Clientele, Information Sales, and Asset Prices

Shiyang Huang, Yan Xiong, and Liyan Yang*

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Abstract

We study the joint determination of information sales and equilibrium asset pricing in an economy with two information sellers and one risky asset with multiple uncertainties. We find that the information-market structure (e.g., whether the sold information is similar or distinct, and how sellers and investors share the trading profit) has important implications for seller-investor clientele formation and asset pricing. First, two sellers could have either orthogonal or overlapping clientele, which depends on the similarity of sellers' information. Second, when two sellers' information is distinct and they have large bargaining power in sharing trading profit, investors' information purchase exhibits complementarity, and multiple equilibria could occur. Third, information-market structure has non-trivial implications on asset-market qualities.

Key words: Clientele formation, information sales, complementarity, market quality

*Huang: The University of Hong Kong; Email: huangsy@hku.hk. Xiong: Rotman School of Management, University of Toronto; E-mail: yan.xiong14@rotman.utoronto.ca. Yang: Rotman School of Management, University of Toronto; E-mail: liyan.yang@rotman.utoronto.ca. Part of this paper was previously circulated under the title "(De)Centralization of Information Sales in the Financial Market," and "Two Sellers of Information in Financial Markets." We thank Pat Akey, Ling Cen, Alexander Chinco, Peter Christoffersen, Diego García (discussant), Tom McCurdy, Francesco Sangiorgi, Chester Spatt, and the audiences at the University of Toronto Rotman Seminar and the 2016 CMU credit ratings conference for their comments and suggestions.

1 Introduction

One central topic in financial economics is how information is distributed and incorporated into asset prices. Information markets undoubtedly play an important role in these processes. Modern information markets always feature multiple sellers. For example, multiple sell-side analysts cover the same firm. And Bloomberg and Thomson Reuters provide trading data or accounting information.[1] In addition to multiple sellers, different information markets are featured with different investor composition. For example, institutional investors typically subscribe to both Bloomberg terminal and Thomson Reuters' Eikon, but sell-side analysts have different clientele.[2]

These observations raise a set of interesting questions: What determines the network of relations between information sellers and their clients? How do information sellers, whose information is alike or different, interact with each other? What are the asset-pricing implications of the interaction between different information sellers and between different informed investors? To answer these questions, it is necessary to develop an information-sale framework with multiple sellers. However, the literature has mainly emphasized the monopolistic information market (e.g., Admati and Pfleiderer, 1986, 1987, 1988). We aim to fill this gap.

We develop an information-sale model with multiple information sellers. Our model builds on Admati and Pfleiderer (1986) and extends it to a setting with two information sellers. Our economy has four dates and two assets: one risk-free asset and one risky asset with multiple uncertainties in the underlying payoff. While both sellers are informed about one dimension of uncertainty, each of them is uniquely endowed with information about one specific dimension of uncertainty in the asset payoff. Thus sellers' information can be different from or similar to each other. At the first date, information sellers choose the quality

[1] The example of different information sellers providing similar information is Bloomberg and Thomson Reuters, both of which provide trading and financial data. Alternative data vendors are the example of different information sellers selling orthogonal information, such as DataMinr's tweets analysis and Thinknum's satellite images. In addition, some information sellers provide correlated information, such as sell-side analysts. Specifically, although analysts cover the same stocks and provide analysis on firms' financial statements, they have distinct information sources and/or expertise in analyzing different aspects of firms, such as operation in different sectors or locations (e.g., Bae et al. (2007), Huang (2015), Bradley et al. (2017)).

[2] For example, sell-side analysts serve distinct groups of institutional investors, which is called franchise management by Groysberg (2010) and Maber, Groysberg, and Healy (2014)): Franchise describes the breadth, depth, and importance of an analyst's client ties.

(precision) for the information potentially sold to investors, and investors choose whether to buy information from either one or two sellers. At the second date, each seller-buyer pair determines the information price via Nash bargaining. At the third date, a financial market opens and investors trade the assets based on their information set. At the last date, investors and information sellers get their payoffs and consume. To be emphasized, sellers' information precision, investor composition (e.g., how many investors purchase from either one or two sellers), and asset prices are endogenously determined.

Our model departs from Admati and Pfleiderer (1986) on two important aspects. First, there are two information sellers and their information endowments can be different from or similar to each other's. When the sold information is distinct, our information market can be viewed as the market featuring information specialists, such as sell-side analysts with different expertise. When the sold information is alike, our information market is the one where different sellers competing with each other to provide similar information; for example, both Bloomberg terminal and Thomson Reuters' Eikon provide trading data and financial statements. Second, Admati and Pfleiderer (1986) assume that the monopolistic information seller extracts all surplus, but we assume that investors have bargaining power when negotiating the information price with the information seller. The Nash bargaining between the information seller and the investor captures the features of financial markets. For example, mutual/hedge funds use broker votes to determine compensation to brokerage firms for their services, particularly information services from sell-side analysts.

Our main results are about how the information-market structure affects the investor composition and the asset pricing. There are two key driving forces of our economy: the relationship between the information sold by the two sellers (e.g., whether one seller's information is a substitute or complement to the other's), and the relationship between the information seller and the investor (e.g., how the seller and the investor split the informed trading rents). The interaction of the two forces delivers novel insights on investor composition and asset prices.

First, when the sold information is complementary, sellers' bargaining power plays a crucial role in shaping the equilibrium investor composition. While investors purchase both information when sellers have small bargaining power, they only purchase one piece of information when sellers have large bargaining power. Why wouldn't investors always purchase both pieces of information given that they are complements? This is due to the "marginal price" charged by information sellers in the Nash Bargaining game. Specifically, when the two pieces of information are complements, the benefit of observing a second piece of information for one investor is larger than that of observing this information in the first place for the same investor, thus the information seller charges a premium price for the investor who has bought the other information. This constitutes the marginal price for the investors who purchase both information. At the same time, to observe both information is at least as good as to know only one piece of information, and this is the marginal gain for the investors who purchase from both sellers. Therefore the investors make the purchase decision by comparing the marginal gain and marginal price. When the sellers have small bargaining power, the marginal gain exceeds the marginal price so that investors find it worth to purchase both information. By contrast, when the sellers have large bargaining power, the marginal price is so high that investors would rather purchase only one piece of information even though the two pieces of information are complements.

Second, when the sold information is complementary, investors' information purchase can be either substitutes or complements. To differentiate the two types of complements/substitutes, we term the one between different information *cross-signal complementarity/substitutability*, which is from the perspective of one investor, and the one between different investors *cross-agent complementarity/substitutability.* To the best of our knowledge, we are the first to study the two types of complementarity/substitutability together, and show that cross-signal complementarity and cross-agent complementarity are inherently connected to each other.[3]

Specifically, when the sold information is complementary, as more investors know the two dimensions of uncertainty, the marginal gain of purchasing the second information decreases due to more intense competition between investors. Similarly, the marginal price decreases as well, because with the shrinking marginal gain the seller can only charge a lower price. When the former (latter) effect prevails investors' information purchase exhibits

[3] To be specific, the cross-signal complementarity/substitutability deals with the following question: If investors are informed about one dimension of uncertainty, will they have the tendency to learn about other dimensions, or are they less willing to do so? And the cross-agent complementarity/substitutability addresses the following question: If some investors are informed about more dimensions of uncertainty, will this encourage other investors to learn more about these dimensions, or discourage them from doing so?

substitutability (complementarity). Particularly, when sellers' bargaining power lies within some intermediate range, there can exist multiple equilibria, which lead to market fragility in the economy.

Third, when the sold information is substitute, investors purchase both information. How come investors purchase two pieces of substitute information? Again, this is because of the "marginal price" charged by information sellers. Unlike the case when the sold information is complementary, when the information is substitute, the benefit of observing a second piece of information for an investor is less than that of observing this information in the first place. Thus the information sellers charges a lower price for the investors who have bought the other information than for those who have not (i.e., the marginal price is negative). Such a price discount plus the marginal gain of observing the second information attract investors to purchase both information.

Moreover, when the sold information is substitute, investors' information purchase always exhibits cross-agent substitutability. The intuition is as follows. As more investors know the two dimensions of uncertainty, the marginal gain of purchasing the second information decreases due to investor competition. However, unlike the case where the sold information is complementary, the marginal price increases since the price discount offered by the seller decreases. Taken together, as more investors know the two dimensions of uncertainty, other investors are less willing to do so.

Fourth, we have shown how the information-market structure affects the investor composition, and one natural question follows: What are the asset-pricing implications of such an information market? The answer depends on whether the two pieces of information are substitutes or complements, and whether the sellers have large or small bargaining power. When the sold information is complementary, comparing the scenarios with high and low sellers' bargaining power, the sold information is more precise when sellers have large bargaining power, but price informativeness is lower. This is because when sellers have large bargaining power, investors only purchase one piece of information as discussed above. Such clientele segmentation prevents information from being leaked via price. Thus, although sellers sell more precise information, the price informativeness is lower. Accordingly, the liquidity is lower, and the cost of capital and the return volatility are higher in the financial market. By contrast, when the sold information is substitute, the competition forces information sellers to sell very precise information, leading to high price informativeness. Consequently, the financial market features high liquidity, low cost of capital and low return volatility.

Related Literature Our paper contributes to the literature on information sales, in particular, direct sales, in the financial market, by proposing an information-sales framework with two sellers, and studying the asset-pricing implications. We here only summarize a few studies that are most closely related to ours.[4] The classical study by Admati and Pfleiderer (1986) models a monopolist information-sale problem in a competitive market and finds that in order to overcome the dilution in the value of information due to its leakage through informative prices, the seller prefers to sell noisier versions of the information he has. Our analysis nests Admati and Pfleiderer (1986) as a special case, because they only consider the case where the single seller has full bargaining power. Simonov (1999) extends the monopolist framework of Admati and Pfleiderer (1986) to a duopoly, but the asset payoff in his setting does not feature multiple dimensions of uncertainty, which is crucial to generating our results through cross-signal substitutability and complementarity.

García and Sangiorgi (2011) extend Admati and Pfleiderer (1986) to non-competitive markets and show that the contracts that arise endogenously as the solution to the information-sale problem resemble the dichotomy observed in actual markets for information, an imprecise newsletter contract or a precise exclusivity contract. They find that the optimal contract depends on the level of noise trading in the financial market. Our study differs from and complements García and Sangiorgi (2011) in three important ways. First, there is only one monopolistic seller in García and Sangiorgi (2011), while we have considered more than one information seller. Second, the trade-off is different. The result in García and Sangiorgi (2011) is driven by a trade-off between profit maximization and risk sharing, while in our setting, the trade-off is between the marginal gain of observing the second information and the marginal price paid for it. Third, in García and Sangiorgi (2011), the seller has full bargaining power in pricing information, while we allow buyers to have bargaining power.

[4]Other information-sale work includes Admati and Pfleiderer (1987, 1988), Allen (1990), Fishman and Hagerty (1995), Naik (1997), and Cespa (2008). Admati and Pfleiderer (1990) and García and Vanden (2009) have considered indirect sales where information sellers set up funds and sell funds shares.

Our paper also contributes to the literature on information complementarity, such as Dow, Goldstein, and Guembel (2017), Froot, Scharfstein and Stein (1992), García and Strobl (2011), Goldstein, Li, and Yang (2013), Goldstein and Yang (2015), Veldkamp (2006), and Huang (2016). In particular, our study is closely related to those studies that generate complementarity through multiple dimensions of uncertainty.[5] Admati and Pfleiderer (1987) define two signals to be complements (substitutes) if the benefit of observing the two signals simultaneously is greater (smaller) than the sum of the benefits of observing the two signals in isolation. Based on whether signals are complements or substitutes, Admati and Pfleiderer (1987) then analyze under what circumstances information will be concentrated in the hands of a few investors rather than spread out across many investors. Ganguli and Yang (2009) show that information about noise supply in the financial market increases coordination possibilities, thereby leading to multiple equilibria in both financial and information markets. Goldstein and Yang (2015) specify that different investors in the financial market have information about different fundamentals in the asset payoff and study how the diverse information among investors affects the trading volume and informational efficiency. In our paper, the two signals observed by investors can be complements (which is similar to Admati and Pfleiderer (1987) and Goldstein and Yang (2015)) or substitutes. Nonetheless, these prior studies do not consider the implications of this complementarity for information sales.

The remaining of the paper is structured as follows. Section 2 introduces the model. Section 3 characterizes the equilibrium. Section 4 examines interplay of cross-signal complementarity/substitutability and cross-agent complementarity/substitutability in the economy. Section 5 studies the asset-market implications. Section 6 concludes. All proofs are relegated to the Appendix.

[5] In the finance literature, it is common to assume that the value of the traded security is affected by more than one fundamental (e.g., Froot, Scharfstein, and Stein (1992), Goldman (2005), Kondor (2012), and Goldstein and Yang (2015), among others).

2 The Model

This section describes an information-sale model with two sellers, based on the standard Admati and Pfleiderer (1986) setting. The economy has four dates, $t = 0, 1, 2$, and 3. At date 0, two information sellers decide their information quality and investors choose their information seller(s). Specifically, date 0 is the formation date of information sellers' clientele network. At date 1, information prices are determined through Nash bargaining between each pair of investor and information seller. At date 2, investors trade on their information in a financial market. At date 3, assets pay off and all agents consume. For ease of reference, main model variables are tabulated and explained in an appendix. Figure 1 sketches the timeline of the economy.

[Figure 1 About Here]

Assets There are two tradable assets in the date-2 financial market: a risk-free asset and a risky asset. The risk-free asset has a constant return of 0 and is in unlimited supply. The risky asset is traded at an endogenous price $\tilde{p}$ per unit in the date-2 financial market, and its total supply is normalized as one share. Specifically, the risky asset pays an uncertain cash flow $\tilde{v}$ at date 3 as follows:

$$\tilde{v} \equiv \tilde{v}_0 + \tilde{v}_1 + \tilde{v}_2, \tag{1}$$

where $\tilde{v}_0 \sim N\left(0, \tau_0^{-1}\right)$, $\tilde{v}_n \sim N\left(0, \tau_v^{-1}\right)$ $(n = 1, 2)$ with $\tau_0, \tau_v \in (0, \infty)$, and $\tilde{v}_0$, $\tilde{v}_1$ and $\tilde{v}_2$ are mutually independent. There is random noise trader demand $\tilde{u}$ for the risky asset, where $\tilde{u} \sim N\left(0, \tau_u^{-1}\right)$ with $\tau_u \in (0, \infty)$. Noisy trading $\tilde{u}$ is independent of all other random variables in the economy, and it serves the usual role of preventing private information from being fully revealed.

Information Sellers There are two information sellers, indexed by $n = 1, 2$. Seller n is endowed with information $\tilde{v}_0 + \tilde{v}_n$ at the start of the economy. Thus, variable $\tilde{v}_0$ represents the overlap between the two sellers' endowed information, while $\tilde{v}_n$ is seller-specific information. In terms of predicting the total asset payoff $\tilde{v}$, we can measure the contribution of the two seller-specific components $\tilde{v}_1$ and $\tilde{v}_2$ as follows:

$$k \equiv \frac{Var(\tilde{v}_1 + \tilde{v}_2)}{Var(\tilde{v})} = \frac{2\tau_0}{\tau_v + 2\tau_0} \in [0, 1].$$

Parameter k measures the degree of product differentiation. Unlike the standard industrial-organization literature which models product differentiation based on consumer preference (e.g., Singh and Vives, 1984), we here capture product differentiation based on the primitive information endowed to information sellers. As k becomes higher (e.g., due to a lower τ_v), information sellers' endowed information overlaps less, and thus the information supplied by the two sellers is more likely to be different.

After observing $\tilde{v}_0 + \tilde{v}_n$, at date 1, seller n sells this information to investors to maximize profits. We follow Admati and Pfleiderer (1986) and García and Sangiorgi (2011) and focus on the sales of personalized information. That is, seller n sells investor i a signal $\tilde{s}_{ni}$ of the form

$$\tilde{s}_{ni} = \tilde{v}_0 + \tilde{v}_n + \tilde{\varepsilon}_{ni}, \tag{2}$$

where $\tilde{\varepsilon}_{ni} \sim N\left(0, \tau_{\varepsilon_n}^{-1}\right)$ with $\tau_{\varepsilon_n} \geq 0$ (for $n = 1, 2$), and $(\tilde{v}_1, \tilde{v}_2, \{\tilde{\varepsilon}_{1i}, \tilde{\varepsilon}_{2i}\}_i)$ are mutually independent. As in Admati and Pfleiderer (1986) and García and Sangiorgi (2011), we focus on the case in which the signals sold by seller n have the same precision τ_{ε_n}. Before selling information at date 1, information seller n chooses τ_{ε_n} at date 0.

The signal form (2) can be justified theoretically as a form of rational inattention. Specifically, as in Myatt and Wallce (2012), investor i needs to pay attention in order to interpret the information sent by seller n. The personalized noise $\tilde{\varepsilon}_{ni}$ corresponds to the "receiver noise," and the precision τ_{ε_n} chosen by seller n reflects the clarity with which the information is imparted to investors. In practice, as Admati and Pfleiderer (1986) and García and Sangiorgi (2011) point out, information sellers can intentionally pass on the data to investors in a vague way, so that investors make independent interpretations of the information.

Investors There exists a continuum of investors, indexed by $i \in [0, 1]$. Investors select information seller(s) at date 0, purchase information at date 1, make financial investments at date 2, and consume at date 3. All investors derive expected utility over their date-3 wealth according to a constant absolute risk aversion (CARA) utility with a common risk aversion coefficient γ.

At date 0, investors and information sellers form networks (termed information-clientele networks). Specifically, investors are categorized into four types: (i) type-0 investors, who do not purchase any information; (ii) type-1 investors, who only purchase information from seller 1; (iii) type-2 investors, who only purchase information from seller 2; and (iv) type-12 investors, who purchase information from both sellers. Let λ_0, λ_1, λ_2, and λ_{12} be the mass of the four types of investors, respectively, where $\lambda_0, \lambda_1, \lambda_2, \lambda_{12} \in [0,1]$ and $\lambda_0+\lambda_1+\lambda_2+\lambda_{12} = 1$. We denote the investor composition by $\Lambda \equiv (\lambda_1, \lambda_2, \lambda_{12})$. We also assume that at date 0, information sellers decide information precision τ_{ε_n} and information-clientele networks form simultaneously.

Once the clientele network is formd at date 0, it cannot be changed. This assumption is motivated by the fact that to establish and maintain a network relation cost resources and time. In our model, we simply assume that the monetary cost is negligible,[6] but time cost is still there: the network can only be formed at the beginning of the economy, and once formed, it can never be altered.

At date 1, sellers and their own clientele engage in long-term bargaining. Specifically, seller n and investor i form a pair, say, denoted by (n, i), where $n \in \{1, 2\}$ and $i \in [0, 1]$. Each pair engages in Nash bargaining and the bargaining outcome yields the equilibrium information price $q^i_{n,n}$ if the investor only purchases one signal and $q^i_{n,12}$ if the investor purchases both signals. Given that investors are idential, the equilibrium price $q^i_{n,n}$ $\left(q^i_{n,12}\right)$ is the same across i; that is, $q^i_{n,n} = q_{n,n}$ and $q^i_{n,12} = q_{n,12}$ for $i \in [0, 1]$. We thus denote information prices $q = [q_{1,1}, q_{1,12}, q_{2,2}, q_{2,12}]'$.

At date 2, investors receive their purchased signals and trade in a financial market. As standard in the rational-expectations equilibrium literature, investors submit demand schedules conditional on their information sets including the risky asset price. Formally, investor i chooses demand x_i for the risky asset to maximize $E\left[-e^{-\gamma x_i(\tilde{v}-\tilde{p})}|\mathcal{F}_i\right]$, where $\mathcal{F}_i$ indicates investor i's information set $\mathcal{F}_i$ (including the asset price $\tilde{p}$).

[6] If we also consider monetary cost, there can be uninformed traders active in equilibrium, which is not the focus of our paper.

Discussion of assumptions We briefly discuss our assumptions. First, our model is featured with multiple dimensions in the asset payoff. Variables $\tilde{v}_0$, $\tilde{v}_1$ and $\tilde{v}_2$ represent different dimensions of uncertainty in the risky asset's payoff. For example, firms' cash flows are affected by their customers, competitors, productivity, macro conditions, different business lines or political environment. Although this multi-dimensional uncertainty structure in (1) is realistic, we notice that it is a modeling device to generate information complementarity and substitutability as in Goldstein and Yang (2015).

Secondly, while the existing literature typically focuses on monopolistic information market (e.g., Admati and Pfleiderer (1986, 1987, 1988)), it is very common to observe multiple information sellers in the financial market, providing similar or differentiated information goods. We thus introduce two information sellers to model the interaction between information sellers. In practice, the overlapped information component $\tilde{v}_0$ represents the financial statements that are accessible to all sell-side analysts, while the seller-specific component $\tilde{v}_n$ ($n = 1, 2$) may arise from individual analyst's expertise. For example, for conglomerate firms operating in multiple industries, some of their sell-side analysts have expertise in one of the industries, while some other analysts specialize in other industries (e.g., Bradley et al. (2017)). Additionally, except for their target firm, some analysts also cover its customers, while other analysts also study its competitors (e.g., Cohen and Frazzini (2008)). For the multinational firms, some analysts are familiar with their business in one country, while some other analysts are familar with their business in other countries (e.g., Bae et al. (2007)).

