Short-sale constraints and credit runs*

Gyuri Venter†

October 2017

Abstract

This paper studies how short-sale constraints affect the informational efficiency of market prices and the link between prices and economic activity. I show that under short-sale constraints security prices contain less information. However, short-sale constraints increase the informativeness of prices to some agents who learn about the quality of an investment opportunity from market prices and have additional private information.

Then I apply this observation when modeling a run on an investment bank by its short-term creditors, who are endowed with dispersed information and also learn from the price of an asset the bank holds. I show that short-selling constraints in the financial market lead to the revival of self-fulfilling beliefs about the beliefs and actions of others, and create multiple equilibria. In the equilibrium where agents rely more on public information (i.e., the price), creditors with high private signals are more lenient to roll over debt, and a bank with lower asset quality remains solvent. This leads to higher allocative efficiency in the real economy. My result thus implies that the decrease in average informativeness due to short-sale constraints can be more than compensated by an increase in informativeness to some agents.

JEL classification: C7, D82, D84, G14, G18, G21

Keywords: asymmetric information, short-sale constraints, coordination, complementarities, efficiency.

---

*I would like to thank Dimitri Vayanos, Péter Kondor, Kathy Yuan, Bruno Biais, Georgy Chabakauri, Jason Donaldson, Bernard Dumas, John Geanakoplos, Denis Gromb, Christian Julliard, Aytek Malkhozov, Emre Ozdenoren, Andrea Vedolin, and seminar and conference participants at CBS, HEC Lausanne, INSEAD, LSE, NHH Bergen, Warwick Business School, EFA, EEA, the ESSFM Gerzensee, the Finance Theory Group London Conference, and the IE-HAS Summer Workshop for helpful comments. Financial support from the Paul Woolley Centre for the Study of Capital Market Dysfunctionality at LSE, the CBS Center for Financial Frictions (FRIC) (grant no. DNRF-102), the Danish Council for Independent Research (grant no. DFF-4091-00247), and the European Research Council (ERC grant no. 312417) at different stages of this project are gratefully acknowledged.

†Copenhagen Business School; email: gv.fi@cbs.dk.

1 Introduction

According to the view of many academics and regulators, short-sale constraints compromise market liquidity and reduce the informativeness of market prices, while preventing value-destroying price manipulation and hence severe economic inefficiencies. A press release of the Securities and Exchange Commission (SEC), issued on the 19th of September, 2008, clearly illustrates this point. They state that "under normal market conditions, short selling contributes to price efficiency and adds liquidity to the markets", but argue in favour of an emergency order that bans short selling, as shorting, observed e.g. after the collapse of Lehman Brothers, can lead to sudden price declines unrelated to true value. Since financial institutions "depend on the confidence of their trading counterparties in the conduct of their core business", if prices can influence how these institutions are perceived by counterparties and clients, low prices can have damaging effects on the value of institutions as well. Thus, providing a floor to asset prices can be beneficial.[1]

In this paper I examine how short-sale constraints affect both the informational efficiency of prices, and the link between prices and economic activity. I show that under short-sale constraints security prices contain less information. This is consistent with previous work, and my contribution is to derive a simple closed-form solution of a rational expectations equilibrium (REE) with short-sale constraints. My main result concerns the feedback to the real economy. I find that although prices contain less information, short-sale constraints increase the informativeness of prices to some agents who have additional private information. This, in turn, yields an equilibrium of the real economy that has higher allocative efficiency. My result thus implies that the decrease in average informativeness due to short-sale constraints can be more than compensated by an increase in informativeness to some agents.

To analyze the informational effects of short-sale constraints, I extend an asset pricing model with information spillover from the financial market to the real economy. I use a noisy rational expectations model of a financial market with asymmetric information, where noise comes from a random demand shock, as in Grossman and Stiglitz (1980), and I introduce short-selling constraints on a subset of informed traders. For the real part of the economy, I consider a distressed financial institution (e.g. investment bank) that requires outside capital from multiple lenders or short-term creditors to support its existing positions.[2] Creditors, endowed with dispersed private information about the

[1] See http://www.sec.gov/news/press/2008/2008-211.htm. A similar point is reached by the Financial Services Authority (FSA) discussion paper on short-selling (Financial Services Authority (2009), p.11-12). In particular, they claim that the negative impact of shorting "reduces the ability of a firm to raise equity capital or to borrow money, and makes it harder for banks to attract deposits."

[2] There is ample anecdotal evidence about Bear Stearns and Northern Rock not being able to secure short-term financing and being the victims of runs by their creditors at the beginning of the 2007-2008 crisis; see, for example, Brunnermeier (2009) and Shin (2009). Ivashina and Scharfstein (2010) show

value of the bank's assets, consider whether to supply capital to this institution. I model bank financing as game with strategic complementarities: the bank survives if the amount of capital provided by creditors is sufficiently large, and creditors' payoff is higher if the bank avoids bankruptcy.[3] Besides their private signals, creditors also observe the price of a traded security. The connection between the security market and the financing is provided by the correlation between the payoff of the security and the unknown quality of the financial institution. Therefore, the price, which gathers information in the security market, constitutes a public signal to capital providers.[4]

The main observation of the model is that even though short-sale constraints decrease the information content of prices, certain creditors endowed with additional private information can learn more from asset prices with short-sale constraints than without the constraints. The idea is that when creditors combine their private signals with the market price to form beliefs about the state of the world, they also have to assess to what extent a high (low) market price reflects a high (low) fundamental value or a high (low) demand shock, i.e. whether informed traders buy or whether they (would) short-sell. In presence of short-sale constraints, a high demand shock increases the price, and informed investors would like to short, but cannot. It leads to a decrease in the aggregate order flow, which is dominated by noise trading. High price realizations are hence more noisy and less informative about the true state of the world, as negative information about fundamentals is less incorporated into prices. Put differently, a given price realization means lower payoff if one believes the constraint binds in the market.

To see the intuition for how prices can provide more information in presence of trading constraints, consider a creditor who receives a private signal realization higher than the price she sees. A high signal means that according to her private information, states when the payoff is lower than the price have low probabilities as they are tail events. She knows that if informed traders (those who can) are shorting, the same price realization corresponds to lower fundamentals compared to the case without short-sale constraints. But lower fundamentals have smaller probabilities according to her private belief. Combining these two observations, she assigns a smaller probability to informed traders shorting the asset than without short-sale constraints, and thus thinks the asset payoff is higher. This

that after the failure of Lehman Brothers in September 2008, there were runs by short-term lenders on financial institutions, making it hard for banks to roll over their short term debt. Moreover, runs on other financial institutions, such as investor withdrawals from hedge funds or mutual funds can be viewed as a coordination game among capital providers, see Shleifer and Vishny (1997) and the vast literature on limited arbitrage.

[3]Indirect and direct evidence of coordination motives among creditors have been shown by Asquith et al. (1994), Brunner and Krahnen (2008) and Hertzberg et al. (2010). Moreover, Chen et al. (2010) document coordination motives among investors of mutual funds.

[4]The financial asset can be interpreted, for example, as a zero-net-supply derivative on the share price of the bank, as an industry index that includes the bank, or the price of a security that the bank has on the asset side of the balance sheet.

reinforces her private signal, and implies that her posterior can be more precise than without short-sale constraints.

Then I study the effect of short-sale constraints in the security market on the bank financing. I show that the presence of short-sale constraints introduces multiple equilibria in the coordination game, even when private information is arbitrarily precise. This result stems from the observation that creditors with high private signal learn more from the market price in presence of short-sale constraints. Indeed, when their posterior variance is smaller, creditors have more precise assessments about both the bank's fundamental and about the information of other creditors. For every level of private noise precision, when short-sale constraints are sufficiently tight, they reinstate common knowledge among the subset of more informed creditors, and lead to self-fulfilling beliefs and two stable equilibria. I refer to the first one, when creditors rely only on their private signals, as the informationally efficient equilibrium, because in the limit when private signals become very precise, agents ignore the market price. This equilibrium is equivalent to the unique equilibrium of the game without short-sale constraints. However, in presence of the constraint there exists a second stable equilibrium, where creditors with high private signals keep relying on the public signal, if they know that other creditors with similarly high signals do as well.

Interestingly, in this second stable equilibrium the bank receives more capital than in the informationally efficient equilibrium, thus they can be called high and low investment equilibrium, respectively. This is because short-sale constraints only improve the precision of agents with signals higher than the market price, so they are the creditors who might react to the 'news' in short-sale constraints. Intuitively, short-sale constraints can only affect the equilibrium outcome if there are some creditors who behave differently in the informationally efficient equilibrium, but due to short-sale constraints learn more about each other's action. Thus it is straightforward that the second equilibrium, whenever it exists, must feature more capital provision than the informationally efficient equilibrium. Short-sale constraints improve the information of some agents who would stay out in absence of the constraint, and create self-fulfilling beliefs and multiplicity in equilibrium actions among these creditors. This leads to more investment, banks with lower asset quality remain solvent, and the equilibrium is closer to the first best. I conclude that short-sale constraints improve allocational efficiency by mitigating the adverse effect of the coordination externality. Therefore, if the gain of short-sale constraints in terms of the increased allocational efficiency of the real economy is higher (or more important) than the loss in terms of informational inefficiency in the financial market, short-sale constraints can be beneficial.

The model presented in this paper is not the first to highlight the impact of trading constraints on the allocational efficiency of the real economy. Panageas (2003) and

Gilchrist et al. (2005) study firms' investment decisions when they raise capital during asset price bubbles, when the cost of capital is low due to short-sale constraints. Both studies rely on the literature initiated by Miller (1977) and Harrison and Kreps (1978), who suggest a link between the level of belief heterogeneity and inflated asset prices (see also Scheinkman and Xiong (2003), and Rubinstein (2004) for many more 'anomalies' associated with short-sale constraints). In contrast to these papers, in my model agents are rational, and short-sale constraints do not inflate the price, following the insights of Diamond and Verrecchia (1987). In particular, in this study security prices in presence of short-sale constraints are lower than without the constraints, and hence according to Panageas (2003) and Gilchrist et al. (2005) investment should be lower. My focus is on the information provided by market prices instead of price levels.

My financial market model is similar to that in Yuan (2005, 2006), who studies a REE with asymmetric information and constraints on borrowing and shorting. She numerically shows that, in presence of borrowing restrictions, a higher market price can reduce uncertainty about the constraint status of informed investors, and that this information effect can be strong enough to cause a backward bending demand curve. In contrast, the first part of this paper provides a simple closed-form solution of a model that is simplified in one dimension but allows for more generality in other dimensions.[5] Finally, Bai et al. (2006) and Marin and Olivier (2008) study the effects of short-sale constraints when investors trade for informational and allocational purposes. In both papers, trading constraints limit the positions of all informed traders. When the constraints bind, asset prices stop reflecting fundamentals, uninformed investors demand a large discount, and prices exhibit large drops. Therefore, in these models high prices are more informative than low prices. In contrast, in models presented here and in Yuan (2005, 2006), only a subset of informed investors are subject to the short-selling constraint, and uninformed investors need to form beliefs about the size of the demand shock, i.e. the constraint status of informed investors. The most important distinction is that short-sale constraints bind for high prices, making them less informative than low prices.

The model also belongs to the literature on coordination games with strategic complementarities, developed by Carlsson and van Damme (1993) and Morris and Shin (1998), and contributes to discussion about the fragile interaction between private and public information (see, for example, Morris and Shin (1999, 2001, 2002, 2003, 2004) and Hellwig (2002)). In particular, Morris and Shin (2001) show that when private information becomes arbitrarily precise, a coordination game has a unique equilibrium. In the discussion of Morris and Shin (2001), Atkeson (2001) highlights the potential role of financial

[5] Also, in a financial market with wealth- and shortsale-constrained risk-neutral agents and an asset supply exponentially distributed, Barlevy and Veronesi (2003) present a theory of stock market crashes, where high asset prices are more informative than low prices.

markets as the source of endogenous public information, formalized by Angeletos and Werning (2006). They show that a unique equilibrium might not prevail, if the precision of the public signal that aggregates private information increases faster than the precision of the private signal. Hellwig et al. (2006) and Tarashev (2007) also study a coordination game with a financial price as the public signal, while Ozdenoren and Yuan (2008) and Goldstein et al. (2013) study coordination among traders in the market. A common element in these papers is that the informational content of the public signal does not vary across equilibria. In contrast to many previous models, in the model presented here the informativeness of the public signal varies across its realizations. This is similar in spirit to Angeletos et al. (2006). However, in their analysis the signal is the equilibrium action of a policy maker, whereas in my study the varying informativeness is the result of the asymmetric nature of short-sale constraints. Finally, there are several papers that highlight the adverse effect of short-selling on allocative efficiency in the economy and hence argue in favour of short-sale constraints, see for example Goldstein and Guembel (2008) or Liu (2015). However, to my knowledge, this is the first model that explicitly studies the informational effect of short-sale constraints on real economic activity.

The remainder of the paper is organized as follows. Section 2 presents the financial market. Section 3 studies the equilibrium of the financial market and examines the effect of short-sale constraints on the equilibrium price. Section 4 analyzes the information content of stock prices with and without short-sale constraints for outside observers. Section 5 embeds the credit run model into the economy, and Section 6 presents the equilibrium of the coordination game with the skewed public signal. Section 7 discusses the results, contrasts the findings with the related literature, and provides some comparative statics and policy implications. Finally, Section 8 concludes.

2 Model

This section introduces the financial market model. I consider a two-period economy with dates $t = 0$ and 1. At date 0 investors trade, and at date 1 assets pay off. The market is populated by three types of agents: informed and uninformed rational investors, and noise traders.

2.1 Assets

There are two securities traded in a competitive market, a risk-free bond and a risky stock. The bond is in perfectly elastic supply and is used as numeraire, with the risk-free rate normalized to 0. The risky asset is assumed to be in net supply of $S \geq 0$, and has final dividend payoff d at date 1, that is the sum of two random components: $d = f + n$. The first risky component of the dividend payoff, f, can be regarded as the fundamental value of the asset. The second component, n, is thought of as additional noise, preventing agents from knowing the exact dividend payoff. The price of the stock at date 0 is denoted by p.

2.2 Traders

I assume that the asset market is populated by a continuum of rational traders in unit mass. Traders do not hold endowments in the risky security. They are risk averse and, for tractability, I assume that they have a mean-variance objective function over terminal wealth.[6] Agent k, for $k \in [0, 1]$, maximizes

$$E\left[W_k|\mathcal{I}_k\right] - \frac{\rho}{2}Var\left[W_k|\mathcal{I}_k\right],$$

where ρ is the risk aversion parameter, common across agents. The final wealth $W_k = W_k^0 + x_k\left(d - p\right)$ is given by the initial wealth W_k^0 plus the number of shares purchased, x_k, multiplied by the profit per share, $d - p$. $\mathcal{I}_k$ is the information set of trader k, and $E\left[.|\mathcal{I}_k\right]$ and $Var\left[.|\mathcal{I}_k\right]$ denote the expectation and variance conditional on the information set $\mathcal{I}_k$, respectively.

Rational investors can be either informed or uninformed. Informed traders, who have a measure of λ and are indexed with $k \in [0, \lambda)$, observe the realization of the fundamental f but not n. The other set of rational traders, with measure $1 - \lambda$, and indexed with $k \in [\lambda, 1]$, are uninformed, and do not observe any (private) signals about f. Instead, all agents of the model observe the market price p. These assumptions imply that the risk associated with n is unlearnable for everyone, thus uninformed traders try to best guess component f. Formally, the information set of informed traders is $\mathcal{I}^i = \{f, p\} = \{f\}$, as the price cannot provide more information about the final payoff than their private observation. The information set of uninformed traders is $\mathcal{I}^{ui} = \{p\}$.

Further, I assume that informed traders might be subject to short-sale constraints.[7] In particular, short-sale constraints mean that trader k's stock position is bounded below by zero, $x_k \geq 0$. Short-sale constraints can be thought of as an extreme case of infinite costs when selling short. I assume that $0 \leq w < 1$ proportion of informed traders are

[6] The mean-variance objective function is equivalent to maximizing exponential (i.e. CARA) utility as long as the uncertainty faced by traders is Gaussian. With short-sale constraints this is not the case for uninformed investors, but the model is nevertheless solvable and yields qualitatively the same result as the one dicussed here.

[7] For simplicity, I assume that uninformed traders are not subject to short-sale constraints. Such an extension would only affect the equilibrium price level by influencing the demand of uninformed traders, but would not change the information content of the market price.

subject to short-sale constraints, and index them by $k \in [0, w\lambda)$, while the remaining, with mass $(1-w)\lambda$, for $k \in [w\lambda, \lambda)$, are unconstrained. When $w = 0$, none of the informed traders are restricted from shorting.[8] Throughout the paper, a higher w can be (broadly) interpreted as higher cost and/or more difficult shorting. This includes regulatory restrictions (such as the short-sale ban of 2008 or the uptick rule), legal restrictions, search costs for lenders, rebate rates, costs of derivative trading, and even the amount of institutional trading in the market.[9] As agents inside the different investor classes are identical, I drop the subscript k from now on.

Finally, there are noise traders in the market, whose trading behavior is not derived from utility maximization. Noise traders simply buy u shares. I will refer to their trade order as demand shock.[10]

Regarding the distribution of random variables, I assume that fundamental f is drawn from an improper uniform distribution on the real line. The unlearnable noise component is given by $n \sim N\left(0, \sigma_n^2 = \tau_n^{-1}\right)$, and the demand shock is given by $u \sim N\left(0, \sigma_u^2 = \tau_u^{-1}\right)$, where σ_x^2 denotes the variance of random variable x, and τ_x denotes its precision. Throughout the paper $\phi(.)$ denotes the probability density function (pdf) of a standard normal distribution, $\Phi(.)$ is its corresponding cumulative distribution function (cdf), and $\Phi^{-1}(.)$ is the inverse of the cdf.

