

# Institutional Investors and Information Acquisition: Implications for Asset Prices and Informational Efficiency\*

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This version: April 29, 2018

## Abstract

We study the joint portfolio and information choice problem of institutional investors who are concerned about their performance relative to a benchmark. Benchmarking influences information choices through two distinct economic mechanisms. First, benchmarking reduces the number of shares in investors' portfolios that are sensitive to private information. Second, benchmarking limits investors' willingness to speculate. Both effects imply a decline in the value of private information. Hence, in equilibrium, investors acquire less information and informational efficiency declines. As a result, return volatility increases and benchmarking can cause a decline in equilibrium stock prices. Moreover, less-benchmarked institutional investors outperform more-benchmarked ones.

*Keywords:* benchmarking, institutional investors, informational efficiency, asset allocation, asset pricing

*JEL:* G11, G14, G23

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\*For useful comments and suggestions to this paper and its previous versions, we thank Jonathan Berk, Andrea Buffa (discussant), Will Cong (discussant), Bernard Dumas, Diego Garcia, Marcin Kacperczyk, Ron Kaniel (discussant), Antonio Miguel (discussant), Stijn van Nieuwerburgh, Anna Pavlova (discussant), Joel Peress, Zacharias Sautner, Larissa Schaefer, Günther Strobl, Raman Uppal, Dimitri Vayanos, Laura Veldkamp, Grigory Vilkov, Jing Zeng and seminar participants at Frankfurt School of Finance and Management, INSEAD, the 2016 European Summer Symposium in Financial Markets, the 2017 Australasian Finance & Banking Conference, the 2017 Annual Meeting of the American Finance Association, the 2017 Adam Smith Workshops in Asset Pricing and Corporate Finance, the CEPR Second Annual Spring Symposium in Financial Economics, the 2017 NBER Conference on New Developments in Long-Term Asset Management Conference, the 2017 World Finance Conference, the 2018 Frontiers of Factor Investing Conference, Collegio Carlo Alberto, Geneva School of Economics and Management, BI Norwegian Business School, University of Piraeus, and Imperial Business School. This research benefited from the support of the Europlace Institute of Finance, the Labex Louis Bachelier and the Lyxor-Dauphine Research Academy. Matthijs Breugem is affiliated with Collegio Carlo Alberto, Piazza Vincenzo Arbarelo 8, 10122 Torino, Italy; E-mail: [matthijs.breugem@carloalberto.org](mailto:matthijs.breugem@carloalberto.org). Adrian Buss is affiliated with INSEAD, Boulevard de Constance, 77305 Fontainebleau, France; E-mail: [adrian.buss@insead.edu](mailto:adrian.buss@insead.edu).

# **Institutional Investors and Information Acquisition: Implications for Asset Prices and Informational Efficiency**

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## **Abstract**

We study the joint portfolio and information choice problem of institutional investors who are concerned about their performance relative to a benchmark. Benchmarking influences information choices through two distinct economic mechanisms. First, benchmarking reduces the number of shares in investors' portfolios that are sensitive to private information. Second, benchmarking limits investors' willingness to speculate. Both effects imply a decline in the value of private information. Hence, in equilibrium, investors acquire less information and informational efficiency declines. As a result, return volatility increases and benchmarking can cause a decline in equilibrium stock prices. Moreover, less-benchmarked institutional investors outperform more-benchmarked ones.

Institutional investors own a majority of U.S. equity and account for most of the transactions and trading volume in financial markets.<sup>1</sup> Notably, the importance of relative performance concerns for institutional investors has grown steadily over the past years. That is, the performance of many institutions is nowadays evaluated relative to a benchmark portfolio (“benchmarking”), either explicitly, for example, through performance fees,<sup>2</sup> or implicitly, for instance, due to the flow-performance relation.

There is now a growing body of literature that studies the asset pricing implications of benchmarking for the case of symmetrically informed institutional investors. In contrast, the focus of this paper is on the impact of benchmarking on information acquisition. In particular, our objective is to understand the economic mechanisms through which benchmarking influences information choices and, hence, how the growth of assets under management by benchmarked institutions affects informational efficiency and asset prices.

For that purpose, we develop an equilibrium model of *joint* portfolio and information choice that explicitly accounts for relative performance concerns. The two key features of the model are (i) that all institutional investors endogenously decide on the precision of their private information, and (ii) that a fraction of institutional investors (“benchmarking investors”) are concerned about their performance relative to a benchmark. Otherwise the framework is kept as simple as possible to illustrate the economic mechanisms in the clearest possible way; for example, for most of the analysis we focus on the case in which investors can trade a risk-free bond and a single risky stock. Varying the fraction of benchmarking investors, that is, the share of assets managed by benchmarking institutions, will be our key comparative statics analysis to illustrate the implications of benchmarking.

We identify two distinct economic channels through which benchmarking influences investors’ information choices: (i) *information scale effects* and (ii) *risk-taking effects*. We illustrate these two effects by means of two economic settings, the first one of which keeps, by design, investors’ risk appetite and, hence, aggregate risk-bearing capacity fixed.

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<sup>1</sup>For evidence on institutional ownership and trading volume, see [French \(2008\)](#), [U.S. Securities and Exchange Commission \(2013\)](#), [Stambaugh \(2014\)](#), and [Griffin, Harris, and Topaloglu \(2003\)](#), respectively.

<sup>2</sup>For example, recently, many fund managers have introduced new performance-linked fee structures, including AllianceBernstein, Allianz Global Investors, Equitile, Fidelity International and Orbis Investment Management. Also, Japan’s Government Pension Investment Fund, the world’s largest retirement fund, introduced a system whereby it pays all active managers a fee based on their return relative to a benchmark.

In particular, we illustrate information scale effects arising from benchmarking in a tractable model in which institutional investors have constant absolute risk-aversion (CARA) but a preference for the early resolution of uncertainty.<sup>3</sup> We show that benchmarked investors’ portfolios can be decomposed into two components. First, the standard mean-variance portfolio, that is, the optimal portfolio of non-benchmarked investors. This component is not affected by benchmarking, but rather driven by investors’ posterior beliefs. Second, a hedging portfolio arising from benchmarking. That is, because benchmarked institutional investors strive to do well when the benchmark performs well, they over-weight the benchmark portfolio. Intuitively, this hedging portfolio is designed to track the benchmark, not to outperform it, and, consequently, it is information-insensitive.

In equilibrium, that is, once the market clears, the aggregate—information-insensitive—hedging demand of the benchmarked institutions reduces the “effective” supply of the stock; that is, the number of shares in the economy that are available for speculation.<sup>4</sup> As a result, for all—benchmarking and non-benchmarking—investors, the expected number of shares in their portfolio that are sensitive to private information declines.<sup>5</sup> Because information is then applied to fewer shares, its marginal value is lower. Accordingly, with no change in information costs, investors acquire less private information, and price informativeness declines as the fraction of benchmarked investors in the economy increases. Benchmarking also affects the equilibrium stock price. In particular, for realistic calibrations, the stock price is increasing and its expected excess return is decreasing in the size of benchmarked institutions. Finally, the decline in price informativeness leads to a higher return volatility since the price tracks the stock’s terminal payoff less closely.

The implications described thus far follow from scale effects in information acquisition. But, benchmarking also affects information and portfolio choice through risk-taking effects which we illustrate using a model with constant relative risk-aversion (CRRA) preferences.

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<sup>3</sup>These preferences are chosen for illustration only. In the case of constant relative risk-aversion, as discussed next, information scale effects are also present. But, the model is less tractable and benchmarking affects the aggregate risk-bearing capacity as well.

<sup>4</sup>Intuitively, the same effect would arise in the presence of “index investors,” that is, investors whose portfolio choice is not driven by (private) information.

<sup>5</sup>This holds for the economically relevant case in which the effective supply is positive which guarantees a positive risk premium for the stock.

Because the equilibrium price function in this model is non-linear, it is considerably less tractable and we rely on a novel numerical solution method to solve it.

Benchmarked investors' portfolios can be (approximately) decomposed into the same two components as in the mean-variance setting discussed above. The key difference is that relative performance concerns now also affect the mean-variance portfolio. In particular, benchmarking limits institutional investors' willingness to speculate;<sup>6</sup> that is, they trade less aggressively on a particular piece of information, so that, in equilibrium, information acquisition and price informativeness decline.

This novel effect has important quantitative, but also qualitative, implications. First, in contrast to the case of mean-variance preferences, the information choices of benchmarked and non-benchmarked institutional investors differ; that is, benchmarked investors choose a lower precision of private information. Consequently, benchmarked investors are less well informed and, hence, earn lower expected portfolio returns than non-benchmarked investors. Moreover, as the assets-under-management of benchmarked institutions increase, less information is revealed through the public stock price, such that the "information gap" between investors widens. Quantitatively, the risk-taking effect considerably strengthens the decline in price informativeness due to benchmarking, because benchmarked investors not only acquire less information but also trade less aggressively. As a result, for realistic calibrations, some of the asset-pricing implications change even quantitatively. For example, the price of the stock can actually decline in the fraction of benchmarked investors, or, equivalently, its expected excess return can increase. Furthermore, the increase in return volatility is considerably more pronounced.

Both mechanisms reduce price informativeness and are driven by the *interaction* between portfolio and information choice. To distinguish between the two effects, recall that the information scale effect is driven by the impact of the aggregate hedging demand in equilibrium and is exclusively due to information acquisition. In contrast, the risk-taking effect is driven by a benchmarked institutional investor's individual portfolio choice and is due to both—information aggregation and information acquisition.

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<sup>6</sup>Technically, benchmarking increases the local coefficient of relative risk-aversion because proportional movements in an investor's wealth have a larger impact in the presence of relative performance concerns.

Finally, we study two extensions of our basic economic framework. First, we extend our model to multiple stocks and document that informational efficiency also deteriorates for stocks that are not part of the benchmark, although to a smaller extent because the information scale effect is weaker (absent) for non-index stocks. Second, we incorporate benchmarking concerns that are non-linear in the benchmark’s performance—in line with asymmetric performance fee structures and implicit in the flow-performance relationship. We demonstrate that asymmetric benchmarking concerns can mitigate the adverse effects of benchmarking on information choice.

We also make a methodological contribution by developing a novel numerical solution approach that allows us to determine the equilibrium in noisy rational expectations models with non-linear price functions. Our approach differs from previous attempts such as [Bernardo and Judd \(2000\)](#) in that it does not rely on a projection of the price law (and the asset demand). It also allows for heterogeneous signal precisions and a continuum of investors. The solution technique is flexible and can also be used to study rational expectation equilibrium models in the presence of constraints or frictions.

The two papers that are closest to our work are [Admati and Pfleiderer \(1997\)](#) and [García and Strobl \(2011\)](#), whose results and implications, however, are distinctly different. [Admati and Pfleiderer \(1997\)](#) study linear benchmarking concerns in the compensation of privately informed portfolio managers, but in a framework with CARA preferences and an exogenous price process. They document that, because each individual manager can use his portfolio choice to “undo” the benchmarking component in his compensation, his optimal information choice is not affected by relative performance concerns. In contrast, taking into account the effect of investors’ portfolio choice on the market-clearing stock price, we illustrate that benchmarking leads to a decline in information acquisition. We also document a novel effect resulting from benchmarked investors’ risk-taking.

[García and Strobl \(2011\)](#) study how relative wealth concerns affect investors’ incentives to acquire information. They demonstrate that, when an investor’s utility depends on the consumption of the average investor, complementarities in information acquisition arise, introducing the possibility of multiple “herding” equilibria. These complementarities can

lead to an increase in informed trading, thereby improving price informativeness. The key difference is that, in our framework, investors have relative performance concerns (with respect to a benchmark), and not relative wealth concerns (with respect to other investors). As a result, an investor's information choice and resulting consumption does not affect other investors' utility. Accordingly, in our framework, no complementarities in information choice arise, and price informativeness declines.

[Kacperczyk, Nosal, and Sundaresan \(2018\)](#) develop a rational expectations model with market power and demonstrate that an increase in the size of passive investors unambiguously reduces price informativeness. While passive investors in their model are, by definition, uninformed, the benchmarked institutional investors in our framework endogenously choose the precision of their private information.

