Abstract

Measuring the value of labor market hires on stock prices, be it the choice of underwriters when firms go public (IPOs) or chief executive officers (CEOs), is difficult due to selection. Opaque firms facing a higher cost of capital might benefit more from a prestigious underwriter, while productive firms might benefit more from a talented CEO. Using an assignment model, we derive a decomposition of talent effects due to agent heterogeneity from selection effects due to firm heterogeneity using only data on wage and firm value distributions associated with firm-agent matches. We use this decomposition to understand changes over time in IPO underpricing and CEO wages.
1. Introduction

Measuring the value of labor market hires for stock prices is a fundamental question in financial economics. Two labor markets are particularly important and widely studied. The first is the market for underwriters when firms issue equity. Firms compete and spend significant resources to hire reputable underwriters (typically investment banks with track records of successful placements) for their initial public offering (IPO) so as to alleviate adverse selection (see, e.g., Allen and Faulhaber (1989), Carter and Manaster (1990), Welch (1989)). Measuring the value of prestigious underwriters is a long-standing goal in the IPO literature (for reviews, see, e.g., Ritter and Welch (2002)). The second is the market for Chief Executive Officers (CEOs). There is a large literature on the value-added of a CEO (see, e.g., Bertrand and Schoar (2003)). In particular, valuing different attributes of a CEO, be it intelligence or other personality attributes, remains a widely researched topic (see, e.g., Kaplan et al. (2012), Graham et al. (2013)).

Regardless of the situation, quantifying these hires’ impact on stock prices is difficult due to selection or sorting in competitive labor markets (Becker (1973) and Rosen (1974)). Firms that hire better underwriters might have more uncertainty or information asymmetry and higher costs of capital to begin with. So correlating a firm’s stock valuation with the status of the underwriter it hired is potentially problematic due to this selection. Indeed, recent empirical work suggests that such competitive selection effects between firms and underwriters (Fernando et al. (2005), Akkus et al. (2013)) might inform the long-running debate on IPO underpricing, the first-day return thought to compensate investors for adverse selection. Prestigious underwriters were associated (correlated) with less underpricing before the Internet period but became associated with more underpricing during and after the Internet period. This change in the sign of this underpricing-prestige correlation remains puzzling.

1Due to CEO wage inequality, this question continues to be important in the popular press and across various disciplines such as strategy and management (see, e.g., "Do CEOs Matter?" by The Atlantic in the June 2009 issue which surveyed a variety of contrasting views of whether CEOs add any value.).
Selection effects arising from assortative matching in a competitive labor market for CEO talent are even more problematic. On the one hand, Terviö (2008) and Gabaix and Landier (2008) point to a positive assortative matching between managerial talent and firm productivity due to complementarities in firm production functions. On the other hand, firms with greater uncertainty and a higher cost of capital might want to hire a talented CEO as well (see, e.g., Lev (2011)). An example is Marissa Mayer of Yahoo, whose goal arguably was not to increase the fundamental value of Yahoo but to increase Yahoo’s stock valuation for a sale. This type of selection would then lead to a negative correlation between stock valuation and managerial talent.

To understand the role of selection from the direct effect of agent talent, we develop an assignment model where firms compete to hire a talented agent, be it an underwriter or a CEO, to raise their stock valuations. Specifically, the value of the match depends on the role of the agent in the asset market. In the labor market for underwriters when firms go IPO, our asset market follows the classical IPO underpricing set-up (Rock (1986), Benveniste and Wilhelm (1990), Habib and Ljungqvist (2001)), in which adverse selection generates underpricing. More prestigious underwriters are assumed to be able to bring in more uninformed investors and hence alleviate the need for underpricing. In the labor market for CEOs, a talented CEO increases firm valuation in two ways. First, talent raises long-term fundamental value as in Terviö (2008) and Gabaix and Landier (2008). Second, talent increases investor confidence by offering more precise public signals regarding a firm’s fundamental value. The price of the stock is determined in a traditional noisy rational expectations framework (Grossman and Stiglitz (1980); Diamond and Verrecchia (1981); Hellwig et al. (2006)).

We characterize the assignment equilibrium that maps (assigns) these multiple dimensions of firms into agent talent by building on Chiappori et al. (2015) and solve for the wage

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2The importance of CEOs for firm cost of capital is also reflected in CEOs of even large companies invariably leading disclosures to reduce investors’ uncertainty. Quarterly conference calls with investors are one of the few managerial tasks that never gets delegated and is a skill that firms value.
function and stock prices. For the labor market for underwriters and IPO underpricing, we create an index of firm opacity (depending on firm investment, issuance size and volatility) that characterizes the positive assortative matching between firms with differing opacity and agents (i.e. underwriters of different prestige). That is, the IPO underpricing model is similar to the CEO assignment model where firms differ only in productivity and surplus of the match multiplicative in firm productivity and agent talent. It turns out that a number of empirical predictions can already be obtained in this setting.

As such, we first focus our discussions on this multiplicative setting before considering the general non-multiplicative one. There are two effects that shape the relationship between firm value, be it the cost of capital (measured with underpricing) or market capitalization and agent talent (prestigious underwriter or talented CEO). The first is the "direct effect" of hiring a more talented agent: all else equal, a firm with a better agent will have a lower cost of capital or higher market value since the more prestigious underwriter will bring more uninformed investors to the IPO or the more talented CEO will increase firm productivity by more. The second is the "selection effect". Firms with greater opacity and hence greater adverse selection and higher cost of capital all else equal, benefit more and pay more for a better agent. Which of these two effects dominates then empirically determines the sign of the correlation between the cost of capital and agent talent. In contrast, productive firms will pay more for a talented CEO due to complementarities which will tend to reinforce or amplify the positive correlation between talent and firm value.

The strength of the selection effect increases with heterogeneity across firms (be it opacity in the IPO setting or firm productivity in the CEO setting) relative to the talent distribution of agents. For instance, if firm opacity is tightly distributed in the population, the highly opaque firms still hire the most talented agent and so the direct effect dominates and firms with a better agent have a lower cost of capital. As dispersion of firm types increases this association flips signs from negative to positive; i.e. firms with a better agent have a higher cost of capital. This selection effect can potentially explain the change in the underpricing-
prestige relationship during Internet period to the extent there is greater dispersion in firm opacity as young Internet firms with little cashflows went IPO.

But dispersion in underlying opacity or productivity and heterogeneity in agent talent are typically unobservable to the econometrician, which results in the challenge of separating selection and from talent effects. A key result of our paper is that we provide a decomposition of these unobservable quantities using observable firm outcomes (underpricing and market value) and agent wages. We show that the slope of the cross-sectional wage distribution, sorted on wage outcomes, normalized by firm value gives us the heterogeneity of agent talent at a given percentile. We can then use the slope of the firm value distribution associated with these worker rankings (normalized by firm value) to back out firm heterogeneity. That is, we can back out the relative strength of talent to selection effects from the slopes of the wage and firm value distributions in any given equilibrium match between firms and agents. This result applies to instances where the surplus generated between firm and agent are multiplicative and there is a one-to-one assignment between some firm index and agent talent.

Given that market value and wage distributions are often observable, empirical researchers can use this decomposition to measure the relative strength of selection versus direct effects. We apply our decomposition to the IPO and CEO settings. The time variation in IPO underpricing patterns has spawned competing explanations of selection effects versus structural changes in underwriter technology or incentives (Loughran and Ritter (2002)). We find using our decomposition a significant decline in talent relative selection effects from the early nineties into dot-com period. That is, the change in signs of underpricing and prestige are driven by the increasing importance of selection effects due to heterogeneity in firm opacity.

