Endogenous Entry and Financial Contagion Across Debt Markets

Hyunsoo Doh*

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Abstract

I develop a short-term debt model with two sectors where the secondary asset markets are partially segmented. In this model, every potential buyer must pay a fixed cost to buy an asset in default in the other sector. I show that policies that reduce the entry cost may decrease welfare. Specifically, when the entry cost is lowered, the gap in the liquidation prices in the two markets shrinks to eliminate cross-market arbitrage opportunities. Then, every firm in a more liquid market that faces a price drop will default earlier to avoid increased rollover losses, triggering more asset liquidation. If this contagion effect dominates positive effects of expedited capital flows, welfare will be decreased.

Keywords: Segmented debt markets, endogenous entry, contagion, liquidity, welfare. (JEL: G01, G20, C72)

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1 Introduction

Market barriers that deter capital flows have been considered as one of the main culprits for a liquidity crisis such as the 2008 financial crisis and the European debt crisis in early 2010. When investors cannot move across the markets flexibly, shocks to a local market cannot be absorbed instantly even if investors in other markets have enough liquidity. As a result, those small regional shocks can actually disrupt the local market. Put differently, market barriers prevent efficient asset allocations.

When markets are well connected to each other, however, shocks to a local market will be spread to other markets, amplifying the initial local shocks. For instance, when assets are traded among a common pool of investors, who are either financially constrained or risk averse, any distresses in some assets can cause a price impact on other assets through the common investors with limited asset-holding capacity. If this transmission effect is so severe, the local shocks can collapse even the entire markets as the US mortgage market caused the turmoil in the global economy during 2008 and 2009; see, for instance, Longstaff (2010) and Gorton and Metrick (2012).

These two opposite effects of facilitating capital flows raise the following questions: Policies that reduce market barriers will increase welfare of the economy? When such policies are not readily implementable, especially because investors are facing informational barriers, injecting liquidity will mitigate a crisis? This paper aims to answer these question. Specifically, I show that, at least in short-term debt markets, such policies may lower welfare if the secondary markets for assets in default are partially segmented from each other.

Two main economic forces will generate this negative outcome. First, when market barriers between the secondary markets are reduced, more investors move across the markets, shrinking the gap in the liquidation prices in the two markets. In general, a relatively more liquid market faces a price drop and the other less liquid market faces a price improvement. Second, although this reduced price gap can bring about more efficient asset allocations, firms issuing short-term debts in the former market will default earlier because the lowered liquidation price increases rollover losses. As a result, more assets will be liquidated in that sector. Put simply, the supply curve in each secondary market is downward sloping in the liquidation price. If this negative contagion effect dominates the efficiency gains caused by more efficiently allocated assets, welfare will be decreased.

To demonstrate this idea, I develop a short-term debt model with two sectors by extending Leland (1994). The two sectors can be interpreted as either two countries or two separate debt markets within a single country, depending on the context. Each primary debt market consists of many individual firms issuing short-term debts and rolling over every retired short-term debt. The key assumption is that the secondary asset markets are partially segmented from each other. Specifically, every potential buyer in one sector is required to pay a fixed cost to buy an asset in default from the other sector. This entry cost can be flexibly interpreted as informational costs, regulatory fees, or asset transaction costs.

In this model, welfare or, more precisely, the total surplus of the economy is determined by two factors: the total outputs from the assets and the total entry cost incurred. Specifically, I assume that the market participants have different productivities. Every firm's incumbent manager has the highest productivity, but the potential buyers are less skillful in managing an asset as in Shleifer and Vishny (1992). Thus, whenever a firm is forced to liquidate its asset upon default, the total surplus will be decreased. But even among the potential buyers, some buyers have relatively high productivity than the other buyers. I will call the former (resp. latter) buyers the high-type (resp. low-type) buyers. As a result, if any asset in default is liquidated to a high-type buyer rather than to a low-type buyer, the efficiency loss will be reduced.

In this environment, how can policies that seek to reduce the entry cost hurt the total surplus? First, note that when the entry cost is reduced, some high-type buyers in a more liquid market will enter the other less liquid market to exploit the price gap. Then, the total surplus in the latter market will increase because those high-type buyers will purchase liquidated assets in that market in place of low-type buyers. However, the outflows of those high-type buyers will push down the liquidation price in the former market, although the liquidation price in the latter market will be pushed up. As a result, the firms issuing short-term debts in the former market will default earlier to avoid increased rollover losses. Hence, more assets will be sold off in that market, increasing efficiency losses. See the bottom-left panel in Figure 9 that provides numerical results supporting these arguments.

In fact, by the same reason, reducing the entry cost can increase the total surplus in other cases. That is, if the size of a price drop in the more liquid market is small, whereas the size of a price improvement in the less liquid market is large, then the total amount of an asset liquidated in the whole economy will be decreased. As a result, the total surplus will be increased.

In general, when the relatively more liquid market is not sufficiently liquid, the policies that reduce the entry cost tend to decrease the total surplus. The reason is that once some high-type buyers in that market move to the other market to exploit the price gap, the price in the former market will drop a lot. This is because the former market now does not have enough remaining high-type buyers to fully absorb the liquidated assets in that market.

On the other hand, if the relatively more liquid market is sufficiently liquid, the same policy will result in the positive outcome. This is because even after some high-type buyers leave that market, the market will still have enough remaining high-type buyers who can purchase all the assets liquidated in that market. As a result, the price in that market will remain unchanged, while the price in the other market will go up. Hence, as discussed above, the total surplus will increase as well.

Importantly, in this model, multiple equilibria may arise because the supply curve is downward sloping. Specifically, when the more liquid market is not sufficiently liquid and the less liquid market is just mildly illiquid, multiple equilibria is likely to emerge. In this case, the policies aimed at reducing the entry cost may generate opposite outcomes, depending on which equilibrium is selected. Suppose that, in one equilibrium, the market participants believe that the price levels will be high enough. Then, even after a more liquid market loses some high-type buyers, the price in that market will remain unchanged, while the price in the other market will be increased, as if the former market is sufficiently liquid. Also, the above beliefs can be actually justified in equilibrium because when the price in the latter market is high, that market does not need many additional high-type buyers to clear the market. As a result, the remaining high-type buyers in the former market can still fully absorb the liquidated assets in that market, justifying the high enough price level. If this scenario obtains, reducing the entry cost will increase the total surplus.

On the other hand, if the market participants believe that the price levels will be low, even after some high-type buyers in that market move to the less liquid market, the price in the latter market will not be improved that much. The worst scenario occurs if the price in that market remains the same even after some new high-type buyers enter that market. In this case, the price in the more liquid market will then drop further; otherwise, the cross-market arbitrage opportunity will be available once the entry cost is reduced. As a result, more assets will be liquidated in that market, decreasing the total surplus further.

Lastly, I also study how injecting more liquidity to the markets affects the economy. Interestingly, this policy can also hurt the total surplus if some special type of equilibrium is selected. That is, when a secondary market receives more liquidity, the liquidation price can actually decrease if the market participants believe that the prices will have intermediate levels. In this case, when the entry cost is reduced, the liquidation prices in both markets will be decreased. In fact, such lowered prices can be sustained in equilibrium because the markets have now more liquidity to absorb the increased amount of liquidated assets. Unless this special type of equilibrium is selected, liquidity injection generally increases the total surplus.

This paper contributes to the literature as follows. One branch of the literature has been studying financial contagion in a variety of markets such as stock markets, debt markets, and interbank markets. For instance, see Kyle and Xiong (2001), Gromb and Vayanos (2002), Dasgupta (2004), Goldstein and Pauzner (2004), Brunnermeier and Pedersen (2008), Oh (2013), Choi (2014), Liu (2016), Gromb and Vayanos (2017), and Bleck and Liu (2018). But most of these papers consider the contagion phenomenon within a single fully integrated market and focus on the contagion effects in that single market. As an exception, Goldstein and Pauzner (2004), Gromb and Vayanos (2017), and Bleck and Liu (2018) consider an economy in which markets are partially segmented. But, in those papers, investors diversify their portfolios according to an exogenously given diversification ratio. Unlike those papers, the present paper studies partially segmented markets in which investors make endogenous entry decisions by taking the prices in the two markets as given.

Another branch of the literature has been studying the implications of segmented markets or limited participations. For instance, see Basak and Cuoco (1998), Gabaix et al. (2007), Duffie (2010), Duffie and Strulovici (2012), He and Krishnamurthy (2013), and Brunnermeier and Sannikov (2014). But these papers mainly focus on asset price dynamics without considering welfare implications. The present paper particularly analyzes the welfare effects of market segmentation, especially in debt markets.

The present paper also contributes to the studies about interactions between the primary debt market and other markets such as secondary markets or interbank markets. He and Milbradt (2014) and Chen et al. (2017) study the feedback effects between the primary debt market and the secondary bond market by analyzing the search frictions in the latter market. Liu (2016) studies the interplay between the primary debt market and the interbank market by examining the role of the interest rate in the latter market. But these papers do not consider the market segmentation issue.

The paper is organized as follows. Section 2 develops a simple static model with two segmented asset markets where the supply curve for an asset in each market is exogenously given. In Section 3, I develop a full credit-risk model. Section 4 solves the model. Section 5 presents the model implications. Section 6 concludes. All technical proofs are included in Appendix.

2 Simple Static Model

In this section, I present a simple static model to illustrate the main idea of the paper. In this model, the supply curve of an asset is exogenously given. In the full-credit risk model, I will generate both supply and demand curves endogenously.

2.1 Setup

Time is discrete and runs over only two dates, $t = 0$ and 1. There are two markets, market A and market B. A certain asset is traded in each market. The assets in the two markets are identical, which I assume for simplicity. Two types of agents exist: original asset holders and potential asset buyers. All the agents are risk neutral and have a zero discount rate. I will use the US dollar as a numeraire for convenience.

Each one unit of the asset will produce 1 dollar at date 1 with certainty. Thus, the present value of the asset is equal to 1. Each market is populated by a continuum of original asset holders of measure 1. Each original asset holder possesses one unit of the asset at date 0. If the asset holder continues to hold the asset until date 1, she will earn 1 dollar for sure. However, some of the asset holders receive a liquidity shock at date 0. Those asset holders are forced to sell off their assets immediately. I will later show that the other asset holders, who are not hit by the shock, do not choose to sell their assets.

Specifically, let p^i denote the asset price in market i. Then, I assume that a fraction $q^i(p^i)$ of the original asset holders in market i are hit by the liquidity shock. Moreover, I also assume that $q^i(p^i)$ decreases in p^i. Put differently, the supply curve is downward sloping. To understand this assumption, imagine the asset as an collateralized asset for short-term debt. Then, note that equityholders issuing short-term debt generally default earlier when the liquidation price of the asset is lower, because the rollover losses get bigger in that case. That is, a lower liquidation price triggers more asset liquidation, producing a downward-sloping supply curve. I take the downward-sloping supply curve $q^i(p^i)$ as given in this section, but I will generate $q^i(p^i)$ endogenously when developing the full credit-risk model.

In each market, there are also potential asset buyers, who can buy an asset at date 0. But the markets are partially segmented from each other. Specifically, each potential buyer in market i has to pay a fixed cost κ to buy an asset from market $-i$, where $-i$ denotes the opposite index of i. I call κ the entry cost. Here, I can certainly assume that the entry cost faced by the potential buyers in market A is different from that faced by the potential buyers in market B. But incorporating this assumption does not provide any significant results at least in this model. So, let me assume that the entry costs are symmetric. Also, for clarification, the original asset holders in market i sell their assets only in market i. This assumption is reasonable enough in many cases, especially when sellers do not have any available tools to disclose or reveal the information about their assets credibly.

To generate non-trivial interactions between the markets, I make the following two assumptions. First, every potential buyer is financially constrained. Specifically, every buyer can purchase at most one unit of the asset. This simple form of financial constraint is widely used in the literature for simplicity; for instance, see Duffie et al. (2005) and He and Milbradt (2014). But then, due to risk neutrality, each potential buyer will buy only 0 or 1 unit of the asset.

Second, the potential buyers have different productivities. Specifically, the high-type buyers can produce α_h from the asset at date 1, while the low-type buyers can produce α_l from the asset, where $\alpha_l < \alpha_h < 1$. Therefore, the high-type (resp. low-type) buyers value one unit of the asset as α_h (resp. α_l) since their discount rate is 0. In this regard, the original asset holders are the best asset users in this model. Put differently, even if an original asset holder sells her asset to a high-type buyer, the economy faces some efficiency losses. Nonetheless, the efficiency losses are lower when the asset is liquidated to a high-type buyer instead of being sold to a low-type buyer. These two points will play an important role when discussing the effect of market integration on welfare. In addition, there are infinitely many low-type buyers in each market. But the number of high-type buyers in each market can be either finite or infinite. Let f^i denote the measure of the high-type buyers in market i, where $f^i \in [0, \infty]$.