The economic framework that we develop builds on two independent strands of research: first, the asset pricing literature on the stock market implications of benchmarking (absent information choice) and, second, the literature on information acquisition in competitive markets (absent benchmarking).<sup>7</sup>

[Brennan \(1993\)](#) derives, in the presence of relative performance concerns, a two-factor model, one of which being the benchmark. [Cuoco and Kaniel \(2011\)](#) study a model of portfolio delegation with CRRA preferences and relative performance concerns, and demonstrate that linear performance fees have an unambiguously positive impact on the price of stocks that are part of the benchmark. Using a tractable specification, [Basak and Pavlova \(2013\)](#) provide closed-form solutions for a dynamic CRRA utility setup with multiple risky assets and institutional investors who care about a benchmark. They document that, in the presence of benchmarking, institutional investors optimally tilt their portfolio toward the benchmark, creating upward price pressure and an amplification of return volatility. [Buffa, Vayanos, and Woolley \(2014\)](#) study a dynamic setup with multiple risky assets and CARA preferences in which, due to agency frictions, asset-management contracts endogenously

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<sup>7</sup>Our work is also related to recent studies that have relaxed the joint CARA-normal assumption (see, e.g., [Barlevy and Veronesi \(2000\)](#), [Peress \(2004\)](#), [Albagli, Hellwig, and Tsyvinski \(2014\)](#), [Breon-Drish \(2015\)](#) and [Chabakauri, Yuan, and Zachariadis \(2016\)](#)).

depend on fund managers' performance relative to a benchmark. [Buffa and Hodor \(2018\)](#) show how heterogeneous benchmarking can result in negative spillovers across assets.

Unlike these papers, we explicitly model the joint information and portfolio choice in the presence of benchmarking and focus on its impact on information acquisition.<sup>8</sup> Notably, allowing for endogenous information choice changes some of the asset pricing implications. For example, stock prices can decline in the fraction of benchmarked investors. We also provide novel predictions regarding investors' expected portfolio returns that arise from the interaction of portfolio and information choice.

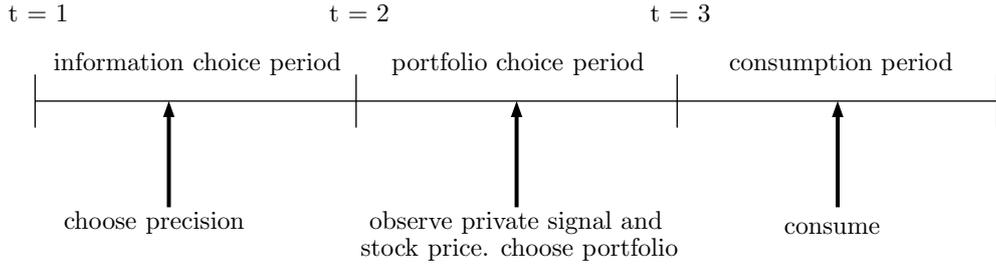
Information scale effects are discussed in [van Nieuwerburgh and Veldkamp \(2009, 2010\)](#), who document a feedback effect between information and portfolio choice through the number of shares investors expect to hold, and [Peress \(2010\)](#), who demonstrates that better risk-sharing lowers the value of information because investors expect to hold fewer shares. [García and Vanden \(2009\)](#) show that competition between fund managers makes prices more informative. [Malamud and Petrov \(2014\)](#) demonstrate that convex compensation contracts lead to equilibrium mispricing, but reduce volatility. [Kacperczyk, van Nieuwerburgh, and Veldkamp \(2016\)](#) and [Farboodi and Veldkamp \(2017\)](#) discuss what data fund managers optimally choose to process. [Bond and García \(2016\)](#) study the consequences of investing solely via the market portfolio and find negative externalities for uninformed investors.

In contrast, we explicitly model institutional investors who are concerned about their relative performance, which allows us to make novel predictions about the relationship between the size of benchmarked institutions and informational efficiency and asset prices.

The remainder of the paper is organized as follows. Section 1 introduces our economic framework and discusses investors' optimization problems. Sections 2 and 3 discuss how benchmarking affects informational efficiency as well as asset prices through information scale effects and risk-taking effects, respectively. Section 4 discusses two extensions of our basic framework. Finally, Section 5 summarizes our key predictions and concludes. Proofs and a description of the numerical solution method are delegated to the Appendix.

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<sup>8</sup>The importance of price informativeness is highlighted in the literature on "feedback effects," which shows that information contained in asset prices affects corporate decisions; see, e.g., [Chen, Goldstein, and Jiang \(2007\)](#), [Bakke and Whited \(2010\)](#), [Bond, Edmans, and Goldstein \(2012\)](#), [Edmans, Goldstein, and Jiang \(2012\)](#), and [Foucault and Frésard \(2012\)](#).



**Figure 1: Timing.** The figure illustrates the sequence of the events.

## 1 The Model

This section introduces our basic economic framework, which explicitly accounts for the information choices of institutional investors who are concerned about their performance relative to a benchmark. In particular, we incorporate benchmarking concerns, as in [Cuoco and Kaniel \(2011\)](#) and [Basak and Pavlova \(2013\)](#), into a competitive rational expectations equilibrium model of joint portfolio and information choice, as in [Verrecchia \(1982\)](#). We also discuss investors' optimization problems and the equilibrium concept.

### 1.1 Economic Framework

#### Information Structure and Timing

We consider a static model, which we break up into three (sub-)periods. Figure 1 illustrates the sequence of the events. In period 1, the information choice period, investors can spend time and resources to acquire private information about a stock. For example, they may study financial statements, gather information about consumers' taste, hire outside financial advisers, or subscribe to proprietary databases. In particular, each investor  $i$  can choose the precision  $q_i$  of his private signal  $Y_i$ . Higher precision will reduce the posterior uncertainty regarding the stock's payoff but will increase the information acquisition costs  $\chi_i \equiv \kappa(q_i)$ .<sup>9</sup>

In period 2, the portfolio choice period, investors observe their private signals (with the chosen precision) and make their investment choice. Prices are set such that the market

<sup>9</sup>The information cost function  $\kappa$  is assumed to be continuous, increasing and strictly convex, with  $\kappa(0) = 0$ . This guarantees the existence of interior solutions and captures the idea that each new piece of information is more costly than the previous one.

clears. In period 3, the consumption period, investors consume the proceeds from their investments.

We denote the expectation and variance conditional on prior beliefs as  $E_1[\cdot]$  and  $Var_1(\cdot)$ . To denote investor  $i$ 's expectation and variance conditional on his time-2 information set  $\mathcal{F}_i = \{Y_i, P\}$ , we use  $E_2[\cdot | \mathcal{F}_i]$  (or,  $E_2[\cdot]$ ) and  $Var_2(\cdot | \mathcal{F}_i)$  (or,  $Var_2(\cdot)$ ).

## Investment Opportunities

There exist two financial securities that are traded competitively in the market: a risk-less asset (the “bond”) and a risky asset (the “stock”). The bond pays an exogenous (gross) interest rate  $R_f$  and is available in perfectly elastic supply. It also serves as the numéraire, with its price being normalized to one. The stock is modeled as a claim to a random payoff  $X$ , which is only observable in period 3. Its price is denoted by  $P$ . The supply of the stock, denoted by  $Z$ , is assumed to be random and unobservable. This prevents the price from fully revealing the information acquired by the investors and, thus, preserves the incentives to acquire private information in the first place.

## Investors

There exists a continuum of atomless investors with mass one that we separate into two groups of institutional investors: (1) a fraction  $\Gamma$  of *benchmarked institutions*, or, short, “benchmarked investors,”  $\mathcal{BI}$ ; and (2) a fraction  $1 - \Gamma$  of non-benchmarked institutions, or, short, “non-benchmarked investors,”  $\mathcal{NI}$ . Each investor  $i \in \{\mathcal{BI}, \mathcal{NI}\}$  is endowed with the same initial wealth  $W_{0,i}$ , which we normalize to 1.

Motivated by recent theoretical contributions,<sup>10</sup> we model the compensation of institutional investors,  $C_i$ , as:

$$C_i(W_i, R_B) = W_i - \gamma_i W_{0,i} R_B - \chi_i. \quad (1)$$

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<sup>10</sup>Basak and Pavlova (2013) demonstrate benchmarking formally using an agency-based argument. In Buffa, Vayanos, and Woolley (2014), investors endogenously—due to agency frictions—make fund managers’ fees sensitive to the performance of a benchmark. Similarly, Sotes-Paladino and Zapatero (2016) show that a linear benchmark-adjusted component in managers’ contracts can benefit investors.

Institutional investors' compensation has three components; first, a “standard” component related to terminal wealth  $W_i$  (i.e., “assets under management”);<sup>11</sup> second, a linear benchmarking component that is related to the performance of the benchmark  $R_B$ ;<sup>12</sup> and, third, information acquisition costs  $\chi_i$ . Consequently,  $\gamma_i$  captures the strength of investor  $i$ 's benchmarking concerns; in particular, while benchmarked institutional investors are concerned about their performance relative to a benchmark, that is,  $\gamma_i > 0, \forall i \in \mathcal{BI}$ , non-benchmarked investors are not, that is,  $\gamma_i = 0, \forall i \in \mathcal{NI}$ .

It is important to highlight that benchmarking is the only source of heterogeneity across the two groups of institutional investors. In particular, benchmarking does not affect the investors' financial wealth but only their utility; that is, both groups of institutional investors have the same capital available for investment in the portfolio choice period.

We consider a general form of investors' preferences over compensation  $C_i$ :

$$U_{1,i} = E_1 [u_1 ( E_2 [u_2(C_i)] )], \quad (2)$$

in which the inner utility function  $u_2$  governs risk-aversion and the outer utility function  $u_1$  governs the preference for the timing of the resolution of uncertainty.<sup>13</sup> Specifically, if  $u_1$  is linear, investors are indifferent about the timing, whereas a convex (concave) function  $u_1$  implies a preference for early (late) resolution of uncertainty. In the next sections, we explore specific examples.

The specification of the benchmarked investors' utility over compensation  $C_i$  exhibits two important characteristics that let them behave differently from non-benchmarked investors. First, benchmarked investors have an incentive to post a high return when the benchmark performs well or, formally, their marginal utility is increasing in  $R_B$ . Second, benchmarked investors' utility function is decreasing in the performance of benchmark, thus

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<sup>11</sup>Considering instead a *fraction* of terminal wealth, e.g.,  $\beta_i W_i$ , has qualitatively no impact on the results. It affects the total amount of information that investors acquire, but not the impact of benchmarking.

<sup>12</sup>These types of benchmarking concerns capture linear (“Fulcrum”) performance fees. The 1970 Amendment of the Investment Advisers Act of 1940 restricts mutual fund fees to be of the Fulcrum type in the U.S. Note, an investor's desire to perform well relative to a benchmark may also be driven by social status, instead of monetary incentives. Note also that these benchmarking concerns do not capture relative wealth concerns, as discussed in [García and Strobl \(2011\)](#).

<sup>13</sup>Technically,  $u_1$  and  $u_2$  are assumed to be continuous, twice differentiable, increasing functions.

affecting information choice. Note that our definition of the investors' compensation scheme in (1) shares many similarities with the specifications in [Cuoco and Kaniel \(2011\)](#) and [Basak and Pavlova \(2013\)](#).<sup>14</sup>

## 1.2 Investors' Optimization Problems and Equilibrium

### Portfolio and Information Choice

In the portfolio choice period ( $t = 2$ ), investor  $i$  chooses the number of shares of the stock,  $\theta_i$ , in order to maximize expected utility; conditional on his posterior beliefs and price  $P$ :

$$U_{2,i} = \max_{\theta_i} E_2[u_2(C_i) | \mathcal{F}_i], \quad (3)$$

with terminal wealth,  $W_i$ , being given by  $W_i = W_{0,i} R_f + \theta_i (X - PR_f)$ .<sup>15</sup>

In the information choice period ( $t = 1$ ), investor  $i$  chooses the precision of his private signal,  $q_i$ , in order to maximize expected utility over all possible realizations of his private signal  $Y_i$  and the public price  $P$ , anticipating his optimal portfolio choice in period 2:

$$\max_{q_i \geq 0} E_1[u_1(U_{2,i})]. \quad (4)$$

### Equilibrium Definition

A rational expectations equilibrium is defined by portfolio choices  $\{\theta_i\}$ , information choices  $\{q_i\}$ , and prices  $\{P\}$  such that:

1.  $\theta_i$  and  $q_i$  solve investor  $i$ 's maximization problems in (3) and (4), taking  $P$  as given.

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<sup>14</sup>Compensation in [Cuoco and Kaniel \(2011\)](#) is composed of a constant "load fee," a fraction of terminal wealth, and a performance-related component. [Basak and Pavlova \(2013\)](#) study a tractable specification similar to ours. However, utility is increasing in the benchmark (which is less important in the absence of information choice because only marginal utility matters). In the case of  $\gamma_i = 1$ , our specification in (1) is comparable to the alternative specification discussed in their Remark 1.

