Seasoned Equity Offerings and Dilution

Mike Burkart†
Hongda Zhong‡

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Abstract

We analyze seasoned equity offerings where some shareholders are informed and can strategically choose to participate, creating informational frictions. When all existing shareholders have wealth to participate in the issue, right offerings achieve the full information outcome, and therefore dominate public offerings which necessarily generate wealth transfers. We show that this ranking may be reversed when some of the existing shareholders are wealth constrained. In rights offerings, outside investors must purchase the rights to buy the underlying shares, rather than only buying these shares as in a public offering. Hence, a positive right price implies a discount in the strike price relative to the public offering price. Therefore, wealth-constrained shareholders become more diluted in a rights offering, and lower strike price increases the wealth transfer from the cash-constrained to the informed cash-rich shareholders. More generally, cash-constrained and cash-rich shareholders have diverging preference over floatation methods and terms.
1 Introduction

Public companies undertake seasoned equity offerings (SEOs) to raise new equity capital from current as well as new shareholders. In SEOs, new investors as well as existing shareholders are concerned about possible mispricing. On one hand, existing shareholders fear that the underpricing of new shares dilutes their holdings (e.g., Myers and Majluf, 1984). On the other hand, prospective investors worry that they fall victim to the winner’s curse problem due to the presence of better informed investors and the practice of giving existing shareholders some priority in the subscription (e.g., Rock, 1986).

How new shares are issued clearly affects the payoffs of each investor. This paper compares two popular floatation methods – public offerings and rights offerings. For either offering method, we assume that informed capital is scarce. In particular, some current shareholders know the value of the firm, respectively, the value of its assets in place, whereas other shareholders as well as outside investors are uninformed. Furthermore, informed shareholders can at most purchase the shares allocated to them on a pro-rata basis, but not purchase any shares or rights from uniformed shareholders or outside investors.

Following Myers and Majluf (1984), we model public offerings as a direct sale in which firms announce the issue size and investors choose to subscribe.\(^1\) In addition, we allow for the possibility that existing shareholders have priority over a fraction of the newly issued shares. The equilibrium issue price is determined by the competitive outside investors, that is, their breakeven constraint. In rights offering, the firm announces the issue size and the strike price of the rights. Existing shareholders receive the rights for free and decide whether to exercise or sell them. Since neither selling nor exercising is (weakly) dominated in our framework, we exclude this option. There is a well-functioning market for rights in which competitive outside investors determine the market value of a right.\(^2\)

We first establish that rights offerings can implement the full information outcome even though some current shareholders are uninformed, provided all shareholders have enough cash to take up their new shares. Intuitively, as long as all shareholders exercise their rights, their fractional ownership in the firm does not change. Accordingly, any dilution of

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\(^1\)Unlike, e.g., Eckbo and Masulis (1992), we abstract from underwriters, their possible certification role, and their possible commitment to purchase all shares not taken by investors.

\(^2\)Since scarcity of informed capital is the only friction in our framework, there is no subscription risk, that is, there are sufficient outside investors to take up any rights current shareholders do not wish to exercise.
existing shares caused by a low strike price is exactly offset by the capital gains on the new
shares. This result extends the intuition from Myers and Majluf (1984) that retained earnings
(cash inside the company) avoids informational friction, and asking current shareholders to
subscribe the offer serves a similar role as financial slack. We show, more generally, that
rights offerings can still achieve the full information outcome even though there information
asymmetries among existing shareholders. In contrast, a public offering always generates
some ex-post wealth transfer between different investors because some new shares are issued
to the outside investors at a pooling price.

This suggests that rights offerings dominate public offerings in the sense that the former
can avoids any wealth transfers. However, empirical evidence shows that rights offerings are
not the predominant floatation method in the world and rarely exist in the U.S. (e.g., Ecbo,
Masulis, and Norli, 2007). We propose that firms may prefer public offerings because current
shareholders are wealth constrained. When some current shareholders cannot exercise and
therefore have to sell their rights, their holdings become more diluted than in a public offering.
Rights have a positive value only if the strike price is more underpriced than the equilibrium
issue price in a public offering. Such a lower strike price implies more new shares must be
issued in a rights offerings. This in turn implies more dilution to the existing holdings, which
is not fully compensated by the price of these rights, as the following example illustrates.

Consider a firm with one share whose the true value is $20, but trades in the market
at $10. Furthermore, the firm needs to raise extra $10 to finance its investment, and the
existing shareholders do not have any extra cash. In a public offering, outside investors are
willing to buy the share at $10. The payoff to the current shareholders is $15, total firm
value $20 + $10 divided by two outstanding shares. Now suppose that the strike price in a
rights offering is $5. Accordingly, the firm issues two rights to the current shareholder who
has no choice but to sell them. The market price of each right must be equal to the market-
perceived share value minus the strike price: Given there are three shares outstanding, the
market-perceived post-issue share value is \( \frac{S_{10}+S_{10}}{1+2} = 6.67 \). Subtracting the strike price of $5
yields a market price of $1.67 for each right. Thus, the payoff to the current shareholder is
\( \frac{S_{20}}{1+2} + 1.67 \times 2 = 13.33 \), the sum of the true value of his one share and the revenue from
selling the two rights. Clearly, public offering dominates because it creates less dilution.