In this setting, I can describe the individual potential buyer's problem as follows. First, a high-type buyer in market i solves the following profit-maximization problem:

$$\max\{0, \alpha_h - p^i, \alpha_h - p^{-i} - \kappa\}. \tag{1}$$

The first term means that the buyer earns no profits if she does not buy any asset. The second term indicates that the buyer earns $\alpha_h - p^i$ if she buys an asset from market i. The third term denotes that the buyer earns $\alpha_h - p^{-i} - \kappa$ if she buys an asset from market $-i$. Note that every high-type buyer has an incentive to buy one unit of the asset if there is an arbitrage opportunity in one of the markets; that is, either $p^i < \alpha_h$ or $p^{-i} - \kappa < \alpha_h$. In that case, the high-type buyer chooses to buy an asset from market $-i$ if the price gap between the markets is larger than the entry cost; that is, $p^i - p^{-i} \geq \kappa$.

Each low-type buyer in market i solves the same problem described in (1) but with α_h being replaced by α_l. Note that both p^A and p^B must lie between α_l and α_h; otherwise, the markets cannot clear. Thus, only when $p^i = \alpha_l$, the buyer is indifferent between buying an asset from market i and not buying any assets. The buyer has no incentive to enter the other market as long as $\kappa > 0$.

Moreover, since p^i lies between α_l and α_h as just mentioned, every original asset holder

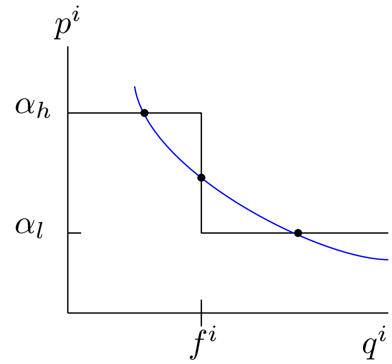

Figure 1: The black lines plot the demand curve and the blue curve plots the supply curve $q(p^i)$ in market i, assuming $\kappa = \infty$.

will keep her asset unless she is hit by the liquidity shock. So, $q^i(p^i)$ indeed becomes the supply curve in market i.

I can now define an equilibrium for this economy as the price pair (p_*^A, p_*^B) that jointly clears both markets. Of course, all the agents maximize their own profits in equilibrium. Now, let us study how this economy works. To this aim, I first consider a case where the markets are fully segmented; that is, $\kappa = \infty$. Next, I will consider a general case in which $\kappa < \infty$.

2.2 Fully Segmented Markets

When $\kappa = \infty$, I can analyze the model by focusing on any single market i because there are no interactions between the markets. To start, I plot one example for the demand and supply curves in market i in Figure 1. That is, the blue curve is the downward-sloping supply curve and the black curve denotes the demand curve. As in the figure, multiple equilibrium may arise in this economy. First, $p_*^i = \alpha_h$ can be an equilibrium if $q^i(\alpha_h) \leq f^i$. In this equilibrium, the marginal buyer is a high-type buyer, but every high-type buyer is indifferent between buying an asset and not buying an asset. Second, a price p^i such that $\alpha_l < p^i < \alpha_h$ can be an equilibrium if $q^i(p_*^i) = f^i$. In this equilibrium, the marginal buyer is still a high-type buyer, but every high-type buyer strictly prefers to buy an asset. Third, $p_*^i = \alpha_l$ can be an equilibrium if $q^i(\alpha_l) \geq f^i$. In this equilibrium, the marginal buyer is now a low-type buyer, but every low-type buyer is indifferent between buying an asset and not buying an asset.

Note that when the equilibrium price is lower than α_h, every high-type buyer earns a positive profit. This arbitrage opportunity is not eliminated because high-type buyers are financially constrained. In addition, as a special case, if the supply curve is perfectly inelastic, then infinitely many equilibria can also emerge.

Also, the above multiple equilibria are commonly called self-fulfilling equilibra. That is, if the market participants believe that the price low be high, then a less amount of the asset will be liquidated and thus, the market can clear even if the market does not have that many high-type buyers, justifying the higher price. But if the market participants believe that the price would be low, then more assets will be liquidated and thus, some low-type buyers must step in as well to absorb the assets, justifying the low price. Lastly, if the market participants believe that the price would be at an intermediate level, the quantity of an asset liquidated will be slightly increased, but the market has a just enough number of high-type buyers who can fully absorb those assets. Interestingly, when f^i is increased, that is, when liquidity is improved, the price will be depressed, if the third type of equilibrium is selected. This is because, again, the supply curve is downward sloping.

2.3 Partially Segmented Markets

In this section, I consider the case of $\kappa < \infty$. Without loss of generality, I can assume $\kappa < \alpha_h - \alpha_l$; otherwise, the price gap cannot be larger than the entry cost and thus, no potential buyers in one market will enter the other market. Also, I hereafter assume that the supply curves in the two markets are identical. That is, $q^A(p) = q^B(p)$ for every p. I will denote this common supply curve by $q(p)$. Although I can certainly relax this assumption, I can present all significant results under this simplified setup. In this regard, I can say the main interest of this paper lies in the demand side of the economy. Now, let us look at two examples to see the model's main implications quickly.

2.3.1 Example 1

Figure 2 describes the first example. I will use this example to show that reducing the entry cost can actually lower welfare. In the left panel, I still assume $\kappa = \infty$ and then plot an

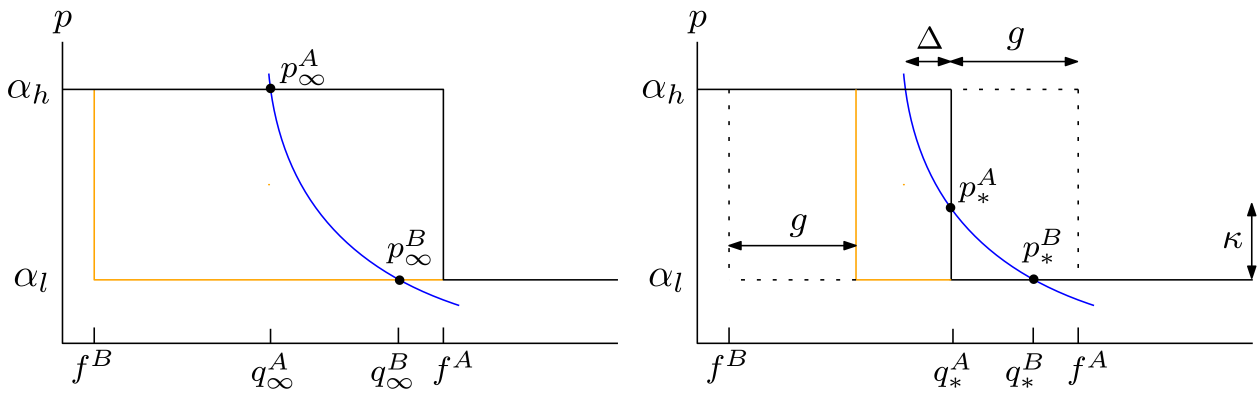

Figure 2: The left panel depicts an equilibrium in case of $\kappa = \infty$. The right panel plots an equilibrium in case of $\kappa < \alpha_h - \alpha_l$. In both panels, the black and orange lines denote the demand curves in market A and market B, respectively.

equilibrium price pair (p^A_∞, p^B_∞). In the right panel, I indeed assume $\kappa < \alpha_h - \alpha_l$ and then plot an equilibrium price pair (p^A_*, p^B_*). I will later compare these two economies when discussing welfare implications. The left panel shows that when $\kappa = \infty$, we have $(p^A_\infty, p^B_\infty) = (\alpha_h, \alpha_l)$. That is, as seen before, all liquidated assets in market A can be absorbed by the high-type buyers in that market, while the number of high-type buyers in market B is not enough to fully absorb the liquidated assets in that market.

However, when κ is smaller than $\alpha_h - \alpha_l$, the price pair (p^A_∞, p^B_∞) cannot be sustained as an equilibrium any more because if so, all high-type buyers in market A will move to market B to exploit the price gap that is larger than the entry cost. The right panel then shows how the new equilibrium looks like. First, some high-type buyers of measure g in market A move to market B. The outflow of these buyers shifts the demand curve in market A (resp. B) to the left (resp. right) by g. The reduced demand in market A pushes down the price in that market to $p^A_* < \alpha_h$. But the price in market B still remains at the lowest level $p^B_* = \alpha_l$, because $f^B + g$ is not big enough to fully absorb the liquidated assets in market B.

To justify (p^A_*, p^B_*) is indeed an equilibrium, note that (g, p^A_*, p^B_*) satisfies

$$\alpha_l < p^A_* < \alpha_h, \quad q(p^A_*) = f^A - g, \quad p^B_* = \alpha_l, \quad q(p^B_*) > f^B + g, \quad p^A_* - p^B_* = \kappa.$$

We can understand the first four conditions as follows. Market A behaves as if being fully isolated but with high-type buyers of measure $f^A - g$ instead of measure f^A. Similarly, market B behaves as if being fully isolated but with high-type buyers with measure $f^B + g$ instead of measure f^B. But, importantly, the last condition implies that every high-type buyer in market A is actually indifferent between buying an asset from market A and buying an asset from market B. Thus, the behaviors of those high-type buyers entering market B and the remaining high-type buyers in market A are equally optimal, justifying (p_*^A, p_*^B) is an equilibrium. In this section, let us put aside any issues about multiple equilibra and focus on a particular equilibrium provided in the examples. I will explain an equilibrium construction algorithm later when discussing the full model.

The key observation from this example is that the quantity of an asset liquidated in market A is increased by Δ, but that quantity in market B remains the same, as shown in the figure. This outcome obtains because market A is liquid but not sufficiently liquid, while market B is severely illiquid. As a result, if some high-type buyers move to market B, the price in market A drops, but the price in market B is not increased. The depressed price in market A then causes more asset liquidation because the supply curve is downward sloping. But then, welfare may go down since the original asset holders are the best asset users. I will discuss the welfare implications in more detail later.

2.3.2 Example 2

Let me discuss another example depicted in Figure 2. Later, I will use this example to illustrate a positive effect of reducing the entry cost on welfare. As before, the left panel plots an equilibrium (p_∞^A, p_∞^B) for the case of $\kappa = \infty$, while the right panel plots an equilibrium (p_*^A, p_*^B) in the case of $\kappa < \alpha_h - \alpha_l$. First, when $\kappa = \infty$, an equilibrium price pair is again given by $(p_\infty^A, p_\infty^B) = (\alpha_h, \alpha_l)$. Yet, when $\kappa < \alpha_h - \alpha_l$, some high-type buyers of measure g in market A move to market B as in the previous example. However, in this example, the price in market A remains at the highest level α_h, while the price in market B is improved to p_*^B. We can quickly justify that (p_*^A, p_*^B) is indeed an equilibrium by observing the following conditions:

$$p_*^A = \alpha_h, \quad q^A(p_*^A) < f^A - g, \quad \alpha_l < p_*^B < \alpha_h, \quad q^B(p_*^B) = f^B + g, \quad p_*^A - p_*^B = \kappa.$$

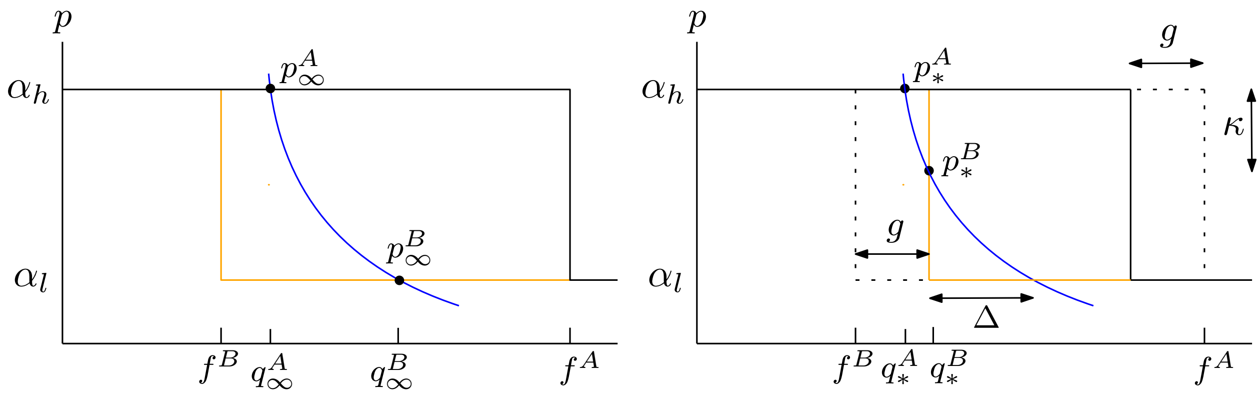

Figure 3: The left panel depicts an equilibrium in case of $\kappa = \infty$. The right panel plots an equilibrium in case of $\kappa < \alpha_h - \alpha_l$. In both panels, the black and orange lines denote the demand curves in market A and market B, respectively.