<sup>15</sup>This follows from the two budget equations  $W_i = \theta_i X + \theta_i^B R_f$  and  $W_0 = \theta_i P + \theta_i^B$  by solving the second equation for  $\theta_i^B$  (number of shares of the bond) and plugging the solution into the first.

2. Expectations are rational; that is, the average precision of private information implied by aggregating investors' precision choices equals the level assumed in investors' optimization problems (3) and (4).
3. Aggregate demand equals aggregate supply.

Note that, in equilibrium, the stock price plays a dual role: It clears the security market and aggregates as well as disseminates investors' private information.

## 2 Benchmarking and Information Scale Effects

In this section, we illustrate how benchmarking affects information choice due to *information scale effects*. For this purpose, we rely on a tractable model that is designed to provide the economic intuition and allows for closed-form solutions of all quantities in the economy.

### 2.1 Setup

In particular, we assume that the payoff of the stock,  $X$ , and its supply,  $Z$ , are normally distributed, with  $X \sim \mathcal{N}(\mu_X, \sigma_X^2)$  and  $Z \sim \mathcal{N}(\mu_Z, \sigma_Z^2)$ . The investors' private signals are given by  $Y_i = X + \varepsilon_i$ , with  $\varepsilon_i \sim \mathcal{N}(0, 1/q_i)$ . Investors have "mean-variance preferences" of the Kreps-Porteus type; that is, their time-1 utility function,  $U_{1,i}$ , is given by:<sup>16</sup>

$$U_{1,i} = E_1 \left[ E_2 [C_i] - \frac{\rho}{2} \text{Var}_2 (C_i) \right], \quad (5)$$

with compensation  $C_i = W_i - \gamma_i (X - PR_f) - \chi_i$ . That is,  $W_{0,i}R_B \equiv X - PR_f$  captures the performance of the benchmark. These preferences lead, in period 2, to the same portfolio that obtains under CARA expected utility, but investors have a preference for the early resolution of uncertainty.<sup>17</sup> They can also arise in a setting with risk-neutral, profit-

<sup>16</sup>This is a special case of the general form in (2), with  $u_1(x) = -\ln(-x)$  and  $u_2(x) = -\exp(-\rho x)$ .

<sup>17</sup>Kreps and Porteus (1978) provide the axiomatic foundations for this class of non-expected utility, which allows to disentangle risk-aversion from the elasticity of inter-temporal substitution. The generalizations of iso-elastic utility in Epstein and Zin (1989) and Weil (1990) are widely used in asset pricing.

maximizing portfolio managers who invest on behalf of clients with CARA expected utility (see, e.g., the discussion in footnote 10 in [van Nieuwerburgh and Veldkamp \(2009\)](#)).

It is important to highlight that, by design, benchmarking does not affect the benchmarked investors' risk appetite. As a result, changes in the fraction of benchmarked institutional investors,  $\Gamma$ , have no impact on aggregate risk-bearing capacity.

## 2.2 Portfolio Choice and the Equilibrium Price

An investor's optimization problem must be solved in two stages, starting with the optimal portfolio choice in period 2, taking information choices as given. The following theorem characterizes an investor's optimal portfolio choice for arbitrary values of his posterior mean  $\hat{\mu}_{X,i} \equiv E_2[X | \mathcal{F}_i]$  and posterior precision  $h_i \equiv Var_2(X | \mathcal{F}_i)^{-1}$ .

**Theorem 1.** *Conditional on an investor's posterior beliefs, described by  $\hat{\mu}_{X,i}$  and  $h_i$ , and the stock price  $P$ , the optimal stock demand equals:*

$$\theta_i = h_i \frac{\hat{\mu}_{X,i} - P R_f}{\rho} + \gamma_i \equiv \theta_i^{MV} + \gamma_i. \quad (6)$$

The optimal demand for the stock has two components. First, for all investors, the standard mean-variance portfolio,  $\theta_i^{MV}$ . Second, for benchmarked investors, a hedging demand  $\gamma_i > 0$ . Intuitively, benchmarked investors have the desire to acquire assets that do well when the benchmark does well, or, equivalently, assets that co-vary positively with the benchmark. In our single-asset economy, the stock, also serving as the benchmark, naturally co-varies positively with the benchmark.

The mean-variance portfolio,  $\theta_i^{MV}$ , is independent of the strength of an investor's benchmarking concerns,  $\gamma_i$ . In contrast, the hedging component is increasing in the investor's benchmarking concerns,  $\gamma_i$ , but is *information-insensitive*, that is, it is not affected by his posterior beliefs. Intuitively, it is designed to closely track or, formally, co-vary with, the benchmark, and not meant for speculation.

Aggregating the demand of both groups of institutional investors and imposing market-clearing delivers a solution for the equilibrium stock price:

**Theorem 2.** *Conditional on investors' information choices, described by  $q_i, \forall i$ , there exists a unique linear rational expectations equilibrium:*

$$P R_f = \frac{1}{\bar{h}} \left( \frac{\mu_X}{\sigma_X^2} + \frac{\mu_Z \bar{q}}{\sigma_Z^2 \rho} + \rho \bar{\gamma} \right) + \frac{1}{\bar{h}} \left( \bar{h} - \frac{1}{\sigma_X^2} \right) X - \frac{1}{\bar{h}} \left( \frac{\bar{q}}{\rho \sigma_Z^2} + \rho \right) Z, \quad (7)$$

$$\text{where} \quad h_0 \equiv \frac{1}{\sigma_X^2} + \frac{\bar{q}^2}{\rho^2 \sigma_Z^2}, \quad \bar{h} \equiv h_0 + \bar{q}, \quad (8)$$

$$\bar{q} \equiv \int_0^\Gamma q_i^{\mathcal{BI}} di + \int_0^\Gamma q_i^{\mathcal{NI}} di, \quad \text{and} \quad \bar{\gamma} \equiv \int_0^\Gamma \gamma_i di. \quad (9)$$

The characterization of the equilibrium price in (7) is standard for this type of economy, and the variables defined in (8) and (9) allow for intuitive interpretations. That is,  $h_0$  equals the sum of the precisions from the prior,  $1/\sigma_X^2$ , and from the price signal,  $\bar{q}^2/(\rho^2 \sigma_Z^2)$ ; that is, it characterizes the precision of public information.  $\bar{q}$  measures average *private* signal precision, that is, the precision of the private information of the average investor. Consequently,  $\bar{h}$  governs average *aggregate* precision, that is, the precision of public and private information of the average investor.

Finally,  $\bar{\gamma}$  captures the degree of benchmarking in the economy, aggregating the strength of the relative performance concerns of benchmarked investors,  $\gamma_i$ , and their size in the economy,  $\Gamma$ . Note that, for ease of exposition, we assume for all graphical illustrations that the strength of the benchmarked investors' relative performance concerns coincide:  $\gamma_i = \gamma, \forall i \in \mathcal{BI}$ . As a result,  $\bar{\gamma}$  simplifies to  $\Gamma \gamma$  and, conditional on  $\gamma$ , all variation in  $\bar{\gamma}$  is driven by the fraction of benchmarked institutions,  $\Gamma$ .

Conditional on information choices, the equilibrium price is given by the price in an economy without benchmarking plus  $(\rho \bar{\gamma})/\bar{h}$ ; hence, the price is increasing in the degree of benchmarking  $\bar{\gamma}$ . Intuitively, conditional on a given supply  $Z$ , the excess demand for the stock,  $\bar{\gamma}$ , resulting from benchmarked investors' hedging demand, drives up the price.<sup>18</sup>

<sup>18</sup>Technically,  $\partial(PR_f)/\partial\bar{\gamma} = \rho/\bar{h}$ . Because the magnitude of an individual investor's hedging demand relative to the mean-variance portfolio is increasing (decreasing) in risk-aversion (aggregate posterior precision), the sensitivity of the price with respect to benchmarking is positively (negatively) related to  $\rho$  ( $\bar{h}$ ).

It is important to note that benchmarking does not limit information aggregation. Price informativeness, defined as the ratio of the price sensitivities with respect to the payoff and the noise, is given by  $\bar{q}^2/(\rho^2 \sigma_Z^2)$ . Hence, conditional on investors' information choices (as aggregated by  $\bar{q}$ ), it is unaffected by  $\bar{\gamma}$ . Intuitively, because the benchmarked investors' hedging demand is fully predictable, it only affects the level of the price but not its sensitivity to the payoff or the noise.

### 2.3 Information Choice

While Theorem 2 takes the information environment as given, information choices are actually an endogenous outcome of the model. That is, at time  $t = 1$ , each investor  $i$  chooses the precision of his private signal,  $q_i$ , in order to maximize utility (5), anticipating his optimal portfolio choice in period 2. In particular, in order to impose rational expectations, we substitute the optimal stock demand (6) into wealth  $W_i$  and, hence, compensation,  $C_i$ , which yields the following time-1 objective function:

$$U_{1,i} = R_f \left( W_{0,i} - \frac{\kappa(q_i)}{R_f} \right) + \frac{1}{2\rho} E_1 [z_i^2], \quad (10)$$

where  $z_i \equiv \sqrt{h_i}(\hat{\mu}_{X,i} - P R_f)$  denotes the investor's time-2 expected Sharpe ratio—a function of  $Y_i$ ,  $q_i$  and  $P$ . Intuitively,  $E_1 [z_i^2]$  governs the squared Sharpe ratio that investor  $i$  expects to achieve in the information choice period ( $t = 1$ ).

The objective function captures the “information choice trade-off.” That is, higher precision  $q_i$  leads to higher information costs  $\kappa(q_i)$ , thereby reducing time-1 utility. On the other hand, higher precision  $q_i$  increases the posterior precision  $h_i$ , the time-1 expected squared Sharpe ratio  $E_1[z_i^2]$  and, thus, time-1 utility.

Notably, benchmarked investors ( $i \in \mathcal{BI}$ ) “undo” the benchmarking component in their compensation; that is, a benchmarked investor's time-1 objective function does not depend on the strength of his *individual* benchmarking concerns,  $\gamma_i$  and, hence, coincides with the objective function of a non-benchmarked investor. That is, the two groups of institutional investors actually face the same information choice problem. Accordingly, in the following,

we discuss the information choice problem of a generic institutional investor  $i \in \{\mathcal{BI}, \mathcal{NI}\}$ , noting that  $q_i = q_i^{\mathcal{BI}} = q_i^{\mathcal{NI}}$ .

Taking the first-order condition of  $U_{1,i}$  with respect to  $q_i$ , delivers an investor's best information choice, given arbitrary precision choices by the other investors:

**Theorem 3.** *Conditional on the average private signal precision,  $\bar{q}$ , investor  $i$ 's optimal signal precision  $q_i(\bar{q})$  is the unique solution of:*

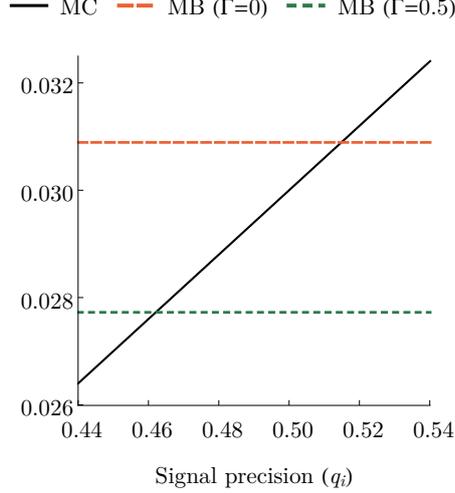
$$2 \kappa'(q_i) = \frac{1}{\rho} \underbrace{\left[ \frac{1}{\bar{h}^2} \left( \rho^2 \left( \sigma_Z^2 + (\mu_Z - \bar{\gamma})^2 \right) + \bar{h} + \bar{q} \right) \right]}_{\equiv A(\bar{\gamma}, \bar{q})}. \quad (11)$$

At the optimum, the marginal cost of private information,  $\kappa'(q_i)$ , equals the marginal benefit, governed by  $A$  and investor  $i$ 's risk aversion  $\rho$ . The marginal value of private information,  $A$ , is related to the degree of benchmarking,  $\bar{\gamma}$ , as follows:

**Lemma 1.** *If  $\mu_Z - \bar{\gamma} > 0$ , an investor's best information choice  $q_i(\bar{q})$  shifts down when the degree of benchmarking in the economy,  $\bar{\gamma}$ , increases. Formally,  $\partial A / \partial \bar{\gamma} < 0$ .*

Intuitively,  $\mu_Z - \bar{\gamma}$  measures the “effective supply” of the stock, that is, the supply after accounting for the aggregate hedging demand. The economically interesting case is the one in which the effective supply is positive, because only this guarantees that the stock's risk premium is positive (confer (A16)). In this case, the value of private information declines in the degree of benchmarking—for benchmarked and non-benchmarked investors. This is illustrated in Figure 2, which depicts the marginal cost of information  $2 \kappa'(q_i)$  and the marginal benefits  $A(\bar{\gamma}, \bar{q}) / \rho$  for two degrees of benchmarking in the economy. In particular, the investor's optimal precision of private information, characterized by the intersection between the marginal cost and the respective marginal benefit, is higher for a low degree of benchmarking ( $\Gamma = 0$ ) than for a high degree of benchmarking ( $\Gamma = 0.5$ ).