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3 The prevalent use of public offerings which have higher direct floating costs than rights offerings has
been referred to as the rights offer paradox (Smith, 1977).
We can rank the floatation methods based on the ex-ante (before the investors learn the firm value) wealth transfer between the cash-rich and cash-poor existing shareholders. The cash-poor shareholders lose more to the cash-rich shareholders in rights offerings. This transfer becomes larger when the strike is smaller, because dilution effect is stronger. A public offerings with full dilution protection is equivalent to a rights offering with a zero rights price. Intuitively, shareholders in such a public offering can maintain their fractional ownership in the firm by subscribing, replicating the outcome of a rights offering. All other public offerings with lower dilution protection creates less wealth transfer between cash-poor and cash-rich shareholders. In the extreme case of no dilution protection, the current shareholders' wealth becomes irrelevant, as they do not receive any allocation even if they subscribe to the issue.

Finally, we offer an explanation why firms may choose different issue methods. If the firm cares more about their cash-poor shareholders, it will choose a public offering with no dilution protection, whereas if the firm cares more about its cash-rich shareholders, it opts for a rights offering with a discounted strike price.

**Literature Review**: to be completed.

## 2 Model Setup and Benchmark

### 2.1 Model

Consider an economy that is populated by publicly traded firms with assets in place $a$ and an unfunded project. The value of the assets in place varies across firms and is distributed on $[a, \bar{a}]$ according to the density function $f(a)$, respectively its distribution function $F(a)$. By contrast, all firms have identical projects, each requiring an investment outlay $I$ and generating a safe payoff $I + b$.\(^4\) The number of existing shares is normalized to 1. To undertake the project, firms have to raise new equity financing $I$. Current shareholders and competitive outside investors are all risk-neutral.

The crucial assumption is the scarcity of informed capital. In particular, among current shareholders only a fraction $(1 - \eta)$ knows the value of the assets in place $a$ while the remaining $\eta$ shareholders merely know its distribution. Like the latter, outside investors are

\(^{4}\)Alternatively, we could the project is risky and $b$ is the expected net present value.
uninformed. In addition and independent of information, a fraction $\pi$ of current shareholders have no spare wealth. The remaining $(1 - \pi)$ shareholders have financial slack to purchase additional shares. However, they cannot trade with either cash-poor shareholders or outside investors. That is, cash-rich shareholders can at most purchase those newly issued shares which are allocated to them on a pro-rata basis. Without this restriction, informed capital would not be scarce.

Throughout the paper, we consider stylized versions of the two most popular equity flotation methods: public offerings (PO) and rights offerings (RO). In public offerings, the firm issues new shares but the current shareholders may receive some dilution protection, that is current shareholders have priority over the same fraction $\lambda \in [0, 1]$ of the new shares on a pro rata basis in all firms. For example, if a current shareholder who owns $\beta$ shares subscribes to the offer, she can purchase $\beta \lambda$ of the new shares. Outside investors get to buy all $(1 - \lambda)$ non-dilution protected shares and those dilution protected shares that current shareholders choose not to take up. Current shareholders and outside investors simultaneously decide whether to subscribe. Finally, the outside investors’ break-even condition determines the price per share $P_{PO}$ and the number of newly issued shares such that $N_{PO} = \frac{I}{P_{PO}}$.

In rights offerings with a strike price $P_{S}$, a total of $N_{RO} = \frac{I}{P_{S}}$ rights are issued to current shareholders at no cost and on a pro rata basis. Each right gives its owner the option to purchase a newly issued share at the strike price $P_{S}$. Cash-rich shareholders can choose between exercising the rights or selling them to outside investors. Cash-poor shareholders have no choice but to sell their rights to outside investors. Doing nothing, that is, shareholders neither exercising nor selling the rights, is weakly dominated by selling the rights as long as $P_{R} \geq 0$. Therefore, we rule out doing nothing as an option and assume that shareholders either exercise or sell the rights. The break-even constraint of the competitive outside investors determines the rights price $P_{R}$ and we exclude negative prices.

Subsequently, we derive the outcome of either offering mode. Instead of letting firms strategically choose type and terms of the offering, we solve for the equilibrium outcome where the uniform offer is fully subscribed, raises the required investment amount $I$, and the competitive outside investors break even in expectation. Then, we compare the equilibrium

\footnote{Thus, we assume a perfectly functioning rights market in the sense that no shareholder keeps her rights but does not exercising them due to inattention, negligence, or some behavioural bias.}

\footnote{If the offer were to fail/be undersubscribed, it would be cancelled and any amount paid for new shares or rights would be returned to shareholders and outside investors.}
outcomes cross different offer types and terms, i.e. \( \lambda \) and \( P_S \). We choose this - possibly - less conventional approach primarily because current shareholders in our framework are heterogeneous and typically will disagree over the optimal flotation mode, extent of dilution protection, or strike price. For the same reason, it is not straightforward to determine whether a firm wants to issue and invest. For the time being, we solve for outcomes where all firm types \( a \in [a, \bar{a}] \) issue and invest. This amounts to assuming that the NPV of the investment \( b \) is sufficiently large relative to discounts that high quality firms may encounter. In addition, solving the current model is also a necessary step to determine how large such discounts are.

