Bank Information Sharing and Liquidity Risk*

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10 Feb 2019

Abstract

This paper proposes a novel rationale for the existence of bank information sharing schemes. We suggest that banks can voluntarily disclose borrowers’ credit history to maintain asset market liquidity. By entering an information sharing scheme, banks face less adverse selection when selling their loans in secondary markets. This reduces the cost of asset liquidation in case of liquidity shocks. Information sharing arises endogenously when the liquidity benefit dominates the concern of reduced profitability due to primary loan market competition. We show banks have incentives to truthfully disclose borrower credit history, even if such information is non-verifiable, and also provide a rationale for promoting public credit registries.

JEL Classification: G21.

Keywords: Information Sharing, Funding Liquidity Risk, Market Liquidity, Adverse Selection in Secondary Market.

*We thank Thorsten Beck, Christoph Bertsch, Sudipto Dasgupta, Hans Degryse, Xavier Freixas, Chong Huang (discussant), Artashes Karapetyan (discussant), Michal Kowalik (discussant), Vasso Ioannidou, Marco Pagano, Ettore Panetti, Francesc Rodriguez Tous (discussant), Kasper Roszbach, Cindy Vojtech (discussant), and Lucy White for insightful comments and discussions. We are also grateful to conference participants at IBEFA 2016, FIRS 2016, AEA 2017, UECE 2017 Lisbon Game Theory meetings, Atlanta Fed workshop on "The role of liquidity in the financial system", 9th Swiss winter conference on financial intermediation in Lenzerheide, Cass Business School 2018 workshop on "Financial System Architecture and Stability" and seminar attendants at Hong Kong University, Riksbank, Tilburg University, University of Bristol, University of Gothenburg, University of Lancaster, University of Warwick for useful comments. The usual disclaimer applies.

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1 Introduction

One of the reasons for the existence of banks is their liquidity transformation service provided by borrowing short-term and lending long-term. The funding liquidity risk is a natural by–product of the banks’ raison d’être (Diamond and Dybvig, 1983). This paper argues that such funding risk can be at the root of the existence of information sharing agreements among banks. The need for information sharing arises because banks in need of liquidity may have to sell their assets in secondary markets, and information asymmetry in such markets can make asset liquidation costly. Information sharing allows banks to reduce adverse selection in secondary loan markets, which in turn reduces the damage of premature liquidation.¹

The benefit of information sharing, however, has to be traded off with its potential cost. Letting other banks learn the credit worthiness of its own borrowers, an incumbent bank sacrifices its market power. As its competitors forcefully compete for the good borrowers, the incumbent bank would face lower profitability due to intensified primary loan market competition. Our paper provides a throughout analysis of this trade-off.

Our theory of bank information sharing is motivated by observations of US consumer credit markets (such as markets for mortgages and credit cards). These markets are competitive and contestable. At the same time, banks are able to securitize and re-sell loans originated in these markets. We argue that the two features are linked and both related to credit information sharing. On the one hand, the shared information on a borrower’s credit history—typically summarized by a FICO score—reduces significantly the asymmetric information about the borrower’s creditworthiness. This enables banks to compete for the borrower with which they have no previous lending relationship. On the other hand, the resulting loan is more marketable because the information contained in the FICO score signals the borrower’s creditworthiness such that the potential buyers of the loan do not fear the winners’ curse. In sum, the shared credit history both intensifies primary credit market competition and promotes secondary market liquidity.

The observation that the information sharing has helped to promote securitization in the US has inspired European regulators. In their effort to revive the securitization market

¹While we mainly model an asset sale, similar arguments can be made for collateralized borrowing and securitization, where the reduced adverse selection will lead to lower haircut and higher prices for securitized assets. We discuss those in Section 5.
in the post-crisis Europe, the European Central Bank and the Bank of England have jointly pointed out that “credit registers could also improve the availability quality of information that could, in principle, also benefit securitization markets by allowing investors to build more accurate models of default and recovery rates” (BoE and ECB, 2014). Our paper provides theoretical supports that credit information sharing schemes can indeed promote asset marketability by reducing information asymmetry, and such schemes are sustainable as it can be in banks’ own interests to share the information.

We consider an economy populated by two banks, one borrower, many depositors and asset buyers. We refer to the first bank as the incumbent bank since it has an existing lending relationship with the borrower. The borrower can be safe or risky, and both types have positive NPV projects. However, a safe borrower’s project will surely succeed while a risky borrower’s does so only with a certain probability. The incumbent bank knows both the type (i.e., the credit worthiness) and the credit history (i.e., any past default) of the borrower. The second bank, which we label the entrant bank, has no lending relationship with the borrower and therefore it does not posses any information about the borrower. The entrant bank can, however, compete for the borrower by offering competitive loan rates. It can still lose from lending if not pricing the loan correctly.

The incumbent bank is subject to liquidity risk, which we model as a possibility of a bank run. When the liquidity need arises, the incumbent bank can sell the loan granted to the borrower on a secondary market. When the quality of the loan is unknown to outsiders, the secondary market for the loan is characterized by adverse selection. Even when holding a loan granted to a safe borrower, the incumbent bank can incur the risk of bankruptcy by selling it at a discount. Ex ante, the incumbent bank then may voluntarily share the information about the borrower to obtain a higher liquidation value for the loan on sale.

We take a progressive approach and characterize the equilibrium under different assumptions on the kind of information that the incumbent bank can share. We start by analyzing the simplest but admittedly unrealistic scenario where the information on the borrower’s type can be shared and is verifiable. This allows us to illustrate the main mechanism our model.

We then consider the more realistic setting in which the soft information on borrower type cannot be communicated and only the hard information on the borrower’s credit history can be shared. When borrower’s credit history is verifiable, we first we pin down
the conditions under which information sharing can sufficiently boost the liquidation value of the loan on sale so as to save the incumbent bank from the run. This result is non-trivial because information sharing has two countervailing effects on the secondary-market price of loans. On the one hand, observing a good credit history, asset buyers perceive a better average quality of the loan on sale since the loan is more likely to the safe borrower. On the other hand, the entrant bank competes more aggressively for the loan for exactly the same reason, which would reduce the face value of the loan. We show that the first effect always dominates and there exists a range of parameters where sharing information is beneficial for the incumbent bank.

We then characterize the conditions when the incumbent bank voluntarily shares information. These conditions coincide with the aforementioned existence conditions if the incumbent bank’s funding liquidity risk is sufficiently high. Otherwise, the parameter constellation in which the incumbent bank chooses to share information is smaller than the region in which it is beneficial. When funding risk is relatively low, the expected benefit of the higher liquidation value of the loan on sale are outweighed by the reduction in expected profits due to intensified competition.

Finally, we analyze the equilibrium by relaxing the common assumption in the literature that the credit history, once shared, is verifiable. When the reported credit history is not verifiable, the incumbent bank may misreport the borrower’s true default history. In particular, we give the incumbent bank the possibility to overstate the borrower’s past repayment performances to get a higher price for the loan on sale. We show that the incumbent bank still has an incentive to truthfully disclose its borrower’s credit history. As expected, however, such incentive exists in a parameter constellation narrower than the one in which the incumbent bank voluntary chooses to share information under the assumption of verifiable credit history. It turns out that a necessary condition for information sharing

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2Such assumption, beside being restrictive from a theoretical point of view, is also questionable from a practical point of view. Giannetti et al. (2017) show that banks manipulated their internal credit ratings of their borrowers before reporting to Argentinian credit registry. On a more casual level, information manipulation can take place in the form of ‘zombie’ lending, like it occurred in Japan with the ever-greening phenomenon or in Spain where banks kept on lending to real estate firms likely to be in distress after housing market crash.

3An alternative and established way to sustain truth telling is to consider a dynamic game where the incumbent bank has some reputation at stake. We show that truth telling can be sustained in equilibrium even in a static game.
to be sustained as a truth-telling equilibrium is when the incumbent bank can capture only 
the borrower with default credit history (but still with positive NPV project) by charging a 
sufficiently high loan rate. This occurs most likely when the primary market is particularly 
competitive, that is the entrant bank competes fiercely for the borrower with no default 
credit history. When, instead, the entrant bank finds not convenient to compete on the 
primary market, the incumbent bank can capture the borrower independently of the credit 
history and then has an incentive to cheat.

We provide interesting normative results. First of all, notice that a social planner 
would implement the first-best by always revealing the borrower’s type. However, when 
the decision about revealing the borrower’s type is left to the incumbent bank, an inefficient 
amount of information sharing is chosen and a public registry is necessary. Indeed, the 
incumbent bank does not find privately optimal to share information on the borrower’s 
type when $\rho$ is too low. Quite counter intuitively even allowing the incumbent bank to 
share borrower’s type does not implement the first best. When the incumbent bank decide 
to share or not the borrower’s credit history again we have inefficiencies. Similarly to the 
previous case, when credit history is verifiable there are cases in which information sharing 
is beneficial to boost the liquidation value of the loan on sale but the incumbent bank does 
not voluntary share such information (note: cases 1 and 2) [note: add briefly the root 
of the inefficiency? which should be the same as when sharing types]. More 
interestingly, when credit history is not verifiable there are cases (note: case 2) where 
information sharing is beneficial and the incumbent bank ex ante does not want to share 
information but ex post it would report the true borrower’s credit history. [note: again 
add intuition]. These are clearly cases in which imposing a public registry is welfare 
improving.

Our model should be interpreted with two caveats. First, historically, governments’ 
goal in creating public credit registries has been to improve SMEs’ access to financing in 
primary loan markets. Our theory does not deny this benefit but shows that an overlooked 
benefit of information sharing is the development of secondary markets for loans. Second, 
it is not our intention to claim that information sharing is the main reason for the explosion 
of the markets for asset-backed securities. It is ultimately an empirical question to what 
extent information sharing had fueled such markets expansion.

Our conjecture that information sharing is driven by market liquidity is novel and com-
plementary to existing rationales. Previous literature has mostly explained the existence of information sharing by focusing on the primary loan market. In their seminal paper, Pagano and Jappelli (1993) rationalize information sharing as a mechanism to reduce adverse selection. Sharing ex-ante more accurate information about borrowers reduces their riskiness and increases banks’ expected profits. Similarly, information sharing can mitigate moral hazard problems (Padilla and Pagano, 1997 and 2000). We see information sharing as stemming also from frictions on the secondary market instead of only on the primary loan market. The two explanations are not mutually exclusive.

Another strand of the literature argues that information sharing allows the incumbent bank to extract more monopolistic rent. When competition for borrowers occurs in two periods, inviting the competitor to enter in the second period by sharing information actually dampens the competition in the first period (Bouckaert and Degryse, 2004; Gehrig and Stenbacka 2007). Sharing information about the past defaulted borrowers deters the entry of competitor, which allows the incumbent bank to capture those unlucky but still good borrowers (Bouckaert and Degryse, 2004). This mechanism is also present in our model and is instrumental in sustain truth-telling when borrower credit history is non-verifiable.

Other than providing explanations for voluntary credit information sharing, the literature also examines how information sharing affects banks’ lending strategies. For example, information sharing can complement collateralization since banks are able to impose high collateral requirement after the high-risk borrowers are identified via information sharing (Karapetyan and Stacescu 2014b). Information sharing can also induce information acquisition: After hard information is communicated, collecting soft information becomes a more urgent task for the bank to boost its profits (Karapetyan and Stacescu 2014a).

There is plenty of micro-evidence supporting our key element of the model: the friction in the secondary loan market. Buyers in this market price the assets depending on hard information (credit history) even though bad loans sometime are sold. For example, Keys et al. (2010) find that delinquency rates are higher for loans with FICO scores just above 620 as compared to loans with FICO scores just below 620. Rajan et al. (2015) report that lenders have an incentive to originate loans that rate high based on characteristics that are reported to investors, even if other unreported variables imply a lower borrower quality.
The empirical literature on information sharing, following the existing theoretical literature, has mostly focused on the impact of credit registries on banks’ risk exposure and firms’ access to bank financing. For example, Djankov, McLiesh and Shleifer (2007) shows how private credit increases after the introduction of credit registries, in particular their positive role is found in developing countries. Brown et al. (2009) show that information sharing improves credit availability and lower cost of credit to firms in transition countries. Houston et al. (2010) find that information sharing is correlated with lower bank insolvency risk and likelihood of financial crisis. Doblas-Madrid and Minetti (2013) provide evidence that information sharing reduces contract delinquencies.

Even if this evidence is broadly supportive of our predictions, our theoretical exposition also opens road for future empirical research. The model implies that information sharing will facilitate banks’ liquidity management and loan securitization. Moreover, the model suggests that information sharing system can be more easily established in countries with competitive banking sector, and in credit market segments where competition is strong. More recent evidence seems supportive of the last prediction. Liberti, Sturgess and Sutherland (2018) show that lenders have incentives to share information when doing so can reduce adverse selection problems that inhibit entry to competitive credit markets.

The remainder of this paper is organized as follows. Section 2 presents the model. In Section 3 we determine the conditions under which information sharing arises endogenously under the assumption that the information shared is on verifiable borrower’s type (3.1), then when the shared information is about verifiable credit history (Section 3.2) and, finally, when the shared credit history is not verifiable (Section 3.3). We also show in Section ?? that even if it is feasible to share borrower types, sharing credit history is a more profitable choice of the incumbent bank. We provide in Section 4 a rationale for public registries: Even if the incumbent bank finds it unprofitable share borrower credit history ex-ante, once it is required to provide such information, the bank will do so truthfully. Section 5 discusses several robustness and modeling choices. Section 6 concludes.

2 The Model

We consider a four-period economy with dates \( t = 0, 1, 2, 3, 4 \). The economy is populated by the following agents: two banks (an incumbent bank and an entrant bank), a borrower,
and many depositors as well as potential buyers of bank assets. All agents are risk neutral. The gross return of the risk-free asset is equal to $r_f$.

We assume that a loan opportunity appears at $t = 2$ and pays off at $t = 4$. The loan requires 1 unit of initial funding, and its return depends on the type of the borrower. The borrower can be either safe ($H$-type) or risky ($L$-type). The ex-ante probability for the safe type is $\alpha$, i.e., $\Pr(H) = \alpha$ and $\Pr(L) = 1 - \alpha$. A safe borrower generates a payoff $R > r_f$ with certainty, whereas a risky borrower generates a payoff that depends on a publicly-observed aggregate state $s \in \{G, B\}$ which realizes at $t = 3$. In the good state $G$, a risky borrower generates the same payoff $R$ as a safe borrower, but the borrower only generates a payoff of 0 in the bad state $B$. The ex-ante probabilities of the two states are $\Pr(G) = \pi$ and $\Pr(B) = 1 - \pi$, respectively. One can interpret the $H$-type being a prime mortgage borrower and the $L$-type being a subprime borrower. While both can pay back their loans in a housing boom ($s = G$), the subprime borrowers will default in a sluggish housing market ($s = B$). We assume

$$\pi R > r_f$$

so that both types of loans have positive NPVs and it is ex ante profitable to lend to both types.\footnote{Suppose the incumbent bank only finds it profitable to finance $H$-type of borrower. Then the fact that the incumbent bank lends a borrower fully reveals that the borrower is an $H$-type, so that credit information sharing schemes play no role, which is inconsistent with the empirical ubiquitous presence of such schemes.}

In the example of mortgage loans, the assumption requires the probability of a housing market boom to be sufficiently large.

The incumbent bank has an ongoing lending relationship with the borrower and privately observes both the borrower’s type (i.e., the credit worthiness) and the credit history (i.e., previous repayments). We analyze the model under the assumption that both kind of information can be shared.\footnote{Notice, however, that credit worthiness can be considered soft information which is difficult to communicate to third parties. Instead credit history has the feature of hard information that can be easily shared with outsiders.} We model the decision to share borrower’s information as a unilateral decision of the incumbent bank at $t = 0$. We denote the information sharing regime as $i \in \{N, S\}$, where $N$ refers to the no information sharing regime and $S$ as the regime with information sharing. When $i = S$, the incumbent bank makes a public announcement in $t = 1$ about either the borrower’s type or the credit history. We indicate
a credit history without previous defaults by $\overline{D}$ and a credit history with defaults by $D$. While the safe borrower has a good credit history $\overline{D}$ with probability 1, the risky borrower has a good credit history $\overline{D}$ with probability $\delta$ and a bad credit history $D$ with probability $1 - \delta$. One may interpret the default as a late repayment on the borrower’s debt (for example, credit card). While the safe type never misses a repayment, the risky type incurs in a late repayment with probability $1 - \delta$.

The entrant bank has no lending relationship with the borrower. It observes no information about the borrower’s type and credit history unless the incumbent bank decides to share such information. The entrant bank can compete in $t = 2$ for the borrower by offering competitive loan rates, but to initiate the new lending relationship it has to pay a fixed cost $c$. Such a cost instead represents a sunk cost for the incumbent bank. We focus on

$$R > c + r_f$$

so that the entrant bank can at least compete for borrower when knowing that the borrower is risk. For future illustration, we define $C \equiv c + r_f$, which is the cost of the entrant bank to finance a risk-free borrower. Throughout the paper, we focus on the case where inequality (2) binds before (1) does, which implies

$$c > \frac{1 - \pi}{\pi} r_f.$$  

Condition (3) satisfies when the probability of State $G$ is relatively high.

The bank that wins the loan market competition will be financed solely by deposits. We abstract from the risk-shifting incentives induced by equity holders’ limited liability. Depositors are assumed to be price-takers who only demand to earn the risk-free rate $r_f$ in expectation. The winner bank holds the market power to set the deposit rate, but we assume perfect market discipline so that deposit rates are determined based on the bank’s

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6This is equivalent to assume a preliminary (i.e., in $t = -1$) round of lending that pays off at $t = 1$ between the incumbent bank and the borrower. In this preliminary round of lending, the safe borrower would generate no default history, and the risky borrower would default with a probability $1 - \delta$.

7Notice that the realization of the aggregate state $s$ is independent of the credit history of the borrower which captures an idiosyncratic risk. In the example of mortgage loans, the probability of a housing market boom is independent of the borrower’s repayment record on, for example, his credit card debt.

8The fixed cost $c$ can be interpreted as the cost that the entrant bank has to bear to establish new branches, to hire and train new staffs, or to comply to any financial regulations. Alternatively, it can represent the borrower’s switching cost that is paid by the entrant bank.
riskiness. Similar to the entrant bank, depositors know the borrower’s type or credit history only if the incumbent bank shares such information. But in the case where the incumbent bank wins the loan market competition, the depositors may also infer the borrower’s type from the deposit rate offered by the incumbent bank. The game between the incumbent bank and the depositors is essentially a signaling game, and throughout the paper we assume that the depositors have an off-equilibrium path belief worse than the prior.\(^9\) To capture the funding liquidity risk, we assume that the incumbent bank faces a run at \(t = 3\) with a probability \(\rho\).\(^10\) We interpret also the risk of a run as an idiosyncratic risk and then independent of the aggregate state \(s\). When the run happens, all depositors withdraw their funds, and the incumbent bank has to raise liquidity to meet the depositors’ withdrawals.\(^11\)

We assume that the loan is indivisible and the bank has to sell it as a whole. Upon the run, the bank can sell the loan on a competitive secondary market to risk-neutral asset buyers who only require to break even in expectation. Asset buyers observe the state \(s\) that realizes in \(t = 3\), but they do not have any information on the borrower. Therefore, while they condition their bids on the state, they can condition on the borrower’s type or credit history only if the incumbent bank shared such information in \(t = 1\).