In short, these conditions imply that all potential buyers behave optimally and both markets clear.

Importantly, in this example, the quantity of an asset supplied in market B is decreased by Δ, while that quantity in market A remains unchanged. This outcome occurs because market A is now sufficiently liquid, while market B is just mildly illiquid. Thus, even after some high-type buyers leave market A, the remaining high-type buyers can still fully absorb the assets liquidated in market A. Moreover, those high-type buyers who move to market B increase the price in that market, reducing the amount of an asset liquidated in that market. Therefore, welfare can be improved by the same reason mentioned above.

2.4 Welfare

In this section, I study how welfare changes when κ decreases from ∞ to some finite number less than $\alpha_h - \alpha_l$, using the previous two examples. I will change κ continuously to conduct more delicate comparative statics analysis when considering the full model.

As usual, I define welfare as the total surplus of the economy in this risk-neutral world. Specifically, let (p^A_*, p^B_*) denote any equilibrium price pair. Then, $q(p^i_*)$ units of the asset in market i are sold to some potential buyers, while $1 - q(p^i_*)$ units of the asset are still held by the original asset holders. Among these $q(p^i_*)$ units of the asset, suppose that q^i_h units are purchased by high-type buyers, some of whom might have come from market $-i$. Then, the remaining $q_l^i := q(p_*^i) - q_h^i$ units are purchased by low-type buyers. Lastly, let g denote the number of potential buyers who leave their own markets. Of course, both q_h^i and g are endogenously determined in equilibrium. Then, the total surplus is equal to

$$\mathrm{TS} = -\kappa g + \sum_{i \in \{A,B\}} (1 - q(p_*^i) + \alpha_h q_h^i + \alpha_l q_l^i). \tag{2}$$

The first term $-\kappa g$ is the total entry cost incurred. The second term $1 - q(p_*^i)$ denotes the total outputs produced by the original asset holders in market i. The third term $\alpha_h q_h^i$ indicates the total outputs in market i produced by high-type buyers. The last term $\alpha_l q_l^i$ denotes the total outputs in market i produced by low-type buyers.

Equivalently, we can discuss welfare effects in terms of efficiency losses instead of the total surplus. Specifically, note that if all potential buyers have the full productivity 1, then the total surplus will be equal to 2 regardless of how many assets are liquidated. Thus, the total efficiency loss is given by

$$2 - \mathrm{TS} = \kappa g + \sum_{i \in \{A,B\}} [(1 - \alpha_h) q_h^i + (1 - \alpha_l) q_l^i]. \tag{3}$$

In particular, the second term $(1 - \alpha_h) q_h^i$ directly corresponds to the efficiency loss caused by asset liquidation to high-type buyers. The third term $(1 - \alpha_l) q_l^i$ denotes the efficiency loss caused by asset liquidation to low-type buyers.

2.4.1 Example 1

Now, I analyze the effect of a change in κ on the total surplus using the example in Figure 2. First, consider the benchmark case in which $\kappa = \infty$. In this case, the left panel in the figure shows that all $q_\infty^A := q(p_\infty^A)$ units of the asset liquidated in market A are fully absorbed by high-type buyers. In market B, f^B units of the asset are sold to high-type buyers, while the remaining $q_\infty^B - f^B$ units of the asset are sold to low-type buyers, where $q_\infty^B := q^B(p_\infty^B)$. Lastly, no potential buyers pay the entry cost when $\kappa = \infty$. Thus, the total efficiency loss is equal to

$$\mathrm{EL}_\infty = (1 - \alpha_h) q_\infty^A + (1 - \alpha_h) f^B + (1 - \alpha_l)(q_\infty^B - f^B).$$

Next, consider the other case in which $\kappa < \alpha_h - \alpha_l$. In this case, as in the right panel, although the quantity of an asset liquidated in market A is increased by Δ, all $q_\infty^A + \Delta$ units of the asset liquidated are still fully absorbed by high-type buyers. This is because the price in market A is still at least larger than the lowest possible level α_l. In market B, due to the inflows of high-type buyers of measure g from market A, $f^B + g$ units of the assets are sold to high-type buyers, while $q_\infty^B - f^B - g$ units of the asset are sold to low-type buyers. Thus, the total efficiency loss is equal to

$$\mathrm{EL} = \kappa g + (1-\alpha_h)(q_\infty^A + \Delta) + (1-\alpha_h)(f^B + g) + (1-\alpha_l)(q_\infty^B - f^B - g),$$

where the first term κg counts the total entry cost.

Hence, the net change in the total efficiency loss is given by

$$\mathrm{EL} - \mathrm{EL}_\infty = (1-\alpha_h)\Delta - (\alpha_h - \alpha_l)g + \kappa g. \tag{4}$$

This short expression is informative. The first term $(1-\alpha_h)\Delta$ denotes the efficiency loss caused by the increased asset liquidation in market A. The second term $-(\alpha_h - \alpha_l)g$ means that due to those entering high-type buyers of measure g, g units of the asset in market B are now liquidated to high-type buyers instead of being sold to low-type buyers. As a result, the efficiency loss is reduced by $(\alpha_h - \alpha_l)g$. The third term is the total entry cost incurred.

Now, note that $-(\alpha_h - \alpha_l)g + \kappa g$ is always negative because $\kappa < \alpha_h - \alpha_l$. Put differently, the efficiency gain derived from the entering high-type buyers always dominates the total entry cost incurred to those buyers. As a result, in case where the supply curve is perfectly inelastic; that is, $\Delta = 0$, the total surplus always increases if the entry cost is reduced. However, if the supply curve is downward sloping, the first term $(1-\alpha_h)\Delta$ may dominate the other terms that measure the net efficiency gain $(\alpha_h - \alpha_l - \kappa)g$. If this outcome occurs, the total surplus will be decreased. The main goal of developing a credit-risk model in a general equilibrium framework is to show that this negative outcome can indeed occur under reasonable market circumstances.

2.4.2 Example 2

Now, let us consider the second example depicted in Figure 3. In the benchmark case where $\kappa = \infty$, the total efficiency loss is again equal to

$$\text{EL}_\infty = (1 - \alpha_h) q_\infty^A + (1 - \alpha_h) f^B + (1 - \alpha_l)(q_\infty^B - f^B).$$

That is, in market A, the entire q_∞^A units of the assets are sold to only high-type buyers. In market B, f^B units of the asset are liquidated to high-type buyers and $q_\infty^B - f^B$ units of the assets are sold to low-type buyers.

In the other case in which $\kappa < \alpha_h - \alpha_l$, some high-type buyers of measure g in market A move to market B. Nonetheless, the outflow of those buyers does not change either the price or the quantity of an asset liquidated in market A because the remaining high-type buyers can still absorb all the liquidated assets in market A. Meanwhile, in market B, due to the inflow of those high-type buyers, the price is increased to p_*^B, reducing the quantity of an asset liquidated to $q_*^B = f^B + g = q_\infty^B - \Delta$. Even better, all these q_*^B units of the asset are now liquidated to high-type buyers. Thus, the total efficiency loss in this economy is equal to

$$\text{EL} = \kappa g + (1 - \alpha_h) q_\infty^A + (1 - \alpha_h)(f^B + g).$$

Hence, the net change in the efficiency loss is equal to

$$\text{EL} - \text{EL}_\infty = -(1 - \alpha_l)\Delta - (\alpha_h - \alpha_l) g + \kappa g.$$

Only the difference from (4) is the first term $-(1 - \alpha_l)\Delta$. Recall that the quantity of an asset liquidated is reduced by Δ when κ is lowered. That is, Δ units of the asset are not held by original asset holders rather than being liquidated to low-type buyers. As a result, the efficiency loss is reduced by $(1 - \alpha_l)\Delta$. But the other two terms that measure the net efficiency gain is negative as discussed above, the total efficiency loss always decreases when κ is lowered in this example. In sum, if market A is sufficiently liquid and market B is mildly illiquid, then reducing the entry cost increases the total surplus.

3 Full Credit-Risk Model

In this section, I develop a credit-risk model with two sectors by extending Leland (1994). I call the two sectors, sector A and sector B, which can be interpreted as either two countries or two local markets within a country. Each sector consists of a primary debt market and a secondary market for the assets of defaulted firms.

The key assumption is that the secondary markets are partially segmented from each other. Specifically, potential asset buyers in each secondary market are required to pay a fixed cost to enter the other secondary market. For simplicity, I assume that the primary debt markets face perfect competition as in Leland (1994). This assumption is reasonable in many cases because the secondary markets generally encounter larger informational barriers than the primary markets do; see, for instance, Wittenberg-Moerman (2008).

Time flows continuously over $[0, \infty)$. The primary debt market in each sector is populated by a continuum of firms, indexed by $(i, j) \in \{A, B\} \times [0, \infty)$. There are three types of agents: equityholders, creditors, and potential asset buyers. All agents are risk neutral and have a discount rate r. Also, all equityholders are ex-ante identical across the two sectors; the same for all creditors. That is, I do not consider ex-ante heterogeneity in the supply sides as in the static model. I will focus on a steady-state equilibrium in what follows. If there is no confusion, I will often refer to sector i as market i for each $i \in \{A, B\}$.

3.1 Firm Assets

Each firm (i, j) has a risky asset that generates an after-tax cash flow $x_t^{ij} dt$ over every time interval $[t, t + dt)$ until the asset dies. I will describe shortly when the asset dies. The cash flow x_t^{ij} evolves according to

$$\frac{dx_t^{ij}}{x_t^{ij}} = \mu dt + \sigma dZ_t^{ij},$$

where μ is the asset's growth rate, σ is the asset volatility, and Z_t^{ij} is a standard idiosyncratic Brownian motion. I hereafter interpret x_t^{ij} as the firm's asset size at time t. The asset does not live forever. Specifically, the asset dies exogenously with a Poisson arrival intensity $\phi > 0$. This exogenous death is not related to a bankruptcy event. Instead, we can simply imagine that a machine ends its life after being used for many years. This assumption is necessary to obtain a steady-state equilibrium as in Miao (2005).

In this setting, the firm's unlevered value at time t is given by

$$F(x_t^{ij}) = E\left[\int_t^\infty \phi e^{-\phi(s-t)} \left(\int_t^s e^{-r(u-t)} x_u^{ij} du\right) ds\right] = \frac{x_t^{ij}}{r+\phi-\mu}.$$

To make sure this value is finite, I assume $r+\phi > \mu$. From now, I will often omit the indexes i, j, or t if there is no confusion.

As mentioned above, I will focus on a steady-state equilibrium throughout the paper. In a steady state, all aggregate variables remain constant over time. Let $m^i(x)$ denote the steady-state distribution of the asset size across all existing firms in sector i. That is, $m^i(x)$ denotes the measure of the firms whose asset size is equal to x. I will pin down this steady-state distribution $m^i(x)$ endogenously later.

3.2 Firm Liability and Default

Each firm has one unit of the bonds outstanding, which provide tax benefit to the firm. Let π denote the corporate tax rate. Each bond pays a coupon cdt over every time period $[t, t+dt)$. The bond matures at a random time that arrives with a Poisson intensity λ. Thus, the average maturity is equal to $\frac{1}{\lambda}$. Put differently, a fraction λdt of the outstanding bonds are retired every time. The case of $\lambda = 0$ corresponds to the perpetuity debt. Also, whenever the bond matures, the firm issues a new bond under the same contract terms and seniority. As a result, the total units of the bonds remain the same over time. Both of these stationary maturity and debt-financing structures are commonly used in the literature for simplicity; see, for instance, Leland (1998), Hackbarth et al. (2006), and He and Xiong (2012).

Let $D(x)$ denote the debt value of a firm holding an asset of size x. Then, the after-tax net cash flow to the firm at time t is given by

$$x_t - (1-\pi)c + \lambda(D(x_t) - P).$$

The first term indicates the after-tax cash flow. The second term is the coupon payment net of the tax subsidy. The third term corresponds to the rollover gains. That is, because λdt units of the bonds are retired every time, the firm needs to pay $P\lambda dt$ as the total principal payments, but receives $D(x_t)\lambda dt$ from issuing new bonds.