### 2.2 Benchmark

To begin we examine the impact that asymmetric information in isolation has on the outcome of the two offering methods. To this end, we consider the case where all current shareholders have cash to purchase new shares, i.e., we set \( \pi \) equal to zero. Still, only the fraction \( (1 - \eta) \) of current shareholders knows the firm type \( a \). Since they cannot buy up the entire issue, informed capital remains scarce.

As shown by Myers and Majluf (1984), selling shares to new investors, invariably leads to wealth transfers. This holds true also in our setting.

**Proposition 1** Any public offering with incomplete dilution protection \( \lambda < 1 \) leads to wealth transfers among informed shareholders and uninformed investors in (almost) all firms.

In public offerings without dilution protection, outside investors purchase all new shares in a successful offering, as in Myers and Majluf (1984). Since they are uninformed, the price \( P_{PO} \) must - in equilibrium - be the same for any and all firms, irrespective of the respective value of the assets in place. Moreover, outside investors only purchase shares if the price is such that they break even on average. Consequently, there is mispricing and redistribution across firm types: For firms whose assets in place have a low (high) value, the new shares are overpriced (underpriced) and outside investors make a loss (profit). Accordingly, current shareholders receive a payoff which is either larger or smaller than \( a + b \), their full information payoff.\(^7\)

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\(^7\)This does not hold for the one firm type whose assets-in place happen to have the value such that the price is fair, i.e., \( a = P_{PO} - b \).
The asymmetric information problem is exacerbated by the dilution protection, since it adds a winner’s curse problem. Informed shareholders take up their allocated quota \( \lambda(1 - \eta) \) only if the issue is underpriced. As a result, outside investors end up buying more shares when the firm is overpriced. Hence, dilution protection leads to additional redistribution among informed shareholders and outside investors. Since uninformed shareholders can always purchase \( \lambda\eta \) new shares but never more, they are not directly exposed to the winner’s curse problem. Given the equilibrium price is set such that outside investors break even, uninformed current shareholders benefit from taking up their allocated quota.

As noted in the literature, rights offering can avoid such wealth transfers if “stockholders can be compelled to exercise their rights and hold the newly issued shares” (Myers and Majluf, 1984 footnote 5). Indeed, rights offerings in our setting also avoid wealth transfers. In addition, they can resolve asymmetric information problems among current shareholders, ensuring that each and every shareholder receives the full information payoff \( a + b \).

**Proposition 2** Given all current shareholders are cash-rich, they all receive a net payoff of \( a + b \) in the unique equilibrium outcome of a rights offering. Moreover, this equilibrium exists only if \( P_S \leq a + b \), and is implemented by all current shareholders exercising their rights.

When a shareholder exercises the rights allocated to her on a pro-rata basis, her payoff does not depend on the strike price. Indeed, exercising the rights implies her proportional ownership stake in the firm does not change. Therefore, any mispricing of the issue (strike price \( P_S \)) is fully offset by a corresponding value change of her “old” shares. However, informed current shareholders of firms types with low asset values may find it more profitable to sell their rights in the market. A sufficiently low strike price makes this an inferior option, thereby ensuring that all current shareholders, informed as well as uninformed, find it in their interest to exercise the rights. As a result, they all receive a net payoff equal to \( a + b \), as they would under complete information.

As shown, rights offerings dominate public offerings since the former but not the latter overcomes informational frictions and avoids redistribution both among shareholders and between shareholders and outside investors. Hence, the use of public offerings cannot be attributed exclusively to asymmetric information problems. There must be at least one additional friction. Subsequently, we (re-)introduce wealth constraints among current shareholders and show how this may reverse the ranking of the two offer methods.
3 Issue Methods with Cash-Constraint Investors

In the remainder of the paper, we shut down the information asymmetry among existing shareholders for simplicity (i.e. \( \eta = 0 \)) and focus on how wealth constraint affects investors payoffs’ under different issuance mechanisms. For each issue mechanism, we characterize the equilibrium and the wealth transfer among different types of investors. We then perform comparative statics analysis with respect to extent of dilution protection (\( \lambda \)) among public offerings and strike price \( P_S \) among rights offerings respectively.

3.1 Public Offerings

We first study the equilibrium outcome in a given public offering with dilution protection \( \lambda \). After issuing the new shares, the firm value is \( I + a + b \), the total number of shares is \( N_{PO} + 1 \), and hence the share price is \( \frac{1}{N_{PO} + 1}(I + a + b) \). If a cash-rich current shareholder with \( \beta \) shares subscribes, he receives \( \lambda \beta N_{PO} \) new shares and invests additional \( \lambda \beta N_{PO} P_{PO} = \lambda \beta I \) in the new company. As a result, his payoff as a function of their true firm type \( a \) is

\[
\beta \left[ \frac{\lambda N_{PO} + 1}{N_{PO} + 1} (I + a + b) - \lambda I \right].
\]

If a current shareholder chooses not to subscribe or has no cash, his payoff is

\[
\beta \frac{1}{N_{PO} + 1}(I + a + b).
\]

Clearly, a cash-poor investor has no strategic action available. The cash-rich investors subscribe to the new shares if and only if the payoff from subscribing (1) is higher than doing nothing (2). The following lemma characterizes the cash-rich investors’ subscribing strategy.