Selection effects due to firm productivity and CEO talent have been argued to explain

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3 Adjusting for selection (see, e.g., Heckman (1977), Roberts and Whited (2012)) is empirically challenging since it generally requires instrumental variables for the selection equation. Moreover, important firm characteristics that might drive the selection, such as investors’ information asymmetry or uncertainty, are also unobservable to econometricians and at a minimum difficult to measure.
the rise of CEO wages since the 1980s (Gabaix and Landier (2008)). The main test is to show that the increase in the average size of the firm in the stock market can explain the rise in CEO wages. But there is debate on whether this coincident trend is causal (Frydman and Jenter (2010), Frydman and Saks (2010)). Changes in the talent distribution of CEOs or in the impact of that talent to matching surplus might also be behind the rise in CEO wages. Our decomposition allows us to separate these two effects. Using CEO wage data and firm value data from 1990-2014, we find that the relative strength of talent effects rose from the early nineties to the early 2000’s but has declined subsequently.

The literatures on underwriters and CEOs have evolved largely separately but they are united, as we have hoped to show, in fundamental ways through the role of selection in labor markets. There are other labor markets we have not covered, such as the labor market for venture capitalists or other types of financial intermediaries (see, e.g., Sørensen (2007)). The matching surplus functions are similar to those considered here. As such, our decomposition likely applies to these settings as well.

But these decompositions can be thought of a diagnostic tests but need to be applied with care depending on the situation. Even in the labor market for CEOs, the selection effect driven by the desire of firms with high fundamental value to hire talented CEOs gives rise to a positive correlation between firm size and CEO talent. But the selection effect driven by the desire to reduce the cost of capital leads to a negative correlation. These off-setting effects can potentially explain empirical findings which show only a modest correlation in the cross-section between firm size and CEO pay, to the extent pay is naturally a proxy for CEO talent (Frydman and Saks (2010)). Once we account for the richer role of CEOs in improving firm value, our decompositions need then to be modified to account for all parameters. We derive these compositions in this paper but leave the empirical implementation to future research.

Our paper proceeds as follows. We describe our assignment model in Section 2. We provide the solution in Section 3. We derive the decomposition of talent versus selection
effects in the standard multiplicative setting in Section 4. We provide some estimates of this decomposition for IPO underpricing and CEO compensation in Section 5. We consider the non-multiplicative setting in Section 6. We conclude in Section 7.

2. Model

The model lasts for three dates. There is a unit measure of heterogeneous firms that issue equity through stock markets. Firms can hire agents (e.g., underwriters or executives) through a competitive labor market at date 0. These agents, who differ in their ability, can increase the share price for the firm. Finally, at date 2, the cash flow is realized, and all players in the economy consume their realized gains.

To proceed, we first introduce a general framework for the labor market. Firms, which differ in multiple dimensions, choose the optimal agent to maximize the firm’s expected payoff. They rationally anticipate how different agents affect their stock price at date 1. We then specify the relationship between agents’ talents and share prices by considering two classical models.

**Agents:** There is a distribution of heterogeneous agents whose ability is indexed by $h \in H \equiv [h^L, h^U]$. Let $G^A(h)$ denote the talent distribution. This ability in the context of underwriters is prestige bringing in more uninformed investors to an IPO. In the context of CEOs, this talent increases fundamental value and investor confidence.

**Firms:** Each firm owns a risky project with capital stock $k$. The payoff of the project for the firm with capital $k$, is given by $k\theta$, where $\theta$ is a firm-specific payoff with mean $\hat{\theta}(h)$. The mean $\hat{\theta}(h)$ aims to capture firms’ long-term fundamental, which can potentially affected by the type of agent a firm hires. The riskiness of the project is indexed by $\sigma$. A firm originally owns $(1 + \psi)$ measure of shares and wants to raise capital by issuing one measure of its equity to investors. The characteristic of a firm is then summarized by $y = (k, \psi, \sigma) \in Y \subseteq \mathbb{R}_+^3$. The types of firms are distributed according to a probability measure $\nu^F$ on $Y$, which is assumed
to be absolutely continuous with respect to the Lebesgue measure.

**Labor Market:** At date 0, each firm can hire at most one agent assuming it hires any at all. The fee paid to the agent is denoted by \(\omega(h)\), which will be determined competitively in equilibrium. The payoff of agent \(h\) is then given by \(\omega(h)\). The end-of-period cash flows for firm \(y\) are then the profit from its project minus the fee that it commits to pay: \(\pi_y = k\theta - \omega(h)\).

At date 0, given the fee required to hire agent \(\omega(h)\), a firm of type \(y\) chooses the optimal agent to maximize its expected payoff. Let \(\tilde{p}_{hy}\) denote the realized share price at date 1 for firm \(y\) if it hires agent \(h\). A firm’s optimization problem yields:

\[
U^*(y) = \max_{h \in H \cup \{\emptyset\}} \mathbb{E} \left[ \left( \frac{1}{1 + \psi} \right) (k\theta - \omega(h)) + \tilde{p}_{hy} \right].
\]  (1)

Given any price function, the equilibrium in the labor market consists of an assignment \(\mu(y) : Y \rightarrow H \cup \{\emptyset\}\) and competitive fee for agents \(\omega(h) : H \rightarrow \mathbb{R}^+\) such that (1) the optimality conditions for both firms and agents are satisfied, i.e. given wage \(\omega(h)\), \(\mu(y)\) is the type of agent that firm \(y\) optimally chooses to hire. So that, \(\mu(y)\) maximizes (1). And (2) the market-clearing condition holds for the labor market.

**Financial Market:** The exact relationship between agents’ talents and share prices depends on the underlying friction and the role of agent in the financial market. Nevertheless, given the price function, the (unconditional) expected asset return for investors is given by

\[
R(y, h) \equiv \mathbb{E}_\theta \left[ \left( \frac{k\theta - \omega(h)}{1 + \psi} \right) - \tilde{p}_{hy} \right].
\]

The payoff of firm \(y\) who hires agent \(h\) can then be rewritten as

\[
U(y, h) \equiv \left\{ k\hat{\theta}(h) - R(y, h) - \omega(h) \right\}.
\]

The first term reflects how talents increase the expected payoff (i.e., firms’ long-term fundamental). The second term can be interpreted as the firm’s cost of capital, which
depends on both the firms’ characteristic and talent effect. That is, the firm’s expected utility when hiring agent $h$ can be conveniently rewritten is simply the value generated by the agent minus the agent’s fee. Note that, although the firm only pays some proportion of the fees at the end of the period (i.e., $\frac{\psi}{1+\psi} \omega(h)$), the reduction in the asset price due to the hiring is $\frac{\omega(h)}{1+\psi}$. Hence, from a firm’s view point, the total cost is simply the agent’s fee.

To fix the ideas, we now consider two classical applications and show how these can be nested in our setup.

**Labor Market for Underwriters:** In the setting of Rock (1986), Benveniste and Wilhelm (1990), and Habib and Ljungqvist (2001), the key friction is the asymmetric information among investors, where informed investors know the quality of the firm, while uninformed investors do not. As the winner’s curse increases in proportion to the fraction of informed investors, so does the necessary amount of underpricing.

The main value of the underwriters is to reduce the cost of capital by attracting uninformed investors. One can thus interpret agents as the underwriters in this setting, who differ in terms of their ability to attract uninformed investors. Specifically, let $\beta(h)$ denote the fraction of uninformed investors that are attracted by underwriter $h$. We assume that the higher is $h$, the more prestigious the underwriter, the higher fraction of investors will be uninformed participating in the IPO: $\beta'(h) > 0$. We further set $\hat{\theta}(h) = \hat{\theta}$, which captures the fact that underwriters do not affect the fundamental and thus only increase the value of forms by reducing the cost of capital.

