_{zm}} \right) - \frac{1}{2} \log \left( \frac{\left( \tau_{im} + \tau_{DAs} + \tau_{s}^2 \tau_{zs} \right) \left( \tau_{im} + \left( \frac{\gamma_{L}}{\lambda_A + \lambda_D - \eta^*} + \tau_{s} \tau_{zs} \right)^2 \right)}{\tau_{zs} \left( \frac{\gamma_{L}}{\lambda_A + \lambda_D - \eta^*} + \tau_{zs} + \tau_{s}^2 \tau_{zs} \right)^2} \right).
\]

(A-17)

Note that \( \tau_m, \tau_s, \tau_{DAm}, \tau_{DAs}, \) and \( \tau_{DIM} \) are all functions of \( \eta \) and \( \eta^* \) solves \( h(\eta) = 0 \). Using the first order conditions, Propositions 3 and 2, it can be shown that \( h'(\eta) > 0 \). Therefore, there exists a unique fraction of DI \( \eta^* \) such that the utility of DA is equal to that of DI, and thus there is a unique equilibrium.
Proof of Proposition 1:

Part 1. Suppose given \( k = k_0 \) the equilibrium endogenous indexing is \( \eta_0^* \). When \( k \) is increased to \( k_1 \), the utility of DA becomes smaller than that of DI because DI investors do not pay the participation cost. Therefore, some DA investors must switch to be DI investors and thus the new equilibrium endogenous indexing \( \eta_1^* \) must be greater than \( \eta_0^* \).

Part 2. Taking derivative in \( h(\eta^*) = 0 \) with respect to \( k \), using the Envelope Theorem for the endogenous precisions (which are all functions of \( r_m, r_s, \) and \( \eta^* \)), we have

\[
\frac{\partial h}{\partial \eta^*} \frac{\partial \eta^*}{\partial k} - \gamma = 0, \tag{A-18}
\]

which implies that

\[
\frac{\partial h}{\partial \eta^*} > 0, \tag{A-19}
\]

because \( \frac{\partial \eta^*}{\partial k} > 0 \) by Part 1.

Taking derivative in \( h(\eta^*) = 0 \) with respect to \( \tau_{zm} \), we have

\[
\frac{\partial h}{\partial \tau_{zm}} + \frac{\partial h}{\partial \eta^*} \frac{\partial \eta^*}{\partial \tau_{zm}} = 0. \tag{A-20}
\]

It can be shown by direct computation that

\[
\text{Sign} \left( \frac{\partial h}{\partial \tau_{zm}} \right) = \text{Sign}(\varphi_D). \tag{A-21}
\]

Therefore, we have

\[
\text{Sign} \left( \frac{\partial \eta^*}{\partial \tau_{zm}} \right) = -\text{Sign}(\varphi_D). \tag{A-22}
\]

Part 3. Taking derivative in \( h(\eta^*) = 0 \) with respect to \( \tau_{zs} \), we have

\[
\frac{\partial h}{\partial \tau_{zs}} + \frac{\partial h}{\partial \eta^*} \frac{\partial \eta^*}{\partial \tau_{zs}} = 0. \tag{A-23}
\]

It can be shown by direct computation that

\[
\frac{\partial h}{\partial \tau_{zs}} < 0. \tag{A-24}
\]
Therefore, we have
\[ \frac{\partial \eta^*}{\partial \tau_{zs}} > 0. \] (A-25)

Part 4. Taking derivative in \( h(\eta^*) = 0 \) with respect to \( \lambda_I \), we have
\[ \frac{\partial h}{\partial \lambda_I} + \frac{\partial h}{\partial r_m} \frac{\partial r_m}{\partial \lambda_I} + \frac{\partial h}{\partial r_s} \frac{\partial r_s}{\partial \lambda_I} + \frac{\partial h}{\partial \eta^*} \frac{\partial \eta^*}{\partial \lambda_I} = 0. \] (A-26)

It can be shown by direct computation that
\[ \frac{\partial h}{\partial \lambda_I} > 0 \] (A-27)

and if \( \varphi_D = 0 \), then
\[ \frac{\partial h}{\partial r_m} = 0, \quad \frac{\partial h}{\partial r_s} < 0, \quad \frac{\partial r_s}{\partial \lambda_I} < 0. \] (A-28)

Therefore, we have
\[ \frac{\partial \eta^*}{\partial \lambda_I} < 0. \] (A-29)

**Proof of Proposition 2:**

Taking derivative with respect to \( r_m \) in (A-13), we have
\[ \lambda_A \frac{\partial \tau_A^*}{\partial r_m} + (\lambda_D - \eta^*) \frac{\partial \tau_{DA}^*}{\partial r_m} + \eta^* \frac{\partial \tau_{DI}^*}{\partial r_m} - (\tau_{DA}^* - \tau_{DI}^*) \frac{\partial \eta^*}{\partial r_m} - \gamma \lambda_L = 0. \] (A-30)

The last term and the first three terms are all negative. This implies that the fourth term must be positive. As shown above \( \tau_{DA}^* - \tau_{DI}^* \) has the same sign as information acquisition externality \( \varphi_D \), therefore \( \frac{\partial \eta^*}{\partial r_m} \) must have the opposite sign of \( \varphi_D \). the claim then follows because \( r_m \) moves in the same direction as the price informativeness \( \rho_m \) holding \( \lambda_L, \tau_{vm} \) and \( \gamma \) constant. Similarly, one can show the second part of the proposition.

**Proof of Proposition 3:**

Because \( \tau_{DA_m} \) solves (note that the information acquisition cost function \( C \) is the same for DA and DI investors):
\[ 2\gamma C_{Dm}(\tau_{DA_m}, \tau_{DA_s}) = \frac{1}{\tau_{DA_m} + \rho_m}. \] (A-31)
If $\varphi_D > 0$, then we have

$$2\gamma C_{Dm}(\tau_{DAm}, 0) > 2\gamma C_{Dm}(\tau_{DAm}, \tau_{DAs}) = \frac{1}{\tau_{DAm} + \rho_m} \quad (A-32)$$

which implies that $\tau_{DAm} > \tau_{DIm}$ because $\tau_{DIm}$ solves

$$2\gamma C_{Dm}(\tau_{DIm}, 0) = \frac{1}{\tau_{DIm} + \rho_m}. \quad (A-33)$$

and $C_{Dm}(\tau_{DAm}, 0)$ increases in $\tau_{DAm}$. The case where $\varphi_D \leq 0$ can be shown similarly.