**Lemma 1** In a public offering, cash-rich current shareholders subscribe to the new shares if and only if

\[
a \geq a^*_{PO} \equiv P_{PO} - b.
\]

The outside investors’ payoff is obviously zero if they do not subscribe. If outsiders
subscribe, their collective payoff is

\[ Pr_{IS} [1 - (1 - \pi)\lambda] \left[ \frac{N_{PO}}{N_{PO} + 1} (I + E_{IS} + b) - I \right] + Pr_{INS} \left[ \frac{N_{PO}}{N_{PO} + 1} (I + E_{INS} + b) - I \right], \]

where \( IS \) (resp. \( INS \)) is the event where cash-rich current shareholders (resp. do not) subscribe. Using Lemma 1, we have

\[ E_{IS} = E(a|a \geq P_{PO} - b), \]

\[ E_{INS} = E(a|a < P_{PO} - b), \]

\[ Pr_{IS} = Pr(a \geq P_{PO} - b), \]

\[ Pr_{INS} = Pr(a < P_{PO} - b). \]

Reorganizing the terms in (4) using the fact that \( N_{PO} = \frac{L}{P_{PO}} \) and factor out \( \frac{N_{PO}}{N_{PO} + 1} \), we have

\[ Pr_{IS} [1 - (1 - \pi)\lambda] [E_{IS} + b - P_{PO}] + Pr_{INS} [E_{INS} + b - P_{PO}], \]

which is

\[ Pr_{IS} [E_{IS} + b - P_{PO}] + Pr_{INS} [E_{INS} + b - P_{PO}] - Pr_{IS} (1 - \pi)\lambda [E_{IS} + b - P_{PO}], \]

which in turn is

\[ E(a) + b - P_{PO} - Pr_{IS} (1 - \pi)\lambda [E_{IS} + b - P_{PO}]. \]

(5)

Outsiders are competitive, so their zero profit condition determines the equilibrium issue price \( P_{PO} \). The next result characterize the full equilibrium in a public offering.

**Proposition 3** In a public offering, there exists a price \( P_{PO} \) such that outside investors break even. In addition, this price implies underpricing (i.e. \( P_{PO} < E(a) + b \)) when there is some dilution protection (\( \lambda > 0 \)). Finally, the issue price \( P_{PO} \) is decreasing in \( \lambda \).

The public offering price \( P_{PO} \) is the lowest when the offer is fully dilution protected, because the preferred allocation to the cash-rich informed existing shareholders causes severe winner curse problem. In contrast, the offering price is the highest when there is no dilution
protection ($\lambda = 0$). In this case, cash endowment of the existing shareholders becomes irrelevant because none of them receives any new shares. The winners curse problem disappears, and the outside investors are willing to purchase the new shares at the unconditional average firm value, that is $P_{PO} = E(a) + b$.

After establishing the equilibrium, we study the payoff to shareholders both before and after the realization of the firm type $a$. Because outside investors on average break even, we can define the ex-ante (before realization of $a$) wealth transfer from cash-poor to cash-rich current shareholders to be the difference between the expected actual payoff and fair payoff $(E(a) + b)$ to cash-poor investors. The next proposition characterizes the realized payoff to the current shareholders and the ex-ante wealth transfer.

**Proposition 4** In a public offering with a given dilution protection $\lambda$, the ex-post payoff to an existing shareholder in a type-$a$ firm equals

$$
\begin{cases}
(a + b) + \frac{1-\lambda}{1+\frac{P_{PO}}{I}}[P_{PO} - (a + b)] & \text{if } a > a^*_PO \text{ and cash rich} \\
\frac{1}{P_{PO}+1}(I + a + b) & \text{if } a \leq a^*_PO \text{ or cash poor}
\end{cases}
$$

Therefore, the ex-ante wealth transfer from cash-rich investors to cash-poor investors is

$$
\frac{I}{I+P_{PO}}[E(a) + b - P_{PO}] \geq 0,
$$

which is decreasing in $P_{PO}$ and increasing in $\lambda$.

When the offer is underpriced ($P_{PO} < a + b$), cash-rich current shareholders subscribe. However, because they cannot maintain their original fractional ownership in the company, their payoff — increasing in $\lambda$ — is strictly less than the full information payoff $a + b$. The cash-poor current shareholders are even worse off because they do not have money to subscribe. As a result, dilution to their holdings is even more severe.

From an ex-ante perspective, the outsiders break even, so the wealth transfer lies between the existing cash-rich and cash-poor investors. The lower the offering price $P_{PO}$ the more shares need to be issued, and the existing shareholders’ ability to subscribe them is consequently more valuable. Hence, wealth transfer is bigger with lower $P_{PO}$. On the other hand,
since better dilution protection $\lambda$ offers higher allocation priority to the informed investors, the winner’s curse problem becomes more severe. The outside investors must offer a lower issue price $P_{RO}$ to break even, resulting in a higher ex-ante wealth transfer.