Specifically, the payoff of the project is given by $\theta \in \{\hat{\theta} - \sigma, \hat{\theta} + \sigma\}$ with equal probability. Informed investors know the realized value of $\theta$, but uninformed investors do not. For any given fraction of uninformed investors, the price at which shares are sold to investors must be such that uninformed investors expect to break even on average. The expression for the
expected return, which is interpreted as underpricing in the IPO literature, yields

\[ R(y, h) \equiv UP(y, h) \equiv \mathbb{E}_\theta \left[ \left( \frac{k\theta - \omega(h)}{1 + \psi} \right) - \tilde{p}_h y \right] = \left( \frac{k\sigma}{1 + \psi} \right) \left( \frac{1 - \beta(h)}{1 + \beta(h)} \right) \] (2)

Note that \( \frac{\partial UP(h, y)}{\partial h} = -\left( \frac{k\sigma}{1 + \psi} \right) \frac{2\beta'(h)}{(1 + \beta(h))^2} < 0 \). Since a better (more prestigious) underwriter can attract more uninformed investors, a better agent helps reduce the amount of underpricing by more than a worse agent.

**Labor Market for CEOs:** CEOs may affect firms’ value through varied channels. One standard channel (Terviö (2008), Gabaix and Landier (2008)) is to increase the firms’ fundamental. To capture this, we assume that the firm-specific payoff \( \theta \) is drawn from a Normal distribution with mean \( \tilde{\theta}(h) = \bar{\theta}h \) and variance \( \sigma^2 \). That is, a better CEO increases the average payoff of the project.

Another aspect of CEO talent is their ability to communicate with investors and thus reduce the cost of capital. To capture this idea, we consider an environment where where risk-averse investors who are imperfectly and heterogeneously informed. And, markets are segmented and thus idiosyncratic risk is priced. Specifically, we assume the agent can produce a report about the firm, which is a noisy, unbiased signal regarding the payoff: \( z = \theta + \sigma_h \eta \), where \( \eta \sim N(0, 1) \), and the variance of the report is given by \( \sigma^2_h = \frac{1}{h} \). Thus, higher values of \( h \) denote more precise agents. All investors observe the public signal \( z \) produced by the agent and know its precision.

The stock price is determined in a noisy REE in the spirit of Grossman and Stiglitz (1980). Each investor receives a private signal \( x_i = \theta + \sigma_x \epsilon_i \), and can submit his demand based on his information set.\(^5\) There are also noise traders in each market. To solve the model in closed form, we assume that noise traders purchase a random quantity \( \Phi(u) \) of stock,

\(^4\)See Appendix A.1 for detailed derivation.

\(^5\)For simplicity, we allow each investor to trade only one stock. We can allow investors to trade multiple stocks as long as markets are incomplete and idiosyncratic risk is priced.
where $u \sim N(0, \sigma_u)$ and $\Phi$ is the standard normal CDF.\(^6\) The (unconditional) expected asset return in this case is then the risk premium as a result of incomplete information, which is effectively the variance conditional on investors’ information:\(^7\)

$$R(y, h) = \frac{\gamma I}{2} \left( \frac{k^2}{(1 + \psi)^2} \left( \frac{1}{\sigma_x^2 + \frac{1}{\sigma_x^2 \sigma_u^2} + \frac{1}{\sigma^2}} + h \right) \right). \quad (3)$$

That is, a higher level of precision $h$ decreases the risk premium charged by investors, since it improves an investor’s estimation of the fundamental pay-off. All things being equal, the risk premium is higher for more opaque firms (i.e., either firms with higher volatility $\sigma$ or investors’ information is more noisy $\sigma_x$), since investors have to bear more risk for those firms. Similarly, the risk premium is higher for firms with a larger scale.

### 3. Labor Market Hiring

Taking into account how the agent affects the price in the asset market, we now characterize the assignment function and wage function. The surplus between firm $y$ and agent $h$, which is the sum of their payoff minus their outside option, yields

$$\Omega(y, h) \equiv U(y, h) - U(y, \emptyset) + \omega(h) = k \left( \hat{\theta}(h) - \hat{\theta}(\emptyset) \right) + R(y, \emptyset) - R(y, h), \quad (4)$$

where $\emptyset$ denotes the case in which a firm hires no agent (i.e., the firm’s autarky value) and the workers’ unemployed value is normalized to zero. The first two terms thus represents the gain of firm $y$ when it hires agent $h$ relative to no hiring. The third term represents the payoff of a worker, which is the fee. Thus, the surplus is simply the change in fundamental plus the reduction in the cost of capital (relative to no hiring).

\(^6\)The specific functional form assumed here is close to that in Hellwig et al. (2006).
\(^7\)See Appendix A.2 for a detailed derivation.
Technically, given the multiple characteristics of a firm, our environment is a multidimensional-to-one matching problem. As established in Chiappori et al. (2016), given our surplus function in (4) and that the measure of firms $\nu^F$ is absolutely continuous with respect to the Lebesgue measure, stable matching exists and the assignment function $\mu(y)$ is unique and pure. That is, each firm hires a unique agent instead of using mixed strategies.

**Sorting** Proposition 1 first establishes the property of the assignment function in terms of firms’ characteristics.

**Proposition 1.** All else equal, a firm with a riskier project $(\sigma)$, a larger size $(k)$, and a higher proportion of issued shares $(\frac{1}{1+\psi})$ hires a more talented agent. That is, $\mu_\sigma(k, \psi, \sigma) > 0$, $\mu_k(k, \psi, \sigma) > 0$ and $\mu_\psi(k, \psi, \sigma) < 0$.

These results can be seen from the firms’ optimization problem. Specifically, given that all firms face the same cost function $\omega(h)$, a firm that has a higher marginal benefit of talents must hire a better agent in equilibrium. Formally, the added value of talent $h$ for firm $y$ is given by

$$\frac{\partial U(y, h)}{\partial h} = k\hat{\theta}(h) - \frac{\partial R(y, h)}{\partial h} - \omega_h(h)$$

For example, in the IPO setting, given the under-pricing expression in Equation (2), the added value of talent $h$ for firm $y$ yields

$$\frac{\partial U(y, h)}{\partial h} = \left(\frac{k\sigma}{1+\psi}\right) \frac{2\beta'(h)}{(1+\beta(h))^2} - \omega_h(h).$$

In other words, there is a complementarity between the prestige of an agent and firms’ scale and riskiness. Thus, firms with a higher scale or more volatility benefit more from hiring a prestigious underwriter.

A similar intuition holds for the CEO setting, where the marginal value of an agent depends on two components. The first term, as in (Terviö (2008), Gabaix and Landier (2008), is the long term valuation, captured by $k\hat{\theta}h$. Hence, a larger firm hires a better
agent. The second term is the effect on risk premium. From Equation (3), one can see that firms with larger scale or with more volatile project has a higher marginal value of a higher precision.

**Characterization**  For the market for underwriters, observe from Equation (5), firms’ marginal value for hiring can be simply summarized by the one-dimensional index

$$q(y) = \left( \frac{k\sigma}{1+\psi} \right).$$

Thus, two different firms will choose the same agent in equilibrium if they have same index $q(y)$. With this particular feature, the model can be solved similarly as in the standard model with one-dimensional heterogeneity, where firms with a higher $q$ is matched with a more talented agent. The assignment function $\mu(y)$ must then satisfies the the familiar market clearing condition: $G^F(q(y)) = G^A(\mu(y))$, where $G^F(\tilde{q})$ denote the measure of firms with an index lower than $q$.

For any agent $h$, his marginal gain in equilibrium (represented by $\omega_h(h)$ ) is his contribution to the surplus within the match, given his optimal assignment:

$$\omega_h(h) = \Omega_h(\mu^{-1}(h), h),$$

where $\mu^{-1}$ denotes the inverse of $\mu$, representing the type of firm assigned to agent $h$.

For the market for CEO, both Terviö (2008) and Gabaix and Landier (2008) consider only the CEO’s impact on the long term valuation, which can the nested in our model by shutting down the risk premium channel ($\gamma = 0$). In that special case, the model is again reduced to one-dimensional heterogeneity, where a larger firm matches with a more talent agent.