It is the incumbent bank’s private information whether it faces a run or not. Therefore, whenever the incumbent bank decides not to share information on the borrower, the loan can be on sale for two reasons: either due to funding liquidity needs, in which case an \(H\)-type loan can be on sale, or due to a strategic sale for arbitrage, in which case only an \(L\)-type loan will be on sale. The possibility of a strategic asset sale leads to adverse selection in the secondary asset market. An \(H\)-type loan will be underpriced in an asset sale, and

\[^9\]This includes the usual assumption that when off-equilibrium action is observed, the players have the worst belief about the loan quality.

\[^10\]While the incumbent bank faces the liquidity risk the entrant bank does not face such risk. Notice however that we give the incumbent bank the possibility to manage liquidity risk by (unilaterally) share information on the borrower. The model setup is symmetric in this respect. If also the entrant bank would face the liquidity risk, to restore symmetry, we should endow also such bank the decision on sharing information on its borrower. We discuss in Section 5 the case in which information sharing may emerge as mutual agreement among banks.

\[^11\]One can think that bank run is triggered by a sun-spot event as in Diamond and Dybvig (1983). This is also a feature of bank runs based on global games with arbitrarily precise private signal. When a run happens, then all depositors run on the bank. We discuss the robustness of the mechanism to endogenous bank run risks in Section 5.5.
even a solvent incumbent bank that owns an $H$-type loan can fail due to illiquidity. In the case of a bank failure, we assume that bankruptcy costs results in zero salvage value. Since the shared information reduces adverse selection and boosts asset liquidity, the funding liquidity risk and costly liquidation may give the incumbent bank the incentive to disclose borrower’s information.

The sequence of events is summarized in Figure 1.

![Insert Figure 1 here]

The timing captures the fact that information sharing is a long-term decision (commitment), while the competition in the loan market and the liquidity risk faced by the bank are shorter-term concerns.\footnote{Notice that it is necessary to assume that information sharing decision ($t = 0$) is made before the incumbent bank acquires the borrower’s information ($t = 1$). Otherwise, information sharing decision itself may serve as incumbent bank’s signaling device.}

## 3 Equilibrium Information Sharing

Notice that, once the incumbent bank has chosen an information sharing regime $i$, we face a well-defined game $g_i$. Therefore, we can determine the incumbent bank’s payoffs in each self-contained game $g_i$, and the incumbent bank then chooses the information sharing regime $i$ that delivers the highest expected payoff.

In Section 3.1, the incumbent bank is assumed to share verifiable information on the borrowers’ type. In Section 3.2, the incumbent bank is assumed to be able to share verifiable credit history of the borrower. We analyze the two scenarios in turn and show that the incumbent bank always has incentives to share credit information. We show in Section 3.3 that even if the incumbent bank can share the borrower’s type, sharing credit history is preferred. Finally, in Section 3.3, we allow the incumbent bank to share unverifiable credit history, so that the bank may overstate the past loan performance

### 3.1 Sharing Borrower Type

To illustrate the main intuition of our model, we start by considering the simplest case, where the incumbent bank can share verifiable information on the borrower type.
Let us first examine the case where the shared information reveals the borrower being an \( H \)-type. The game \( g_S(H) \) features complete-information, and we solve for its SPE by backward induction. First, we determine the secondary market loan price \( P_i^s \) in the aggregate state \( s \in \{G, B\} \) and under the information sharing regime \( i \in \{N, S\} \). Second, we compute the deposit rate \( r_i \) at which depositors supply their funds to the banks under Regime \( i \). Finally, we determine the loan rate \( R_i \) at which the bank offers credit to the borrower under Regime \( i \).\(^{13}\)

In the secondary loan market, asset buyers’ competitive bidding drives the price of the loan on sale up to its face value, because an \( H \)-type never defaults. Denoting with \( P_S^B(H) \) and \( P_S^G(H) \) the price of an \( H \)-type loan under regime \( i = S \) in state \( s = B \) and \( s = G \), respectively, we have
\[
P_S^B(H) = P_S^G(H) = R^*_S(H),
\]
where \( R^*_S(H) \) denotes the equilibrium loan rate for the \( H \)-type borrower under regime \( i = S \).

Depositors understand that a loan given to an \( H \)-type borrower does not default. Thus, they perceive lending to a bank that finances such a loan safe, and are willing to accept the risk-free rate. Letting \( r^I_S(H) \) and \( r^E_S(H) \) denote the deposit rates that the incumbent bank and the entrant bank, respectively, need to offer, we have
\[
r^I_S(H) = r^E_S(H) = r_f.
\]

The entrant bank also understands that an \( H \)-type borrower is risk-free and can break even by offering a loan rate \( R^E_S(H) \) that equals its cost of deposits \( r^E_S(H) \) plus its entry cost \( c \). The equilibrium loan rate \( R^*_S(H) \) is determined by the value of the loan return \( R \): If \( R \geq r_f + c = R^E_S(H) \), then the incumbent bank has to match the entrant bank’s loan rate, resulting in an equilibrium loan rate \( r_f + c \). Whereas if \( R < r_f + c \), the entrant bank does not find profitable to bid for the borrower and the equilibrium loan rate will hit a corner solution of \( R \). Therefore the equilibrium loan rate for an \( H \)-type borrower is as follows
\[
R^*_S(H) = \min\{R, r_f + c\}.
\]

\(^{13}\)Recall that both the deposit rate and the loan rate are set before the aggregate state \( s \) realizes. Therefore, they cannot be conditional on \( s \).
The incumbent bank’s profit, under \(i = S\) and when the borrower is \(H\)-type, is

\[
\Pi_S(H) = R^*_S(H) - r^I_S(H).
\]

Consider now the shared information reveals the borrower being an \(L\)-type. The game \(g_S(L)\) still features complete information and its SPE can be solved by backward induction. Since the \(L\)-type succeeds in state \(s = G\) but it defaults in state \(s = B\), the loan price on the secondary market will be state-dependent. It equals zero when \(s = B\) and the face value of the loan when \(s = G\). That is,

\[
P^B_S(L) = 0 \quad \text{and} \quad P^G_S(L) = R^*_S(L),
\]

where \(R^*_S(L)\) denotes the equilibrium loan rate for the \(L\)-type borrower.

Depositors understand that the price of an \(L\)-type loan is zero when \(s = B\), so that they will be repaid only in the favorable state \(s = G\) which occurs with probability \(\pi\). The depositors consider their lending to a bank that finances such a loan risky, and would accept the following deposit rates

\[
r^I_S(L) = r^E_S(L) = \frac{r_f}{\pi} > r_f.
\]

Expecting to recoup the investment only when \(s = G\), the entrant bank breaks even by offering a loan rate \(R^E_S(L)\) so that the expected payoff equals the lending cost. That is

\[
\pi \cdot [R^E_S(L) - r^E_S(L)] + (1 - \pi) \cdot 0 = c \quad \text{which implies} \quad R^E_S(L) = \frac{r_f + c}{\pi}.
\]

Again, depending on whether \(R^E_S(L)\) is higher or lower than \(R\), the equilibrium loan rate for the \(L\)-type borrower can be written as

\[
R^*_S(L) = \min \left\{ R, \frac{r_f + c}{\pi} \right\}.
\]

Provided that the borrower is an \(L\)-type, the incumbent bank’s expected profit is

\[
\Pi_S(L) = \pi \cdot [R^*_S(L) - r^I_S(L)].
\]

When the information sharing decision is made, the type of the borrower is unknown. The incumbent bank earns an ex-ante expected profit

\[
V^{Type}_S = \alpha \Pi_S(H) + (1 - \alpha) \Pi_S(L) = \alpha R^*_S(H) + (1 - \alpha) \pi R^*_S(L) - r_f.
\]
We now turn to the case where the bank shares no information. The dynamic game $g_N$ features incomplete information game. All outsiders (the entrant bank, depositors and asset buyers) need to form beliefs about the quality of the borrower, and we solve the game using the concept of PBE.\footnote{We refer for the formal definition of the equilibrium notion adopted in game $g_N$ to Appendix.}

The secondary market for loans, for example, is characterized by adverse selection: asset buyers face uncertainty on the type of loan they are buying. Under no information sharing we then employ PBE.

With no information on borrower’s type and a belief that both types of borrowers are financed, asset buyers maintain the prior belief that the loan on sale is an $H$-type loan with probability $\alpha$. Moreover, the incumbent bank will sell the loan generated by the $H$-type borrower only if it is hit by the liquidity shock. Therefore, in State $B$ the loan will be repaid with a probability lower than one, and asset buyer is only willing to pay a price lower than the face value of the loan. Whereas in State $G$, the asset buyers understand that both $H$- and $L$-type borrowers can repay the loan. As a result, the loan price in the secondary market in $s = B$ and $s = G$ will be, respectively,

$$P^B_N < R^*_N$$ and $$P^G_N = R^*_N,$$

where $R^*_N$ denotes the equilibrium loan rate with no information sharing.

Suppose $P^B_N < r_f$ (i.e., the price of the loan on sale in state $B$ is not sufficient to cover the risk-free rate). Then depositors at the incumbent bank understand that their bank is risky in state $B$. Only if the incumbent bank holds an $H$-type loan and experiences no run can repay its deposit funding. Whereas the incumbent bank can always repay its deposit funding in state $G$. Therefore, we have

$$r^I_N = \frac{r_f}{\pi + (1 - \pi)\alpha(1 - \rho)}.$$

On the other hand, the depositors at the entrant bank are willing to accept a deposit rate

$$r^E_N = \frac{r_f}{\pi + (1 - \pi)\alpha}$$

because the entrant bank is assumed to face no liquidity shock.

Given the funding cost $r^E_N$ and a belief that both $H$- and $L$-type borrowers participate in the market, the entrant bank will break even by offering a loan rate $R^E_N$ such that

$$\alpha \left( R^E_N - r^E_N \right) + (1 - \alpha)\pi \left( R^E_N - r^E_N \right) = c \quad \text{which implies} \quad R^E_N = \frac{r_f + c}{\alpha + (1 - \alpha)\pi}.$$
Again, depending on whether the loan rate $R^E_N$ of the entrant bank is higher or lower than $R$, the equilibrium loan rate $R^*_N$ can be written as

$$R^*_N = \min \left\{ R, \frac{r_f + c}{\alpha + (1 - \alpha)\pi} \right\}.$$  

We establish in Lemma 1 that when $P^B_N < r_f$, the game $g_N$ has a unique PBE.

**Lemma 1** When $P^B_N < r_f$, there exists a unique PBE of the game $g_N$. The incumbent bank offers to the borrower a pooling loan rate

$$R^*_N = \min \{ R, R^E_N \},$$

regardless of his type or credit history. Here

$$R^E_N = \frac{r_f + c}{\alpha + (1 - \alpha)\pi}.$$  

The incumbent bank offers a risky deposit rate

$$r^I_N = \frac{r_f}{\pi + (1 - \pi)\alpha(1 - \rho)}$$

that allows depositors to break even. The incumbent bank sells its $H$-type loan only if hit by liquidity shock in $s = B$. Upon the loan sale, buyers offer state-contingent prices

$$P^G_N = R^*_N \quad \text{and} \quad P^B_N = \frac{\alpha \rho}{(1 - \alpha) + \alpha \rho} R^*_N.$$  

**Proof.** See Appendix. ■

Lemma 1 shows there exist a set of parameters in which the unique pure-strategy PBE involves the incumbent bank financing the borrower regardless of his type. This implies that, on the equilibrium path, the secondary market for the loan features adverse selection and the incumbent bank fails due to the run in the state $B$—even when holding a loan granted to an $H$-type borrower.

The incumbent bank’s expected profit when sharing no information can be written as

$$V_N = \pi \left( R^*_N - r^I_N \right) + (1 - \pi)\alpha(1 - \rho) \left( R^*_N - r^I_N \right) = [\pi + (1 - \pi)\alpha(1 - \rho)] \cdot R^*_N - r_f. \quad (5)$$

Comparing expressions (4) and (5) leads to the following proposition.

**Proposition 1** There exists a critical $\hat{\rho}$, such that when $\rho > \hat{\rho}$ and $P^B_N < r_f$, the incumbent bank prefers to share the information on borrowers’ type to no information sharing.
Proof. See Appendix. ■

To gain the intuition, notice that the proposition is easily verified when all equilibrium loan rates take either the interior or the corner solutions. Indeed, in these two cases, the proposition holds independent of the level of \( \rho \). Specifically, consider the all-corner-solution case: \( R^*_S(L) = R^*_S(H) = R^*_N = R \). We have

\[
V^{Type}_S = \left[ \alpha + (1 - \alpha)\pi \right] R - r_f > E(\Pi_N) = \left[ \pi + (1 - \pi)\alpha (1 - \rho) \right] R - r_f.
\]

The difference in payoffs, \( V^{Type}_S - V_N = (1 - \pi)\alpha \rho \cdot R > 0 \), captures that by sharing the borrower’s type, the bank of an \( H \)-type loan avoids the failure when facing a run in State \( B \). Intuitively, when all the loan rates are equal to \( R \), sharing information does not negatively affect the market power of the incumbent bank, who only benefits from the reduced adverse selection, making it preferred by the incumbent bank. On the other hand, when the equilibrium loan rates are instead all interior solutions (i.e., the equilibrium loan rate is dictated by the entrant bank), we have \( V^{Type}_S = c > V_N = 0 \). Intuitively, all loan rates being interior solutions implies that the primary market is always contestable, independent of whether the entrant bank knows the borrower’s type. As a result, the incumbent bank can only earn the entry cost \( c \), so again, sharing borrower’s type only brings the benefit of reduced liquidity risk. In sum, in these two cases, sharing borrower’s type always dominates no information sharing. For intermediate cases, the result still holds for sufficiently high liquidity risk \( \rho \).

The simple case where the incumbent bank can share directly the borrower’s type illustrates the main intuition of the paper. Information sharing reduces adverse selection and boosts the price of \( H \)-type loans. In state \( B \), the price increases from \( P^B_N < r_f \) to \( P^B_S(H) > r_f \), rescuing the incumbent bank with an \( H \)-type loan from the run. On the other hand, the incumbent bank can lose market power as the loan rate charged to the borrower drops from \( R^*_N \) to \( R^*_S(H) \). Intuitively, the benefit of information sharing is more prominent when the liquidity risk is high so that the benefit of information sharing exceeds its cost.

### 3.2 Sharing Verifiable Credit History

We now analyze the more realistic scenario in which borrower’s type cannot be shared but only borrower’s credit history can be communicated to third parties. This assumption
makes a clear distinction between soft and hard information: While the borrower’s credit history can be hard information to be shared with outsiders, the borrower’s type represents soft information that cannot be shared.

Clearly, when the incumbent bank does not share information on the borrower credit history we have again the game $g_N$, whose PBE is characterized in Lemma 1. Under the information sharing regime, we first consider the case in which the incumbent bank discloses the no default history $\overline{D}$. This announcement only partially reveals the borrower’s type, as the borrower can still be either an $H$- or $L$-type. Game $g_S(\overline{D})$ therefore features incomplete information. Parallel to Lemma 1, we characterize a pure strategy PBE of the incomplete information game $g_S(\overline{D})$ in Lemma 2.

**Lemma 2** When $P_S^B(\overline{D}) > r_f$, there exists a unique PBE of the game $g_S(\overline{D})$. The incumbent bank offers to the borrower with no default $\overline{D}$-history a loan rate

$$R^*_S(\overline{D}) = \min \{ R, R^E_S(\overline{D}) \},$$

regardless of the borrower’s type. Here

$$R^E_S(\overline{D}) = \frac{\alpha + (1 - \alpha)\delta}{\alpha + (1 - \alpha)\delta} (c + r_f).$$

The incumbent bank offers a risk-free deposit rate $r^I_S(\overline{D}) = r_f$ to depositors who will accept it. It sells the $H$-type loan only if hit by liquidity shock. Upon the loan sale, asset buyers offer state-contingent prices

$$P^G_S(\overline{D}) = R^*_S(\overline{D}) \quad \text{and} \quad P^B_S(\overline{D}) = \frac{\alpha \rho}{(1 - \alpha)\delta + \alpha \rho} R^*_S(\overline{D}).$$

**Proof.** See Appendix. □

Lemma 2 shows that in equilibrium the incumbent bank finances the borrower no matter the type at $t = 1$ and it survives the possible run at $t = 3$.

When the incumbent bank announces that the borrower has a previous default (i.e., a $D$-history). Then the shared information perfectly reveals the borrower as an $L$-type. Game $g_S(D)$ therefore features complete information and can be solved with SPE, similarly to the case where the incumbent bank announces the borrower being an $L$-type as analyzed in Section 3.1. We summarize the result in Lemma 3.
Lemma 3  When the borrower has defaults history $D$, there exists a unique SPE of the game $g_S(D)$. The incumbent bank offers to the borrower with defaults history $D$ a loan rate

$$R^*_S(D) = \min \{ R, R^E_S(D) \}, \text{ with } R^E_S(D) = \frac{c + r f}{\pi}.$$  

The incumbent bank offers a risky deposit rate $r^*_S(D) = r f / \pi$ to depositors, who will accept.

Upon the loan sale, asset buyers offer state-contingent prices

$$P^C_S(D) = R^*_S(D) \quad \text{and} \quad P^B_S(D) = 0.$$  

Proof. See Appendix. ■

Lemma 3 characterizes the equilibrium in which the incumbent bank finances the loan at $t = 1$ and it fails at $t = 3$ when the state $B$ occurs.

Comparing the equilibrium loan rates in Lemma 1, 2 and 3, we can rank them as follows

$$R^*_S(\overline{D}) \leq R^*_N \leq R^*_S(D). \quad (6)$$

The equalities hold only when all the equilibrium loan rates hit the corner solution $R$ (i.e., the entrant bank does not find profitable to bid for the borrower). Intuitively, the borrower with a default ($D$-history) is identified as an $L$-type and charged the highest loan rate accordingly. On the other hand, the borrower with no previous default ($\overline{D}$-history) is more likely to be an $H$-type. Correspondingly, the loan rate drops. When no information is shared, the equilibrium loan rate will be set according to the prior probabilities of borrower types, resulting in an intermediate loan rate.

Depending on how the project income $R$ relates to the ranking of the equilibrium loan rates in (6), we have four cases $j = \{0, 1, 2, 3\}$:

- Case 0: $R \in \mathbb{R}_0 \equiv [c + r f, R^E_S(\overline{D})]$ then $R^*_S(\overline{D}) = R^*_N = R^*_S(D) = R$
- Case 1: $R \in \mathbb{R}_1 \equiv [R^E_S(\overline{D}), R^E_N]$ then $R^*_S(\overline{D}) = R^E_S(\overline{D})$ and $R^*_N = R^*_S(D) = R$
- Case 2: $R \in \mathbb{R}_2 \equiv [R^E_N, R^E_S(D)]$ then $R^*_S(\overline{D}) = R^E_S(\overline{D}), R^*_N = R^E_N$ and $R^*_S(D) = R$
- Case 3: $R \in \mathbb{R}_3 \equiv [R^E_S(D), +\infty)$ then $R^*_S(\overline{D}) = R^E_S(\overline{D}), R^*_N = R^E_N$ and $R^*_S(D) = R^E_S(D)$.

Note that we index each case with the number of interior solutions (i.e., when the equilibrium loan rates is determined by the bid of the entrant bank). Each case shows a different degree of loan market contestability. In Case 0, the project payoff $R$ is so low
that the entrant bank finds it unprofitable to enter the market even if the borrower has no previous default. In Case 3, on the other hand, \( R \) is so high that the entrant bank competes even for the borrower who previously defaulted. The higher \( R \), the more contestable the primary loan market. The four mutually exclusive cases are illustrated in Figure 2.

[Insert Figure 2 here]

**The benefit of information sharing.** We show now that, in each of the four cases, there exists a set of parameters where the incumbent bank holding a loan with a \( D \)-history survives a run in state \( B \) under the information sharing regime and fails under the no information sharing regime. That is, information sharing is beneficial since it saves the incumbent bank from illiquidity.

As we pointed out in Section 3.1, information sharing has a positive effect because it boosts the secondary-market loan price in state \( B \). Indeed in such state when there is no information sharing asset buyers hold their prior probability about the quality of the loan on sale. The incumbent bank can therefore fail in a run even if it holds a safe \( H \)-type loan. When sharing the credit history, the perceived loan quality is higher for a loan with a no default \( D \) history mitigating the adverse selection and boosting the price of the loan on sale in the secondary market.