Thirdly, we use Nash bargaining to determine information prices, which is very relevant in the sell-side analyst setting. In most cases the buy-side compensates the sell-side indirectly for the information service, either by commission fee through broker votes, or by trading allocation to the brokerage firms, which suggests the bargaining power of the buy-side in the information market. For example, Maber et al. (2014) demonstrate that institutional investors can exert great influence on the sell-side analysts in the form of broker votes. In our model, after the information-clientele network is formed, the seller and the client bargain over the information price. Nash bargaining is thus applied to reflect the long-term relationship between information seller and buyer to split the informed trading rents.

In addition, information prices in our model depend not only on the information quality, but also on investors' identity. In other words, information sellers are able to discriminate their clients.[7] In practice, fund size (i.e., assets under management) can serve as a "discrimination device" for information sellers. For example, large funds typically charge the information price as if they are served by a lot of sell-side analysts or purchase a variety of data sources.

3 Equilibrium Characterization

The equilibrium in our economy is defined as follows.

Definition 1 *The equilibrium in the economy is characterized by investor composition* $\Lambda = \{\lambda_1, \lambda_2, \lambda_{12}\}$*, information precision* $\{\tau_{\varepsilon_1}, \tau_{\varepsilon_2}\}$*, signal prices* $q(\tau_{\varepsilon_1}, \tau_{\varepsilon_2}, \Lambda)$*, a linear asset price function* $\tilde{p} = \alpha_\emptyset + \alpha_0\tilde{v}_0 + \alpha_1\tilde{v}_1 + \alpha_2\tilde{v}_2 + \alpha_u\tilde{u}$*, and demand functions* x_i *(*$i \in [0, 1]$*) such that:*

1. At date 0*, given* τ_{ε_1} *and* τ_{ε_2}*, each investor optimally chooses whether or not to purchase information, and if yes, how many signals to purchase to maximize ex-ante expected utility, and given* Λ*, information seller* n *(*$n = 1, 2$*) optimally chooses the information precision* τ_{ε_n} *(i.e., how noisy the sold information is) and both sellers act simultaneously, to maximize the expected profit;*

2. At date 1*, information prices* $q_{n,n}$ *and* $q_{n,12}$ $(n = 1, 2)$ *are determined in Nash bargaining games;*

3. At date 2*, (i) the demand function* x_i *maximizes investor* i*'s expected utility; (ii) the linear price function* $\tilde{p}$ *clears the financial market:*

$$\int_{i=0}^{1} x_i di + \tilde{u} = 1. \tag{3}$$

We use the backward induction to solve this model. At date 2, given investors composition Λ, information precision $\{\tau_{\varepsilon_1}, \tau_{\varepsilon_2}\}$ and signal prices $q(\tau_{\varepsilon_1}, \tau_{\varepsilon_2}, \Lambda)$, we solve for investors' demand schedules and price function. At date 1, given Λ, and information precision

[7] This has the similar feature to the cross-referential contracts. For example, in many settings, firms do not charge a fixed price, but instead make their prices explicitly conditional on the prices offered by competitors. Examples include meet-the-competition clauses, price-beating promises, and lowest fare guarantees (Szentes, 2014).

$\{\tau_{\varepsilon_1}, \tau_{\varepsilon_2}\}$, we derive information prices through Nash Bargaining games between each pair of information seller and investor. Next we solve for equilibrium investor composition Λ and information precision $\{\tau_{\varepsilon_1}, \tau_{\varepsilon_2}\}$ simultaneously at date 0. We focus on symmetric equilibrium. That is, both information sellers choose the same information quality (i.e., $\tau_{\varepsilon_1} = \tau_{\varepsilon_2}$), and have the same amount of clients (i.e., $\lambda_1 = \lambda_2$) in equilibrium.

3.1 Financial-Market Equilibrium

In the financial market, the equilibrium concept we use is the noisy rational-expectation equilibrium (NREE), as in Grossman and Stiglitz (1980) and Hellwig (1980). Following the literature, we conjecture that the price is a linear function of all random variables in this economy: $\tilde{v}_0, \tilde{v}_1, \tilde{v}_2$ and $\tilde{u}$. The conjectured price function is given by:

$$\tilde{p} = \alpha_{\emptyset} + \alpha_0 \tilde{v}_0 + \alpha_1 \tilde{v}_1 + \alpha_2 \tilde{v}_2 + \alpha_u \tilde{u}, \tag{4}$$

where the α coefficients are endogenously determined in equilibrium.

Investor i chooses the demand schedule x_i to maximize his expected utility conditional on the information set $\mathcal{F}_i$. Given the CARA-normal setup, the corresponding demand schedules for different types of investors are as follows:

$$\text{Type-0 Investors} \quad : \quad x(\tilde{p}) = \frac{E(\tilde{v}|\tilde{p}) - \tilde{p}}{\gamma Var(\tilde{v}|\tilde{p})}, \tag{5}$$

$$\text{Type-1 Investors} \quad : \quad x(\tilde{s}_{1i}, \tilde{p}) = \frac{E(\tilde{v}|\tilde{s}_{1i}, \tilde{p}) - \tilde{p}}{\gamma Var(\tilde{v}|\tilde{s}_{1i}, \tilde{p})} \tag{6}$$

$$\text{Type-2 Investors} \quad : \quad x(\tilde{s}_{2i}, \tilde{p}) = \frac{E(\tilde{v}|\tilde{s}_{2i}, \tilde{p}) - \tilde{p}}{\gamma Var(\tilde{v}|\tilde{s}_{2i}, \tilde{p})}, \tag{7}$$

$$\text{Type-12 Investors} \quad : \quad x(\tilde{s}_{1i}, \tilde{s}_{2i}, \tilde{p}) = \frac{E(\tilde{v}|\tilde{s}_{1i}, \tilde{s}_{2i}, \tilde{p}) - \tilde{p}}{\gamma Var(\tilde{v}|\tilde{s}_{1i}, \tilde{s}_{2i}, \tilde{p})}, \tag{8}$$

where $E(\tilde{v}|\cdot)$, $Var(\tilde{v}|\cdot)$ are the conditional expectations and variances given the corresponding information set. The market-clearing condition is:

$$\int_0^{\lambda_1} x(\tilde{s}_{1i}, \tilde{p})\, di + \int_{\lambda_1}^{\lambda_1+\lambda_2} x(\tilde{s}_{2i}, \tilde{p})\, di + \int_{\lambda_1+\lambda_2}^{\lambda_1+\lambda_2+\lambda_{12}} x(\tilde{s}_{1i}, \tilde{s}_{2i}, \tilde{p})\, di + \int_{\lambda_1+\lambda_2+\lambda_{12}}^{1} x(\tilde{p})\, di + \tilde{u} = 1. \tag{9}$$

Because the equilibrium price aggregates all investors' information, investors could infer information from the equilibrium price $\tilde{p}$. Specifically, with $\alpha_u \neq 0$ (which we verified in equilibrium), the price $\tilde{p}$ is equivalent to the following signal in predicting the fundamental value $\tilde{v}$:

$$\tilde{s}_p \equiv \frac{\tilde{p} - \alpha_\emptyset}{\alpha_u} = a_0\tilde{v}_0 + a_1\tilde{v}_1 + a_2\tilde{v}_2 + \tilde{u}, \tag{10}$$

where

$$a_n = \frac{\alpha_n}{\alpha_u}, \text{ for } n = 0, 1, 2,$$

is the ratio between the sensitivity of the price to fundamentals $\tilde{v}_n$ and the sensitivity of the price to noise $\tilde{u}$. Higher a_n indicates that the price is more informative about the fundamental component $\tilde{v}_n$.

We then apply Bayes' rule to compute the conditional moments $E(\tilde{v}|\cdot)$ and $Var(\tilde{v}|\cdot)$, which are in turn plugged into the demand functions (5)-(8) and the market-clearing condition (9), to compute the price $\tilde{p}$ as a function of $\tilde{v}_0$, $\tilde{v}_1$, $\tilde{v}_2$ and $\tilde{u}$. Comparing with the conjectured price function in equation (4), we obtain a system defining the unknown α coefficients. Solving this system yields the following proposition characterizing the equilibrium in the asset market.

Proposition 1 (Financial-market equilibrium) *Given symmetric investor composition $\lambda_1 = \lambda_2 \equiv \lambda$, there exists a linear price $\tilde{p} = \alpha_\emptyset + \alpha_0\tilde{v}_0 + \alpha_1\tilde{v}_1 + \alpha_2\tilde{v}_2 + \alpha_u\tilde{u}$ to clear the asset market. The α coefficients are as follows*

$$\begin{aligned}
\alpha_\emptyset &= -\frac{\gamma}{\frac{\lambda_{12}}{Var(\tilde{v}|\tilde{s}_{1i},\tilde{s}_{2i},\tilde{p})} + \frac{\lambda}{Var(\tilde{v}|\tilde{s}_{1i},\tilde{p})} + \frac{\lambda}{Var(\tilde{v}|\tilde{s}_{2i},\tilde{p})} + \frac{\lambda_0}{Var(\tilde{v}|\tilde{p})}}, && (11)\\
\alpha_u &= \frac{\gamma + \frac{\lambda_{12}\Theta_{12}}{Var(\tilde{v}|\tilde{s}_{1i},\tilde{s}_{2i},\tilde{p})} + \frac{\lambda\Theta_1}{Var(\tilde{v}|\tilde{s}_{1i},\tilde{p})} + \frac{\lambda\Theta_2}{Var(\tilde{v}|\tilde{s}_{2i},\tilde{p})} + \frac{\lambda_0\Theta_0}{Var(\tilde{v}|\tilde{p})}}{\frac{\lambda_{12}}{Var(\tilde{v}|\tilde{s}_{1i},\tilde{s}_{2i},\tilde{p})} + \frac{\lambda}{Var(\tilde{v}|\tilde{s}_{1i},\tilde{p})} + \frac{\lambda}{Var(\tilde{v}|\tilde{s}_{2i},\tilde{p})} + \frac{\lambda_0}{Var(\tilde{v}|\tilde{p})}}, && (12)\\
\alpha_1 &= \alpha_u a_1, && (13)\\
\alpha_2 &= \alpha_u a_2, && (14)\\
\alpha_0 &= \alpha_u (a_1 + a_2) && (15)
\end{aligned}$$

where $\Theta_0, \Theta_1, \Theta_2, \Theta_{12}$ and conditional moments $Var(\tilde{v}|\cdot)$ are shown in the Appendix, and a_1 and a_2 are the solution to the following equation system:

$$\left\{\begin{array}{l} a_1 = \frac{\tau_{\varepsilon_1}(\tau_0+\tau_v)}{\gamma}\left(\begin{array}{c} \frac{\lambda[\tau_v+a_2\tau_u(a_2-a_1)]}{a_1^2\tau_u(\tau_0+\tau_v)+a_2^2\tau_u\left(\tau_0+\tau_v+\tau_{\varepsilon_1}\right)+(\tau_0+\tau_v)\left(\tau_v+\tau_{\varepsilon_1}\right)+\tau_0\tau_v-2a_1a_2\tau_0\tau_u} \\ +\frac{\lambda_{12}\left[\tau_v+\tau_{\varepsilon_2}+a_2\tau_u(a_2-a_1)\right]}{a_1^2\tau_u\left(\tau_0+\tau_v+\tau_{\varepsilon_2}\right)+a_2^2\tau_u\left(\tau_0+\tau_v+\tau_{\varepsilon_1}\right)+\left(\tau_v+\tau_{\varepsilon_1}\right)\left(\tau_v+\tau_{\varepsilon_2}\right)+\tau_0\left(2\tau_v+\tau_{\varepsilon_1}+\tau_{\varepsilon_2}\right)-2a_1a_2\tau_0\tau_u} \end{array}\right) \\ a_2 = \frac{\tau_{\varepsilon_2}(\tau_0+\tau_v)}{\gamma}\left(\begin{array}{c} \frac{\lambda[\tau_v+a_1\tau_u(a_1-a_2)]}{a_1^2\tau_u\left(\tau_0+\tau_v+\tau_{\varepsilon_2}\right)+a_2^2\tau_u(\tau_0+\tau_v)+(\tau_0+\tau_v)\left(\tau_v+\tau_{\varepsilon_2}\right)+\tau_0\tau_v-2a_1a_2\tau_0\tau_u} \\ +\frac{\lambda_{12}\left[\tau_v+\tau_{\varepsilon_1}+a_1\tau_u(a_1-a_2)\right]}{a_1^2\tau_u\left(\tau_0+\tau_v+\tau_{\varepsilon_2}\right)+a_2^2\tau_u\left(\tau_0+\tau_v+\tau_{\varepsilon_1}\right)+\left(\tau_v+\tau_{\varepsilon_1}\right)\left(\tau_v+\tau_{\varepsilon_2}\right)+\tau_0\left(2\tau_v+\tau_{\varepsilon_1}+\tau_{\varepsilon_2}\right)-2a_1a_2\tau_0\tau_u} \end{array}\right) \end{array}\right. . \tag{16}$$

In particular, given the symmetric equilibrium ($\tau_{\varepsilon_1} = \tau_{\varepsilon_2}$), when $\lambda = 0$ or $\lambda_{12} = 0$, there exists a unique linear price function.

Proof. See the Appendix. ■

3.2 Information-Price Equilibrium

We follow Admati and Pfleiderer (1987) and define the "value of information" as the maximum amount (in terms of foregone consumption) an investor would pay for the opportunity to see the information before trading. That is, the information value is the difference between the certainty equivalent of an investor with the information and that of an investor without it. So, the value of the signal $\tilde{s}_{1i}$ would be the difference between the certainty equivalent of a type-1 investor and that of an uninformed investor who can only observe price $\tilde{p}$. We can define the value of observing the signal $\tilde{s}_{2i}$ and of observing the two signals $\{\tilde{s}_{1i}, \tilde{s}_{2i}\}$ similarly. Denote the value of information $\{\tilde{s}_{1i}\}$, $\{\tilde{s}_{2i}\}$ and $\{\tilde{s}_{1i}, \tilde{s}_{2i}\}$ respectively ϕ_1, ϕ_2 and ϕ_{12}. The expressions of information values are given by:

$$\phi_{12}\left(\tau_{\varepsilon_1}, \tau_{\varepsilon_2}; \Lambda\right) = \frac{1}{2\gamma}\ln\left[\frac{Var\left(\tilde{v}|\tilde{p}\right)}{Var\left(\tilde{v}|\tilde{s}_{1i}, \tilde{s}_{2i}, \tilde{p}\right)}\right], \tag{17}$$

$$\phi_{1}\left(\tau_{\varepsilon_1}, \tau_{\varepsilon_2}; \Lambda\right) = \frac{1}{2\gamma}\ln\left[\frac{Var\left(\tilde{v}|\tilde{p}\right)}{Var\left(\tilde{v}|\tilde{s}_{1i}, \tilde{p}\right)}\right], \tag{18}$$

$$\phi_{2}\left(\tau_{\varepsilon_1}, \tau_{\varepsilon_2}; \Lambda\right) = \frac{1}{2\gamma}\ln\left[\frac{Var\left(\tilde{v}|\tilde{p}\right)}{Var\left(\tilde{v}|\tilde{s}_{2i}, \tilde{p}\right)}\right], \tag{19}$$

where the conditional variances $Var\left(\tilde{v}|\tilde{p}\right)$, $Var\left(\tilde{v}|\tilde{s}_{1i}, \tilde{s}_{2i}, \tilde{p}\right)$, $Var\left(\tilde{v}|\tilde{s}_{1i}, \tilde{p}\right)$, and $Var\left(\tilde{v}|\tilde{s}_{2i}, \tilde{p}\right)$ are given in (B1)-(B4) in the Appendix.

Recall that information prices are set through Nash bargaining games between a seller and an investor independently, which happen after the information-clientele network is formed. The bargaining outcome depends on agents' utility in the events of agreement versus no agreement. Seller n bargains with each investor given the investor identity, the information precision τ_{ε_n} and the other seller's choice $\{\tau_{\varepsilon_{n'}}, q_{n',n'}, q_{n',12}\}$ ($n \neq n'$ and $n, n' \in \{1, 2\}$). For the type-n investor, the benefit of the investor in agreement with seller n of information price $q_{n,n}$ is $\phi_n - q_{n,n}$. If no agreement is reached, the investor's outside option is to invest as an uninformed investor. Thus, the additional benefit of the investor staying in the agreement is $\phi_n - q_{n,n}$. Meanwhile, the seller n obtains $q_{n,n}$ after reaching the agreement with one type-n investor, while he obtains nothing if the agreement is not reached. Thus, the marginal profit of seller n in one specific agreement is $\pi_n = q_{n,n}$. The bargaining outcome maximizes the product of the gains from agreement, i.e.,

$$q_{n,n} = \arg\max_{q_{n,n}} \left(q_{n,n}\right)^{\beta} \left(\phi_n - q_{n,n}\right)^{1-\beta}. \tag{20}$$

Solving the problem in (20) yields the information price charged by seller n for the type-n investor:

$$q_{n,n} = \beta\phi_n. \tag{21}$$

For the type-12 investor, given the existence of the other signal, the benefit of the investor in agreement with seller n of information price $q_{n,12}$ is $\phi_{12} - q_{n,12} - q_{n',12}$. If no agreement is reached, the investor's outside option is to invest only knowing $\tilde{s}_{n'i}$ with the benefit $\phi_{n'} - q_{n',12}$. Thus, the additional benefit of the investor staying in the agreement is $\phi_{12} - \phi_{n'} - q_{n,12}$. The marginal profit of seller n in the agreement is $\pi_n = q_{n,12}$. The bargaining outcome maximizes the product of the gains from agreement, i.e.,

$$q_{n,12} = \arg\max_{q_{n,12}} \left(q_{n,12}\right)^{\beta} \left(\phi_{12} - \phi_{n'} - q_{n,12}\right)^{1-\beta}. \tag{22}$$

Solving the problem in (22) yields the information price charged by seller n for the type-12 investor:

$$q_{n,12} = \beta\left(\phi_{12} - \phi_{n'}\right). \tag{23}$$

Note that the equilibrium information price for the type-12 investor is equal to the "marginal value" of the additional signal given that the investor is already informed of the signal offered by the other seller. The following proposition summarizes the equilibrium information prices.

Proposition 2 (Information-price equilibrium) *Seller n sells investor i signal $\tilde{s}_{ni}$ at the price $q_{n,n}$ if the investor only purchases information from seller n, and at the price $q_{n,12}$ if the investor purchases information from both sellers, where $q_{n,n}$ and $q_{n,12}$ are respectively given by (21) and (23).*

3.3 Clientele Equilibrium

With information prices (21) and (23), the profit of seller n is:

$$\pi_n = \lambda_n q_{n,n} + \lambda_{12} q_{n,12} = \lambda_n \beta \phi_n + \lambda_{12} \beta \left(\phi_{12} - \phi_{n'} \right). \tag{24}$$

Obviously, π_n is a function of $(\tau_{\varepsilon_1}, \tau_{\varepsilon_2})$, since information precision affects information values ϕs. At date 0, equilibrium information precision $\left(\tau^*_{\varepsilon_1}, \tau^*_{\varepsilon_2}\right)$ is determined by the intersections of the best-response functions of both sellers. Specifically, given $\tau^*_{\varepsilon_{n'}}$ ($n' \neq n$ and n, $n' \in \{1, 2\}$), seller n chooses τ_{ε_n} to maximize profits $\pi_n \left(\tau_{\varepsilon_n}, \tau^*_{\varepsilon_{n'}}\right)$, where π_n is given by equation (24). Differentiating $\pi_n \left(\tau_{\varepsilon_n}, \tau^*_{\varepsilon_{n'}}\right)$ with respect to τ_{ε_n} and setting it to zero lead to the best response function of seller n, $\tau^*_{\varepsilon_n} = \tau_{\varepsilon_n} \left(\tau^*_{\varepsilon_{n'}}\right)$. These two best-response functions jointly determine the values of $\left(\tau^*_{\varepsilon_1}, \tau^*_{\varepsilon_2}\right)$. We consider symmetric equilibrium, i.e., $\tau^*_{\varepsilon_1} = \tau^*_{\varepsilon_2} \equiv \tau^*_{\varepsilon}$.

At the same time when information sellers determine the information precision, investors select their seller(s), and thus investor composition Λ is endogenously determined. Specifically, we solve for the equilibrium investor composition by comparing the benefit of each type of investors. For type-12 investors, as we derived above, the cost of purchasing the two signals is $q_{1,12} + q_{2,12} = \beta \left(2\phi_{12} - \phi_1 - \phi_2\right)$, and thus the equilibrium benefit of type-12 investors is

$$B_{12} \equiv \phi_{12} - \left(q_{1,12} + q_{2,12}\right) = \phi_{12} - \beta \left(2\phi_{12} - \phi_1 - \phi_2\right). \tag{25}$$

For type-n investors, they only purchase one signal $\tilde{s}_{ni}$ from seller n. The information price resulting from the Nash bargaining between seller n and a type-n investor is $\beta\phi_n$, and the value of $\tilde{s}_{ni}$ is ϕ_n. Thus the equilibrium benefit of a type-n investor is

$$B_n \equiv \phi_n - q_{n,n} = (1-\beta)\,\phi_n. \tag{26}$$

Of course, a type-0 investor has the benefit B_0 of 0. Given that $\beta \in [0,1]$, $B_{12} \geq 0$ and $B_n \geq 0$; that is, investors will not get hurt by purchasing the information. Thus to purchase information is a dominant strategy for investors, and in equilibrium all investors are informed (i.e., $\lambda_0^* = 0$). We formally present this result in the following lemma.