Intuitively, this result is driven by *information scale effects*. That is, when making information choices in period 1, investors take into account the *expected number of shares in their portfolios that are sensitive to private information*. In particular, one can re-write



**Figure 2: Information demand.** The figure illustrates an investor’s optimal information demand. It plots the marginal cost of information  $MC = 2\kappa'(q_i)$  as a function of signal precision  $q_i$  together with the marginal benefit of information  $MB = A/\rho$  in the absence of benchmarked investors ( $\Gamma = 0$ ) and if they make up half of the population ( $\Gamma = 0.5$ ). The intersection between the marginal cost and benefit characterize an investor’s optimal information choice. The graph is based on the framework described in Section 2.1, with the following parameter values:  $\mu_X = 1.05$ ,  $\sigma_X^2 = 0.25$ ,  $\mu_Z = 1.0$ ,  $\sigma_Z^2 = 0.2$ ,  $\rho = 3$ ,  $\gamma_i = \gamma = 1/3 \forall i \in \mathcal{BM}$ ,  $\bar{q} = 0.35$ , and an information cost function  $\kappa(q_i) = \omega q_i^2$ , with  $\omega = 0.015$ .

investor  $i$ ’s time-2 expected squared Sharpe ratio, scaled by  $\rho$ , as:

$$\frac{z_i^2}{\rho} = h_i \frac{\hat{\mu}_{X,i} - PR_f}{\rho} \times (\hat{\mu}_{X,i} - PR_f) = \theta_i^{MV} \times (E_2[X] - PR_f).$$

Hence, another way to express the time-1 expected squared Sharpe ratio, governing the benefit of private information, is as the product of the mean-variance portfolio (the information-sensitive part of the portfolio) and the time-2 expected stock return. In equilibrium, that is, taking into account market-clearing, the—information-insensitive—aggregate hedging demand reduces the expected number of shares in investors’ portfolios that are sensitive to private information.<sup>19</sup> Because returns to information are increasing (that is, one piece of information can be used for many shares), while costs are not, this implies a decline in the marginal value of information for all investors.

Theorem 3 takes the information choices of the other investors as given. However, in equilibrium, the information choice of each investor,  $q_i$ , affects aggregate average precision,

<sup>19</sup>In particular, for each investor, the time-1 expectation of the number of shares in his portfolio that are sensitive to private information is lowered by  $\bar{\gamma}$  relative to absence of benchmarking.

$\bar{q}$ , which, in turn, affects each investor's information choice. Therefore, the equilibrium value of the average private signal precision,  $\bar{q}$ , is determined by plugging  $q_i$  in (11) into its definition in (9):

**Theorem 4.** *The average private signal precision,  $\bar{q}$ , is the unique solution to:*

$$\bar{q} = \int_0^\Gamma q_i^{\text{BI}}(\bar{q}) di + \int_\Gamma^1 q_i^{\text{NI}}(\bar{q}) di = \int_0^1 q_i(\bar{q}) di. \quad (12)$$

In particular, one can derive the following corollary:

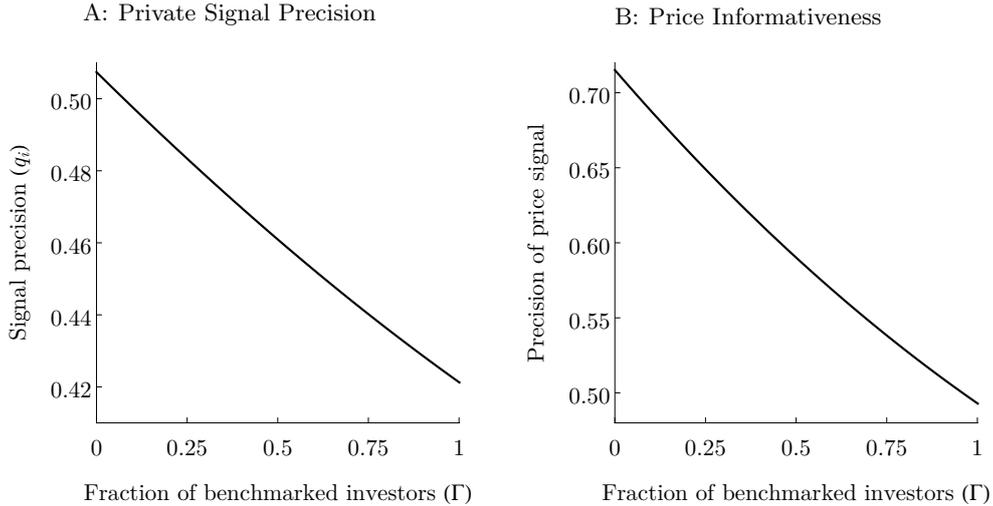
**Corollary 1.** *If  $\mu_Z - \bar{\gamma} > 0$ , the average private signal precision,  $\bar{q}$ , is declining in the degree of benchmarking,  $\bar{\gamma}$ . Formally,  $d\bar{q}/d\bar{\gamma} < 0$ .*

That is, for the economically relevant case of  $\mu_Z - \bar{\gamma} > 0$ , an increase in the degree of benchmarking in the economy,  $\bar{\gamma}$ , leads to a decline in the precision of the private information of all investors. Consequently, equilibrium price informativeness,  $\bar{q}^2/(\rho^2 \sigma_Z^2)$  also declines in the degree of benchmarking,  $\bar{\gamma}$ .

Both effects are illustrated in Figure 3. In particular, while Panel A shows that the precision of all investors' private signals,  $q_i$ , is declining in the fraction of benchmarked investors in the economy, Panel B depicts the corresponding decline in price informativeness.

*REMARK 1.* It is instructive to consider the case of a single (small) benchmarked institutional investor with constant absolute risk-aversion and a preference for the early resolution of uncertainty. First, recall that the marginal value of private information  $A$  does not depend on the strength of the investor's *individual* benchmarking concerns,  $\gamma_i$ , but only on the *aggregate* degree of benchmarking  $\bar{\gamma}$ . Accordingly, a single benchmarked investor's information choice is not affected by benchmarking. Intuitively, the expected number of shares in his portfolio that are sensitive to private information do not decline in the absence of an aggregate hedging demand.

*REMARK 2.* It is also instructive to study the case of CARA expected utility. The optimal stock demand and the resulting price function are identical to the case of CARA utility with a preference for the early resolution of uncertainty and are given by (6) and (7). However,



**Figure 3: Equilibrium information choice.** The figure shows the optimal private signal precision of benchmarked and non-benchmarked investors,  $q_i$ , in equilibrium (Panel A) and the resulting price informativeness, that is, the precision of the public price signal  $\bar{q}^2/(\rho^2 \sigma_Z^2)$  (Panel B), as a function of the fraction of benchmarked investors in the economy. The graphs are based on the framework described in Section 2.1, with the following parameter values:  $\mu_X = 1.05$ ,  $\sigma_X^2 = 0.25$ ,  $\mu_Z = 1.0$ ,  $\sigma_Z^2 = 0.2$ ,  $\rho = 3$ ,  $\gamma_i = \gamma = 1/3 \forall i \in \mathcal{BM}$ , and an information cost function  $\kappa(q_i) = \omega q_i^2$ , with  $\omega = 0.015$ .

at the information choice stage, an investor's best information choice differs. In particular, given arbitrary precision choices by the other investors, his optimal information choice is characterized by the following theorem:

*Theorem 5. Assume that investors have CARA expected utility. Conditional on the average private signal precision,  $\bar{q}$ , investor  $i$  chooses a signal of precision  $q_i(\bar{q})$  such that:*

$$2 \kappa'(q_i) = \frac{1}{\rho} \frac{1}{h_0 + q_i}.$$

That is, the marginal benefit of information is governed by the investor's posterior precision  $h_0 + q_i$  and his risk-aversion  $\rho$ . In particular, his information choice and the average equilibrium signal precision  $\bar{q}$ , do *not* depend on the degree of benchmarking  $\bar{\gamma}$ ; that is information scale effects are absent.

It is important to highlight that, with CRRA preferences, one obtains results similar to those with mean-variance preferences (see Section 3). Thus, it appears that the results for CARA expected utility are an exception rather than the rule and depend on a combination

of the absence of wealth effects and the investors' indifference to the timing of the resolution of uncertainty.

## 2.4 Unconditional Asset Prices and Return Moments

Benchmarking also affects the unconditional stock price and return moments. The following theorem summarizes a first set of results:

**Theorem 6.** *The total derivative of the unconditional stock price  $S \equiv E_1[PR_f]$  and of the unconditional expected excess return  $M \equiv E_1[X - PR_f]$  with respect to the degree of benchmarking,  $\bar{\gamma}$ , are given by:*

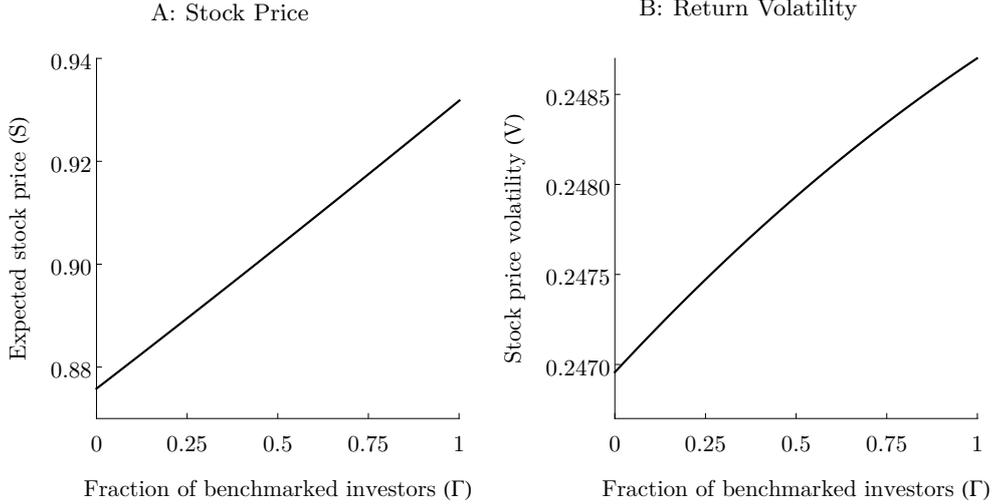
$$\frac{dS}{d\bar{\gamma}} = \frac{1}{\bar{h}} \rho + \frac{1}{\bar{h}^2} \rho (\mu_Z - \bar{\gamma}) \frac{d\bar{h}}{d\bar{\gamma}}, \quad \text{and} \quad \frac{dM}{d\bar{\gamma}} = \frac{-\rho}{\bar{h}} + \frac{-\rho}{\bar{h}^2} (\mu_Z - \bar{\gamma}) \frac{d\bar{h}}{d\bar{\gamma}}.$$

The stock price is affected directly through the excess demand and indirectly by the induced change in price informativeness. The direct effect (first component) leads to an increase in the price. The indirect effect (second component) is of the opposite sign (if  $\mu_Z - \bar{\gamma} > 0$ ), that is, lowers the price. Intuitively, with lower price informativeness risk-averse investors command a price discount. In equilibrium, the total impact on the stock price depends on the relative importance of the two effects. For realistic parameter values the first effect dominates; that is, the unconditional price increases, as illustrated in Panel A of Figure 4. The impact of benchmarking on the expected excess return follows accordingly but is of opposite sign. That is, the direct (indirect) effect reduces (increases) the expected return because it increases (reduces) the stock price.

Finally, benchmarking also affects the unconditional stock return variance:

**Theorem 7.** *The total derivative of the unconditional return variance  $V^2 \equiv Var_1(X - PR_f)$  with respect to the degree of benchmarking,  $\bar{\gamma}$ , is given by:*

$$\frac{dV^2}{d\bar{\gamma}} = -\frac{1}{\bar{h}^3} \frac{d\bar{h}}{d\bar{\gamma}}.$$



**Figure 4: Equilibrium price and return volatility.** The figure shows the unconditional expected equilibrium stock price  $S$  (Panel A) and the unconditional stock return volatility  $V$  (Panel B), as functions of the fraction of benchmarked investors in the economy,  $\Gamma$ . The graphs are based on the framework described in Section 2.1, with the following parameter values:  $\mu_X = 1.05$ ,  $\sigma_X^2 = 0.25$ ,  $\mu_Z = 1.0$ ,  $\sigma_Z^2 = 0.2$ ,  $\rho = 3$ ,  $\gamma_i = \gamma = 1/3 \forall i \in \mathcal{BM}$  and an information cost function  $\kappa(q_i) = \omega q_i^2$ , with  $\omega = 0.015$ .