However, when sharing a no default credit history \( D \), the incumbent bank can charge a loan rate lower than the one under the no information sharing regime. Indeed, from condition (6), we have \( R_S^*(D) \leq R_N^* \). Then information sharing may result in \( P_{B_S}(D) < P_{B_N} \). The following Lemma shows however that the positive effect of mitigating adverse selection dominates the negative one of higher competition. We have

**Lemma 4** The equilibrium prices of the loan on sale are such that \( P_{S_B}(D) > P_{N_B} \). That is, in State \( B \), the price in the secondary market for a loan with \( D \) history is higher than the price for a loan under no information sharing. There always exist a range of parameters such that \( P_{S_B}(D) > r_f > P_{N_B} \), such that in State \( B \), the incumbent bank with a loan with \( D \) history under information sharing is saved from a run while it fails under no information sharing.

**Proof.** See Appendix. ■
To provide the intuition, we discuss here Case 2, a core case upon which the general proof builds, and provide the complete proof is in Appendix.\(^\text{15}\) Recall that in Case 2 the equilibrium loan rates are \(R^*_N = R^*_E\) and \(R^*_S(D) = R^*_E(D)\). Substituting them into the expressions that characterize the equilibrium prices \(P^B_N\) and \(P^B_S(D)\), as given in Lemma 1 and 2 respectively, we have

\[
\frac{P^B_N}{P^B_S(D)} = \left( \frac{Pr(L|D) + Pr(H|D) Pr(\text{run})}{Pr(L) + Pr(H) Pr(\text{run})} \right) \left( \frac{Pr(H|D) + Pr(L|D) Pr(G)}{Pr(H) + Pr(L) Pr(G)} \right)
\]

\(=\) \(\frac{(1 - \alpha)\delta + \alpha\rho}{(1 - \alpha) + \alpha\rho}\) \(\frac{\alpha + (1 - \alpha)\delta\pi}{(\alpha + (1 - \alpha)\pi)(\alpha + (1 - \alpha)\delta)}\) \(\text{(A)}\) \(\text{(B)}\).

This ratio between \(P^B_N\) and \(P^B_S(D)\) can be decomposed into the product of two elements. Expression (A) reflects how information sharing affects the information asymmetry in the secondary loan market, and expression (B) captures the impact of information sharing on the information asymmetry in the primary loan market. Specifically, expression (A) is the ratio of the expected quality of a borrower with a \(D\)-history to that of a borrower with unknown credit history.\(^\text{16}\) This ratio is smaller than 1, implying an increase in the expected quality given that the borrower has no previous default. Expression (B) is the ratio between the probability of no fundamental credit risk for a borrower with a \(D\)-history to that of a borrower of unknown credit history.\(^\text{17}\) This ratio is greater than 1, implying a decline in the perceived credit risk (since the credit history \(D\) more likely the loan is granted to the \(H\)-type borrower) and the corresponding drop in the loan rate in the primary market.

The information asymmetry in both primary and secondary credit markets is rooted in the uncertainty of the borrower’s type. These two markets, however, differ in two aspects. First, the strategic asset sale by the incumbent bank is only relevant for the information asymmetry in the secondary market. Second, the uncertainty about the aggregate state \(s\) has resolved when the secondary market opens, whereas when the primary market operates there is still uncertainty about the state \(s\). Such differences disappear when parameters \(\rho\) and \(\pi\) approach limit. In particular, the strategic asset sale disappears when the incumbent bank is selling the loan because it is facing a run for sure, i.e. when \(\rho = 1\). In addition, the difference in the uncertainty about the state \(s\) disappears when \(\pi = 0\). Therefore, the

\(^\text{15}\) Some cases are straightforward. In Case 0, for example, the equilibrium loan rates are \(R^*_S(D) = R^*_N = R\), making it straightforward to verify \(P^B_S(D) > P^B_N\).

\(^\text{16}\) The expected quality is defined as the probability that the loan is granted to an \(H\)-type borrower.

\(^\text{17}\) No fundamental credit risk means that either the borrower is an \(H\)-type or an \(L\)-type in the \(G\) state.
primary and secondary loan markets have the same level of information asymmetry only when both $\rho = 1$ and $\pi = 0$, in which case the impact of information sharing is symmetric in the two markets. As a result, the price ratio $P^B_N/P^B_S(\overline{D})$ equals 1.

The price ratio $P^B_N/P^B_S(\overline{D})$ is smaller than 1 for either $\rho < 1$ or $\pi > 0$, or both. To see so, notice that expression (A) increases in $\rho$. Intuitively, as the probability of a run decreases from 1, it becomes more likely that the loan is on sale for strategic reasons. As a result, the adverse selection in the secondary market aggravates, and the gap in the expected qualities widens across the two information sharing regimes, leading to a lower value for expression (A). On the other hand, expression (B) decreases in $\pi$. Intuitively, as $\pi$ increases, the difference between $H$- and $L$-type borrower diminishes. The credit history becomes less relevant as an informative signal of the borrower’s type, and the gap between the two loan rates narrows, leading to a lower value of expression (B). Therefore, whenever $\rho < 1$ or $\pi > 0$, or both, information sharing’s positive impact of increasing the price of the loan on sale in the secondary market dominates its negative effect of decreasing the loan rate in the primary market.

Given $P^B_S(\overline{D}) > P^B_N$, a continuity argument ensures that there must exist a set of parameters where the risk-free rate $r_f$ lies between the two prices. In such a case, the incumbent bank that lends to an $H$-type borrower survives the run under the information sharing regime but fails under the no information sharing regime. We denote by $\mathbb{F}_j$ the set of parameters where the condition $P^B_S(\overline{D}) > r_f > P^B_N$ holds in each Case $j = \{0, 1, 2, 3\}$. We establish non-empty sets $\Psi_j \equiv \mathbb{R}_j \cap \mathbb{F}_j$ with $j = \{0, 1, 2, 3\}$.

- $\Psi_0 \equiv \mathbb{R}_0 \cap \mathbb{F}_0$ with $\mathbb{F}_0 \equiv \{(C, R) | R < R < \overline{R}\}$.
- $\Psi_1 \equiv \mathbb{R}_1 \cap \mathbb{F}_1$ with $\mathbb{F}_1 \equiv \{(C, R) | R < \overline{R}$ and $C > \overline{C}\}$.
- $\Psi_2 \equiv \mathbb{R}_2 \cap \mathbb{F}_2$ with $\mathbb{F}_2 \equiv \{(C, R) | \overline{C} < C < \overline{C}\}$.
- $\Psi_3 \equiv \mathbb{R}_3 \cap \mathbb{F}_3$ with $\mathbb{F}_3 \equiv \mathbb{F}_2$.

Notice that the prices $P^B_N$ and $P^B_S(\overline{D})$ are the same in Cases 2 and 3. This is because the payoff of the loan $R$ is sufficiently high that the entrant bank competes with the incumbent bank both for a loan under no information sharing and for a loan with no default credit history $\overline{D}$. Therefore, we have $\mathbb{F}_3 = \mathbb{F}_2$. We provide the expressions for the cutoff values (i.e., $R, \overline{R}, C, \overline{C}$) in the Proof of Lemma 4.
Figure 3 gives the graphic representation of the sets $\Psi_j$. The shaded area corresponds the set of parameters that satisfy the inequality $P^B_S(D) > r_f > P^B_N$.

Information sharing can endogenously emerge only inside those shaded areas, because for all other possible parametric combinations information sharing does not reduce the incumbent bank’s liquidity risk, but only leads to higher competition from the entrant bank, and therefore the incumbent bank loses the information rent. In the rest of the paper, we will focus on the parametric configuration given by the set $\Psi_j$ in each region $j = \{0, 1, 2, 3\}$.

**Ex ante decision on information sharing.** We are now in a position to determine when information sharing emerges as an equilibrium of our game. At $t = 0$, the incumbent bank decides whether to choose the information sharing regime $S$ or the no information sharing regime $N$. The choice is made by comparing the expected profits in those two regimes. Let us denote with $V_i$ the incumbent bank’s expected profits at $t = 0$ under the information sharing regime $i = \{N, S\}$, and with $\varphi_j$, $j = \{0, 1, 2, 3\}$, the set of parameters in which the condition $V_S > V_N$ holds. We have

**Proposition 2**  The incumbent bank chooses voluntarily to share information on $\varphi_j = \Psi_j$ with $j = \{0, 3\}$ and on $\varphi_j \subseteq \Psi_j$ with $j = \{1, 2\}$. When $\rho > (1 - \alpha)(1 - \delta)$, the incumbent bank always choose to share information on the borrower credit history, i.e. we have $\varphi_j = \Psi_j, \forall j \in \{0, 1, 2, 3\}$.

**Proof.** See Appendix.

To illustrate the main trade-off, we decompose the difference between the incumbent bank’s expected profits in the two regimes as follows

$$V_S - V_N = \left[\alpha + (1 - \alpha)\delta \pi\right] (R^*_S(D) - R^*_N) + \left[(1 - \alpha)\pi (R^*_S(D) - R^*_N) + \alpha (1 - \pi)\rho R^*_N \right].$$

Term (1) represents the competition effect, and it has a negative consequence for the adoption of the information sharing regime since $R^*_S(D) \leq R^*_N$. Sharing information encourages the entrant bank to compete for the borrower with a good credit history $D$. The expected profits of the incumbent bank is reduced because the entrant bank undercuts the
loan rate. Term (2) is understood in the literature as the 
*capturing* effect, and it has positive impact on the decision to share information since \( R^*_S(D) \geq R^*_N \). Sharing information about the borrower with bad credit history \( D \) deters the entry of the entrant bank. The incumbent bank can capture such borrower by charging higher loan rate. Finally, Term (3) denotes the new effect that our model features. We refer to it as the *liquidity* effect and it highlight the positive consequence of the adoption of the information sharing regime. Sharing information on a borrower with good credit history \( D \) reduces the adverse selection in the secondary market and the incumbent bank will be saved from a run in the bad state \( B \).

Information sharing is chosen by the incumbent bank if and only if \( V_S - V_N > 0 \), which crucially depends if the capturing effect together with the liquidity effect dominate the competition effect. In Cases 0 and 3 this condition is always met and the set of parameters \( \varphi_j \) coincides with the set \( \Psi_j \). The reason is that there is no cost for the incumbent bank to share information in both cases. In Case 0 the distant bank never competes for the borrower, therefore \( R^*_S(D) = R^*_N = R^*_S(D') = R \) and only the liquidity effect is present.

In Case 3 the entrant bank always competes for the borrower, therefore under information sharing expected profits for the incumbent bank are \( V_S = c \). That is, the entry cost \( c \) of the entrant bank. Under no information sharing the expected profits are \( V_S < c \) since under such regime the incumbent bank can fail in the state \( B \) when it experiences a run.

In Cases 1 and 2 the competition effect could instead overcome the sum of the capturing and the liquidity effect. This possibility reduces the set of parameters \( \varphi_j \) in which sharing information is actually chosen versus the set of parameters \( \Psi_j \) in which is actually beneficial. However, when the probability of a bank run is sufficiently high the incumbent bank always finds it convenient to share information. Figure 4 reproduces again Cases 0, 1, 2 and 3 where the double-shaded area corresponds to the set of parameters \( \varphi_j \).

[Insert Figure 4 here]

Clearly the double-shaded areas in Cases 0 and 3 correspond to the shaded areas in Figure 3. When \( \rho \) is low, the double-shaded areas in Cases 1 and 2 are smaller than the corresponding areas in Figure 3. The red line depicts indeed the boundary of the double-shaded area in which the incumbent bank voluntarily chooses to share information. When \( \rho \) is sufficiently high the shaded area in Figure 3 and the double-shaded area in Figure 4 coincide.
3.3 Sharing Unverifiable Credit History

We now relax the assumption of verifiable credit history. If the reported borrower’s credit history is not verifiable, the incumbent bank may have an incentive to overstate the borrower’s credit history. That is, we consider the possibility to report as a no default credit history $\overline{D}$ instead of the true credit history with defaults $D$.\(^1\) Since the pre-requisite for the incumbent bank to manipulate the reported credit history is that it must choose the information sharing regime in the first place, we restrict the analysis on the set of parameters that define the regions $\varphi_j$, with $j = \{0, 1, 2, 3\}$, determined in Section 3.2.

We have the following

**Proposition 3** The incumbent bank truthfully discloses the borrower’s credit history only if it leads to an increase in the loan rate for borrowers who have a credit history with default $D$ - that is, $R_S^*(D)$. Moreover, truth telling cannot be sustained in regions $\varphi_j$ with $j = \{0, 1\}$. It can be sustained in region $\varphi_2$ when $\rho$ is sufficiently low, and it is always sustained in region $\varphi_3$.

**Proposition 4** The incumbent bank truthfully discloses the borrower’s credit history with default $D$ only if the loan rate for borrowers with a $D$-history - that is, $R_S^*(D)$ - is sufficiently high. Moreover,...

The proof is in the Appendix. The intuition is as follows. In order to sustain truthful credit reporting, a necessary condition is that the incumbent bank must suffer a sufficiently high loss when deviating from the equilibrium strategy $D$. If the incumbent bank truthfully reveals the credit history $D$, it can charge the loan rate $R_S^*(D)$. Nevertheless, the incumbent bank will survive only in the good state $G$ (i.e., with probability $\pi$) because a loan with a credit history $D$ has been surely granted to an $L$-type borrower which generates zero payoff in state $B$. So the expected loss of the incumbent bank by deviating from the equilibrium strategy is $\pi R_S^*(D)$. What is the gain from the deviation? If the incumbent bank lies about the credit history the loan rate charged would be $R_S^*(\overline{D})$, which can be lower than $R_S^*(D)$ due to the higher competition from the entrant bank. The return $R_S^*(\overline{D})$ is then earned.

\(^1\)We assume that the incumbent bank cannot report a credit history with default $D$ instead of the true no default history $\overline{D}$. This is because borrowers have means to correct it or act against it (e.g., under Fair Credit Act in the US). According to the documentations in www.doingbusiness.com, borrowers can access their own credit record. A false report in this case can result in a legal dispute.
when the state is $G$ (the state in which also the $L$-type borrower succeed). However, when the state is $B$, cheating makes the incumbent bank more resilient against the possible run since it can sell the loan for the price $P^B_S(D) > r_f$. Therefore the necessary condition to truthfully revealing the credit history $D$ is

$$\pi R^*_S(D) > \pi R^*_S(\bar{D}) + (1 - \pi) P^B_S(\bar{D}) = \left[ \pi + \frac{(1 - \pi) \alpha \rho}{(1 - \alpha) \delta + \alpha \rho} \right] R^*_S(\bar{D}). \quad (7)$$

Consider now Case 0. The loan market is the least contestable and we have $R^*_S(D) = R^*_S(\bar{D}) = R$. Condition (7) is not satisfied and then the incumbent bank always has incentive to misreport the true $D$-history as $\bar{D}$-history in the parameters space that determines region $\varphi_0$. Ex-ante, assuming truthful reporting, the incumbent bank finds it more profitable to adopt the information sharing regime in the region $\varphi_0$. However, ex-post, when the incumbent bank observes a credit history $D$, it will incur no profit loss by misreport the credit history as $\bar{D}$ because $R^*_S(D) = R^*_S(\bar{D})$. In Case 0 truthfully reporting can not be sustained as an equilibrium.

In the other Cases the loan market is more contestable and we have $R^*_S(D) > R^*_S(\bar{D})$. Then condition (7) could be satisfied. However, even if ex-post the incumbent bank has an incentive to tell the truth, it is possible that ex-ante it is not willing to share information. That is, the set of parameters that guarantees truth telling, and therefore satisfies condition (7), have to be consistent with the set of parameters that makes information sharing ex-ante profitable and therefore are given by the regions $\varphi_i$ with $i = \{1, 2, 3\}$.

Consider Case 1. Ex-ante, assuming truthful reporting, the incumbent bank prefers the information sharing regime when $R$ is sufficiently low. This is because the expected profit with no information sharing is increasing in $R$ (recall $R^*_N = R$), while the expected profit with information sharing is increasing in $R$ only if the loan is granted to an $L$-type borrower. Ex-post the incumbent bank reports the true credit history when $R$ is sufficiently high. Indeed in Case 1 we have $R^*_S(D) = R$, so the LHS in condition (7) is increasing in $R$, while $R^*_S(\bar{D}) = R^*_S(\bar{D})$ that does not depend on $R$ (see Lemma 2). It turns out that the ex-ante and ex-post conditions on $R$ determine an empty set and, therefore, truthful reporting cannot be sustained as an equilibrium in the parameters space that characterizes region $\varphi_1$.

Consider Case 2. Ex-ante, assuming truthful reporting, the incumbent bank chooses the information sharing regime when $R$ is sufficiently high. The expected profit with no information sharing does not depend on $R$ (recall, $R^*_N = R^*_N$), while the expected profit
with information sharing is increasing in $R$ (recall $R^*_S(D) = R$). Ex-post, the incumbent bank reports the true credit history when $R$ is sufficiently high. Indeed, like in Case 1, we have $R^*_S(D) = R$ and $R^*_S(D) = R^E_S(D)$. It turns out that the ex-ante condition on $R$ is more restrictive than the ex-post condition if $\rho$ is lower than a critical value $\bar{\rho}$. Under this condition, whenever the incumbent bank finds ex-ante convenient to share information it will also report ex-post the true credit history, and truthful reporting can be sustained as an equilibrium in the parameters space that determines region $\varphi_2$.

Finally, consider Case 3. Ex-ante, assuming truthful reporting, the incumbent bank prefers information sharing irrespective of $R$. Ex-post the incumbent bank does not have incentive to misreport the credit history. Indeed in this Case we have $R^*_S(D) = \frac{c+\tau l}{\pi}$ and $R^*_S(D) = R^E_S(D)$, so both loan rates do not depend on $R$, but they make condition (7) satisfied. Accordingly, truthful reporting is sustained as an equilibrium in the parameter space $\varphi_3$.

To sum up, in Cases 0 and 1 the market is less contestable and therefore the deviation’s expected loss $\pi R^*_S(D)$ is bounded above by the loan’s relatively low return $R$. Thus the gain of the deviation, mostly represented by the higher price at which the loan is sold saving the bank in state $B$, dominates the loss. However, in Case 2 the return $R$ becomes larger and the loss of a deviation from the equilibrium tends to dominate the benefit. Truthfully reporting the credit history can be sustained as equilibrium in this Case. In Case 3 the return $R$ is sufficiently high that truth telling is always sustained.

Figure 5 shows again Cases 0, 1, 2 and 3. With respect Figure 4, it adds in each Case a dark-blue area corresponding to the set of parameters in which truth-telling is sustained as an equilibrium.

In Cases 0 and 1 there is no dark-blue area depicted since truth-telling is not sustainable under these Cases. In Case 2 we show a situation where truth-telling can be sustained in a subset of $\varphi_2$, that is when $\rho < \min[\bar{\rho}, (1 - \alpha)(1 - \delta)]$. The dark-blue area is then smaller than the double-shaded area. In Case 3 truth-telling is always sustained in the entire region $\varphi_3$ therefore the dark-blue area coincides with the double-shaded area in Figure 4.

The analysis also reveals why it is important that the information sharing should be an ex-ante arrangement. If the incumbent bank only shares information when the liquidity
risk strikes, no truthful information sharing can be sustained. This is because...