The equityholders can default on the debt payments at any point in time. Especially, when the net cash flow is negative, the equityholders choose whether to inject more money to cover the operating losses or to default. But once the firm defaults, the control right over the firm asset is entirely transferred to the creditors. That is, the equityholders receive nothing upon default. As a result, the equityholders optimally default when the equity value hits zero; otherwise, the equityholders inject more money or, equivalently, issue new equity. Here, I assume that the equityholders have deep pockets.

In this setting, I can reasonably focus on a threshold-type default strategy. That is, I will find a threshold pair, (x_D^A, x_D^B), such that each firm (i, j) in sector i optimally defaults when its cash flow x_t^{ij} hits x_D^i. I will pin down both x_D^A and x_D^B endogenously later.

3.3 Secondary Markets

Once a firm in sector i defaults, its creditors take over the firm's entire asset and then liquidate the asset immediately in the secondary market in sector i. The creditors sell off the asset immediately because they are generally not skilled enough to reorganize the asset by their own. In this regard, I can literally interpret the liquidity shock in the previous model as the default event in the present model.

The secondary markets behave almost the same way as in the static model. That is, each buyer in one sector is required to pay a fixed cost κ to enter the other sector; each buyer can buy only one unit of the asset throughout her lifetime; there are high-type buyers and low-type buyers with productivities α_h and α_l, respectively. The only difference is that some high-type buyers of measure $f^i dt$ are newly born in each sector i over every time interval $[t, t + dt)$, where $f^i \in [0, \infty]$. I need this minor adjustment to obtain a steady-state equilibrium. An initial population of the high-type buyers will be irrelevant in a steady-state equilibrium. Put differently, only how many high-type buyers are newly born per unit time matters in a steady-state equilibrium. There are still infinitely many low-type buyers in each sector.

Let me clarify some of the above descriptions. First, each potential buyer can at least choose when to buy an asset and when to enter the other market. Second, every new potential buyer is endowed with some amount of cash that can be used to buy at most one unit of the asset throughout her lifetime. The potential buyer does not receive any additional incomes after being born.[1] I also implicitly assume that the potential buyer consumes all the cash flows from the asset immediately. Thus, after purchasing one unit of the asset, the buyer indeed does not have any additional money to buy another asset. Nonetheless, I can at least allow the potential buyer to resell her asset and buy another asset afterwards. But, even in this case, any potential buyer will do so if the asset prices in the two markets stay constant over time. That is, every buyer will optimally use a buy-and-hold strategy in a steady state. Third, each high-type buyer can produce a cash flow $\alpha_h x$ from any asset of size x. Thus, the high-type buyer values one unit of the asset as $\frac{\alpha_h}{r+\phi-\mu}$. Similarly, each low-type buyer values one unit of the asset as $\frac{\alpha_l}{r+\phi-\mu}$. I will often use ρ to denote $r+\phi-\mu$ for notational simplicity.

In this environment, let p^i be the liquidation price per one unit of the asset in sector i. That is, any failed asset in sector i is traded at the price of p^i per unit. For clarification, the entire asset of any failed firm in sector i is liquidated at the price of $p^i x_D^i$ because each firm defaults when its asset size hits the default threshold x_D^i. In this regard, I implicitly assume that any firm asset is infinitely divisible.

3.4 New Entrants

In this economy, every firm exits the markets if one of the two events occurs. First, when the firm's asset itself dies, the firm exits the markets. Second, once the firm defaults, the firm hands over the asset to its creditors and then exits the markets. But, to obtain a steady-state equilibrium, I need to keep the number of existing firms constant. Thus, whenever one of those events occurs, I assume that a new firm is born to replace the exiting firm. Further, each new firm is endowed with a new asset of size x_N, where x_N is exogenously given. I can

[1] Again, imposing some financial constraints in terms of a limit on the asset size is widely used in the literature; see, for instance, Duffie et al. (2005) and He and Milbradt (2014).

always choose x_N so that $x_N > x_D^i$ for every i, because I can compute the maximum possible default threshold in closed form.

3.5 Equilibrium

An equilibrium in this economy is defined as a liquidation price pair (p_*^A, p_*^B) such that (i) all the agents, including the equityholders, creditors, and potential buyers, maximize their own profits and (ii) the price pair (p_*^A, p_*^B) jointly clears both secondary markets. Specifically, the quantity of an asset liquidated in each secondary market is solely determined by the default decisions of the equityholders in that sector. However, the demand curves in the two secondary markets are jointly determined by the asset-purchase and entry decisions of the potential buyers in the two sectors. The price pair (p_*^A, p_*^B) must clear both secondary markets.

4 Model Solutions

In this section, I characterize an equilibrium. I first compute the equity value and debt value for any given liquidation price in each sector. Second, I generate the supply curve in each secondary market. Then, I solve for an equilibrium price pair.

4.1 Equity Value and Debt Value

Let $E(x; p^i)$ and $D(x; p^i)$ denote the equity value and debt value of an individual firm in sector i, respectively, given any liquidation price p^i. Here, I do not need to use the notation $E^i(x; p^i)$ or $D^i(x; p^i)$ because there is no ex-ante heterogeneity in the primary debt markets across the two sectors. Also, I will derive $E(x; p^i)$ and $D(x; p^i)$ by solving for an optimal default threshold x_D^i as well. In this regard, I will use $x_D(p^i)$ whenever I want to emphasize its dependence on p^i. Again, note that I do not need to use the notation $x^i(p^i)$.

I will start with computing the debt value. To this aim, let x_D^i be any given default threshold. Also, note that since the creditors face perfect competition in the risk-neutral world, the return on the bond must be the same as the risk-free return. Then, a standard continuous-time method implies that $D(x) := D(x; p^i, x^i_D)$ satisfies the following Hamilton-Jacobi-Bellman (HJB) equation:

$$rD(x) = c + \lambda(P - D(x)) - \phi D(x) + \mu x D_x(x) + \frac{\sigma^2}{2} x^2 D_{xx}(x), \tag{5}$$

subject to $D(x^i_D) = p^i x^i_D$. The term in the left-hand side denotes the required return for the bond. In the right-hand side, the first term denotes the coupon payment. The second term corresponds to the event in which the bond is retired. The third term corresponds to the exogenous death event. The remaining terms explain how the debt value changes due to the fluctuations in the cash flows. The boundary condition indicates the liquidation value of the failed asset of size x^i_D.

The HJB equation in (5) has the following closed-form solution:

$$D(x) = \frac{c + \lambda P}{r + \lambda + \phi} + \left(p^i x^i_D - \frac{c + \lambda P}{r + \lambda + \phi}\right)\left(\frac{x}{x^i_D}\right)^{\xi},$$

where

$$\xi = \frac{-\mu + \frac{\sigma^2}{2} - \sqrt{\left(\mu - \frac{\sigma^2}{2}\right)^2 + 2\sigma^2(r + \lambda + \phi)}}{\sigma^2} < 0.$$

We can derive this formula using a standard ordinary differential equation method; thus, let me omit the derivation.

Now, I will compute the equity value $E(x; p^i)$ together with an optimal default threshold x^i_D. Similarly as above, $E(x) := E(x; p^i)$ satisfies the following HJB equation:

$$rE(x) = x - (1 - \pi)c + \lambda(D(x) - P) - \phi E(x) + \mu x E_x(x) + \frac{\sigma^2}{2} x^2 E_{xx}(x), \tag{6}$$

subject to $E(x^i_D) = 0$ and $E_x(x^i_D) = 0$. The term in the left-hand side is the required return for the equity. In the right-hand side, the first three terms indicate the after-tax net cash flows as described above. The fourth term corresponds to the exogenous death event. The remaining terms explain how the equity value changes according to the fluctuations in the cash flows. The first boundary condition means that the equity value becomes worthless upon default. The second boundary condition indicates the so-called smooth-pasting condition that is necessarily satisfied by an optimal default threshold.

In Appendix 7.1, I obtain both $E(x)$ and x_D^i in closed form. Here, let me present the formula only for x_D^i:

$$x_D^i = x_D(p^i) = \frac{(r+\phi-\mu)(\eta\pi c(r+\lambda+\phi) - \xi(c+\lambda P)(r+\phi))}{(1-\eta+(\eta-\xi)(r+\phi-\mu)p^i)(r+\phi)(r+\lambda+\phi)}, \tag{7}$$

where

$$\eta = \frac{-\mu + \frac{\sigma^2}{2} - \sqrt{\left(\mu - \frac{\sigma^2}{2}\right)^2 + 2\sigma^2(r+\phi)}}{\sigma^2} \in (\xi, 0). \tag{8}$$

To ensure that $x_D^i > 0$, I assume $(c+\lambda P)(r+\phi) > \pi c(r+\lambda+\phi)$. Importantly, the formula in (7) implies that $x_D^i(p^i)$ decreases in p^i. This result is almost straightforward because if the liquidation price goes down, then the debt value goes down as well and thus, the equityholders choose to default earlier to avoid the increased rollover losses. I will use this simple but crucial result to generate a downward-sloping supply curve.

4.2 Supply Curves

In this section, given any liquidation price p^i, I will pin down the quantity of an asset liquidated in sector i per unit time, denoted by $q(p^i)$. In what follows, I will simply use x_D^i to denote the optimal default threshold $x_D(p^i)$ obtained in the previous section.

To start, I need to pin down $m^i(x)$ first. According to Stokey (2009) for instance, the density function $m^i(x)$ in a steady state satisfies the following Kolmogorov forward equation:

$$0 = -\phi m^i(x) - \frac{\partial}{\partial x}[\mu x m^i(x)] + \frac{\partial^2}{\partial x^2}\left[\frac{\sigma^2}{2}x^2 m^i(x)\right], \quad \forall x \neq x_N, \tag{9}$$

subject to $m^i(x_D^i) = 0$ and up to some normalization. The first term in the right-hand side denotes the number of the firms exiting the market because of the exogenous death of their assets. The other two terms indicate how the distribution changes due to the fluctuations in the individual firms' asset sizes. In a steady state, the sum of all those terms must be zero. Equation (9) does not hold at $x = x_N$ because new firms enter the market with a new asset of size x_N. The boundary condition $m^i(x_D^i) = 0$ means that every firm exits the market immediately upon default.

I will normalize $m^i(x)$ as follows. In fact, without normalization, any multiple of $m^i(x)$ still satisfies equation (9). Recall that, in the static model, I have normalized the maximum possible total surplus in each market to 1. Similarly, in the present model, the maximum possible total surplus is obtained if all potential buyers have the full productivity 1. In this regard, I will also normalize the maximum possible total surplus in sector i per unit time to 1 as follows:

$$\int_{x_D^i}^{\infty} x m^i(x)dx + \frac{\sigma^2}{2}(x_D^i)^2 m_x^i(x_D^i)F(x_D^i) = 1. \tag{10}$$

The first term in the left-hand side denotes the total outputs from all existing firms in sector i per unit time. Regarding the second term, $\frac{\sigma^2}{2}(x_D^i)^2 m_x^i(x_D^i)$ counts the total measure of defaulted firms in sector i per unit time.[2] Given that all potential buyers have full productivity, the present value of all those defaulted firms' assets is equal to $\frac{\sigma^2}{2}(x_D^i)^2 m_x^i(x_D^i)F(x_D^i)$. Thus, the sum of the two terms in the left-hand side in (10) is equal to the maximum possible total surplus in sector i per unit time, given any p^i. In Appendix 7.2, I provide a closed-form solution for $m^i(x)$ satisfying the above boundary and normalization conditions.

Next, I can now compute the supply curve $q(p^i)$. Recall that the total measure of defaulted firms in sector i per unit time is equal to $\frac{\sigma^2}{2}(x_D^i)^2 m_x^i(x_D^i)$. Then, because each failed asset is of size x^iD, the quantity of an asset liquidated $q(p^i)$ must be equal to

$$q(p^i) = \frac{\sigma^2}{2}(x_D^i)^3 m_x^i(x_D^i). \tag{11}$$

Importantly, using the closed-form solution for $m^i(x)$, I can show that $q(p^i)$ increases as p^i decreases; see Appendix 7.2 for the proof. Intuitively, as already discussed above, when p^i goes down, all equityholders in sector i choose to default earlier to avoid the increased rollover losses, triggering more asset liquidation. In this way, I have indeed generated a downward-sloping supply curve endogenously.