In particular, an increase in the degree of benchmarking,  $\bar{\gamma}$ , unambiguously leads to an increase in the unconditional return variance, due to the decline in price informativeness ( $d\bar{h}/d\bar{\gamma} < 0$ ). This is illustrated in Panel B of Figure 4.

### 3 Benchmarking and Risk-Taking Effects

In this section, we discuss a second channel through which benchmarking affects information choice; that is, benchmarking limits an investor's willingness to speculate. To illustrate this effect, we rely on a model with CRRA-preferences which, however, is considerably less tractable.<sup>20</sup>

#### 3.1 Setup

In particular, we assume that the payoff,  $X \geq 0$ , is binomially distributed, with equally likely realizations  $X_H \equiv \mu_X + \sigma_X$  and  $X_L \equiv \mu_X - \sigma_X$ . Similarly, the investors' private

<sup>20</sup>The equilibrium price function is non-linear, and, hence, the model has to be solved numerically. Appendix B provides the technical details of our novel numerical solution approach that delivers the exact equilibrium. See also Breugem (2016) for more technical details.

signals,  $Y_i \in \{Y_H, Y_L\}$ , are binomially distributed. Intuitively, a higher precision of the private signal,  $q_i$ , increases an investor's posterior precision by increasing the correlation between the payoff,  $X$ , and his private signal,  $Y_i$ ; formally,  $\mathbb{P}[X_o | Y_i = Y_o] = \frac{1}{2} + \frac{1}{2} \sqrt{\frac{q_i}{q_i+4}}$ ,  $o \in \{H, L\}$ . The supply of the stock,  $Z$ , is log-normally distributed, with  $\ln Z \sim \mathcal{N}(\mu_Z, \sigma_Z^2)$ , guaranteeing a positive supply of the asset. Investors have CRRA preferences; that is, their time-1 utility function  $U_{1,i}^{\text{CRRA}}$  is given by:<sup>21</sup>

$$U_{1,i}^{\text{CRRA}} = E_1 \left[ E_2 \left[ \frac{C_i^{1-\rho}}{1-\rho} \right] \right] = E_1 \left[ \frac{C_i^{1-\rho}}{1-\rho} \right], \quad (13)$$

where  $\rho$  denotes the curvature parameter in utility and  $C_i = W_i - \gamma_i W_{0,i}(X/PR_f) - \chi_i$  denotes the investor's compensation. That is,  $R_B \equiv (X/PR_f)$  captures the performance of the benchmark.<sup>22</sup>

It is important to highlight that, in this setting, the strength of an individual investor's benchmarking concerns,  $\gamma_i$ , has an impact on his risk-aversion:

**Lemma 2.** *The local curvature of the CRRA-utility function (13) with respect to terminal wealth,  $W_i$ ; that is, the local coefficient of relative risk-aversion,  $\hat{\rho}_i$ , is given by:*

$$\hat{\rho}_i = \rho \frac{W_i}{W_i - \gamma_i R_B - \chi_i}. \quad (14)$$

If  $\gamma_i > 0$ , it holds that  $\hat{\rho}_i \geq \rho$  and  $\partial \hat{\rho}_i / \partial \gamma_i \geq 0$ , with strict inequalities for the case  $R_B > 0$  (or, equivalently,  $X > 0$ ).

Lemma 2 shows that the risk-aversion of a benchmarked investor,  $\hat{\rho}_i$ , exceeds the risk-aversion of a non-benchmarked investor (equal to  $\rho$ ) and is increasing in the strength of his benchmarking concerns  $\gamma_i$ .<sup>23</sup> Intuitively, keeping  $\gamma_i R_B$  fixed, a given proportional movement in wealth  $W_i$  has a stronger impact on utility (13) if the wealth in excess of the

<sup>21</sup>This is a special case of the general form in (2), with  $u_1(x) = x$  and  $u_2(x) = x^{1-\rho}/(1-\rho)$ .

<sup>22</sup>We assume that  $\gamma_i \ll 1$ , so that buying the benchmark is always a feasible strategy that yields a strictly positive compensation and, hence, strictly positive marginal utility. In particular, if  $\phi_i = 1$ , an investor's compensation is given by  $C_i = (1 - \gamma_i) R_B - \chi_i$ .

<sup>23</sup>Risk-aversion is also increasing in the benchmark portfolio's return; formally,  $\partial \hat{\rho}_i / \partial R_B > 0$ .

benchmarking component,  $W_i - \gamma_i R_B$ , is low.<sup>24</sup> Consequently, variations in the fraction of benchmarked investors,  $\Gamma$ , imply changes in the aggregate risk-bearing capacity in the economy; in particular, an increase in the fraction of benchmarked institutions leads to a *decline* in the aggregate risk-bearing capacity.

Note that, for all graphical illustrations, we again assume that the strength of the benchmarked investors' benchmarking concerns coincide:  $\gamma_i = \gamma > 0, \forall i \in \mathcal{BI}$ , while non-benchmarked investors have no relative performance concerns:  $\gamma_i = 0, \forall i \in \mathcal{NI}$ , so that variations in the degree of benchmarking in the economy are driven by changes in the fraction of benchmarked investors,  $\Gamma$ .

### 3.2 Portfolio Choice

At time  $t = 2$ , an investor must choose his optimal portfolio in order to maximize (3), conditional on his posterior beliefs, described by  $\mathcal{F}_i$ , and taking the price  $P$  as given. For illustrative purposes, one can derive the following approximate solution for the optimal fraction of wealth invested into the stock,  $\phi_i \equiv (\theta_i P)/W_{0,i}$ .<sup>25</sup>

$$\phi_i \approx \frac{E_2[r^e | \mathcal{F}_j]}{\frac{\rho}{1-\gamma_i} \text{Var}_2[r^e | \mathcal{F}_j]} + \gamma_i \equiv \phi_i^{MV} + \gamma_i, \quad (15)$$

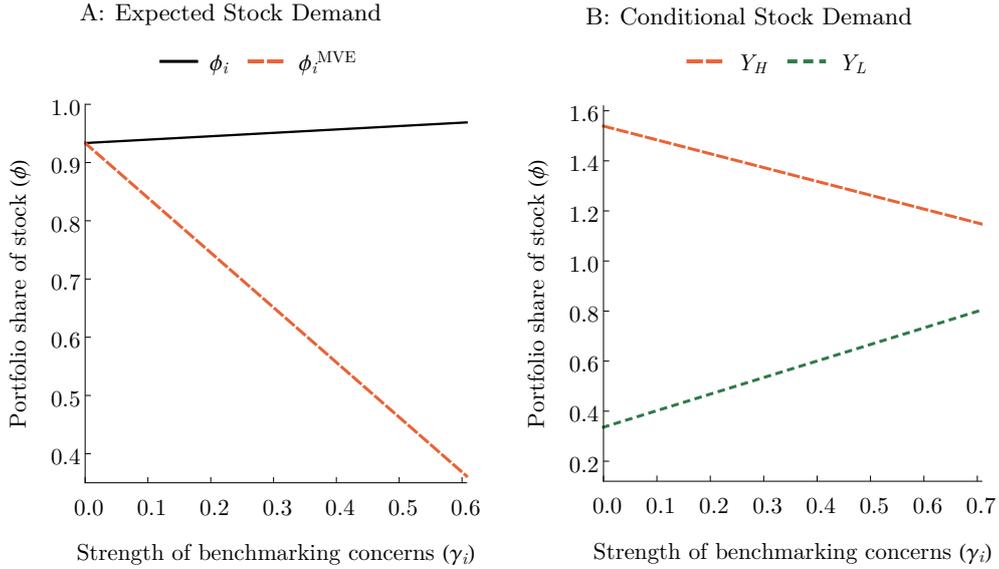
where  $r^e$  denotes the stock's excess return,  $r^e \equiv X/P - R_f$ .

The optimal demand for the stock has the same two components as in the case of constant absolute risk-aversion. First, the mean-variance portfolio,  $\phi_i^{MV}$ , but for an investor with *local risk-aversion*  $\rho/(1 - \gamma_i)$ .<sup>26</sup> As a result, this component is now declining in the strength of an investor's benchmarking concerns,  $\gamma_i$ . Second, the hedging portfolio,  $\gamma_i$ , which is increasing in an investor's benchmarking concerns, but is information-insensitive.

<sup>24</sup>In that regard, benchmarking concerns are akin to a subsistence level or background risk.

<sup>25</sup>The approximation is valid if  $(1/(1 - \gamma))(W_{0,i}R_f + (\phi_i - \gamma_i)r^e - \gamma_i - \kappa(q_i))$  is around one and, e.g., quite accurate if the risk-free rate is below 5% and the expected excess return is below 10%.

<sup>26</sup>This is consistent with Lemma 2. In particular, a benchmarked investor's risk-aversion  $\hat{\rho}_i$  can be rewritten as  $\rho \left(1 - \gamma_i \frac{R_B - \chi_i/\gamma_i}{W_i}\right)^{-1}$ . In a one-stock economy and ignoring the (small) information cost,  $\chi_i$ , the last part is given by  $(1 + r^e)/(R_f + \phi r^e)$ , which, for realistic values of  $R_f$  (around 1.0) and  $\phi$  (around 1.0), is close to 1, such that  $\hat{\rho}_i$  reduces to  $\rho/(1 - \gamma_i)$ .

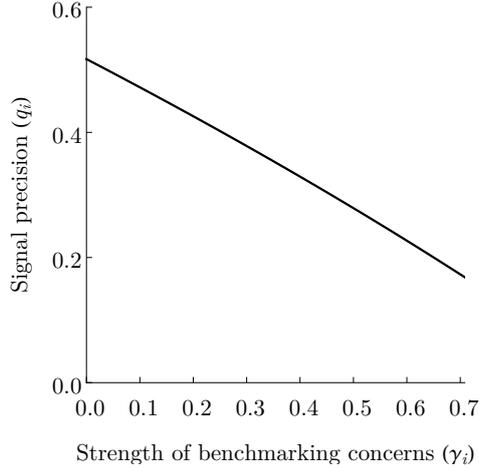


**Figure 5: Stock demand.** The figure illustrates a benchmarked investor’s portfolio choice for various levels of the strength of his benchmark concerns,  $\gamma_i$  and conditional on a given signal precision  $q_i$ . Panel A shows the expected stock demand and the mean-variance portfolio component. Panel B depicts the stock demand conditional on a specific signal realization  $Y_i \in \{Y_H, Y_L\}$ . The graphs are based on the CRRA framework described in Section 3.1, with the following parameter values:  $\mu_X = 1.05$ ,  $\sigma_X^2 = 0.25$ ,  $\mu_Z = 1.0$ ,  $\sigma_Z^2 = 0.2$ ,  $\rho = 3$ ,  $q_i = 0.1667$ , and an information cost function  $\kappa(q_i) = \omega q_i^2$ , with  $\omega = 0.015$ .

Naturally, for non-benchmarked investors ( $\gamma_i = 0$ ), the optimal demand reduces to the standard mean-variance portfolio of an investor with risk-aversion  $\rho$ .

Figure 5 illustrates how the optimal stock demand of a benchmarked investor varies with the strength of his benchmarking concerns,  $\gamma_i$ . In particular, Panel A shows that the overall portfolio share of the stock,  $\phi_i$ , is increasing in an investor’s benchmarking concerns  $\gamma_i$ —comparable to the case of constant absolute risk-aversion. But, due to the decline in the mean-variance portfolio,  $\phi_i^{MV}$ , the overall increase is substantially smaller.