4 Welfare and Policy Implication

Casual observations show us that banks may have private incentives to establish credit bureau, for example in the USA. However, some countries, like Spain, have imposed banks to participate in public credit registry. We use our theoretical framework to address a fundamental, yet overlooked, question: when is a public registry needed? when it may outweigh a private bureau in facilitating information sharing? We first need to identify what the socially optimal level of information sharing is. We focus our welfare analysis on the main friction of our model, that is the adverse selection in the secondary loan market.\(^{19}\)

When coarse information about the quality of the loan is available, a loan granted to an \(H\)-type borrower will be liquidated at discount because buyers can not differentiate it from an \(L\)-type loan. Suppose now the social planner knows the borrower’s type. Then the first best can be implemented by directly communicating the borrower’s type to the market participants. The inefficient liquidation of \(H\)-type loan can be always avoided. However, proposition 1 establishes that when \(\rho\) is sufficiently low there are cases in which it is convenient for the incumbent bank to not share the borrower type. Even if the incumbent bank mostly values information sharing for its role of boosting the liquidation value of the \(H\)-type loan, there are cases in which the reduced market power versus the \(H\)-type borrower induces the incumbent bank to not share information. The incumbent bank’s private choice of information sharing is lower than the efficient one, and then a public registry improves welfare.

The assumption that the planner can observe directly types is however quite demanding.\(^{20}\) We then move from a first-best approach to a, more realistic, constrained first-best approach. That is, the social planner has the same information constraints of the market participants. That is, only the credit history can be communicated and the planner cannot

\(^{19}\)Notice that another friction in the model is the market power of the incumbent bank over the borrower. A planner that wants to implement Pareto improving policies does not take into account such distributional friction.

\(^{20}\)The recent financial crisis showed us that central banks (supposedly the most informed actor in place) need to extract information from private financial institutions. For instance, regulators now conduct various kinds of stress testing to check banks’ resiliency.
force the incumbent bank to tell the truth about the reported information. In this case, a public registry is considered to be welfare improving if it generates and communicate the credit history better than a private bureau (which is identified with the incumbent bank’s information sharing choice). That is, if the public registry allows less early liquidation of the loan generated by an $H$-type borrower than a private bureau.

Our model highlighted two possible inefficiencies when the incumbent bank chooses the information sharing regime and the quality of information to report. First, a incumbent bank may refuse to enter into a private bureau anticipating the loss of lucrative informational rent extracted from the borrower although it is of the society’s best interest not to liquidate prematurely the loan.\footnote{This corresponds to the regions $\Psi_j - \varphi_j$ in Cases $j = 1, 2$. In both cases information sharing would help the incumbent bank to avoid the inefficient liquidation of its $H$-type loan.} Second, a relationship bank may whitewash its borrower’s credit history during the information disclosure process.\footnote{Proposition 3 establishes that the incumbent bank always lie ex-post in Cases 0 and 1, and it may lie in Case 2.}

To tackle the first inefficiency, the planner could impose a public credit registry by forcing the incumbent bank to disclose credit information. However, the power of this public authority may be compromised as the incumbent bank can manipulate the credit reports to the registry. The following analysis then formally address the question when a public registry can is welfare improving compared to a private bureau. We focus on the interesting case where it is not always of the incumbent bank’s best interest to participate in the sharing scheme, i.e., we have $0 < \rho < (1 - \alpha)(1 - \delta)$. We revisit our four cases.

Consider Case 0 where $c + r_0 < R < R^E_S(D)$, then we have $R^*_N = R^*_S(D) = R^*_S(D) = R$. Suppose there is no ex-post manipulation at first, Proposition 2 tells us the incumbent bank will voluntarily participate in information sharing whenever inefficiently liquidation is avoided, i.e., $\Psi_0 = \varphi_0$. Then there is no role for the public registry as it has already been the incumbent bank’s private optimal choice to form a private bureau. When ex-post manipulation is considered, Proposition 3 tells us the incumbent bank will always misreport the credit performance of default borrowers even if it decides to participate in a sharing scheme ex-ante. In this situation, an information sharing scheme has no use at all no matter it has been formed privately or publicly. Indeed, a public registry has the authority to force information sharing. However, facing the same information constraint as the outside private parties, it can not authenticate the unverifiable credit information.
shared by the incumbent bank. Rationally anticipating incumbent bank’s manipulation in either form of schemes, the outside parties always evaluate the loan quality with their ex-ante priors. The situation is equivalent to case without information sharing.

For illustrative simplicity, we analyze Case 3 next where \( R > R_E^S(D) \). In this case, the equilibrium loan rates are interior solutions. Recall Proposition 2, we have \( \Psi_3 = \varphi_3 \) when the ex-post manipulation constraint is neglected first. Again, there is no need to establish a public registry as a private bureau will emerge voluntarily. When the ex-post manipulation constraint is imposed, Proposition 3 shows the incumbent bank will truthfully communicate the credit history in all parameter region of Case 3. So the incumbent bank will privately establish a bureau and communicate credit information truthfully. Different from Case 0, a registry can induce truthfully credit information sharing as well, but it does not outperform its private counterparty.

We then turn to Case 1 and 2. It will become evident that a public registry will add social welfare for certain parameters in those cases. Consider Case 1 where \( R^E(D) < R < R_E^N \), the only interior solution is the equilibrium loan rate for \( D \)-history loan. To avoid inefficient liquidation, information sharing is socially desirable in the parametric set \( \Psi_1 \). The incumbent bank nevertheless will voluntarily establish such scheme only in \( \varphi_1 \), a smaller set of the constellation of \( \Psi_1 \).\(^{23}\) In the region \( \Psi_1 - \varphi_1 \), a \( H \)-type of loan will be inefficiently liquidated in the bad state. So if ex-post manipulation constraint is not a concern, a public registry could improve social welfare by forcing the incumbent bank to share credit information in the entire region \( \Psi_1 \). When ex-post manipulation is a concern, truthful reporting can only be sustained in a subset \( \chi_1 \) of \( \Psi_1 \) assuming the incumbent bank has already entered into a sharing scheme, where

\[
\chi_1 := \left\{ (c + r_0, R) \mid R > R_3 \equiv \left[ \frac{\alpha \rho + (1 - \alpha) \delta \pi}{\alpha \rho + (1 - \alpha) \delta} \cdot \frac{\alpha + (1 - \alpha) \delta}{\alpha + (1 - \alpha) \delta \pi} \right] \cdot \frac{c + r_f}{\pi} \right\} \subset \Psi_1.
\]

However, Proposition 3 also shows that \( \chi_1 \cap \varphi_1 = \emptyset \) the ex-ante voluntary information sharing and the ex-post truth-telling cannot be satisfied at the same time. With this result, a private bureau will never be established in the first place. Indeed, outside market participants rationally anticipating the incumbent bank’s manipulation of credit history will always use the prior beliefs to evaluate the loan quality. Costly liquidation can never be avoided in \( \Psi_1 \).

\(^{23}\)Note that \( \varphi_1 \subset \Psi_1 \) if \( \rho < (1 - \alpha)(1 - \delta) \). When \( \rho = 0 \), the incumbent bank never voluntarily participates in the information sharing scheme.
Instead, suppose the social planner now commands the incumbent bank to disclose credit information to a public registry when the parameters are in the region $\chi_1$. Then the incumbent bank’s ex-post truth reporting IC is also satisfied in this region. As a result, the incumbent bank with a $H$-type loan survives from a bank run in this region through the information truthfully communicated to the public registry.

Lastly, we analyze Case 2 where $R_N^F < R < R^S(D)$. In this case, we again have $\varphi_2 \subset \Phi_2$ when $\rho < (1 - \alpha)(1 - \delta)$. So the incumbent bank’s private optimal decision differs from the socially efficient choice in region $\Psi_2 - \varphi_2$. Then Proposition 3 tells us when $0 < \rho < \rho_2$, the parameter set that ex-post truthful reporting being sustained

$\chi_2 := \{(c + r_0, R) | R > R_3 \equiv \left[ \frac{\alpha \rho + (1 - \alpha) \delta \pi}{\alpha \rho + (1 - \alpha) \delta} \frac{\alpha + (1 - \alpha) \delta}{\alpha + (1 - \alpha) \delta \pi} \right] \frac{c + r_f}{\pi} \} \supset \varphi_2$.

Consequently, by commanding information communication to a public registry in the region $\chi_2 - \varphi_2$, the inefficient liquidation in this region is avoided.

On the other hand, when $\rho > \rho_2$, we have $\chi_2 < \varphi_2$. Truthful information sharing can only be attained in the region $\chi_2$ where the incumbent bank has an incentive to voluntarily participate in a private bureau. So a public registry does not improve social efficiency in this last situation.

We summarize the above discussion in the following Proposition

**Proposition 5** A public registry can improve social welfare compared to the situation with a private bureau only in Cases 1 and 2. In Case 1, public registry outperforms private bureau when the parameters are in the set $\chi_1$. In Case 2, it outperforms private bureau when the parameters are in the set $\chi_1$ as well as $\rho < \rho_2$.

[Insert Figure 6 here]

5 Robustness and Discussion

We check now the robustness of our main assumptions, and we also provide discussions of possible extensions of the model.

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24 Forcefully disclosure outside $\chi_1$ is subject to the same comment as Case 0.

25 Recall that the mathematical form of $\chi_2$ coinsides with that of $\chi_1$. But these two sets are actually mutually exclusive depending on the value of $R$. 

30
5.1 Risky $H$-type borrowers

We first show that our results are robust to the introduction of risky $H$-type borrowers. Instead of assuming an $H$-type borrower who never defaults, we now relax the assumption to allow an $H$-type borrower only to succeed with probability $q \in (0, 1)$ in State $B$. We show that the main trade-off between secondary-market liquidity and primary-market rent extraction remains, when the incumbent bank decides on information sharing. We illustrate with the case where the information on type is verifiable for simplicity, and the result can also be extended to the case where the incumbent bank shares only past credit history.

In the absence of information sharing, the asset price of an $H$-type loan will be $P^B_s(H) = q R^*_s(H)$ in State $B$, and $P^G_s(H) = R^*_s(H)$ in state $G$. The price of an $L$-type loan remains $P^B_s(L) = 0$ and $P^G_s(L) = R^*_s(L)$, in State $B$ and $G$ respectively. Now move one step backwards and consider the incumbent bank’s funding. The deposit rate to finance an $L$-type loan remains as $r_f/\pi$. However, compared to the setup where $q = 1$, the depositors’ break-even rate for an $H$-type loan increases from $r_f$ to $r_f/[(\pi + q(1 - \pi))]$, reflecting the fact that the incumbent bank can now default in state $G$ with a probability $1 - q$. Moving one step further backwards, one can show that the entrant bank’s break-even loan rate is $R^E_s(H) = (c + r_f)/[(\pi + (1 - \pi)q]$ for an $H$-type borrower and $R^E_s(L) = (c + r_f)/\pi$ for an $L$-type borrower. As a result, the incumbent bank’s equilibrium loan rate will be $R^*_s(H) = \min\{R^E_s(H), R\}$ and $R^*_s(L) = \min\{R^E_s(L), R\}$ for the respective $H$ and $L$ type borrower.

With information sharing, the equilibrium secondary market asset price for $H$-type loan can be expressed explicitly as

$$P^B_s(H) = q \cdot \min\{R^E_s(H), R\} \quad \text{if} \quad s = B, \quad P^G_s(H) = \min\{R^E_s(L), R\} \quad \text{if} \quad s = G.$$ 

On the other hand, one can follow the proof of Lemma 1 to compute the equilibrium secondary-market asset prices in the absence of information sharing as

$$P^B_N = \frac{q \cdot \alpha \rho}{(1 - \alpha) + \alpha \rho} \min\{R^E_N, R\}, \quad P^G_N = \min\{R^E_N, R\},$$

where the loan rate offered by entrant bank $R^E_N = (c + r_f)/[\alpha(\pi + (1 - \pi)q) + (1 - \alpha)\pi]$. 

31
It is straightforward to establish $R_S^*(H) \leq R_N^*$, so that the incumbent bank does lose rent from intensified primary loan market competition.\footnote{The inequality is strict when both loan rates are interior solutions lower than $R$.} Furthermore, $P^B_N < P^B_S(H)$ is always true, which indicates that information sharing boosts the secondary-market price for bank’s asset. As a result, there exists $r_f \in (P^B_N, P^B_S(H))$, such that the incumbent bank is saved from runs in State $B$ because of information sharing.

Therefore, the incumbent bank still trades off the benefit of increasing secondary market liquidity against the loss of monopolistic rent when making ex-ante information sharing decisions. Thus, introducing a risky $H$-type borrower will not qualitatively change our main results. One can find similar result to Proposition 1 where there exist a critical value of $\rho$ and values of $r_f$ such that informations sharing endogenously arises when $\rho$ is less than this value and $P^B_N < r_f < P^B_S(H)$.

\subsection*{5.2 Unfairly priced deposits}

ToWhen the incumbent bank’s deposits are subsidized, it can be shown that the incumbent bank’s incentive to share information decreases. The reason is that unfairly priced deposits give the bank incentive to engage in risk-shifting. Yet, the main trade off of our model remains.

We envisage a situation where the deposit insurance is provided to the incumbent bank free of charge. So the bank’s funding cost is $r_f$, the risk free interest rate. We illustrate with sharing information on type case and stick to the parametric assumption that $P^B_N < r_f < P^B_S(H)$. Consider case 1 in Proposition 1 where $c + r_f < R < (c + r_f)/(\alpha + (1 - \alpha)\pi)$. We can express the incumbent bank’s expected profit without information sharing as $E(\Pi'_N) = [\pi + (1 - \pi)\alpha(1 - \rho)](R - r_f)$. Its expected profit with information sharing is $E(\Pi'_S) = \alpha(c + r_f - r_f) + (1 - \alpha)\pi(R - r_f)$. $E(\Pi')$ denotes incumbent bank’s expected profit when there is free deposit insurance. In Proposition 1 our main text, those expected profits with fairly priced deposits are $E(\Pi_N) = [\pi + (1 - \pi)\alpha(1 - \rho)]R - r_f$ and $E(\Pi_S) = \alpha(c + r_f) + (1 - \alpha)\pi R - r_f$. Thus, we can compute the following

$$E(\Pi'_S) - E(\Pi'_N) = E(\Pi_S) - E(\Pi_N) - \alpha(1 - \pi)\rho.$$ 

Due to the additional cost $-\alpha(1 - \pi)\rho$, unfairly priced deposits reduce the incumbent bank’s
incentive to share information.\textsuperscript{27} Note that $-\alpha(1 - \pi)\rho$ is actually the additional subsidy from the state where the incumbent fails due to bank runs when there is no information sharing.

Without information sharing, the incumbent bank’s (ex-post) success probability can be decomposed into $(\pi + (1 - \pi)(1 - \rho))$ when it holds an $H$-type loan, and $\pi$ when it holds an $L$-type loan. With information sharing, the success probability is 1 when it holds an $H$-type loan, and $\pi$ when it holds an $L$-type loan. Ex ante, $Pr(H) = \alpha = 1 - Pr(L)$. This means, the incumbent bank has a riskier cash flow distribution without information sharing. The unfairly priced deposits bring into the incumbent bank’s risk shifting incentive, which undermines its incentive to choose a safer distribution of its cash flows. However, as long as the benefit is larger enough, the incumbent bank still chooses information sharing because it has higher mean of cash flows. One can find similar results when information sharing is about the previous credit history.

### 5.3 Mutual information sharing

We do not rely on our asymmetric banks setup to establish our main results either. In fact, one can easily introduce the mutual information sharing agreement a-la Pagano and Jappelli (1993) into our model. In spirit of the classic approach, one can assume two towns, each of which has a bank and a borrower who needs 1 unit of funding. In each town, the bank has an ongoing lending relationship with the borrower. At $t = 0$, both banks decide to enter into a mutual information sharing agreement or not. At $t = 1$, a bank shares its borrower’s type/previous credit history if has chosen information sharing, and discloses no information otherwise. At $t = 2$, banks compete for borrowers in both towns, and the winner issues deposits to finance the loan(s) as in our main model. At $t = 3$, both banks could undergo liquidity shocks and need to conduct a loan sale to outside asset buyers.

With the assumption that a bank needs to pay an additional $c$ to extend credit to the borrower who lives in the other town,\textsuperscript{28} there would exist an equilibrium where the local bank always wins the local borrower. While continuing to stay with the incumbent bank,

\textsuperscript{27}One can also check for similar result in case 2 Proposition 1 where $(c + r_f)/(\alpha + (1 - \alpha)\pi) < R < (c + r_f)/\pi$. For case 0 and 3, we still have that the incumbent bank always chooses to share information.

\textsuperscript{28}The original setup of Pagano and Jappelli (1993) also features the same assumption, with $c$ being interpreted as the cost for a borrower to switch from the local lender to a distant competitor.
bank, a borrower benefits from the lower loan rate under information sharing, thanks to the intensified primary market competition.

Our existing analyses easily extends to this symmetric-bank setup. When it is profitable to share information, i.e., \( E(\Pi_S) > E(\Pi_N) \), the incumbent bank will do so, regardless of the competing bank’s decision on information sharing decision. In other words, information sharing is the bank’s dominant strategy when \( E(\Pi_S) > E(\Pi_N) \), and our existing results are robust to alternative setups where banks are symmetric and information sharing appears to be a mutual agreement.

### 5.4 Diversification in loan portfolio and securitization

Our results are robust to the diversification of idiosyncratic risks. As long as there is systematic risk and the scope of adverse selection, the main result will remain. We choose one-loan setup because its tractability. The model can be recast into a setup where the incumbent bank has private information on the quality of the loan portfolio.

Suppose now the incumbent bank inherits a unit loan portfolio consisting of \( \alpha \) proportion of \( H \)-type loans and \( 1 - \alpha \) proportion of \( L \)-type loans. There is uncertainty about the portfolio composition, i.e., \( \tilde{\alpha} \in \{\alpha_H, \alpha_L\}, \alpha_H > \alpha_L \). The common prior is \( Pr(\tilde{\alpha} = \alpha_H) = l = 1 - Pr(\tilde{\alpha} = \alpha_L), l \in (0, 1) \). Namely, the incumbent bank can either have a high quality loan portfolio (\( \alpha_H \) proportion of \( H \)-type borrowers) or a low quality loan portfolio (\( \alpha_L \) proportion of \( H \)-type borrowers). We allow partial liquidation of the loan portfolio but forbid the incumbent bank’s cherry picking by selling those \( L \) loans in the portfolio first.

We still use the information sharing on type case to illustrate that sharing information increases the incumbent bank’s secondary market liquidity, so all our results remain be true in this framework. To see this, consider first the case without information sharing. The unit asset price \( P_N^B \) for the incumbent’s loan portfolio without information sharing. Now it is

\[
P_N^B = \left( \frac{l\alpha_H}{l\alpha_H + (1-l)\alpha_L} \right) R_N^*. \]

Note that \( \alpha_H R_N^* \) and \( \alpha_L R_N^* \) are the unit returns of the high quality portfolio and low quality portfolio in state \( B \). With \( P_N^B \) is less than \( \alpha_H R_N^* \), the incumbent bank still sells its high quality loan portfolio only if it faces the liquidity shock. When there is information sharing on the type, the incumbent bank directly communicate to the outside players whether each loan in its portfolio is an \( H \)-type or an \( L \)-type. Then the outside players can simply figure out whether the incumbent bank holds an high quality
portfolio or not. The unit asset prices are $P_S^B(HP) = \alpha_H R_S^*(H)$ and $P_S^B(LP) = \alpha_L R_S^*(H)$ respectively, where $HP$ denotes “high quality portfolio”. Note that $R_S^*(H)$, $R_S^*(L)$ and $R_N^*$ are the unit loan rates, again determined by the entrant bank’s break-even condition and the market contestability. It is clear that $R_S^*(H)$, $R_S^*(L)$ are the same as before. However, we have now $R_N^* = \min\left\{ R, \left( \frac{l \alpha_H + (1 - \alpha_H) \pi}{\alpha_H + (1 - \alpha_H) \pi} + \frac{1 - l}{\alpha_L + (1 - \alpha_L) \pi} \right) (c + r_f) \right\}$. When then entrant offers loan rate to a borrower, it believes that with probability $l$ the incumbent has an high quality portfolio, in which case the the borrower is an $H$-type with probability $\alpha_H$. So we still have $R_N^* \geq R_N^* \geq R_S^*(L)$. We can still prove that $P_S^B(HP) > P_N^B$. So there exist cases such that $P_S^B(HP) > r_f > P_N^B$, without information sharing the incumbent bank with an high quality portfolio will be forced into costly liquidation, while with information sharing it can sell a fraction $\beta = r_f/P_S^B(HP)$ to meet the depositors’ withdrawal $r_f$.