Lemma 1 *In equilibrium investors purchases information from at least one informtion seller. That is,* $\lambda_0^* = 0$.

Proof. See the above discussion. ■

Recall we consider the symmetric equilibrium for investor composition: $\lambda_1^* = \lambda_2^* \equiv \lambda^*$. Given that $\lambda_0^* = 0$, the fraction of investors who purchase both signals is $\lambda_{12}^* = 1 - 2\lambda^*$. With symmetric equilibrium, type–1 and type-2 investors have the same benefit: $B_1 = B_2$. The corner solution where all investors purchase both signals (i.e., $\lambda^* = 0$) can be supported if $B_{12} \geq B_1 = B_2$, and the corner solution where no investor purchases both signals (i.e., $\lambda^* = \frac{1}{2}$) will be supported in equilibrium if $B_{12} \leq B_1 = B_2$. The interior equilibrium (i.e., $\lambda^* \in \left(0, \frac{1}{2}\right)$) holds only when $B_{12} = B_1 = B_2$.

General Findings. The main message is that the information-clientele equilibrium characterization in the economy depends on the interaction between two parameters: product-differentiation parameter k and bargaining-power parameter β. We use a numerical example to illustrate the general findings. Without loss of generality, we let $\gamma = 3$, $\tau_u = 15$ and $2/\tau_v + 1/\tau_0 = 1$, and plot the equilibrium investor composition against k and β in Figure 2. The numerical results are robust across different sets of parameters. Note that the constraint regarding τ_v and τ_0 guarantees that the total unconditional volatility of the risky asset remains constant when k and (or) β change.

[Figure 2 About Here]

As shown in Figure 2, the whole space is divided into four regions. In Region I, the equilibrium investor composition features that every investor purchases information from both sellers (i.e., $\lambda_1^* = \lambda_2^* = 0$ and $\lambda_{12}^* = 1$). This occurs more often when information sellers sell different information (i.e., a small k). In Region II, in equilibrium all investors are split into two groups, each of which only purchases information from one seller (i.e., $\lambda_1^* = \lambda_2^* = \frac{1}{2}$ and $\lambda_{12}^* = 0$). This happens when the two sellers' information is different (i.e., a large k) and they have large bargaining power (i.e., a large β). In Region III, in equilibrium some investors purchase both signals while some only purchase one signal (i.e., $0 < \lambda_1^* = \lambda_2^* < \frac{1}{2}$ and $0 < \lambda_{12}^* < 1$). Finally, multiple equilibrium is possible in our economy. In Area IV, when the signals sold are differentiated and information sellers' bargaining power lies within some range, in equilibrium it could be that all investors purchase both signals as in Area I, or that all investors only choose one seller as in Area II, or that some investors purchase both signals and some purchase only one, as in Region III.

We next present some analytical results in extreme cases.

Sellers Selling Orthogonal Information. When product-differentiation parameter k is 1, the two sellers sell orthogonal signals: $\tilde{s}_{ni} = \tilde{v}_n + \tilde{\varepsilon}_{ni}$. Recall that according to Admati and Pfleiderer (1987), complementarity between signals $\tilde{s}_{1i}$ and $\tilde{s}_{2i}$ refers to that the value of the joint information set is higher than the sum of the value of the information set containing each signal (i.e., $\phi_{12} \geq \phi_1 + \phi_2$). We show that there exists such complementarity in our economy when information sellers sell orthogonal information, and we term this *cross-signal complementarity.*

Lemma 2 (Cross-signal complementarity) *When product-differentiation parameter k is 1, with symmetric equilibrium $\tau_{\epsilon_1} = \tau_{\epsilon_2}$ the two signals $\tilde{s}_{1i}$ and $\tilde{s}_{2i}$ are complements: $\phi_{12} > \phi_1 + \phi_2$.*

Proof. See the Appendix. ■

When sellers sell orthognal information, the information-clientele equilibrium depends on the bargaining-power parameter β. Specifically, if sellers have small bargaining power (i.e., β close to 0), the benefit of the type-12 investor is $B_{12} = \phi_{12} - \beta\,(2\phi_{12} - \phi_1 - \phi_2)$, which is close to ϕ_{12} when β moves towards 0. The benefit of the type-n investor is $B_n = (1-\beta)\,\phi_n$, which moves to ϕ_n as β gets close to 0. Thus $B_{12} - B_n = \phi_{12} - \phi_n \geq 0$. The inequality follows because the value of two signals is at least as high as the value of one signal. In equilibrium all investors purchase both signals (i.e., $\lambda_1^* = \lambda_2^* = 0$ and $\lambda_{12}^* = 1$). However, if sellers have large bargaining power (i.e., β close to 1), the benefit of the type-12 investor is $B_{12} = \phi_{12} - \beta(2\phi_{12} - \phi_1 - \phi_2)$, which becomes negative as β gets close to 1 because $-(\phi_{12} - \phi_1 - \phi_2) < 0$, and the benefit of the type-n investor is $B_n = (1 - \beta)\phi_n$, which declines to 0 as β moves towards 1. Therefore $B_{12} - B_n =- (\phi_{12} - \phi_1 - \phi_2) < 0$, and investors only purchase one signal. Due to symmetry, in equilibrium half of the investors end with only knowing $\tilde{s}_{1i}$ and the other half only $\tilde{s}_{2i}$ (i.e., $\lambda_1^* = \lambda_2^* = \frac{1}{2}$ and $\lambda_{12}^* = 0$).

Intuitively, the two orthognal signals provide cross-signal complementarity, and sellers materialize such complementarity by discriminating against type-12 investors through Nash bargaining. Specifically, note that the price of signal $\tilde{s}_{ni}$ charged for type-12 investors is higher than that for type-n investors (i.e., $q_{n,12} = \beta(\phi_{12} - \phi_{n'}) > q_{n,n} = \beta\phi_n$ when $k = 1$). When information sellers' bargaining power is small, the discrimination is mild, investors purchase both signals to enjoy the cross-signal complementarity. By contrast, when information sellers own large bargaining power, the discrimination against type-12 investors is so severe that no investors want to purchase the two signals at the same time.

Given the equilibrium investor composition, equilibrium information precision is derived from the interactions of the best-response functions of the two sellers. We find a unique information precision equilibrium when sellers have sufficiently small bargaining power (i.e., β close to 0). However, when sellers have sufficiently large bargaining power (i.e., β close to 1), the equilibrium information precision depends on investors' risk aversion. Specifically, when investors are less risk averse, there exists a unique information precision equilibrium, and when investors are very risk averse, both sellers just sell their information "as is."

The intuition behind these results is as follows. As analyzed above, when informaiton sellers have sufficiently small bargaining power, all investors are informative of both signals. Now information leakage is a big concern for information sellers, and they control it by adding more noise to the sold information. Similarly, there is concern for information leakage when sellers own large bargaining power but investors are less risk averse: though all investors purchase only one signal, they are aggressive traders so that information can be leaked, and sellers respond to it by adding noise to the sold information. However, when sellers have large bargaining power and the investors are very risk averse, not much information is leaked through price because all investors only know one signal and they do not trade aggressively on it. Thus information sellers can afford to sell the most precise information.

Sellers Selling Almost Identical Information. When product-differentiation parameter k is sufficiently small (i.e., k close to 0), the two sellers sell almost identical signals: $\tilde{s}_{ni} = \tilde{v}_0$. We find that the two signals $\tilde{s}_{1i}$ and $\tilde{s}_{2i}$ are substitutes as defined by Admati and Pfleiderer (1987). That is, the value of the joint information set is lower than the sum of the value of the information set containing each signal: $\phi_{12} \leq \phi_1 + \phi_2$. We term this *cross-signal substitutability.*

Lemma 3 (Cross-signal substitutability) *When the product-differentiation parameter k is sufficiently small (i.e., k close to 0), with symmetric equilibrium $\tau_{\epsilon_1} = \tau_{\epsilon_2}$ the two signals $\tilde{s}_{1i}$ and $\tilde{s}_{2i}$ are substitutes: $\phi_{12} < \phi_1 + \phi_2$.*

Proof. See the Appendix. ■

Since the two signals are very alike, the two sellers compete intensively by making information as precise as possible. Unlike the case where the two signals are orthognal, now the equilibrium investor composition is independent of information sellers' bargaining power. Recall that the benefit of the type-12 investor is $B_{12} = \phi_{12} - \beta (2\phi_{12} - \phi_1 - \phi_2)$, and the benefit of the type-n investor is $B_n = (1-\beta)\phi_n$. Thus $B_{12} - B_n = (1-\beta)(\phi_{12} - \phi_n) + \beta(\phi_1 + \phi_2 - \phi_{12}) \geq 0$. The inequality follows from the fact that $\phi_{12} \geq \phi_n$ and Lemma 3. Therefore in equilibrium, all investors are informed of both signals (i.e., $\lambda_1^* = \lambda_2^* = 0$ and $\lambda_{12}^* = 1$).

Intuitively, if the two signals are substitutes as defined by Admati and Pfleiderer (1987), the marginal pricing scheme of the two sellers actually favors type-12 investors (i.e., $q_{n,12} = \beta(\phi_{12} - \phi_{n'}) < q_{n,n} = \beta\phi_n$). This unambiguously encourages investors to purchase both signals.

We find that when the two signals are almost identical, in equilibrium sellers sell their information "as is" regardless of the value of the other parameter β. Intuitively, when sellers own almost identical information, they compete with each other by making the information as precise as possible. The competition keeps driving sellers' profit margin down so that in the end sellers sell their information "as is." Unlike the above mentioned information-leakage concern in the case where sellers sell orthogonal information, competition is the dominant driving force behind information precision when the information sold is alike.

The following proposition summarizes the equilibrium at date 0 in extreme cases.

Proposition 3 (Information-clientele equilibrium in extreme cases) *(1) When product-differentiation parameter k is 1, the information-clientele equilibrium characterization depends on information sellers' bargaining power. (i) If information sellers' bargaining-power parameter β is sufficiently small (i.e., β close to 0), all investors purchase both signals; that is, $\lambda_1^* = \lambda_2^* = 0$ and $\lambda_{12}^* = 1$. There exists symmetric equilibrium signal precision τ_ε^*, which is uniquely determined by the following equation*

$$-\tau_u \tau_\varepsilon^3 - 2\tau_v \tau_u \tau_\varepsilon^2 + 4\gamma^2 \tau_v \tau_\varepsilon + 4\gamma^2 \tau_v^2 = 0. \tag{27}$$

(ii) If information sellers' bargaining-power parameter β is sufficiently large (i.e., β close to 1), investors are split into two groups each of which purchases one signal; that is, $\lambda_1^* = \lambda_2^* = \frac{1}{2}$ and $\lambda_{12}^* = 0$. When investors' risk aversion is small (i.e., $2\gamma^2 < \tau_u \tau_v$), there exists a unique information precision equilibrium

$$\tau_\varepsilon^* = \frac{2\left(2\gamma^2 \tau_v + \gamma \tau_v \sqrt{2\tau_u \tau_v}\right)}{\tau_u \tau_v - 2\gamma^2}, \tag{28}$$

and when investors' risk aversion is large (i.e., $2\gamma^2 \geq \tau_u \tau_v$), sellers sell their information "as is."

(2) When the product-differentiation parameter k is sufficiently small (i.e., k close to 0), all investors purchase both signals: $\lambda_1^* = \lambda_2^* = 0$ and $\lambda_{12}^* = 1$. Sellers sell their information "as is."

Proof. See the Appendix. ■

Based on Proposition 3, we further show how the equilibrium information precision varies across different sets of k and β. When $k = 1$, we find that when information sellers have sufficiently small bargaining power, in equilibrium they sell less precise signals than when they have sufficiently large bargaining power. In the context of information sales, selling more precise information can lower the information value through information leakage of the asset price, because investors can freely extract information from the asset price which aggregates investors' private signals (Admati and Pfleiderer (1986)). As analyzed above, when information sellers have sufficiently small bargaining power, all investors purchase both signals, and the information leakage effect is so strong that sellers control it by constraining the information precision. By contrast, when information sellers have large bargaining power, investors only purchase one signal. Thus information leakage is less of a concern, and when the investors are risk averse enough, sellers even sell the information "as is". When k is close to 0, sellers always sell their information "as is," which is independent of the value of β. The following corollary formally presents the results.

Corollary 1 *(1) When product-differentiation parameter k is 1, information sellers choose more precise signal when they have sufficiently large bargaining power (i.e., β close to 1) than when they have sufficiently small bargaining power (i.e., β close to 0). (2) When product-differentiation parameter k is sufficiently small (i.e., k close to 0), the equilibrium information precision is not affected by sellers' bargaining power.*

4 Substitute, Complement, and Multiplicity

One crucial question in understanding the information market, the financial market, and their interplay is what is the interaction between different information and between different types of informed investors. For example, if investors are informed about one dimension of uncertainty, will they have the tendency to learn about other dimensions, or are they less willing to do so? Additionaly, if some investors are informed about more dimensions of uncertainty, will this encourage other investors to know more about other dimensions, or discourage them from doing so? The former question speaks to the *cross-signal complementarity/substitutability* as mentioned in Lemma 2 and 3, while the latter question concerns the strategic complementarity/substitutability between the different investors. To differentiate between these two types of interactions, we term the latter *cross-agent complementarity/substitutability.*

In this section, we discuss in details how the two types of interactions interact with each other. Similar to Section 3.3, our discussion centers on the extreme cases, where we have analytical results.

To study the cross-agent interaction between different investors in purchasing information, we compute the marginal increase in the benefit of knowing the second signal ($\Delta B_n \equiv B_{12} - B_n$, where $n = \{1, 2\}$) and its change with respect to the population of investors purchasing the two signals (λ_{12}). Specifically, as more investors purchase both signals (i.e., λ_{12} increases), if the marginal benefit of the investor switching from type-n to type-12 increases (i.e., ΔB_n increases), then there is a cross-agent complementarity among investors in purchasing information. Otherwise, if the marginal benefit decreases (i.e., ΔB_n decreases), there is a cross-agent substitutability.

First we look at the case when sellers sell orthogonal information (i.e., $k = 1$). We find that whether there is a cross-agent complementarity or substitutability in information purchase depends on the bargaining-power parameter β. To clearly show the mechanism, we decompose the marginal benefit of purchasing the second signal into the following two terms:

$$\Delta B_n \equiv B_{12} - B_n = \underbrace{(1-\beta)(\phi_{12} - \phi_n)}_{>0,\text{ marginal gain}} - \underbrace{\beta(\phi_{12} - \phi_1 - \phi_2)}_{>0,\text{ marginal price}}. \tag{29}$$

The first term $(1-\beta)(\phi_{12} - \phi_n) > 0$ captures the marginal gain of an investor switching from type-n to type-12. It is the additional information value of knowing two signals $(\phi_{12} - \phi_n)$ times the buyers' bargaining power $(1-\beta)$. When sellers sell orthogonal information, there exists a cross-signal complementarity as shown in Lemma 2, so the second term $\beta(\phi_{12} - \phi_1 - \phi_2) > 0$. Note it also equals to the marginal price of signal $\tilde{s}_n$ for type-12 over that for type-n investors: $\beta(\phi_{12} - \phi_1 - \phi_2) = q_{n,12} - q_{n,n}$, where $q_{n,12}, q_{n,n}$ are respectively given in (23) and (21). Because of information sellers' "marginal pricing" scheme for type-12 investors (see equation (23)), for the same signal type-12 investors need to pay more than what type-n investors do, and this constitutes the marginal loss to the investors who purchase the two signals. To sum, the investors' decision to switch from type-n to type-12 is a trade-off between the marginal gain and marginal price.

Now we examine the change of the marginal benefit as more investors purchase both signals to study the cross-agent interaction. Specifically, we take derivative of ΔB_n with respect to λ_{12}:

$$\frac{\partial \Delta B_n}{\partial \lambda_{12}} = (1-\beta)\underbrace{\frac{\partial\left(\phi_{12}-\phi_n\right)}{\partial \lambda_{12}}}_{<0} - \beta\underbrace{\frac{\partial\left(\phi_{12}-\phi_1-\phi_2\right)}{\partial \lambda_{12}}}_{<0}. \tag{30}$$

As more investors purchase the two signals and trade on them, there is more competition among type-12 investors so the marginal gain declines: $\frac{\partial(\phi_{12}-\phi_n)}{\partial \lambda_{12}} < 0$. At the same time, the cross-signal complementarity between the two signals (i.e., $\phi_{12}-\phi_1-\phi_2$) decreases. This implies that the marginal price charged for type-12 investors becomes lower. This is good news for the type-12 investors because now the discriminatory price is less unfavorable to them (i.e., $q_{n,n}$ and $q_{n,12}$ are closer). When $\frac{\partial \Delta B_n}{\partial \lambda_{12}} > (<)\, 0$ there is a cross-agent complementarity (substitutability). Therefore, whether there is a cross-agent complementarity or substitutability in the investors' information purchase depends on the trade-off of the two countervailing effects, which crucially depends on the bargaining-power parameter β.

Specifically, when information sellers have small bargaining power (i.e., small β), the marginal gain is more pronounced (i.e., the right-hand side of (30) is mainly determined by the first term). Thus the increasing population of type-12 investors discourages other investors from purchasing the two signals. However, when information sellers have large bargaining power (i.e., large β), the marginal price dominates (i.e., the right-hand side of (30) is mainly determined by the second term). So more investors are willing to purchase the two signals.

These results establish that the cross-signal complementarity can result in either cross-agent complementarity or cross-agent substitutability, depending on information sellers' bargaining power. We analytically show the mechanism when λ_{12} gets close to 1 and the variance of noise trader demand is sufficiently low (i.e., $1/\tau_u$ close to 0) in the following proposition.

Proposition 4 (Cross-signal complementarity and cross-agent complementarity/substitutability) *Product-differentiation parameter $k = 1$. Assume λ_{12} is sufficiently large (i.e., λ_{12} close to 1), and the variance of noise trader demand is sufficiently low (i.e., $1/\tau_u$ close to 0).*

(i) When bargaining-power parameter is sufficiently small (i.e., β close to 0), as more investors purchase both signals, the marginal investor is less willing to purchase both signals. That is, $\frac{\partial \Delta B_n}{\partial \lambda_{12}} \equiv \frac{\partial(B_{12}-B_n)}{\partial \lambda_{12}} < 0$.

(ii) When bargaining-power parameter is sufficiently large (i.e., β close to 1), as more in-

vestors purchase both signals, the marginal investor is more willing to purchase both signals. That is, $\frac{\partial \Delta B_n}{\partial \lambda_{12}} \equiv \frac{\partial (B_{12}-B_n)}{\partial \lambda_{12}} > 0$.

Proof. See the Appendix. ■

Next we show that under some conditions, the cross-agent complementarity in information purchase can generate multiplicity in equilibrium. This suggests that complementarity can lead to large effects of changes in the underlying environment. The following proposition summarizes the result.

Proposition 5 (Multiplicity) *When product-differentiation parameter* $k = 1$*, and the variance of noise trader demand is sufficiently low (i.e.,* $1/\tau_u$ *close to* 0*), there exist multiple equlibria in the economy if* $\beta \in \left[\underline{b}, \bar{b}\right]$*; that is, there exist at least three equilibria of investors composition (i)* $\lambda_1^* = \lambda_2^* = 0$ *and* $\lambda_{12}^* = 1$*, (ii)* $\lambda_1^* = \lambda_2^* = \frac{1}{2}$ *and* $\lambda_{12}^* = 0$*, and (iii)* $0 < \lambda_1^* = \lambda_2^* < \frac{1}{2}$ *and* $0 < \lambda_{12}^* < 1$*, where* $\underline{b} \leq \bar{b}$*,* $\underline{b}$ *and* $\bar{b}$ *are shown in the Appendix.*

Proof. See the Appendix. ■

When sellers sell almost identical information (i.e., k close to 0), investors' information purchase always exhibits a strategic substitutability: the more investors are informative about the two signals, the less willing other investors are to do so. Similar to the above analysis, we can decompose the marginal benefit of knowing the second signal ΔB_n into the following two terms

$$\Delta B_n \equiv B_{12} - B_n = \underbrace{(1-\beta)\left(\phi_{12} - \phi_n\right)}_{>0,\text{ marginal gain}} - \underbrace{\beta\left(\phi_{12} - \phi_1 - \phi_2\right).}_{<0,\text{ marginal price}} \tag{31}$$

Similar to the first term in equation (29), purchasing the second signal yields more information value (i.e., $\phi_{12} > \phi_n$). But unlike the case when sellers sell orthogonal information, when sellers sell almost identical information there exists a cross-signal substitutability as shown in Lemma 3, so the second term $\beta\left(\phi_{12} - \phi_1 - \phi_2\right) < 0$. Note now information sellers' discriminative prices are actually favourable to type-12 investors: as in equation (23), information sellers only charge marginal price of signal $\tilde{s}_{ni}$ for the type-12 investors: $q_{n,12} = \beta\left(\phi_{12} - \phi_{n'}\right)$, which is lower than that for the type-n investors: $q_{n,n} = \beta\phi_n$. The difference of the prices $q_{n,n} - q_{n,12} = \beta\left(\phi_1 + \phi_2 - \phi_{12}\right) > 0$ is the "discount" offered to type-12 investors, and thus the marginal price for the type-12 investors ($\phi_{12} - \phi_1 - \phi_2$) is negative. Taken together, with the positive marginal gain and negative marginal price, to purchase the two signals always yields more payoff for the investors (i.e., $\Delta B_n > 0$).