Panel B further illustrates that effect. It shows an investor’s stock demand, conditional on a specific signal realization  $Y_i \in \{Y_H, Y_L\}$ . As expected, the investor, in general, overweights the stock following a high signal,  $Y_H$ , and vice versa for a low signal,  $Y_L$ . Most importantly, however, the “spread” between a benchmarked investor’s stock demand following a positive and a negative signal narrows as his benchmarking concerns,  $\gamma_i$ , strengthen. That is, in the presence of benchmarking, the investor’s willingness to speculate declines (due to his higher risk-aversion). This result is in clear contrast to the case of constant absolute risk-aversion (with or without a preference for the early resolution of uncertainty), for



**Figure 6: Information demand.** The figure depicts the optimal information choice, that is, optimal signal precision  $q_i$  of a single (small) benchmarked investor, as function of the strength of his benchmark concerns,  $\gamma_i$ . The graph is based on the CRRA framework described in Section 3.1, with the following parameter values:  $\mu_X = 1.05$ ,  $\sigma_X^2 = 0.25$ ,  $\mu_Z = 1.0$ ,  $\sigma_Z^2 = 0.2$ ,  $\rho = 3$ ,  $\gamma_i = 1/3$ , and an information cost function  $\kappa(q_i) = \omega q_i^2$ , with  $\omega = 0.015$ .

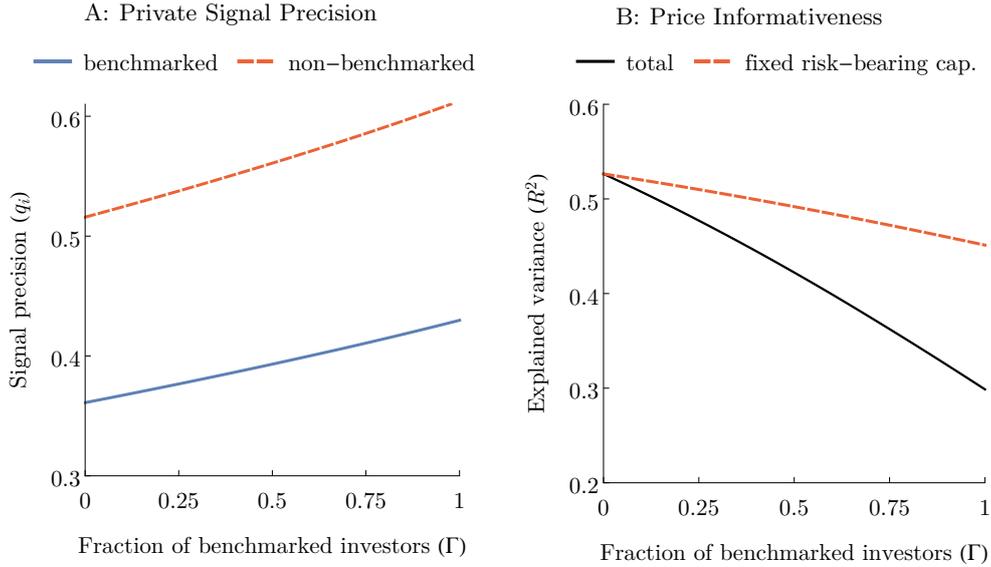
which the mean-variance portfolio component is independent of the strength of an investor’s benchmarking concerns.

### 3.3 Information Choice

To build the basic intuition for the incremental impact of the risk-taking effect on information choice in the clearest possible way, consider first the case of a single (small) benchmarked investor. Hence, benchmarking does not affect the equilibrium price. Recall from Remark 1, that, in this case, with mean-variance preferences, the strength of the investor’s individual benchmarking concerns,  $\gamma_i$ , does not affect his information choice.

As Figure 6 depicts, this result does not hold with CRRA preferences. In particular, the optimal signal precision of a single benchmarked investor is declining in the strength of his benchmarking concerns,  $\gamma_i$ . That is, the investor anticipates that his time-2 portfolio choice will be less sensitive to the realization of his private signal because he trades less aggressively and, consequently, the value of private information declines.

Intuitively, the lower value of information for benchmarked investors implies that benchmarked institutions endogenously collect less information than non-benchmarked institu-



**Figure 7: Equilibrium information choice.** The figure illustrates the equilibrium signal precision, as a function of the fraction of benchmarked investors in the economy,  $\Gamma$ . Panel A shows the optimal signal precision of the two groups of investors and Panel B depicts the resulting price informativeness, that is, the precision of the public price signal. Precision is measured as  $R^2$ , that is, the fraction of the variance of the payoff  $X$  that is explained by the investors' private information and the stock price, respectively. The graphs are based on the CRRA framework described in Section 3.1, with the following parameter values:  $\mu_X = 1.05$ ,  $\sigma_X^2 = 0.25$ ,  $\mu_Z = 1.0$ ,  $\sigma_Z^2 = 0.2$ ,  $\rho = 3$ ,  $\gamma_i = \gamma = 1/3 \forall i \in \mathcal{BI}$ , and an information cost function  $\kappa(q_i) = \omega q_i^2$ , with  $\omega = 0.015$ .

tions in equilibrium—in stark contrast to the earlier results (in which both groups of market participants chose the same signal precision). This is illustrated in Panel A of Figure 7.

The figure also shows the novel result that *both groups of investors choose a higher precision* for their private signals as the share of benchmarked investors,  $\Gamma$ , increases. To understand this result, note that, all else equal, an increase in the fraction of benchmarked investors implies a shift toward less-informed investors and, in turn, a decline in aggregate information acquisition. Accordingly, the marginal benefit of private information goes up, increasing the incentives of all investors to choose a more precise signal.

However, as shown in Panel B of Figure 7, price informativeness, that is, the amount of private information aggregated and revealed by the stock price, is still declining in the fraction of benchmarked investors. This result is driven by two distinct economic forces: average information acquisition and information aggregation. First, an increase in the share of benchmarked investors implies that better-informed non-benchmarked investors are replaced by less-informed benchmarked investors (as illustrated in Panel A). Second, it

implies a shift toward a group of investors that trades less aggressively based on available private information, because of their lower risk appetite. Consequently, less information can be aggregated into the price. Both effects imply a decline in price informativeness.

To (quantitatively) disentangle this effect from the information scale effect, Panel B of Figure 7 also shows how price informativeness varies with the fraction of benchmarked investors, if one would keep the aggregate risk-bearing capacity fixed.<sup>27</sup> While the majority of the decline is due to the reduction in aggregate risk-bearing capacity, the effect from information scale effects is non-negligible and can explain about 25% of the decline for the parameters used for illustration.

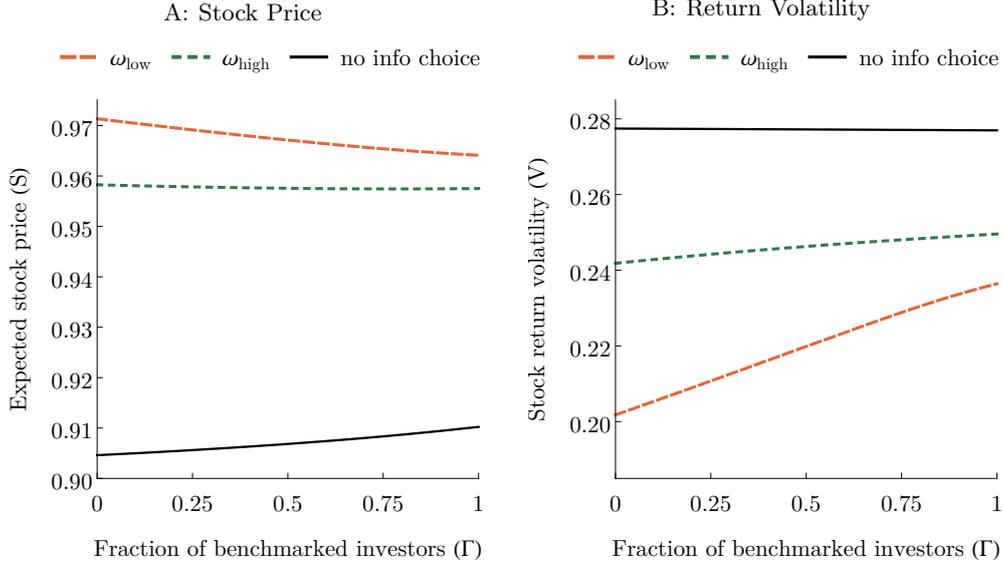
### 3.4 Unconditional Asset Prices and Return Moments

The unconditional stock price is affected through two channels. On the one hand, an increase in the fraction of benchmarked investors increases the aggregate hedging demand. Because the stock's supply is fixed, the price goes up. On the other hand, price informativeness declines in the fraction of benchmarked investors. This, in turn, increases posterior uncertainty, and, hence, risk-averse investors command a lower price. These two effects are in conflict, such that the net effect depends on their relative importance.

As shown in Panel A of Figure 8, for low information costs ( $\omega_{low}$ ), the *price of the stock declines in the fraction of benchmarked investors*. Intuitively, because of the substantial decline in price informativeness, the second effect dominates. In contrast, for higher information costs ( $\omega_{high}$ ), the price is almost flat. That is, the negative price effect resulting from a decline in price informativeness is effectively offset by the positive price effect induced by the aggregate hedging demand. In both cases, the impact of benchmarking is distinctly different from the setting with mean-variance preferences in which the price practically always increases. The implications for the expected excess return are simply of the opposite sign. The stock's return volatility is unambiguously increasing in the share of benchmarked investors, as illustrated in Panel B, because of the higher posterior uncertainty.

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<sup>27</sup>For that purpose, we reduce the curvature parameter of benchmarked investors' utility,  $\rho^{BI}$ , such that their risk-aversion, after accounting for their benchmarking concerns, equals that of non-benchmarked in-



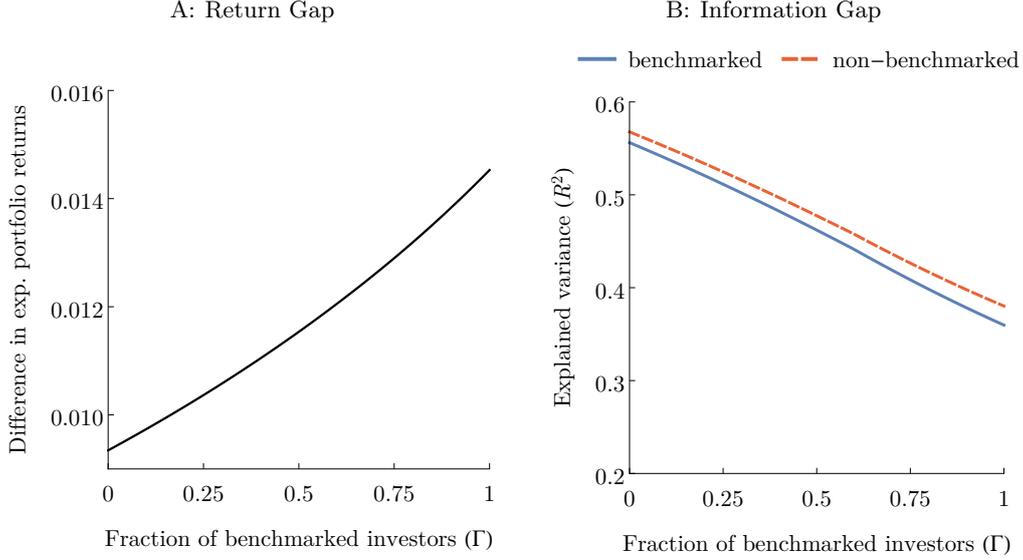
**Figure 8: Equilibrium price and return volatility.** The figure shows the unconditional expected equilibrium stock price (Panel A) and the unconditional stock return volatility (Panel B), as functions of the fraction of benchmarked investors in the economy,  $\Gamma$ . The graphs are based on the CRRA framework described in Section 3.1, with the following parameter values:  $\mu_X = 1.05$ ,  $\sigma_X^2 = 0.25$ ,  $\mu_Z = 1.0$ ,  $\sigma_Z^2 = 0.2$ ,  $\rho = 3$ ,  $\gamma_i = \gamma = 1/3 \forall i \in \mathcal{BI}$  and an information cost function  $\kappa(q_i) = \omega q_i^2$ , with  $\omega_{low} = 0.015$ , and  $\omega_{high} = 0.045$ .

### 3.5 Investors' Portfolio Returns

The differences in the investors' information choices also have an impact on their expected portfolio returns. In particular, as shown in Panel A of Figure 9, the difference between the expected portfolio return of the non-benchmarked and the benchmarked investors is increasing with the fraction of benchmarked investors. That is, as a bigger share of wealth is managed by benchmarked institutions, non-benchmarked institutional investors can generate higher expected portfolio returns relative to their benchmarked peers.

To understand this effect, recall that the precision of the private information of benchmarked investors is lower than that of non-benchmarked investors (see Panel A of Figure 7). Moreover, as the fraction of benchmarked investors,  $\Gamma$ , increases, less information is revealed through the public stock price. Consequently, the importance of private information increases, and the “information gap” between investors' information sets  $\mathcal{F}_i$  widens, as illustrated in Panel B of Figure 9. This has two effects for portfolio returns. First,

investors. In particular, we choose the curvature parameter such that a single benchmarked investor's mean-variance portfolio component matches that of a non-benchmarked investor.