Our results are robust to the assumption that the incumbent bank raises liquidity through securitization instead of direct sale of its loan portfolio. One can think out the credit enhancement needed is higher when the incumbent bank does not share information about its loan portfolio.

### 5.5 Endogenous liquidity risk

In the main text, we interpret the incumbent bank’s liquidity risk as the risk of bank runs. We assume that runs occur with an exogenous probability $\rho$ and that the event is not related to the incumbent bank’s fundamental, i.e., the type of loans and State $s$. In other words, the runs in our model are sun-spot events. We believe, though, that our main results should be robust to alternative modeling approaches that endogenize the risk of runs. A popular approach that links the liquidity risk to banks’ fundamental is to use global games to refine the multiple equilibria of bank run games. The approach typically predicts a threshold value of bank’s cash flow: Bank run occurs if and only if the realized fundamental is lower than the threshold. A common feature of the threshold equilibrium is that the threshold decreases in the liquidation value of banks’ assets.\(^{29}\)

As illustrated in the main model, the price for bank’s asset increases with information sharing,\(^{30}\) as the adverse selection mitigates in the secondary loan market. The higher asset market liquidity of the incumbent bank reassures the depositors that the bank would not

\(^{29}\)For example, see...

\(^{30}\)Recall that
incur liquidation losses as high as in the case without information sharing. Introducing a global-games setup actually strengthens our main trade off: the benefit of sharing information comes from the increase of both bank’ market liquidity as well as its funding liquidity. The argument is, again, true when we consider information sharing on credit history.

Our main result should be robust to this alternative modeling approach of bank run. Suppose now there is uncertainty on the loan’s return: An \( H \)-type loan has a lower maximum return than an \( L \)-type loan, but former has a better distribution of returns than the latter in the sense of first order stochastic dominance. Depositors after financing the incumbent bank at \( t = 2 \), receive private signals about the realization of loan return and make the simultaneous bank run decision at \( t = 3 \).

6 Conclusion

This paper formally analyzes the conjecture according to which banks’ decision to share information about the credit history of their borrowers is driven by the needs for market liquidity. To meet urgent liquidity needs, banks have to make loan sale in the secondary market. However, the information friction in loan markets makes this sale costly and good loans can be priced below their fundamental value. This concern became very evident during the financial crisis started in the summer of 2007. Several potentially solvent banks risk to fail because they could not raise enough short term liquidity.

This basic observation implies that banks could find convenient to share information on their loans in order to reduce the information asymmetry about their quality in case they have to sell them in the secondary market. Information sharing can be a solution to reduce the cost of urgent liquidity needs so to make banks more resilient to funding risk. Clearly, sharing information makes banks to lose the rent they extract if credit information were not communicated. Banks may be no longer able to lock in their loan applicants because competing banks also know about the quality of those loans. Eventually, the benefit of a greater secondary market liquidity has to be traded off with the loss in information rent.

We show that it possible to rationalize information sharing as such device. We show under which conditions information sharing is feasible, and when is actually chosen by the banks.
in equilibrium.

We also show that our rationale for information sharing is robust to truth telling. A common assumption in the literature is that when banks communicate the credit information, they share it truthfully. We allow banks to manipulate the information they release by reporting bad loans as good ones. The reason is for the banks to increase the liquidation value in the secondary market. We show that when banks lose too much in information rent from good borrowers with bad credit history, then information sharing is a truth telling device.

Coherently with previous theoretical model of information sharing, the existing empirical literature has mostly focused on the impact of information sharing on bank risks and firms’ access to bank financing. Our theoretical contribution generates new empirical implications. In particular, information sharing should facilitate banks liquidity management and loan securitization. The model also suggests that information sharing can be more easily established, and work more effectively, in countries with competitive banking sector, and in credit market segments where competition is strong.

References


Appendix: Proofs of Lemmas and Propositions

Proof of Lemma 1. When the incumbent bank does not share any information, the game features incomplete information. Therefore, we apply the solution concept of PBE. We show that the unique pure-strategy equilibrium is a pooling equilibrium, as the incumbent bank offers a unified loan rate $R^I \in [r_f, R]$ and a unified deposit rate $r^I \in [r_f, r_f/\pi]$, independent of the type of borrower it finances.\footnote{Notice that the incumbent bank holds private information about the borrower’s type as well as the previous credit history. We consider the incumbent bank directly conditions its loan rates on the true types.}

Definition 1. A pure strategy pooling PBE of the game $g_N$ is characterized as follows.
(i) An equilibrium strategy profile: Based on its knowledge of the borrower’s type, the incumbent bank at $t = 2$ sets a loan rate $R^I_N$ for the borrower and offers a take-it-or-leave-it deposit rate $r^I_N$ to depositors. When having financed the borrower, the incumbent bank decides at $t = 3$ whether to sell the loan, according to the loan quality, State $s$, and its own liquidity position. The entrant bank offers a competing loan rate $R^E$ without knowing the borrower’s type or credit history. Depositors choose to provide funding or not based on the offered deposit rate. Asset buyers bid $P^G_N$ in State $G$ and $P^B_N$ is State $B$ to purchase any loan on sale.
(ii) A system of beliefs: The entrant bank holds the prior belief about the borrower’s type. The depositors Bayesian update their beliefs according to the deposit rate offered by the incumbent bank. The asset buyers Bayesian update their beliefs according to the aggregate state and observation of asset sale.
(iii) The strategy profile in (i) is sequential rational given the beliefs in (ii).

The proof consists of two parts. In Part I, we show that the strategy profile and belief system described in Lemma 1 indeed constitute a PBE. In Part II, we prove that no other pure-strategy PBE exists.

Part I: The existence of a pure-strategy pooling PBE.

To establish the equilibrium described in Lemma 1, we solve the game backwards.

Step 1. We start by analyzing the secondary loan market in State $G$ and $B$ respectively. In State $G$, the incumbent bank sells its loan only when facing a run, and in that case sells its loan regardless of the borrower’s type. The asset buyers, therefore, believe the
loan on sale to be an $H$-type with probability $\alpha$ and an $L$-type with probability $1 - \alpha$. Moreover, both $H$- and $L$-type borrowers will repay $R_N^*$ in State $G$. As a result, asset buyers’ competitive bidding leads to the following break-even condition

$$\alpha(R_N^* - P_N^G) + (1 - \alpha)(R_N^* - P_N^G) = 0,$$

which implies

$$P_N^G = R_N^* \quad (8)$$

In state $B$, the incumbent bank always sells its $L$-type loan and sells an $H$-type loan only when facing a run, and asset buyers Bayesian update their belief accordingly.

$$Prob(H|\text{loan sale}) = \frac{Pr(\text{run})Pr(H)}{Pr(\text{run})Pr(H) + Pr(L)} = \frac{\alpha \rho}{\alpha \rho + (1 - \alpha)}$$

Since only an $H$-type borrower will repay $R_N^*$ in state $B$, the asset buyers’ competitive bidding leads to the following break-even condition

$$\frac{\alpha \rho}{\alpha \rho + (1 - \alpha)} (R_N^* - P_N^B) + \frac{1 - \alpha}{\alpha \rho + (1 - \alpha)} (0 - P_N^B) = 0,$$

which implies

$$P_N^B = \frac{\alpha \rho}{(1 - \alpha) + \alpha \rho} R_N^* < R_N^* \quad (9)$$

One can verify that asset buyers’ beliefs are consistent with the incumbent bank’s equilibrium strategy. In state $B$, an $L$-type loan is doomed to fail, so the incumbent bank optimally sells an $L$-type loan for $P_N^B > 0$. If the loan is an $H$-type, the incumbent bank optimally holds it to maturity unless it experiences runs, because $P_N^B < R_N^*$. In state $G$, the incumbent bank is indifferent between holding the loan to maturity and selling the loan in the secondary market when facing no run, as both actions generate the same revenue $R_N^*$. Given an innocuous assumption that the incumbent bank holds its loan to maturity when facing no run in State $G$, asset buyers’ belief will remain the same as the prior.

Step 2: We now move to the stage where the incumbent bank raises its funding. The strategic interaction between the incumbent bank and its depositors can be considered as a signaling game. The incumbent bank (sender) has private information about its loan quality and offers a deposit rate (a message) to the depositors (receivers). The depositors may infer the bank’s type when deciding on accepting the bank’s offer or not. The bank raises 1 unit of funding from depositors if its offer is accepted, and receives a zero payoff otherwise, as in the latter case no loan will be financed.
We first analyze depositors’ belief and strategy on the equilibrium path described by Lemma 1. As the incumbent bank offers a pooling deposit rate \( r_I^N \), the depositors belief about the bank’s loan quality remains the same as the prior. 

\[
Pr(H|r_I^N) = Pr(H) = \alpha, \quad Pr(L|r_I^N) = Pr(L) = 1 - \alpha
\]

Furthermore, under condition 

\[
P_B^B = \frac{\alpha \rho}{\alpha \rho + (1 - \alpha)} R_N^* < r_f,
\]

the incumbent bank cannot raise enough liquidity from the loan sale when a run happens. The depositors therefore anticipate the incumbent bank to fully repay its debt in State \( G \) and to be able to repay in state \( B \) only when it holds an \( H \)-type of loan and faces no bank run. Given the depositors’ belief about the loan quality,

\[
r_I^N = \frac{r_f}{\pi + (1 - \pi)\alpha(1 - \rho)} \tag{10}
\]

is the minimum rate that depositors are willing to accept, as the rate allows them to break even given the subsequent equilibrium strategies of the incumbent bank and asset buyers.

\[
Pr(G)r_I^N + Pr(B)Pr(H|r_I^N)Pr(\text{no run})r_I^N = [\pi + (1 - \pi)\alpha(1 - \rho)]r_I^N = r_f.
\]

The incumbent bank’s equilibrium deposit rate as in Lemma 1 can be sustained by depositors’ off-equilibrium path belief 

\[
Pr(H|r_I^N \neq r_I^N) < \alpha \quad \text{and} \quad Pr(L|r_I^N \neq r_I^N) > 1 - \alpha.
\]

In other words, depositors hold a worse-than-prior belief about the incumbent bank’s loan quality when receiving an off-equilibrium deposit rate \( r_I^N \neq r_I^N \). Given the off-equilibrium path beliefs, the depositors’ break-even deposit rate can be computed as

\[
\hat{r} = \frac{r_f}{\pi + (1 - \pi)Pr(H|r_I^N \neq r_I^N)(1 - \rho)} > r_I^N.
\]

Given the depositors’ break-even rate, it is indeed sequentially rational for the incumbent bank to offer pooling deposit rate \( r_I^N \). In particular, when the incumbent bank wins an \( H \)-type loan, it earns an expected profit

\[
\Pi^I(H) = \pi(R_N^* - r_I^N) + (1 - \pi)(1 - \rho)(R_N^* - r_I^N) = [\pi + (1 - \pi)(1 - \rho)](R_N^* - r_I^N) > 0
\]

by offering the equilibrium deposit rate \( r_I^N \).\(^{32}\) Suppose, instead, the incumbent bank deviates by offering a deposit rate \( r_I^N \neq r_I^N \), then its expected profit is either

\[
\hat{\Pi}^I(H) = 0 < \Pi^I(H) \quad \text{if} \quad r_I^N < \hat{r},
\]

\(^{32}\)When \( R_N^* = R, R_N^* > r_I^N \) is guaranteed by our assumption \( \pi R > r_f \). When \( R_N^* = R_N^E = \frac{c + r_f}{\pi + (1 - \alpha)\pi} \), the inequality is guaranteed by our assumption \( c > \frac{1 - \pi}{\pi} r_f \).
or
\[ \hat{\Pi}^I(H) = [(\pi + (1 - \pi)(1 - \rho))(R^*_N - \hat{r}) < \Pi^I(H) \quad \text{if} \quad r^I > \hat{r}. \]

Thus, the incumbent bank has no profitable deviation by offering a deposit rate \( r^I \neq r^I_N \).

Similarly, when the incumbent bank wins an \( L \)-type loan, it earns an expected profit
\[ \Pi^I(L) = \pi(R^*_N - r^I_N) > 0 \]
by offering the equilibrium rate \( r^I_N \). While its expected profit from deviation is either
\[ \hat{\Pi}^I(L) = 0 < \Pi^I(L) \quad \text{if} \quad r^I < \hat{r}, \]
or
\[ \hat{\Pi}^I(L) = \pi(R^*_N - \hat{r}) < \Pi^I(L) \quad \text{if} \quad r^I > \hat{r}. \]

Therefore, the incumbent bank has no profitable deviation when holding an \( L \)-type loan either. In sum, the worse-than-prior off-equilibrium belief is a sufficient condition for \( r^I_N \) to be part of the PBE.\(^{33}\)

\( \text{Step 3:} \) We now analyze the primary loan market competition between the incumbent and entrant banks.

Given the entrant bank’s belief \( \text{Prob}(H) = \alpha \) and \( \text{Prob}(L) = 1 - \alpha \), the minimum loan rate that satisfies the participation constraint of the entrant bank is
\[ R^E_N = \frac{c + r_f}{\alpha + (1 - \alpha)\pi}. \] (11)

Otherwise, the entrant bank will be better off holding risk-free assets. That is,
\[ [\alpha + (1 - \alpha)\pi] R^E - c < r_f, \forall R^E < R^E_N. \]

We now show that given the subsequent equilibrium strategies characterized in Step 1 and 2, the incumbent bank’s equilibrium strategy in the primary loan market is to offer a pooling rate
\[ R^*_N = \min\{R, R^E_N\} \] (12)
regardless of the type of the borrower.

\(^{33}\)The worse-than-prior off-equilibrium belief is also a necessary condition for \( r^I_N \) to be part of the PBE. If the off-equilibrium beliefs are more optimistic than the prior, the incumbent will have the incentive to deviate from offering \( r^I_N \). Therefore, any pooling equilibrium must be associated with worse-than-prior off-equilibrium beliefs.
First, consider the interior solution $R_N^* = R_E^N < R$.\footnote{We assume the borrower sticks to the incumbent bank when there is a tie in competing loan rates $R^I(H) = R_E^N$.} For an $H$-type borrower, the incumbent bank’s expected profit when offering the loan rate $R_N^* = R_E^N$ is

$$\Pi^I(H) = [\pi + (1 - \pi)(1 - \rho)](R_N^E - r_N^I) > 0,$$

with $\Pi^I(H) > 0$ guaranteed by Inequality (3).

The bank has no profitable deviation. If the incumbent bank deviates by charging a loan rate $R^I(H) > R_N^E$, it will lose the loan competition and realizes a zero profit. If the incumbent bank deviates by charging a loan rate $R^I(H) < R_N^E$, it still wins the $H$-type borrower but only earns a lower expected profit.

$$\hat{\Pi}^I(H) = [\pi + (1 - \pi)(1 - \rho)](R^I(H) - r_N^I) < \Pi^I(H)$$

For an $L$-type loan, the incumbent bank earns an expected profit

$$\Pi^I(L) = \pi(R_N^E - r_N^I) > 0,$$

by offering the equilibrium rate $R_N^E$. Again, the bank has no profitable deviation. If the incumbent bank deviates by charging a loan rate $R^I(L) > R_N^E$, it will lose the loan competition and realizes a zero profit. If the incumbent bank deviates by charging a loan rate $R^I(L) < R_N^E$, it still wins the $L$-type borrower but only earns a lower expected profit.

$$\hat{\Pi}^I(L) = \pi(R^I(L) - r_N^I) < \Pi^I(L)$$

Consider now the corner solution $R_N^* = R < R_N^E$. The incumbent bank charges a loan rate $R$ independent of the borrower’s type and earns an expected profit

$$\Pi^I(H) = [\pi + (1 - \pi)(1 - \rho)](R - r_N^I)$$

and

$$\Pi^I(L) = \pi(R - r_N^I)$$

on an $H$- and $L$-type borrower respectively. Both $\Pi^I(H)$ and $\Pi^I(L)$ are guaranteed to be positive by assumption (??).
Similar to the case of interior solution, one can verify the incumbent bank having no profitable deviation for both types.

Lastly, we show that, given its belief about the loan quality and the incumbent bank’s strategy $R^*_N$, the entrant bank has no profitable deviation either. Consider first the interior case $R^*_N < R$. By offering a slightly higher loan rate $R^*_N + \varepsilon$, the entrant bank loses the loan market competition regardless of the borrower’s type and realizes a profit 0. By offering a slightly lower loan rate $R^*_N - \varepsilon$, the entrant bank wins the borrower but would be better off in expectation by investing in the risk-free asset.

\[ [\alpha + (1 - \alpha)\pi](R^*_N - \varepsilon) - (c + r_f) = -[\alpha + (1 - \alpha)\pi]\varepsilon < 0 \]

Consider next the corner case where $R^*_N > R$. If the entrant bank offers a loan rate higher than $R$ but below $R^*_N$, it will not be able to attract the borrower. If it offers a loan rate that is even higher than $R^*_N$, the entrant bank also attracts no borrower. If it offers a rate that is lower than $R$, the entrant bank wins the borrower regardless of its type but only realizes a negative expected profit.

To summarize, we have established that the strategy profile and belief system described in Lemma 1 is indeed a pure-strategy PBE.

**Part II: The uniqueness of the pure-strategy pooling PBE.**

To establish the uniqueness of the PBE in Lemma 1, we consider all possible alternative strategy profiles from the loan market competition and prove by contradiction that none can be part of a PBE. Let $R^E$ denote the loan rate offered by the entrant bank; $R^I(H)$ and $R^I(L)$ the loan rated offered by the incumbent bank to an $H$- and $L$-type borrower respectively. We have the following four alternatives scenarios.

**Scenario 1**: $R^E < \min\{R^I(H), R^I(L)\}$, so that the incumbent bank loses loan market competition irrespective of the borrower’s type. Suppose these loan rates had indeed been a part of a PBE. Then the incumbent bank offering a deposit rate to raise funding will be off equilibrium path. We now show this cannot be a PBE, because the incumbent bank has profitable deviations for any off-equilibrium belief of depositors or asset buyers.

Consider most pessimistic beliefs of depositors and asset buyers. That is, they believe that the incumbent bank must have financed an $L$-type loan if the bank had ever offered a deposit rate $r^I$ or entered a loan sale. Given the belief, the asset buyers will offer a price equal to 0 in State $B$, and the depositors will accept a rate $r_f/\pi$ to break even. Now, consider the loan competition between given the incumbent bank’s cost of funding. Note
that when the entrant bank wins the loan competition irrespective of the borrower’s type, \( R^E \) must satisfy
\[
R^E \geq \frac{c + r_f}{\alpha + (1 - \alpha)\pi}
\]
so that the entrant bank’s participation constraint is satisfied. Consequently, the incumbent bank has profitable deviation for an \( L \)-type borrower. By undercutting \( R^E \), the incumbent bank earns an expected profit
\[
\pi \left( R^E - \frac{r_f}{\pi} \right) \geq \pi \left( \frac{c + r_f}{\alpha + (1 - \alpha)\pi} - \frac{r_f}{\pi} \right) > 0.
\]
The profit is guaranteed to be positive by inequality (3). It is straightforward to see that profitable deviation also exists for an \( H \)-type borrower and for any more optimistic off-equilibrium-path beliefs of depositors and asset buyers. Therefore, \( R^E < \min\{R^I(H), R^I(L)\} \) cannot be part of a PBE.