2.3 Equilibrium concept

I define an equilibrium of the financial market as follows.

Definition 1 *A rational expectations equilibrium (REE) of the asset market is a collection of a price function $P(f, u)$, and individual strategies for constrained informed, unconstrained informed and uninformed traders, $x^c(f, p)$, $x^{uc}(f, p)$, and $x^{ui}(p)$, respectively, such that*

1. demand is optimal for informed traders:

$$x^c(f, p) \in \arg\max_{x \in \mathbb{R}^+} E\left[W^c | f\right] - \frac{\rho}{2} Var\left[W^c | f\right], \tag{1}$$

[8]The qualitative results of the model do not depend on the exact proportions of the three different trader classes. The cardinal question is whether $w = 0$ or $w > 0$. As discussed later, the assumption $w < 1$ impies that there are always unconstrained informed traders, the stock price always reflects the fundamental f up to some noise, and the equilibrium stock price does not exhibit a jump.

[9]See the Securities and Exchange Commission Rule 10a-1, Almazan et al. (2004), Duffie et al. (2002), Jones and Lamont (2002), Ofek and Richardson (2003), and Nagel (2005), respectively, for these proxies on the difficulty of short-selling.

[10]As it is standard in models with informational heterogeneity, the presence of noise trading u makes sure that the price does not reveal f perfectly, and hence the Grossman-Stiglitz paradox does not apply.

and

$$x^{uc}(f,p) \in \arg\max_{x\in\mathbb{R}} E\left[W^{uc}|f\right] - \frac{\rho}{2} Var\left[W^{uc}|f\right]; \tag{2}$$

2. demand is optimal for uninformed traders:

$$x^{ui}(p) \in \arg\max_{x\in\mathbb{R}} E\left[W^{ui}|P(f,u)=p\right] - \frac{\rho}{2} Var\left[W^{ui}|P(f,u)=p\right]; \tag{3}$$

3. market clearing:

$$w\lambda x^{c}(f,p) + (1-w)\lambda x^{uc}(f,p) + (1-\lambda)x^{ui}(p) + u = S, \tag{4}$$

Conditions (1)-(4) define a competitive noisy rational expectations equilibrium for the trading round. In particular, condition (1) states that individual asset demands are optimal for informed traders subject to short-sale constraints, conditioned on their private information. Similarly, condition (2) states that individual asset demands are optimal for informed traders with no restriction on shorting, given their private information. Also, condition (3) states that individual asset demands are optimal for uninformed traders, conditioned on any information they infer from the price. Finally, (4) imposes that the asset market clears: aggregate demand equals supply.

3 Equilibrium in the financial market

This section solves for the equilibrium of the trading round and studies the informational effects of short-sale constraints on market prices. The model is solved in the general case with $w \geq 0$, then I contrast the results for $w = 0$ and $w > 0$, that is in absence and presence of short-selling constraints, respectively.

Given the optimization problems (1), (2) and (3), optimal demands are the following: an unconstrained informed trader submits demand function

$$x^{uc}(f,p) = \frac{f-p}{\rho\sigma_n^2}, \tag{5}$$

a short-sale constrained informed trader demands

$$x^{c}(f,p) = \max\left\{\frac{f-p}{\rho\sigma_n^2}, 0\right\} = 1_{f\geq p}\frac{f-p}{\rho\sigma_n^2}, \tag{6}$$

and an uninformed trader demands

$$x^{ui}(p) = \frac{E\left[f|P=p\right]-p}{\rho\left(Var\left[f|P=p\right]+\sigma_n^2\right)}. \tag{7}$$

Solving for equilibrium requires three fairly standard steps. First, I postulate a REE price function. Given the price, I derive the optimal demand of uninformed traders. Finally, I show that the market indeed clears at the conjectured price.

I conjecture the equilibrium price of the form

$$P = f + \begin{cases} A(u - C) & \text{if } u \leq C \\ B(u - C) & \text{if } u > C \end{cases}, \tag{8}$$

with constants A, B and C to be determined in equilibrium, where $A, B > 0$.

Uninformed agents' information set is given by $\mathcal{I} = \{P(f, u) = p\}$. They observe neither f, nor u, only the price realization p; and they know that in equilibrium this is a piecewise linear function of the two unknown variables, described in (8). From the price realization p they form a probabilistic estimate about the fundamental f, while also guessing whether short-sale restrictions bind for constrained traders. Given the conjectured price function (8) and the Gaussian distribution of u, uninformed investors' posterior is characterized by the conditional probability density function

$$\begin{aligned} g(f|P = p) &= 1_{f<p} g(f|P = p) + 1_{f \geq p} g(f|P = p) \\ &= \frac{1}{B\sigma_u} 1_{f<p} \phi\left(\frac{f - (p + BC)}{B\sigma_u}\right) + \frac{1}{A\sigma_u} 1_{f \geq p} \phi\left(\frac{f - (p + AC)}{A\sigma_u}\right), \end{aligned} \tag{9}$$

where I use that for any X random variable with density function $g_X(x)$ and a $\varphi(.)$ continuous, differentiable, and injective transformation, the density function of $Y = \varphi(X)$ is given by

$$g_Y(y) = g_X\left(\varphi^{-1}(y)\right) \left|\varphi^{-1\prime}\right|.$$

The above density function in turn allows uninformed traders to compute the conditional expectation and variance of payoff f given p:

$$E[f|P = p] = p + D \text{ and } Var[f|P = p] = E - D^2, \tag{10}$$

where

$$\begin{aligned} D &\equiv -A \int_{-\infty}^{C} (u - C) \phi(u) \, du - B \int_{C}^{\infty} (u - C) \phi(u) \, du, \text{ and} \\ E &\equiv A^2 \int_{-\infty}^{C} (u - C)^2 \phi(u) \, du + B^2 \int_{C}^{\infty} (u - C)^2 \phi(u) \, du. \end{aligned}$$

The conjectured REE price function must equate demand and supply for each possible resolution of f and u. Substituting the optimal demands $(6) - (7)$ into the market clearing condition (4) gives

$$w\lambda 1_{f\geq p}\frac{f-p}{\rho\sigma_n^2}+(1-w)\,\lambda\frac{f-p}{\rho\sigma_n^2}+(1-\lambda)\,\frac{p+D-p}{\rho\left(E-D^2+\sigma_n^2\right)}+u=S,$$

where the resulting coefficients must equal the conjectured A, B and C, which leads to the following result:

Theorem 2 *A piecewise linear REE of the model exists in the form*

$$P=f+\begin{cases} A\,(u-C) & \text{if } u<C \\ B\,(u-C) & \text{if } u\geq C \end{cases}, \tag{11}$$

where

$$A=\frac{\rho\sigma_n^2}{\lambda}\text{ and } B=\frac{\rho\sigma_n^2}{(1-w)\,\lambda},$$

and C is the solution of

$$0=L\,(C)\equiv C+\frac{1-\lambda}{\rho}\frac{D\,(C)}{E\,(C)-D^2\,(C)+\sigma_n^2}-S. \tag{12}$$

The omitted technicalities are provided in the appendix.

To see why the model has such an elegant solution, regardless of the distributional assumptions on n and u, notice that the demand of uninformed traders is constant, independent of the price p:

$$x^{ui}=\frac{E\,[f|P=p]-p}{\rho\,(Var\,[f|P=p]+\sigma_n^2)}=\frac{D}{\rho\left(E-D^2+\sigma_n^2\right)}.$$

It is due to the diffuse prior assumption, which implies that uninformed traders have only one source of information, namely the market price. Thus, a change in the price p is fully offset by a change in their expectation $E\,[f|p]$, while the precision of their information, given by $Var\,[f|p]$, remains constant. Hence, the diffuse prior assumption makes the the inference problem of uninformed traders trivial, and simplifies the analysis relative to Yuan (2005, 2006).

To see the intuition behind the piecewise linear structure and the presence of a kink at $u=C$, consider the aggregate demand of informed investors, given by $wx^c\,(f,p)+(1-w)\,x^{uc}\,(f,p)$, for the price p being close to fundamental payoff f. As long as $p\leq f$, the short-selling constraint does not bind, and a unit mass of informed investors are present in the market, submitting a total demand of $(f-p)\,/A$. However, for price and fundamental realizations such that $p>f$, some informed traders are barred from the market, and informed investors' aggregate demand is $(1-w)\,(f-p)\,/A$, less in absolute terms. It implies that when $p>f$, a less aggressive informed demand meets the residual

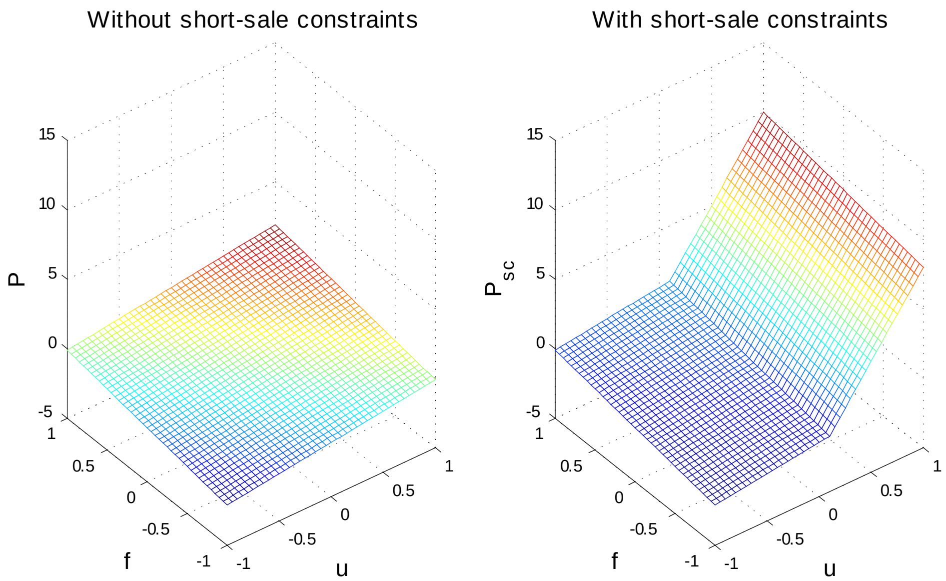

Figure 1: **The equilibrium price as a function of the fundamental and demand shock, in absence and presence of short-sale constraints.** The demand shock (u) is on the x axis and the payoff (f) is on the y axis. The left panel shows the price when informed investors are not short-sale constrained, and the right panel shows the price when $w > 0$ proportion of informed investors are subject to short-sale constraints. The parameters are set to $S = 0$, $\sigma_n^2 = 1$, $\lambda = 0.5$ and $w = 0.9$, which imply $A = 1$ and $B = 10$. The equilibrium value of C depends on the assumption about the demand shock distribution g_u; without making any distributional assumptions I set $C = 0.1$.

demand, defined as the demand of uninformed traders, plus the demand of noise traders, minus the asset supply, i.e. $u + x^{ui} - S$, which is simply a linear function of the demand shock u. Therefore, the equilibrium price is more sensitive to large demand shocks, implying $B \geq A$, and is a linear function of state variables f and u, conditional on both the constraint binding or not.

Figure 1 illustrates the main result of this section. The graph shows the asymmetric change in the equilibrium price due to the presence of short-sale constraints. When shorting is allowed (left panel), the slope of the price p as a function of demand shock u is the same for every realization of the shock. When shorting is prohibited (right panel), the price function is steeper for large demand shocks than for small (negative) demand shocks. It means that when the constraint binds for some speculators, a small increase in the demand shock has a larger upward price impact. Thus, the price reveals information about the payoff f at different rates in the two regions: it provides more information when the constraint does not bind, i.e. the demand shock is low, and less information, when the constraint does bind, i.e. the demand shock is high.

3.1 Properties of the equilibrium price

The rest of the section illustrates how certain properties of the equilibrium price change due to short-sale constraints. In order to determine the direct effect of introducing short-sale constraints in a market, one can compare conditional moments of the fundamental f. My main focus is on the information content of the price, illustrated by the conditional variance and skewness. All the results are proven in the Appendix.

Notice first that $A = \rho\sigma_n^2/\lambda$ does not depend on w. Let B_w, C_w, D_w and E_w denote the equilibrium constants B, C, D and E as a function of the w proportion of short-sale-constrained informed traders. Similarly, one can define P_w to be the equilibrium price function as a function of w, for given f and u realizations and with the corresponding equilibrium coefficients B_w and C_w. The absence of short-sale constraints, i.e. $w = 0$, implies $A = B_0 = \rho\sigma_n^2/\lambda$, and solving for the equilibrium price, (12) yields

$$C_0 = \frac{\lambda + \rho A\sigma_u^2}{1 + \rho A\sigma_u^2} S.$$

If the asset is in positive net supply, $S > 0$, C_0 is positive, and $D_0 = AC_0 > 0$, which means that uninformed investors' expectation about the asset payoff is above the market price, $E[f|P_0 = p] = p + D_0 > p$, and they demand a discount of $D_0 > 0$ to hold the asset. The equilibrium price thus becomes

$$P_0(f, u) = f + A(u - C_0), \tag{13}$$

which implies a conditional second moment of

$$Var[f|P_0 = p] = A^2\sigma_u^2. \tag{14}$$

As (9) shows, in the presence of short-sale constraints, the conditional distribution changes because of the different impact of the demand shock on the price for high and low prices, that is when the constraint binds or not. The following proposition compares the informativeness of market prices with and without short-sale constraints:

Proposition 3 *Short-sale constraints lead to a decrease in price informativeness, which is defined as the inverse of the conditional variance of the payoff. Formally, for any price realization p,*

$$Var[f|P_w = p] > Var[f|P_0 = p]. \tag{15}$$

Condition (15) shows that short-sale constraints increase uninformed traders' perceived uncertainty about the asset payoff, because they decrease the information content of the market price for high demand shock realizations.[11] Uninformed investors demand a larger discount for this increase in uncertainty, implying $D_w > D_0$.

It is also interesting to see the implications of short-sale constraints on the equilibrium price volatility. From equations (11) and (13) one can obtain

$$Var\left[P_w|f\right] = E_w - D_w^2 \text{ and } Var\left[P_0|f\right] = A^2\sigma_u^2.$$

Comparing the volatility with and without short-sale constraints gives the following result:

Proposition 4 *Short-sale constraints lead to an increase in price volatility:*

$$Var\left[P_w|f\right] > Var\left[P_0|f\right].$$

This finding is in line with previous empirical results. Indeed, Ho (1996) finds an increase in stock return volatility when short sales were restricted during the Pan Electric crisis in the Singapore market in 1985-1986. Boehmer et al. (2013) document a sharp increase in intraday volatility during the September 2008 emergency order.

The asymmetric effect of short-sale constraints on prices and price informativeness can be easily tested by analyzing return skewness. In the static model presented here, one can define two returns. Following Bai et al. (2006), I define the *announcement-day return* of the stock as the dollar return made between the trading round, date 0, and the final date 1, and the *market return* as the return made between a hypothetical date -1, before trading commences, and date 0. For simplicity, I assume that the price at this date -1, denoted by p^{-1} is constant. Formally, the announcement-day return is given by $r\left(f,u\right) = f - P\left(f,u\right)$, and the market return is given by $R\left(f,u\right) = P\left(f,u\right) - p^{-1}$.

Hong and Stein (2003) argue that short-sale constraints can lead to negative skewness in stock returns, which they relate to market crashes. On the empirical side, Reed (2007) documents that under short-sale constraints, the distribution of announcement day stock returns is more left-skewed. He also reports that returns have larger absolute values, when short-selling is constrained. Calculating properties of the announcement-day return with and without short-sale constraints gives the following results:

Proposition 5 *Short-sale constraints lead to more negatively skewed announcement-day returns:*

$$Skew\left[r_w\left(f,u\right)\right] < Skew\left[r_0\left(f,u\right)\right],$$

[11] The increase in the conditional variance is present in Bai et al. (2006) and Marin and Olivier (2008) as well, but, as discussed shortly, in those models short-sale constraints decrease the information content of the market price for low price realizations.

and an increase in the absolute value of returns:

$$E\left[\left|r_w\left(f,u\right)\right|\right] > E\left[\left|r_0\left(f,u\right)\right|\right].$$

The intuition for the negative skewness and is that short-sale constraints impede the negative information to be incorporated into the price, which leads to larger realized losses when the final payoff becomes public knowledge. Market prices reflect positive information more, and hence announcement day returns are smaller in this case. Moreover, absolute returns increase simply because losses become larger.

Regarding empirical evidence, Bris et al. (2007) find that in markets where short-selling is either prohibited or not practiced, market returns display significantly less negative skewness. Analyzing market returns with and without short-sale constraints gives the following result:

Proposition 6 *Short-sale constraints lead to less negatively skewed market returns:*

$$Skew\left[R_w\left(f,u\right)\right] > Skew\left[R_0\left(f,u\right)\right].$$

Because of short sale constraints, negative information is less incorporated to the market price and hence downward price movements and negative market returns are smaller in markets where shorting is prohibited.

To conclude this section with a technical sidenote, it is interesting to mention that there are differences in the asset pricing implications of two branches of asymmetric information models with portfolio constraints. The first type of these models includes Bai et al. (2006) and Marin and Olivier (2008). In both of these papers, noise in the market (from the point of view of uninformed traders) comes from the unknown endowment of insiders, and trading constraints limit the positions of all informed traders. These assumptions have two implications: the constraint status of informed investors can be directly inferred from the equilibrium price, and the constraint for insiders is binding for low prices. Therefore, in these models, high prices are more informative than low prices. The model presented here belongs to the other branch, together with Barlevy and Veronesi (2003), and Yuan (2005, 2006). In these studies noise arrives to the market from noise traders' demand or random supply, and only a subset of informed investors are subject to the short-selling constraint. Importantly, uninformed traders have to guess the probability that the constraint binds, and the constraint binds for high prices. Therefore, low prices are more informative than high prices.[12]

[12] The model presented in this paper here does not cover the $w = 1$ case, which is the subject of Bai et al. (2006) and Marin and Olivier (2008). When the constraint binds, the aggregate demand of all rational traders would be price-inelastic, and hence no price could clear the market with the random noise trading.