More generally, different firms’ characteristics matter for CEO hiring when one takes into account aspects of CEO’s ability. In Appendix A.3, we thus provide an algorithm to
characterize the assignment function and wages when type spaces are multidimensional.\textsuperscript{8}

Intuitively, similar to one-dimensional assignment model, the equilibrium wage $\omega(h)$ must be such that it is indeed for firm $y$ to hire agent $\mu(y)$.

Thus, the wage schedule schedule $\omega(h)$ together with the assignment function $\mu(y)$ must be such that firms’ first-order condition is satisfied: if firm $y$ chooses to match with agent $\mu(y)$ in equilibrium, then his marginal benefit of precision must equal the marginal cost. That is, $U_h(y, \mu(y)) = \omega_h(\mu(y))$. As illustrated in Figure 1, each line represents the set of firms such that $U_h(y, h) = \omega_h(h)$.

Furthermore, to make sure that the market-clearing condition is satisfied, $\omega_h(h)$ must be chosen so that the measure of firms below the line coincides with $G_A(h)$. That is, the wage schedule must be constructed in a way so that the measure of firms that find a particular talent type $h$ too expensive coincides with the measure of agents below $h$.

4. Selection and Talent Effects in Multiplicative Setting

Consider an environment where the output function $V(a, b) = ab$ is multiplicatively separable and a firm’s type can be summarized by one aggregate index $a$ and an agent’s type is given by $b$. As we discussed below, this relation holds for both the underpricing and the standard CEO literature. Let $a[j]$ represents the $j$th percentile firm sorted based on the one-dimensional index $a$ and let $b[i]$ represents the value generated by the $i$th percentile agent, where $a'[j] > 0$ and $b'[i] > 0$.\textsuperscript{9}

Due to sorting and labor market clearing, an $i$th percentile agent then must match with an $i$th percentile firm. In other words, positive sorting thus implies that $i$th percentile agent thus match with $i$th percentile firm: $j^*(i) = i$. Hence, the output created by the $i$th percentile

\textsuperscript{8}As established in Chiappori et al. (2016), the constructed algorithm works as long as the underlying distribution satisfies certain conditions. See Appendix A.3 for detailed discussion.

\textsuperscript{9}As explained in Terviö (2008), a central feature of the assignment is that the characteristics $a$ and $b$ are essentially ordinal. It is thus without loss of generality to consider a simple multiplicative function $V(a, b) = ab$. Any separable function, for example, $Aa^{\gamma}b^{1-\gamma}$, can be nested in this expression.
agent can be expressed as
\[ V[i] = a[j^*(i)]b[i] = a[i]b[i]. \] 

In our underpricing environment, for example, this value is measured by underpricing \( V[i] = -UP[i] \), where \( a[i] = q[i] \) represents the the \( i \)th percentile firm sorted based on the one-dimensional index of firm opaqueness given by Equation (6), and \( b[i] = -\left(\frac{1-\beta(h[i])}{1+\beta(h[i])}\right) \) represents the value generated by underwriter of prestige \( h[i] \). In the standard CEO literature (e.g., Gabaix and Landier (2008)), \( a[i] \) represents the firm size and \( b[i] \) represents the ability of the \( i \)th percentile CEO. Specifically, this can be nested in our framework by shutting down the risk premium channel (i.e., setting \( \gamma = 0 \)). Thus, \( a[i] = k[i] \) represents the firm productivity and \( b[i] = \hat{\theta}(h[i]) \) represents the payoff of the project generated by \( i \)th percentile agent.

In either case, given that the output function and the wages depend on the characteristics of firms and agent, it is thus difficult to tell whether the change in output/income is driven by either talent or selection effects. Formally, the difference in the output yields
\[ \frac{V'[i]}{V[i]} = \frac{a'[i]}{a[i]} + \frac{b'[i]}{b[i]} . \] 

Thus, the first (second) term is the selection (talent) effect, which captures the characteristics of firms (talent).

**Comparison to the No-Sorting Benchmark** To highlight the selection effect, it is useful to consider a counterfactual environment where agents and firms are matched randomly. That is, an agent is hired by a firm randomly drawn from the distribution. Hence, \( a[i] \) then represents the expected type of firm that hires the \( i \)th agent, which will be the same across all agents (i.e., \( a'[i] = 0 \)). In other words, the first term in Equation (9) no longer exists. That is, the expected value generated by the \( i \)th agent is driven by the talent effect only.

This thus shows that, when the output depends positively on both firms’ and talents’
characteristics, the existence of the selection effect reinforce/amplify the positive correlation of firm value and talent. This is true, for example, in the CEO setting, where the value of firm increases in both firms’ size and CEO’s ability.

On the other hand, in the underpricing environment, the existence of selection effect leads to an offsetting effects. In particular, underpricing is higher for more opaque firms but is lower for more prestigious underwriters. Formally, according to Equation (9), by setting $V[i] = -UP[i]$, the percentage change of underpricing for the $i$th quantile underwriter yields:

$$
\frac{UP'[i]}{UP[i]} = \frac{q'[i]}{q[i]} - \frac{2\beta'(h[i])h'[i]}{(1 - \beta(h[i])^2)}
$$

(10)

The first term is the selection effect and is positive: more prestigious underwriters match with firms that are more opaque (i.e., $q'[i] \geq 0$). The second term is the direct effect of prestige (which is also often referred to as the certification effect) and is negative: all else being equal, more prestigious underwriters lead to lower underpricing, since they can attract more uninformed investors (which is captured by the fact that $\beta'(h) > 0$). This relationship can be captured in data by the correlation of firm underpricing and the prestige of the underwriter. When the selection effect is large, the correlation is positive in our model. When the selection effect is small, the correlation is negative.

**Proposition 2.** (Prestige vs. Under-pricing) Under a no-sorting benchmark, underpricing is always negatively correlated with underwriter prestige. Under a competitive sorting benchmark, underpricing can either be positively or negatively correlated with underwriter prestige. It is positively correlated with prestige when the selection effect dominates.

4.1. Decomposition

In either event, accurately assessing worker contribution to the surplus is challenging because of selection effects. To separate selection from talent effects, we make use of the wage
distribution. Let ω[i] denote the wage for the ith percentile agent. According to Equation (7), the marginal wage increase for the ith percentile agent is given by his contribution to the surplus within the match, given his optimal assignment with the ith firm. The wage profile as a function of the ith percentile agent can then be rewritten as

\[ \omega'[i] = V_b(a[i], b[i])b'[i] = a[i]b'[i]. \]  

(11)

From Equation (8) and (11), we thus have

\[ \frac{\omega'[i]}{V'[i]} = \frac{b'[i]}{b[i]}, \]

which measures talent heterogeneity. Furthermore, according to Equation (9), the change in firm value depends on both firm and talent heterogeneity.

**Proposition 3.** Hence, the proportion driven by talent’s heterogeneity can then be measured by:

\[ \gamma[i] \equiv \frac{\omega'[i]}{V'[i]} = \frac{a[i]h'[i]}{a'[i]b[i] + a[i]b'[i]} = \frac{\frac{b'[i]}{b[i]}}{\frac{a'[i]}{a[i]} + \frac{b'[i]}{b[i]}}. \]  

(12)

That is, a higher γ[i] means that underlying talent’s heterogeneity matters more for the change for the value of ith percentile firm and thus a weaker selection effect.

It is this quantity that we will estimate in the data. But before we turn to implementing this decomposition, it is worth highlighting what drives this quantity.

### 4.2. Determinants of the proportion of talent to selection effects

Intuitively, the strength of the selection vs. talent effects depends on the underlying heterogeneity of firms and talents. We now formalize how the underlying distribution changes the observable wages, output, and the strengthen of the selection effect, captured by γ[i].