**Scenario 2:** \( R^I(H) \leq R^E < R^I(L) \), so that the incumbent bank only wins the \( H \)-type borrower in loan market competition. Suppose these loan rates had indeed been a part of a PBE and that the incumbent bank offers a rate \( r^I(H) \) to depositors. The depositors’ on-equilibrium-path belief will be
\[
Pr(H|r^I(H)) = 1.
\]
Furthermore, the depositors expect asset buyers to Bayesian update the incumbent bank’s loan quality according to the equilibrium strategy and to purchase the incumbent bank’s asset on sale at a price \( P^G = P^B = R^I(H) > r_f \). Therefore, it is sequentially rational for the depositors to provide financing for \( r^I(H) = r_f \).

We now analyze the primary loan market. In such a separating equilibrium where the entrant bank only finances the \( L \)-type, the entrant bank’s participation constraint requires
\[
R^E \geq \frac{c + r_f}{\pi}.
\]
Given its funding cost \( r_f \), the incumbent bank can profitably deviates by increasing \( R^I(H) \) to \( R^E \) for the \( H \)-type, i.e.,
\[
(R^E - r_f) - (R^I(H) - r_f) = R^E - R^I(H) \geq 0
\]
and by decreasing \( R^I(L) \) to \( R^E \) for the \( L \)-type, i.e.,
\[
\pi(R^E - r_f) > \pi \left( \frac{c + r_f}{\pi} - r_f \right) > 0.
\]
Consequently, \( R^I(H) \leq R^E < R^I(L) \) cannot be part of a PBE either.

**Scenario 3:** \( R^I(L) \leq R^E < R^I(H) \), so that the incumbent bank only wins the \( L \)-type borrower in loan market competition. Suppose these loan rates had indeed been a part of a PBE. Similar to Scenario 2, the depositors would hold an on-equilibrium-path belief

\[
Pr(L|r^I(L)) = 1.
\]

and expect that the asset buyers to offer \( P^G = R^I(L) > r_f \) and \( P^B = 0 \) for the incumbent bank’s asset on sale in State \( G \) and \( B \) respectively. Therefore, it is sequentially rational for the depositors to provide financing for \( r^I(L) = r_f/\pi \).

In a separating equilibrium where the entrant bank now wins the \( H \)-type borrower, the bank’s participation constrain requires

\[
R^E \geq c + r_f.
\]

Given its funding cost \( r_f/\pi \), the incumbent bank can profitably deviate by increasing \( R^I(L) \) to \( R^E \) for the \( L \)-type, i.e.,

\[
\pi \left( R^E - \frac{r_f}{\pi} \right) - \pi \left( R^I(L) - \frac{r_f}{\pi} \right) = \pi (R^E - R^I(L)) \geq 0
\]

and by decreasing \( R^I(H) \) to \( R^E \) for the \( H \)-type

\[
R^E - \frac{r_f}{\pi} \geq c + r_f - \frac{r_f}{\pi} = c - \frac{1 - \pi}{\pi} r_f > 0.
\]

The last inequality holds due to our parametric assumption (3). Therefore, we establish that \( R^I(L) \leq R^E < R^I(H) \) cannot be part of a PBE.

**Scenario 4:** \( \max\{R^I(H), R^I(L)\} \leq R^E \), so that the incumbent bank still wins the loan market competition regardless of the borrower’s type. But instead of a pooling equilibrium as described in Lemma 1, the incumbent bank sustain a separating PBE where either \( R^I(H) \neq R^I(L) \), or \( r^I(H) \neq r^I(L) \), or both. We prove by contradiction that the rates cannot be part of a PBE because the incumbent bank will have incentives to deviate. In particular, an incumbent bank with an \( L \)-type loan will mimic a bank of an \( H \)-type loan.

Suppose the incumbent bank offers separating deposit rates \( r^I(H) \neq r^I(L) \) in equilibrium. The depositors’ on-equilibrium-path belief will be

\[
Pr \left( H|r^I(H) \right) = 1, \quad \text{and} \quad Pr \left( L|r^I(L) \right) = 1.
\]
and they will provide funding if \( r^I(L) \geq r_f/\pi \) and \( r^I(H) \geq \frac{r_f}{\pi + (1-\pi)(1-\rho)} \). As a result, the incumbent bank can profitably deviate by offering \( r^I(L) = r^I(H) = \frac{r_f}{\pi + (1-\pi)(1-\rho)} \). That is, to secure a lower cost of funding, the incumbent bank always prefers to claim having lend to an H-type borrower given the depositors’ on-equilibrium-path beliefs. Therefore, offering separating deposit rates must not be a part of PBE.

We now move one step backwards to analyze the primary loan market competition. Suppose separating loan rates \( R^I(H) \neq R^I(L) \) had indeed been a part of a PBE. Note that the incumbent bank’s funding cost for a loan will be \( r_N^I \) according to our previous discussion. Moreover, recall that the entrant bank’s minimum required loan rate is \( R_N^E \), so we must have \( R^E \geq R_N^E \). Given such competing loan rate \( R^E \) and funding cost \( r_N^I \), the incumbent bank has profitable deviations by increasing both \( R^I(H) \) and \( R^I(L) \) to \( R^E \) if \( \max\{R^I(H), R^I(L)\} < R^E \) or increasing the lower one to \( R^E \) if there is an equality. To see this, note that

\[
R^E - r_N^I \geq \min\{R, R_N^E\} - r_N^I > 0.
\]

Consequently, offering the above separating loan rates can not be a sequential rational action for the incumbent bank.

To summarize, we establish the strategy profile and beliefs in Lemma 1 as a unique pure-strategy PBE. •

**Proof of Proposition 1.** Depending on the value of \( R \), we have four cases (note that the index given to each case represents the number of the interior solutions):

- **Case 0:** \( R \in [0, r_f + c) \) so that \( R_S^*(H) = R_N^* = R_S^*(L) = R \);
- **Case 1:** \( R \in \left[ r_f + c, \frac{r_f + c}{\alpha + (1-\alpha)\pi} \right) \) so that \( R_S^*(H) = r_f + c \) and \( R_N^* = R_S^*(L) = R \);
- **Case 2:** \( R \in \left[ \frac{r_f + c}{\alpha + (1-\alpha)\pi}, \frac{r_f + c}{\pi} \right) \) so that \( R_S^*(H) = r_f + c \), \( R_N^* = \frac{r_f + c}{\alpha + (1-\alpha)\pi} \) and \( R_S^*(L) = R \);
- **Case 3:** \( R \in \left[ \frac{r_f + c}{\pi}, \infty \right) \) so that \( R_S^*(H) = r_f + c \), \( R_N^* = \frac{r_f + c}{\alpha + (1-\alpha)\pi} \) and \( R_S^*(L) = \frac{r_f + c}{\pi} \).

For Cases 0 and 3, in the text we have shown \( E(\Pi_S) \geq E(\Pi_N) \), independent of \( \rho \), with the inequality being strict as long as \( \rho > 0 \). It remains to show that in Cases 1 and 2 there exists a \( \hat{\rho} \in (0, 1) \) such that \( E(\Pi_S) > E(\Pi_N) \) for \( \rho > \hat{\rho} \).

Consider Case 1. The expected profit \( E(\Pi_N) \) can be written as

\[
E(\Pi_N) = [\pi + (1-\pi)\alpha(1-\rho)]R - r_f,
\]

which monotonically decreases in \( \rho \). At the boundaries we have

\[
\lim_{\rho \to 0} E(\Pi_N) = [\pi + (1-\pi)\alpha]R - r_f \quad \text{and} \quad \lim_{\rho \to 1} E(\Pi_N) = \pi R - r_f.
\]

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On the other hand, the expected profits $E(\Pi_S)$ can be written as

$$E(\Pi_S) = \alpha (r_f + c) + (1 - \alpha) \pi R - r_f$$

$$= \pi R + \alpha [(r_f + c) - \pi R] - r_f$$

The last expression is strictly smaller than $[\pi + (1 - \pi) \alpha] R - r_f$ because under Case 1 we have $R > r_f + c$, and it is greater than $\pi R - r_f$ because $\pi R < (r_f + c)$ (a condition implied by the upper bound on $R$ under Case 1). Since $E(\Pi_N)$ is continuous in $\rho$, there must exist a $\hat{\rho}_1 \in (0, 1)$ such that $E(\Pi_S) > E(\Pi_N)$ for $\rho > \hat{\rho}_1$.

Consider Case 2. The expected payoff $E(\Pi_N)$ becomes

$$E(\Pi_N) = \frac{\pi + (1 - \pi) \alpha (1 - \rho)}{\alpha + (1 - \alpha) \pi} (r_f + c) - r_f,$$

which still monotonically decreases in $\rho$. At the boundaries we have

$$\lim_{\rho \to 0} E(\Pi_N) = c \quad \text{and} \quad \lim_{\rho \to 1} E(\Pi_N) = \frac{\pi (r_f + c)}{\alpha + (1 - \alpha) \pi} - r_f.$$

On the other hand, the expression of $E(\Pi_S)$ remains the same as in Case 1, that is

$$E(\Pi_S) = \alpha (r_f + c) + (1 - \alpha) \pi R - r_f < c$$

because $\pi R < (r_f + c)$. Meanwhile, we know

$$E(\Pi_S) > \alpha (r_f + c) + (1 - \alpha) \pi \frac{r_f + c}{\alpha + (1 - \alpha) \pi} - r_f = \left[\alpha + \frac{(1 - \alpha) \pi}{\alpha + (1 - \alpha) \pi}\right] (r_f + c) - r_f,$$

where the inequality holds because under Case 2 we have $R > \frac{r_f + c}{\alpha + (1 - \alpha) \pi}$. The last expression is greater than the lower bound of $E(\Pi_N)$ because

$$\alpha + \frac{(1 - \alpha) \pi}{\alpha + (1 - \alpha) \pi} > \frac{\pi}{\alpha + (1 - \alpha) \pi}.$$

Therefore, since $E(\Pi_N)$ is continuous in $\rho$, there must exist a $\hat{\rho}_2 \in (0, 1)$ such that $E(\Pi_S) > E(\Pi_N)$ for $\rho > \hat{\rho}_2$. Letting $\hat{\rho} \leq \min\{\hat{\rho}_1, \hat{\rho}_2\}$, we have proven Proposition 1. ■

**Proof of Lemma 2.** When the incumbent shares the borrower’s credit history and lends to a borrower with no previous default ($\overline{D}$), the game features incomplete information, because such a borrower can either be an $H$- or $L$-type. Therefore, we apply the solution concept of PBE. Yet, different from the previous case without information sharing, the
outsiders can update their beliefs about the borrower’s type according to Bayesian rule. Their posterior beliefs \( \mu(\overline{D}) \) at the start of game \( g_S(\overline{D}) \) is as follows.

\[
Pr(H|\overline{D}) = \frac{Pr(H, \overline{D})}{Pr(\overline{D})} = \frac{\alpha}{\alpha + (1 - \alpha)\delta} > \alpha
\]

\[
Pr(L|\overline{D}) = \frac{Pr(L, \overline{D})}{Pr(\overline{D})} = \frac{(1 - \alpha)\delta}{\alpha + (1 - \alpha)\delta} < 1 - \alpha
\] (13)

(14)

We denote by \( R^I(\theta, \overline{D}) \) and \( r^I(\theta, \overline{D}) \) the loan rate and deposit rate offered by the incumbent bank on a \( \theta \)-type borrower with a \( \overline{D} \)-history. Let \( R^E(\overline{D}) \) denote the loan rate offered by the entrant bank. We show that the unique pure-strategy equilibrium is, again, a pooling equilibrium, as the incumbent bank offers a unified loan rate \( R^I \in [r_f, R] \) and a unified deposit rate \( r^I \in [r_f, r_f/\pi] \), independent of the type of borrower it finances.

**Definition 2.** A pure strategy pooling PBE of the game \( g_S(\overline{D}) \) is characterized as follows. (i) An equilibrium strategy profile: Based on its knowledge of the borrower’s type, the incumbent bank at \( t = 2 \) sets a loan rate \( R^I_S(\overline{D}) \) for the borrower with no default history and offers a take-it-or-leave-it rate \( r^I_S(\overline{D}) \) to depositors. When having financed the borrower, the incumbent bank decides at \( t = 3 \) whether to sell the loan, according to the loan quality, State \( s \), and its own liquidity position. The entrant bank offers a competing loan rate \( R^E(\overline{D}) \) to the borrower with no previous default. Depositors choose to provide funding or not based on the offered deposit rate and the borrower’s credit history. Asset buyers bid \( P^G_S(\overline{D}) \) in state \( G \) and \( P^B_S(\overline{D}) \) in state \( B \) to purchase the loan if it is on sale. (ii) A system of beliefs: The entrant bank holds the posterior belief \( \mu(\overline{D}) \) about loan quality. Seeing that the borrower has no previous default, the depositors hold a posterior belief \( \mu(\overline{D}, r^E(\overline{D})) \) about loan quality when receiving incumbent’s deposit rate \( r^I_S(\overline{D}) \). Asset buyers Bayesian update their beliefs \( \mu(\overline{D}, s) \) for the borrower who has no default history, according to both the aggregate state and observation of asset sale. (iii) The strategy profile in (i) is sequential rational given the beliefs in (ii).

The proof again consists of two parts, with Part I establishing the strategies and beliefs in Lemma 2 as a PBE, and Part II establishing the uniqueness of the PBE.

**Part I: The existence of a pure-strategy pooling PBE.**

To derive the equilibrium described in Lemma 2, we solve the game backwards.

**Step 1.** We start by analyzing the secondary loan market in state \( G \) and \( B \) respectively. Given the shared borrower credit history \( \overline{D} \) and the incumbent bank’s equilibrium strategy
described in Lemma 2, the asset buyers’ beliefs are characterized by equation (13) and (14). Since both $H$- and $L$-type borrowers will repay $R^*_S(D)$ in state $G$, asset buyers’ competitive bidding leads to the following break-even condition

$$Pr(H|D) \left[ R^*_S(D) - P^G_S(D) \right] + Pr(L|D) \left[ R^*_S(D) - P^G_S(D) \right] = 0,$$

which implies

$$P^G_S(D) = R^*_S(D).$$

(15)

In state $B$, the incumbent bank always sells its $L$-type loan and sells an $H$-type loan only when facing a run, and asset buyers Bayesian update their belief accordingly.

$$Pr(H|\text{loan sale}, D) = \frac{Pr(\text{run}) Pr(H|D)}{Pr(\text{run}) Pr(H|D) + Pr(L|D)} = \frac{Pr(H|D)\rho}{Pr(H|D)\rho + Pr(L|D)}$$

Since only an $H$-type borrower will repay $R^*_S(D)$ in state $B$, the asset buyers’ competitive bidding leads to the following break-even condition

$$\frac{Pr(H|D)\rho}{Pr(H|D)\rho + Pr(L|D)} \left[ R^*_S(D) - P^B_S(D) \right] + \frac{Pr(L|D)}{Pr(H|D)\rho + Pr(L|D)} \left[ 0 - P^B_S(D) \right] = 0.$$ 

Inserting the posterior beliefs, we obtain

$$P^B_S(D) = \frac{\alpha\rho}{(1 - \alpha)\delta + \alpha\rho} R^*_S(D).$$

(16)

Similar to the proof of Lemma 1, one can verify that asset buyers’ beliefs are consistent with the incumbent bank’s equilibrium strategy, and the incumbent bank’s loan sale decision is indeed sequentially optimal, given the asset buyers’ equilibrium bidding.

**Step 2.** We move to the stage when the incumbent bank raises its funding, and analyze again the signaling game between the incumbent bank and its depositors.

We again start with depositors’ belief and strategy on the equilibrium path described by Lemma 2. As the incumbent bank offers a pooling deposit rate $r^I_S(D)$, the depositors belief about the bank’s loan quality remains the same as the posterior $\mu(D)$. That is,

$$Pr(H|r^I_S(D), D) = Pr(H|D) \quad \text{and} \quad Pr(L|r^I_S(D), D) = Pr(L|D).$$

Furthermore, under condition

$$P^B_S(D) = \frac{\alpha\rho}{(1 - \alpha)\delta + \alpha\rho} R^*_S(D) > r_f,$$
the incumbent bank can raise enough liquidity from the loan sale when a run happens in State B. The depositors therefore anticipate the bank to fully repay its debt in both states. Therefore, depositors are willing to accept the risk-free rate,

\[ r^I_S(\mathcal{D}) = r_f, \]  

as the rate allows them to break even given the subsequent equilibrium strategies of the incumbent bank and asset buyers.

The incumbent bank’s equilibrium deposit rate as in Lemma 2 can be sustained if the depositors’ off-equilibrium path belief is worse than the prior.\(^{35}\) Given \( P^B_S > r_f \) the depositors would still consider themselves breaking even, as the bank does not fail even if facing a run in State B.

Given the depositors’ on- and off-the equilibrium path break-even rates, it is indeed sequentially rational for the incumbent bank to offer the pooling equilibrium deposit rate \( r^I_S(\mathcal{D}) \). In particular, when the incumbent bank finances an \( H \)-type loan, it earns an expected profit

\[
\Pi^I_S(H, \mathcal{D}) = \pi \left( R^*_S(\mathcal{D}) - r^I_S(\mathcal{D}) \right) + \left(1 - \pi\right) \left[ \rho \left( P^S_B(\mathcal{D}) - r^I_S(\mathcal{D}) \right) + \left(1 - \rho\right) \left( R^*_S(\mathcal{D}) - r^I_S(\mathcal{D}) \right) \right] \\
= \left[ \pi + \left(1 - \pi\right)(1 - \rho) \right] \left( R^*_S(\mathcal{D}) - r_f \right) + \left(1 - \pi\right) \rho \left( P^S_B(\mathcal{D}) - r_f \right) > 0
\]  

(18)

by offering the deposit rate \( r^I_S(\mathcal{D}) \).\(^{36}\) Suppose, instead, the incumbent bank deviates by offering a deposit rate \( r^I(\mathcal{D}) \neq r^I_S(\mathcal{D}) \), then its expected profit is either

\[ \hat{\Pi}(H, \mathcal{D}) = 0, \quad \text{if} \quad r^I(\mathcal{D}) < r^I_S(\mathcal{D}) = r_f, \]

or

\[ \hat{\Pi}(H, \mathcal{D}) = \left[ \pi + (1 - \pi)(1 - \rho) \right] \left( R^*_S(\mathcal{D}) - r^I(\mathcal{D}) \right) + \left(1 - \pi\right) \rho \left( P^S_B(\mathcal{D}) - r^I(\mathcal{D}) \right) \\
< \Pi^I_S(H, \mathcal{D}), \quad \text{if} \quad r^I(\mathcal{D}) > r^I_S(\mathcal{D}) = r_f. \]

Similarly, when the incumbent bank wins an \( L \)-type loan, it earns an expected profit

\[
\Pi^I_S(L, \mathcal{D}) = \pi \left( R^*_S(\mathcal{D}) - r_f \right) + \left(1 - \pi\right) \left( P^S_B(\mathcal{D}) - r_f \right) > 0
\]  

(19)

\(^{35}\)In fact, the assumption can be further relaxed. It is sufficient if the depositors have an off-equilibrium path belief \( Pr(H| r^I(\mathcal{D}) \neq r^I_S(\mathcal{D}), \mathcal{D}) < Pr(H| \mathcal{D}) \) and \( Pr(L| r^I(\mathcal{D}) \neq r^I_S(\mathcal{D}), \mathcal{D}) > Pr(L| \mathcal{D}) \), which states that, depositors hold a worse belief about the incumbent bank’s loan quality when receiving an off-equilibrium deposit rate \( r^I(\mathcal{D}) \neq r^I_S(\mathcal{D}) \).