### 3.2 Rights Offerings

In this section, we consider the equilibrium outcome in a given rights offering with a strike price $P_S$. The post-issue firm value is $I + a + b$, the total number of shares is $N_{RO} + 1$, and hence the share price is $\frac{I + a + b}{N_{RO} + 1}$. If a cash-rich current shareholder with $\beta$ shares exercise his rights, he receives $\beta N_{RO}$ new shares and invests additional $\beta N_{RO} P_S = \beta I$ in the new company. As a result, his payoff as a function of their true firm type $a$ is

$$ I + a + b \frac{1}{N_{RO} + 1} (N_{RO} \beta + \beta) - \beta N_{RO} P_S = \beta (a + b). \quad (7) $$

If a current shareholder chooses not to exercise the rights, the payoff is

$$ I + a + b \frac{1}{N_{RO} + 1} \beta + P_R \beta N_{RO}. \quad (8) $$

Cash-rich current shareholders exercise the rights if and only if (7) is weakly greater than (8). The next lemma characterizes the cash-rich current shareholders’ rights exercising strategy. Similar to the public offering, cash-rich shareholders exercise their rights if they have a good company.

**Lemma 2** In a rights offering, cash-rich current shareholders exercise the rights if and only if:

$$ a \geq a_{RO}^* \equiv P_S + P_R (N_{RO} + 1) - b. \quad (9) $$

Outside investors who purchase one right has the following payoff after exercising the right:

$$ I + a + b \frac{1}{N_{RO} + 1} - P_S - P_R. $$

If the firm is good (i.e. $a \geq a_{RO}^*$), only the cash-poor investors (fraction $\pi$) sell the rights, totaling $\pi N_{RO}$ rights, while if the firm is bad (i.e. $a < a_{RO}^*$), all rights are sold, totaling $N_{RO}$ rights.
rights. Therefore, the expected payoff to outsiders is

\[ P_{\text{rob}}(a \geq a^*_{\text{RO}})N_{\text{RO}} \left( \frac{I + E(a|a \geq a^*_{\text{RO}}) + b}{N_{\text{RO}} + 1} - P_S - P_R \right) + P_{\text{rob}}(a < a^*_{\text{RO}})N_{\text{RO}} \left( \frac{I + E(a|a < a^*_{\text{RO}}) + b}{N_{\text{RO}} + 1} \right) \]

Outsiders break even if

\[ P_S + P_R = \frac{I + \pi P(a \geq a^*_{\text{RO}})E(a|a \geq a^*_{\text{RO}}) + P(a < a^*_{\text{RO}})E(a|a < a^*_{\text{RO}}) + \pi E(a)}{N_{\text{RO}} + 1}. \]  

(10)

In a rights offering with a given strike price \( P_S \), the equilibrium outcome is characterized by a pair of \( (a^*_{\text{RO}}, P_R) \) such that (9) and (10) hold. The next proposition shows that counterintuitively, the cutoff type \( a^*_{\text{RO}} \) does not depend on the strike price of the rights \( P_S \).

**Proposition 5** For any \( P_S \), the equilibrium cutoff type \( a^*_{\text{RO}} \) solves

\[ a^*_{\text{RO}} = \frac{(1 - \pi)P(a < a^*_{\text{RO}})E(a|a < a^*_{\text{RO}}) + \pi E(a)}{(1 - \pi)P(a < a^*_{\text{RO}}) + \pi}, \]

which is independent of \( P_S \). In addition, such an \( a^*_{\text{RO}} \) exists and lies in \((a, E(a))\). Finally, for any \( 0 < P_S < a^*_{\text{RO}} + b \), an rights offering equilibrium exists in the sense that there is a \( P_R \geq 0 \) such that (9) and (10) hold.

At first glance, the proposition seems to be very counterintuitive. For example, consider the worst firm type \( a \), if the strike price \( P_S > a + b \), then the strike price is overvalued and informed shareholders would naturally sell the rights. However, Proposition 5 implies that regardless of how small the strike price is \( (P_S \ll a + b) \), the informed investors still sell the rights. Why do they not take advantage of the low strike price and exercise the option? The reason lies in the fact selling the rights generates a higher payoff. Exercising the rights is equivalent to selling the rights as if it is fairly priced for the \( a \)-type firm. Since the rights are priced by outsiders’ average belief conditional on rights being sold, so the worst company effectively receive a subsidy from the uninformed market. Regardless of the strike price, the firms above or below the conditional average belief are still the same, therefore the cutoff firm type is not affected.

Finally, we study the wealth transfer in a rights offering as the firm chooses different strike prices \( P_S \). The next result shows that lower strike price generates more wealth transfer from
cash-poor to cash-rich investors.

**Proposition 6** In a rights offering with a strike price $P_S$, the ex-post payoff to a current shareholder of a type-$a$ firm (with a unit holding) equals:

$$
\begin{cases}
  a + b & \text{if } a > a^*_\text{RO} \text{ and cash rich} \\
  a + b + \frac{I}{I+P_S}(a^*_\text{RO} - a) & \text{if } a \leq a^*_\text{RO} \text{ or cash poor}
\end{cases}
$$

Therefore, the ex-ante wealth transfer from cash-poor to cash-rich shareholders is

$$
\frac{I}{I+P_S}[E(a) - a^*_\text{RO}] > 0,
$$

(12)

which is decreasing in $P_S$ and increasing in $\lambda$.