Furthermore, both terms in equation (31) decrease as more investors purchase the second signal:

$$\frac{\partial \Delta B_n}{\partial \lambda_{12}} = (1-\beta)\underbrace{\frac{\partial(\phi_{12}-\phi_n)}{\partial \lambda_{12}}}_{<0} - \beta\underbrace{\frac{\partial(\phi_{12}-\phi_1-\phi_2)}{\partial \lambda_{12}}}_{>0}. \tag{32}$$

The first term $\frac{\partial(\phi_{12}-\phi_n)}{\partial \lambda_{12}} < 0$ follows the same intuition as in the case where sellers sell orthogonal information. For the second term, as more investors purchase the second signal, the prices of signal $\tilde{s}_{ni}$ charged for type-n and for type-12 investors are closer, so the "discount" $q_{n,n} - q_{n,12}$ diminishes. Note the marginal price (the reverse of the "discount" offered to type-12 investors) is a negative term, and thus it will increases: $\frac{\partial(\phi_{12}-\phi_1-\phi_2)}{\partial \lambda_{12}} > 0$. To sum, that more investors purchasing the second signal discourages other investors from doing so when sellers sell almost identical information; that is, $\frac{\partial \Delta B}{\partial \lambda_{12}} < 0$.

These results indicate that unlike cross-signal complementarity, cross-signal substitutability can only lead to cross-agent substitutability. The following proposition formally presents the result.

Proposition 6 (Cross-signal substitutability and cross-agent substitutability) *When product-differentiation parameter k is sufficiently small (i.e., k clost to 0), as more investors purchase the two signals, the marginal investor is less willing to do so. That is, $\frac{\partial \Delta B_n}{\partial \lambda_{12}} \equiv \frac{\partial(B_{12}-B_n)}{\partial \lambda_{12}} < 0$.*

Proof. See the Appendix. ■

We next graphically illustrate the cross-agent complementarity and substitutability in Figure 3 and 4. Unless otherwise specificied, the same set of parameters is applied as in Figure 2.

[Figure 3 About Here]

In Figure 3 Panel A, we fix $k = 1$, and plot the marginal benefit of purchasing the second signal (i.e., $\Delta B_n \equiv B_{12} - B_n$) against λ_{12} for different levels of information sellers' bargaining power (β). We find that when β is small, ΔB_n is a downward-sloping curve, meaning that when more investors are informative of the two signals, to purchase them will be less appealing, which is consistent with Part (i) in Proposition 4. As β increases after some point, ΔB_n starts to have a positive slope, which indicates that there can exist a cross-agent complementarity in investors' information purchase. This is consistent with Part (ii) in Proposition 4. More interestingly, when β takes certain values (i.e., $\beta = 0.65$ in the example), the upward-sloping curve ΔB_n crosses the horizontal line. This suggests that multiple equilibria can arise in our economy: either that all investors only purchase one signal as indicated by the left-most of the curve (i.e., $\lambda_1^* = \lambda_2^* = \frac{1}{2}$ and $\lambda_{12}^* = 0$), or that some investors only purchase one signal while others purchase both signals as shown by the intersection of the curve and X-axis (i.e., $0 < \lambda_1^* = \lambda_2^* < \frac{1}{2}$ and $0 < \lambda_{12}^* = 1$), or that all investos purchase the two signals as indicated by the right-most of the curve (i.e., $\lambda_1^* = \lambda_2^* = 0$ and $\lambda_{12}^* = 1$). The multiplicity result is consistent with Proposition 5.

However, as shown in Proposition 6, when product-differentiation parameter k is sufficienty small (i.e., k close to 0), as more investors purchase the two signals, the marginal investor is less willing to purchase the second signal. Consistently, we only observe downward-sloping curves in Figure 3 Panel B when $k = 0.0001$.

[Figure 4 About Here]

To further illustrate the interplay between cross-signal interaction and cross-agent interaction, in Figure 4 we plot the equilibrium fraction of the investors purchasing both signals (i.e., λ_{12}^*) against β given different values of k. When k is small, there is a unique information-clientele equilibrium: $\lambda_1^* = \lambda_2^* = 0$ and $\lambda_{12}^* = 1$. As k increases, when information sellers' bargaining power β is large, the unique equilibrium switches to the one where investors only purchase one signal: $\lambda_1^* = \lambda_2^* = \frac{1}{2}$ and $\lambda_{12}^* = 0$. As k continues to increase, within some range of β, we indeed observe multiple information-clientele equilibria. These results are again consistent with Proposition 4, 5 and 6.

5 Market-quality Implications

We have shown in Section 4 how the interplay of the cross-signal interaction and the cross-agent interaction shape the equilibrium information-clientele network and information precision. We demonstrate that the cross-signal complementarity is the necessary condition to generate the cross-agent complementarity. In other words, in a financial market where the sold information is differentiated, there is a possibility of a cross-signal complementarity between investors in purchasing information, which can create amplification of shocks in the economy. Furthermore, the multiplicity of equilibria in some parameter range suggests a potential market crash, in which the literature has a long standing interest. In this section, we take a closer look at the implications of the cross-signal and the cross-agent interactions on the asset market.

We measure asset market quality using the four variables: price efficiency, market liquidity, cost of capital, and return volatility (e.g., O'Hara (2003)). As in the literature, price efficiency (or informational efficiency and price informativeness) is measured by the inverse of the variance of the asset payoff conditional on the equilibrium price, $Var^{-1}(\tilde{v}|\tilde{p})$. Market liquidity can be inversely captured by coefficient α_u in the price function (4): More liquid markets have a smaller α_u. Coefficient α_u is the Kyle (1985)'s lambda, which measures the effect of noise trading on prices, and so it is an inverse measure of market depth. In addition, the cost of capital is captured by $E(\tilde{v} - \tilde{p})$, which is the expected difference between the cash flow generated by the risky asset and its price. This difference arises from the compensation required by risk-averse investors to hold the risky asset. Finally, in the financial market, the price is $\tilde{p}$. When the asset cash flow $\tilde{v}$ is realized, the asset price is simply equal to its cash flow $\tilde{v}$. Thus, the asset return is $\tilde{v} - \tilde{p}$, and return volatility is given by $\sigma(\tilde{v} - \tilde{p})$.

For different values of k, Figure 5 plots the equilibrium price efficiency, liquidity, cost of capital and return volatility against β respectively in Panel A, B, C and D. We find that the two parameters - k and β - jointly shape the equilibrium information-clientele network and information precision, and further determine the asset market quality in equilibrium. Specifically, for large k, the bargaining-power parameter β affects the market quality: price effciciency and market liquidity tend to be low, and cost of capital and return volatility tend to be high when information sellers have large bargaining power. With large k and certain values of β where multiplicity occurs, we indeed observe multiple market qualities, which suggests the importance of studying information market and financial market together. By contrast, for small k, price effciency is low, and consistently we observe low market liquidity, high cost of capital and high return volatility.

[Figure 5 About Here]

We next discuss the market quality in the two extreme cases as in Section 3.3, where we can obtain analytical results.

Sellers selling orthogonal information When $k = 1$, the sold information is orthogonal. According to Corollary 1, when information sellers have sufficiently large bargaining power, the sold information is more precise. However, the equilibrium investor composition will negatively affect the price efficiency. Specifically, when sellers' bargaining power is sufficiently small (i.e., β close to 0), all investors are informative of the two signals (i.e., $\lambda_1^* = \lambda_2^* = 0$ and $\lambda_{12}^* = 1$), whereas when sellers' bargaining power is sufficiently large (i.e., β close to 1), the investors are only informative of one signal (i.e., $\lambda_1^* = \lambda_2^* = \frac{1}{2}$ and $\lambda_{12}^* = 0$). In equilibrium, though the sold information is more precise when sellers have sufficiently large bargaining power, the price efficiency ends up to be lower because the investors only purchase and trade on one signal.

As for the market liquidity, we show that α_u is higher when information sellers have sufficiently large bargaining power, which suggests lower market liquidity. In addition, with less informative price when sellers own sufficiently large bargaining power, cost of capital is higher since investors face more uncertainty in trading the risky asset, and return volatility is higher. We summarize the above results in the following proposition.

Proposition 7 (Market quality with cross-signal complementarity) *Assume product-differentiation parameter $k = 1$. Compared with the case where information sellers' bargaining power is sufficiently small (i.e., β close to 0), price efficiency and liquidity are lower, and cost of capital* *and return volatility are higher when information sellers' bargaining power is sufficiently large (i.e., β close to 1).*

Proof. See the Appendix. ■

Sellers selling almost identical information When the sold information is almost identical (i.e., k close to 0), as shown in Proposition 3 the competition forces sellers to sell their information "as is," which results in fully-revealing asset price. Consequently, the asset market is sufficiently liquid, and cost of capital and the return volatility are sufficiently low. The following proposition presents the results.

Proposition 8 (Market quality with cross-signal substitutability) *When product-differentiation parameter k is sufficiently small (i.e., k close to 0), the asset price is fully revealing. The asset market is sufficiently liquid, and cost of capital and return volatility are sufficiently low. The results are independent of the value of β.*

Proof. See the Appendix. ■

6 Conclusion

We propose an information-sale framework to capture the features of the modern information market: (i) there are multiple information sellers selling similar or distinct information; and (ii) information sellers and investors share the informed trading rents. Our model is based on Admati and Pfleiderer (1986), but deviate from theirs to capture the above features which are absent in the literature: (i) there are two information sellers, each of which is endowed with related but different information about asset fundamentals, and (ii) the information prices are set through Nash bargaining.

We find that the information-market structure - information differentiation k and information sellers' bargaining power β - have nontrivial implications for the equilibrium investor composition and asset pricing. When the two sellers' information is complementary, investors purchase from both sellers (only one seller) when the sellers have small (large) bargaining power. If the sellers have large bargaining power, investors' information purchase exhibits cross-agent complementarity and multiple equilibria could occur. Furthermore, the asset-market quality is higher when the sellers have small bargaining power; for example, price efficiency and liquidity are higher, the cost of capital and return volatility are lower. By contrast, when the two sellers sell substitute information, investors always purchase both pieces of information, and their information-purchase decisions are always substitutes. The asset market features high price efficiency and liquidity, low cost of capital and return volatility.

More importantly, we firstly examine the interplay of the cross-signal and the cross-agent interaction, and show that the cross-signal complementarity can lead to either cross-agent complementarity or substitutability, while the cross-signal substitutability is always associated with cross-agent substitutability.

Appendix A: List of variables

Variables	Description
Introduced in Section 2	
	Exogenous Variables
$\tilde{v}$	Risky asset value
$\tilde{v}_0$	The common information of two sellers: $\tilde{v}_0 \sim N\left(0, \tau_0^{-1}\right)$
$\tilde{v}_n$	Seller n's unique information about dimension n: $\tilde{v}_n \sim N\left(0, \tau_v^{-1}\right)$
$\tilde{u}$	Noise-trader demand with prior precision τ_u
k	Production-differentiation parameter
β	Information sellers' bargaining power, $\beta \in [0,1]$
γ	Investors' coefficient of risk aversion
	Endogenous Variables
$\tilde{p}$	Price of risky asset
$\tilde{s}_{ni}$	Signal sold to investor i by seller n: $\tilde{s}_{ni} = \tilde{v}_0 + \tilde{v}_n + \tilde{\varepsilon}_{ni}$
$\tilde{\varepsilon}_{ni}$	Personalized noise added by seller n, with precision τ_{ε_n}
Λ	Investor composition $\Lambda = \{\lambda_1, \lambda_2, \lambda_{12}\}$
x_i	Investor i's demand for the risky asset
λ_n, λ_{12}	Mass of different types of investors
π_n	Net profit of seller n
q	Prices of information: $q = [q_{1,1}, q_{1,12}, q_{2,2}, q_{2,12}]'$
Introduced in Section 3	
$\alpha_\emptyset, \alpha_n, \alpha_u$	Coefficients of price function
a_n	$a_n = \alpha_n / \alpha_u$
ϕ_n, ϕ_{12}	Information value
B_n, B_{12}	Marginal benefit of investors
ΔB_n	Marginal benefit of purchasing the second signal for type-n

Appendix B: Proofs

Proof of Proposition 1

By Bayes' rule, we can compute the conditional moments: $E\left(\tilde{v}|\tilde{s}_{1i},\tilde{s}_{2i},\tilde{p}\right)$, $E\left(\tilde{v}|\tilde{s}_{1i},\tilde{p}\right)$, $E\left(\tilde{v}|\tilde{s}_{2i},\tilde{p}\right)$, $Var\left(\tilde{v}|\tilde{s}_{1i},\tilde{s}_{2i},\tilde{p}\right)$, $Var\left(\tilde{v}|\tilde{s}_{1i},\tilde{p}\right)$, $Var\left(\tilde{v}|\tilde{s}_{2i},\tilde{p}\right)$ and $Var\left(\tilde{v}|\tilde{p}\right)$. The conditional moments are as follows:

$$\begin{aligned}
E\left(\tilde{v}|\tilde{s}_{1i},\tilde{s}_{2i},\tilde{p}\right) &= \frac{\left(\tau_0+\tau_v\right)\begin{pmatrix}\tau_{\varepsilon_1}\left(a_2^2\tau_u+\tau_v+\tau_{\varepsilon_2}-a_1a_2\tau_u\right)\tilde{s}_{1i}\\ +\tau_{\varepsilon_2}\left(a_1^2\tau_u+\tau_v+\tau_{\varepsilon_1}-a_1a_2\tau_u\right)\tilde{s}_{2i}\\ +\tau_u\left(a_1\left(\tau_v+\tau_{\varepsilon_2}\right)+a_2\left(\tau_v+\tau_{\varepsilon_1}\right)\right)\tilde{s}_p\end{pmatrix}}{\begin{array}{c}a_1^2\tau_u\left[\tau_0\left(\tau_v+\tau_{\varepsilon_2}\right)+\tau_v\left(\tau_v+2\tau_{\varepsilon_2}\right)\right]+a_2^2\tau_u\left[\tau_0\left(\tau_v+\tau_{\varepsilon_1}\right)+\tau_v\left(\tau_v+2\tau_{\varepsilon_1}\right)\right]\\ +\tau_0\left(\tau_v+\tau_{\varepsilon_1}\right)\left(\tau_v+\tau_{\varepsilon_2}\right)+\tau_v\left[2a_1a_2\tau_u\tau_v+2\tau_{\varepsilon_1}\tau_{\varepsilon_2}+\tau_v\left(\tau_{\varepsilon_1}+\tau_{\varepsilon_2}\right)\right]\end{array}},\\
E\left(\tilde{v}|\tilde{s}_{1i},\tilde{p}\right) &= \frac{\left(\tau_0+\tau_v\right)\begin{pmatrix}\tau_{\varepsilon_1}\left(\tau_v+a_2\tau_u\left(a_2-a_1\right)\right)\tilde{s}_{1i}\\ +\tau_u\left(a_1\tau_v+a_2\left(\tau_v+\tau_{\varepsilon_1}\right)\right)\tilde{s}_p\end{pmatrix}}{a_1^2\tau_u\tau_v\left(\tau_0+\tau_v\right)+a_2^2\tau_u\left[\tau_0\left(\tau_v+\tau_{\varepsilon_1}\right)+\tau_v\left(\tau_v+2\tau_{\varepsilon_1}\right)\right]+\tau_0\tau_v\left(\tau_v+\tau_{\varepsilon_1}\right)+\tau_v^2\left(2a_1a_2\tau_u+\tau_{\varepsilon_1}\right)},\\
E\left(\tilde{v}|\tilde{s}_{2i},\tilde{p}\right) &= \frac{\left(\tau_0+\tau_v\right)\begin{pmatrix}\tau_{\varepsilon_2}\left(\tau_v+a_1\tau_u\left(a_1-a_2\right)\right)\tilde{s}_{2i}\\ +\tau_u\left(a_2\tau_v+a_1\left(\tau_v+\tau_{\varepsilon_2}\right)\right)\tilde{s}_p\end{pmatrix}}{a_1^2\tau_u\left[\tau_0\left(\tau_v+\tau_{\varepsilon_2}\right)+\tau_v\left(\tau_v+2\tau_{\varepsilon_2}\right)\right]+a_2^2\tau_u\tau_v\left(\tau_0+\tau_v\right)+\tau_0\tau_v\left(\tau_v+\tau_{\varepsilon_2}\right)+\tau_v^2\left(2a_1a_2\tau_u+\tau_{\varepsilon_2}\right)},\\
E\left(\tilde{v}|\tilde{p}\right) &= \frac{\tau_u\left(\tau_0+\tau_v\right)\left(a_1+a_2\right)\tilde{s}_p}{\tau_0\tau_v+\tau_u\tau_0\left(a_1^2+a_2^2\right)+\tau_v\tau_u\left(a_1+a_2\right)^2};
\end{aligned}$$

$$Var\left(\tilde{v}|\tilde{s}_{1i},\tilde{s}_{2i},\tilde{p}\right)=\frac{\begin{array}{c}a_1^2\tau_u\left(\tau_0+\tau_v+\tau_{\varepsilon_2}\right)+a_2^2\tau_u\left(\tau_0+\tau_v+\tau_{\varepsilon_1}\right)\\ +\left(\tau_v+\tau_{\varepsilon_1}\right)\left(\tau_v+\tau_{\varepsilon_2}\right)+\tau_0\left(2\tau_v+\tau_{\varepsilon_1}+\tau_{\varepsilon_2}\right)-2a_1a_2\tau_0\tau_u\end{array}}{\begin{array}{c}a_1^2\tau_u\left[\tau_0\left(\tau_v+\tau_{\varepsilon_2}\right)+\tau_v\left(\tau_v+2\tau_{\varepsilon_2}\right)\right]+a_2^2\tau_u\left[\tau_0\left(\tau_v+\tau_{\varepsilon_1}\right)+\tau_v\left(\tau_v+2\tau_{\varepsilon_1}\right)\right]\\ +\tau_0\left(\tau_v+\tau_{\varepsilon_1}\right)\left(\tau_v+\tau_{\varepsilon_2}\right)+\tau_v\left[2a_1a_2\tau_u\tau_v+2\tau_{\varepsilon_1}\tau_{\varepsilon_2}+\tau_v\left(\tau_{\varepsilon_1}+\tau_{\varepsilon_2}\right)\right]\end{array}},\tag{B1}$$

$$Var\left(\tilde{v}|\tilde{s}_{1i},\tilde{p}\right)=\frac{a_1^2\tau_u\left(\tau_0+\tau_v\right)+a_2^2\tau_u\left(\tau_0+\tau_v+\tau_{\varepsilon_1}\right)+\tau_v\left(\tau_v+\tau_{\varepsilon_1}\right)+\tau_0\left(2\tau_v+\tau_{\varepsilon_1}\right)-2a_1a_2\tau_0\tau_u}{a_1^2\tau_u\tau_v\left(\tau_0+\tau_v\right)+a_2^2\tau_u\left[\tau_0\left(\tau_v+\tau_{\varepsilon_1}\right)+\tau_v\left(\tau_v+2\tau_{\varepsilon_1}\right)\right]+\tau_0\tau_v\left(\tau_v+\tau_{\varepsilon_1}\right)+\tau_v^2\left(2a_1a_2\tau_u+\tau_{\varepsilon_1}\right)},\tag{B2}$$

$$Var\left(\tilde{v}|\tilde{s}_{2i},\tilde{p}\right)=\frac{a_1^2\tau_u\left(\tau_0+\tau_v+\tau_{\varepsilon_2}\right)+a_2^2\tau_u\left(\tau_0+\tau_v\right)+\tau_v\left(\tau_v+\tau_{\varepsilon_2}\right)+\tau_0\left(2\tau_v+\tau_{\varepsilon_2}\right)-2a_1a_2\tau_0\tau_u}{a_1^2\tau_u\left[\tau_0\left(\tau_v+\tau_{\varepsilon_2}\right)+\tau_v\left(\tau_v+2\tau_{\varepsilon_2}\right)\right]+a_2^2\tau_u\tau_v\left(\tau_0+\tau_v\right)+\tau_0\tau_v\left(\tau_v+\tau_{\varepsilon_2}\right)+\tau_v^2\left(2a_1a_2\tau_u+\tau_{\varepsilon_2}\right)},\tag{B3}$$

$$Var\left(\tilde{v}|\tilde{p}\right)=\frac{1}{\tau_v}\frac{(a_1-a_2)^2\tau_0\tau_u+\left(a_1^2+a_2^2\right)\tau_u\tau_v+\tau_v\left(2\tau_0+\tau_v\right)}{\left(a_1^2+a_2^2\right)\tau_0\tau_u+\left(a_1+a_2\right)^2\tau_u\tau_v+\tau_0\tau_v}.\tag{B4}$$

Inserting these conditional moments into the demand functions (5)-(8), and using the market-clearing condition (9), we obtain the equilibrium price as shown in Proposition 1. Θ_{12}, Θ_1 and Θ_2 are as follows:

$$
\begin{aligned}
\Theta_{12} &= \frac{\tau_u(\tau_0+\tau_v)\left[a_2(\tau_v+\tau_{\varepsilon_1})+a_1(\tau_v+\tau_{\varepsilon_2})\right]}{\begin{array}{c} a_1^2\tau_u\left[\tau_0(\tau_v+\tau_{\varepsilon_2})+\tau_v(\tau_v+2\tau_{\varepsilon_2})\right]+a_2^2\tau_u\left[\tau_0(\tau_v+\tau_{\varepsilon_1})+\tau_v(\tau_v+2\tau_{\varepsilon_1})\right] \\ +\tau_0(\tau_v+\tau_{\varepsilon_1})(\tau_v+\tau_{\varepsilon_2})+\tau_v\left[2a_1a_2\tau_u\tau_v+2\tau_{\varepsilon_1}\tau_{\varepsilon_2}+\tau_v(\tau_{\varepsilon_1}+\tau_{\varepsilon_2})\right]\end{array}}, \\
\Theta_1 &= \frac{\tau_u(\tau_0+\tau_v)\left[a_1\tau_v+a_2(\tau_v+\tau_{\varepsilon_1})\right]}{a_1^2\tau_u\tau_v(\tau_0+\tau_v)+a_2^2\tau_u\left[\tau_0(\tau_v+\tau_{\varepsilon_1})+\tau_v(\tau_v+2\tau_{\varepsilon_1})\right]+\tau_0\tau_v(\tau_v+\tau_{\varepsilon_1})+\tau_v^2(2a_1a_2\tau_u+\tau_{\varepsilon_1})}, \\
\Theta_2 &= \frac{\tau_u(\tau_0+\tau_v)\left[a_1(\tau_v+\tau_{\varepsilon_2})+a_2\tau_v\right]}{a_1^2\tau_u\left[\tau_0(\tau_v+\tau_{\varepsilon_2})+\tau_v(\tau_v+2\tau_{\varepsilon_2})\right]+a_2^2\tau_u\tau_v(\tau_0+\tau_v)+\tau_0\tau_v(\tau_v+\tau_{\varepsilon_2})+\tau_v^2(2a_1a_2\tau_u+\tau_{\varepsilon_2})}, \\
\Theta_0 &= \frac{\tau_u(\tau_0+\tau_v)(a_1+a_2)}{\tau_0\tau_u(a_1^2+a_2^2)+\tau_u\tau_v(a_1+a_2)^2+\tau_0\tau_v}.
\end{aligned}
$$

To show the existence of solution (a_1, a_2) to Equation (16), we let $\Phi(a_1, a_2)$ be a vector valued function with the following form:

$$
\Phi(a_1, a_2) = \begin{pmatrix} g_1(a_1, a_2) \\ g_2(a_1, a_2) \end{pmatrix},
$$

where $g_1(a_1, a_2)$ is the right-hand side (RHS) of the first equation in (16) and $g_2(a_1, a_2)$ is the RHS of the second equation in (16). Therefore, solution to (16) is a fixed point of $\Phi(a_1, a_2)$. Since $\Phi : R^2 \to R^2$ and it is continuous, the existence proof directly follows Brouwer fixed-point theorem.