**Proposition 4.** When all firms are scaled by some constant λ > 1, that is, \( \tilde{a}[i] = \lambda a[i] \), both
wages and outputs also changes by the same multiple: \( \tilde{\omega}[i] = \lambda \omega[i] \) and \( \tilde{V}[i] = \lambda V[i] \), and the strength of the selection effect, captured by \( \gamma[i] \), remains the same.

In the underpricing setting, for example, this means that when all firms become uniformly more opaque, underpricing is scaled up by the same constant. Intuitively, the demand for prestige increases when all firms become more opaque. However, since the increase is uniform across firms, it will not change the matching pattern. That is, all firms match with exactly the same underwriter, but the matching surplus within each pair is simply scaled up by the same constant \( \lambda \).

This also implies that such a uniform change will not affect the strength of the selection effect. Thus, the correlation between prestige and under-pricing (i.e., \( \frac{\partial \text{UP}'[i]}{\partial \text{UP}[i]} \)) must remain the same. In other words, the scaling effect cannot explain why the sign of the underpricing-prestige correlation changed signs from negative pre-Internet to positive during and after the Internet period.

Thus, the change of the signs must be driven by the change of underlying distribution of talents or firms, as it affects the strength of the selection effect. In particular, the strength of the selection effect depends on \( a'[i] \). Intuitively, a steep \( a'[i] \) means that an \( i \)-th quantile firm has a higher \( a \) index relative to the competitor right below it. Similarly, the strength of the direct effect depends on \( b'[i] \). As a result, which effect dominates crucially depends on the ratio of these two, which can be mapped onto the ratio of the density function:

\[
\frac{a'[i]}{b'[i]} = \frac{dG^A(b[i])}{dG^F(a[i])}.
\]

A higher (lower) ratio thus represents that firms are relatively dispersed (homogeneous) relative to the talent distribution, resulting in a stronger selection effect. For the sake of illustration, consider a simple case where both firms and agents’ effective types (summarized by \( a \) and \( b \), respectively) follow a uniform distribution. This term is then a constant ratio of the density function (i.e., \( \frac{a'[i]}{b'[i]} = \frac{(a^U - a_L)}{(b^U - b_L)} \)). Hence, this shows that the selection effect is
stronger whenever firms are dispersed relative to the talent. The same intuition holds for
more general distributions: for any given distribution of talent, the selection effect is stronger
when firms are more heterogeneous in the sense that there is a smaller mass for a given $a$.
The proposition below formalizes this effect.

**Proposition 5.** Consider two distributions, where the heterogeneity of firms is higher under
$\tilde{a}[i]$ in the sense that $\tilde{a}'[i] \geq a'[i]$ for $i$ and $\tilde{a}[1] = a[1]$. For any $a[i] \geq 0$ and $b[i] \geq 0$, the
selection effect is stronger under $\tilde{a}[i]$. And thus, a lower $\gamma[i]$. On the other hand, an increase
in the heterogeneity of agents leads to a weaker selection effect and thus a higher $\gamma[i]$.

5. **Estimates**

5.1. **Time Variation in IPO Underpricing-Prestige Relationship**

We now implement our decomposition to understand time variation in the IPO underpricing-
prestige relationship. The stylized facts are well summarized in various review papers (see,
e.g. Ritter and Welch (2002), Loughran and Ritter (2002)). First, underpricing before the
Internet era of the late nineties averaged a few percent and firms that hired prestigious un-
derwriters had lower underpricing. Second, the underpricing became much larger after the
late nineties, averaging nearly 20% and coverage by a prestigious underwriter is associated
more underpricing. Explanations have typically centered on structural changes in firm ob-
jective functions, such as firms underpricing to benefit friends or family in the late nineties,
and changes in underwriter strategies, such as underpricing to pay buy-side clients.

We use our model to examine the extent to which selection effects induced by competitive
sorting in the labor market for underwriters can account for these stylized facts. Given that
the Internet period produced a sizeable Internet (new technology) sector that had little
cashflows before going IPO, a plausible hypothesis is that the $q$ distribution most likely
has a higher mean, i.e. the average firm becomes more opaque, and also more dispersed
or heterogeneous. Indeed, as shown in Ritter and Welch (2002), the composition of firms
that go IPO changed during the Internet period. In particular, during this period, firms are much younger and many of them have negative earnings, which thus leads to a higher level of heterogeneity in opacity. While opacity is itself difficult to measure, we can evaluate the importance of each of these changes to the firm distributions using our decomposition, i.e. to estimate the $\gamma$.

We use the IPO underpricing data set kindly provided by Jay Ritter from 1990 through 2015. There are several years during that period when there are a very small number of IPOs, so we only include during that period years when there are at least 100 IPOs. This gives us 18 years of IPOs. For each IPO in our sample, we can calculate the underpricing for each IPO offering following the literature as:

$$\text{Underpricing} = (\text{ClosingPrice of Offering} - \text{OfferPrice}) \times \text{NumberOfShares}$$

To implement our decomposition, we ideally need the underpricing data and the wages of the banks that the firm hired. While such wages are not readily available, we propose the following work-around. The dataset includes IPO underpricing and the Carter-Manaster ranking of investment bank prestige. Carter-Manaster rankings are coarse measures of bank prestige and are correlated with higher wages (??). We then collapse the data by Carter-Manaster Ranking and year. That is, for each year/Carter-Manaster bucket, we calculate the median $\text{Underpricing}$ of the IPOs in that bucket. For Carter-Manaster Rankings, we lump together all the rankings based on the whole digit of the ranking, i.e. rankings of 7 and 7.001 are lumped together.

With this data set with unit of observation Carter-Manaster Ranking year, we can then calculate the change in the $\text{Underpricing}$ by ranking. That is, we sort the data within year by Carter-Manaster ranking. Then $V'$ is:

$$V' = \text{MediumUnderpricing}(n) - \text{MedianUnderpricing}(n + 1)$$
where \( n \) corresponds to the Carter-Manaster rank of the observation within the year (lower \( n \) corresponds to higher Carter-Manaster rankings). We then take the median value of the \( V' \) calculated above for each year.

To get the wages of employees of these banks, we recognize that an important component of underwriting services is security analysts who issue research on companies going IPO. We can look to the wage distribution of such analysts to gauge the extent to which selection effects are important. We can gather financial analysts wages data for each year \( t \) using the American Community Survey produced by the Census Bureau. We get information on the income of analysts from the March supplements of the Current Population Survey. People sampled in that supplement are asked to report their yearly income the previous calendar year. We gather together the CPS data sets from 1991 to 2016. This gives us income information for the years 1990 through 2015. The income variable we use is total personal wage and salary income. We only look people with non-zero income.

We limit our sample in three ways. First, we look at people classified as being in the occupation "Securities and financial services sales occupations" using the 1990 Census occupation codes. Next, we look at only at men. Finally, we only look at observations that have income above the median for the year. We only think the analysts that deal in any way with IPOs would be at the upper half of the income distribution, suggesting that we should not look below the median.

With this sample, we collapse the data into quartiles each year. That is, we take the median total income of analysts by quartile each year. This gives us a data set with four observations each year. We can then use this data set to calculate the \( \omega' \). That is, sorting on income quartile we calculate:

\[
\omega' = \text{MedianIncome}(n) - \text{MedianIncome}(n + 1)
\]

where \( n \) corresponds to the income quartile. (Lower \( n \) is higher quartile.) Then we take the
median of \( \omega' \) by year.

We can put together the year by year calculations we make of \( \omega' \) and \( V' \) from above into one data set. We take the ratio of the two to create our estimate of \( \gamma \) for each year. The graph of this ratio is presented below in Figure 2. Notice that \( \gamma \) is is highest in the early nineties, with a \( \gamma \) equal to 0.04 in 1992. But there is a decline from the early 1990s to the mid 2000's. During the peak of the Internet period of 1998, 2000, \( \gamma \) is low at 0.005 when underpricing is highest and the correlation between underpricing and prestige becomes positive. That is, the flip in the sign of the IPO underpricing-prestige is associated with much lower talent effects or equivalently, much large selection effects. There are a couple of years where talent effects regain some importance, such as 1997, right before the Internet boom and 2004, right after the Internet boom. But our \( \gamma \) estimates are consistent overall with a significant shift in the importance of selection effects to talent effects.