[2] See page 607 in Luttmer (2012) for heuristic derivation of this general formula. This formula actually says that the default size is large if the asset volatility is large and the slope of $m^i(x)$ at x_D^i is large, both of which are intuitive enough. The term $(x_D^i)^2$ appears because the cash flow follows a geometric Brownian motion.

4.3 Liquidation Prices

In this section, I will finally derive an equilibrium price pair (p_*^A, p_*^B) by producing the demand curves in both markets. Actually, this task does not require any additional work because, in a steady state, the potential buyers behave the same way as in the static model. The reason is that when there is an arbitrage opportunity, every potential buyer takes that opportunity as soon as she is born. For instance, suppose for a moment that the two sectors are fully segmented and $p^i < \frac{\alpha_h}{\rho}$. Then, any newly born high-type buyer in market i will purchase an asset from market i immediately. The buyer will never delay purchasing an asset because if she does so, the present value of the profits will be lowered due to the time discount. Even when the two sectors are partially segmented, every high-type buyer in market i will buy an asset from market $-i$ right after being born if $p^i - p^{-i} > \kappa$. In this manner, in a steady state, every potential buyer will either buy an asset from one of the two markets as soon as she is born or never buys any assets during her lifetime. Hence, I can indeed pin down an equilibrium price pair (p_*^A, p_*^B) as if the economy has only two dates.

I now provide an equilibrium construction algorithm, which I have omitted before. In particular, when the two sectors are completely segmented, that is, $\kappa \geq \frac{\alpha_h - \alpha_l}{\rho}$, I can obtain an equilibrium by simply replacing α_h and α_l in Section 2.2 by $\frac{\alpha_h}{\rho}$ and $\frac{\alpha_l}{\rho}$, respectively. Thus, let me focus on the other case where $\kappa < \frac{\alpha_h - \alpha_l}{\rho}$

4.3.1 Equilibrium Construction

Without loss of generality, I look for an equilibrium in which $p_*^A \geq p_*^B$. If this type of equilibrium exists, some high-type buyers of measure g in market A move to market B per unit time, where $g \in [0, f^A]$, while no potential buyers in market B move to market A. To find such g, I follow the two steps as follows.

First, I guess that $g = 0$ in equilibrium. Then, each sector must behave as if it is fully segmented from the other sector. Let p_*^i be an equilibrium price in sector i, assuming this sector is fully isolated. Then, I examine whether the price pair (p_*^A, p_*^B) satisfies $p_*^A - p_*^B \leq \kappa$. If this condition holds, no potential buyers in market A have incentive to enter market B, justifying $g = 0$ can be sustained in equilibrium.

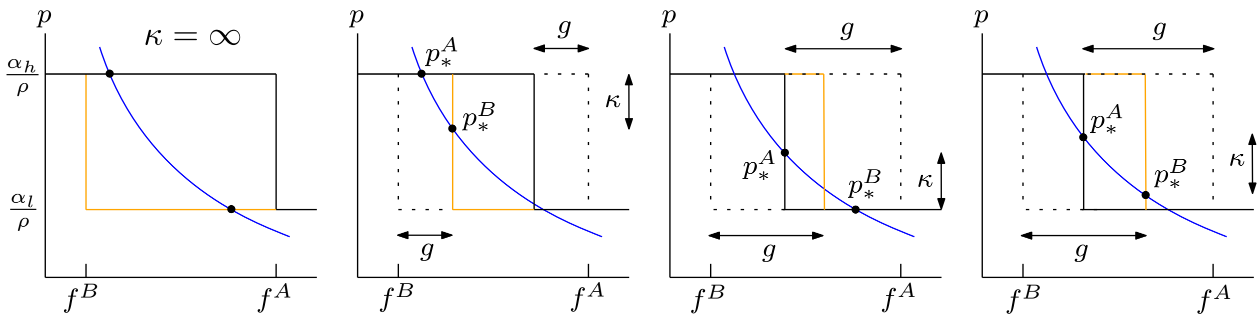

Figure 4: This figure illustrates an example in which three equilibria can arise, in each of which $g > 0$. The left panel plots the demand and supply curves by assuming $\kappa = \infty$ for a moment. The other three panels depict all three equilibria in this economy.

Second, I now guess that $g > 0$. Then, sector A behaves as if it is a fully isolated sector but with high-type buyers of measure $f^A - g$ instead of measure f^A. Similarly, sector B behaves as if it is a fully isolated sector but with high-type buyers of measure $f^B + g$ instead of measure f^B. Let p^i_* be an equilibrium price in sector i under this assumption. I then examine whether the price pair (p^A_*, p^B_*) satisfies $p^A_* - p^B_* = \kappa$. If this condition holds, every high-type buyer in market A is indifferent between buying an asset from market A and from market B, justifying $g > 0$ can be sustained in equilibrium. For clarification, g cannot be equal to f^B because if so, the price in market A will drop to the lowest level $\frac{\alpha_l}{\rho}$. In this case, no high-type buyers in market A have strong incentive to enter market B and therefore, $g = f^B$ cannot be sustained in equilibrium.

Using these two steps, I can obtain all possible equilibria. Although closed-form solutions for those equilibria are no longer available, I can compute all equilibria numerically. For completeness, I prove that this economy has at least one equilibrium in Appendix 7.3.

In particular, the economy described in Figure 4 has three so-called self-fulfilling equilibria in each of which $g > 0$. Let me examine each of the equilibria because this example will be useful when discussing policy implications of the model. First, as in the left panel, $g = 0$ cannot be sustained in equilibrium because if so, an equilibrium price pair must be $(\frac{\alpha_h}{\rho}, \frac{\alpha_l}{\rho})$, but the difference between these two prices is larger than κ. The remaining three panels indeed present three different equilibria. In equilibrium in the second panel, the price pair is given by $p^A_* = \frac{\alpha_h}{\rho}$ and $p^B_* = \frac{\alpha_h}{\rho} - \kappa$. To understand this equilibrium, suppose that all market participants believe that p^A_* stays at the highest level $\frac{\alpha_h}{\rho}$. Then, p^B_* must be given by $\frac{\alpha_h}{\rho} - \kappa$, which is larger than $\frac{\alpha_l}{\rho}$, because otherwise, the markets cannot clear. Then, the quantity of an asset liquidated in market B will be lower than that in the case of $\kappa = \infty$. Nonetheless, the high-type buyers in market B cannot still fully absorb those liquidated assets. Thus, for market B to clear at the price larger than $\frac{\alpha_l}{\rho}$, some high-type buyers of measure g in market A must move to market B. Fortunately, even after those buyers left market A, the market still has enough remaining high-type buyers who can absorb all liquidated assets in that market. As a result, the price in market A still stays at the highest level, justifying the beliefs of the market participants.

In equilibrium in the third panel, the price pair is given by $p_*^A = \frac{\alpha_l}{\rho} + \kappa$ and $p_*^B = \frac{\alpha_l}{\rho}$. Similarly as above, suppose that all market participants believe that p_*^B stays at the lowest level $\frac{\alpha_l}{\rho}$. Then, p_*^A must be equal to $\frac{\alpha_l}{\rho} + \kappa$ because otherwise, the markets cannot clear. Then, the quantity of an asset liquidated in market A will be larger than that in the case of $\kappa = \infty$. Nonetheless, that quantity is still lower than the number of high-type buyers in market A. Thus, for market A to clear at the price lower than $\frac{\alpha_h}{\rho}$, all the residual high-type buyers must move to market B. Yet, even with the inflow of those high-type buyers, the liquidated assets in market B cannot be absorbed by high-type buyers only because market B is illiquid. Thus, the price in market B still remains at the lowest level, justifying the above beliefs.

The fourth panel plots another equilibrium such that $\frac{\alpha_l}{\rho} < p_*^B < p_*^A < \frac{\alpha_h}{\rho}$. In this equilibrium, the market participants do not have extreme beliefs as in the above. When the markets are not either sufficiently liquid or severely illiquid, this intermediate type of equilibrium can be also obtained. This equilibrium will provide intriguing policy implications regarding liquidity injection.

In this example, multiple equilibria arise because market A is not sufficiently liquid, while market B is just mildly illiquid. I will discuss soon that when multiple equilibria emerge, policy implications will be very different, depending on which equilibrium is selected. Lastly, this model economy can have even more than three equilibria in some other cases. But let me omit to present all different scenarios because the main goal of this paper is not to exhibit all possible equilibria, but to extract important policy implications from the model.

5 Model Implications

In this section, I will discuss welfare implications of the model. I will particularly examine the role of the entry cost and the amount of liquidity in the markets. That is, I will study how a change in κ, f^A, or f^B will affect welfare in this economy. I will not pay much attention to other parameters because economic mechanisms related to those parameters have been intensively studied in the literature.

5.1 Welfare

As in the static model, I define welfare as the total surplus in the economy. Specifically, let (p_*^A, p_*^B) be any equilibrium price pair. Without loss of generality, I can assume that $p_*^A \geq p_*^B$. Then, let g be the number of high-type buyers in market A who move to market B. Also, suppose that $q(p_*^i)$ units of the asset are liquidated in market i per unit time. Among those $q(p_*^i)$ units of the asset, suppose that q_h^i units are sold to high-type buyers and the remaining $q_l^i = q(p_*^i) - q_h^i$ units are liquidated to low-type buyers. More explicitly,

$$q_h^A = \begin{cases} q(p_*^A), & \text{if } \frac{\alpha_h}{\rho} < p_*^A \leq \frac{\alpha_h}{\rho} \\ f^A - g, & \text{if } p_*^A = \frac{\alpha_l}{\rho}, \end{cases} \qquad q_h^B = \begin{cases} q(p_*^B), & \text{if } \frac{\alpha_h}{\rho} < p_*^B \leq \frac{\alpha_h}{\rho} \\ f^B + g, & \text{if } p_*^B = \frac{\alpha_l}{\rho}, \end{cases}$$

The total surplus per unit time in this economy is then given by

$$\text{TS} = -\kappa g + \sum_{i \in \{A,B\}} \left[\int_{x_D^i}^{\infty} x m^i(x) dx + \frac{\alpha_h q_h^i}{\rho} + \frac{\alpha_l q_l^i}{\rho} \right]. \tag{12}$$

That is, the first term $-\kappa g$ is the total entry cost incurred per unit time. The second term $\int_{x_D^i}^{\infty} x m^i(x) dx$ is the total outputs produced from all existing firms in market i per unit time. The third term $\frac{\alpha_h q_l^i}{\rho}$ is the time-t present value of the assets liquidated to high-type buyers in market i at time t. Here, t can be any number between 0 and ∞. The fourth term $\frac{\alpha_l q_l^i}{\rho}$ is the time-t present value of the assets liquidated to low-type buyers in market i at time t.

Also, similarly as in the static model, I can define the total efficiency loss as

$$2 - \mathrm{TS} = \kappa g + \sum_{i \in \{A,B\}} \left[\frac{(1-\alpha_h)q_h^i}{\rho} + \frac{(1-\alpha_l)q_l^i}{\rho} \right]. \tag{13}$$

In particular, the first term in the bracket is the efficiency loss caused by asset liquidation to high-type buyers. The second term in the bracket has a similar meaning.

When there are multiple equilibria, I will rank those equilibria according to their corresponding total surplus. For instance, if three equilibria arise as in the example in Figure 4, I will use the terms the best equilibrium, the intermediate equilibrium, and the worst equilibrium to call those three equilibria accordingly. Under most parameter values, the equilibrium in the second panel in Figure 4 corresponds to the best equilibrium and the equilibrium in the third panel corresponds to the worst equilibrium.