4 Short-sale constraints and conditional variance

According to the prevailing view, the introduction of short-sale constraints reduces the informativeness of the market price, which is confirmed by the analysis of the previous section. Indeed, (15) states that the perceived uncertainty of uninformed traders increases with a partial ban on shorting. This section investigates the effect of short-sale constraints, when an outside observer (e.g. a creditor from Section 5) with additional private information tries to learn from the market price. I show that in presence of short-sale constraints the information content of the market price (which constitutes a public signal) varies with the private signal of this agent. In particular, if this information content is measured by the variance conditional on the private and the public signal, then it is a non-monotonic function of the private signal. Moreover, for some private signal realizations the conditional variance is lower in presence of short-sale constraints than for the same private signal in absence of the constraint.

First, I restate the equilibrium price provided in (11), with the emphasis on the information content of the price, characterized by the pdf of the payoff, conditional on observing only the market price p:

Proposition 7 *A piecewise linear REE of the financial market exists with*

$$P = f + \begin{cases} A(u-C) & \text{if } u < C \\ B(u-C) & \text{if } u \geq C, \end{cases}$$

where the equilibrium constants A, $B \geq A$, and C are uniquely determined. Moreover, conditional on the price observation, the payoff f is only 'locally' Gaussian, with a jump around the price realization p:

$$g(f|P=p) = \frac{1}{B\sigma_u} 1_{f<p} \phi\left(\frac{f-(p+BC)}{B\sigma_u}\right) + \frac{1}{A\sigma_u} 1_{f\geq p} \phi\left(\frac{f-(p+AC)}{A\sigma_u}\right). \tag{16}$$

When $w=0$, that is $A=B$, the conditional distribution simplifies to a normal distribution:

$$g(f|P_0=p) = \frac{1}{A\sigma_u} \phi\left(\frac{f-(p+AC_0)}{A\sigma_u}\right).$$

Figure 2 illustrates the distribution of f conditional on the market-clearing price p in absence and presence of short-sale constraints. The left panel shows that the distribution without short-sale constraints is normally distributed with mean $p+AC_0$ and precision $\tau_{Au} \equiv 1/\left(A^2\sigma_u^2\right)$. The right panel shows that under short-sale constraints the distribution is only locally normal, but not globally. For states of the world when the constraint does not bind, i.e. $f \geq p$, it is normally distributed with mean $p+AC$ and precision τ_{Au}. For states of the world when the constraint binds in the financial market, i.e. $f < p$, it is

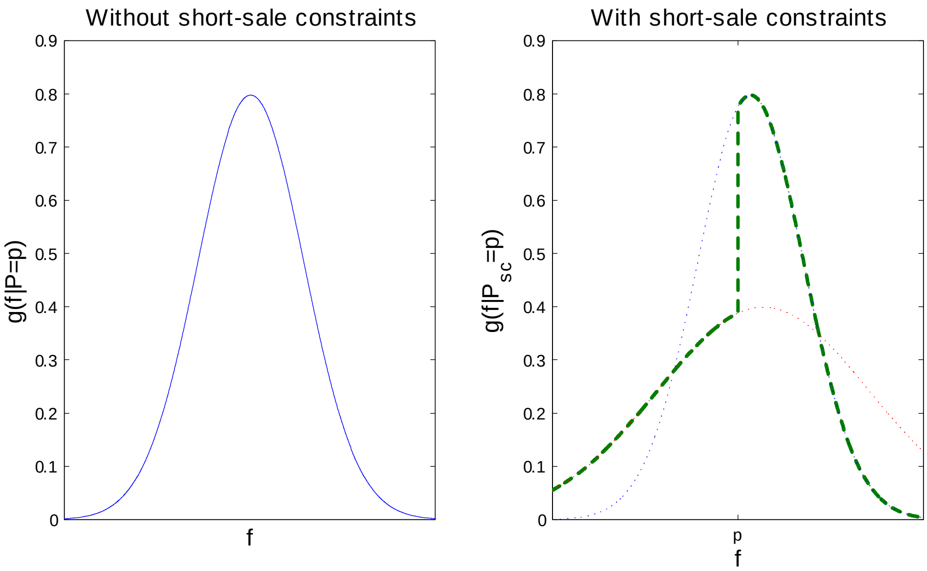

Figure 2: **Distribution of f conditional on p, in absence and presence of short-sale constraints.** The left panel shows that the distribution without short-sale constraints is Gaussian. The right panel shows that in presence of short-sale constraints (dashed line), the distribution is only locally normal, with different means and variances on the two segment, and with a jump at the price p. For comparison, the continuous line represents the conditional distribution without the constraint.

normally distributed with mean $p + BC$ and precision $\tau_{Bu} \equiv 1/\left(B^2\sigma_u^2\right)$. The variance increases, that is the precision decreases, because in this case there is less informed trading in the market.

Consider now a creditor who, besides observing the market price realization p, is also endowed with private signal t. For the tractability of the analysis, I assume that this private signal is given by $t = f + \xi$, where $\xi \sim N\left(0, \sigma_t^2 = \tau_t^{-1}\right)$.

Suppose first that there are no short-sale constraints in the market. Due to the jointly Gaussian distribution of f, t and p, the inference problem of the agent is simple: her posterior about the f is normally distributed with mean $\frac{\tau_t}{\tau_t+\tau_{Au}}t + \frac{\tau_{Au}}{\tau_t+\tau_{Au}}\left(p + AC_0\right)$ and precision $\tau_t + \tau_{Au}$. That is, her conditional pdf is given by

$$g\left(f|t, P_0 = p\right) = \frac{1}{\left(\tau_t + \tau_{Au}\right)^{-1}}\phi\left(\frac{f - \left[\frac{\tau_t}{\tau_t+\tau_{Au}}t + \frac{\tau_{Au}}{\tau_t+\tau_{Au}}\left(p + AC_0\right)\right]}{\left(\tau_t + \tau_{Au}\right)^{-1/2}}\right), \qquad (17)$$

and due to the characteristics of normal distributions, her conditional variance is independent of the private signal realization t:

$$Var\left[f|t, P_0 = p\right] = \left(\tau_t + \tau_{Au}\right)^{-1}.$$

Consider now the case with short-sale constraints. A creditor must combine her private signal t, which is normally distributed, with the public signal p, whose distribution is only locally normal, given in (16). A simple application of Bayes' rule implies that her posterior becomes

$$g(f|t, P = p) = \pi \cdot g(f|t, P = p, f < p) + (1 - \pi) \cdot g(f|t, P = p, f \geq p), \qquad (18)$$

where $\pi \equiv \Pr(f < p|t, p)$ and $1-\pi \equiv \Pr(f \geq p|t, p)$ are the probabilities the agent assigns to the constraint binding or not, respectively, and the conditional pdfs $g(f|t, P = p, f < p)$ and $g(f|t, P = p, f \geq p)$ belong to truncated normal distributions on the respectable ranges $f < p$ and $f \geq p$. In particular,

$$g(f|t, P = p, f < p) = 1_{f<p} \frac{1}{(\tau_t + \tau_{Bu})^{-1/2}} \frac{\phi\left(\frac{f - \frac{\tau_t t + \tau_{Bu}(p+BC)}{\tau_t + \tau_{Bu}}}{(\tau_t + \tau_{Bu})^{-1/2}}\right)}{\Phi\left(\frac{p - \frac{\tau_t t + \tau_{Bu}(p+BC)}{\tau_t + \tau_{Bu}}}{(\tau_t + \tau_{Bu})^{-1/2}}\right)}$$

is the pdf of a truncated normal distribution with mean $\frac{\tau_t}{\tau_t + \tau_{Bu}} t + \frac{\tau_{Bu}}{\tau_t + \tau_{Bu}} (p + CB)$ and precision $\tau_t + \tau_{Bu}$, because if the short-sale constraint binds in the financial market, the price equals $p = f + B(u - C)$. Similarly,

$$g(f|t, P = p, f \geq p) = 1_{f \geq p} \frac{1}{(\tau_{Au} + \tau_t)^{-1/2}} \frac{\phi\left(\frac{f - \frac{\tau_t t + \tau_{Au}(p+AC)}{\tau_t + \tau_{Au}}}{(\tau_t + \tau_{Au})^{-1/2}}\right)}{1 - \Phi\left(\frac{p - \frac{\tau_t t + \tau_{Au}(p+AC)}{\tau_t + \tau_{Au}}}{(\tau_t + \tau_{Au})^{-1/2}}\right)}$$

is the pdf of a truncated normal distribution with mean $\frac{\tau_t}{\tau_t + \tau_{Au}} t + \frac{\tau_{Au}}{\tau_t + \tau_{Au}} (p + AC)$ and precision $\tau_t + \tau_{Au}$, because if the short-sale constraint does not bind in the financial market, the price equals $p = f + A(u - C)$.

The conditional variance of a creditor in presence of short-sale constraints, as a function of the private signal t is illustrated on Figure 3. In general, the computation of this conditional variance becomes analytically intractable, but Figures 4 and 5 help to understand the intuition behind it.

Let us fix p, and consider two special cases. First, suppose that the creditor receives a much higher private signal, i.e. $t \to \infty$. As it implies

$$\lim_{t \to \infty} \pi = \Pr(f < p|t, p) = 0 \text{ and } \lim_{t \to \infty} \Pr(f \geq p|t, p) = 1,$$

the creditor is sure that the constraint does not bind in the financial market, which implies that all informed traders trade, and hence the precision of the price signal is τ_{Au}. Therefore, the posterior precision of the information available to her is given by $\tau_t + \tau_{Au}$,

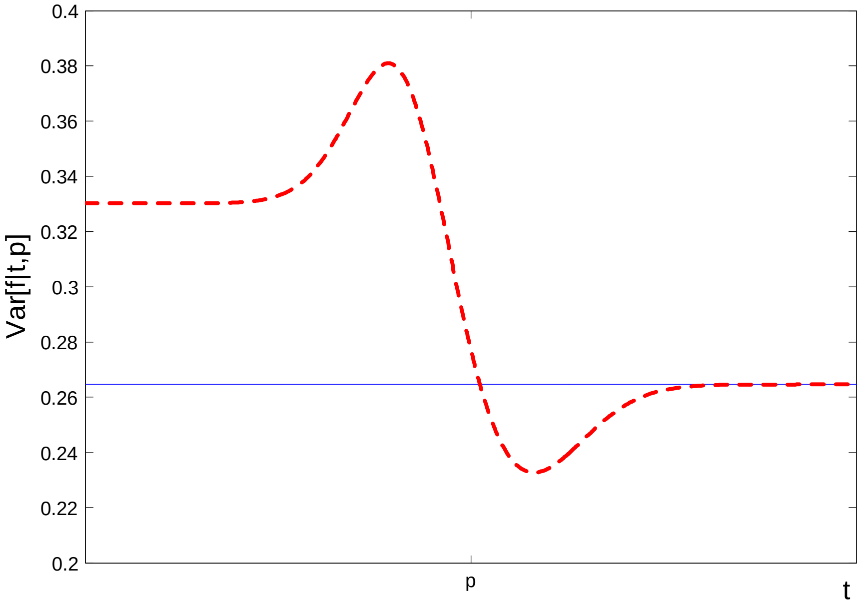

Figure 3: **Variance of f, conditional on the private signal t and the price p, without and with short-sale constraints.** The solid line shows the variance of f conditional on the private signal t and the price p with no short-sale constraints. The dashed line shows the same in presence of short-sale constraints. When $t \to \infty$, short-sale constraints do not change price informativeness. When $t \to -\infty$, short-sale constraints, intuitively, decrease price informativeness. For intermediate t values, creditors can actually learn more from prices under short-sale constraints. The parameters are set to $\sigma_t = 0.2$, $\sigma_u = 0.5$, $\sigma_n = 1$, $\rho = 1$, $\lambda = 0.5$ and $w = 0.9$, implying $A = 2$ and $B = 20$.

as if there were no short-sale constraints in the market at all. Figure 3 illustrates that in this case the conditional variances do not differ with and without the constraint.

Suppose now that the creditor receives a private signal much lower than the market price, i.e. $t \to -\infty$. It implies that

$$\lim_{t \to -\infty} \pi = \Pr(f < p|t, p) = 1 \text{ and } \lim_{t \to -\infty} \Pr(f \geq p|t, p) = 0,$$

hence she is sure that the constraint binds in the financial market, which implies that only a subset of informed traders trade, and hence the precision of the price signal is τ_{Bu}, lower than without the short-sale constraints. Therefore, the posterior precision of the information available to her is given by $\tau_t + \tau_{Bu}$, again lower than without the constraint. Put it differently, her posterior variance, as illustrated on Figure 3, increases. Thus, short-sale constraints decrease price informativeness for agents with private signals much smaller than the price realization.

Finally, consider the cases when the two signals are close to each other. Figures 4 and 5 illustrate the change in the conditional distribution due to short-sale constraints

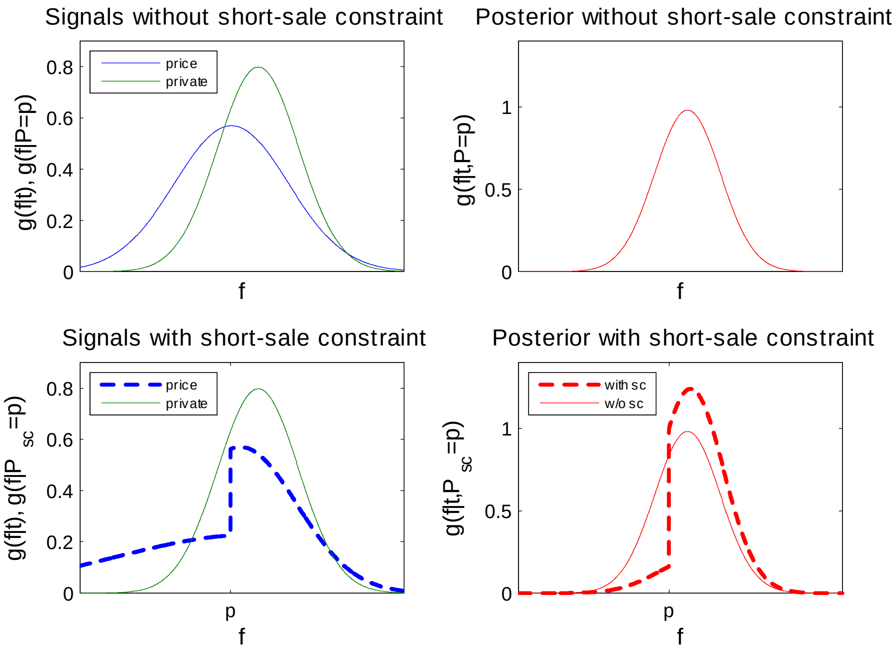

Figure 4: **Conditional distribution of f based on t, p, and both t and p, in absence and presence of short-sale constraints, when $t > p$.** The top left panel shows $g(f|t)$ and $g(f|p)$, and the top right panel shows $g(f|t,p)$, without short-sale constraints. The bottom left panel shows $g(f|t)$ and $g(f|p)$, and the bottom right panel shows $g(f|t,p)$, with short-sale constraints. The parameters are set to $\sigma_t = 0.5$, $\sigma_u = 0.7$, $\sigma_n = 1$, $\rho = 1$, $\lambda = 0.5$, and $w = 0.6$. Signal realizations are $t = -0.2$ and $p = 0.14$.

for a creditor with a private signal greater than the price, $t > p$, and for a creditor with a private signal smaller that the price, $t < p$, respectively. Contrasting the signal distributions without and with short-sale constraints, it is easy to see that when $t > p$, the introduction of short-sale constraints means that the creditor puts smaller weights on low payoff states of the world that she would consider unlikely based only on her private signal. The reason for this is that when constraint binds in the security market, i.e. when $f < p$, the demand shock is amplified due to the short-sale constraint. Therefore, the same price realization means a lower fundamental. However, if $t > p$, the private signal of the creditor suggests that the fundamental is high, thus low fundamental states are even more improbable. The agent knows that the market price is more likely to contain more positive information in general, therefore she becomes more certain about the payoff being high. Short-sale constraints hence confirm and strengthen her private information, as Figure 4 suggests. When $t \to \infty$, this effect gets weaker, and in the limit disappears. Hence, for $t \to \infty$, short-sale constraints do not alter the conditional distribution, and thus the conditional variance is not affected either.