\(^{36}\)Note that our presumption of Lemma 2 is \( P^B_S(\mathcal{D}) > r_f \). The inequality is guaranteed by this condition.
by offering the equilibrium rate $r^l_S(D)$. While its expected profit from deviation is either
\[ \hat{\Pi}(L, D) = 0 < \Pi^l_S(L, D), \quad \text{if} \quad r^l(D) < r_f, \]
or
\[ \hat{\Pi}(L, D) = \pi \left( R^*_{S} (D) - r^l(D) \right) + (1 - \pi) \left( P^S_B(D) - r^l(D) \right) < \Pi^l_S(L, D), \quad \text{if} \quad r^l(D) > r_f. \]
Therefore, the incumbent bank has no profitable deviation when holding an $L$-type loan either. In sum, the worse-than-posterior off-equilibrium is a sufficient condition for $r^l_S(D)$ to be part of the PBE.

**Step 3.** We now analyze the primary loan market competition between the incumbent bank and entrant bank.

Given the entrant bank’s posterior belief $\mu(D)$, the minimum loan rate that satisfies the participation constraint of the entrant bank is
\[ R^E_S(D) = \frac{\alpha + (1 - \alpha)\delta}{\alpha + (1 - \alpha)\delta \pi} (c + r_f). \]
(20)

Otherwise, the entrant bank will be better off holding risk-free assets.
\[ [Pr(H|D) + Pr(L|D)\pi] R^E(D) - c = \frac{\alpha + (1 - \alpha)\delta \pi}{\alpha + (1 - \alpha)\delta} R^E - c < r_f, \quad \forall R^E(D) < R^E_S(D) \]

We now show that given the subsequent equilibrium strategies characterized in Step 1 and 2, the incumbent bank’s equilibrium strategy in the primary loan market is to offer a pooling rate
\[ R^*(D) = \min\{R, R^E_S(D)\} \]
(21)
regardless of the type of the borrower.

First, consider the interior solution $R^*_S(D) = R^E_S(D) < R$. Our presumption $P^B_S(D) > r_f$ ensures a positive expected profit for the incumbent bank, regardless the type of the borrower it finances.\(^{37}\)

Suppose the incumbent bank deviates by charging a loan rate $R^l(H, D) > R^E_S(D)$. The bank will lose the loan competition for the $H$-type borrower and realizes a zero-profit
\[ \hat{\Pi}^l(H, D) = 0. \]

\(^{37}\)Recall that we have established $\Pi^l_S(H, D) > 0$ and $\Pi^l_S(L, D) > 0$ in equation (18) and (19).
If the bank deviates by charging a loan rate $R_I^t(H, \bar{D}) < R^E_S(\bar{D})$, it will still win the $H$-type borrower but will reduce its expected profit.\footnote{Note that in the expression of $\hat{\Pi}(H, \bar{D})$, the only deviation from $\Pi^I(H, \bar{D})$ is loan rate $R_I^t(H, \bar{D})$. The asset price $P^B_S(\bar{D})$ is not affected as we assume that the asset buyers can not observe the loan rate charged by the incumbent bank. Thus, the buyers still believe the loan rate is the equilibrium rate $R^E_S(\bar{D})$.}

$$\hat{\Pi}(H, \bar{D}) = [\pi + (1 - \pi)(1 - \rho)](R^I(H, \bar{D}) - r_f) + (1 - \pi)\rho(P^B_S(\bar{D}) - r_f) < \Pi^I_S(H, \bar{D})$$

For an $L$-type borrower, the incumbent bank’s expected profit by deviating is either

$$\hat{\Pi}^I(L, \bar{D}) = 0 < \Pi^I_S(L, \bar{D}) \quad \text{if} \quad R_I^t(L, \bar{D}) > R^E_S(\bar{D}),$$

or

$$\hat{\Pi}^I(L, \bar{D}) = \pi(R^I(L, \bar{D}) - r_f) + (1 - \pi)(P^B_S(\bar{D}) - r_f) < \Pi^I(L, \bar{D}) \quad \text{if} \quad R_I^t(L, \bar{D}) < R^E_S(\bar{D}).$$

Therefore, the incumbent has no profitable deviation for either an $H$- or $L$-type borrower.

For the corner solution $R^*_S(\bar{D}) = R > R^E_S(\bar{D})$, the incumbent bank charges the entire cash flow the loan $R$. Under condition $P^B_S(\bar{D}) = \frac{\alpha \rho}{(1 - \alpha)\beta + \alpha \rho} R > r_f$, the incumbent bank’s expected profit for either an $H$- or an $L$-type loan is positive. One can verify following the same argument that the incumbent bank has no profitable deviation in the corner solution case.

Lastly, one can follow exactly the same argument in Lemma 1 to show that the entrant bank has no profitable deviation by offering any loan rate other than $R^E_S(\bar{D})$ given its posterior $\mu(\bar{D})$.

To summarize, we establish the strategy profile and belief system described in Lemma 1 is indeed a pure-strategy PBE.

**Part II: The uniqueness of the pure-strategy pooling PBE.**

We again consider all possible alternative pure-strategy profiles from the loan market competition and prove by contradiction that none can be part of a PBE. Similar to the discussion in Lemma 1, we still have the following four alternative scenarios *Scenario 1*: $R^E(\bar{D}) < \min\{R^I(H, \bar{D}), R^I(L, \bar{D})\}$, so that the incumbent loses loan market competition for the borrower with $\bar{D}$ credit history irrespective of the borrower’s type. Suppose these loan rates had indeed been a part of a PBE. Then the incumbent bank offering a deposit rate to raise funding will be off equilibrium path.
We again consider the most pessimistic beliefs of depositors and asset buyers. That is, they again believe the incumbent bank must have financed an \( L \)-type loan if the bank had ever offered a deposit rate \( r^I \) or entered a loan sale. Given the belief, the asset buyers will offer a price equal to 0 in state \( B \), and the depositors will accept a rate \( r_f/\pi \) to break even. Consider then the loan market competition given the incumbent bank’s cost of funding.

Note that when the entrant bank wins the loan competition irrespective of the borrower’s type, its loan rate \( R^E(D) \) must satisfy
\[
R^E(D) \geq \frac{\alpha + (1 - \alpha)\delta}{\alpha + (1 - \alpha)\delta\pi}(c + r_f) = R^E_S(D).
\]

Note that now the entrant bank’s break-even rate \( R^E_S(D) \) is lower than \( R^E_N \), since a borrower with \( D \) credit history is less likely to be an \( H \)-type, i.e., posterior probability is lower than \( 1 - \alpha \).

The incumbent bank again has profitable deviation for an \( L \)-type borrower. By undercutting \( R^E(D) \), it earns an expected profit
\[
\pi \left( R^E(D) - \frac{r_f}{\pi} \right) \geq \pi \left( \frac{\alpha + (1 - \alpha)\delta}{\alpha + (1 - \alpha)\delta\pi}(c + r_f) - \frac{r_f}{\pi} \right) > 0.
\]

One can check that the last inequality is equivalent to \( c > \frac{\alpha(1 - \pi)}{\alpha\pi + (1 - \alpha)\delta\pi}r_f \), which is more strict than the one in Lemma 1. Yet, it is still implied by Inequality (1). It is again straightforward to see that profitable deviation also exists for an \( H \)-type and for any more optimistic off-equilibrium-path beliefs of depositors and asset buyers. Therefore, \( R^E(D) < \min\{R^I(H, D), R^I(L, D)\} \) cannot be part of a PBE.

**Scenario 2**: \( R^I(H, D) \leq R^E(D) < R^I(L, D) \), so that the incumbent bank only wins the \( H \)-type borrower in loan market competition. Suppose these loan rates had indeed been a part of a PBE and that the incumbent bank offers a rate \( r^I(H, D) \) to depositors. The depositors’ on-equilibrium-path belief will be
\[
Pr(H|r^I(H, D)) = 1.
\]

Furthermore, the depositors expect asset buyers to Bayesian update the incumbent bank’s loan quality according to the equilibrium strategy and to purchase the incumbent bank’s asset on sale at a price \( P^G = P^B = R^I(H, D) > r_f \). Therefore, it is sequentially rational for the depositors to provide financing for \( r^I(H, D) = r_f \).
We again analyze the primary loan market. In such a separating equilibrium where the entrant bank only finances the $L$-type, the entrant bank’s participation constraint requires

$$R^E(D) \geq \frac{c + r_f}{\pi}.$$  

Given its funding cost $r_f$, the incumbent bank can profitably deviates by increasing $R^I(H, D)$ to $R^E(D)$ for the $H$-type, i.e.,

$$\left( R^E(D) - r_f \right) - \left( R^I(H, D) - r_f \right) = R^E(D) - R^I(H, D) \geq 0$$

and by decreasing $R^I(L, D)$ to $R^E(D)$ for the $L$-type, i.e.,

$$\pi \left( R^E(D) - r_f \right) > \pi \left( \frac{c + r_f}{\pi} - r_f \right) > 0.$$  

Consequently, $R^I(H, D) \leq R^E(D) < R^I(L, D)$ cannot be part of a PBE either.

*Scenario 3*: $R^I(L, D) \leq R^E(D) < R^I(H, D)$, so that the incumbent bank only wins the $L$-type borrower in loan market competition. Suppose these loan rates had indeed been a part of a PBE. Similar to Scenario 2, the depositors would hold an on-equilibrium-path belief

$$Pr(L|r^I(L, D)) = 1,$$

and expect that asset buyers to offer $P^G = R^I(L, D) > r_f$ and $P^B = 0$ for the incumbent bank’s asset on sale in state $G$ and $B$ respectively. Therefore, it is sequentially rational for the depositors to provide financing for $r^I(L, D) = r_f/\pi$.

In such a separating equilibrium where the entrant bank now wins the $H$-type borrower, its participation constraint requires

$$R^E(D) \geq c + r_f$$

Given its funding cost $r_f/\pi$, the incumbent bank can profitably deviates by increasing $R^I(L, D)$ to $R^E(D)$ for the $L$-type, i.e.,

$$\pi \left( R^E(D) - \frac{r_f}{\pi} \right) > \pi \left( R^I(L, D) - \frac{r_f}{\pi} \right) = \pi \left( R^E(D) - R^I(L, D) \right) \geq 0,$$

and by decreasing $R^I(H, D)$ to $R^E(D)$ for the $H$-type

$$R^E(D) - \frac{r_f}{\pi} \geq c + r_f - \frac{r_f}{\pi} > 0.$$  

Consequently, $R^I(L, D) \leq R^E(D) < R^I(H, D)$ cannot be part of a PBE.
Scenario 4: max{R^l(H, D), R^l(L, D)} ≤ R^E(D), so that the incumbent bank still wins the loan market competition for the borrower with credit history D regardless of its true type. But instead of a pooling equilibrium as described in Lemma 2, the incumbent bank sustain a separating PBE where either R^l(H, D) ≠ R^l(L, D), or r^l(H, D) ≠ r^l(L, D), or both.

Suppose the incumbent bank offers separating deposit rates r^l(H, D) ≠ r^l(L, D) in equilibrium. The depositors’ on-equilibrium-path belief will be

\[ Pr(H|r^l(H, D)) = 1, \quad \text{and} \quad Pr(L|r^l(L, D)) = 1. \]

In this case, they will provide funding either if r^l(L, D) ≥ r_f and r^l(H, D) ≥ r_f when they anticipate the competitive equilibrium asset price in state B \( P^B \geq r_f \), or if r^l(L, D) ≥ r_f/\pi and r^l(H, D) ≥ r_f/\pi when they anticipate the competitive equilibrium asset price in state B \( P^B < r_f \). As a result, to secure a lower cost of funding, the incumbent bank always prefer to claim having lend to the type of borrower with lower deposit rate (not necessarily H-type in this case). Therefore, offering separating deposit rates must not be a part of any PBE.

We now move to primary loan market competition stage. Suppose separating loan rates R^l(H, D) ≠ R^l(L, D) had indeed been a part of a PBE. Then there must be profitable deviation for the incumbent bank to increase the lower loan rate at least equal to the higher one when it is profitable to finance both type borrower at the original separating rates.

To summarize, we establish the strategy profile and beliefs in Lemma 2 as a unique pure-strategy PBE for subgame \( g_S(D) \).

**Proof of Lemma 3.** When the incumbent shares the borrower’s credit history and lends to a borrower with previous default, the game features complete information, because an H-type borrower is assumed to be risk-free and the outsiders Bayesian update their beliefs as \( Pr(L|D) = 1 \). Therefore, we apply the solution concept of SPE and define a pure-strategy SPE of the game \( g_S(D) \) as follows.\(^{39}\)

**Definition 3:** In a pure-strategy SPE of the game \( g_S(D) \), the incumbent bank at \( t = 2 \) sets a loan rate \( R^l_S(D) \) and offers a take-it-or-leave-it deposit rate \( r^l_S(D) \). When having financed the borrower, the incumbent bank decides at \( t = 3 \) whether to sell the loan, according to the aggregate state \( s \) and its own liquidity position. Knowing that the borrower is an

\(^{39}\)For notation, we express the conditionality in terms of credit history \( D \) instead of the true type \( L \).
The entrant bank sets a loan rate $R^E_S(D)$. Also knowing the loan is of an $L$-type, depositors decide to provide funding or not for the deposit rate offered. When the $L$-type loan is on sale, the asset buyers bid $P^G_S(D)$ in state $G$ and $P^B_S(D)$ in state $B$.

We solve the game backwards in three steps.

*Step 1.* We first derive the secondary-market price for a $D$-history loan in State $G$ and $B$ respectively. In State $B$, asset buyers understand that the loan will produce a 0 return with certainty, which leads to a unique price

$$P^B_S(D) = 0.$$ 

In State $G$, asset buyers’ competitive bidding leads to a unique price

$$P^G_S(D) = R^*_S(D),$$

because an $L$-type loan does not default in State $G$.

*Step 2.* We move to the stage when the incumbent bank raises its funding. Different from the previous cases in Lemma 1 and 2, now the depositors are perfectly informed about the loan quality. Their unique break-even rate is

$$r^I_S(D) = \frac{r_f}{\pi},$$

since the bank defaults with certainty in state $B$. So the optimal strategy for the incumbent bank is to offer $r^I_S(D)$ to make the depositors just break even.

*Step 3.* We now analyze the primary loan market competition between the incumbent and entrant banks.

Given the entrant bank’s posterior belief $Pr(L|D) = 1$, the minimum loan rate that satisfies the bank’s participation constraint of the entrant bank is

$$R^E_S(D) = \frac{c + r_f}{\pi}.$$ 

Otherwise, the entrant bank will be better off holding risk-free assets.

By the standard argument of the price competition, the unique equilibrium in the primary loan market involves the incumbent bank offering

$$R^*_S(D) = \min \left\{ R, R^E_S(D) \right\}$$

and the entrant bank offering rate $R^E_S(D)$. Given the incumbent bank’s funding cost $r_f/\pi$, it makes positive profit

$$\Pi(D) = \min \left\{ R, \frac{c + r_f}{\pi} \right\} - \frac{r_f}{\pi} > 0.$$
as guaranteed by inequality (3).

To summarize, we establish the strategy profile in Lemma 3 as a unique pure-strategy SPE for subgame \( g_S(D) \). □

**Proof of Lemma 4.** The equilibrium prices of the loan on sale in the secondary market are

\[
P_N^B = \frac{\alpha \rho}{(1 - \alpha) + \alpha \rho} R_N^* \]

and

\[
P_S^B(\overline{D}) = \frac{\alpha \rho}{(1 - \alpha) \delta + \alpha \rho} R_S^*(\overline{D}),
\]

where \( R_N^* \) and \( R_S^*(\overline{D}) \) are the equilibrium loan rates under the no information sharing and information sharing regime, respectively. The equilibrium loan rates depend on the loan’s final payoff \( R \) that determines which Case \( j \), with \( j = \{0, 1, 2, 3\} \), we have to analyze.

Consider Case 0. The return \( R \) is so low that the entrant bank does not compete for any loan even if the incumbent bank shared a no default \( \overline{D} \) credit history of the borrower. The incumbent bank extracts the entire payoff of the loan irrespective of the information sharing regime, that is \( R_S^*(\overline{D}) = R_N^* = R \). Information sharing solely brings in the benefit from boosting liquidity of the loan on sale with a \( \overline{D} \)-history. Consequently, \( P_S^B(\overline{D}) > P_N^B \).

Consider Case 2 (for the easy of exposition it is convenient to analyze this case first). The value of \( R \) is sufficiently high that the entrant bank competes both under information sharing (and the borrower has no default history \( \overline{D} \)) and under no information sharing. The equilibrium loan rates are therefore

\[
R_N^* = R_E^N = \frac{c + r_f}{\alpha + (1 - \alpha) \pi} < \frac{\alpha + (1 - \alpha) \delta}{\alpha + (1 - \alpha) \pi \delta} (c + r_f) = R_E^S(\overline{D}) = R_S^*(\overline{D}).
\]

We want to show that

\[
P_N^B = \frac{\alpha \rho}{(1 - \alpha) + \alpha \rho} \frac{c + r_f}{\alpha + (1 - \alpha) \pi} < \frac{\alpha \rho}{(1 - \alpha) \delta + \alpha \rho \alpha + (1 - \alpha) \pi \delta} (c + r_f) = P_S^B(\overline{D}),
\]

which can be rewritten as

\[
\frac{\alpha \rho + (1 - \alpha) \delta}{\alpha \rho + (1 - \alpha) \pi \delta} \frac{\alpha + (1 - \alpha) \pi \delta}{\alpha + (1 - \alpha) \delta \pi} < 1.
\]

To show that the inequality in (22) holds, notice that the ratio \( \frac{(1 - \alpha) \delta + \alpha \rho}{(1 - \alpha) + \alpha \rho} \) is increasing in \( \rho \), so its maximum value is reached when \( \rho = 1 \) and it equal to \( \alpha + (1 - \alpha) \delta \). Therefore, a bigger value of the LHS in (22) can be written as

\[
\frac{\alpha + (1 - \alpha) \pi \delta}{\alpha + (1 - \alpha) \delta \pi} = \frac{\alpha + (1 - \alpha) \pi \delta}{\alpha + (1 - \alpha) \pi},
\]
which is smaller than 1 since \( \delta \in (0, 1) \). Thus, \( P^B_S(D) > P^B_N \).

Consider Case 1. The entrant bank only competes for the loan when no default history \( D \) is announced under the information sharing regime. The equilibrium loan rate \( R^*_S(D) \) is determined by the entrant bank in this case. With no information sharing, the incumbent bank can charge the borrower \( R^*_N = R > R^S_S(D) \) since the distant does not bid for the borrower. The negative effect from competition is clearly smaller than in Case 2, therefore the inequality \( P^B_S(D) > P^B_N \) necessarily holds also in Case 1.

Consider Case 3. The loan payoff \( R \) is so high, that the entrant bank competes even when the loan is granted to a borrower with a default credit history \( D \). The relevant equilibrium loan rates \( R^*_N \) and \( R^S_S(D) \) do not change with respect Case 2, then also in Case 3 we have \( P^B_S(D) > P^B_N \).

Since we have that always we have \( P^B_S(D) > P^B_N \), by continuity when the risk-free \( r_f \) is located between the two prices the incumbent bank survives the run under the information sharing regime (since the price at which the incumbent bank sells the loan is higher than the risk-free deposit rate) and it fails under no information sharing regime (since the price of the loan on sale is lower even of the risk-free deposit rate).