When $\tau_{\varepsilon_1} = \tau_{\varepsilon_2} \equiv \tau_\varepsilon$, Equation (16) can be simplified as

$$
a = \frac{1}{\gamma}\tau_\varepsilon(\tau_0+\tau_v)\left[\frac{\lambda_{12}}{2\tau_0+\tau_v+\tau_\varepsilon+2a^2\tau_u}+\frac{\lambda\tau_v}{\tau_v(\tau_v+\tau_\varepsilon)+\tau_0(2\tau_v+\tau_\varepsilon)+a^2\tau_u(2\tau_v+\tau_\varepsilon)}\right], \tag{B5}
$$

where $a_1 = a_2 \equiv a$. Equation (B5) be rewritten as the following polynomial function about a:

$$
\begin{aligned}
f(a) \equiv\; & 2\gamma\tau_u^2(2\tau_v+\tau_\varepsilon)a^5+\gamma\tau_u\left[4\tau_v^2+5\tau_v\tau_\varepsilon+\tau_\varepsilon^2+4\tau_0(2\tau_v+\tau_\varepsilon)\right]a^3 \\
& -\tau_u\tau_\varepsilon(\tau_0+\tau_v)\left[2\tau_v(\lambda+\lambda_{12})+\lambda_{12}\tau_\varepsilon\right]a^2 \\
& +\gamma\left[\tau_v(\tau_v+\tau_\varepsilon)^2+2\tau_0^2(2\tau_v+\tau_\varepsilon)+\tau_0(\tau_v+\tau_\varepsilon)(4\tau_v+\tau_\varepsilon)\right]a \\
& -\tau_\varepsilon(\tau_0+\tau_v)\left[\tau_v(2\tau_0+\tau_v+\tau_\varepsilon)\lambda+\lambda_{12}(\tau_v(\tau_v+\tau_\varepsilon)+\tau_0(2\tau_v+\tau_\varepsilon))\right] \\
=\;& 0.
\end{aligned}
$$

Since $f(0) = -\tau_\varepsilon(\tau_0+\tau_v)\left[\tau_v(2\tau_0+\tau_v+\tau_\varepsilon)\lambda+\lambda_{12}(\tau_v(\tau_v+\tau_\varepsilon)+\tau_0(2\tau_v+\tau_\varepsilon))\right] < 0$ and

$\lim_{a \to +\infty} f(a) \to +\infty$, by intermediate value theorem there exists solution to (B5). When $\lambda = 0$, Equation $f(a)$ can be rewritten as follows $f(a) = [\tau_u(2\tau_v + \tau_\varepsilon)a^2 + \tau_v(\tau_v + \tau_\varepsilon) + \tau_0(2\tau_v + \tau_\varepsilon)] f_1(a)$, where

$$f_1(a) = 2\gamma\tau_u a^3 + \gamma(2\tau_0 + \tau_v + \tau_\varepsilon)a - \lambda_{12}\tau_\varepsilon(\tau_0 + \tau_v). \tag{B6}$$

Since $f_1'(a) > 0$, the solution is unique. When $\lambda_{12} = 0$, Equation $f(a)$ can be rewritten as $f(a) = [2\tau_u a^2 + 2\tau_0 + \tau_v + \tau_\varepsilon] f_2(a)$, where

$$f_2(a) = \gamma\tau_u(2\tau_v + \tau_\varepsilon)a^3 + \gamma[\tau_v(\tau_v + \tau_\varepsilon) + \tau_0(2\tau_v + \tau_\varepsilon)]a - \lambda_1\tau_v\tau_\varepsilon(\tau_0 + \tau_v). \tag{B7}$$

Since $f_2'(a) > 0$, the solution is unique. QED.

Note the proof of Lemma (1) is omitted since we prove it in the main text.

Proof of Equation (17)-(19)

Suppose investors' initial wealth is W_0. Following Grossman and Stiglitz (1980), we can compute the ex-ante expected utility of type-12, type-1, type-2 and type-0 investors as follows respectively

$$\begin{aligned}
V_{12} &= -\exp(-\gamma W_0)\sqrt{\frac{Var(\tilde{v}|\tilde{s}_{1i}, \tilde{s}_{2i}, \tilde{p})}{Var(\tilde{v} - \tilde{p})}}\exp\left(-\frac{1}{2}\frac{[E(\tilde{v} - \tilde{p})]^2}{Var(\tilde{v} - \tilde{p})}\right), \\
V_1 &= -\exp(-\gamma W_0)\sqrt{\frac{Var(\tilde{v}|\tilde{s}_{1i}, \tilde{p})}{Var(\tilde{v} - \tilde{p})}}\exp\left(-\frac{1}{2}\frac{[E(\tilde{v} - \tilde{p})]^2}{Var(\tilde{v} - \tilde{p})}\right), \\
V_2 &= -\exp(-\gamma W_0)\sqrt{\frac{Var(\tilde{v}|\tilde{s}_{2i}, \tilde{p})}{Var(\tilde{v} - \tilde{p})}}\exp\left(-\frac{1}{2}\frac{[E(\tilde{v} - \tilde{p})]^2}{Var(\tilde{v} - \tilde{p})}\right), \\
V_0 &= -\exp(-\gamma W_0)\sqrt{\frac{Var(\tilde{v}|\tilde{p})}{Var(\tilde{v} - \tilde{p})}}\exp\left(-\frac{1}{2}\frac{[E(\tilde{v} - \tilde{p})]^2}{Var(\tilde{v} - \tilde{p})}\right).
\end{aligned}$$

Thus their certainty equivalent are respectively

$$
\begin{aligned}
CE_{12} &= -\frac{1}{\gamma}\log\left(V_{12}\right)-W_0=-\frac{1}{2\gamma}\log\left(\frac{Var\left(\tilde{v}|\tilde{s}_{1i},\tilde{s}_{2i},\tilde{p}\right)}{Var\left(\tilde{v}-\tilde{p}\right)}\right)+\frac{1}{2\gamma}\frac{\left[E\left(\tilde{v}-\tilde{p}\right)\right]^2}{Var\left(\tilde{v}-\tilde{p}\right)},\\
CE_{1} &= -\frac{1}{\gamma}\log\left(V_{1}\right)-W_0=-\frac{1}{2\gamma}\log\left(\frac{Var\left(\tilde{v}|\tilde{s}_{1i},\tilde{p}\right)}{Var\left(\tilde{v}-\tilde{p}\right)}\right)+\frac{1}{2\gamma}\frac{\left[E\left(\tilde{v}-\tilde{p}\right)\right]^2}{Var\left(\tilde{v}-\tilde{p}\right)},\\
CE_{2} &= -\frac{1}{\gamma}\log\left(V_{2}\right)-W_0=-\frac{1}{2\gamma}\log\left(\frac{Var\left(\tilde{v}|\tilde{s}_{2i},\tilde{p}\right)}{Var\left(\tilde{v}-\tilde{p}\right)}\right)+\frac{1}{2\gamma}\frac{\left[E\left(\tilde{v}-\tilde{p}\right)\right]^2}{Var\left(\tilde{v}-\tilde{p}\right)},\\
CE_{0} &= -\frac{1}{\gamma}\log\left(V_{0}\right)-W_0=-\frac{1}{2\gamma}\log\left(\frac{Var\left(\tilde{v}|\tilde{p}\right)}{Var\left(\tilde{v}-\tilde{p}\right)}\right)+\frac{1}{2\gamma}\frac{\left[E\left(\tilde{v}-\tilde{p}\right)\right]^2}{Var\left(\tilde{v}-\tilde{p}\right)}.
\end{aligned}
$$

Therefore $\phi_{12}=CE_{12}-CE_0$, $\phi_1=CE_1-CE_0$, and $\phi_2=CE_2-CE_0$. QED.

Proof of Lemma 2

With symmetric equilibrium (i.e., $\tau_{\varepsilon_1}=\tau_{\varepsilon_2}\equiv\tau_\varepsilon$ and $a_1=a_2\equiv a$), (B1)-(B4) can be simplified to be

$$
\begin{aligned}
Var\left(\tilde{v}|\tilde{s}_{1i},\tilde{s}_{2i},\tilde{p}\right) &= \frac{2a^2\tau_u+2\tau_0+\tau_v+\tau_\varepsilon}{2a^2\tau_u\left(\tau_0+2\tau_v\right)+\tau_0\tau_v+\tau_0\tau_\varepsilon+2\tau_v\tau_\varepsilon}, && \text{(B8)}\\
Var\left(\tilde{v}|\tilde{s}_{1i},\tilde{p}\right) &= Var\left(\tilde{v}|\tilde{s}_{2i},\tilde{p}\right)=\frac{a^2\tau_u\left(2\tau_v+\tau_\varepsilon\right)+\tau_v\left(\tau_v+\tau_\varepsilon\right)+\tau_0\left(2\tau_v+\tau_\varepsilon\right)}{a^2\tau_u\left(\tau_0+2\tau_v\right)\left(2\tau_v+\tau_\varepsilon\right)+\tau_v^2\left(\tau_\varepsilon+\tau_0\right)+\tau_0\tau_\varepsilon\tau_v}, && \text{(B9)}\\
Var\left(\tilde{v}|\tilde{p}\right) &= \frac{2a^2\tau_u+2\tau_0+\tau_v}{2a^2\tau_u\left(\tau_0+2\tau_v\right)+\tau_0\tau_v}. && \text{(B10)}
\end{aligned}
$$

Given that the total variance of the risky asset is limited, when $k=1$, we know $\tau_0\to+\infty$. With $\tau_0\to+\infty$, (B8)-(B10) can be further simplified to be

$$
\begin{aligned}
Var\left(\tilde{v}|\tilde{s}_{1i},\tilde{s}_{2i},\tilde{p}\right) &= \frac{2}{2a^2\tau_u+\tau_v+\tau_\varepsilon}, && \text{(B11)}\\
Var\left(\tilde{v}|\tilde{s}_{1i},\tilde{p}\right) &= Var\left(\tilde{v}|\tilde{s}_{2i},\tilde{p}\right)=\frac{2\tau_v+\tau_\varepsilon}{a^2\tau_u(2\tau_v+\tau_\varepsilon)+\tau_v\left(\tau_v+\tau_\varepsilon\right)}, && \text{(B12)}\\
Var\left(\tilde{v}|\tilde{p}\right) &= \frac{2}{2a^2\tau_u+\tau_v}. && \text{(B13)}
\end{aligned}
$$

Thus based on (B11)-(B13) we can compute $Var\left(\tilde{v}|\tilde{s}_{1i},\tilde{p}\right)Var\left(\tilde{v}|\tilde{s}_{2i},\tilde{p}\right)-Var\left(\tilde{v}|\tilde{s}_{1i},\tilde{s}_{2i},\tilde{p}\right)Var\left(\tilde{v}|\tilde{p}\right)$ as follows:

$$
\begin{aligned}
& Var\left(\tilde{v}|\tilde{s}_{1i},\tilde{p}\right)Var\left(\tilde{v}|\tilde{s}_{2i},\tilde{p}\right)-Var\left(\tilde{v}|\tilde{s}_{1i},\tilde{s}_{2i},\tilde{p}\right)Var\left(\tilde{v}|\tilde{p}\right)\\
= & \frac{\tau_\varepsilon^2\left[\tau_v\left(\tau_v+\tau_\varepsilon\right)+2a^2\tau_u\left(2\tau_v+\tau_\varepsilon\right)\right]}{\left(2a^2\tau_u+\tau_v\right)\left(2a^2\tau_u+\tau_v+\tau_\varepsilon\right)\left[\tau_v\left(\tau_v+\tau_\varepsilon\right)+a^2\tau_u\left(2\tau_v+\tau_\varepsilon\right)\right]^2}>0.
\end{aligned}
$$

Therefore, $\frac{Var(\tilde{v}|\tilde{s}_{1i},\tilde{p})Var(\tilde{v}|\tilde{s}_{2i},\tilde{p})}{Var(\tilde{v}|\tilde{s}_{1i},\tilde{s}_{2i},\tilde{p})Var(\tilde{v}|\tilde{p})} > 1$, which implies that $\phi_{12}-\phi_1-\phi_2 = \frac{1}{2}\ln\left[\frac{Var(\tilde{v}|\tilde{s}_{1i},\tilde{p})Var(\tilde{v}|\tilde{s}_{2i},\tilde{p})}{Var(\tilde{v}|\tilde{s}_{1i},\tilde{s}_{2i},\tilde{p})Var(\tilde{v}|\tilde{p})}\right] > 0$ according to (17)-(19). QED.

Proof of Lemma 3

The proof is similar to that of Lemma 2. With limited total variance of the risky asset, when $k \to 0$, we know $\tau_v \to +\infty$; that is, $\lim_{k\to 0}\tau_v = +\infty$. With $\tau_0 \to +\infty$, (B8)-(B10) can be further simplified to be

$$\begin{aligned} Var\left(\tilde{v}|\tilde{s}_{1i},\tilde{s}_{2i},\tilde{p}\right) &= \frac{1}{\tau_0+2\tau_\varepsilon+4a^2\tau_u}, &\text{(B14)}\\ Var\left(\tilde{v}|\tilde{s}_{1i},\tilde{p}\right) &= Var\left(\tilde{v}|\tilde{s}_{2i},\tilde{p}\right) = \frac{1}{\tau_0+\tau_\varepsilon+4a^2\tau_u}, &\text{(B15)}\\ Var\left(\tilde{v}|\tilde{p}\right) &= \frac{1}{\tau_0+4a^2\tau_u}. &\text{(B16)} \end{aligned}$$

With (B14)-(B16) we can compute $Var\left(\tilde{v}|\tilde{s}_{1i},\tilde{p}\right)Var\left(\tilde{v}|\tilde{s}_{2i},\tilde{p}\right) - Var\left(\tilde{v}|\tilde{s}_{1i},\tilde{s}_{2i},\tilde{p}\right)Var\left(\tilde{v}|\tilde{p}\right)$ as follows

$$\begin{aligned} & Var\left(\tilde{v}|\tilde{s}_{1i},\tilde{p}\right)Var\left(\tilde{v}|\tilde{s}_{2i},\tilde{p}\right) - Var\left(\tilde{v}|\tilde{s}_{1i},\tilde{s}_{2i},\tilde{p}\right)Var\left(\tilde{v}|\tilde{p}\right) \\ = & -\frac{\tau_\varepsilon^2}{\left(\tau_0+a^2\tau_u+\tau_\varepsilon\right)^2\left(\tau_0+a^2\tau_u\right)\left(\tau_0+a^2\tau_u+2\tau_\varepsilon\right)} < 0. \end{aligned}$$

Therefore, $\frac{Var(\tilde{v}|\tilde{s}_{1i},\tilde{p})Var(\tilde{v}|\tilde{s}_{2i},\tilde{p})}{Var(\tilde{v}|\tilde{s}_{1i},\tilde{s}_{2i},\tilde{p})Var(\tilde{v}|\tilde{p})} < 1$, which implies that $\phi_{12}-\phi_1-\phi_2 = \frac{1}{2}\ln\left(\frac{Var(\tilde{v}|\tilde{s}_{1i},\tilde{p})Var(\tilde{v}|\tilde{s}_{2i},\tilde{p})}{Var(\tilde{v}|\tilde{s}_{1i},\tilde{s}_{2i},\tilde{p})Var(\tilde{v}|\tilde{p})}\right) < 0$. QED.

Proof of Proposition 3

(i) Sellers selling orthogonal information. With limited total variance of the risky asset, we have $\tau_0 \to +\infty$ when $k = 1$. Before calculating the information value and seller's objective function, we need prepare serveral derivatives: $\frac{\partial a_1}{\partial \tau_{\varepsilon_1}}, \frac{\partial a_1}{\partial \tau_{\varepsilon_2}}, \frac{\partial a_2}{\partial \tau_{\varepsilon_1}}, \frac{\partial a_2}{\partial \tau_{\varepsilon_2}}$. With $\lambda_0 = 0$ from Lemma 1 (thus $\lambda_{12} = 1-2\lambda$) and $\tau_0 \to +\infty$, the Equation system (16) can be rewritten follows:

$$\left\{\begin{aligned} g_1\left(\tau_{\varepsilon_1},\tau_{\varepsilon_2},a_1,a_2\right) &= \frac{\tau_{\varepsilon_1}}{\gamma}\left(\frac{\lambda[\tau_v+a_2\tau_u(a_2-a_1)]}{(a_1-a_2)^2\tau_u+2\tau_v+\tau_{\varepsilon_1}} + \frac{\lambda_{12}\left[\tau_v+\tau_{\varepsilon_2}+a_2\tau_u(a_2-a_1)\right]}{\tau_u(a_1-a_2)^2+2\tau_v+\tau_{\varepsilon_1}+\tau_{\varepsilon_2}}\right) - a_1 \\ g_2\left(\tau_{\varepsilon_1},\tau_{\varepsilon_2},a_1,a_2\right) &= \frac{\tau_{\varepsilon_2}}{\gamma}\left(\frac{\lambda[\tau_v+a_1\tau_u(a_1-a_2)]}{\tau_u(a_1-a_2)^2+2\tau_v+\tau_{\varepsilon_2}} + \frac{\lambda_{12}\left[\tau_v+\tau_{\varepsilon_1}+a_1\tau_u(a_1-a_2)\right]}{\tau_u(a_1-a_2)^2+2\tau_v+\tau_{\varepsilon_1}+\tau_{\varepsilon_2}}\right) - a_2 \end{aligned}\right. . \quad \text{(B17)}$$

Then applying implicit function theorem, we can compute $\frac{\partial a_1}{\partial \tau_{\varepsilon_1}}, \frac{\partial a_1}{\partial \tau_{\varepsilon_2}}, \frac{\partial a_2}{\partial \tau_{\varepsilon_1}}, \frac{\partial a_2}{\partial \tau_{\varepsilon_2}}$ as follows:

$$\begin{pmatrix} \frac{\partial a_1}{\partial \tau_{\varepsilon_i}} \\ \frac{\partial a_2}{\partial \tau_{\varepsilon_i}} \end{pmatrix} = -\begin{pmatrix} \frac{\partial g_1}{\partial a_1} & \frac{\partial g_1}{\partial a_2} \\ \frac{\partial g_2}{\partial a_1} & \frac{\partial g_2}{\partial a_2} \end{pmatrix}^{-1} \begin{pmatrix} \frac{\partial g_1}{\partial \tau_{\varepsilon_i}} \\ \frac{\partial g_2}{\partial \tau_{\varepsilon_i}} \end{pmatrix}, \; i = \{1, 2\} \tag{B18}$$

where $\frac{\partial g_1}{\partial a_1}, \frac{\partial g_1}{\partial a_2}, \frac{\partial g_2}{\partial a_1}, \frac{\partial g_2}{\partial a_2}, \frac{\partial g_1}{\partial \tau_{\varepsilon_1}}, \frac{\partial g_1}{\partial \tau_{\varepsilon_2}}, \frac{\partial g_2}{\partial \tau_{\varepsilon_1}}$ and $\frac{\partial g_2}{\partial \tau_{\varepsilon_2}}$ are calculated based on (B17).