**Figure 9: Return and information gap.** The figure illustrates the return and information gap between the two groups of institutional investors, as a function of the fraction of benchmarked investors in the economy,  $\Gamma$ . Panel A shows the difference between the expected portfolio return of the non-benchmarked and the benchmarked institutions. Panel B shows the precision of their information sets,  $\mathcal{F}_i$ , measured as a fraction of the variance of the payoff  $X$  that they can explain. The graphs are based on the CRRA framework described in Section 3.1, with the following parameter values:  $\mu_X = 1.05$ ,  $\sigma_X^2 = 0.25$ ,  $\mu_Z = 1.0$ ,  $\sigma_Z^2 = 0.2$ ,  $\rho = 3$ ,  $\gamma_i = \gamma = 1/3 \forall i \in \mathcal{BI}$ , and an information cost function  $\kappa(q_i) = \omega q_i^2$ , with  $\omega = 0.015$ .

non-benchmarked investors hold, on average, more of the stock, because their superior information renders the investment less risky. Due to the stock's positive risk premium, this increases their expected portfolio return. Second, conditional on a signal realization, non-benchmarked investors' trades are more profitable. Both effects lead to unambiguously higher expected portfolio returns for non-benchmarked investors and are not present in the case of CARA preferences.

### 3.6 Robustness

The results are robust to changes in the parameter values. Intuitively, the increase in the benchmarked investors' risk-aversion, as discussed in Lemma 2, does not rely on specific parameter values. Hence, benchmarked investors always decide to collect less information. As a result, both economic mechanisms—information scale effects and wealth effects—work in the same direction and cause a decline in price informativeness. Consequently, the difference in the expected portfolio returns between benchmarked and non-benchmarked investors in-

creases in the fraction of benchmarked investors and so does return volatility. The only quantities for which the impact of benchmarking depends on the choice of parameter values are, as discussed above, the unconditional expected stock price and excess return. In both cases, there are two economic forces that are in conflict, with the net effect depending on their magnitudes.

Also, note that the assumption of binomial payoff and signals distributions is only for numerical convenience; that is, the results are robust to incorporating continuous distributions. In particular, the results are qualitatively unchanged if one relies on log-normal distributions for the payoff and the private signals, as discussed in Appendix C and illustrated in Figure A1 therein.

## 4 Extensions

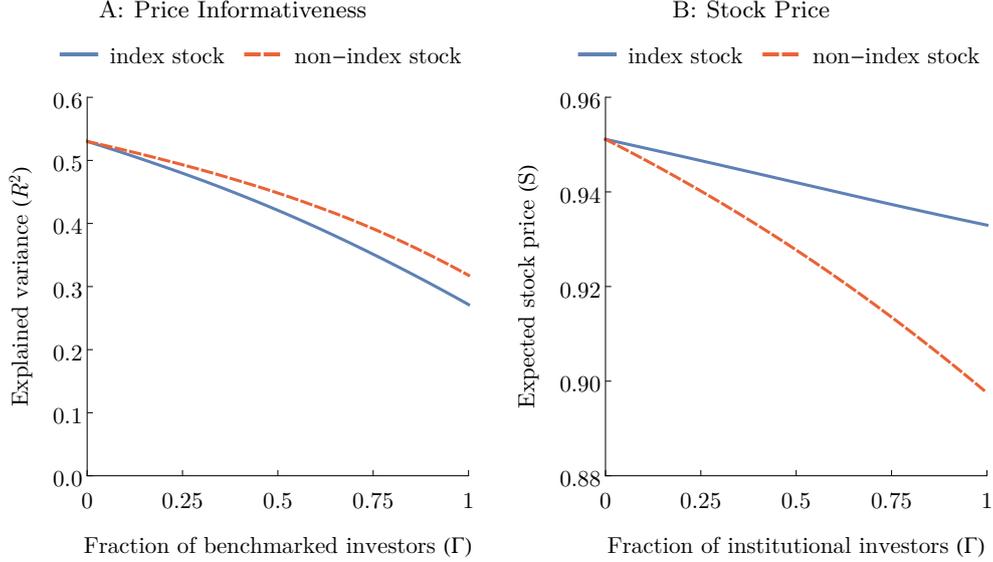
This section introduces two extensions of our basic economic framework. First, an extension to multiple stocks and, second, an extension to asymmetric benchmarking concerns. In both cases, we rely on CRRA preferences, capturing information scale and risk-taking effects.

### 4.1 Multiple Stocks

Our objective in this section is to study how the results generalize in an economy with multiple stocks. In particular, we consider an economy with two symmetric stocks,  $k \in \{1, 2\}$ , in which each investor has to decide simultaneously on the signal precision for the two stocks,  $q_{i,k}$ . Similar to the one-stock setup described in Section 3.1, the stocks' payoffs,  $X_k$ , and signals,  $Y_{i,k}$ , are assumed to be binomially distributed, and the supply  $Z_k$  is assumed to be log-normal. In addition, we assume that payoffs, signals, and supplies are independent across assets.<sup>28</sup> The first stock (the “index stock”) also serves as the benchmark; that is, the compensation of the benchmarked investors is declining in its (gross) return:  $\gamma_{i,1} > 0, \forall i \in \mathcal{BI}$ . In contrast, the second stock (the “non-index stock”) is not part of the benchmark; that is,  $\gamma_{i,2} = 0, \forall i$ .

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<sup>28</sup>We focus on the case of stocks with symmetric distributions of independent fundamentals and noise, so that all differences between the two stocks arise exclusively from benchmarking.



**Figure 10: Multiple stocks.** The figure shows price informativeness and the unconditional expected stock price of the index and non-index stock, as functions of the fraction of benchmarked investors in the economy. Panel A depicts price informativeness, that is, the precision of the public price signal, measured the fraction of the variance of the payoff  $X_k$  that is explained by the corresponding stock price ( $R^2$ ). Panel B shows the unconditional expected equilibrium stock price. The graphs are based on the symmetric two-stock CRRRA framework described in Section 4.1, with the following parameter values:  $\mu_{X,k} = 1.05$ ,  $\sigma_{X,k}^2 = 0.25$ ,  $\mu_{Z,k} = 1.0$ ,  $\sigma_{Z,k}^2 = 0.2$  for  $k \in \{1, 2\}$ ,  $\rho = 3$ ,  $\gamma_{i,1} = \gamma = 1/3 \forall i \in \mathcal{BM}$ ,  $\gamma_{i,2} = 0 \forall i$ , and an information cost function  $\kappa(q_{i,k}) = \omega q_{i,k}^2$ ,  $k \in \{1, 2\}$ , with  $\omega = 0.015$ .

Panel A of Figure 10 illustrates that price informativeness is declining for both stocks in the fraction of benchmarked investors,  $\Gamma$ . However, the decline in price informativeness is more pronounced for the index stock. In particular, the increase in the benchmarked investors' risk-aversion affects both stocks and, thus, limits their speculative activities in the non-index stock as well.<sup>29</sup> However, because of the absence of an information-insensitive aggregate hedging demand, the expected number of shares of the non-index stock in investors' portfolios that are sensitive to private information does not decline. Consequently, a piece of information can be applied to more shares, and, hence, the value of private information and price informativeness are higher.

Panel B shows that the price of the index stock is always higher than that of the non-index stock, with the price gap widening as the fraction of benchmarked investors increases. Intuitively, the aggregate hedging demand for the index stock increases its price relative to

<sup>29</sup>Technically, conditional on a benchmarked investor's posterior beliefs, his optimal demand for the index stock is characterized by (15), with  $\gamma_{i,1} > 0, \forall i \in \mathcal{BL}$ . Moreover, his demand for the non-index stock is also described by (15), but with  $\gamma_{i,2} = 0$ .

the non-index stock. At the same time, the stronger decline in price informativeness for the index stock implies more of a discount. In equilibrium, the first effect is qualitatively stronger, explaining the observed price pattern. The results for the expected stock return follow accordingly. For both stocks, the unconditional return volatility is increasing because price informativeness declines. However, due to the more pronounced decline in the index stock's price informativeness, the increase in its return volatility is stronger as well. Finally, non-benchmarked investors' private information in both assets is more precise, so that the results regarding their superior expected portfolio returns carry over.

## 4.2 Asymmetric Benchmarking Concerns

Second, we consider an extension to asymmetric benchmarking concerns. In particular, while U.S. mutual funds are limited to use linear (Fulcrum) performance fees, asymmetric performance fees for mutual funds can be used outside the United States and are also much more standard in other asset classes (such as hedge funds or private equity). Moreover, implicit benchmarking concerns at the fund manager's level or arising from the flow-performance relation are typically also rather asymmetric.<sup>30</sup>

The easiest way to incorporate benchmarking concerns that are non-linear in the benchmark's return is to model a benchmarked investor's compensation,  $C_i$ , as:

$$C_i(W_i, R_B) = W_i - W_{0,i} R_B (\gamma_i^+ \mathbb{1}_{R_B > 1} + \gamma_i^- \mathbb{1}_{R_B \leq 1}) - \chi_i,$$

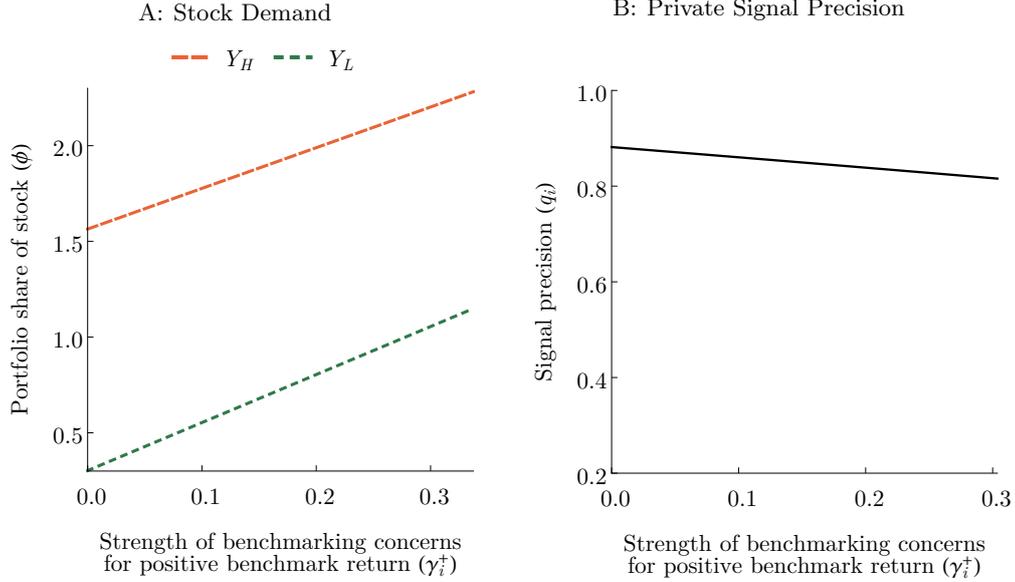
where  $\mathbb{1}$  denotes the indicator function. In particular, with this specification, the sensitivity of the investor's compensation with respect to positive (gross) benchmark returns,  $\gamma_i^+$ , can differ from the sensitivity with respect to negative (gross) benchmark returns,  $\gamma_i^-$ .<sup>31</sup>

As a full analysis of the equilibrium with asymmetric benchmarking concerns is outside the scope of this paper, we concentrate on the case of a single (small) benchmarked investor with asymmetric benchmarking concerns; that is, benchmarking does not affect the equilib-

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<sup>30</sup>See, e.g., [Ibert, Kaniel, van Nieuwerburgh, and Vestman \(2017\)](#) for empirical evidence on Swedish mutual fund managers and [Sirri and Tufano \(1998\)](#), [Chevalier and Ellison \(1997\)](#) and [Huang, Wei, and Yan \(2007\)](#) for evidence on an asymmetric flow-performance relation.

<sup>31</sup>Technically, this introduces a kink at  $R_B = 1$ , which further complicates the solution.