When $t < p$, the opposite effect arises. Comparing the signal distributions without and with short-sale constraints, when $t < p$, the introduction of short-sale constraints

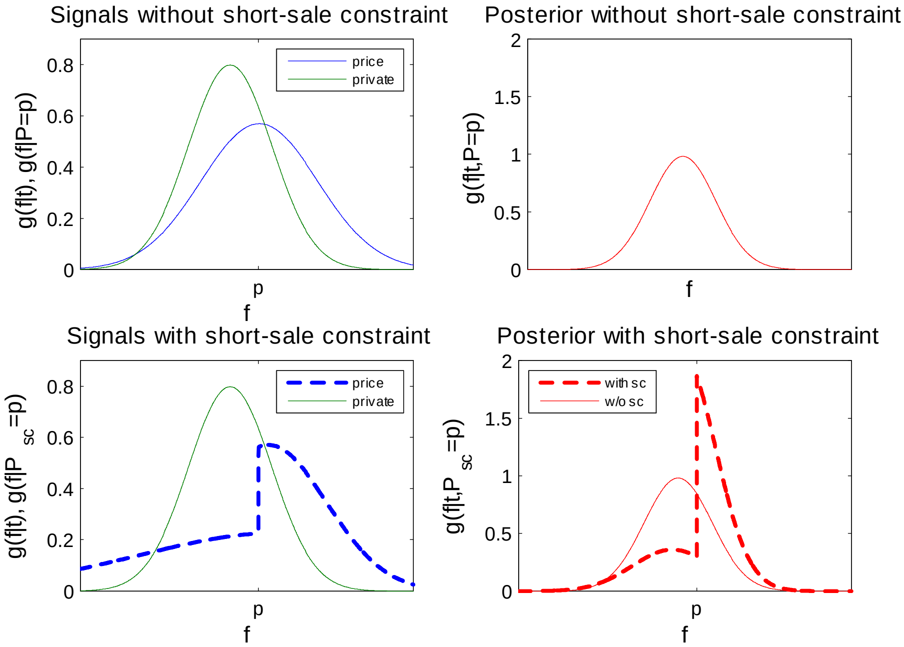

Figure 5: **Conditional distribution of f based on t, p, and both t and p, in absence and presence of short-sale constraints, when $t < p$.** The top left panel shows $g(f|t)$ and $g(f|p)$, and the top right panel shows $g(f|t,p)$, without short-sale constraints. The bottom left panel shows $g(f|t)$ and $g(f|p)$, and the bottom right panel shows $g(f|t,p)$, with short-sale constraints. Signal realizations are $t = 0.2$ and $p = -0.14$.

means that the creditor puts larger weights on states of the world that she thought to be unlikely based on her private signal. That is, short-sale constraints force the agent to consider some previously irrelevant states of the world. She knows that the market price is more likely to contain more positive information in general, therefore when her private signal is below the price realization, her uncertainty about whether the constraint binds in the financial market increases, and hence her uncertainty about the payoff increases, too. Short-sale constraints dispute her private information, and hence weaken her posterior precision, as Figure 5 suggests. When $t \to -\infty$, this effect gets weaker, and in the limit disappears. However, the agent becomes certain that the constraint binds, and in this case the precision of the price signal is lower. Hence, for $t \to -\infty$, short-sale constraints alter the conditional distribution by affecting its precision, and thus the conditional variance increases.

As the following proposition states, short-sale constraints can increase the information content of the price for high enough private signal realizations, measured by the variance conditional on the private and the public signal:

Proposition 8 *There exists constant $\bar{t}$ such that the posterior variance of the asset payoff conditional on observing price p and private signal t, $Var[f|t,p]$, is lower under short-*

sale constraints (i.e. when $w > 0$ or $B > A$), if and only if $t - p \geq \bar{t}$. The threshold $\bar{t}$ is a decreasing function of w, and $\lim_{\sigma_t} \bar{t} = 0$.

The following section studies how this non-monotonic change in the conditional variance due to short-sale constraints affects coordination in a game with strategic complementarities.

5 Economy with a financial market and creditors

This section extends the previous setup by embedding a coordination game between dates 0 and 1. Suppose that a financial institution (e.g. investment bank, or bank, for short) is financed through a combination of short-term and long-term debt. Long-term debt holders are passive - in the past they have decided to provide capital that cannot be withdrawn. Short-term debt matures at date $t = 1$, on which occasion it can be renewed.

The state of fundamentals is characterized by θ that is interpreted as the cash-flow the bank's assets generate at date 1. Higher values of θ correspond to higher quality/liquidity projects. I assume that the bank has outstanding debt with size normalized to 1, from which the short-term debt amounts to ω and the long-term debt is $1 - \omega$. Short-term debt holders (creditors, for short from now on) can decide to roll over their debt. For simplicity, I assume that the bank's assets generate sufficiently large cash-flows in the long run, but they only have θ to pay out creditors who demand capital payoff at date 1. Therefore, the bank remains solvent if and only if $\theta \geq \omega (1 - I)$, where I denotes the proportion of creditors who roll over, and hence $\omega (1 - I)$ is the amount to be paid out to creditors who recall their loans.

Creditors are a continuum of risk-neutral agents with measure one, and indexed by $j \in [0, 1]$.[13] Each creditor can choose between two actions. They either provide capital (i.e. roll over the short-term debt), $i_j = 1$, the risky action, or refrain from doing so (i.e. recall the loan or withdraw money), $i_j = 0$, the safe action. The net payoff from withdrawing is normalized to zero. The net payoff from lending to the bank is $1 - c$ if the bank remains solvent and $-c$ otherwise, where $c \in (0, 1)$ parametrizes the private costs of lending, which can be interpreted, for example, as transaction costs, administrative fees,

[13] The security market and debt market (i.e. the capital provision environment of creditors) are assumed to be segmented markets, that is the asset price is fully exogenous from the point of view of creditors, and hence it does not incorporate their private information, as in Angeletos and Werning (2006). See the discussion later.

or taxes.[14] It follows that the payoff of creditor j is

$$U(i_j, I, \theta) = i_j \left(1_{\theta \geq \omega(1-I)} - c\right), \tag{19}$$

where $1_{\theta \geq \omega(1-I)}$ is the indicator of the bank remaining solvent, and takes the value of 1 if $\theta \geq \omega(1-I)$ and 0 otherwise.[15]

If creditors know the value of θ perfectly before making their decision, there exist a tripartite classification of the state, in the spirit of Obstfeld (2004). Based on this, the optimal strategy of creditors is as follow: If $\theta \leq 0$, then the dominant strategy is to withdraw deposits from the bank, irrespective of what other capital providers do, because the bank always fails. In turn, if $\theta \geq \omega$, then the dominant strategy is to give money to the bank, irrespective of what other creditors do, because it always remains solvent. When the bank asset value θ lies in the interval $(0, \omega)$, there is a coordination problem among capital providers. On one hand, if every other creditor rolls over the debt, the bank survives, and lending yields more than withdrawing: $1 - c > 0$. On the other hand, if every other creditor withdraws, the bank fails, and withdrawing yields more than financing the bank: $0 > -c$. Therefore, both $I = 1$ and $I = 0$ is an equilibrium whenever $\theta \in (0, \omega)$: the former outcome represents the first best, while the latter is considered a coordination failure. In this interval the bank's future depends on the size of the credit run.[16]

Following standard global game setups in the spirit of Carlsson and van Damme (1993) and Morris and Shin (1998), I assume that information is imperfect, so that the state θ is not common knowledge. In the beginning of the game, nature draws θ from a diffuse uniform distribution over the real line, which constitutes the agents' initial common prior about the state of the world. Investor j then receives a private signal $t_j = \theta + \xi_j$, where ξ_j has a Gaussian distribution with mean 0 and standard deviation σ_t, ξ_j is independent of θ, and independently and identically distributed across short-term debt holders. The

[14] As creditors are assumed to be risk-neutral, this setting is equivalent to any set of payoffs $\{\pi_H, \pi_L, \pi_0\}$, where providing capital pays either π_H in case of the bank remaining solvent and $\pi_L < \pi_H$ in case of failure, while recalling the loan gives a sure payoff π_0 that satisfies $\pi_L < \pi_0 < \pi_H$. The utility of a creditor in this setting would simply be a linear function of the utility given in (19), and hence would lead to the same optimal action.

[15] The coordination setup presented here is a simplified version of models on bank runs, e.g. Diamond and Dybvig (1983), Rochet and Vives (2004) and Goldstein and Pauzner (2005); or Morris and Shin (2004), who study coordination among creditors of a distressed borrower. In contrast to those papers, I choose to work with a parsimonious model, as my aim is to analyze the effect of short-sale constraints on coordination, instead of providing a more realistic setting. In particular, I will abstract away from the first mover's advantage and demand-deposit insurance, emphasized by Diamond and Dybvig (1983) and Goldstein and Pauzner (2005), or the price at which the debt is issued, as in Morris and Shin (2004).

[16] There exist other interpretations of coordination, here presented using the terminology of creditor runs. In models of currency crises, as in Obstfeld (1986, 2004) or Morris and Shin (1998), speculators decide whether to attack a currency by shorting it. Chamley (1999), Morris and Shin (1999) and Dasgupta (2007) consider investment complementarities.

precision of the private signal is given by $\tau_t = 1/\sigma_t^2$.

To connect the security trading and the credit run, I assume that the payoff of the asset, f, correlates with the quality of the banks' assets. Thus, the price of the financial asset, p, can provide additional (public) information regarding the state of the world beyond the private signals, and hence can facilitate or hurt coordination among capital providers. For simplicity, I assume $f = \theta$, and think about the traded asset as the (only) security that the bank has on the asset side of the balance sheet, a zero-net-supply financial derivative on the bank's equity, or as an industry index that includes the bank. Thus, the price is an exogenous signal from the viewpoint of creditors, in the sense that the , but is nevertheless correlated with the fundamental θ.

Because the two parts of the economy are segmented, with an information spillover from the financial market to the credit run in the form of the price p, without the outcome of the coordination game affecting the market price, the equilibrium of the whole economy is also separable into two parts. In fact, it is just a simple conjugate of the equilibrium of the trading round, defined in Definition 1 and discussed in Section 3, and the equilibrium of the run, conditional on the realization of the market price. Thereby, I only define the equilibrium of the coordination game:

Definition 9 *Let p denote the price of the asset with payoff $f = \theta$ emerging from the financial market. A perfect Bayesian equilibrium of the credit run consists of individual strategies for investing, $i(t_j, p)$, and the corresponding aggregate, $I(\theta, p)$, such that*

1. *decision is optimal for creditors:*

$$i(t_j, p) \in \arg\max_{i \in \{0,1\}} E\left[U(i, I(\theta, p), \theta) \,|t_j, p\right] \text{ for } j \in [0,1]; \tag{20}$$

2. *proportion of short-term debt rolled over is*

$$I(\theta, p) = \int_0^1 i(t_j, p)\, dj; \tag{21}$$

3. *agents update their beliefs according to Bayes' rule.*

This section hence only solves for the equilibrium of a standard global game setup with private and public information, fully characterized by conditions (20) and (21). Combining the equilibrium of the credit run with the equilibrium of the financial market would provide an equilibrium of the whole economy.

I restrict my attention to monotone equilibria, defined as perfect Bayesian equilibria such that, for a given realization p of the public signal, a creditor provides capital to the bank if and only if the realization of her private signal is at least some threshold $t^*(p)$; that is $i(t_j, p) = 1$ iff $t_j \geq t^*(p)$. It implies that the bank can be characterized in a similar way: the bank with asset quality θ survives if and only if this quality is higher that some threshold $\theta^*(p)$; formally if $\theta \geq \theta^*(p)$.[17]

6 Credit runs and portfolio constraints

After trading in the financial market has taken place, but before the payoff at date 1 happens, creditors decide whether to roll over short-term debt, thereby providing capital to the bank in need of liquidity, or to withdraw it. Since the payoff of the financial asset f and the bank asset value θ are correlated, the equilibrium price of the financial market, p, provides an observable public signal regarding the unknown parameter θ, and creditors can coordinate their actions based on it. In the following subsections, I solve the coordination model, first without constraints on short-selling, then with the short-sale constraints.

6.1 Equilibrium analysis with no short-sale constraints

In this section I provide a solution to the coordination game among capital providers when short-selling is allowed for everyone. To pin down the equilibrium of the model, characterized by the pair $\{t^*, \theta^*\}$, I solve for the optimal θ^* while taking t^* as given, and for the optimal t^* if θ^* is assumed to be given. The joint solutions of these two conditions describe the equilibria of the credit run.

In a monotone equilibrium described above, creditors with private signals $t_j \geq t^*$ provide capital. Based on the joint distribution of θ and the private signals t_j, the aggregate proportion of creditors who roll over is given by

$$I(\theta, p) = \Pr(t \geq t^*(p) | \theta) = 1 - \Phi\left(\sqrt{\tau_t}\left(t^*(p) - \theta\right)\right).$$

The right hand side of this equation increases in θ, therefore a better bank receives more capital rolled over. The bank avoids bankruptcy if and only if $\theta \geq \theta^*(p)$, where $\theta^*(p)$ is the quality of the marginal bank that solves $\theta = \omega(1 - I(\theta, p))$. Therefore,

$$t^*(p) = \theta^*(p) + \frac{1}{\sqrt{\tau_t}}\Phi^{-1}\left(\frac{\theta^*(p)}{\omega}\right). \tag{22}$$

Condition (22) characterizes the banks that survives withdrawals for a given switching strategy $t^*(p)$. There are several remarks to be made about this equation. First, notice

[17] My results concerning multiple equilibria are obtained even within this restricted class. Moreover, in absence of short-sale constraints, uniqueness within this class implies overall uniqueness, see Morris and Shin (1998, 1999).

that the right-hand side of (22) is strictly increasing in $\theta^*(p)$, therefore there is a unique $\theta^*(p)$ that satisfies the equation for a given $t^*(p)$. Secondly, the bank survival threshold $\theta^*(p)$ is an increasing function of the creditor cutoff $t^*(p)$, as a lower switching strategy from creditors implies more capital rolled over, and hence a bank with lower asset payoff surviving. Thirdly, as

$$\frac{dt^*(p)}{d\theta^*(p)} = 1 + \frac{1}{\sqrt{\tau_t}\omega} \frac{1}{\phi\left(\Phi^{-1}\left(\frac{\theta^*(p)}{\omega}\right)\right)} > 1,$$

it must be that $d\theta^*/dt^* < 1$. The presence of strategic complementarities implies that any increase in the cutoff $t^*(p)$ results in a smaller increase in the marginal bank's value, because no creditor can be certain about the signals received by others and hence the strategy of others. Finally, in the limit when private signals become arbitrarily precise, $\tau_t \to \infty$, creditors become certain about others' signals as well, and the bank survival threshold $\theta^*(p)$ becomes exactly the individual capital provision threshold $t^*(p)$.

Next, consider the derivation of the equilibrium cutoff strategy $t^*(p)$ as a function of the threshold $\theta^*(p)$. Creditors receive payoff 1 if the bank avoids distress, and 0 if not, while paying a cost c. Because they do not observe the state θ directly, the payoff from rolling over the loan must be calculated from the posterior distribution over the states, conditional on the private and public signal. If creditor j knows that the bank solvency threshold is $\theta^*(p)$, she assigns probability $\Pr(\theta \geq \theta^*(p) | t_j, p)$ to the bank surviving, based on all her information, which implies that the expected payoff from rolling over is $\Pr(\theta \geq \theta^*(p) | t_j, p) - c$. As withdrawing yields a payoff normalized to 0, the signal of the marginal agent, who is indifferent between withdrawing or not, must solve the indifference condition $\Pr(\theta \geq \theta^*(p) | t_j, p) = c$.

In absence of short-sale constraints, the market price is $p = \theta + A(u - C_0)$. As u is normally distributed with mean zero and precision $\tau_u = 1/\sigma_u^2$, the precision of the price signal is $\tau_{Au} \equiv \tau_u / A^2$. Therefore, the posterior of agent j about θ is normally distributed with mean $\frac{\tau_t}{\tau_t + \tau_{Au}} t_j + \frac{\tau_{Au}}{\tau_t + \tau_{Au}} (p + AC_0)$ and precision $\tau_t + \tau_{Au}$. Thus, the indifference condition becomes

$$\Phi\left(\sqrt{\tau_t + \tau_{Au}}\left(\theta^*(p) - \frac{\tau_t}{\tau_t + \tau_{Au}} t^*(p) - \frac{\tau_{Au}}{\tau_t + \tau_{Au}} (p + AC_0)\right)\right) = 1 - c,$$

which is equivalent to

$$\theta^*(p) = \frac{\tau_t}{\tau_t + \tau_{Au}} t^*(p) + \frac{\tau_{Au}}{\tau_t + \tau_{Au}} (p + AC_0) + \frac{1}{\sqrt{\tau_t + \tau_{Au}}} \Phi^{-1}(1 - c), \tag{23}$$

and implies a linear relationship between θ^* and t^*. Figure 6 illustrates the the critical mass condition, (22), and the individual optimality condition without short-sale con-

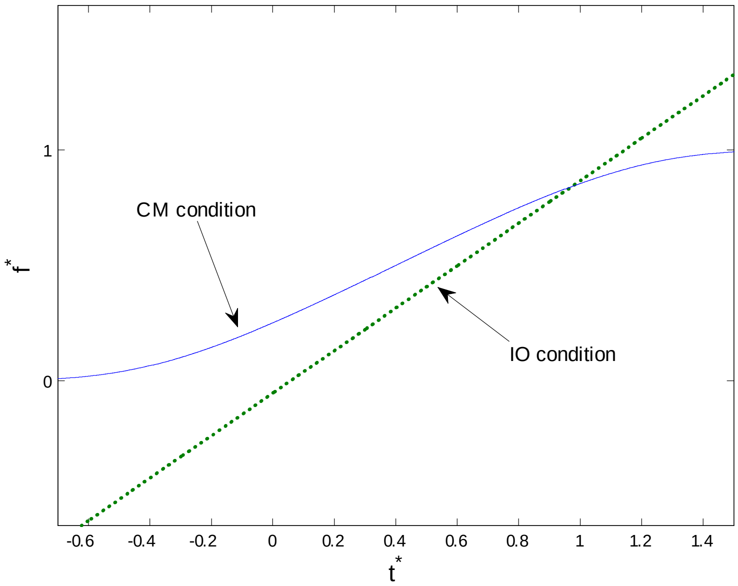

Figure 6: **IO and CM conditions without short-sale constraints.** This figure plots the critical mass condition, (22), thin line, and the individual optimality condition without short-sale constraints, (23), dotted line. The cutoff strategy for investment, t^*, is shown on the x axis, and the success threshold, θ^*, is on the y axis. The parameters and variable realizations used here are $\sigma_n = 1$, $\sigma_u = 0.5$, $\sigma_t = 0.3$, $\omega = 0.8$, $c = 0.7$ and $p = 0.25$.

straints, (23), respectively.