Next, plugging equations (17)-(19) into (24), we obtain the profit of seller n: π_n. Given $\tau_{\varepsilon_{n'}}$, $\lambda_1 = \lambda_2 \equiv \lambda$ and $\lambda_{12} = 1 - 2\lambda$, taking partial derivatives of π_n with respect to τ_{ε_n} yields the best response function of seller n, $\tau_{\varepsilon_n} = \tau_{\varepsilon_n}\left(\tau_{\varepsilon_{n'}}\right)$. Imposing symmetric equilibrium $\tau_{\varepsilon_1} = \tau_{\varepsilon_2} \equiv \tau_\varepsilon$, we obtain the FOC $\frac{\partial \pi_n}{\partial \tau_{\varepsilon_n}}\left(\tau_{\varepsilon_1} = \tau_{\varepsilon_2} \equiv \tau_\varepsilon\right)$, which is positively related to the following polynomial function $h\left(\tau_\varepsilon\right)$ about τ_ε of order 13.

$$\begin{aligned}
& h\left(\tau_\varepsilon\right) \equiv 16\gamma^6 \tau_v^2 \left(2\tau_v + \tau_\varepsilon\right)^4 \left(2\tau_v^2 + 3\tau_v\tau_\varepsilon + \tau_\varepsilon^2\right)^2 \\
& +4\gamma^4 \tau_u \tau_v \tau_\varepsilon^2 \left(\tau_v + \tau_\varepsilon\right)\left(2\tau_v + \tau_\varepsilon\right)^4 \begin{bmatrix} 8\left(1-\lambda\right)^2 \tau_v^3 + 12\left(1-2\lambda\right)\left(1-\lambda\right)\tau_v^2\tau_\varepsilon \\ +2\left(1-2\lambda\right)\left(3\lambda+5\right)\tau_v\tau_\varepsilon^2 + \left(2\lambda-1\right)\left(4\lambda-1\right)\tau_\varepsilon^3 \end{bmatrix} \\
& -\tau_u \tau_\varepsilon^6 \left[2\left(1-\lambda\right)\tau_v + \left(1-2\lambda\right)\tau_\varepsilon\right]^3 \begin{bmatrix} 16\left(1-\lambda\right)^3 \tau_v^4 - 16\left(1-\lambda\right)^2\left(3\lambda-2\right)\tau_v^3\tau_\varepsilon \\ -4\left(14\lambda^3 - 33\lambda^2 + 25\lambda - 6\right)\tau_v^2\tau_\varepsilon^2 \\ +4\left(1-2\lambda\right)^2\left(\lambda-2\right)\tau_v\tau_\varepsilon^3 + \left(1-2\lambda\right)^2\tau_\varepsilon^4 \end{bmatrix} \\
& -2\gamma^2 \tau_u^2 \tau_\varepsilon^4 \left(2\tau_v + \tau_\varepsilon\right)^2 \left[2\left(1-\lambda\right)\tau_v + \left(1-2\lambda\right)\tau_\varepsilon\right] \begin{bmatrix} -16\left(1-\lambda\right)^3\tau_v^5 + 8\left(1-\lambda\right)^2\left(6\lambda-5\right)\tau_v^4\tau_\varepsilon \\ +4\left(24\lambda^3 - 56\lambda^2 + 43\lambda - 11\right)\tau_v^3\tau_\varepsilon^2 \\ +2\left(56\lambda^3 - 100\lambda^2 + 62\lambda - 13\right)\tau_v^2\tau_\varepsilon^3 \\ +4\left(14\lambda^3 - 21\lambda^2 + 11\lambda - 2\right)\tau_v\tau_\varepsilon^4 - \left(1-2\lambda\right)\tau_\varepsilon^5 \end{bmatrix} \\
& = 0.
\end{aligned} \tag{B19}$$

Denote the optimal signal precision as a function of the two key parameters: $\tau_\varepsilon\left(k, \beta\right)$.

When $\beta \to 0$, based on (25) and (26), we know that $B_{12} > B_n$ as discussed in the paper, and thus $\lambda^* = 0$. Substituting $\lambda^* = 0$ into (B19) yields $h\left(\tau_\varepsilon\right) = h_1\left(\tau_\varepsilon\right) \times PositiveConstant$, where

$$h_1\left(\tau_\varepsilon\right) \equiv -\tau_u\tau_\varepsilon^3 - 2\tau_v\tau_u\tau_\varepsilon^2 + 4\gamma\tau_v\tau_\varepsilon + 4\gamma^2\tau_v^2. \tag{B20}$$

Setting $h_1(\tau_\varepsilon) = 0$ yields $\tau_\varepsilon^*(1,0)$. Note $h_1(\tau_\varepsilon) = 0$ is equivalent to

$$\frac{4\gamma\tau_v}{\tau_u\tau_\varepsilon^2 + 2\tau_v\tau_u\tau_\varepsilon} + \frac{4\gamma^2\tau_v^2}{\tau_u\tau_\varepsilon^3 + 2\tau_v\tau_u\tau_\varepsilon^2} = 1. \tag{B21}$$

Now we prove there exists a unique positive solution to the equation. First the LHS of the formula is decreasing with τ_ε. Second, when $\tau_\varepsilon = +\infty$, the LHS goes to zero, while when $\tau_\varepsilon = 0$, the LHS goes to infinity. Thus, there exists a unique positive $\tau_\varepsilon^*(1,0)$.

When $\beta \to 1$, based on Lemma 2, equations (25) and (26), we know that $B_{12} < 0$ and $B_n > 0$, and thus $\lambda^* = \frac{1}{2}$. Substituting $\lambda^* = \frac{1}{2}$ into (B19) yields $h(\tau_\varepsilon) = h_1(\tau_\varepsilon) \times PositiveConstant,$ *where*

$$h_2(\tau_\varepsilon) \equiv \left(2\gamma^2 - \tau_u\tau_v\right)\tau_\varepsilon^2 + 8\gamma^2\tau_v\tau_\varepsilon + 8\gamma^2\tau_v^2. \tag{B22}$$

When $2\gamma^2 < \tau_u\tau_v$, there exists a unique positive root to $h_2(\tau_\varepsilon) = 0$ as shown in equation (28). When $2\gamma^2 \geq \tau_u\tau_v$, based on equation (B19) the FOC $\frac{\partial\pi_n}{\partial\tau_{\varepsilon_n}}(\tau_{\varepsilon_1} = \tau_{\varepsilon_2} \equiv \tau_\varepsilon)= 0$ can be rewritten in the following term:

$$\begin{aligned} 0 \;=\; & \tau_\varepsilon^{13} f_{13}\left(\lambda_{12}^5\right) + \tau_\varepsilon^{12} f_{12}\left(\lambda_{12}^6\right) + \tau_\varepsilon^{11} f_{11}\left(\lambda_{12}^6\right) + \tau_\varepsilon^{10} f_{10}\left(\lambda_{12}^6\right) + \tau_\varepsilon^9 f_9\left(\lambda_{12}^6\right) \\ & + \tau_\varepsilon^8 f_8\left(\lambda_{12}^6\right) + \tau_\varepsilon^7 f_7\left(\lambda_{12}^6\right) + \tau_\varepsilon^6 f_6\left(\lambda_{12}^6\right) + \tau_\varepsilon^5 f_5\left(\lambda_{12}^4\right) + \tau_\varepsilon^4 f_4\left(\lambda_{12}^4\right) \\ & + \tau_\varepsilon^3 f_3\left(\lambda_{12}^2\right) + \tau_\varepsilon^2 f_2\left(\lambda_{12}^2\right) + 5120\gamma^6\tau_v^9\tau_\varepsilon + 1024\gamma^6\tau_v^{10}, \end{aligned} \tag{B23}$$

where $\lambda_{12} = 1 - 2\lambda$, and $f_i\left(\lambda_{12}^j\right)$ is a polynomial function in front of τ_ε^i in which the highest order of λ_{12} is j. As $\beta \to 1$ we know that $\lambda_{12} \to 0$. To make sure that equation (B23) hold, we need $\tau_\varepsilon \sim O\left(\lambda_{12}^{-1.6}\right)$. Thus $\tau_\varepsilon^* \to +\infty$; that is, sellers do not add any noise to the sold information.

We then compare the equilibrium information precision as $\beta \to 1$ to that as $\beta \to 0$, and find that in the former case the sold information is more precise. The following corollary summarizes the result.

(ii) Sellers selling almost identical information. First given that the total variance of the risky asset is limited, $\lim_{k\to 0}\tau_v = +\infty$. As discussed in the paper, when $k \to 0$ we have $\lambda^* = 0$.

When $\tau_v = +\infty$, based on (16) we can compute $a_i = \frac{\tau_{\varepsilon_i}}{\gamma}$, denoted by $a_i(0)$. Similar to the procedure in Case (i), applying implicit function theorem to Equation system (16) we can obtain $\frac{da_i}{d(1/\tau_v)}$. Using Taylor series to expand a_i around $\frac{1}{\tau_v} = 0$ we obtain $a_i \approx a_i(0) + \frac{da_i}{d(1/\tau_v)} \frac{1}{\tau_v}$, which can be simplified to the following form

$$\begin{cases} a_1 \approx \frac{\tau_{\varepsilon_1}}{\gamma} - \frac{\tau_{\varepsilon_1}}{\gamma^3}\left[\gamma^2\left(\tau_0 + \tau_{\varepsilon_1}\right) + \tau_u \tau_{\varepsilon_1}\left(\tau_{\varepsilon_1} + \tau_{\varepsilon_2}\right)\right]\frac{1}{\tau_v} \\ a_2 \approx \frac{\tau_{\varepsilon_2}}{\gamma} - \frac{\tau_{\varepsilon_2}}{\gamma^3}\left[\gamma^2\left(\tau_0 + \tau_{\varepsilon_2}\right) + \tau_u \tau_{\varepsilon_2}\left(\tau_{\varepsilon_1} + \tau_{\varepsilon_2}\right)\right]\frac{1}{\tau_v} \end{cases}.$$

With symmetric $\tau_{\varepsilon_1} = \tau_{\varepsilon_2} \equiv \tau_\varepsilon$ this can be written as:

$$a \approx \frac{\tau_\varepsilon}{\gamma} - \frac{\tau_\varepsilon}{\gamma^3}\left[\gamma^2 \tau_0 + \tau_\varepsilon\left(\gamma^2 + 2\tau_u \tau_\varepsilon\right)\right]\frac{1}{\tau_v}. \tag{B24}$$

Plugging equations (17)-(19) into equation (24), we obtain the profit of seller n: π_n. Given $\tau_{\varepsilon_{n'}}$, $\lambda_1 = \lambda_2 \equiv \lambda$, taking partial derivatives of π_n with respect to τ_{ε_n} yields the best-response function of seller n, $\tau_{\varepsilon_n} = \tau_{\varepsilon_n}\left(\tau_{\varepsilon_{n'}}\right)$. After imposing symmetric equilibrium $\tau_{\varepsilon_1} = \tau_{\varepsilon_2} \equiv \tau_\varepsilon$ and $\lambda = 0$, and inserting (B24) into the first-order condition, we know that to set the first-order condition to be zero is equivalent to set the following complicated polynomial function to be zero:

$$\begin{aligned} 0 \; = \; & x^{20} f_{20}\left(\tau_\varepsilon^{33}\right) + x^{19} f_{19}\left(\tau_\varepsilon^{33}\right) + x^{18} f_{18}\left(\tau_\varepsilon^{33}\right) + x^{17} f_{17}\left(\tau_\varepsilon^{33}\right) + x^{16} f_{16}\left(\tau_\varepsilon^{32}\right) + x^{15} f_{15}\left(\tau_\varepsilon^{31}\right) \\ & + x^{14} f_{14}\left(\tau_\varepsilon^{30}\right) + x^{13} f_{13}\left(\tau_\varepsilon^{28}\right) + x^{12} f_{12}\left(\tau_\varepsilon^{26}\right) + x^{11} f_{11}\left(\tau_\varepsilon^{24}\right) + x^{10} f_{10}\left(\tau_\varepsilon^{22}\right) + x^9 f_9\left(\tau_\varepsilon^{20}\right) \\ & + x^8 f_8\left(\tau_\varepsilon^{18}\right) + x^7 f_7\left(\tau_\varepsilon^{16}\right) + x^6 f_6\left(\tau_\varepsilon^{14}\right) + x^5 f_5\left(\tau_\varepsilon^{12}\right) + x^4 f_4\left(\tau_\varepsilon^{10}\right) + x^3 f_3\left(\tau_\varepsilon^{8}\right) + x^2 f_2\left(\tau_\varepsilon^{6}\right) \\ & + x f_1\left(\tau_\varepsilon^{4}\right) + \beta\gamma^2 \tau_0 + \beta\gamma^2 \tau_\varepsilon + 8\beta \tau_u \tau_\varepsilon^2, \end{aligned} \tag{B25}$$

where $x = \frac{1}{\tau_v}$ and $f_i\left(\tau_\varepsilon^j\right)$ is a polynomial function in front of x^i in which the highest order of τ_ε is j. When $\frac{1}{\tau_v} \to 0$ (i.e., $\tau_v \to +\infty$), based on (B25) we know that to make sure that the FOC $\frac{\partial \pi_n}{\partial \tau_{\varepsilon_n}}\left(\tau_{\varepsilon_1} = \tau_{\varepsilon_2} \equiv \tau_\varepsilon\right) = 0$ we need $\tau_\varepsilon \sim O\left(\tau_v^{1/2}\right)$. Thus when $\frac{1}{\tau_v} \to 0$, $\tau_\varepsilon^* \to +\infty$; that is, sellers do not add any noise to the sold information. QED.

Proof of Corollary 1

When $2\gamma^2 \geq \tau_u \tau_v$, as $\beta \to 1$ the sellers sell the information "as is," which is larger than the limited precision when $\beta \to 0$. Now we only consider the case when $2\gamma^2 < \tau_u \tau_v$. Denote

$\tau_{\varepsilon}^{*}(1,1)$ the precision as in (28). Substituting $\tau_{\varepsilon}^{*}(1,1)$ into LHS of (B21) yields

$$\frac{4\gamma\tau_v\tau_{\varepsilon}^{*}(1,1)+4\gamma^2\tau_v^2}{\tau_u\left[\tau_{\varepsilon}^{*}(1,1)\right]^2+2\tau_v\tau_u\tau_{\varepsilon}^{*}(1,1)}=\frac{\frac{1}{2}\left(\tau_u\tau_v-2\gamma^2\right)\left[\tau_{\varepsilon}^{*}(1,1)\right]^2}{\tau_u\left[\tau_{\varepsilon}^{*}(1,1)\right]^3+2\tau_v\tau_u\left[\tau_{\varepsilon}^{*}(1,1)\right]^2}<\frac{1}{4}.$$

Note the first equality follows because $h_2\left(\tau_{\varepsilon}^{*}(1,1)\right)=0$ as in (B22). Therefore $h_1\left(\tau_{\varepsilon}^{*}(1,1)\right)<0$, which suggests that $\tau_{\varepsilon}^{*}(1,1)>\tau_{\varepsilon}^{*}(1,0)$, where $\tau_{\varepsilon}^{*}(1,1)$ is the equilibrium precision when $k\to 1$, $\beta\to 1$, and $2\gamma^2<\tau_u\tau_v$, and $\tau_{\varepsilon}^{*}(1,0)$ is the equilibrium precision when $k\to 1$ and $\beta\to 0$. QED.

Proof of Proposition 4

Note for ease of exposition we use the notation $X|_{Y\to 1}$ to indicate $\lim_{Y\to 1}X$. In this proof, we discuss how λ_{12} affects investors' purchase of two signals when $k=1$ in the extreme condition where $\lambda\to 0$ and $1/\tau_u\to 0$. That is, we compute the sign of $\frac{\partial(B_{12}-B_n)}{\partial\lambda_{12}}|_{\beta\to 0}$ and $\frac{\partial(B_{12}-B_n)}{\partial\lambda_{12}}|_{\beta\to 1}$ when $\lambda\to 0$ and $1/\tau_u\to 0$. Without loss of generality, we focus on $\frac{\partial(B_{12}-B_1)}{\partial\lambda_{12}}|_{\beta\to 0}$ and $\frac{\partial(B_{12}-B_1)}{\partial\lambda_{12}}|_{\beta\to 1}$.

When $\lambda\to 0$, based on (B19) and implicit function theorem we obtain $\frac{d\tau_\varepsilon}{d\lambda}$ around $\lambda\to 0$, denoted by $\frac{d\tau_\varepsilon}{d\lambda}|_{\lambda\to 0}$:

$$\frac{d\tau_{\varepsilon_1}}{d\lambda}|_{\lambda\to 0}=\frac{2\tau_u^2\tau_\varepsilon^2\begin{pmatrix}32\gamma^4\tau_v^5+8\gamma^2\tau_v^4\tau_\varepsilon\left(13\gamma^2-4\tau_u\tau_\varepsilon\right)-\tau_u\tau_\varepsilon^6\left(8\gamma^2+5\tau_u\tau_\varepsilon\right)\\-4\tau_v^3\tau_\varepsilon^2\left[6\tau_u^2\tau_\varepsilon^2+18\gamma^2\tau_u\tau_\varepsilon-29\gamma^4\right]\\-2\tau_v^2\tau_\varepsilon^3\left[23\tau_u^2\tau_\varepsilon^2+40\gamma^2\tau_u\tau_\varepsilon-28\gamma^4\right]\\+\tau_v\tau_\varepsilon^4\left[12\gamma^4-\tau_u\tau_\varepsilon\left(42\gamma^2+29\tau_u\tau_\varepsilon\right)\right]\end{pmatrix}}{\left(2\tau_v+\tau_\varepsilon\right)\begin{pmatrix}-\tau_u^2\tau_\varepsilon^6\left(24\gamma^2+13\tau_u\tau_\varepsilon\right)+32\tau_v^4\left(5\gamma^6+\gamma^4\tau_u\tau_\varepsilon\right)\\+8\gamma^2\tau_v^3\tau_\varepsilon\left[36\gamma^4+\tau_u\tau_\varepsilon\left(17\gamma^2-4\tau_u\tau_\varepsilon\right)\right]\\-2\tau_u\tau_v\tau_\varepsilon^4\left[\tau_u\tau_\varepsilon\left(34\gamma^2+19\tau_u\tau_\varepsilon\right)-20\gamma^4\right]\\+4\tau_v^2\tau_\varepsilon^2\left[32\gamma^6+\tau_u\tau_\varepsilon\left[35\gamma^4-2\tau_u\tau_\varepsilon\left(10\gamma^2+3\tau_u\tau_\varepsilon\right)\right]\right]\end{pmatrix}} \tag{B26}$$

$$\to_{\text{when }\frac{1}{\tau_u}\to 0}\frac{2\tau_\varepsilon\left(24\tau_v^3+46\tau_v^2\tau_\varepsilon+29\tau_v\tau_\varepsilon^2+5\tau_\varepsilon^3\right)}{\left(2\tau_v+\tau_\varepsilon\right)^2\left(12\tau_v+13\tau_\varepsilon\right)}.$$

Next for ease of exposition we let

$$\omega\equiv\frac{1}{\tau_u}. \tag{B27}$$

Thus we now discuss the case when $\omega\to 0$. When $\lambda\to 0$, τ_ε is solved from (B20). With

(B27), Equation (B20) can be rewritten as

$$h_3 \equiv -\tau_\varepsilon^3 - 2\tau_v \tau_\varepsilon^2 + 4\gamma^2 \tau_v (\tau_v + \tau_\varepsilon) \omega = 0. \tag{B28}$$

It is easy to verify that when $\omega \to 0$, $\tau_\varepsilon \to 0$ from (B28). Using implicit function theorem on (B28) we obtain

$$\frac{\partial \tau_\varepsilon}{\partial \omega}|_{\lambda \to 0} = -\frac{\partial h_3/\partial \omega}{\partial h_3/\partial \tau_\varepsilon} = -\frac{4\gamma^2 \tau_v (\tau_v + \tau_\varepsilon)}{-3\tau_\varepsilon^2 - 4\tau_v \tau_\varepsilon + 4\gamma^2 \tau_v \omega}. \tag{B29}$$

Now since that as $\omega \to 0$, $\tau_\varepsilon \to 0$, we want to find the τ_ε's infinitesimal of the same order as ω, so that we can obtain the approximation term of τ_ε as $\omega \to 0$. Suppose $\tau_\varepsilon = O(\omega^y)$, where $y > 0$ and is a constant. Then we can write $\tau_\varepsilon \approx A\omega^y$ as $\omega \to 0$, where A is a constant. Then we know that $\lim_{\omega \to 0} \frac{\tau_\varepsilon}{A\omega^y} = 1$, based on which we can solve for y and A.

$$\begin{aligned}
\lim_{\omega = \frac{1}{\tau_u} \to 0} \frac{\tau_\varepsilon}{A\omega^y} &= \lim_{\omega = \frac{1}{\tau_u} \to 0} \frac{\partial \tau_\varepsilon / \partial \omega}{Ay\omega^{y-1}} \\
&= -\frac{4\gamma^2 \tau_v (\tau_v + \tau_\varepsilon)}{-3\tau_\varepsilon^2 - 4\tau_v \tau_\varepsilon + 4\gamma^2 \tau_v \omega} \frac{1}{Ay\omega^{y-1}} \\
&\approx -\frac{4\gamma^2 \tau_v (\tau_v + A\omega^y)}{-3A^2\omega^{2y} - 4\tau_v A\omega^y + 4\gamma^2 \tau_v \omega} \frac{1}{Ay\omega^{y-1}} \\
&= -\frac{1}{Ay} \frac{4\gamma^2 \tau_v A\omega^y + 4\gamma^2 \tau_v^2}{-3A^2\omega^{3y-1} - 4\tau_v A\omega^{2y-1} + 4\gamma^2 \tau_v \omega^y} \\
&\approx -\frac{1}{Ay} \frac{4\gamma^2 \tau_v^2}{-4\tau_v A\omega^{2y-1}}.
\end{aligned}$$