We now determine the cut off value that the condition \( P^B_S(D) > r_f > P^B_N \) implies in each of the four cases. In Case 0, we have \( R^*_N = R^*_S(D) = R \) then the condition can be written as

\[
\frac{\alpha \rho}{(1 - \alpha) \delta + \alpha \rho} R > r_f > \frac{\alpha \rho}{(1 - \alpha) + \alpha \rho} R
\]

which implies

\[
R > \frac{\alpha \rho + (1 - \alpha) \delta}{\alpha \rho} r_f \equiv \bar{R}
\]

and

\[
R < \frac{\alpha \rho + (1 - \alpha) \delta}{\alpha \rho} r_f \equiv \underline{R}
\]  \( \text{(23)} \)

In Case 1, we have \( R^*_S(D) = R^E_S(D) \) and \( R^*_N = R \) then the condition becomes

\[
\frac{\alpha \rho}{(1 - \alpha) \delta + \alpha \rho \alpha + (1 - \alpha) \delta \pi} (c + r_f) > r_f > \frac{\alpha \rho}{(1 - \alpha) + \alpha \rho} R
\]

which implies inequality (23) and

\[
C \equiv (c + r_f) > \frac{\alpha \rho + (1 - \alpha) \delta \alpha + (1 - \alpha) \delta \pi}{\alpha \rho \alpha + (1 - \alpha) \delta \pi} r_f \equiv C
\]  \( \text{(24)} \)

In Case 2, we have \( R^*_S(D) = R^E_S(D) \) and \( R^*_N = R^E_N \) then the condition becomes

\[
\frac{\alpha \rho}{(1 - \alpha) \delta + \alpha \rho \alpha + (1 - \alpha) \delta \pi} (c + r_f) > r_f > \frac{\alpha \rho}{(1 - \alpha) + \alpha \rho \alpha + (1 - \alpha) \pi} c + r_f
\]
which implies inequality (24) and

\[ C \equiv (c + r_f) < [\alpha + (1 - \alpha) \pi] \frac{\alpha \rho + (1 - \alpha)}{\alpha \rho} r_f \equiv C \quad (25) \]

In Case 3, the relevant equilibrium loan rates \( R_S^*(\overline{D}) \) and \( R_N^* \) are the same as in Case 2. Therefore we obtain the same cutoff values as in (24) and (25).

**Proof of Proposition 2.** Let us determine the expected profits \( V_i \) of the incumbent bank in each information sharing regime \( i = \{N, S\} \). Under no information sharing regime we have

\[ V_N = [\Pr(G) + \Pr(B) \Pr(H) \Pr(\text{no run})](R_N^* - r_N^f) = [\pi + (1 - \pi)\alpha(1 - \rho)]R_N^* - r_f. \]

In the good state \( G \), the incumbent bank will survive irrespective of the type of its borrower. However, in the bad state \( B \) the incumbent bank holding a loan granted to a \( H \)-type borrower will survive only if there is no bank run. With no information sharing scheme, the equilibrium deposit rate \( r_I^f \) is risky and given in Lemma 1. For an easier comparison, we can write the expected profit \( V_N \) as follows

\[ V_N = [\alpha + (1 - \alpha)\delta \pi]R_N^* + (1 - \alpha)(1 - \delta)\pi R_N^* - \alpha(1 - \pi)\rho R_N^* - r_f. \]

When the incumbent bank participates in the information sharing regime, its expected profits are

\[ V_S = \Pr(\overline{D})[\Pr(H|\overline{D})V^H_S(\overline{D}) + \Pr(L|\overline{D})V^L_S(\overline{D})] + \Pr(D)V^L_S(D), \quad (26) \]

where \( V^H_S(\overline{D}) \) and \( V^L_S(\overline{D}) \) are the expected profits of financing an \( H \)-type and an \( L \)-type borrower, respectively, when they generate a no default credit history \( \overline{D} \). While \( V^L_S(D) \) is the expected profit of financing an \( L \)-type borrower with a default credit history \( D \). Notice that when a loan has a credit history \( \overline{D} \), with posterior probability \( \Pr(H|\overline{D}) \) it is an \( H \)-type loan. Moreover, \( \Pr(D) = \Pr(L) \Pr(B) = (1 - \alpha)(1 - \pi) \) and \( \Pr(\overline{D}) = 1 - \Pr(D) = \alpha + (1 - \alpha)\pi. \)

The expected profit of financing an \( H \)-type borrower with credit history \( \overline{D} \) is

\[ V^H_S(\overline{D}) = [\Pr(G) + \Pr(B) \Pr(\text{no run})]R^*_S(\overline{D}) + \Pr(B) \Pr(\text{run}) P^B_S(\overline{D}) - r_f. \]

Notice that, given that we focus on the case in which information sharing saves the incumbent bank from illiquidity, then \( r^*_S(\overline{D}) = r_f. \) Moreover, the incumbent bank holds
an $H$-type loan to maturity if no bank run occurs because $P^R_S(\mathcal{D}) < R^*_S(\mathcal{D})$ (see Lemma 2). Similarly, the expected profit of financing an $L$-type borrower with credit history $\mathcal{D}$ is given by

$$V^L_S(\mathcal{D}) = \Pr(G)R^*_S(\mathcal{D}) + \Pr(B)P^B_S(\mathcal{D}) - r_f.$$ 

When the incumbent bank holds an $L$-type loan, it will sell it on the secondary market in state $B$, even without facing a run. Finally, a borrower that generates a default credit history $D$ must be an $L$-type borrower. The equilibrium deposit rate is risky, that is $r^*_S(D) = r_f/\pi$. The expected profit of financing such a loan is

$$V^L_S(D) = \Pr(G)[R^*_S(D) - r^*_L(D)] = \Pr(G)R^*_S(D) - r_f.$$

Inserting the expressions of $V^H_S(\mathcal{D})$, $V^L_S(\mathcal{D})$ and $V^L_S(D)$ into equation (26) and, after rearranging, we get

$$V_S = \alpha + (1 - \alpha)\delta\pi R^*_S(\mathcal{D}) + (1 - \alpha)(1 - \delta)\pi R^*_S(D) - r_f.$$

The difference between the expected profits in the two regimes can be rewritten as

$$V_S - V_N = \alpha + (1 - \alpha)\delta\pi(R^*_S(\mathcal{D}) - R^*_N) + (1 - \alpha)(1 - \delta)\pi(R^*_S(D) - R^*_N) + \alpha(1 - \pi)\rho R^*_N.$$

We now evaluate the difference $V_S - V_N$ in each region $\Psi_j$ with $j = \{0, 1, 2, 3\}$. We indicate with $\varphi_j$ the set of parameters that satisfy the condition $V_S - V_N > 0$ in Case $j$.

Consider Case 0. We have $R^*_S(\mathcal{D}) = R^*_N = R^*_S(D) = R$ therefore

$$V_S - V_N = \alpha(1 - \pi)\rho R > 0.$$ 

The condition $V_S > V_N$ is always satisfied in the region $\Psi_0$, then $\varphi_0 = \Psi_0$.

Consider Case 1. We have $R^*_S(\mathcal{D}) = R^*_E(\mathcal{D}) = \frac{\alpha + (1 - \alpha)\delta}{\alpha + (1 - \alpha)\delta\pi} (c + r_f)$ and $R^*_N = R^*_S(D) = R$, therefore

$$V_S - V_N = \alpha + (1 - \alpha)\delta(c + r_f) - [(1 - \alpha)\delta\pi + \alpha - \alpha(1 - \pi)\rho]R.$$

Notice that $(1 - \alpha)\delta\pi + \alpha - \alpha(1 - \pi)\rho > 0$, then $V_S - V_N > 0$ if and only if

$$R < \frac{\alpha + (1 - \alpha)\delta}{\alpha - \alpha(1 - \pi)\rho + (1 - \alpha)\delta\pi} (c + r_f) \equiv R_1.$$ 

In Case 1 then the region $\varphi_1$ may not coincide with $\Psi_1$, and we have

$$\varphi_1 = \Psi_1 \cap \{R | R < R_1\} \subseteq \Psi_1.$$
Recall that the upper bound for $R$ that defines Case 1 is given by $R^E_N$. If $R_1 > R^E_N$, the condition $V_S > V_N$ is satisfied in the entire region $\Psi_1$. We have

$$R_1 = \frac{\alpha + (1 - \alpha)\delta}{\alpha - \alpha(1 - \pi)\rho + (1 - \alpha)\delta \pi} (c + r_f) > \frac{1}{\alpha + (1 - \alpha) \pi} (c + r_f) = R^E_N$$

which implies

$$\rho > (1 - \alpha)(1 - \delta).$$

Therefore, whenever the last inequality holds the region $\varphi_1$ coincides with $\Psi_1$. Otherwise, we have $\varphi_1 \subset \Psi_1$. Notice that, given the definition of $\Psi_1$, there is always a non-empty set $\varphi_1 = \Psi_1 \cap \{R| R < R_1\}$ for any value of $\rho \in (0, 1)$.

Consider Case 2. We have $R^*_S(D) = R^E_S(D)$, $R^*_N = R^E_N$ and $R^*_S(D) = R$, therefore

$$V_S - V_N = \left[\alpha + (1 - \alpha)\delta - 1 + \frac{\alpha(1-\pi)\rho}{\alpha + (1 - \alpha) \pi}\right] (c + r_f) + (1 - \alpha)(1 - \delta)\pi R.$$

The condition $V_S - V_N > 0$ holds if and only if

$$R > \left[1 - \frac{1 - \pi}{(1 - \delta)} \frac{\alpha \rho}{(1 - \alpha)[\alpha + (1 - \alpha) \pi]}\right] \frac{(c + r_f)}{\pi} \equiv R_2.$$

Also in Case 1 then the region $\varphi_2$ may not coincide with $\Psi_2$, and we have

$$\varphi_2 = \Psi_2 \cap \{R| R < R_2\} \subseteq \Psi_2.$$

The lower bound for $R$ that defines Case 2 is given again by $R^E_N$. If $R_2 < R^E_N$, the condition $V_S > V_N$ is satisfied in the entire region $\Psi_2$. That is, if

$$R_2 = \left[1 - \frac{1 - \pi}{(1 - \delta)} \frac{\alpha \rho}{(1 - \alpha)[\alpha + (1 - \alpha) \pi]}\right] \frac{(c + r_f)}{\pi} < \frac{c + r_f}{\alpha + (1 - \alpha) \pi} = R^E_N$$

which again implies

$$\rho > (1 - \alpha)(1 - \delta),$$

we have $\varphi_2 = \Psi_2$. Otherwise, we have $\varphi_2 \subset \Psi_2$. Given the definition of $\Psi_2$, there is always a non-empty $\varphi_2$ for any $\rho \in (0, 1)$.

Consider Case 3. We have $R^*_S(D) = R^E_S(D)$, $R^*_N = R^E_N$ and $R^*_S(D) = R^E_S(D)$, and it is easy to verify that $V_S = c$ and $V_N = c - \alpha(1 - \pi)\rho \frac{c + r_f}{\alpha + (1 - \alpha) \pi}$. Therefore difference in expected profits is strictly positive

$$V_S - V_N = \alpha(1 - \pi)\rho \frac{c + r_f}{\alpha + (1 - \alpha) \pi} > 0.$$
Like in Case 1, also in Case 3 the condition \( V_S > V_N \) is satisfied in the entire region \( \Psi_3 \). Therefore \( \varphi_3 = \Psi_3 \) and information sharing regime is always chosen by the incumbent bank. ■

**Proof of Proposition 3.** Suppose the entrant bank, depositors and asset buyers hold the belief that the incumbent bank tell the truth about the credit history of the borrower. We check if there are profitable deviation for the incumbent bank to announce a false credit history without default \( \overline{D} \) instead of the true defaulted history \( D \) under such belief. We focus our analysis on the set of parameters that define the regions \( \varphi_j \) with \( j = \{0, 1, 2, 3\} \) in which the incumbent bank finds convenient to choose ex-ante the information sharing regime.

Consider Case 0. We first compute the incumbent bank’s expected profit \( V(D) \) at \( t = 1 \) of truthfully reporting a loan with credit history \( D \). Recall that \( R^*_S(D) = R \) in this Case, so we have

\[
V(D) = \pi R^*_S(D) - r_f = \pi R - r_f. \tag{27}
\]

The expected profit of reporting the false \( \overline{D} \)-history is (recall that the incumbent bank does not fail in state \( B \) by misreporting the credit history since it can sell the loan at price \( P^B_S(\overline{D}) > r_f \))

\[
V(D, \overline{D}) = \Pr(G)R^*_S(\overline{D}) + \Pr(B)P^B_S(\overline{D}) - r_f, \tag{28}
\]

and given the expression for \( P^B_S(\overline{D}) \) we have

\[
V(D, \overline{D}) = \pi R + \frac{(1 - \pi)\alpha \rho}{\alpha \rho + (1 - \alpha)\pi} R - r_f.
\]

Clearly we have \( V(D) - V(D, \overline{D}) < 0 \) and the incumbent bank finds it profitable to misreport the borrower’s credit history. In this Case, the belief of outsiders can not be rationalized and truthful information sharing can not be sustained as a PBE in the region \( \varphi_0 \).

Consider Case 1. Like in Case 0, the relevant equilibrium loan rate is \( R^*_S(D) = R \). Then reporting the true default history gives the same expected profit as in (27). The expected benefit of reporting the false \( \overline{D} \)-history is also given by (28). Since in Case 1 we have \( R^*_S(\overline{D}) = \frac{\alpha + (1 - \alpha)\delta}{\alpha + (1 - \alpha)\delta \pi} (c + r_f) \), the expected gain from deviation can be expressed as

\[
V(D, \overline{D}) = \frac{\alpha \rho + (1 - \alpha)\delta \pi}{\alpha \rho + (1 - \alpha)\pi} \frac{\alpha + (1 - \alpha)\delta}{\alpha + (1 - \alpha)\delta \pi} (c + r_f) - r_f. \tag{29}
\]
Then the ex-post incentive compatibility constraint to tell the truth is
\[ V(D) - V(D, D) = \pi R - \frac{\alpha \rho + (1 - \alpha) \delta \pi}{\alpha \rho + (1 - \alpha) \pi} \alpha + (1 - \alpha) \delta (c + r_f) > 0, \]
which can be simplified as
\[ R > \left[ \frac{\alpha \rho + (1 - \alpha) \delta \pi}{\alpha \rho + (1 - \alpha) \delta} \alpha + (1 - \alpha) \delta \pi \right] \frac{c + r_f}{\pi} \equiv R_3. \tag{30} \]
Ex-ante information sharing is chosen in Case 1 when (recall the proof of Proposition 2)
\[ R < \frac{\alpha + (1 - \alpha) \delta}{\alpha - \alpha(1 - \alpha) \rho + (1 - \alpha) \delta \pi} (c + r_f) \equiv R_1. \]
It can be calculated that
\[ \frac{1}{R_1} - \frac{1}{R_3} = \frac{1}{\alpha + (1 - \alpha) \delta} \frac{\alpha^2(1 - \rho)(1 - \pi)}{\alpha \rho + (1 - \alpha) \delta \pi} \frac{1}{c + r_f} > 0. \]
Consequently, \( R_1 < R_3 \) and there exists no \( R \) such that the incumbent bank will ex-ante participate in information sharing scheme and ex-post report the true default credit history. The belief of outsiders can not be rationalized and truthful information sharing can not be sustained as a PBE in region \( \varphi_1 \).

Consider Case 2. Again we have \( R^*_S (D) = R \) therefore reporting the true default history gives the same expected profit as in (27). The expected profit of misreporting the true credit history is the same as in expression (29), since \( R^*_S (D) \) has the same expression like in Case 1. Therefore the condition on \( R \) to ensure ex-post the incumbent bank tells the truth is the same as in (30). Information sharing is ex-ante chosen in Case 2 when (recall proof of Proposition 2)
\[ R > \left[ 1 - \frac{1 - \pi}{1 - \delta (1 - \alpha) [\alpha + (1 - \alpha) \pi]} \right] \frac{\alpha \rho}{\alpha + (1 - \alpha) \delta \pi} \alpha + (1 - \alpha) \delta \pi (c + r_f) \equiv R_2. \tag{31} \]
We find a region of parameters in which both conditions (30) and (31) are satisfied. That is, whenever it is ex-ante optimal for the incumbent bank to share information it is also ex-post convenient to disclose the true credit history. This implies to impose the following restriction
\[ 1 - \frac{1 - \pi}{1 - \delta (1 - \alpha) [\alpha + (1 - \alpha) \pi]} \frac{\alpha \rho}{\alpha + (1 - \alpha) \delta \pi} > \frac{\alpha \rho + (1 - \alpha) \delta \pi}{\alpha \rho + (1 - \alpha) \delta} \alpha + (1 - \alpha) \delta. \tag{32} \]
Let us define the function
\[ F(\rho) = 1 - \frac{1 - \pi}{1 - \delta (1 - \alpha) [\alpha + (1 - \alpha) \pi]} \frac{\alpha \rho}{\alpha + (1 - \alpha) \delta \pi} - \frac{\alpha \rho + (1 - \alpha) \delta \pi}{\alpha \rho + (1 - \alpha) \delta} \alpha + (1 - \alpha) \delta. \]
It can be checked that
\[
F'(\rho) = -\frac{1 - \pi}{1 - \delta (1 - \alpha)[\alpha + (1 - \alpha)\pi]} - \frac{\alpha(1 - \alpha)\delta(1 - \pi)}{[\alpha\rho + (1 - \alpha)\delta]^2} \alpha + (1 - \alpha)\delta \pi < 0.
\]
Moreover, we can take the limits
\[
\lim_{\rho \to 0} F'(\rho) = 1 - \frac{\alpha\pi + (1 - \alpha)\delta\pi}{\alpha + (1 - \alpha)\delta\pi} > 0
\]
\[
\lim_{\rho \to 1} F'(\rho) = -\frac{1 - \pi}{1 - \delta (1 - \alpha)[\alpha + (1 - \alpha)\pi]} < 0.
\]
Thus, there exists a unique \( \bar{\rho} \) such that \( F(\bar{\rho}) = 0 \). Whenever \( 0 < \rho < \bar{\rho} \), we have \( F(\rho) > 0 \) and expression (32) holds. Then truth telling can be sustained as a PBE in the region \( \varphi_2 \). Recall from Proposition 2 that \( \varphi_2 \) is non-empty for all \( \rho \in (0, 1) \).

Consider Case 3. In this Case we have \( R^*_S(D) = (c + r_f)/\pi \) since the entrant bank competes also for the loan with a default history \( D \). Reporting the true default history gives an expected profit equal to
\[
V(D) = \pi R^*_S(D) - r_f = c.
\]
The expected profit of misreporting the credit history is the same as in (29), and since
\[
\frac{\alpha\rho + (1 - \alpha)\delta\pi}{\alpha\rho + (1 - \alpha)\pi} \frac{\alpha + (1 - \alpha)\delta}{\alpha + (1 - \alpha)\delta\pi} < 1
\]
we have \( V(D, \overline{D}) < c \) and therefore \( V(D, \overline{D}) - V(D) < 0 \). The belief of outsiders can be rationalized, and truthful information sharing can be sustained as a PBE in the region \( \varphi_3 \).
1. The incumbent bank inherits a lending relationship from history.
2. The incumbent bank decides whether to share borrower’s credit history or not.

1. The borrower’s type and credit history realize.
2. The information is privately observed by the incumbent bank.
3. The incumbent bank announces the borrower’s type or credit history if it chooses to share such information in the previous stage.

1. The incumbent bank and entrant bank compete to finance the borrower by offering loan rates.
2. The winning bank extends the loan to the borrower and is financed by fairly priced deposits.

1. State s realizes and is publicly observed.
2. The incumbent bank’s liquidity risk realizes, and is privately observed by the bank.
3. The secondary loan market opens where the loan can be sold to competitive asset buyers.

The bank loan pays off.
Figure 2: Equilibrium loan rates: Interior and corner solutions

\[ R \equiv c + r_f \]
Figure 3: Regions where information sharing can save the incumbent bank from illiquidity

\[ R \equiv c + r_f \]
Figure 4: Regions where information sharing leads to a greater value for the incumbent bank
Figure 5: Regions where truthful information sharing can be sustained in a perfect Bayesian equilibrium

\[ C \equiv c + r_f \]
Figure 6: Regions where public registry can improve allocation