5.2 Parameter Values

Table 1 summarizes the baseline parameter values for this model. I choose $r = 5.0\%$, which is close to the annualized one-month Treasury rate over the periods from 2001 to 2008. I choose $\lambda = 1$ as in He and Xiong (2012), meaning that the average debt maturity is 1. According to Custódio et al. (2013), the average maturity of long-term debt is around 6 years. But the average maturity of the asset-backed commercial papers in the first half of 2007 is 33 days, according to Covitz, Liang, and Suarez (2013). Thus, $\lambda = 1$ can be used as a reasonable benchmark for debt maturity. I normalize P to 100 and then choose $c = 9$, which is close to the coupon rate 10.3% chosen by He and Xiong (2012). He and Xiong (2012) choose this coupon rate to target the average spread of BB-rated bonds. I choose $\phi = 4\%$ as in Miao (2005). According to Dunne et al. (1988), an annual turnover rate in the US manufacturing industry is around 7%. In the present model, firms go bankrupt endogenously as well. Thus, $\phi = 4\%$ is a reasonable choice. I appropriately choose $\mu = 3\%$, which is close to the values commonly used in the literature. Hackbarth et al. (2006) use $\mu = 5\%$. I choose $\sigma = 20\%$. According to Zhang et al. (2009), the average asset volatility of BB-rated firms is 21%. I choose $\alpha_h = 70\%$ and $\alpha_l = 30\%$ because the recovery rate during booms is around 80%, while the recovery rate during recessions is around 40%, according to Chen (2010). I choose

Parameter	Value	Meaning
r	4%	discount rate
λ	1	maturity intensity
ϕ	4%	exogenous death rate
μ	3%	asset growth rate
σ	20%	asset volatility
P	100	principal payment
c	9	coupon payment
α_h	0.7	high-type buyer's productivity
α_l	0.3	low-type buyer's productivity
x_N	8.7	asset size of a newly born firm
κ	1.5	entry cost
f^A	0.045	number of high-type buyers in market A
f^B	0.0035	number of high-type buyers in market B

Table 1: Baseline Parameter Values

$x_N = 8.7$. Using this value, I can generate a reasonable annual default rate in this model. Specifically, under the above parameter choices, the highest possible liquidation price is equal to $\frac{\alpha_h}{r+\phi-\mu} = 10.77$ and the lowest possible liquidation price is equal to $\frac{\alpha_l}{r+\phi-\mu} = 4.62$. Then, using $x_N = 8.7$, I can generate a supply curve $q(p)$ with $q(4.62) = 0.041$ and $q(10.77) = 0.005$. Moreover, in this model, for any given p, the annual value-weighted default rate in each sector i is given as

$$\frac{q(p)}{\int_{x_D^i}^{\infty} x m^i(x) dx}.$$

Based on this formula, the high possible default rate is given by 12.6% and the lowest possible default rate is given by 0.56%. Both of these numbers are reasonable enough if we take into account the exogenous turnover rate $\phi = 4\%$. I appropriately choose $\kappa = 1.5$, which corresponds to 24% of the largest possible price gap that is equal to $\frac{\alpha_h - \alpha_l}{r+\phi-\mu} = 6.15$. Lastly, I also appropriately choose $f^A = 0.045$ and $f^B = 0.0035$ to consider a situation in which market A is relatively more liquid but not sufficiently liquid, whereas market B is relatively less liquid. This market condition will provide fruitful enough insights and policy implications that can be applied to other situations as well.

Under the above parameter values, the supply and demand curves are plotted in Figure 5, assuming $\kappa = \infty$ for a moment. When some parameters are changed, this economy will have at most three equilibria as in the example in Figure 4. I will consider all these equilibria

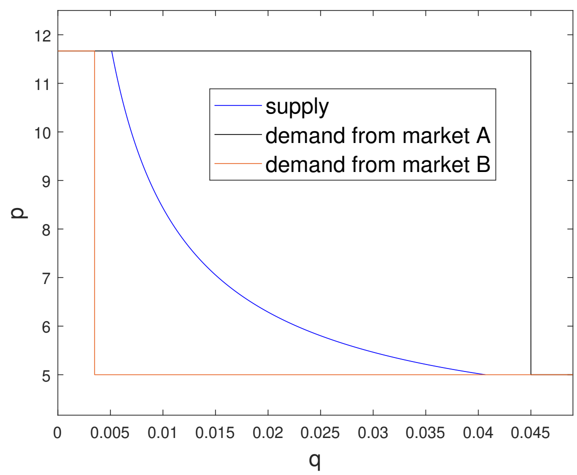

Figure 5: This figure plots both demand and supply curves under the baseline parameter values, assuming $\kappa = \infty$.

whenever doing is so is meaningful.

5.3 The Effect of the Entry Cost

This section studies how a change in the entry cost affects the total surplus. In general, reducing the entry cost increases the total surplus if a relatively more liquid market is sufficiently liquid and the other market is just mildly illiquid, similarly as in Figure 3. In contrast, such a policy may hurt the total surplus when the more liquid market is not sufficiently liquid and the less liquid market is severely illiquid, similarly as in Figure 2. In fact, in these two cases, the economy is likely to have a unique equilibrium. But, similar arguments can be applied to analyze other cases. Recall that when the more liquid market is not sufficiently liquid and the less liquid market is just mildly illiquid, multiple equilibria are likely to emerge. In this case, the above positive outcome obtains if the best equilibrium is selected, while the negative outcome obtains if the worst equilibrium is selected. When the intermediate equilibrium is selected, both positive and negative effects will be mechanically mixed together. In this sense, I will not pay that much attention to the intermediate equilibrium in this section. However, the outcomes in the intermediate equilibrium will provide intriguing implications about the policies aimed at injecting liquidity to the markets.

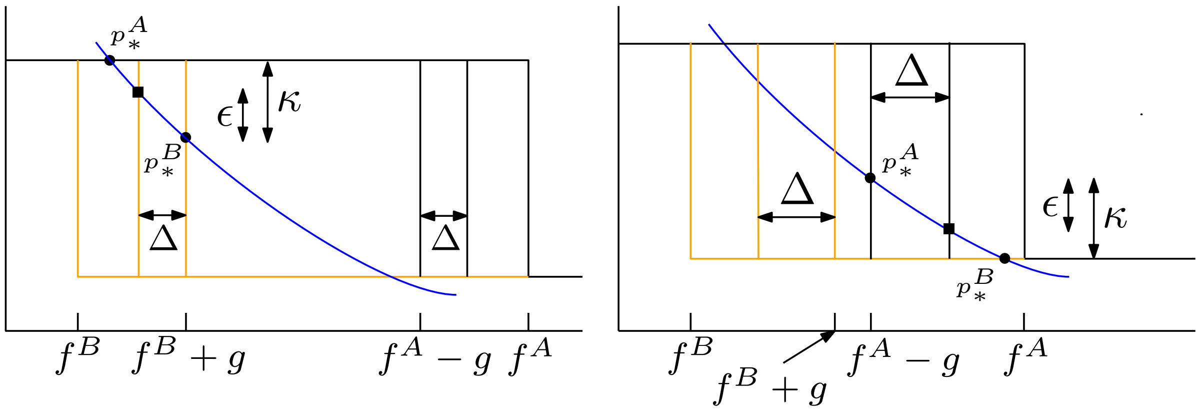

Figure 6: This figure illustrates the effect of a change in κ.

5.3.1 Positive Outcomes

Let me first explain the positive outcomes using the left panel in Figure 6. In this figure, the two circle dots denote p_*^A and p_*^B in the case where the entry cost is κ. Also, g denotes the number of high-type buyers in market A moving to market B in that case. But when κ is reduced by ϵ, the price in market A still remains the same at the highest level $\frac{\alpha_h}{\rho}$, while the price in market B is pushed up by ϵ. The square dot denotes the new equilibrium price in market B. This outcome generally occurs if either market A is sufficiently liquid or the best equilibrium is selected in the sense mentioned above. As a result, the quantity of an asset liquidated in market B is lowered by Δ, whereas the quantity of an asset liquidated in market A stays unchanged. Because of the former effect, the number of high-type buyers in market A moving to market B is also reduced by the same amount Δ. Hence, the net change in the total efficiency loss is equal to

$$(\kappa-\epsilon)(g-\Delta)-\kappa g-\frac{(1-\alpha_h)\Delta}{\rho}=-\epsilon g-\kappa\Delta-\frac{(1-\alpha_h)\Delta}{\rho},$$

where the term $\epsilon\Delta$ is not included because it is negligible. The first term in the right-hand side denotes the direct benefit from the decrease in the entry cost. The second term means that the total entry cost is further reduced by $\kappa\Delta$ because the number of moving high-type buyers is decreased by Δ. The third term indicates the efficiency gain caused by the reduction in the quantity of an asset liquidated in market B. Because all the above terms are negative, the total surplus certainly increases.

The top three panels in Figure 9 plot numerical results supporting all the above arguments. Specifically, the top-left panel shows that the total surplus indeed increases as the entry cost is reduced. Moreover, even when κ is decreased from some number larger than $\frac{\alpha_h - \alpha_l}{\rho} = 6.7$ to another number less than 6.7, the total surplus also increases as already discussed in Section 2.4.2. For clarification, when κ is above 4.1, a unique equilibrium obtains in this example.

5.3.2 Negative Outcomes

Let me now discuss the above-mentioned negative effects of a reduction in the entry cost, using the right panel in Figure 6. Again, in this figure, when the entry cost is κ, the two circle dots denote p_*^A and p_*^B. But when the entry cost is lowered by ϵ, the price in market B still remains at the lowest level $\frac{\alpha_l}{\rho}$, while the price in market A is pushed down by ϵ. The square dot denotes the new equilibrium price in market A. This outcome tends to arises if either market B is severely illiquid or the worst equilibrium is selected in the above-mentioned sense. As a result, the quantity of an asset liquidated in market A increases by Δ. But then, the number of high-type buyers in market A moving to market B is decreased by the same amount Δ. As a result, the net change in the total efficiency loss is equal to

$$(\kappa - \epsilon)(g - \Delta) - \kappa g + \frac{(1 - \alpha_h)\Delta}{\rho} + \frac{(\alpha_h - \alpha_l)\Delta}{\rho} \tag{14}$$

The first two terms correspond to the reduction in the total entry cost. The third term denotes the efficiency loss caused by the increased amount of an asset liquidated in market A. The fourth term means that since the number of high-type buyers in market A moving to market B is decreased by Δ, the same units of the asset in market B are now liquidated to low-type buyers rather than to high-type buyers, increasing the efficiency loss by $\frac{(\alpha_h - \alpha_l)\Delta}{\rho}$. The expression in (14) is then reduced to

$$-\epsilon g + \frac{(1 - \alpha_h)\Delta}{\rho} + \left(\frac{\alpha_h - \alpha_l}{\rho} - \kappa\right)\Delta. \tag{15}$$

Importantly, the third term is always positive. The reason is that although the total entry cost is saved by $\kappa\Delta$ when the number of moving high-type buyers is reduced by Δ, this outcome increases the efficiency loss by more than $\kappa\Delta$ because the same units of the asset in market B are now liquidated to low-type buyers. Thus, both second and third terms in (15) become positive. As a result, the expression in (15) implies that reducing the entry cost may actually decrease the total surplus, especially when g is sufficiently small. But, as mentioned above, g decreases when κ is decreased. Accordingly, the negative outcome of that policy is more likely to occur when κ is small.

The bottom three panels in Figure 9 show numerical results supporting all these arguments. In particular, the bottom-left panel demonstrates that the total surplus is indeed decreased as κ is lowered, particularly when κ is small. Moreover, the total surplus is 0.76 when $\kappa = 7$, in which a unique equilibrium arises, as in the top-left panel. But the total surplus drops to 0.70 when κ is reduced to 0.1 and the bad equilibrium is selected. This result supports the results in Section 2.4.1 that covers the case in which κ jumps down from a number larger than $\frac{\alpha_h - \alpha_l}{\rho}$ to a number less than $\frac{\alpha_h - \alpha_l}{\rho}$.

5.3.3 Longer Debt Maturity

In this section, I will examine the effect of the entry cost when firms issue debt with longer maturity. Extending debt maturity causes two effects. First, if the firms lengthen debt maturity, the default threshold will be changed. As a result, the quantity of an asset liquidated will be also changed for any given liquidation price. Whether the default threshold increases or decreases depends on the model parameter values. Second, when the firms extend debt maturity, the equityholders become less sensitive to the changes in the liquidation price. This is because long-term debt issuers are not very much concerned about the rollover risks. As a result, the supply curve becomes more inelastic. But recall that the negative effect of reducing the entry cost arises because the supply curve is downward sloping. The argument used to show this result actually then implies that if the supply curve becomes less elastic, the negative effect must be weaker. This result suggests that the policies that reduce the entry cost will be more effective when the firms issue long-term debt.