5.2. Time Variation in CEO Wages

There is debate on whether selection effects due to firm productivity and CEO talent can explain the rise of CEO wages since the 1980s in the CEO literature. The main test in the literature thus far is to show that the increase in the average size of the firm in the stock market can explain the rise in CEO wages (Gabaix and Landier (2008)). But Frydman and Saks (2010)) find that in the pre-1980s period, wages of CEOs did not rise eventhough the average size of the firm in the stock market rose. They point out concerns regarding coincident trends. Alternative explanations such as managerial talent might also be part of the story (see Frydman and Jenter (2010) for a review). Our model offers an alternative test. That is, we can calculate our decomposition using using the firm size and CEO wage distributions.

The data set with CEO compensation runs from 1993 to 2014. In 1993 we have about 1140 observations. In 2014, we have about 1719 observations. The largest observation count is in 2007 with 2062. I drop observations from this data set if observations have missing
compensation information, market equity value, total assets and book value of equity.\footnote{Like Gabaix and Landier (2008), if the observation has a missing value for deferred taxes, we set the value to zero.} We lose about 2194 observations over the entire sample period because of these cuts.

We classify firms by their size. We use the Gabaix and Landier (2008) definition of firm size: market equity plus the book value of debt. For each year, we classify the firms with the 1000 highest values of and drop the other firm observations. That is, we start with a data set of the 1000 biggest firms each year from 1993 to 2014.

Then, for each year, we sort the remaining 1000 observations by smoothed net income. That is, for each firm/year observation, we average their firm’s net income for the five previous years including the year of the observation.\footnote{If the firm does not have 5 years of net income information, we use four years. If it does not have four years, then we use three years.} That is, we construct:

\[
\text{SmoothedNetIncome}_{i,t} = \frac{1}{5} \sum_{j=t-4}^{t} \text{NetIncome}_{i,j}
\]

Define \( V_t(n) \) as the smoothed net income of the firm with the \( n \)-th highest smoothed net income in year \( t \). We sort from highest to lowest compensation; that is, \( n = 1 \) designates the firm with the highest net income that year. Smoothed net income needs to be positive; so we drop observations where net income is either missing or non-positive.

So we can define \( V'/V \) as:

\[
\frac{V'}{V}_t(n) = \frac{V_t(n) = V_t(n + 1)}{.5 \times [V_t(n) + V_t(n + 1)]}
\]

We next create our measure of \( \frac{V'}{V} \). Define \( \omega_t(n) \) as the as the total compensation in dollars of the CEO with the \( n \)-the highest total compensation in year \( t \). Again this is the same sort on smoothed net income as before. \( \gamma \) is then:

\[
\gamma = \frac{\omega_t(n) - \omega_t(n + 1)}{.5 \times [V_t(n) + V_t(n + 1)]}
\]
We have built a data set that has information on $\gamma$ for a sample of firms each year between 1993 and 2014. We then take the median value of these two variables each year and then plot them in Figure 3. We find that the relative strength of talent effects are increasing for the median firm in our sample between 1993 and 2002. The peak is in 2002. It then declines from 2002 to the current period. Our decomposition suggests that part of the rise in CEO wages is due to time variation in talent effects but clearly selection effects play a large role as CEO wages have kept rising over the entire sample period even as talent effects have decreased from their peak in 2002. Estimates for the 75th percentile and the 25th percentile are also shown each year. We can see that the firms with the highest talent proportions exhibit a similar time series pattern as the median firm. The 25th percentile firms have zero or negative estimates, which we attribute to noise and set to zero as well given that our model imposes zero as a lowerbound for $\gamma$.

6. Non-Multiplicative Surplus Function and Multidimensional Sorting

The theoretical literature on CEO wages has mostly focused on the sorting between skill and firm size. It predicts a positive correlation between firm valuation or size and CEO talent (as in Gabaix and Landier (2008)). Such a force is captured by $\hat{\theta}(h) = h\bar{\theta}$ in our model, where a better agent increases a firm’s long-run fundamental. Our model highlights a neglected aspect of CEO talent, which is their ability to reduce the cost of capital. In this case, fixing firm size, a more talented CEO may work for a more volatile firm with a lower valuation.

Thus, based on the same intuition behind Proposition 2, the selection effect under the cost of capital channel generates a negative correlation between firm valuation and CEO talent. To see this clearly, the risk premium for firm $y$ who hires an agent with ability $\mu(y)$
is given by:

\[
R(y, \mu(y)) = \frac{\gamma I}{2} \left( \frac{k^2}{(1 + \psi)^2} \right) \left( \frac{1}{\sigma_x^2 + \sigma_u^2 + \sigma_y^2} + \mu(y) \right),
\]

where, similar as before, \(q(y)\) represents the opacity of the firm. Thus, firms with higher volatility \(\sigma^2\) or more market noise, captured by \(\sigma_x^2\) and \(\sigma_u^2\) will have a higher opacity index \(q(y)\). Conditional on scale \(\kappa(y)\), more opaque firms will hire a more talented agent, who gets a higher wage. This thus immediately shows that the correlation between wage and firm value can go either way.

Formally, taking into account both channels, the solution of these two dimensional sorting for the CEO labor market can be characterized by \(\mu(\kappa, q)\), where \(\kappa(y)\) represents the effective scale for firm \(y\) and \(q(y)\) represents firm’s opacity. Conditional on \(k\), let \(q[i|k]\) denote the opacity for \(i\)th quantile firm within the subsample with scale \(\kappa\). We then rank all agents who work for firms with scale \(k\) by their wages (denoted by \(\omega[i|k]\)), which maps to their ability (denoted by \(h[i|k]\)). The performance of the \(i\)th quantile agent is measured as the risk premium of his firm \(RP[i|k] = \left( \frac{k^2}{2 (1 + \psi)} \right) - \frac{1}{q[i|k] + h[i|k]} \). Thus, the correlation between the agent’s ability and his performance yields

\[
RP'[i|k] = \left( \frac{k^2}{2 (1 + \psi)} \right) \frac{q'[i|k] - h'[i|k]}{(-q[i|k] + h[i|k])^2}.
\]

That is, the selection effect leads to a higher risk premium or lower firm valuation to be associated with agent talent. Without taking into account the selection effect on this dimension, one may mistakenly conclude that these hiring have little impact (or even a negative impact) on the firm value. This negative correlation can have an offset the positive correlation predicted under the traditional theory. Such effect thus potentially rationalizes empirical findings which show only a modest correlation in the cross-section between firm
size and CEO pay, to the extent pay is naturally a proxy for CEO talent (Frydman and Saks (2010)).

Furthermore, observe from Equation (3), conditional on firms’ opacity $q(y)$, risk premium is then again separable in talents $h$ and firms’ scale $\kappa(y)$. Thus, one can apply the logic of our decomposition method developed in Section 4 but conditional on firms’ informational environment. The only difference in this case, however, is that the underlying distribution of talents now describes the subset of talents that work for firms for a given opacity. But one does need to take a stand that one can have a proxy for opacity in contrast to the multiplicative setting where there is no such need to take a stand on underlying firm characteristics.

7. Conclusion

Firms hire in labor markets, be it underwriters when going public or executives on an ongoing basis, to improve firm value. But assessing the value of these hires for stock prices is challenging because of selection since more opaque firms with higher costs of capital and lower valuations to begin with might hire better agents. By developing an assignment model, where matching surpluses, firm valuation, and wages emerge from a stock-market equilibrium, we derive a decomposition of selection from the direct effect of agent talent. We use this decomposition to understand changes over time in IPO underpricing and the surge in CEO wages since the 1980s.
References


Loughran, T. and J. R. Ritter (2002). Why has IPO underpricing changed over time?