Note the first equality holds because of L'Hôspital's Rule, and we plug in (B29) to obtain the second equality. The third line follows when we subtitute $\tau_\varepsilon \approx A\omega^y$ into it. The fifth line follows because $\lim_{\omega \to 0} \frac{4\gamma^2 \tau_v A\omega^y}{4\gamma^2 \tau_v^2} = 0$ and $\lim_{\omega \to 0} \frac{-3A^2\omega^{3y-1} + 4\gamma^2 \tau_v \omega^y}{-4\tau_v A\omega^{2y-1}} = 0$. By observing the equation, the solution to $\lim_{\omega \to 0} \frac{\tau_\varepsilon}{A\omega^y} = 1$ are $A = \gamma\sqrt{2\tau_v}$ and $y = \frac{1}{2}$. Therefore as $\omega \to 0$,

$$\tau_\varepsilon \approx \gamma\sqrt{2\tau_v}\omega^{1/2}. \tag{B30}$$

Next we discuss the sign of $\frac{\partial(B_{12} - B_1)}{\partial \lambda_{12}}|_{\beta \to 0}$. When $\beta \to 0$, $\Delta B_1 = B_{12} - B_1 = \phi_{12} - \phi_1$. From (B20) we can solve for $\tau_\varepsilon^*(\lambda)$, which is a function of λ. Plugging $\tau_\varepsilon^*(\lambda)$ into $\phi_{12} - \phi_1$ we obtain $\Delta B_1(\tau_\varepsilon^*(\lambda)) = \phi_{12}^* - \phi_1^*$. Taking derivative of $\Delta B_1(\tau_\varepsilon^*(\lambda))$ with respect to λ we obtain

$$
\frac{\partial \Delta B_1(\tau_\varepsilon^*(\lambda))}{\partial \lambda} = \gamma \frac{\begin{array}{c} 4\gamma^2\tau_v\left(\tau_v+\tau_\varepsilon\right)^2\left(2\tau_v+\tau_\varepsilon\right)^2\frac{\partial \tau_\varepsilon}{\partial \lambda} + \tau_u\tau_\varepsilon^2\left[2\left(1-\lambda\right)\tau_v+\left(1-2\lambda\right)\tau_\varepsilon\right] \\ \times\left[4\tau_\varepsilon\left(\tau_v+\tau_\varepsilon\right)^2\left(2\tau_v+\tau_\varepsilon\right) - \left(\begin{array}{c} 4\left(1-\lambda\right)\tau_v^3+6\left(1-2\lambda\right)\tau_v^2\tau_\varepsilon \\ +2\left(2-5\lambda\right)\tau_v\tau_\varepsilon^2+\left(1-2\lambda\right)\tau_\varepsilon^3 \end{array}\right)\frac{\partial \tau_\varepsilon}{\partial \lambda}\right] \end{array}}{\begin{array}{c} \left[\begin{array}{c} 8\gamma^2\tau_v^3+\left(1-2\lambda\right)^2\tau_u\tau_\varepsilon^4+4\tau_v^2\tau_\varepsilon\left[3\gamma^2+\left(1-\lambda\right)^2\tau_u\tau_\varepsilon\right] \\ +4\tau_v\tau_\varepsilon^2\left[\gamma^2+\left(1-\lambda\right)\left(1-2\lambda\right)\tau_u\tau_\varepsilon\right] \end{array}\right] \\ \times\left[\begin{array}{c} 8\gamma^2\tau_v^3+\tau_\varepsilon^3\left[2\gamma^2+\left(1-2\lambda\right)^2\tau_u\tau_\varepsilon\right]+4\tau_v^2\tau_\varepsilon\left[4\gamma^2+\left(1-\lambda\right)^2\tau_u\tau_\varepsilon\right] \\ +2\tau_v^2\tau_\varepsilon^5\left[5\gamma^2+2\left(1-\lambda\right)\left(1-2\lambda\right)\tau_u\tau_\varepsilon\right] \end{array}\right] \end{array}}
$$

$$
=^{\lambda\to 0} \gamma \frac{\tau_u\tau_\varepsilon^2\left[2\tau_v^2\left(2\tau_\varepsilon-\frac{\partial \tau_\varepsilon}{\partial \lambda}|_{\lambda\to 0}\right)+2\tau_v\tau_\varepsilon\left(4\tau_\varepsilon-\frac{\partial \tau_\varepsilon}{\partial \lambda}|_{\lambda\to 0}\right)+\tau_\varepsilon^2\left(4\tau_\varepsilon-\frac{\partial \tau_\varepsilon}{\partial \lambda}|_{\lambda\to 0}\right)+4\gamma^2\tau_v(\tau_v+\tau_\varepsilon)^2\frac{\partial \tau_\varepsilon}{\partial \lambda}|_{\lambda\to 0}\right]}{(2\tau_v+\tau_\varepsilon)[2\gamma^2\tau_v+\tau_\varepsilon(2\gamma^2+\tau_u\tau_\varepsilon)][4\gamma^2\tau_v^2+\tau_u\tau_\varepsilon^3+2\tau_v\tau_\varepsilon(2\gamma^2+\tau_u\tau_\varepsilon)]}
$$

Plugging (B29) and (B30) into the above equation, we obtain

$$
\frac{\partial \Delta B_1\left(\tau_\varepsilon^*\left(\lambda\right)\right)}{\partial \lambda} \approx \frac{\sqrt{2}\tau_v^{3/2}}{4\gamma^4\omega^{3/2}} > 0.
$$

Therefore $\frac{\partial(B_{12}-B_1)}{\partial \lambda_{12}}|_{\beta\to 0} = \frac{\partial(B_{12}-B_1)}{\partial(1-2\lambda)}|_{\beta\to 0} = -\frac{1}{2}\frac{\partial(B_{12}-B_1)}{\partial \lambda}|_{\beta\to 0} < 0$; that is, there is cross-agent substitutability in investors' information purchase.

Similarly, for the sign of $\frac{\partial(B_{12}-B_1)}{\partial \lambda_{12}}|_{\beta\to 1}$. When $\beta \to 1$, $B_{12}-B_1 = \phi_1+\phi_2-\phi_{12}$. Plugging $\tau_\varepsilon^*\left(\lambda\right)$ into $\phi_1+\phi_2-\phi_{12}$ we obtain $\Delta B_1\left(\tau_\varepsilon^*\left(\lambda\right)\right) = \phi_1^*+\phi_2^*-\phi_{12}^*$. Taking derivative of $\Delta B_1\left(\tau_\varepsilon^*\left(\lambda\right)\right)$ with respect to λ we obtain

$$
\frac{\partial\left(B_{12}-B_1\right)}{\partial \lambda}|_{\beta\to 1} \approx -\frac{3\tau_v}{2\gamma^3\omega} < 0.
$$

Therefore $\frac{\partial(B_{12}-B_1)}{\partial \lambda_{12}}|_{\beta\to 1} = \frac{\partial(B_{12}-B_1)}{\partial(1-2\lambda)}|_{\beta\to 1} = -\frac{1}{2}\frac{\partial(B_{12}-B_1)}{\partial \lambda}|_{\beta\to 0} > 0$; that is, there is cross-agent complementarity in investors' information purchase. QED.

Proof of Proposition 5

Without loss of generality, we focus on $\Delta B_1\left(\lambda\right)$.

$$
\begin{aligned}
\Delta B_1\left(\lambda\right) &\equiv B_{12}-B_1 \qquad\qquad \text{(B31)} \\
&= \frac{1}{\gamma}\left[\frac{3}{2}\ln\left(1+\frac{\tau_v}{2\tau_v+\tau_\varepsilon}\frac{2\gamma^2\tau_\varepsilon}{2\gamma^2\tau_v+\tau_u\tau_\varepsilon^2}\right)-\ln\left(1+\frac{2\gamma^2\tau_\varepsilon}{2\gamma^2\tau_v+\tau_u\tau_\varepsilon^2}\right)\right]\beta \\
&\quad +\frac{1}{2\gamma}\left[\ln\left(1+\frac{2\gamma^2\tau_\varepsilon}{2\gamma^2\tau_v+\tau_u\tau_\varepsilon^2}\right)-\ln\left(1+\frac{\tau_v}{2\tau_v+\tau_\varepsilon}\frac{2\gamma^2\tau_\varepsilon}{2\gamma^2\tau_v+\tau_u\tau_\varepsilon^2}\right)\right].
\end{aligned}
$$

To show that there exist multiple equilibria: $\lambda = 0$, $\lambda = \frac{1}{2}$, and $\lambda \in \left(0, \frac{1}{2}\right)$, we need to figure out the range of β so that $\Delta B_1\left(\frac{1}{2}\right) < 0$, and $\Delta B_1\left(0\right) > 0$. We can prove that there exists such β when $k \to 1$ and τ_u is sufficiently large (i.e., $\frac{1}{\tau_u} \to 0$). Again we let $\omega = \frac{1}{\tau_u}$ as in (B27).

(i) The case when $\lambda \to 0$. When $\lambda \to 0$, τ_ε is solved from (27). Based on (B30) we know that as $\omega = \frac{1}{\tau_u} \to 0$, in equation (B31) $\frac{\tau_v}{2\tau_v+\tau_\varepsilon}\frac{2\gamma^2\tau_\varepsilon}{2\gamma^2\tau_v+\tau_u\tau_\varepsilon^2} \to 0$ and $\frac{2\gamma^2\tau_\varepsilon}{2\gamma^2\tau_v+\tau_u\tau_\varepsilon^2} \to 0$. Using the fact $\lim_{x\to 0}\ln(1+x) = x$, $\Delta B_1(0) > 0$ can be simplified as follows:

$$\beta \leq \frac{\tau_v + \tau_\varepsilon^*(1,0)}{\tau_v + 2\tau_\varepsilon^*(1,0)} \equiv \bar{b} \in [0,1],$$

where $\tau_\varepsilon^*(1,0)$ is the equilibrium signal precision when $k \to 1$ and $\beta \to 0$, which is the solution to (27).

(ii) The case when $\lambda \to \frac{1}{2}$. When $\lambda \to \frac{1}{2}$ and $2\gamma^2 < \tau_u\tau_v$, τ_ε is shown in (28), which can be rearranged to $\tau_\varepsilon^*(1,1) = \frac{2\left(2\gamma^2\tau_v\omega+\gamma\tau_v\sqrt{2\tau_v}\sqrt{\omega}\right)}{\tau_v-2\gamma^2\omega}$. As $\omega \to 0$, we know that $\tau_\varepsilon \to 0$. Similar to the proof in Proposition 4, we want to find ω's infinitesimal of the same order as τ_ε, so that we can obtain the approximation term of τ_ε as $\omega \to 0$. By observing $\tau_\varepsilon^*(1,1)$ we know that $\tau_\varepsilon = O\left(\omega^{1/2}\right)$. Thus

$$\begin{aligned}\lim_{\omega\to 0}\frac{\tau_\varepsilon}{\omega^{1/2}} &= \frac{2\left(2\gamma^2\tau_v\omega+\gamma\tau_v\sqrt{2\tau_v}\sqrt{\omega}\right)}{\sqrt{\omega}\left(\tau_v-2\gamma^2\omega\right)} \\ &\approx 2\gamma\sqrt{2\tau_v}.\end{aligned}$$

Therefore, as $\omega = \frac{1}{\tau_u} \to 0$, $\tau_\varepsilon \approx 2\gamma\sqrt{2\tau_v}\omega$. Using the fact $\lim_{x\to 0}\ln(1+x) = x$, $\Delta B_1\left(\frac{1}{2}\right) < 0$ can be simplified as follows:

$$\beta \geq \frac{\tau_v + \tau_\varepsilon^*(1,1)}{\tau_v + 2\tau_\varepsilon^*(1,1)} \equiv \underline{b} \in [0,1],$$

where $\tau_\varepsilon^*(1,1)$ is the equilibrium signal precision when $k \to 1$, $\beta \to 1$ and $2\gamma^2 < \tau_u\tau_v$, as shown in (28). Note that $\frac{\partial\left(\frac{\tau_v+\tau_\varepsilon}{\tau_v+2\tau_\varepsilon}\right)}{\partial\tau_\varepsilon} = -\frac{\tau_v}{(\tau_v+2\tau_\varepsilon)^2} < 0$. Thus based on Corollary 1, $\tau_\varepsilon^*(1,1) > \tau_\varepsilon^*(1,0)$, we have $\bar{b} > \underline{b}$ and so $\left[\underline{b}, \bar{b}\right]$ is not an empty set.

When $\lambda \to \frac{1}{2}$ and $2\gamma^2 \geq \tau_u\tau_v$, we know that $\tau_\varepsilon \to +\infty$. Based on (B31) $\Delta B_1\left(\frac{1}{2}\right) < 0$ can be simplified to be $\beta \geq \frac{\tau_v+\tau_\varepsilon}{\tau_v+2\tau_\varepsilon} \to \frac{1}{2} \equiv \underline{b}$. It is easy to verify that $\bar{b} > \frac{1}{2}$. Again, $\left[\underline{b}, \bar{b}\right]$ is not an empty set.

Taken together, there exist multiple equilibria when $\beta \in \left[\underline{b}, \bar{b}\right]$, where $\bar{b} = \frac{\tau_v+\tau_\varepsilon^*(1,0)}{\tau_v+2\tau_\varepsilon^*(1,0)}$ and $\underline{b} = \frac{\tau_v+\tau_\varepsilon^*(1,1)}{\tau_v+2\tau_\varepsilon^*(1,1)}1_{\{2\gamma^2<\tau_u\tau_v\}} + \frac{1}{2}1_{\{2\gamma^2\geq\tau_u\tau_v\}}$, where $1_{\{2\gamma^2<\tau_u\tau_v\}} = 1$ only when $2\gamma^2 < \tau_u\tau_v$ and $1_{\{2\gamma^2\geq\tau_u\tau_v\}} = 1$ only when $2\gamma^2 \geq \tau_u\tau_v$. QED.

Proof of Proposition 6

First given that the total variance of the risky asset is limited, $k \to 0$ means that $\tau_v \to +\infty$. Based on (17)-(19) and (25)-(26), we have $\frac{\partial(B_{12}-B_n)}{\partial\lambda_{12}} = -\frac{1}{2}\frac{\partial(B_{12}-B_n)}{\partial\lambda}$
$= -\frac{1}{2}\frac{4\gamma(1-\lambda)\tau_u\tau_\varepsilon^3\left[(1+\beta)\gamma^2\tau_0+4\tau_\varepsilon\left(\beta\gamma^2+(1+\beta)(1-\lambda)^2\tau_u\tau_\varepsilon\right)\right]}{\left[\gamma^2\tau_0+4(1-\lambda)^2\tau_u\tau_\varepsilon^2\right]\left[\gamma^2\tau_0+2\tau_\varepsilon\gamma^2+4\tau_u\tau_\varepsilon^2(1-\lambda)^2\right]\left[\gamma^2\tau_0+\tau_\varepsilon\gamma^2+4\tau_u\tau_\varepsilon^2(1-\lambda)^2\right]} < 0$. QED.

Proof of Proposition 7

First given that the total variance of the risky asset is limited, when $k = 1$, we know $\tau_0 \to +\infty$. We discuss the following two cases: when $2\gamma^2 < \tau_u\tau_v$ and when $2\gamma^2 \geq \tau_u\tau_v$.

Case 1: $2\gamma^2 < \tau_u\tau_v$. Denote $\tau_\varepsilon^*(1,0)$ the equilibrium signal precision when $k \to 1$ and $\beta \to 0$, and $\tau_\varepsilon^*(1,1)$ the equilibrium signal precision when $k \to 1$, $\beta \to 1$ and $2\gamma^2 < \tau_u\tau_v$.

(1) Price efficiency. Based on (B4), when $\frac{1}{\tau_0} \to 0$, price efficiency can be simplified to

$$Var(\tilde{v}|\tilde{p})^{-1} = a^2\tau_u + \frac{1}{2}\tau_v. \tag{B32}$$

When $\beta \to 0$, we have $\lambda \to 0$. Based on (16) $a \to \frac{\tau_\varepsilon}{2\gamma} \equiv a_{\lambda=0}$. Similarly, when $\beta \to 1$, we have $\lambda \to \frac{1}{2}$, and $a \to \frac{\tau_v}{2\gamma}\frac{\tau_\varepsilon}{\tau_\varepsilon+2\tau_v} \equiv a_{\lambda=\frac{1}{2}}$. Inserting $a_{\lambda=0}$, $\tau_\varepsilon^*(1,0)$ into the price efficiency formula to calculate the price efficiency when $\beta \to 0$, inserting $a_{\lambda=\frac{1}{2}}$ and $\tau_\varepsilon^*(1,1)$ as in (28) when $\beta \to 1$ to calculate the price efficiency, and taking the difference yield

$$Var(\tilde{v}|\tilde{p})^{-1}|_{\beta\to 0} - Var(\tilde{v}|\tilde{p})^{-1}|_{\beta\to 1} = \frac{1}{4}\left(\frac{\tau_u\left[\tau_\varepsilon(1,0)\right]^2}{\gamma^2} - 2\tau_v\right). \tag{B33}$$

Substituting $\tau_\varepsilon = \gamma\sqrt{\frac{2\tau_v}{\tau_u}}$ into (B20) generates $h_1(\tau_\varepsilon) > 0$. Therefore $\tau_\varepsilon^*(1,0) > \gamma\sqrt{\frac{2\tau_v}{\tau_u}}$. Inserting this inequality into (B33) we obtain a positive number in the right-hand side. Thus $Var(\tilde{v}|\tilde{p})^{-1}|_{\beta\to 0} > Var(\tilde{v}|\tilde{p})^{-1}|_{\beta\to 1}$: price efficiency is higher when $\beta \to 0$.

(2) Liquidity. With symmetric equilibrium equation (12) can be simplified to

$$\alpha_u = \frac{a\tau_u + \gamma}{\frac{2\gamma}{Var(\tilde{v}|\tilde{s}_{1i},\tilde{p})} + \frac{1-2\gamma}{Var(\tilde{v}|\tilde{s}_{1i},\tilde{s}_{2i},\tilde{p})}}. \tag{B34}$$

Inserting $a_{\lambda=\frac{1}{2}}$ and $\tau_\varepsilon^*(1,1)$ as in (28) into (B34) when $\beta \to 1$, inserting $a_{\lambda=0}$, $\tau_\varepsilon^*(1,0)$ into

(B34) when $\beta \to 0$, and taking the difference yield

$$\alpha_u|_{\beta\to 1} - \alpha_u|_{\beta\to 0} \propto^+ \left(\begin{array}{c} \left(6\gamma^2\sqrt{2\tau_u\tau_v} + 4\gamma^3 + 6\gamma\sqrt{\tau_v}\tau_u + \sqrt{2\tau_u\tau_v}\tau_u\tau_v\right)\tau_u^{\frac{3}{2}}\left[\tau_\varepsilon(1,0)\right]^2 \\ +4\gamma\sqrt{\tau_u}\left(\gamma^2 - \tau_u\tau_v\right)\left(\tau_u\tau_v + 2\gamma^2 + 2\gamma\sqrt{2\tau_u\tau_u}\right)\left[\tau_\varepsilon(1,0)\right] \\ +2\gamma^2\sqrt{\tau_v}\left(\sqrt{2}\tau_u^2\tau_v^2 + 2\gamma\sqrt{\tau_u\tau_u}\tau_u\tau_v - 4\sqrt{2}\gamma^4 - 4\sqrt{2}\gamma^2\tau_u\tau_v - 12\gamma^3\sqrt{\tau_u\tau_v}\right) \end{array} \right). \tag{B35}$$

Substituting $\tau_\varepsilon(1,0) > \gamma\sqrt{\frac{2\tau_v}{\tau_u}}$ into (B35) yields a positive RHS. Therefore $\alpha_u|_{\beta\to 1} > \alpha_u|_{\beta\to 0}$: liquidity is higher when $\beta \to 0$.

(3) Cost of capital. Inserting $a_{\lambda=\frac{1}{2}}$ and $\tau_\varepsilon^*(1,1)$ as in (28) into $E(\tilde{v} - \tilde{p}) = -\alpha_0$ (where α_0 is in (15)) when $\beta \to 1$, inserting $a_{\lambda=0}$, $\tau_\varepsilon^*(1,0)$ into $E(\tilde{v} - \tilde{p})$ when $\beta \to 0$, and taking the difference yield

$$E(\tilde{v} - \tilde{p})|_{\beta\to 1} - E(\tilde{v} - \tilde{p})|_{\beta\to 0} \propto^+ \left(\begin{array}{c} 2\gamma\sqrt{\tau_u}\left(2\gamma^2 + \tau_u\tau_v + 2\gamma\sqrt{2\tau_u\tau_v}\right)\left(\tau_u\left[\tau_\varepsilon(1,0)\right]^2 + 2\gamma^2\left[\tau_\varepsilon(1,0)\right]\right) \\ -4\gamma^3\left(2\sqrt{2\tau_v}\gamma^3 + 6\gamma^2\tau_v\sqrt{\tau_u} + 3\sqrt{2\tau_v}\gamma\tau_u\tau_v + \tau_v^2\tau_u\sqrt{\tau_u}\right) \end{array} \right). \tag{B36}$$

Substituting $\tau_\varepsilon(1,0) > \gamma\sqrt{\frac{2\tau_v}{\tau_u}}$ into (B36) yields a positive RHS. Therefore $E(\tilde{v} - \tilde{p})|_{\beta\to 1} > E(\tilde{v} - \tilde{p})|_{\beta\to 0}$: the cost of capital is lower when $\beta \to 0$.