An equilibrium is the joint solution to conditions (22) and (23), which lead to

$$\frac{\tau_{Au}}{\sqrt{\tau_t}}\theta^*(p) - \Phi^{-1}\left(\frac{\theta^*(p)}{\omega}\right) = \sqrt{1 + \frac{\tau_{Au}}{\tau_t}}\Phi^{-1}(1-c) + \frac{\tau_{Au}}{\sqrt{\tau_t}}(p + AC_0).$$

As the left-hand side of the equation is a continuous function of θ^*, which takes the value $-\infty$ for $\theta^* = \omega$ and ∞ for $\theta^* = 0$, the equation always has a solution. Moreover, the solution is unique for every p_0 if and only if the left-hand side of the equation is a strictly decreasing function of θ^*, that is if and only if $\tau_{Au} \leq \sqrt{2\pi\tau_t}$.[18]

The following proposition states the above result:

Proposition 10 (Morris and Shin) *In absence of short-sale constraints, the equilibrium is unique if and only if the private noise is small relative to the price noise, that is for $\sigma_t \leq \sqrt{2\pi}A^2\sigma_u^2$. Moreover, in the limit as private noise vanishes so that $\sigma_t \to 0$, a creditor with private signal below $t^*(p) = c\omega$ recalls her loan, and the bank with asset quality below $\theta^*(p) = c\omega$ fails.*

Proposition 10 confirms the uniqueness result of Morris and Shin (1999, 2001). For any

[18] As Figure 6 suggests, this is equivalent to the slope of the critical mass condition (22) always being below the slope of the individual optimality condition, (23).

positive level of noise in the public signal, $\sigma_u > 0$, uniqueness is ensured by sufficiently small noise in the private signal. The intuition is that as the private signal becomes much more precise than the public signal, creditors stop relying on the public signal and use only their private information. This implies that the equilibrium dependence on the common noise component u vanishes, and makes it harder to predict the actions of others, heightening strategic uncertainty. When strategic uncertainty is strong enough, multiplicity breaks down. It is interesting to note that the equilibrium run size and outcome outcome does not depend on public signal p (or common noise component u), which is the second finding of Morris and Shin (1999). In what follows, I will refer to this equilibrium as the informationally efficient equilibrium,. It is important to mention that this informationally efficient equilibrium is different from the first best or allocationally efficient equilibrium, i.e. $I^* = \omega$ and $\theta^* = 0$. This difference is due to the presence of the coordination externality.

6.2 Equilibrium analysis with short-sale constraints

As shown in Section 3, the introduction of short-sale constraints has an adverse effect on the market price. The fact that the price reveals information about the payoff at different rates for high and low realizations of the demand shock implies that short-sale constraints notably change the inference problem of creditors, as presented in Section 4.

To solve for the equilibrium in presence of short-sale constraints, one needs to repeat the steps of the previous subsection. First, given that the joint distribution of the state θ and the private signals does not change, the critical mass condition (22) that determines the quality of the marginal bank as a function of individual strategies, does not change either. However, short-sale constraints do affect the posterior of creditors after observing both the price and the private signal. The public signal p is now only locally Gaussian, but not globally, as given in (16) and illustrated on Figure 2.

As shown in Section 4, in presence of short-sale constraints the posterior pdf can be given in the following way (see (18)):

$$g(\theta|t, P = p) = \pi \cdot g(\theta|t, P = p, \theta < p) + (1 - \pi) \cdot g(\theta|t, P = p, \theta \geq p).$$

In this equation, $\pi \equiv \Pr(\theta < p|t, P = p)$ and $1 - \pi \equiv \Pr(\theta \geq p|t, P = p)$ denote probabilities that the creditor with private signal t associates with the short-sale constraint binding in the financial market or not, respectively. Moreover, the conditional pdfs $g(\theta|t, P = p, \theta < p)$ and $g(\theta|t, P = p, \theta \geq p)$ belong to the class of truncated normal distributions, with means $\frac{\tau_t}{\tau_t+\tau_{Bu}}t + \frac{\tau_{Bu}}{\tau_t+\tau_{Bu}}(p + CB)$ and $\frac{\tau_t}{\tau_t+\tau_{Au}}t + \frac{\tau_{Au}}{\tau_t+\tau_{Au}}(p + CA)$, and precisions $\tau_t + \tau_{Bu}$ and $\tau_t + \tau_{Au}$, respectively, because in the first case the creditor

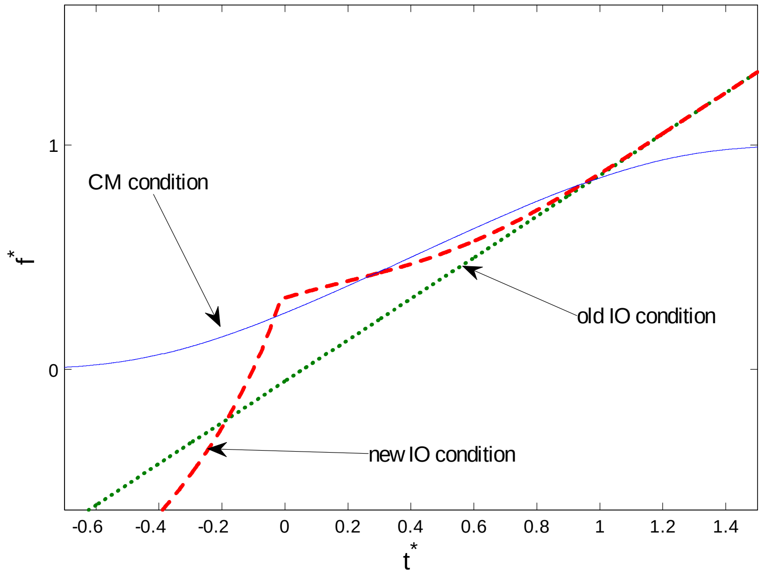

Figure 7: **IO and CM conditions with short-sale constraints.** This figure plots the critical mass condition, (22), thin line, and the individual optimality condition with short-sale constraints, (24), dashed line. The cutoff strategy for investment, t^*, is shown on the x axis, and the success threshold, θ^*, is on the y axis. The parameters and variable realizations used here are $\sigma_n = 1$, $\sigma_u = 0.5$, $\sigma_t = 0.3$, $\omega = 0.8$, $c = 0.7$ and $p = 0.25$. For comparison, the dotted line shows the individual optimality condition without short-sale constraints, (23).

knows the short-sale constraint binds in the financial market, and hence the price equals $p = \theta + B(u - C)$, while in the second case this creditor knows the constraint does not bind, and hence the price equals $p = \theta + A(u - C)$.

As before, the expected net payoff of agent j from providing capital to the bank, for a fixed success threshold θ^*, is $\Pr(\theta > \theta^* | t_j, P = p) - c$ and hence t^* must solve the indifference condition $\Pr(\theta \geq \theta^* | t_j, P = p) = c$, which is equivalent to

$$\theta^*(p) = \begin{cases} \frac{\tau_t}{\tau_t + \tau_{Bu}} t^*(p) + \frac{\tau_{Bu}}{\tau_t + \tau_{Bu}}(p + BC) + \frac{1}{\sqrt{\tau_t + \tau_{Bu}}} \Phi^{-1}\left((1 - c)\frac{\pi_B^*}{\pi^*}\right) & \text{if } \theta^*(p) \leq p \\ \frac{\tau_t}{\tau_t + \tau_{Au}} t^*(p) + \frac{\tau_{Au}}{\tau_t + \tau_{Au}}(p + AC) + \frac{1}{\sqrt{\tau_t + \tau_{Au}}} \Phi^{-1}\left(1 - c\frac{1 - \pi_A^*}{1 - \pi^*}\right) & \text{if } \theta^*(p) > p, \end{cases} \tag{24}$$

where $\pi^* \equiv \Pr(\theta < p | t^*, p)$ is the probability the marginal agent assigns to the short-sale constraint binding in the market, $\pi_B^* \equiv \Pr(\theta < p | t^*, p = \theta + B(u - C))$ is the probability that the marginal agent assigns to informed traders shorting/selling in a market with no constraints but volatility τ_{Bu}^{-1}, and $\pi_A^* \equiv \Pr(\theta < p | t^*, p = \theta + A(u - C))$ is the probability that the marginal agent assigns to informed traders shorting/selling in a market with no constraints but volatility τ_{Au}^{-1}. It is easy to see that when there are no short-sale constraints, i.e. $B = A$, $\pi^* = \pi_A^* = \pi_B^*$, and (24) is equivalent to (23).

Figure 7 illustrates the critical mass condition, (22), and the individual optimality condition in presence of short-sale constraints, (24), respectively. The former displays the quality of the marginal bank, θ^*, given that creditors follow the threshold strategy with t^*, that is a capital provider leaves her money in the bank if and only if she receives private signal $t_j \geq t^*$. As the θ^* threshold is determined only by the joint distribution of the fundamental and the private signals, short-selling constraints do not alter it.

What changes is the optimal switching strategy of creditors for a fixed θ^* bank solvency threshold. However, as the distributions are not jointly Gaussian, the posterior (18) is not Gaussian any more, and hence it is not possible to simplify condition (24) further more, and to provide a simple necessary and sufficient condition for the number of equilibria. The reason for this is that, as seen on Figure 7, the slope of (24) is not monotonic. Instead, short-sale constraints create a hump shape on the individual optimality condition, with the slope $d\theta^*/dt^*$ of the individual optimality condition taking values between the upper slope at the kink, $d\theta^*/dt^*|_{\theta^* \to p+}$, where it is clearly the smallest, and when $\theta^* \to -\infty$, where it is the largest, $\frac{\tau_t + \tau_{Bu}}{\tau_t}$. Thus, a sufficient condition for uniqueness would be that

$$\frac{d\theta^*}{dt^*}|_{\theta^* \to p+} > \frac{1}{1 + \frac{1}{\sqrt{\tau_t}\omega}\sqrt{2\pi}}. \tag{25}$$

However, as shown in Appendix C, there exists a constant $\overline{\sigma}_t > 0$ such that for every $0 < \sigma_t < \overline{\sigma}_t$, there is a price realization p such that (25) does not hold, and hence the individual optimality condition and the critical mass condition have three intersections. Thereby, there are multiple equilibria of the system of equations (22) and (24). The following proposition formally states this result:

Proposition 11 *In presence of short-sale constraints, there are multiple equilibria in investment strategies when σ_t is sufficiently small. Moreover, multiplicity remains as private noise vanishes so that $\sigma_t \to 0$: the switching strategies become*

$$t^*(p) = \theta^*(p) = \begin{cases} c\omega & \text{for all } p \\ p & \text{if } \beta c\omega < p < c\omega, \end{cases} \tag{26}$$

and hence for every $\theta \in [p, c\omega)$ both the informationally efficient equilibrium and a 'high capital provision' equilibrium exist whenever $\beta c\omega \leq p \leq c\omega$, where $\beta = \frac{A^2}{(1-c)B^2 + cA^2} < 1$.

The technical bits of the proof are in Appendix C.

The informationally efficient equilibrium is the same as the unique equilibrium of the unconstrained economy: a bank with asset quality above $c\omega$ remains solvent. However, there exists an equilibrium with more capital provision: creditors also finance banks with lower asset quality, between the public signal realization p and $c\omega$. This is an informationally inefficient equilibrium, as agents put excessive weight on the public signal. It is characterized by overinvestment compared to the informationally efficient equilibrium, because agents with lower signals provide capital too, hence I refer to it as the 'high investment' equilibrium.

7 Discussion

In this section I discuss my results on multiplicity, allocational efficiency, provide comparative statics, and refer to some policy implications.

7.1 Multiplicity

Canonical papers in the literature on transparency show that releasing more information is not necessarily good. Indeed, in Morris and Shin, while without a public signal the market may be in a uniqueness region, by adding a precise enough public signal, the economy has multiple equilibria.

Since Morris and Shin (2001), several authors have considered ways that reinstate multiplicity in coordination games. The existing literature mainly focuses on the endogenous nature of the public signal. For example, Angeletos and Werning (2006) study financial market prices, which aggregate the dispersed information of agents, or direct noisy signals about others' activity. Information aggregation can overturn the Morris and Shin (1998) uniqueness result and lead to multiplicity if the precision of public information increases faster than the precision of the private information. Hellwig et al. (2006) and Tarashev (2007) also study coordination games with financial prices being endogenous public signals. Because all these papers stay in the class of jointly Gaussian distributions, the informational content of the public signal does not vary for its different realizations and hence across the multiple equilibria.

In contrast, the model presented here provides a fundamentally different setting. What is cardinal for the analysis is that agents with different private signals interpret the same public signal in different ways. In particular, the information they infer from the public signal changes with the distance of their private and public signal. Holding the price constant and increasing the private signal can provide more information about the composition of the market price: a high price is more likely to be the result of a high demand shock than a low price to be the result of a low demand shock, because in the first case the fewer informed traders have a smaller corrective effect on the market price.

According to the prevailing view, the introduction of short-sale constraints reduces the informativeness of the market price, i.e. decrease its precision, and hence, following the Morris and Shin logic, should not lead to coordination failures. Indeed, (15) states that the perceived uncertainty of uninformed traders increases with a ban on shorting. The surprising finding of this model is that, in contrast to the existing literature, I show that short-sale constraints can make asset prices contain more information for some creditors with additional information, as demonstrated in Proposition 8.[19]

Although both the setup and the motivation are different, the results of the paper are close in spirit to Angeletos et al. (2006). They examine the informational role of policy decisions in a coordination setting. They show that policy interventions create endogenous public information and can lead to multiple equilibria.[20] There are two differences though. First, in their paper the public signal reveals that the state of the world is neither too high, nor too low. In contrast, short-sale constraints 'help' to rule out only lower states of the world by making creditors' posterior distributions more left-skewed. This has strong implications on allocational efficiency, discussed below. Second, in their analysis the signal is the equilibrium action of a policy maker, whereas the present article takes the constraint as given. I show that, even abstracting from signaling and analyzing the constraint on short-selling as an endogenous decision of regulators, short-sale constraints are nevertheless capable of suggesting that prices, influenced by demand shocks, are lower than economic fundamentals would imply. It would be interesting to see how introducing signaling (i.e. endogenizing the authority's decision to introduce short-sale constraints in a security market) would influence the results of the model.

7.2 Efficiency

An interesting result of the analysis is that in the second equilibrium creditors always provide more capital than in the informationally efficient equilibrium. It is shown in 26: the second equilibrium only exists for $\beta c\omega < p < c\omega$.

The intuition is the following. As shown in Proposition 8, asset prices under short-sale constraints provide more information to creditors with high private signals. First, consider the case when $p > c\omega$. Without short-sale constraints, in the informationally efficient (unique) equilibrium, creditors rely only on their private signals, and based on

[19]It is interesting to refer back to similarities and differences with Bai et al. (2006), and Marin and Olivier (2008). What is crucial in the analysis is that short-sale or other trading constraints result in a varying information content across different price levels. Therefore, even if the asset pricing implications of the two types of models are different, qualitative results, such as the increasing information content of the price under short-sale constraints for some agents with additional private information, and the possibility of multiple equilibria would not be affected. However, with a financial market model, where high asset prices are more informative than low prices, the informationally efficient equilibrium would be allocationally more efficient as well.

[20]The two types of equilibria that they identify are also in line with the findings of this paper. Their inactive-policy equilibrium, where agents coordinate on a strategy that is insensitive to the policy, is analogous to my informationally efficient level of creditor run, and their continuum of active-policy equilibria correspond to equilibria when capital provision depends on the price p.

their assessments about the bank's asset value, agents with private signals $t_j \geq c\omega$ provide capital to the bank. In presence of short-sale constraints, agents with private signals above p will become more informed about both the fundamental θ and hence about other creditors' beliefs. Therefore short-sale constraints weaken strategic uncertainty among these creditors, leading to the possible multiplicity of equilibria. In one equilibrium they all provide capital, but as they all provide capital in the informationally efficient equilibrium, it would not change bankruptcy outcomes. In the other equilibrium they all refrain from doing so, which implies that only agents with medium signals (between $c\omega$ and p) would invest, which is not a monotone equilibrium. Therefore when $p > c\omega$, the uniqueness of the equilibrium survives.

Consider now the case when $p < c\omega$. In this case, agents with signals above p become more informed due to short-sale constraints. Those with signals above c will become more certain about what others do, which only reinforces their willingness to invest. The main difference is that now creditors with signals $p < t_j < c\omega$, who would have stayed out in absence of the constraint, obtain more precise information. Therefore strategic uncertainty weakens among these creditors, and they all become more informed about both the fundamental θ and about the beliefs of other creditors who have signal realizations between p and $c\omega$. Self-fulfilling beliefs and the resulting multiplicity hence arise in this group of creditors with medium realizations of the private signal. If they all stay out, we obtain an equilibrium equivalent to the informationally efficient equilibrium. However, there exist another equilibrium in which they all provide capital. In this second equilibrium creditors rely more on the public signal.