Under the benchmark parameter values, when λ is reduced from 1 to 0.8, the supply

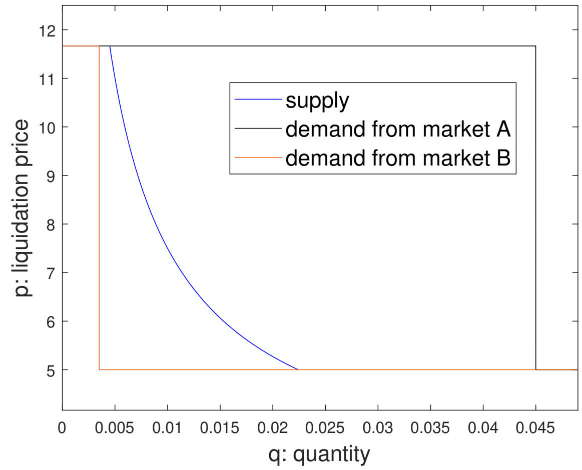

Figure 7: This figure plots both demand and supply curves when $\lambda = 0.8$, assuming $\kappa = \infty$. All other parameter values are chosen from the benchmark parameter values.

curve shifts to the left as shown in Figure 7. That is, extending the debt maturity lowers the default probability for any given liquidation price. Moreover, in this example, since this effect is so strong, the economy now has only one equilibrium of the best type. In other words, the bad type of equilibrium does not arise when the supply shifts to the left and becomes more inelastic. As a result, reducing the entry cost will unambiguously increase the total surplus. Figure 10 shows numerical results for this experiment. Similar results obtain if I choose $\lambda = 0.2$ or 0.1, which corresponds to the average maturity of 5 years and 10 years, respectively. Instead, if λ is very close to 1, say, $\lambda = 0.95$, I can check the second effect mentioned above. But reporting the result for this minor case does not seem to be important.

5.4 The Effect of Liquidity Injection

In this section, I will study how the policies that inject liquidity to one of the markets affect the total surplus. That is, I will examine the effect of a change in f^A and f^B. But the changes in these two parameters will generate qualitatively similar results and the underlying mechanisms for those results are also similar. Thus, I will focus on the parameter f^A in this section. On the one hand, when market A is sufficiently liquid or market B is severely illiquid, injecting liquidity to market A always increases the total surplus. However, recall that multiple equilibria tend to arise if market A is not sufficiently liquid and market B is just mildly illiquid. In this case, such a policy may hurt the total surplus if the intermediate equilibrium is selected. If the best or worst equilibrium selected, injecting liquidity always weakly increases the total surplus.

5.4.1 Positive Outcomes

Let me first explain the positive effects of the above policy. When market A is sufficiently liquid or the best equilibrium is selected in the above sense, injecting liquidity to market A does not make any changes in the economy. To see why, in this case, the price in market A remains at the highest level. Thus, if some additional high-type buyers in market A move to market B, the price in market B will go down, making the price gap bigger than κ. This price gap cannot be sustained in equilibrium. Hence, even after some high-type buyers are relocated to market A from outside the model, those buyers will not take any actions and thus, simply make a zero profit. The top three panels in Figure 11 shows numerical results confirming these arguments.

Now, suppose that market B is severely illiquid or the worst equilibrium is selected in the above sense. Specifically, in this case, the price in market B remains at the lowest level and the price in market A remains at $\frac{\alpha_l}{\rho} + \kappa$. Thus, when f^A is increased by ϵ, all those additional high-type buyers will move to market B; otherwise, market A cannot clear at that price level. As discussed many times above, the inflow of those high-type buyers reduces the total efficiency loss by $\left(\frac{\alpha_h - \alpha_l}{\rho} - \kappa\right)\epsilon$. That is, ϵ units of the asset in market B are now liquidated to high-type buyers rather than to low-type buyers. This efficiency gain dominates the increased total entry cost. Hence, injecting liquidity to market A always increases the total surplus. The bottom three panels in Figure 11 numerically support these arguments.

5.4.2 Negative Outcomes

Interestingly, if the intermediate equilibrium is selected in the above sense, injecting liquidity to market A will lower the total surplus. Let me explain this result using Figure . In this case, when f^A is increased by ϵ, some of those new high-type buyers stay in market A and the remaining high-type buyers move to market B. This is because otherwise, the price in only

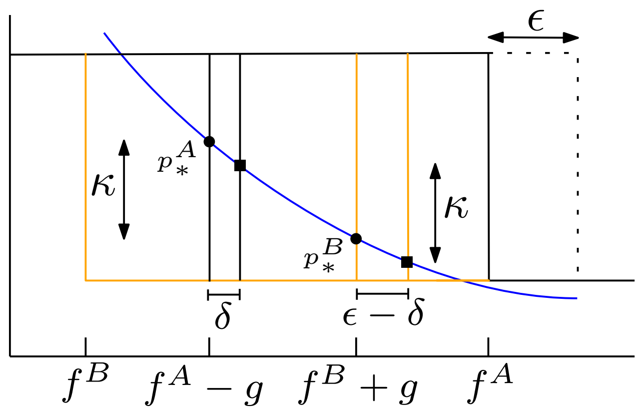

Figure 8: This figure illustrates the effect of a change in f^A.

one market will change, making the price gap either lower or larger than κ, which cannot be sustained in equilibrium. Thus, among the new high-type buyers of measure ϵ, suppose that measure δ of the buyers stay in market A. As a result, the quantity of an asset liquidated in market A increases by δ and the quantity of an asset liquidated in market B increases by $\epsilon - \delta$. Thus, the total efficiency loss is changed by

$$\kappa(\epsilon - \delta) + \frac{(1-\alpha_h)\delta}{\rho} + \frac{(1-\alpha_h)(\epsilon - \delta)}{\rho}.$$

That is, the first term denotes the increment in the total entry cost. The second term is the efficiency loss caused by the increased amount of asset liquidation in market A. The third term is the efficiency loss caused by the increased quantity of asset liquidation in market B. Since all these terms are positive, injecting liquidity to market B actually lowers the total surplus. The middle three panels in Figure 11 presents numerical results supporting these arguments.

6 Conclusion

In this paper, I consider short-term debt markets with two sectors where the secondary markets for assets in default are partially segmented. In this economy, every potential buyer in one secondary market is required to pay a fixed cost to enter the other secondary market. I show that policies aimed at reducing the entry cost may hurt the total surplus. Intuitively, a reduction in the entry cost shrinks the gap in the liquidation prices in the two markets by facilitating more capital flows between the markets. That is, a more liquid market faces a price drop, whereas the price in the other market is lifted up. Thus, every firm in the former market defaults earlier to avoid increased rollover risks. If this negative contagion effect is so severe, the total surplus will be decreased.

7 Appendix

7.1 Equity Value

The HJB equation in (6) has the following closed-form solution:

$$E(x) = \frac{\pi c}{r+\phi} + \frac{x}{r+\phi-\mu} + Ax^{\eta} - \frac{c+\lambda P}{r+\lambda+\phi} - \left(p^i x_D^i - \frac{c+\lambda P}{r+\lambda+\phi}\right)\left(\frac{x}{x_D^i}\right)^{\xi},$$

where η is given in (8). The coefficient A and the default threshold x_D^i must satisfy the following value-matching and smooth-pasting conditions:

$$\frac{\pi c}{r+\phi} + \frac{x_D^i}{r+\phi-\mu} + A(x_D^i)^{\eta} = p^i x_D^i, \quad \frac{x_D^i}{r+\phi-\mu} + A\eta(x_D^i)^{\eta} = \xi\left(p^i x_D^i - \frac{c+\lambda P}{r+\lambda+\phi}\right).$$

These two conditions lead to

$$x_D^i = \frac{(r+\phi-\mu)(\eta\pi c(r+\lambda+\phi) - \xi(c+\lambda P)(r+\phi))}{(1-\eta+(\eta-\xi)(r+\phi-\mu)p^i)(r+\phi)(r+\lambda+\phi)}.$$

We can then compute A as well from the above conditions.

7.2 Supply Curves

In this section, I first provide a closed-form solution for $m^i(x)$. I will then show that

$$q^i(p^i) = \frac{\sigma^2}{2}(x_D^i)^3 m_x^i(x_D^i)$$

decreases in p^i, where $x_D^i = x_D^i(p^i)$. To start, a standard method for ordinary differential equations implies that $m^i(x)$ is equal to

$$m^i(x) = \begin{cases} m^{i,1}(x) = A_1 x^{\eta_1} + A_2 x^{\eta_2}, & \text{if } x \in [x_D^i, x_N] \\ m^{i,2}(x) = A_3 x^{\eta_2}, & \text{if } x \in [x_N, \infty), \end{cases}$$

where

$$\eta_1 = \frac{\mu - \frac{3\sigma^2}{2} + \sqrt{(\mu - \frac{\sigma^2}{2})^2 + 2\sigma^2\phi}}{\sigma^2} > -1$$
$$\eta_2 = \frac{\mu - \frac{3\sigma^2}{2} - \sqrt{(\mu - \frac{\sigma^2}{2})^2 + 2\sigma^2\phi}}{\sigma^2} < -1.$$

The coefficients $\{A_1, A_2, A_3\}$ satisfy the following conditions:

$$m^{i,1}(x_D^i) = 0, \quad m^{i,1}(x_N) = m^{i,2}(x_N), \quad \int_{x_D^i}^{\infty} x m^i(x)dx + \frac{\sigma^2}{2}(x_D^i)^2 F(x_D^i) = 1.$$

The first conditions is the boundary condition at x_D^i. The second condition is the value-matching condition at x_N. The third condition is the normalization condition. To make sure the left-hand side in the third condition is finite, I impose a parameter restriction to have $\eta_2 < -2$. Also, note that the smooth-pasting condition does not hold at x_N because new firms enter the market with new assets of size x_N. More explicitly,

$$A_1(x_D^i)^{\eta_1} + A_2(x_D^i)^{\eta_2} = 0, \tag{16}$$

$$A_1 x_N^{\eta_1} + A_2 x_N^{\eta_2} = A_3 x_N^{\eta_2}, \tag{17}$$

$$\frac{A_1 x_N^{2+\eta_1}}{2+\eta_1} + \frac{A_2 x_N^{2+\eta_2}}{2+\eta_2} - \frac{A_1(x_D^i)^{2+\eta_1}}{2+\eta_1} - \frac{A_2(x_D^i)^{2+\eta_2}}{2+\eta_2} - \frac{A_3 x_N^{2+\eta_2}}{2+\eta_2} + \frac{\sigma^2(A_1\eta_1(x_D^i)^{2+\eta_1} + A_2\eta_2(x_D^i)^{2+\eta_2})}{2(r+\phi-\mu)} = 1. \tag{18}$$

We can solve this system of linear equations explicitly.

Now, I will show that $q^i(p^i)$ decreases in p^i. But since x_D^i decreases in p^i, it suffices to show that $\frac{\sigma^2}{2}(x_D^i)^3 m_x^i(x_D^i)$ increases in x_D^i. The conditions in (16) and (17) imply that

$$A_2 = -A_1(x_D^i)^{\eta_1-\eta_2} \quad \text{and} \quad A_3 = A_1 x_N^{\eta_1-\eta_2} - A_1(x_D^i)^{\eta_1-\eta_2}. \tag{19}$$

These conditions imply that

$$\frac{\sigma^2}{2}(x_D^i)^3 m_x^i(x_D^i) = \frac{\sigma^2}{2} A_1 (x_D^i)^{2+\eta_1}(\eta_1 - \eta_2).$$

Thus, to show the above claim, it suffices to show that $A_1(x_D^i)^{2+\eta_1}$ increases in x_D^i. Now, plugging the conditions in (19) into (18), I can derive

$$A_1(x_D^i)^{2+\eta_1}\left[\frac{(\eta_2-\eta_1)}{(2+\eta_1)(2+\eta_2)}\left(\left(\frac{x_N}{x_D^i}\right)^{2+\eta_1} - 1\right) + \frac{\sigma^2(\eta_1-\eta_2)}{2(r+\phi-\mu)}\right] = 1.$$

Then, the conditions such that $\eta_1 > -1$, $\eta_2 < -2$, and $x_D^i < x_N$ imply that $A_1 > 0$. But then, since

$$\frac{(\eta_2-\eta_1)}{(2+\eta_1)(2+\eta_2)}\left(\left(\frac{x_N}{x_D^i}\right)^{2+\eta_1} - 1\right) + \frac{\sigma^2(\eta_1-\eta_2)}{2(r+\phi-\mu)}$$

decreases in x_D^i, $A_1(x_D^i)^{2+\eta_1}$ must increase in x_D^i. I have therefore shown that $q^i(p^i)$ decreases in p^i.

7.3 Existence of Equilibrium

In this section, I prove that the economy has at least one equilibrium. Without loss of generality, I can assume $f^A > f^B$. First, let $p_1^i = \frac{\alpha_h}{\rho}$ for each i. Then, let q_1^i denote the quantity of an asset liquidated in market i, which is given by formula (11). Here, $q_1^A = q_1^B$ because $p_1^A = p_1^B$. I then consider the following three cases: (i) $q_1^A \leq f^B$, (ii) $f^B < q_1^A \leq f^A$, and (iii) $f^A < q_1^A$. In case (i), because $q_1^A = q_1^B \leq f^B < f^A$, one equilibrium can be given by $(\frac{\alpha_h}{\rho}, \frac{\alpha_h}{\rho})$ in which $g = 0$. In case (iii), because $f^B < f^A < q_1^A = q_1^B$, one equilibrium can be given by $(\frac{\alpha_l}{\rho}, \frac{\alpha_l}{\rho})$ in which $g = 0$. Now, let me consider case (ii). In this case, I further consider the following two cases: (a) $q_1^A - f^B > f^A - q_1^A$ and (b) $q_1^A - f^B \leq f^A - q_1^A$.