A. Appendix

A.1. Detailed Derivation for Underpricing

For any given fraction of uninformed investors, the price at which shares are sold to investors must be such that uninformed investors expect to break even on average. Thus $\tilde{p}_{hy}$ must solve

$$0 = \frac{1}{2} \beta(h) \left( \frac{k(\bar{\theta} + \sigma) - \omega(h)}{1 + \psi} - \tilde{p}_{hy} \right) + \frac{1}{2} \left( \frac{k(\bar{\theta} - \sigma) - \omega(h)}{1 + \psi} - \tilde{p}_{hy} \right).$$

To interpret this break-even condition, if $\beta(h) = 1$, and all investors are uninformed, there is no winner’s curse since the uninformed investor will with 50% chance get the asset when the valuation is low and 50% chance get the asset when the valuation is high. But if $\beta(h) < 1$ and hence some fraction of the investors are informed, the informed investors will only buy when the valuation is good. That is, from the viewpoint of an uninformed investors, when the project has a high valuation, the probability that an order is filled is $\beta(h) < 1$. Hence, the share price for firm $y$ that hires an underwriter with ability $h$ is then given by

$$\tilde{p}_{hy} = \frac{k \left( \beta(h)(\bar{\theta} + \sigma) + (\bar{\theta} - \sigma) \right)}{(1 + \beta(h))(1 + \psi)} - \frac{\omega(h)}{1 + \psi}.$$

This then gives us the expression in 2.

A.2. Detailed Derivation for CEO REE Setting

We now provide detailed derivation for the market clearing price. When firm $y$ hires agent $h$, investors thus obtain a public signal with precision $h = \frac{1}{\sigma_h^2}$. Aggregating the demand decisions of all investors in market $(h, y)$, market clearing then implies

$$\int D(x_i, z, p|h, y)dF(x_i|\theta) + \Phi(\tilde{u}) = 1. \quad (14)$$
From the investor’s optimization problem,

\[ D(x_i, z, p|h, y) \in \arg \max_{d \in \{0, 1\}} \left\{ dE\left[ \frac{k\theta - \omega(h)}{1 + \psi} - p|x_i, z, p\right] - \frac{\gamma_d}{2}d^2\text{Var}\left(\frac{k\theta - \omega(h)}{1 + \psi} - p|x_i, z, p\right) \right\} = \arg \max_{d \in \{0, 1\}} \left\{ dE\left[ \frac{k\theta - \omega(h)}{1 + \psi} - p|x_i, z, p\right] - \frac{\gamma_d}{2} d^2(1 + \psi)^2\text{Var}(\theta|x_i, z, p) \right\}, \]

Since \( \text{Var}(\theta|x_i, z, p) \) is constant over the realization of \((x_i, z, p)\), the demand \( D(x_i, z, p|h, y) \in \{0, 1\} \) can be characterized by a cutoff \( \hat{x}(z, p) \), such that \( D(x_i, z, p|h, y) = 1 \) if and only if \( x_i > \hat{x}(z, p) \).

Recall that each investor receives a private signal \( x_i = \theta + \sigma_x \epsilon_i \), where \( \epsilon_i \sim N(0, 1) \). This cut-off equilibrium then implies that only investors with good signals will buy, i.e. those investors with \( \epsilon_i > \frac{\hat{x}(z, p) - \theta}{\sigma_x} \). With our specifications, the market-clearing condition can then be conveniently rewritten as

\[ 1 - \Phi\left( \frac{\hat{x}(z, p) - \theta}{\sigma_x} \right) + \Phi(u) = 1. \] (15)

For the market to clear,

\[ \hat{x} = \theta + \sigma_x u. \]

Hence, observing price in our model is informationally equivalent to a public signal (i.e. this cut-off value \( \hat{x} \)) with the precision \( \frac{1}{\sigma_x^2 \sigma_u^2} \). 12 An investor’s information set can be summarized by \( I_i = (x_i, z, \hat{x}) \). Thus, the conditional expectation of the fundamental is then given by

\[ \mathbb{E}[\theta|I_i] = \frac{\sigma_{\theta}^{-2}\theta + \sigma_x^{-2}x_i + (\sigma_x \sigma_u)^{-2}\hat{x} + hz}{\sigma_\theta^{-2} + \sigma_x^{-2} + (\sigma_x \sigma_u)^{-2} + h}. \] (16)

For the cut-off investor \( \hat{x} \), the price must be equalized to the payoff of holding one share.

12In general, as shown in Albagli et al. (2011), there exists a random variable that is only a function of \( \theta \) and \( \tilde{u} \), and contains the same information as the price.
Hence,

\[ P(\theta, z, u|h, y) = \mathbb{E} \left[ \left( \frac{k\theta - \omega(h)}{1 + \psi} \right) | x_i = \hat{x}, z, \hat{\theta} \right] - \frac{\gamma l}{2} \frac{k^2}{(1 + \psi)^2} \text{Var}(\theta|z, p) \]

(17)

\[ = - \frac{\omega(h)}{1 + \psi} + \frac{k}{1 + \psi} \mathbb{E}[\theta|x_i = \hat{x}, z, \hat{\theta}] - \frac{\gamma l}{2} \frac{k^2}{(1 + \psi)^2} \text{Var}(\theta|z, p), \]

The expression for the risk premium yields:

\[ \text{Var}(\theta|z, p) = \sigma^2_{\theta} - \frac{\sigma^2_{\theta}}{1 + \psi} \left[ \begin{array}{c} \sigma^2_{\theta} \\ \sigma^2_{\theta} \end{array} \right] \left[ \begin{array}{c} \sigma^2_{\theta} \\ \sigma^2_{\theta} \end{array} \right] \Sigma_{dd}^{-1} \left[ \begin{array}{c} \sigma^2_{\theta} \\ \sigma^2_{\theta} \end{array} \right] = \frac{1}{(\tau_m + h + \frac{1}{\sigma^2_{\theta}})} \]

where \( \Sigma_{dd} \equiv \left[ \begin{array}{ccc} \sigma^2_{\theta} + \sigma^2_{\theta} & \sigma^2_{\theta} & \frac{k (h+\tau_m)\sigma^2_{\theta}}{1 + \psi (h+\tau_m+h)} \\ \frac{k (h+\tau_m)\sigma^2_{\theta}}{1 + \psi (h+\tau_m+h)} & \sigma^2_{\theta} + \sigma^2_{\theta} & \frac{k (h+\tau_m)\sigma^2_{\theta} + h\sigma^2_{\theta}}{1 + \psi (h+\tau_m+h)} \\ \frac{k (h+\tau_m)\sigma^2_{\theta}}{1 + \psi (h+\tau_m+h)} & \frac{k (h+\tau_m)\sigma^2_{\theta} + h\sigma^2_{\theta}}{1 + \psi (h+\tau_m+h)} & \frac{k (h+\tau_m)\sigma^2_{\theta} + h\sigma^2_{\theta}}{(\tau_m + h + \frac{1}{\sigma^2_{\theta}})^2} \end{array} \right] \]

Hence, the expression for the risk premium is given by

\[ R(y, h) = \gamma l k^2 \frac{\text{Var}(\theta|x, z, p)}{2(1 + \psi)^2} = \frac{\gamma l^2 k^2}{2(1 + \psi)^2 \left( \frac{1}{\sigma^2_{\theta}} + \frac{1}{\sigma^2_{\theta}} + \frac{1}{\sigma^2_{\theta}} + h \right)} \]

A.3. Characterization for Multidimensional Sorting

As discussed in Chiappori et al. (2016), when type spaces are multidimensional, it is generally not possible to derive a closed-form solution for the assignment function. Nevertheless, one can see that the characteristics of firms can be further reduced to aggregated indices in our setting, thereby simplifying our characterization. Facing equilibrium fee \( \omega(h) \), \( \omega_{h}(h) \) represents the marginal cost of a particular precision from the viewpoint of firms. From the first-order condition, if firm \( y \) chooses to match with agent \( \mu(y) \) in equilibrium, then his marginal benefit of precision must equal the marginal cost. That is, \( \Omega_{h}(y, \mu(y)) = \omega_{h}(\mu(y)) \).