As the second equilibrium only emerges when $p < c\omega$, the second equilibrium, with bankruptcy threshold $\theta^* = p$ is closer to the first best ($\theta^* = 0$) than the informationally efficient equilibrium ($\theta^* = c\omega$). I conclude that short-sale constraints improve economic efficiency by mitigating the adverse effect of the coordination externality. In contrast to Morris and Shin (2002), who show that an increase in transparency might decrease welfare, short-sale constraints provide a 'good type' of transparency, recreating multiplicity only when it is desirable.[21]

7.3 Comparative statics and policy implications

The two main parameters of the coordination game are the proportion of informed investors barred from shorting, w, and creditors' private cost of providing capital, c.

As motivated in Section 2, the interpretation of w is quite broad. Here I focus mainly

[21] This is clearly not a welfare analysis of the whole economy, which would have to take into account that short-sale constraints compromise market liquidity and price discovery, and certainly make constrained informed investors worse off.

on regulatory restrictions such as a short-sale ban, the uptick rule, or legal restrictions on institutional trading. As shown in Section 6, w only affects the lower threshold for existence of the high investment equilibrium, through influencing

$$\beta = \frac{A^2}{(1-c)\, B^2 + cA^2},$$

which simplifies to

$$\beta = \frac{(1-w)^2}{(1-c) + c\,(1-w)^2}.$$

It is easy to verify that

$$\frac{\partial \beta}{\partial w} = -\frac{2\,(1-w)\,(1-c)}{\left[(1-c) + c\,(1-w)^2\right]^2} < 0,$$

i.e. tighter short-selling constraints lead to a higher probability of multiple equilibria. One interpretation of this multiplicity in the bankruptcy outcome is an increase in ex ante uncertainty about the outcome of the coordination, which can be interpreted as undesirable excess volatility. Clearly, to make predictions about the impact of certain policy measures, one needs to be able to find robust patterns across certain equilibria, as in Angeletos and Pavan (2013).

The other crucial parameter of the coordination model is the private cost of capital provision, c. Parameter c affects the net benefit or loss for creditors if they choose the risky action. Clearly, a higher c makes capital provision less desirable from the point of view of creditors, which implies that in the informationally efficient equilibrium, with $\theta^* = c\omega$ in the limit, banks receive less capital and hence they need a higher asset quality to remain solvent. The effect on the lower threshold for a high investment equilibrium, $\beta c\omega$, is more subtle. After some simple algebra one finds that

$$\frac{\partial \beta}{\partial c} = (1-w)^2\, \frac{1-(1-w)^2}{\left[1-c+c\,(1-w)^2\right]^2} \geq 0,$$

which also implies that as long as short-selling is restricted, i.e. $w > 0$, $\beta c\omega$ increases in c.

Moreover, one can characterize the benefit of short-sale constraints by the proportion of additional banks that get financed, $c\omega - \beta c\omega = (1-\beta)\, c\omega$, which can be interpreted as the ex ante probability of multiple equilibria. Here the lower bound, $\beta c\omega$ decreases in w, hence tighter short-sale constraints increase the benefits in the real economy. Furthermore, it satisfies

$$\frac{\partial}{\partial c}\left(\left(1-\beta\right)c\omega\right)=\left[\left(\frac{1-c}{c}\right)^{2}-\left(1-w\right)^{2}\right]\omega\frac{1-\left(1-w\right)^{2}}{\left[\frac{1-c}{c}+\left(1-w\right)^{2}\right]^{2}},$$

which implies an inverse U-shaped relationship. For small c values the derivative is positive, hence the ex ante probability of multiplicity, or the potential benefit of short-sale constraints, increases, while for c close to 1, this benefit decreases.

Finally, as the analysis of the previous section shows that tighter short-sale constrains can promote allocational efficiency, one can reflect on the short-sale bans around the globe in late 2008 and the following year. In fact, a sudden jump in c, implied for example by news that the investment opportunity worsens, increases the bankruptcy threshold for the bank. Introducing strict enough shorting restrictions, by increasing w, can create a second equilibrium and hence partly offset the increase in c. Empirical studies about the effect of short-sale bans in and after 2008, such as Boehmer et al. (2013) and Beber and Pagano (2013), conclude that if the SEC's and other regulators' goal with the short-sale ban was to artificially raise prices on financial stocks, they failed, and in the meantime compromised market quality. However, the SEC might have just been trying to avert a credit run on the largest investment banks. My model shows that while short-sale constraints increase market volatility, they can also affect the information that agents learn from prices, and can lead to outcomes where creditors do not withdraw money from low quality banks. Washington Mutual and Wachovia did go bankrupt during the 3-week shorting ban, collapsing under the weight of their bad loans, suggesting that their fundamentals were below the threshold $\beta c\omega$. But, while it is now clear that other financial firms such as Citigroup had extremely troubled fundamentals, the introduction of short-sale constraints could have contributed to their survival.

8 Concluding remarks

The model presented in this paper examines the informational effects of short-sale constraints when asset prices provide guidance for decisions made in a coordination environment. I present a model that shows although short-selling constraints make asset prices more volatile and decrease price informativeness, they can provide more information for certain agents of the economy, who are endowed with additional private information too. Due to learning more in presence of short-sale constraints, creditors with moderate private signals are willing to lend more, if they think others with similar signals lend as well, which leads to a second equilibrium with higher allocational efficiency. My result thus implies that the decrease in average informativeness is more than compensated by an increase in informativeness to some agents.

The existing literature studying the effects of short-sale constraints identifies both benefits and detriments of these restrictions. The first group include prevention from speculative shorting that otherwise could lead to bear raids. On the other hand, introducing a ban on short-selling has been shown to decrease market liquidity and reduce price informativeness. In this paper, I show that, allowing for a richer structure than in previous models, short-sale constraints can increase the information content of market prices. Although it leads to informational inefficiency in capital provision, it can increase allocative efficiency and prevent financial institutions from collapsing in uncertain times, when a fear of distress prevents creditors to roll over short-term debt. In particular, short-sale constraints improve the information of creditors with private signals above the market price realization. If this increase in precision is strong enough, short-sale constraints can create a second equilibrium in which creditors provide more capital, leading to less severe credit runs. My model hence suggests that emergency orders such as the one in September 2008 can increase efficiency even in absence of manipulative shorting, if the foregone costs of a potential collapse of part of the banking industry and systemic risk (i.e. the increase in allocational efficiency in the real economy) are large enough to dominate the costs of compromised market quality (i.e. the fall in informational efficiency in the financial market) in troubled times.

The model considered here studies information aggregation in a coordination game, with an external public signal emerging from a market subject to trading constraints. A more straightforward way to study information aggregation and portfolio restrictions would be to assume that investors with dispersed information are actually participants in the market, and hence the market price aggregates their information in presence of the short-selling constraint. Such a model must be more complicated because of the dual role of the price (for the inference and market clearing), but the present analysis suggests that it could shed more light on the interaction between asymmetric information and portfolio constraints.

References

Almazan, A., K. C. Brown, M. Carlson, and D. A. Chapman (2004). Why constrain your mutual fund manager? *Journal of Financial Economics 73*, 289–321.

Angeletos, G.-M., C. Hellwig, and A. Pavan (2006). Signaling in a global game: coordination and policy traps. *Journal of Political Economy 114*, 452–484.

Angeletos, G.-M. and A. Pavan (2013). Selection-free predictions in global games with endogenous information and multiple equilibria. *Theoretical Economics 8*, 883–938.

Angeletos, G.-M. and I. Werning (2006). Crises and prices: information aggregation, multiplicity, and volatility. *American Economic Review 96*, 1720–1736.

Asquith, P., R. Gertner, and D. Scharfstein (1994). Anatomy of financial distress: an examination of junk-bond issuers. *Quarterly Journal of Economics 109*, 625–658.

Atkeson, A. (2001). Discussion on Morris and Shin. *NBER Macroeconomics Annual (2000)*, 162–171.

Bai, Y., E. C. Chang, and J. Wang (2006). Asset prices under short-sale constraints. Working paper, MIT.

Barlevy, G. and P. Veronesi (2003). Rational panics and stock market crashes. *Journal of Economic Theory 110*, 234–263.

Beber, A. and M. Pagano (2013). Short-selling bans around the world: evidence from the 2007-09 crisis. *Journal of Finance 68*, 343–381.

Boehmer, E., C. M. Jones, and X. Zhang (2013). Shackling short sellers: the 2008 shorting ban. *Review of Financial Studies 26*, 1363–1400.

Bris, A., W. N. Goetzmann, and N. Zhu (2007). Efficiency and the bear: short sales and markets around the world. *Journal of Finance 62*, 1029–1079.

Brunner, A. and J. P. Krahnen (2008). Multiple lenders and corporate distress: evidence on debt restructuring. *Review of Economic Studies 75*, 415–442.

Brunnermeier, M. K. (2009). Deciphering the liquidity and credit crunch 2007-2008. *Journal of Economic Perspectives 23*, 77–100.

Carlsson, E. and E. van Damme (1993). Global games and equilibrium selection. *Econometrica 61*, 989–1018.

Chamley, C. (1999). Coordinating regime switches. *Quarterly Journal of Economics 114*, 869–905.

Chen, Q., I. Goldstein, and W. Jiang (2010). Payoff complementarities and financial fragility - evidence from mutual fund outflows. *Journal of Financial Economics 97*, 239–262.

Dasgupta, A. (2007). Coordination and delay in global games. *Journal of Economic Theory 134*, 195–225.

Diamond, D. W. and P. H. Dybvig (1983). Bank runs, deposit insurance, and liquidity. *Journal of Political Economy 91*, 401–419.

Diamond, D. W. and R. E. Verrecchia (1987). Constraints on short-selling and asset price adjustment to private information. *Journal of Financial Economics 18*, 277–311.

Duffie, D., N. Garleanu, and L. H. Pedersen (2002). Securities lending, shorting, and pricing. *Journal of Financial Economics 66*, 307–339.

Financial Services Authority (2009). Short selling. Discussion paper.

Gilchrist, S., C. P. Himmelberg, and G. Huberman (2005). Do stock price bubbles influence corporate investment? *Journal of Monetary Economics 52*, 805–827.

Goldstein, I. and A. Guembel (2008). Manipulation and the allocational role of prices. *Review of Economic Studies 75*, 133–164.

Goldstein, I., E. Ozdenoren, and K. Yuan (2013). Trading frenzies and their impact on real investment. *Journal of Financial Economics 109*, 566–582.

Goldstein, I. and A. Pauzner (2005). Demand-deposit contracts and the probability of bank runs. *Journal of Finance 60*, 1293–1327.

Grossman, S. J. and J. E. Stiglitz (1980). On the impossibility of informationally efficient markets. *American Economic Review 70*, 393–408.

Harrison, J. M. and D. M. Kreps (1978). Speculative investor behavior in a stock market with heterogeneous expectations. *Quarterly Journal of Economics 92*, 323–336.

Hellwig, C. (2002). Public information, private information, and the multiplicity of equilibria in coordination games. *Journal of Economic Theory 107*, 191–222.

Hellwig, C., A. Mukherji, and A. Tsyvinski (2006). Self-fulfilling currency crises: the role of interest rates. *American Economic Review 96*, 1769–1787.

Hertzberg, A., J. M. Liberti, and D. Paravisini (2010). Public information and coordination: evidence from a credit registry expansion. *Journal of Finance 66*, 379–412.

Ho, K. W. (1996). Short-sales restrictions and volatility: the case of the stock exchange of singapore. *Pacific-Basin Finance Journal 4*, 377–391.

Hong, H. and J. Stein (2003). Differences of opinion, short-sales constraints, and market crashes. *Review of Financial Studies 16*, 377–391.

Ivashina, V. and D. S. Scharfstein (2010). Bank lending during the financial crisis of 2008. *Journal of Financial Economics 97*, 319–338.

Jones, C. M. and O. A. Lamont (2002). Short sale constraints and stock returns. *Journal of Financial Economics 97*, 319–338.

Liu, X. (2015). Short-selling attacks and creditor runs. *Management Science 61*, 814–830.

Marin, J. M. and J. P. Olivier (2008). The dog that did not bark: insider trading and crashes. *Journal of Finance 63*, 2429–2476.

Miller, E. M. (1977). Risk, uncertainty, and divergence of opinion. *Journal of Finance 32*, 1151–1168.

Morris, S. and H. S. Shin (1998). Unique equilibrium in a model of self-fulfilling currency attacks. *American Economic Review 88*, 587–597.

Morris, S. and H. S. Shin (1999). Private versus public information in coordination problems. Working paper, Yale University and University of Oxford.

Morris, S. and H. S. Shin (2001). Rethinking multiple equilibria in macroeconomics. *NBER Macroeconomics Annual (2000)*, 139–161.

Morris, S. and H. S. Shin (2002). The social value of public information. *American Economic Review 92*, 1521–1534.

Morris, S. and H. S. Shin (2003). Global games: theory and applications. In *Advances in Economics and Econometrics: Theory and Applications, Eighth World Congress.* Cambridge University Press, Cambridge, UK.

Morris, S. and H. S. Shin (2004). Coordination risk and the price of debt. *European Economic Review 48*, 133–153.

Nagel, S. (2005). Short sales, institutional investors and the cross-section of stock returns. *Journal of Financial Economics 75*, 277–309.

Obstfeld, M. (1986). Rational and self-fulfilling balance-of-payments crises. *American Economic Review 76*, 72–81.

Obstfeld, M. (2004). Models of currency crises with self-fulfilling features. *European Economic Review 40*, 1037–1047.

Ofek, E. and M. Richardson (2003). DotCom mania: the rise and fall of internet stock prices. *Journal of Finance 58*, 1113–1137.

Ozdenoren, E. and K. Yuan (2008). Feedback effects and asset prices. *Journal of Finance 63*, 1939–1975.

Panageas, S. (2003). Speculation, overpricing, and investment - theory and empirical evidence. Working paper, MIT.

Reed, A. V. (2007). Costly short selling and stock price adjustment to earnings announcements. Working paper, University of North Carolina.

Rochet, J.-C. and X. Vives (2004). Coordination failures and the lender of last resort: was bagehot right after all? *Journal of the European Economic Association 2*, 1116–1147.

Rubinstein, M. E. (2004). Great moments in financial economics: III. Short-sales and stock prices. *Journal of Investment Management 2*.

Scheinkman, J. A. and W. Xiong (2003). Overconfidence and speculative bubbles. *Journal of Political Economy 111*, 1183–1219.

Shin, H. S. (2009). Reflections on northern rock: the bank run that heralded the global financial crisis. *Journal of Economic Perspectives 23*, 101–119.

Shleifer, A. and R. W. Vishny (1997). The limits of arbitrage. *Journal of Finance 52*, 35–55.

Tarashev, N. A. (2007). Speculative attacks and the information role of the interest rate. *Journal of the European Economic Association 5*, 1–36.

Yuan, K. (2005). Asymmetric price movements and borrowing constraints: a REE model of crises, contagion, and confusion. *Journal of Finance 60*, 247–292.

Yuan, K. (2006). The price impact of borrowing and short-sale constraints. Working paper, University of Michigan.

A REE in the financial market

Optimal demands. Investor k's optimization problem is given by

$$\begin{aligned}\max_{x_k} U(W_k) &= E[W_k|\mathcal{I}_k] - \frac{\rho}{2} Var[W_k|\mathcal{I}_k] \\ &= x_k(E[f|\mathcal{I}_k] - p) - \frac{\rho}{2} x_k^2 \left(Var[f|\mathcal{I}_k] + \sigma_n^2\right).\end{aligned}$$

Solving the FOC without short-sale constraints, one obtains

$$x_k = \frac{E[f|\mathcal{I}_k] - p}{\rho(Var[f|\mathcal{I}_k] + \sigma_n^2)}.$$

From here the optimal demands for all three types of traders are straightforward. ■

Proof of Theorem 2. The derivation in the main text provides the step-by-step solution to the problem. There are three issues left for this appendix: (i) to derive the conditional expectations $E[f|p]$ and $E[f^2|p]$, and the conditional variance $Var[f|p]$, (ii) to prove the existence of the equilibrium, and (iii) to analyze uniqueness.

(i) The conditional distribution (9) implies that the expectation simply becomes

$$E[f|P=p] = \int_{-\infty}^{p} \frac{1}{B\sigma_u} f\phi\left(\frac{f-(p+BC)}{B\sigma_u}\right) df + \int_{p}^{\infty} \frac{1}{A\sigma_u} f\phi\left(\frac{f-(p+AC)}{A\sigma_u}\right) df = p + D,$$

where

$$\begin{aligned}D \equiv B\sigma_u \int_{-\infty}^{-\frac{C}{\sigma_u}} \left(v + \frac{C}{\sigma_u}\right)\phi(v)\,dv + A\sigma_u \int_{-\frac{C}{\sigma_u}}^{\infty} \left(v + \frac{C}{\sigma_u}\right)\phi(v)\,dv \\ = AC - (B-A)\sigma_u \int_{\frac{C}{\sigma_u}}^{\infty} \left(w - \frac{C}{\sigma_u}\right)\phi(w)\,dw,\end{aligned}$$

and similarly

$$E\left[f^2|p\right] = \int_{-\infty}^{C} [p - A(u-C)]^2 \phi(u)\,du + \int_{C}^{\infty} [p - B(u-C)]^2 \phi(u)\,du = p^2 + 2pD + E,$$

with

$$E \equiv A^2 \int_{-\infty}^{C} (u - C)^2 \phi(u)\, du + B^2 \int_{C}^{\infty} (u - C)^2 \phi(u)\, du$$

$$= A^2 \left[\sigma_u^2 + C^2\right] + \left(B^2 - A^2\right) \int_{C}^{\infty} (u - C)^2 \phi(u)\, du.$$

Therefore

$$Var\left[f|p\right] = E\left[f^2|p\right] - E^2\left[f|p\right] = E - D^2,$$

that is independent of p. From here, the calculations end in the main text, and C solves

$$0 = L(C) \equiv C + \frac{1-\lambda}{\rho} \frac{D(C)}{E(C) - D^2(C) + \sigma_n^2} - S. \tag{27}$$

Consider the case when $w = 0$, that is $B_0 = A$; it implies that

$$D_0 = AC_0 \text{ and } E_0 = A^2\left[\sigma_u^2 + C_0^2\right],$$

therefore

$$Var\left[f|p_0\right] = E_0 - D_0^2 = A^2\sigma_u^2,$$

and hence

$$C_0 = \frac{\lambda + \rho A \sigma_u^2}{1 + \rho A \sigma_u^2} S = kS,$$

where $0 \leq k \leq 1$. As $S \geq 0$, we also get that $C_0 \geq 0$. In particular, if the asset is in positive net supply, $S > 0$, C_0 is positive, and $D_0 = AC_0 > 0$, which means that uninformed investors demand a discount of $D_0 > 0$ to hold the asset.