In case (a), let

$$p_2^A = \frac{\alpha_l}{\rho} + \kappa \quad \text{and} \quad p_2^B = \frac{\alpha_l}{\rho}. \tag{20}$$

Then, for each i, let q_2^i be the quantity of an asset liquidated in market i for that given price p_2^i. Since $p_2^A > p_2^B$, I have $q_2^A < q_2^B$. Then, first, if $f^A < q_2^A$, then because $f^B < f^A < q_2^A < q_2^B$, one equilibrium can be given by $(\frac{\alpha_l}{\rho}, \frac{\alpha_l}{\rho})$ in which $g = 0$. Second, if $q_2^A \leq f^A$, then since

$0 \leq f^A - q_2^A < q_2^B - f^B$, one equilibrium can be given by $(\frac{\alpha_l}{\rho} + \kappa, \frac{\alpha_l}{\rho})$ in which $g = f^A - q_2^A$.

In case (b), let

$$p_2^A = \frac{\alpha_h}{\rho}, \quad p_2^B = \frac{\alpha_h}{\rho} - \kappa, \quad g = q_1^A - f^B.$$

Then, for each i, let q_2^i be the quantity of an asset liquidated in market i for that given price p_2^i. Since $p_1^A = p_2^A > p_2^B$, I have $q_2^A = q_2^A < q_2^B$. Then, first, if $q_2^B - f^B \leq f^A - q_2^A$, then one equilibrium can be given by $(\frac{\alpha_h}{\rho}, \frac{\alpha_h}{\rho} - \kappa)$ in which $g = q_2^B - f^B$. Second, if $q_2^B - f^B > f^A - q_2^A$, then I can go back to case (a) and then find an equilibrium as done in that case. Hence, I have obtained at least one equilibrium.

In fact, I can similarly apply the above procedure to find another equilibrium, starting with $(\frac{\alpha_l}{\rho}, \frac{\alpha_l}{\rho})$. This equilibrium can be the same as the above equilibrium. Lastly, the economy can have more than two equilibria, depending on the parameter values. As mentioned above, I can find all equilibrium numerically, using the equilibrium construction algorithm described in Section 4.3.1.

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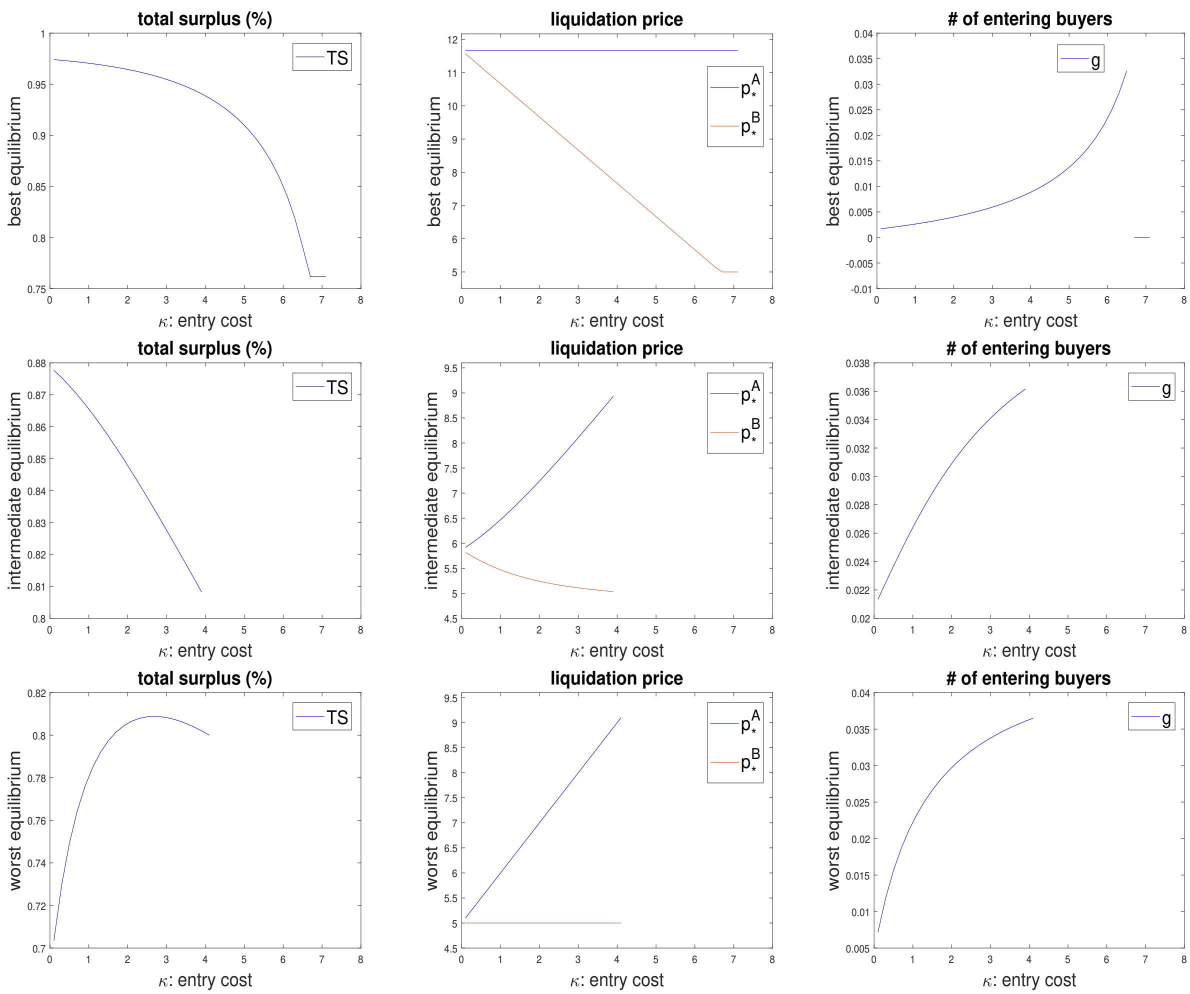

Figure 9: The left three panels show the effect of a change in κ on the total surplus divided by 2, which is the maximum possible total surplus. The middle three panels show the effect of a change in κ on the liquidation prices. The right three panels show the effect of a change in κ on g. The top three panels correspond to the best equilibrium. The middle three panels correspond to the intermediate equilibrium. The bottom three panels correspond to the worst equilibrium.

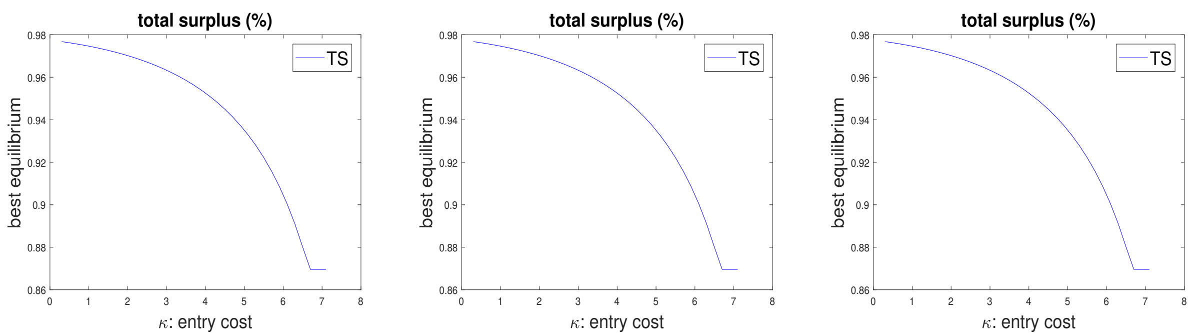

Figure 10: This figure shows the effect of a change in κ when λ is changed to 0.8. Under this value of λ, a unique equilibrium obtains. The left panel shows the effect on the total surplus. The middle panel shows the effect on the liquidation prices. The right panel shows the effect on g.

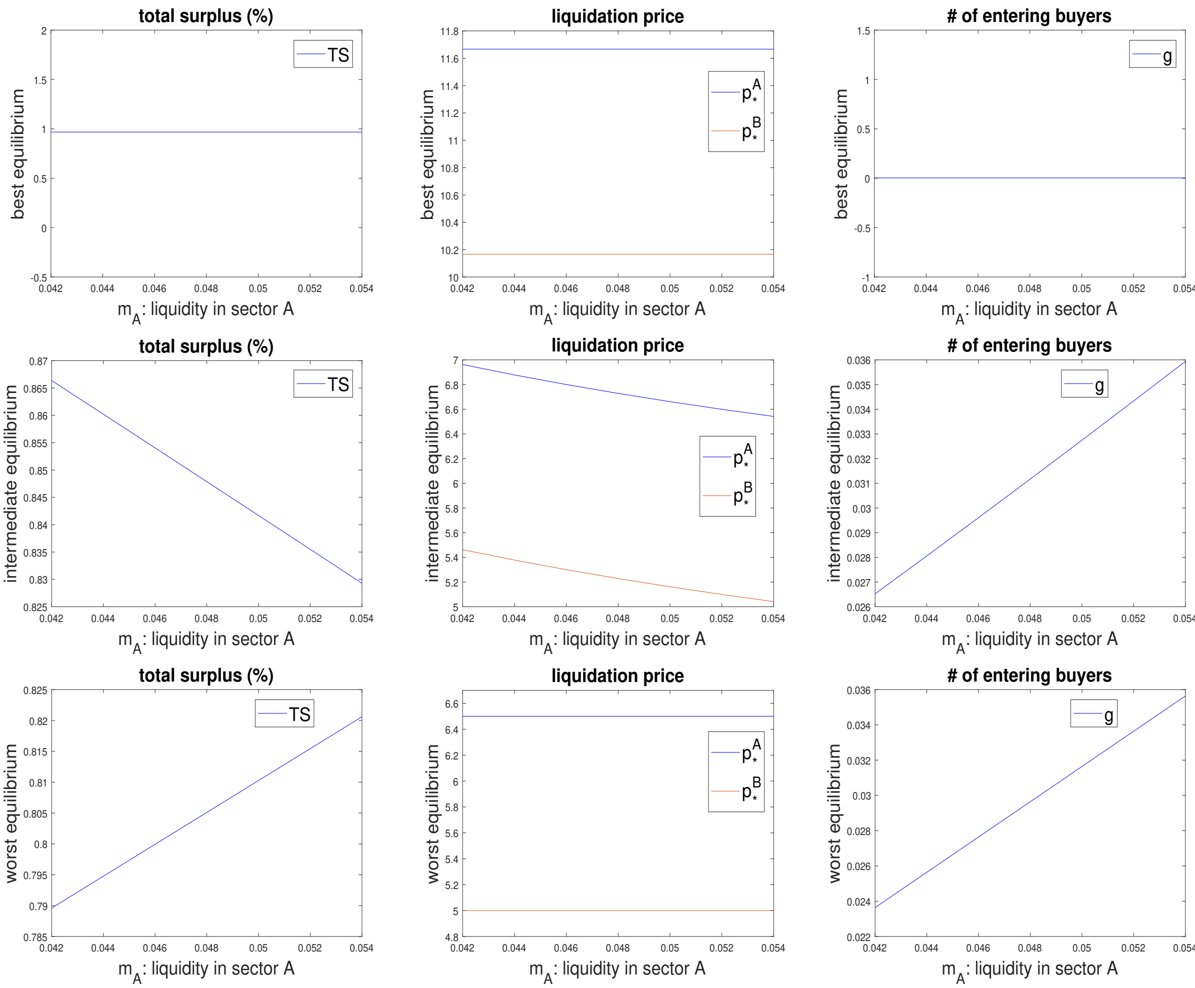

Figure 11: The left three panels show the effect of a change in f^A on the total surplus divided by 2, which is the maximum possible total surplus. The middle three panels show the effect of a change in f^A on the liquidation prices. The right three panels show the effect of a change in f^A on g. The top three panels correspond to the best equilibrium. The middle three panels correspond to the intermediate equilibrium. The bottom three panels correspond to the worst equilibrium.