(4) Return volatility. With symmetric equilbrium, return volatility is shown as below

$$\sigma(\tilde{v} - \tilde{p}) = \sqrt{\frac{2(1 - a\alpha_u)}{\tau_v} + \frac{\alpha_u^2}{\tau_u}}. \tag{B37}$$

We first compare $a\alpha_u$. Inserting $a_{\lambda=\frac{1}{2}}$ and $\tau_{\varepsilon,\lambda=\frac{1}{2}}$ as in (28) into $a\alpha_u$ when $\beta \to 1$, inserting $a_{\lambda=0}$, $\tau_{\varepsilon,\lambda=0}$ into $a\alpha_u$ when $\beta \to 0$,, and taking the difference yield

$$a\alpha_u|_{\beta\to 0} - a\alpha_u|_{\beta\to 1} \propto^+ \left(\begin{array}{c} \sqrt{\tau_u\tau_v}\left(2\gamma^2 + \tau_u\tau_v + 2\gamma\sqrt{2\tau_u\tau_v}\right)\left(\tau_u\left[\tau_\varepsilon(1,0)\right]^2 + 2\gamma^2\left[\tau_\varepsilon(1,0)\right]\right) \\ -2\gamma^2\tau_v\left(2\sqrt{2}\gamma^3 + 6\gamma^2\sqrt{\tau_u\tau_v} + 3\sqrt{2}\gamma\tau_u\tau_v + \tau_v\tau_u\sqrt{\tau_v\tau_u}\right) \end{array} \right). \tag{B38}$$

Substituting $\tau_\varepsilon(1,0) > \gamma\sqrt{\frac{2\tau_v}{\tau_u}}$ into (B38) yields a positive RHS. Therefore $a\alpha_u|_{\beta\to 0} > a\alpha_u|_{\beta\to 1}$. Since $\alpha_u|_{\beta\to 1} > \alpha_u|_{\beta\to 0}$, $\sigma(\tilde{v} - \tilde{p})|_{\beta\to 0} < \sigma(\tilde{v} - \tilde{p})|_{\beta\to 1}$: return volatility is lower when $\beta \to 0$.

Case 2: $2\gamma^2 \geq \tau_u\tau_v$. When $k \to 1$, $\beta \to 1$ and $2\gamma^2 \geq \tau_u\tau_v$, we know $\tau_\varepsilon^* \to +\infty$. With $\tau_\varepsilon^* \to +\infty$ and $\frac{1}{\tau_0} \to 0$, based on (16) we have $a \to \frac{\tau_v}{2\gamma}$.

(1) Price efficiency. The difference of price efficiency when $\beta \to 0$ and when $\beta \to 1$ can

be computed as follows:

$$Var\left(\tilde{v}|\tilde{p}\right)^{-1}|_{\beta\to 0} - Var\left(\tilde{v}|\tilde{p}\right)^{-1}|_{\beta\to 1} = \tau_u \frac{\left[\tau_\varepsilon\left(1,0\right)\right]^2 - \tau_v^2}{4\gamma^2} > 0.$$

Substituting $\tau_\varepsilon = \tau_v$ into (B20) generates $h_1\left(\tau_\varepsilon\right) > 0$. Thus $\tau_\varepsilon\left(1,0\right) > \tau_v$, which verifies the above inequality. Therefore, price efficiency is higher when $\beta \to 0$, which is the same as the conclusion in Case 1.

(2) Liquidity. Similar to Case 1 analysis, we can compute

$$\alpha_u|_{\beta\to 1} - \alpha_u|_{\beta\to 0} \propto^+ 2\gamma\left[\tau_\varepsilon\left(1,0\right) - \tau_v\right]\left[4\gamma^4 + \tau_\varepsilon\left(1,0\right)\tau_u\left(2\gamma^2 + \tau_u\tau_v\right)\right] > 0.$$

Thus liquidity is higher when $\beta \to 0$, which is the same as the conclusion in Case 1.

(3) Cost of capital. Similar to Case 1 analysis, we compute

$$E\left(\tilde{v} - \tilde{p}\right)|_{\beta\to 1} - E\left(\tilde{v} - \tilde{p}\right)|_{\beta\to 0} \propto^+ \left[\tau_\varepsilon\left(1,0\right) - \tau_v\right]\left[2\gamma^2 + \tau_u\left(\tau_v + \tau_\varepsilon\left(1,0\right)\right)\right] > 0.$$

Thus cost of capital is lower when $\beta \to 0$, which is the same as the conclusion in Case 1.

(4) Return volatility. Similar to Case 1 analysis, we can ompute

$$a\alpha_u|_{\beta\to 0} - a\alpha_u|_{\beta\to 1} \propto^+ \left[\tau_\varepsilon\left(1,0\right) - \tau_v\right]\left[2\gamma^2 + \tau_u\left(\tau_v + \tau_\varepsilon\left(1,0\right)\right)\right] > 0.$$

Since $\alpha_u|_{\beta\to 1} > \alpha_u|_{\beta\to 0}$, based on (B38) we know that return volatility is lower when $\beta \to 0$, which is the same as the conclusion in Case 1. QED.

Proof of Proposition 8

Given that the total variance of the risky asset is limited, $\lim_{k\to 0}\frac{1}{\tau_v} = 0$. Based on Proposition 3, we know that $\lambda^* = \frac{1}{2}$, $\tau_\varepsilon^* \to +\infty$ and $\tau_\varepsilon^* \sim O\left(\tau_v^{1/2}\right)$. (1) Price efficiency. Based on (B7), when $\frac{1}{\tau_v} \to 0$ and $\tau_\varepsilon \sim O\left(\tau_v^{1/2}\right)$, $a = \frac{\tau_\varepsilon}{2\gamma} \sim O\left(\tau_v^{1/2}\right)$ which goes to infinity. Thus (B4) goes to zero and price efficiency is sufficiently high. (2) Liquidity. When $k \to 0$, plugging $\lambda^* = \frac{1}{2}$, $\tau_\varepsilon^* \sim O\left(\tau_v^{1/2}\right)$ and $a = \frac{\tau_\varepsilon}{2\gamma}$ into (12), we obtain $\alpha_u = \frac{2\gamma\left(1+2\sqrt{\tau_v}\right)\left(2\gamma^2+\tau_u\sqrt{\tau_v}\right)}{\tau_v\left[4\gamma^2\left(1+\sqrt{\tau_v}\right)+\tau_u\left(1+2\sqrt{\tau_v}\right)\right]}$. As $\frac{1}{\tau_v} \to 0$, α_u is sufficiently close to zero, and thus the liquidity is sufficiently high. (3) Cost of capital. When $k \to 0$, plugging $\lambda^* = \frac{1}{2}$, $\tau_\varepsilon^* \sim O\left(\tau_v^{1/2}\right)$ and $a = \frac{\tau_\varepsilon}{2\gamma}$ into (15) α_0 can be simplified to $\alpha_0 = \frac{4\gamma^3\left(1+2\sqrt{\tau_v}\right)}{\tau_v\left[4\gamma^2\left(1+\sqrt{\tau_v}\right)+\tau_u\left(1+2\sqrt{\tau_v}\right)\right]}$. As $\frac{1}{\tau_v} \to 0$, α_0 is sufficiently close to zero, and thus cost of capital $E\left(\tilde{v} - \tilde{p}\right) = -\alpha_0$ is sufficiently low. (4) Return volatility. When $k \to 0$, with $\lambda^* = \frac{1}{2}$, $\tau_\varepsilon^* \sim O\left(\tau_v^{1/2}\right)$ and $a = \frac{\tau_\varepsilon}{2\gamma}$ we have $\alpha_u a = \frac{\left(1+2\sqrt{\tau_v}\right)\left(2\gamma^2+\tau_u\sqrt{\tau_v}\right)}{\left(4\gamma^2+\tau_u\right)\sqrt{\tau_v}+2\left(2\gamma^2+\tau_u\right)\tau_v} \to 1$ as $\frac{1}{\tau_v} \to 0$. Recall $\alpha_u \to 0$ as $\frac{1}{\tau_v} \to 0$. Therefore based on (B37) as $1/\tau_v \to 0$, $\sigma\left(\tilde{v} - \tilde{p}\right)$ is sufficiently close to zero: return volatility is sufficiently low. QED.

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Figure 1. Timeline

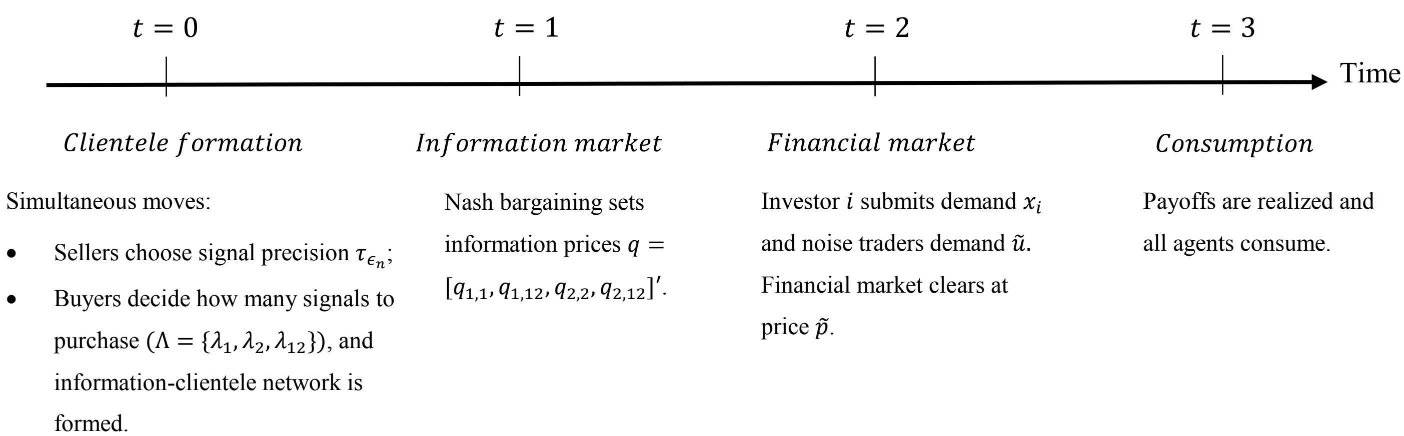
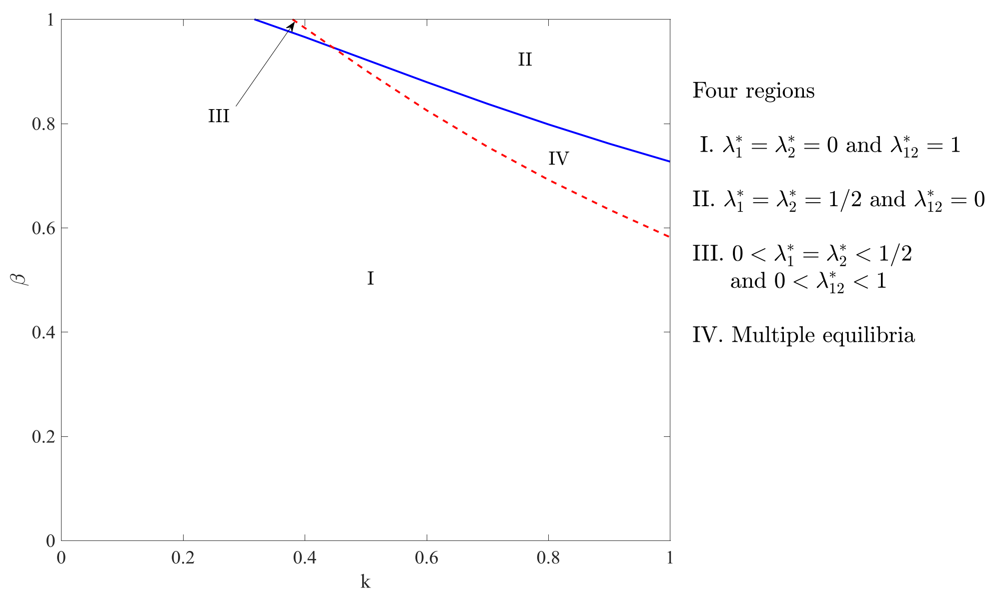


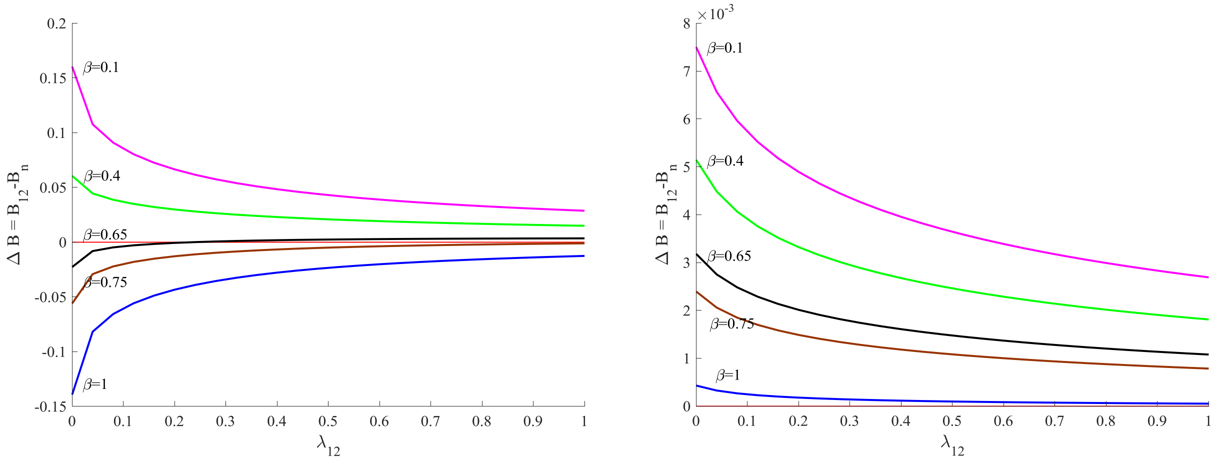
Figure 2. Equilibrium investor composition



The figure plots the equilibrium investor composition (Λ^*) against different values of (k, β). The other parameters are $\frac{2}{\tau_v} + \frac{1}{\tau_0} = 1, \tau_u = 15, \gamma = 3$. There are four regions of investor composition as shown in the right column. (1) In region I, all investors purchase both signals. (2) In region II, all investors only purchase one signal. (3) In region III, some investors purchase both signals, while other purchase only one signal. (4) In region IV, the equilibria in region I, II, III can coexist.

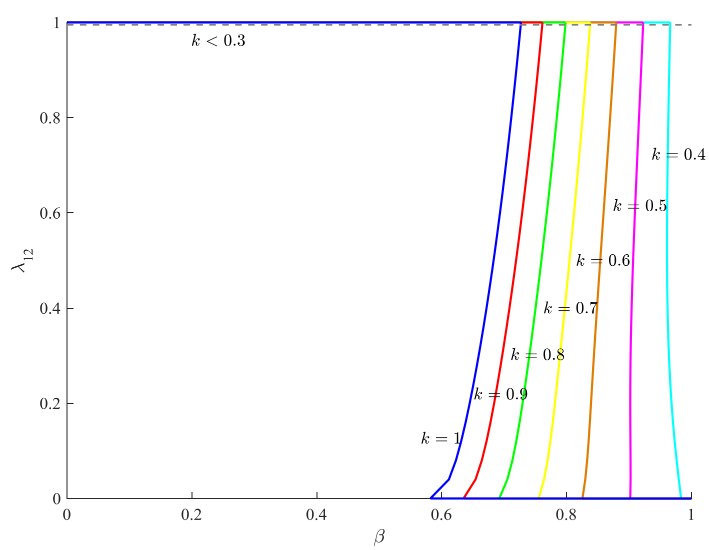
Figure 3. Marginal benefit of purchasing the second signal ($\boldsymbol{B_{12} - B_n}$)

Panel A: $k = 1$ Panel B: $k = 0.0001$



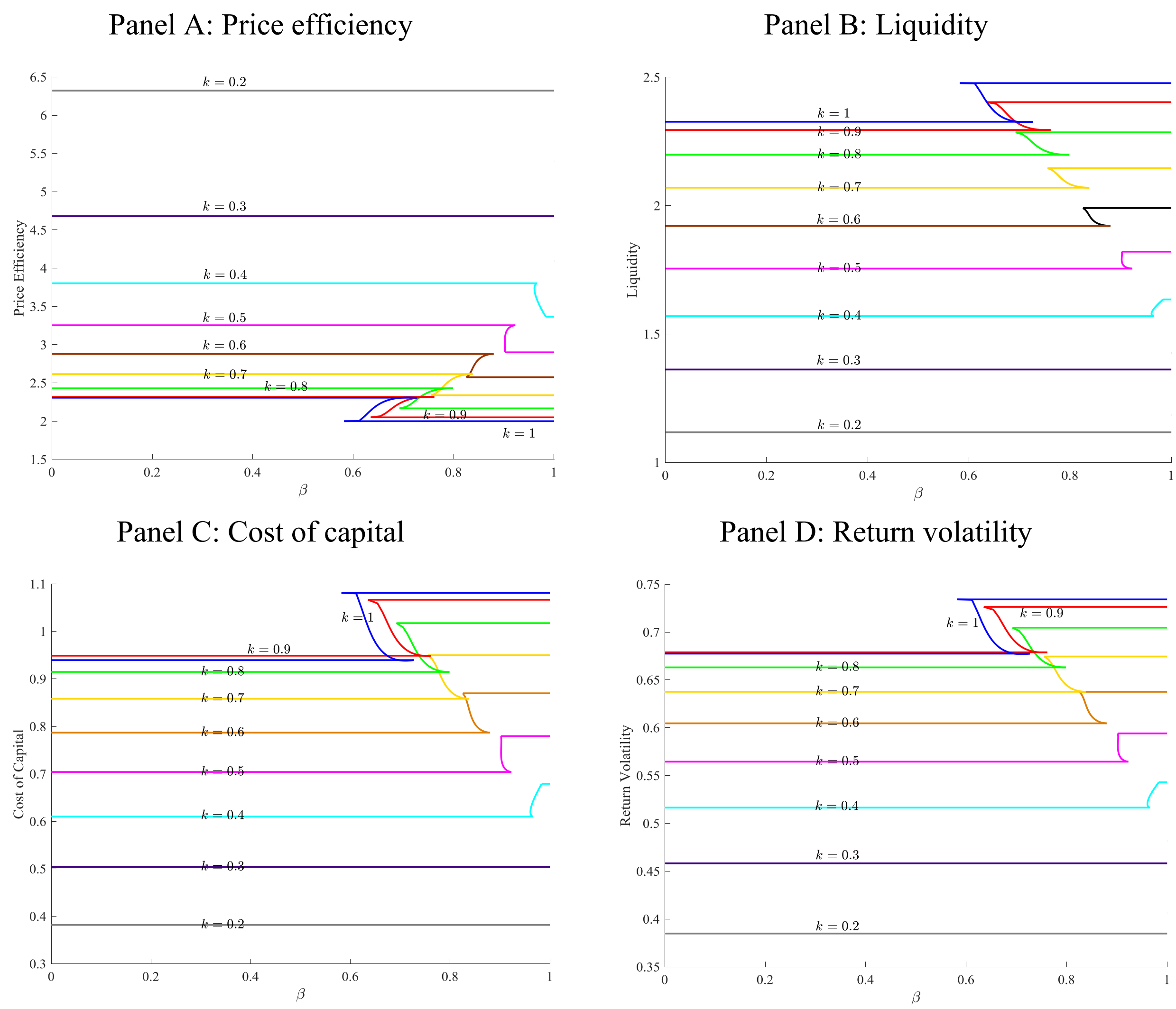
The figure plots the marginal benefit of knowing the second ($B_{12} - B_n$) against λ_{12}. We set $k = 1$ in Panel A and $k = 0.0001$ in Panel B. The other parameters are the same as in Figure 2. In Panel A when $k = 1$, if β is large (small), there is a cross-agent complementarity (substitutability) in investors' information purchase. In Panel B when k close to 0, there is always a cross-agent substitutability in investors' information purchase.

Figure 4. Equilibrium investor composition ($\boldsymbol{\lambda_{12}^*}$) against $\boldsymbol{\beta}$ with varying $\boldsymbol{k}$



The figure plots the equilibrium fraction of the investors purchasing both signals (λ_{12}^*) against β when k takes different values. The other parameters are the same as in Figure 2. When k is large, if β is large (small), $\lambda_{12}^* = 0(1)$, and if β lies within some intermediate range, the three equilibria coexist: $\lambda_{12}^* = 1$, $\lambda_{12}^* = 0$ and $0 < \lambda_{12}^* < 1$. As k decreases, λ_{12}^* is always 1.

Figure 5. Market quality



The figure plots equilibrium market-quality implications against different values of (k, β): price efficiency in Panel A, liquidity in Panel B, cost of capital in Panel C, and return volatility in Panel D. The other parameters are the same as in Figure 2. When k is large, if β is large (small), price efficiency and liquidity are low (high), and cost of capital and return volatility are high (low). And if β lies within some intermediate range, there exist multiple equilibria. As k decreases, price efficiency and liquidity increase, while cost of capital and return volatility decrease.