(ii) To show the existence of a real C that satisfies $L(C) = 0$, notice that when $B > A$, for $C = C_0$,

$$L(C_0) = C_0 + \frac{1-\lambda}{\rho} \frac{D(C_0)}{E(C_0) - D^2(C_0) + \sigma_n^2} - S$$

$$< -\frac{1-\lambda}{\rho} \frac{B - A}{E(C_0) - D^2(C_0) + \sigma_n^2} \int_{C_0}^{\infty} (u - C_0)\phi(u)\, du < 0.$$

Moreover, from (27) one can rewrite $L(C)$ as

$$L(C) = C + \frac{1-\lambda}{\rho}\frac{D(C)}{E(C) - D^2(C) + \sigma_n^2} - S$$
$$= \frac{\lambda Var[f|p] + \sigma_n^2}{\lambda Var[f|p] + \lambda\sigma_n^2}C - \frac{1-\lambda}{\rho}\frac{B-A}{Var[f|p] + \sigma_n^2}\int_C^\infty (u-C)\,\phi(u)\,du - S,$$

where S is constant, $Var[f|p]$ is finite (see below in the proof of Proposition 3), and for $C \to \infty$, $\int_C^\infty (u-C)\,\phi(u)\,du \to 0$. Therefore,

$$\lim_{C\to\infty} L(C) = \infty. \tag{28}$$

As $L(C)$ is continuos, combining it with $L(C_0) < 0$ and (28), it must have a real root above C_0.

(iii) For the proof of uniqueness, notice first that for every $C < C_0$, $D(C) < D(C_0)$ and $Var[f|p] > Var[f|p_0]$, therefore

$$L(C) < L(C_0) < 0.$$

Thus, there is no such $C < C_0$ that satisfies $L(C) = 0$. Regarding the case $C > C_0$, simple algebra shows that

$$\frac{dD}{dC} = A - (B-A)\frac{d}{dC}\int_C^\infty (u-C)\,\phi(u)\,du > A > 0,$$

and

$$E - D^2 = A^2\left[\sigma_u^2 + C^2\right] + \left(B^2 - A^2\right)\int_C^\infty (u-C)^2\,\phi(u)\,du$$
$$= A^2\sigma_u^2 + 2(B-A)A\int_C^\infty u(u-C)\,\phi(u)\,du + (B-A)^2\,Var[\max\{0, u-C\}],$$

hence

$$\frac{d}{dC}\left[E - D^2\right] < 0.$$

Therefore, $\frac{D(C)}{E(C)-(D(C))^2+\sigma_n^2}$ is increasing in C, and

$$\frac{d}{dC}L(C) = 1 + \frac{d}{dC}\frac{D(C)}{\rho\left[E(C) - (D(C))^2\right] + A} > 1.$$

As $L(C)$ is strictly increasing and continuous, the $L(C)=0$ equation must have a unique solution. ■

Proof of Proposition 3. From (10),

$$
\begin{aligned}
Var[f|p] &= E - D^2 \\
&= A^2\sigma_u^2 + \left(B^2 - A^2\right)\left[\int_C^\infty (u-C)^2\,\phi(u)\,du - \left(\int_C^\infty (u-C)\,\phi(u)\,du\right)^2\right] \\
&\quad + 2(B-A)AC\int_C^\infty (u-C)\,\phi(u)\,du + 2A(B-A)\left(\int_C^\infty (u-C)\,\phi(u)\,du\right)^2,
\end{aligned}
$$

where the second term of the RHS is nonnegative due to Jensen's inequality applied on the random variable $w \equiv \max\{0, u-C\}$ and the convex function $x \mapsto x^2$: $E[w^2] \geq E^2[w]$, and the third and fourth components are trivially non-negative too. Therefore

$$Var[f|p] \geq A^2\sigma_u^2 = Var[f|p_0]. \tag{29}$$

■

Proof of Proposition 4. Due to the improper prior assumption, it is easy to see that $Var[p|f] = Var[f|p] = E - D^2$ and $Var[p_0|f] = Var[f|p_0] = A^2\sigma_u^2$, hence (29) also implies that $Var[p|f] \geq Var[p_0|f]$. ■

Proof of Proposition 5. As $r_0 = f - p_0 = A(u - C_0)$, where u has a symmetric distribution around 0, which implies $Skew[r_0] = 0$. Therefore, to have $Skew[r] < Skew[r_0]$, it is sufficient to show $E\left[(r - E[r])^3\right] < 0$. From its definition,

$$r = f - p = -\begin{cases} A(u-C) & \text{if } u \leq C \\ B(u-C) & \text{if } u > C \end{cases},$$

and

$$E\left[(r-E[r])^3\right] = E\left[r^3\right] - 3E\left[r^2\right]E[r] + 2E^3[r]$$

where

$$\begin{aligned}
E[r] &= D = AC - (B-A)\int_C^{\infty}(u-C)\,\phi(u)\,du, \\
E\left[r^2\right] &= E = A^2\left(\sigma_u^2 + C^2\right) + \left(B^2 - A^2\right)\int_C^{\infty}(u-C)^2\,\phi(u)\,du, \text{ and} \\
E\left[r^3\right] &= A^3C\left(3\sigma_u^2 + C^2\right) - \left(B^3 - A^3\right)\int_C^{\infty}(u-C)^3\,\phi(u)\,du.
\end{aligned}$$

After some tedious algebra, the negativity of the skewness follows from the fact that the skewness of the random variable $w = \max\{0, u-C\}$ is positive.

To prove the second part of the proposition, notice that

$$E[|r_0|] = E[A|u|] = 2A\int_0^{\infty}\max\{u, C_0\}\,\phi(u)\,du$$

and

$$\begin{aligned}
E[|r|] &= B\int_C^{\infty}(u-C)\,\phi(u)\,du - A\int_{-\infty}^{C}(u-C)\,\phi(u)\,du \\
&= (B+A)\int_C^{\infty}u\phi(u)\,du - (B-A)\,C\int_C^{\infty}\phi(u)\,du + 2AC\int_0^{C}\phi(u)\,du,
\end{aligned}$$

thus simple algebra yields

$$\begin{aligned}
E[|r|] - E[|r_0|] = (B-A)\int_C^{\infty}(u-C)\,\phi(u)\,du& \\
+\, 2A\int_0^{\infty}\left(\max\{u, C\} - \max\{u, C_0\}\right)\phi(u)\,du&.
\end{aligned}$$

On the right-hand side both components are non-negative, therefore $E[|r|] \geq E[|r_0|]$. ■
Proof of Proposition 6. As $R = p - p^{-1} = f - r - p^{-1}$, $Skew[R|f] = Skew[-r]$ and $Skew[R_0|f] = Skew[-r_0]$, hence $Skew[R|f] > Skew[R_0|f]$ is straightforward from $Skew[r] < Skew[r_0]$. ■

B Information content of the price under short-sale constraints

The posterior of a creditor with private signal t and price signal p that comes from a financial market with short-sale constraints is given by

$$g\left(f|t,P=p\right)=\pi\cdot g\left(f|t,P=p,f<p\right)+\left(1-\pi\right)\cdot g\left(f|t,P=p,f\geq p\right),$$

where, using the simplifying notation

$$\Phi_1=\Phi\left(\frac{p-\frac{\tau_t t+\tau_{Bu}(p+BC)}{\tau_t+\tau_{Bu}}}{\left(\tau_{Bu}+\tau_t\right)^{-1/2}}\right)\text{ and }\Phi_2=1-\Phi\left(\frac{p-\frac{\tau_t t+\tau_{Au}(p+AC)}{\tau_t+\tau_{Au}}}{\left(\tau_{Au}+\tau_t\right)^{-1/2}}\right),$$

the variable

$$\begin{aligned}\pi &= \Pr\left(f<p|t,P=p\right)\\ &= \frac{\frac{\left(\tau_{Bu}+\tau_t\right)^{-1/2}}{\tau_t^{-1}\tau_{Bu}^{-1}}\phi\left(\frac{t-(p+BC)}{\left(\tau_{Bu}^{-1}+\tau_t^{-1}\right)^{1/2}}\right)\Phi_1}{\frac{\left(\tau_{Bu}+\tau_t\right)^{-1/2}}{\tau_t^{-1}\tau_{Bu}^{-1}}\phi\left(\frac{t-(p+BC)}{\left(\tau_{Bu}^{-1}+\tau_t^{-1}\right)^{1/2}}\right)\Phi_1+\frac{\left(\tau_{Au}+\tau_t\right)^{-1/2}}{\tau_t^{-1}\tau_{Au}^{-1}}\phi\left(\frac{t-(p+AC)}{\left(\tau_{Au}^{-1}+\tau_t^{-1}\right)^{1/2}}\right)\Phi_2}\end{aligned}$$

gives the probability that the creditor assigns to the constraint binding in the financial market, $\Pr\left(f\geq p|t,P=p\right)=1-\pi$, and the conditional distributions are given by

$$g\left(f|t,P=p,f<p\right)=1_{f<p}\frac{1}{\left(\tau_{Bu}+\tau_t\right)^{-1/2}\Phi_1}\phi\left(\frac{f-\frac{\tau_t t+\tau_{Bu}(p+BC)}{\tau_t+\tau_{Bu}}}{\left(\tau_{Bu}+\tau_t\right)^{-1/2}}\right)$$

and

$$g\left(f|t,P=p,f\geq p\right)=1_{f\geq p}\frac{1}{\left(\tau_{Au}+\tau_t\right)^{-1/2}\Phi_2}\phi\left(\frac{f-\frac{\tau_t t+\tau_{Au}(p+AC)}{\tau_t+\tau_{Au}}}{\left(\tau_{Au}+\tau_t\right)^{-1/2}}\right).$$

When no informed trader is subject to the short-sale constraint, i.e. $w=0$ or $B=A$, both truncated normal pdfs belong to the same normal distribution, with mean $\frac{\tau_t}{\tau_t+\tau_{Au}}t+\frac{\tau_{Au}}{\tau_t+\tau_{Au}}\left(p+AC_0\right)$ and precision $\tau_t+\tau_{Au}$, and the probabilities simplify to

$$\begin{aligned}\Pr\left(f<p|t,P_0=p\right) &= \Phi\left(\frac{p-\frac{\tau_t t+\tau_{Bu}(p+AC_0)}{\tau_t+\tau_{Au}}}{\left(\tau_{Bu}+\tau_t\right)^{-1/2}}\right)\text{ and}\\ \Pr\left(f\geq p|t,P_0=p\right) &= 1-\Phi\left(\frac{p-\frac{\tau_t t+\tau_{Bu}(p+AC_0)}{\tau_t+\tau_{Au}}}{\left(\tau_{Bu}+\tau_t\right)^{-1/2}}\right).\end{aligned}$$

C Global game solution under short-sale constraints

C.1 General notations

As shown in Section 4, in presence of short-sale constraints the posterior pdf becomes

$$g(\theta|t, P = p) = \pi \cdot g(\theta|t, P = p, \theta < p) + (1 - \pi) \cdot g(\theta|t, P = p, \theta \geq p),$$

hence a simple integration yields that

$$G(x|t, P = p) = \pi \cdot G(x|t, P = p, \theta < p) \text{ if } x \leq p,$$

and

$$G(x|t, P = p) = \pi \cdot G(p|t, P = p, \theta < p) + (1 - \pi) \cdot G(x|t, P = p, \theta \geq p) \text{ if } x > p.$$

Combining it with the indifference condition $\Pr(\theta \geq \theta^* | t^*, P = p) = c$ gives

$$1 - c = \pi \cdot G(\theta^* | t^*, P = p, \theta < p) \text{ if } \theta^* \leq p,$$

and

$$1 - c = \pi \cdot G(p|t^*, P = p, \theta < p) + (1 - \pi) \cdot G(\theta^* | t^*, P = p, \theta \geq p) \text{ if } \theta^* > p,$$

which leads to (24) with the notation

$$\pi_A^* = \Phi\left(\frac{p - \frac{\tau_t t^* + \tau_{Au}(p + AC)}{\tau_t + \tau_{Au}}}{(\tau_t + \tau_{Au})^{-1/2}}\right) \text{ and } \pi_B^* = \Phi\left(\frac{p - \frac{\tau_t t^* + \tau_{Bu}(p + BC)}{\tau_{Bu} + \tau_t}}{(\tau_t + \tau_{Bu})^{-1/2}}\right)$$

and

$$\begin{aligned}
\pi^* &\equiv \Pr(\theta < p | t^*, p) \\
&= \frac{\frac{(\tau_{Bu}+\tau_t)^{-1/2}}{\tau_t^{-1}\tau_{Bu}^{-1}} \phi\left(\frac{t^* - (p+BC)}{\left(\tau_{Bu}^{-1} + \tau_t^{-1}\right)^{1/2}}\right) \pi_B^*}{\frac{(\tau_{Bu}+\tau_t)^{-1/2}}{\tau_t^{-1}\tau_{Bu}^{-1}} \phi\left(\frac{t^* - (p+BC)}{\left(\tau_{Bu}^{-1} + \tau_t^{-1}\right)^{1/2}}\right) \pi_B^* + \frac{(\tau_{Au}+\tau_t)^{-1/2}}{\tau_t^{-1}\tau_{Au}^{-1}} \phi\left(\frac{t^* - (p+AC)}{\left(\tau_{Au}^{-1} + \tau_t^{-1}\right)^{1/2}}\right) [1 - \pi_A^*]}.
\end{aligned}$$

C.2 Multiplicity

First, I characterize the critical mass and individual optimality curves, (22) and (24), respectively. It is easy to see that both of them imply θ^* is a continuous and strictly increasing function of t^*.

Condition (22) yields that the slope of the critical mass curve (CM, for simplicity) is given by

$$\frac{dt^*}{d\theta^*} = 1 + \frac{1}{\sqrt{\tau_t}\omega} \frac{1}{\phi\left(\Phi^{-1}\left(\frac{\theta^*}{\omega}\right)\right)},$$

or by its inverse

$$\delta_{CM} \equiv \frac{d\theta^*}{dt^*} = \frac{1}{1 + \frac{1}{\sqrt{\tau_t}\omega} \frac{1}{\phi\left(\Phi^{-1}\left(\frac{\theta^*}{\omega}\right)\right)}}$$

for the θ^* that solves

$$t^* = \theta^* + \frac{1}{\sqrt{\tau_t}} \Phi^{-1}\left(\frac{\theta^*}{\omega}\right).$$

Thus, δ_{CM} can be interpreted as a function of t^*. In particular, as $0 < \phi(x) < 1/\sqrt{2\pi}$ for every $x \in R$, we have a lower and an upper threshold for the slope:

$$0 < \delta_{CM} < \overline{\delta}_{CM} \equiv \frac{1}{1 + \frac{1}{\sqrt{\tau_t}\omega}\sqrt{2\pi}},$$

and it reaches its maximum for

$$\theta^* = \frac{\omega}{2},$$

or for the t^* value of

$$t^* = \frac{\omega}{2}.$$

Moreover, as $\phi(x)$ is strictly increasing for $x < 0$ and strictly decreasing for $x > 0$, and θ^* is an increasing function of t^*, the slope δ_{CM} is strictly increasing in t^* for $t^* < \frac{\omega}{2}$ and strictly decreasing for $t^* > \frac{\omega}{2}$. Therefore, δ_{CM} takes every value in $\left(0, \overline{\delta}_{CM}\right]$ for $t^* \leq \frac{\omega}{2}$, and every value between $\overline{\delta}_{CM}$ and 0, when t^* increases from $\frac{\omega}{2}$ to ∞, where $\overline{\delta}_{CM} < 1$.