The intuition for this result is very similar to Proposition 4. A lower strike price $P_S$ requires that more rights $N_{RO}$ must be issued. Since rights are on average underpriced due to winner’s curse problem. The ability to exercise the rights becomes more important with higher $N_{RO}$, which leads to higher ex-ante wealth transfer from the cash-poor and cash-rich existing shareholders.

4 Comparison between Public Offerings and Rights Offerings

In Section 3, we have analyzed public offerings and rights offerings separately. In this section, we explicitly compare the dilution effects between two types of offerings. As a starting point, we first show that a public offering with full dilution protection ($\lambda = 1$) is equivalent to a rights offering with a zero price of rights ($P_R = 0$).

**Proposition 7** A rights offering with a strike price $P_S$ which entails a zero rights price $P_R = 0$ is equivalent to a public offering with $\lambda = 1$.

The intuition of the result is simple. In a public offering, if existing shareholders are fully dilution protected, then they can maintain their proportional stake in the company.
by subscribing to the newly offered shares — the same outcome as in a rights offering. Alternatively, if current shareholders cannot or choose not to subscribe the new shares, they can only keep their original shares in a public offerings by definition and do not receive any extra payoff in a rights offering because $P_R = 0$, again achieving the same outcome. Because the terminal allocation of shares is identical across two offering mechanisms, the corresponding issue prices $P_{PO}$ and $P_S$ must be the same as well. As a result, a public offering with full dilution protection is equivalent to a rights offering with zero price of rights.

Using this equivalence result, we can rank all share floatation methods based on the wealth transfer from cash-poor to cash-rich investors. From Proposition 4, we know that this wealth transfer is increasing in the dilution protection $\lambda$. Therefore, among all public offerings with different dilution protections, full dilution protection maximize the wealth transfer, whereas no dilution protection eliminates the ex-ante wealth transfer. From Proposition 6, the wealth transfer in rights offerings decreases with the strike price $P_S$. The strike price is the highest, minimizing the wealth transfer, when the value of a right $P_R$ is zero. All other rights offerings with lower strike price creates more wealth transfer. The following corollary summarizes the discussion.

**Corollary 1** Among rights offerings, the smallest wealth transfer from cash-poor to cash-rich existing shareholders is attained by having a highest strike price $P_S$ such that $P_R = 0$, which is equivalent to a public offering with full dilution protection $\lambda = 1$. All other rights offerings generate higher wealth transfer, which increases with lower $P_S$; all other public offerings create less wealth transfer, which decreases with lower $\lambda$.

Finally, based on the above finding on wealth transfer, we propose one channel that may affect firms’ choice of share issuance mode — their preference for cash-poor and cash-rich investors. When firms favor their cash-rich shareholders, they use rights offerings with low strike price to create wealth transfer from the cash-poor investors. On the contrary, if firms favor the cash-poor shareholders, public offering with no dilution protection minimizes the wealth transfer.

**Proposition 8** Firms who weigh more heavily on cash-poor investors choose public offerings with $\lambda = 0$. Assume there is an exogenous lower bound on the strike price $P_S \geq P$. Firms who weigh more heavily on cash-rich investors choose rights offering with $P_S = P$. 

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5 Conclusion

We analyze seasoned equity offerings where some shareholders are informed and can strategically choose to participate, creating informational frictions. When all existing shareholders have wealth to participate in the issue, right offerings achieve the full information outcome, and therefore dominate public offerings which necessarily generate wealth transfers. We show that this ranking may be reversed when some of the existing shareholders are wealth constrained. In rights offerings, outside investors must purchase the rights to buy the underlying shares, rather than only buying these shares as in a public offering. Hence, a positive right price implies a discount in the strike price relative to the public offering price. Therefore, wealth-constrained shareholders become more diluted in a rights offering, and lower strike price increases the wealth transfer from the cash-constrained to the informed cash-rich shareholders. More generally, cash-constrained and cash-rich shareholders have diverging preference over floatation methods and terms.

References

[1] to be completed

A Omitted Proofs

Proof of Proposition 1: Given \( \lambda < 1 \), outside investors must purchase some shares in equilibrium. Since their purchase decision cannot depend on \( a \), their break-even condition implies a unique \( P_{PO} \). The per-share payoff to outside investors is

\[
\frac{I + a + b}{1 + N_{PO}} - P_{PO},
\]

which is linear in \( a \) and has a unique root at \( a = P_{PO} - b \). Hence, the outside investors’ payoff is non-zero for any firm type \( a \neq P_{PO} - b \), implying wealth transfers. \( \blacksquare \)

Proof of Proposition 2: In any equilibrium, the informed insiders can secure a net payoff of at least \( a + b \) by exercising the rights. Similarly, the uninformed insiders must receive at
least a net payoff $E(a) + b$. Outsiders must on average at least break even. Because the total firm value net of investment $I$ is $a + b$, the above payoffs are exactly the equilibrium payoffs for each type of investors.