In other words, once we have figured out the value of \( \omega_{h}(h) \), one can then find the set of
firms matched to agent $h$. Note that when firms differ in multiple dimensions, two different types of firms may have the same marginal value of $h$. To facilitate the analysis, define the set of firms $y$ whose marginal benefit of $h$ is given by a value of $m$:

$$\Upsilon(h, m) \equiv \{y \in Y \mid \Omega_h(y, h) = m\}.$$  

That is, if the marginal cost of hiring agent $h$ is given by $\omega_h(h)$, then $\Upsilon(h, \omega_h(h))$ is the set of firms matched to agent $h$.

Clearly, $\omega_h(h)$ is an equilibrium object that depends on the underlying distribution. We now consider the following algorithm that allows us to construct an explicit solution for this multi-dimensional environment. The basic idea of the equilibrium construction is the following.

First, for each $h$, we will need to choose some level $m \in \mathbb{R}$ that satisfies the following condition:

$$\Upsilon(h, m) \equiv \nu^F (\{y \in Y \mid \Omega_h(y, h) \leq m\}) = G^A(h).$$  \hspace{1cm} (18)

By choosing $m$ properly for each agent $h$, the measure of firms whose marginal benefit of $h$ is lower than $m$ exactly coincides with the measure of agents below $h$. Intuitively, if $m$ were the price for ability $h$, all firms within (outside of) the set $y \in \Upsilon(h, m)$ find this type of agent to be too expensive (cheap).

Choosing $m$ for each $h$ is thus as if we are choosing the price for any given ability. Equation (18) requires that, in equilibrium, the price for any particular ability must be chosen in a way so that the measure of firms that find this type of agent to be too expensive exactly coincides the measure of agents below this agent.

As established in Chiappori et al. (2015), this algorithm works only in the environment where the constructed $\omega_h(h)$ in the above procedure satisfies the following nested condition:

$$\Upsilon(h, \omega_h(h)) \subset \Upsilon(h', \omega_h(h')) \forall h' > h.$$  \hspace{1cm} (19)
The construction of the fee schedule is such that if a firm finds that hiring agent $h$ is too expensive, then it must find a more better agent $h' > h$ to be too expensive as well.

Observe that Condition (19) together with Condition (18) guarantee that (1) the set of firms that found $h$ to be too expensive are always matched to firms below agent $h$ and (2) the market clears in the sense that the measure that firms that hire agents below $h$ coincides with the measure of agents below $h$. As a result, the optimality condition of firms and market-clearing condition are satisfied. The following Lemma summarizes the characterization.

**Lemma 1.** Let $\omega_h(h)$ be the value that solves $Y(h, \omega_h(h)) = G^A(h)$. Under nested matching (i.e., if condition (19) holds), the optimal assignment is characterized by $\mu^{-1}(h) = \Upsilon(h, \omega_h(h))$.

**Proof.** By construction, Equations (18) and (19) guarantee that the market clearing condition is satisfied: the measure of agents below $h$ is the same as the measure of firms that hire agents whose precision is lower than $h$. We now examine firms’ optimality condition.

Recall that $\Upsilon(h, \omega_h(h))$ is the set of firms that are matched to agent $h$,

$$ \Upsilon(h, \omega_h(h)) \equiv \{ y \in Y \mid \Omega_h(y, h) = \omega_h(h) \}. $$

Since $\Omega_h(y, h) = U_h(y, \mu(y))$, it thus shows that the FOC of firms is satisfied as

$$ U_h(y, \mu(y)) = \omega_h(\mu(y)). $$

We now show that $\mu(y)$ is indeed the maximum of $U(y, h)$. Condition (19) suggests that, for any $h' > \mu(y)$, $y \in Y(\mu(y), \omega_h(\mu(y))) \subset Y(h', \omega_h(h'))$. That is, the marginal cost of a hiring a better agent $h'$ is too high:

$$ U_h(y, h') < \omega_h(h) \forall h' > \mu(y). $$

That is, $U(y, h)$ decreases with $h$ for $h > \mu(y)$. Similarly, hiring an agent with lower precision
is too cheap:

\[ U_h(y, h') > \omega_h(h) \forall h' < \mu(y). \]

Thus, \( U(y, h) \) increases with \( h \) for \( h < \mu(y) \). Hence, the constructed \( \mu(y) \) solves the firm’s optimization problem.

\[ \square \]

A.4. Omitted Proofs

A.4.1. Proof for Proposition 1

Proof. From Equation (5) and (3), observe that \( U_h(k, \psi, \sigma, h) \) increases with \( k, \sigma, \) and \( \frac{1}{1+\psi} \). According to Milgrom and Segal (2002), \( \mu(k, \psi, \sigma) \), the solution to Equation (1), must increase with \( k, \sigma, \) and \( \frac{1}{1+\psi} \). Since \( U_h(y, h) = \Omega_h(y, h) \), this is equivalent to looking at the complementarity of the surplus function, as is standard in matching models.

\[ \square \]

A.4.2. Proof for Proposition 2

Proof. From Equation (10), under random matching \( q'[i] = 0 \), and thus \( UP'[i] \) < 0 for all distributions. Under competitive sorting, we have have \( q'[i] > 0 \). And thus \( UP'[i] > 0 \) if the selection effect dominates (i.e., \( \frac{q'[i]}{q[i]} \) is larger enough).

\[ \square \]

A.4.3. Proof for Proposition 4

Proof. Given that \( \tilde{a}[i] = \lambda a[i] \), \( \tilde{V}[i] = \tilde{a}[i]b[i] = \lambda V[i] \) and \( \tilde{\omega}[i] = \int \tilde{a}[j]b[j]dj = \lambda \omega[i] \). Furthermore, given that

\[ \frac{\tilde{a}'[i]}{\tilde{a}[i]} = \frac{\lambda a'[i]}{\lambda a[i]} = \frac{a'[i]}{a[i]}, \]

according to Equation (12), \( \tilde{\gamma}[i] = \gamma[i] \forall i \).

\[ \square \]

A.4.4. Proof for Proposition 5
Proof. From Equation (12), $\gamma[i]$ decreases in $\frac{a'[i]}{a[i]}$ and increases in $\frac{b'[i]}{b[i]}$. Hence, for a change in the distribution that implies a higher heterogeneity (i.e., a higher $a'[i]$) with the same maximum, it thus implies that $\tilde{a}[i] < a[i] \forall i < 1$. And hence, for any $a[i] \geq 0$, $\frac{\tilde{a}'[i]}{\tilde{a}[i]} > \frac{a'[i]}{a[i]}$, and thus a stronger selection effect (lower $\gamma[i]$). Similarly, for any $b[i] \geq 0$, an increase in $b'[i]$ leads to a higher $\frac{b'[i]}{b[i]}$ and thus higher $\gamma[i]$.

\qed
Figure 1: Figure 1 illustrates the set of firms that hire an agent with ability $h$, where $h_3 > h_2 > h_1$. Each line is given by $U_h(y, h) = \theta + \gamma \frac{k^2}{(1+y)^2} \frac{1}{(r+h)^2} = \omega(h)$. 
Figure 2: Proportion of Talent Effect ($\gamma$) for IPO Underpricing
Figure 3: Proportion of Talent Effect ($\gamma$) for CEOs