Now I turn my attention to the individual optimality curve (IO, for simplicity), given in (24), which, for tractability, is restated here:

$$\theta^* = \begin{cases} \frac{\tau_t}{\tau_t + \tau_{Bu}} t^* + \frac{\tau_{Bu}}{\tau_t + \tau_{Bu}} (p + BC) + \frac{1}{\sqrt{\tau_t + \tau_{Bu}}} \Phi^{-1}\left((1-c)\frac{\pi_B^*(p,t^*)}{\pi^*(p,t^*)}\right) & \text{if } \theta^* \leq p \\ \frac{\tau_t}{\tau_t + \tau_{Au}} t^* + \frac{\tau_{Au}}{\tau_t + \tau_{Au}} (p + AC) + \frac{1}{\sqrt{\tau_t + \tau_{Au}}} \Phi^{-1}\left(1 - c\frac{1-\pi_A^*(p,t^*)}{1-\pi^*(p,t^*)}\right) & \text{if } \theta^* > p. \end{cases}$$

The first observation I make is that this condition can be rewritten with the introduction of $\Delta\theta \equiv \theta^* - p$ and $\Delta t \equiv t^* - p$:

$$\Delta\theta = \begin{cases} \frac{\tau_t}{\tau_t + \tau_{Bu}} \Delta t + \frac{\tau_{Bu}}{\tau_t + \tau_{Bu}} BC + \frac{1}{\sqrt{\tau_t + \tau_{Bu}}} \Phi^{-1}\left((1-c)\frac{\pi_B^*(\Delta t)}{\pi^*(\Delta t)}\right) & \text{if } \Delta\theta \leq 0 \\ \frac{\tau_t}{\tau_t + \tau_{Au}} \Delta t + \frac{\tau_{Au}}{\tau_t + \tau_{Au}} AC + \frac{1}{\sqrt{\tau_t + \tau_{Au}}} \Phi^{-1}\left(1 - c\frac{1-\pi_A^*(\Delta t)}{1-\pi^*(\Delta t)}\right) & \text{if } \Delta\theta > 0, \end{cases} \tag{30}$$

where

$$\pi_A^*(\Delta t) = \Phi\left(-\frac{\tau_t \Delta t + \tau_{Au} AC}{(\tau_t + \tau_{Au})^{1/2}}\right) \text{ and } \pi_B^*(\Delta t) = \Phi\left(-\frac{\tau_t \Delta t + \tau_{Bu} BC}{(\tau_t + \tau_{Bu})^{1/2}}\right), \tag{31}$$

and

$$\pi^*(\Delta t) = \\ = \frac{\frac{(\tau_{Bu}+\tau_t)^{-1/2}}{\tau_t^{-1}\tau_{Bu}^{-1}}\phi\left(\frac{\Delta t - BC}{\left(\tau_{Bu}^{-1}+\tau_t^{-1}\right)^{1/2}}\right)\pi_B^*(\Delta t)}{\frac{(\tau_{Bu}+\tau_t)^{-1/2}}{\tau_t^{-1}\tau_{Bu}^{-1}}\phi\left(\frac{\Delta t - BC}{\left(\tau_{Bu}^{-1}+\tau_t^{-1}\right)^{1/2}}\right)\pi_B^*(\Delta t) + \frac{(\tau_{Au}+\tau_t)^{-1/2}}{\tau_t^{-1}\tau_{Au}^{-1}}\phi\left(\frac{\Delta t - AC}{\left(\tau_{Au}^{-1}+\tau_t^{-1}\right)^{1/2}}\right)\left[1-\pi_A^*(\Delta t)\right]}. \tag{32}$$

It means that on the (t^*, θ^*) plane every solution-pair $(t^*(p), \theta^*(p))$ of (24) is given by an appropriate shift of the point $(\Delta t, \Delta\theta)$ along the 45-degree line, where Δt and $\Delta\theta$ solve $(30)-(32)$, and are only functions of the parameters of the model and the equilibrium constants of the financial market, and do not depend on p. It also implies that the slope and convexity attributes of the curve do not depend on p either.

The characterization of the solution to $(30)-(32)$ is as follows. The slope of the curve is given by

$$\delta_{IO}(\Delta t) = \begin{cases} \frac{\tau_t}{\tau_t+\tau_{Bu}} + \frac{1}{\sqrt{\tau_t+\tau_{Bu}}}\frac{d}{d(\Delta t)}\Phi^{-1}\left((1-c)\frac{\pi_B^*(\Delta t)}{\pi^*(\Delta t)}\right) & \text{if } \Delta\theta \le 0 \\ \frac{\tau_t}{\tau_t+\tau_{Au}} + \frac{1}{\sqrt{\tau_t+\tau_{Au}}}\frac{d}{d(\Delta t)}\Phi^{-1}\left(1-c\frac{1-\pi_A^*(\Delta t)}{1-\pi^*(\Delta t)}\right) & \text{if } \Delta\theta > 0, \end{cases}$$
$$= \begin{cases} \frac{\tau_t}{\tau_t+\tau_{Bu}} + \frac{(1-c)}{\sqrt{\tau_t+\tau_{Bu}}\phi\left(\Phi^{-1}\left((1-c)\frac{\pi_B^*(\Delta t)}{\pi^*(\Delta t)}\right)\right)}\frac{d}{d(\Delta t)}\left(\frac{\pi_B^*(\Delta t)}{\pi^*(\Delta t)}\right) & \text{if } \Delta t \le \Delta t_0 \\ \frac{\tau_t}{\tau_t+\tau_{Au}} - \frac{c}{\sqrt{\tau_t+\tau_{Au}}\phi\left(\Phi^{-1}\left(1-c\frac{1-\pi_A^*(\Delta t)}{1-\pi^*(\Delta t)}\right)\right)}\frac{d}{d(\Delta t)}\left(\frac{1-\pi_A^*(\Delta t)}{1-\pi^*(\Delta t)}\right) & \text{if } \Delta t > \Delta t_0. \end{cases}$$

where Δt_0 is the unique solution to

$$\pi^*(\Delta t_0) = 1-c,$$

i.e. Δt_0 is the Δt value that gives $\Delta\theta = 0$ in (31). Moreover, after some demanding calculations, omitted here, it is possible to show that:

1. $\frac{d}{d(\Delta t)}\left(\frac{\pi_B^*(\Delta t)}{\pi^*(\Delta t)}\right) > 0$ for $\Delta t \le \Delta t_0$ and $\frac{d}{d(\Delta t)}\left(\frac{1-\pi_A^*(\Delta t)}{1-\pi^*(\Delta t)}\right) < 0$ for $\Delta t > \Delta t_0$;
2. δ_{IO} is increasing in Δt for both $(-\infty, \Delta t_0)$ and $(\Delta t_0, \infty)$;
3. When $\Delta t \to \pm\infty$ we have $\lim_{\Delta t\to-\infty}\delta_{IO}(\Delta t) = \frac{\tau_t}{\tau_t+\tau_{Bu}}$ and $\lim_{\Delta t\to\infty}\delta_{IO}(\Delta t) = \frac{\tau_t}{\tau_t+\tau_{Au}}$;

4. There is a 'kink' at Δt_0, and thus the IO curve is not differentiable: on the two sides of $\Delta t = \Delta t_0$, the slopes are finite but different. In particular, I define $\alpha(\Delta t_0)$ and $\beta(\Delta t_0)$ such that $\frac{\tau_t}{\tau_t+\tau_{Bu}} < \frac{1}{\beta(\Delta t_0)} \equiv \lim_{\Delta t \to \Delta t_0 -} \delta_{IO}(\Delta t) < \infty$ and $0 < \alpha(\Delta t_0) \equiv \lim_{\Delta t \to \Delta t_0 +} \delta_{IO}(\Delta t) < \frac{\tau_t}{\tau_t+\tau_{Au}}$, are well-defined, and satisfy $\alpha(\Delta t_0) < \frac{1}{\beta(\Delta t_0)}$;

5. The slope $\delta_{IO}(\Delta t)$ is increasing in Δt and takes every value in $\left(\frac{\tau_t}{\tau_t+\tau_{Bu}}, \frac{1}{\beta(\Delta t_0)}\right)$ for $\Delta t < \Delta t_0$, and is increasing and takes every value in $\left(\alpha(\Delta t_0), \frac{\tau_t}{\tau_t+\tau_{Au}}\right)$ for $\Delta t > \Delta t_0$.

In what follows, for simplicity, I refer to this $(\Delta t_0, 0)$ point as the 'kink' of the IO curve.

The next observation is that the uniqueness or multiplicity of the solutions for a given parameter set depends only on which part of the CM curve the 'kink' of the IO condition would get shifted to. In particular, if the 'kink' is shifted to a part of the CM curve where its slope is sufficiently small such that $\delta_{CM} \leq \alpha(\Delta t_0)$, there is a unique solution. This is because both before and after the kink the IO curve is steeper than the CM curve, and hence there cannot be any more intersections. However, if at this point the slope satisfies $\delta_{CM} > \alpha(\Delta t_0)$, multiplicity can be ensured by choosing the appropriate p: if the kink is before $t^* = \frac{\omega}{2}$, as δ_{IO} starts to increase from $\alpha(\Delta t_0)$, and as δ_{CM} decreases, a slight increase in p would ensure that they have multiple equilibria, and if the kink is after $t^* = \frac{\omega}{2}$, as δ_{IO} increases from $\alpha(\Delta t_0)$, and as δ_{CM} increases too, a slight decrease in p would ensure that they have multiple equilibria. Therefore, in what follows, I solve for the point where the 'kink' gets shifted to, and determine the relationship of the two slopes δ_{CM} and δ_{IO}.

First, suppose that $\alpha(\Delta t_0) \geq \overline{\delta}_{CM}$; in this case there is a unique solution. This condition hence requires

$$\alpha(\Delta t_0) \geq 1 - \frac{\sqrt{2\pi}}{\sqrt{2\pi} + \sqrt{\tau_t}\omega}.$$

Second, suppose that $0 < \alpha(\Delta t_0) < \overline{\delta}_{CM}$. It means that there are two points of the CM curve such that the slope is exactly $\alpha(\Delta t_0)$: they are pinned down by

$$\begin{aligned}
\underline{\theta}^* &= \omega\left(1 - \Phi\left[\phi^{-1}\left(\frac{1}{\omega\sqrt{\tau_t}}\frac{\alpha(\Delta t_0)}{1-\alpha(\Delta t_0)}\right)\right]\right) < \frac{\omega}{2} \text{ and} \\
\overline{\theta}^* &= \omega\Phi\left[\phi^{-1}\left(\frac{1}{\omega\sqrt{\tau_t}}\frac{\alpha(\Delta t_0)}{1-\alpha(\Delta t_0)}\right)\right] > \frac{\omega}{2},
\end{aligned}$$

where $\phi^{-1}(y)$ denotes the unique non-negative x such that $\phi(x) = y$ for $y \leq 1/\sqrt{2\pi}$. Therefore, if the shifted kink has $\theta^* \leq \underline{\theta}^*$ or $\theta^* \geq \overline{\theta}^*$, the solution is unique, and if $\underline{\theta}^* < \theta^* < \overline{\theta}^*$, there are multiple equilibria.

As the 'kink' has coordinates $(\Delta t_0, 0)$ on the (t^*, θ^*) plane, shifting it is equivalent to moving it to the point $(p + \Delta t_0, p)$. It is thus on the CM curve if and only if

$$p + \Delta t_0 = p + \frac{1}{\sqrt{\tau_t}} \Phi^{-1} \left(\frac{p}{\omega} \right),$$

that is if it is shifted by $\widehat{p} \equiv \omega \Phi \left(\sqrt{\tau_t} \Delta t_0 \right)$ to $(\widehat{p} + \Delta t_0, \widehat{p})$. Therefore, there are multiple equilibria if and only if

$$\underline{\theta}^* < \omega \Phi \left(\sqrt{\tau_t} \Delta t_0 \right) < \overline{\theta}^*,$$

that is

$$\frac{1}{\omega \sqrt{\tau_t}} \frac{\alpha (\Delta t_0)}{1 - \alpha (\Delta t_0)} < \phi \left(\sqrt{\tau_t} \Delta t_0 \right) < \frac{1}{\sqrt{2\pi}},$$

or, equivalently,

$$\alpha (\Delta t_0) < 1 - \frac{1}{1 + \omega \sqrt{\tau_t} \phi \left(\sqrt{\tau_t} \Delta t_0 \right)}.$$

But what is exactly $\alpha (\Delta t_0) \equiv \lim_{\Delta t \to \Delta t_0 +} \delta_{IO} (\Delta t)$? Using the relevant part of the δ_{IO} function, and the fact that $\pi^* (\Delta t_0) = 1 - c$, the (upper) slope δ_{IO} close to the 'kink' becomes

$$\begin{aligned}
\alpha (\Delta t_0) = & \lim_{\Delta t \to \Delta t_0 +} \delta_{IO} (\Delta t) \\
& = (1 - c) \frac{\tau_t}{\tau_t + \tau_{Au}} + c \frac{\tau_t}{\tau_t + \tau_{Bu}} \frac{\tau_{Bu}}{\tau_{Au}} \\
& - c \frac{\tau_{Bu}}{\tau_{Au}} \frac{1}{\sqrt{\tau_t + \tau_{Bu}}} \frac{\phi \left(\frac{\Delta t_0 - BC}{\left(\tau_t^{-1} + \tau_{Bu}^{-1} \right)^{1/2}} \right)}{\phi \left(\frac{\Delta t_0 - AC}{\left(\tau_t^{-1} + \tau_{Au}^{-1} \right)^{1/2}} \right)} \frac{\left(\frac{\Delta t_0 - AC}{\tau_t^{-1} + \tau_{Au}^{-1}} - \frac{\Delta t_0 - BC}{\tau_t^{-1} + \tau_{Bu}^{-1}} \right) \Phi \left(- \frac{\tau_t \Delta t_0 + \tau_{Bu} BC}{\sqrt{\tau_t + \tau_{Bu}}} \right)}{\phi \left(- \frac{\tau_t \Delta t_0 + \tau_{Au} AC}{\sqrt{\tau_t + \tau_{Au}}} \right)}.
\end{aligned}$$

Because of the elaborate expression above, it is impossible to characterize the number of equilibria as a function of the precisions τ_t and τ_u in the general case. Instead, I only consider the special case, when τ_u is held constant and $\tau_t \to \infty$, because this is the case where Morris and Shin provide uniqueness. In fact, it is easy to show that $\pi^* (\Delta t_0) = 1 - c$ implies $\Delta t_0 = 0$,and hence

$$\alpha \equiv \lim_{\tau_t \to \infty} \alpha (\Delta t_0) = (1 - c) + c \frac{\tau_{Bu}}{\tau_{Au}} = \frac{(1 - c) B^2 + c A^2}{B^2} < 1.$$

For multiplicity it must be that

$$\alpha (\Delta t_0) < 1 - \frac{1}{1 + \omega \sqrt{\tau_t} \phi \left(\sqrt{\tau_t} \Delta t_0 \right)},$$

but when $\tau_t \to \infty$, the RHS converges to 1, and in the limit indeed

$$\alpha = \frac{(1-c)\, B^2 + cA^2}{B^2} < 1.$$

Therefore, when the private signal becomes arbitrarily precise, there are still multiple equilibria of the coordination problem. Similarly one can show that in the limit $\tau_t \to \infty$ the 'lower' slope of the IM curve becomes

$$\frac{1}{\beta} \equiv \lim_{\tau_t \to \infty} \frac{1}{\beta\,(\Delta t_0)} = (1-c) + c\frac{\tau_{Au}}{\tau_{Bu}} = \frac{(1-c)\, B^2 + cA^2}{A^2} > 1.$$

The next question is what the exact thresholds are in these multiple equilibria. First, one derived intersection is at the kink, which provides an equilibrium of the model. By its definition, the kink satisfies $\Delta\theta_0 = 0$, which in the limit $\tau_t \to \infty$ implies $\Delta t_0 = 0$, and thus the shifting by p gives the solution $t^* = \theta^* = p$.

For a second intersection to be derived, one needs to find the joint solution of equations (22) and (24) in the limit when $\tau_t \to \infty$. Instead of solving for the explicit joint solutions, I instead guess and verify that the solution is $t^* = \theta^* = c\omega$, and only when $p < c\omega$. Indeed, assuming that $\theta^* = c\omega$, the CM equation gives

$$t^* = \theta^* + \frac{1}{\sqrt{\tau_t}}\Phi^{-1}\left(\frac{\theta^*}{\omega}\right) = c\omega + \frac{1}{\sqrt{\tau_t}}\Phi^{-1}\,(c)\,,$$

and hence when $\tau_t \to \infty$, the RHS converges to $c\omega$, therefore we have $\lim_{\tau_t \to \infty} t^* = c\omega$.

Suppose now that $t^* = c\omega$, and plug it in the IM equation. First, if $\theta^* > p$,

$$\lim_{\tau_t \to \infty} \frac{1 - \pi_A^*\,(p, t^* = c\omega)}{1 - \pi^*\,(p, t^* = c\omega)} = 1 + \left[\frac{A^2}{B^2}\frac{\phi\left(-\frac{(p-c\omega)+BC}{\tau_{Bu}^{-1/2}}\right)}{\phi\left(-\frac{(p-c\omega)+AC}{\tau_{Au}^{-1/2}}\right)} - 1\right] \lim_{\tau_t \to \infty} \Phi\left(\sqrt{\tau_t}\,(p - c\omega)\right),$$

where $\lim_{\tau_t \to \infty} \Phi\left(\sqrt{\tau_t}\,(p - c\omega)\right)$ is bounded and hence always finite. Therefore, in the limit it satisfies

$$\lim_{\tau_t \to \infty} \frac{1}{\sqrt{\tau_t + \tau_{Au}}}\Phi^{-1}\,(.) = 0,$$

and hence $\lim_{\tau_t \to \infty} \theta^* = c\omega$. Therefore, in the limit the other intersection satisfies $t^* = \theta^* = c\omega$.

Second, suppose that $\theta^* \leq p$. When $t^* = c\omega$, in the limit $\tau_t \to \infty$ again

$$\lim_{\tau_t \to \infty} \frac{\pi_B^* \left(p, t^* = c\omega\right)}{\pi^* \left(p, t^* = c\omega\right)} = \lim_{\tau_t \to \infty} \Phi \left(\sqrt{\tau_t} \left(p - c\omega\right)\right) + \frac{B^2}{A^2} \frac{\phi\left(-\frac{(p-c\omega)+AC}{\tau_{Au}^{-1/2}}\right)}{\phi\left(-\frac{(p-c\omega)+BC}{\tau_{Bu}^{-1/2}}\right)} \left(1 - \lim_{\tau_t \to \infty} \Phi \left(\sqrt{\tau_t} \left(p - c\omega\right)\right)\right),$$

which is always finite, and hence in the limit (24) simplifies to $\lim_{\tau_t \to \infty} \theta^* = c\omega$. Therefore, in the limit the other intersection satisfies $t^* = \theta^* = c\omega$.