Next, we show uninformed insiders receive exactly $a + b$ net of investment in equilibrium as well. Suppose otherwise, then some uninformed insiders can earn an ex-post net payoff strictly higher than $a + b$. The informed insiders in the same firm would deviate to this strategy to earn a strictly higher payoff. Contradiction! Thus all insiders receive exactly $a + b$, which can be implemented by exercising the rights.

We now prove that no equilibrium exists if the strike price $P_S > a + b$. Consider the informed insiders with a firm type $a \in [a, P_S - b)$, which implies $P_S > a + b$. If they choose not to exercise, their payoff

$$\frac{I + a + b}{1 + \frac{I}{P_S}} > \frac{I + a + b}{1 + \frac{I}{a+b}} = a + b.$$ 

Thus these informed insiders’ equilibrium payoff must be strictly higher than $a + b$. Contradiction! Therefore, no equilibrium exists.

To complete the proof, we finally show that when the strike price $P_S \leq a + b$, all insiders have incentive to exercise the rights. The equilibrium is supported by the outside investors’ belief that if rights are sold, they come from the worst firm type $a$. The associated price of the rights is therefore

$$P_R = \frac{I + a + b}{1 + \frac{I}{P_S}} - P_S.$$ 

The payoff to shareholders in a firm with type $a$, should they choose to sell the rights, is

$$\frac{I + a + b}{1 + \frac{I}{P_S}} + P_R N_{RO}.$$ 

Since $P_R \leq \frac{I + a + b}{1 + \frac{I}{P_S}} - P_S$ for any $a \geq a$, the above payoff is bounded by

$$\frac{I + a + b}{1 + \frac{I}{P_S}} + \left( \frac{I + a + b}{1 + \frac{I}{P_S}} - P_S \right) \frac{I}{P_S} = a + b,$$

which can be achieved by exercising the rights. Therefore, all insiders have incentive to
exercise the rights. ■

**Proof of Lemma 1:** Cash-rich insiders have incentive to subscribe if and only if:

\[(\lambda N_{PO} + 1)(I + a + b) - \lambda I(N_{PO} + 1) \geq (I + a + b),\]

which is equivalent to

\[\lambda N_{PO}(I + a + b) \geq \lambda I(N_{PO} + 1),\]

which in turn equivalent to

\[N_{PO}(a + b) \geq I.\]

Together with the fact that \(P_{PO}N_{PO} = I\), we have condition (3). ■

**Proof of Proposition 3:** When \(P_{PO} = a + b\), (3) always holds, and all cash-rich insiders subscribe. In this case, \(Pr_{IS} = 1\) and \(E_{IS} = E(a)\). Therefore, outsiders’ payoff (5) becomes

\[\left[1 - (1 - \pi)\lambda\right] \left[E(a) + b - P_{PO}\right],\]

which is strictly positive.

For any \(P_{PO} \geq E(a) + b\), by definition

\[E_{IS} = E(a | a \geq P_{PO} - b) \geq P_{PO} - b,\]

with strict inequality whenever \(P_{PO} < a + b\). Therefore, (5) is strictly negative. Therefore, by continuity, expression (5) as a function of \(P_{PO}\) has a root and all roots lie in \((a + b, E(a) + b)\).

Finally, we show \(P_{PO}\) is decreasing in \(\lambda\). Suppose \(\lambda_1 < \lambda_2\). Denote by \(P_{PO,i}\) the corresponding solution to (5) for \(\lambda_i\) \((i = 1, 2)\). Also denote by \(Pr_{IS,i}\) and \(E_{IS,i}\) the corresponding values for \(\lambda_i\). Since \(P_{PO,i} < E(a) + b < E_{IS,i} + b\), we have

\[E(a) + b - P_{PO,1} - Pr_{IS,1}(1 - \pi)\lambda_2 [E_{IS,1} + b - P_{PO,1}] < E(a) + b - P_{PO,1} - Pr_{IS,1}(1 - \pi)\lambda_1 [E_{IS,1} + b - P_{PO,1}] = 0.\]

As we have shown, (5) is positive when \(P_{PO} = a + b\). So there must exist a \(P_{PO,2} \in \)
Proof of Proposition 4: From (1), if the insiders subscribe, their payoffs are:

\[
\frac{\lambda N_{PO}+1}{P_{PO}+1} (I + a + b) - \lambda I = (a + b) + \frac{1-\lambda}{N_{PO}+1} I - \frac{(1-\lambda)N_{PO}}{N_{PO}+1} (a + b)
\]

\[
= (a + b) + \frac{1-\lambda}{N_{PO}+1} [I - N_{PO}(a + b)]
\]

\[
= (a + b) + \frac{1-\lambda}{1+P_{PO}} [P_{PO} - (a + b)].
\]

From (2), if the insiders do not subscribe, their payoffs are:

\[
\frac{1}{1+P_{PO}} (I + a + b),
\]

which is clearly increasing in \(P_{PO}\). Thus, non-subscribing insiders favor higher offering price.

Because outsiders break even, so the ex-ante wealth transfer is

\[
E(a) + b - \frac{1}{1+P_{PO}} (I + E(a) + b).
\]

Simple algebraic manipulation gives (6).

Proof of Lemma 2: Cash-rich insiders exercise the rights if it yields a (weakly) higher payoff (7) than selling the rights (8):

\[
a + b \geq \frac{I + a + b}{N_{RO} + 1} + P_{R} N_{RO},
\]

which implies

\[
N_{RO}(a + b) \geq I + P_{R} N_{RO}(N_{RO} + 1),
\]

which in turn implies

\[
a + b \geq \frac{I}{N_{RO} + 1} + P_{R} (N_{RO} + 1).
\]

Using the fact that \(\frac{I}{N_{RO}} = P_{S}\), condition (9) follows immediately.
**Proof of Proposition 5:** From (9) and (10), we have
\[ a_{RO}^* = P_S + P_S \left( \frac{I + (1-\pi)P(a<a_{RO}^*)E(a|a<a_{RO}^*) + \pi E(a)}{I + P_s} + b \right) - 1 \left( \frac{I}{P_S} + 1 \right) - b \]
\[ = P_S + P_S \left( \frac{(1-\pi)P(a<a_{RO}^*)E(a|a<a_{RO}^*) + \pi E(a)}{I + P_s} + b \right) - 1 \left( \frac{I}{P_S} + 1 \right) - b \]
\[ = P_S + (1-\pi)P(a<a_{RO}^*)E(a|a<a_{RO}^*) + \pi E(a) \]
\[ = (1-\pi)P(a<a_{RO}^*)E(a|a<a_{RO}^*) + \pi E(a), \]
which is the expression in the statement.

Next, we show that \( a_{RO}^* \in (a, E(a)) \). At \( a_{RO}^* = a \), the RHS of (11) is \( E(a) > a \). Because \( E(a|a < a_{RO}^*) < E(a) \) whenever \( a_{RO}^* < \pi \), so RHS of (11) is in turn dominated by \( E(a) \). Therefore, \( a_{RO}^* \) must exist and lie in \((a, E(a))\).

Finally, from (9), we can solve for the unique \( P_R \):
\[ P_R = \frac{a_{RO}^* + b - P_S}{\frac{I}{P_S} + 1} > 0. \]

\[ \blacksquare \]

**Proof of Proposition 6:** In a rights offering, cash-rich insiders with \( a > a_{RO}^* \) exercise the rights and their payoff is therefore \( a + b \). Insiders with \( a < a_{RO}^* \) or cash poor insiders sell the rights and their payoffs are
\[ \frac{I + a + b}{N_R + 1} + P_R N_R = P_S \frac{I + a + b}{P_S + I} + P_S \left( \frac{I + (1-\pi)P(a<a_{RO}^*)E(a|a<a_{RO}^*) + \pi E(a)}{(1-\pi)P(a<a_{RO}^*) + \pi} + b \right) - 1 \left( \frac{I}{P_S} \right) \]
which in turn equal

\[
P_S \frac{I+a+b}{P_S+I} + \left( \frac{(1-\pi)P(a<\ast_{RO})E(a|a<\ast_{RO})+\pi E(a)}{(1-\pi)P(a<\ast_{RO})+\pi} + b-P_S \right) I
\]

\[
= \frac{(a+b)P_S}{P_S+I} + \left( \frac{(1-\pi)P(a<\ast_{RO})E(a|a<\ast_{RO})+\pi E(a)}{(1-\pi)P(a<\ast_{RO})+\pi} + b \right) I
\]

\[
= a + b + \frac{I}{I+P_S} \left( \frac{(1-\pi)P(a<\ast_{RO})E(a|a<\ast_{RO})+\pi E(a)}{(1-\pi)P(a<\ast_{RO})+\pi} - a \right)
\]

Therefore, wealth transfer from cash-poor to cash-rich investors is \( \frac{I}{I+P_S} [E(a) - a_{RO}^*] \). ■

Proof of Proposition 7: When \( \lambda = 1 \), from (5) and the condition \( a_{PO}^* = P_{PO} - b \), we have the following condition for \( a_{PO}^* \):

\[
E(a) - a_{PO}^* - P(a \geq a_{PO}^*) (1-\pi) [E(a|a \geq a_{PO}^*) - a_{PO}^*] = 0.
\]

Solving for \( a_{PO}^* \), we have

\[
a_{PO}^* = \frac{E(a) - (1-\pi)P(a \geq a_{PO}^*)E(a|a \geq a_{PO}^*)}{1 - P(a \geq a_{PO}^*)(1-\pi)}.
\] (13)

Together with the fact that

\[
E(a) = P(a < a_{PO}^*)E(a|a < a_{PO}^*) + P(a \geq a_{PO}^*)E(a|a \geq a_{PO}^*),
\]

condition (13) is equivalent to the condition for \( a_{RO}^* \) in a rights offering (11). Therefore \( a_{PO}^* = a_{RO}^* \), immediately implying that \( P_{PO} = P_S \) and \( N_{PO} = N_{RO} \) as well. One can simply verify in both types of offerings, existing shareholders receive \( a + b \) if they participate (subscribe or exercise the rights) and the same payoff \( \frac{I+a+b}{N_{RO}+1} \) if not participate. In all, everyone’s payoff in a public offering is exactly the same as in a rights offering. ■

Proof of Proposition 8: Simple corollary of Corollary 1. ■