**Figure 11: Asymmetric benchmarking concerns.** The figure illustrates a single (small) benchmarked investor’s stock and information demand, as functions of the strength of his benchmarking concerns for positive benchmark returns  $\gamma_i^+$ . Panel A shows the stock demand conditional on a specific signal realization  $Y_i \in \{Y_H, Y_L\}$ , with exogenous signal precision  $q_i = 0.1667$ . Panel B depicts the optimal, endogenous signal precision  $q_i$ . The graphs are based on the framework described in Section 4.2, with the following parameter values:  $\mu_X = 1.05$ ,  $\sigma_X^2 = 0.25$ ,  $\mu_Z = 1.0$ ,  $\sigma_Z^2 = 0.2$ ,  $\rho = 3$ ,  $\gamma_i^- = 0$ , and an information cost function  $\kappa(q_i) = \omega q_i^2$ , with  $\omega = 0.015$ .

rium stock price. For illustration, consider the case of varying the strength of the investor’s benchmarking concerns for positive benchmark returns,  $\gamma_i^+$ , while setting the strength of his benchmarking concerns for negative benchmark returns,  $\gamma_i^-$ , to zero.<sup>32</sup>

Panel A of Figure 11 shows the investor’s stock demand conditional on a specific signal realization  $Y_i \in \{Y_H, Y_L\}$ . Interestingly, the investor’s willingness to speculate, measured by the “spread” between his stock demand following a high and a low signal, is declining substantially less than in the case of symmetric benchmarking concerns (see Panel B of Figure 5). On the one hand, when the investor observes a signal indicating a low payoff and, hence, a negative benchmark return, the aggressiveness with which he trades is essentially unaffected because  $\gamma_i^- = 0$ . On the other hand, when the investor observes a signal indicating a high payoff and, hence, a positive benchmark return, the aggressiveness with which he trades is only marginally affected relative to the no-benchmarking case (although

<sup>32</sup>The results are qualitatively unchanged if one varies the strength of the investor’s benchmarking concerns for negative benchmark returns,  $\gamma_i^-$ , while setting the strength of his benchmarking concerns for positive benchmark returns,  $\gamma_i^+$ , to zero.

$\gamma_i^+ > 0$ ). Intuitively, because the investor over-weights the benchmark following a high signal (i.e., the stock’s portfolio share is above 1), his expected final wealth,  $W_i$ , relative to the benchmark component  $\gamma_i^+ R_B$ , is high, limiting the increase in local risk-aversion (14).

Consequently, the sensitivity of the investor’s portfolio with respect to private information declines only marginally as the strength of the investor’s benchmarking concerns for positive benchmark returns,  $\gamma_i^+$ , increases. Hence, as shown in Panel B of Figure 11, the under-provision of private information seems to be considerably weaker than in the case of symmetric benchmarking concerns. That is, the investor chooses a substantially higher signal precision than in the symmetric case (see Figure (6))—although the optimal signal precision is still declining in the strength of his benchmarking concerns.

## 5 Key Predictions and Conclusion

Relative performance concerns play a key role in the decisions of institutional investors who are in the business of acquiring information and using that information for portfolio management. In this paper, we develop an economic framework that explicitly accounts for benchmarking *and* investors’ joint portfolio and information choice.

We identify two distinct economic channels through which benchmarking affects information acquisition. Key to both economic mechanisms is the *interaction* between investors’ portfolio and information choices. First, the information-insensitive aggregate hedging demand of benchmarked investors reduces, in equilibrium, the expected number of shares in investors’ portfolios that are sensitive to private information. Second, an investor’s individual benchmarking concerns limit his willingness to speculate, so that the sensitivity of his portfolio with respect to private information declines. Both effects lower the value of private information, and, hence, informational efficiency deteriorates. Notably, incorporating risk-taking effects leads to quantitatively, but also qualitatively different implications. For example, benchmarked and non-benchmarked investors differ in their information choice and, hence, in their portfolio returns. Moreover, the equilibrium stock price can decline in the fraction of benchmarked investors.

The model generates a rich set of predictions, which are empirically refutable. For example, similar to our results, the literature on “index effects” has found a higher price and a lower Sharpe ratio for index stocks relative to non-index stocks. While these results are not unique to our model (see, e.g., [Cuoco and Kaniel \(2011\)](#) and [Basak and Pavlova \(2013\)](#)), the joint analysis of portfolio and information choices also leads to novel and unique predictions. In particular, our model predicts a lower price informativeness for index stocks. In line with this prediction, [Israeli, Lee, and Sridharan \(2017\)](#) document that an increase in ETF ownership is associated with less-informative security prices. Also, our model makes unique predictions about the expected portfolio returns of benchmarked and non-benchmarked investors that would not arise in models with symmetric information and can be confronted with the data.

In this paper, the benchmarking concerns of the institutional investors are exogenous. However, in the presence of agency frictions, they might be an endogenous outcome; that is, in the presence of benchmarking the incentives of institutional investors (fund managers) and their investors (see, e.g., [Buffa, Vayanos, and Woolley \(2014\)](#) or [Sotes-Paladino and Zapatero \(2016\)](#)) as well as between fund managers within an asset management firm (see, e.g., [van Binsbergen, Brandt, and Koijen \(2008\)](#)) might be better aligned. In this regard, our paper highlights a novel tension between benchmarking as a tool to align incentives and its adverse effects on managers’ information acquisition and, hence, their portfolio returns.

Consequently, a logical extension of our work would be to study optimal asset management contracts in a setting with agency frictions *and* information acquisition.<sup>33</sup> In particular, allowing for nonlinear contracts might lead to many new insights regarding the optimal compensation in the asset management industry. Also, extensions of our framework can be used to study the optimal size of benchmarked investors in the economy ([Pástor and Stambaugh \(2012\)](#)) or the costs of passive investing.

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<sup>33</sup>Intuitively, because monitoring information acquisition is difficult, fund managers have an incentive to lie, which, naturally, creates an agency conflict.

# Appendix

## A Proofs for Section 2

### Proof of Theorem 1

Plugging wealth  $W_i$  into an investor's compensation  $C_i = W_i - \gamma_i (X - PR_f) - \chi_i$  and substituting into the time-2 objective function  $E_2 [C_i] - \rho/2 \text{Var}_2 (C_i)$  yields:

$$\begin{aligned} & E_2 [W_{0,i} R_f + (\theta_i - \gamma_i) (X - PR_f) - \chi_i] - \frac{\rho}{2} \text{Var}_2 (W_{0,i} R_f + (\theta_i - \gamma_i) (X - PR_f) - \chi_i) \\ &= W_{0,i} R_f - \chi_i + (\theta_i - \gamma_i) (\hat{\mu}_{X,i} - PR_f) - \frac{\rho}{2} (\theta_i - \gamma_i)^2 \frac{1}{h_i}. \end{aligned}$$

Hence, the first-order condition with respect to  $\theta_i$  is given by:

$$(\hat{\mu}_{X,i} - PR_f) - \rho (\theta_i - \gamma_i) \frac{1}{h_i} \triangleq 0,$$

which, after re-arranging, yields Theorem 1.

### Proof of Theorem 2

We conjecture (and later verify) that the equilibrium stock price is a linear function of the (unobservable) payoff and the (unobservable) aggregate stock supply:

$$PR_f = a + bX - dZ. \tag{A1}$$

As a result, conditional on a signal  $Y_i$  and the realized price  $PR_f$ , investor  $i$ 's posterior precision and mean are given by:

$$h_i = \underbrace{\frac{1}{\sigma_X^2} + \frac{b^2}{d^2 \sigma_Z^2}}_{\equiv h_0} + q_i = \frac{(1 + q_i \sigma_X^2) d^2 \sigma_Z^2 + b^2 \sigma_X^2}{\sigma_X^2 d^2 \sigma_Z^2}, \tag{A2}$$

$$\begin{aligned} \hat{\mu}_{X,i} &= \mu_X + \frac{1}{h_i} q_i (Y_i - \mu_X) + \frac{1}{h_i} \frac{b^2}{d^2 \sigma_Z^2} \left( \frac{PR_f - a + d\mu_Z}{b} - \mu_X \right) \\ &= \frac{d^2 \sigma_Z^2 (q_i \sigma_X^2 Y_i + \mu_X) + b (PR_f - a + d\mu_Z) \sigma_X^2}{(1 + q_i \sigma_X^2) d^2 \sigma_Z^2 + b^2 \sigma_X^2}, \end{aligned} \tag{A3}$$

where we substituted the posterior precision  $h_i$  with (A2) in the last step.

Intuitively, investors have three pieces of information that they aggregate to form their expectation of the asset's payoff: their prior beliefs, their private signals and the public stock price. The posterior mean is simply the weighted average of the three signals' realizations, while the posterior precision is weighted average of the signals' precisions.

Plugging the posterior mean and precision into the optimal stock demand in (6) yields:

$$\theta_i = \frac{1}{\rho} \left( \frac{\mu_x}{\sigma_x^2} + q_i Y_i + \frac{b(-a + d\mu_Z)}{d^2\sigma_Z^2} - PR_f \left( \frac{1}{\sigma_x^2} + q_i + \frac{b}{d^2\sigma_Z^2}(b-1) \right) \right) + \gamma_i. \quad (\text{A4})$$

Market clearing requires that aggregate demand, that is, demand (A4) integrated over all investors, equals (random) supply:

$$\begin{aligned} & \int_0^\Gamma \left[ \frac{1}{\rho} \left( \frac{\mu_x}{\sigma_x^2} + q_i Y_i + \frac{b(-a + d\mu_Z)}{d^2\sigma_Z^2} - PR_f \left( \frac{1}{\sigma_x^2} + q_i + \frac{b}{d^2\sigma_Z^2}(b-1) \right) \right) + \gamma_i \right] di \\ & + \int_\Gamma^1 \left[ \frac{1}{\rho} \left( \frac{\mu_x}{\sigma_x^2} + q_i Y_i + \frac{b(-a + d\mu_Z)}{d^2\sigma_Z^2} - PR_f \left( \frac{1}{\sigma_x^2} + q_i + \frac{b}{d^2\sigma_Z^2}(b-1) \right) \right) \right] di \triangleq Z. \end{aligned}$$

Substituting the private signals by  $Y_i = X + \varepsilon_i$  and using that investors are, on average, unbiased ( $\int_0^\Lambda \varepsilon_i = 0$ ,  $\int_\Lambda^1 \varepsilon_i = 0$ ), this can be simplified to:

$$\frac{1}{\rho} \left( \frac{\mu_x}{\sigma_x^2} + \bar{q} X + \frac{b(-a + d\mu_Z)}{d^2\sigma_Z^2} - PR_f \left( \frac{1}{\sigma_x^2} + \bar{q} + \frac{b(b-1)}{d^2\sigma_Z^2} \right) \right) + \bar{\gamma} \triangleq Z, \quad (\text{A5})$$

with the precision of private information of the average agent,  $\bar{q}$ , and the degree of benchmarking in the economy,  $\bar{\gamma}$ , being defined in (9).

Solving the market-clearing condition (A5) for the price  $PR_f$  yields:

$$PR_f = \frac{d^2\sigma_Z^2}{\bar{h} d^2\sigma_Z^2 - b} \left( \frac{\mu_x}{\sigma_x^2} + \rho \bar{\gamma} + \frac{b(-a + d\mu_Z)}{d^2\sigma_Z^2} \right) + \frac{\bar{q} d^2\sigma_Z^2}{\bar{h} d^2\sigma_Z^2 - b} X - \frac{\rho d^2\sigma_Z^2}{\bar{h} d^2\sigma_Z^2 - b} Z,$$

with  $\bar{h}$  being defined in (8). Moreover, it implies a ratio of the price sensitivities with respect to the payoff and the noisy supply  $b/d$  of  $\bar{q}/\rho$ .

Finally, matching the coefficients of this price function to the ones of the conjecture (A1) and solving the resulting equation system for  $a$ ,  $b$  and  $d$  yields the price function (7). Finally, plugging the coefficients into  $h_0$  defined in (A2) yields its expression in (8).

### Proof of Theorem 3 and Lemma 1

Plugging the optimal portfolio  $\theta_i$  in (6) into compensation  $C_i$  and computing its expectation and variance, yields:

$$E_2[C_i] = W_0 R_f + h_i \frac{(\hat{\mu}_{X,i} - P R_f)^2}{\rho} - \kappa(q_i) = R_f \left( W_0 - \frac{\kappa(q_i)}{R_f} \right) + \frac{1}{\rho} z_i^2, \quad (\text{A6})$$

$$\text{Var}_2(C_i) = h_i^2 \left( \frac{\hat{\mu}_{X,i} - P R_f}{\rho} \right)^2 \frac{1}{h_i} = h_i \frac{(\hat{\mu}_{X,i} - P R_f)^2}{\rho^2} = \frac{1}{\rho^2} z_i^2, \quad (\text{A7})$$

with  $z_i$  being defined as  $z_i \equiv \sqrt{h_i}(\hat{\mu}_{X,i} - P R_f)$ . Substituting the expectation and variance into the investor's utility function (5) gives the investor's time-1 objective function (10).

To ease the computations of  $E_1[z_i^2]$ , introduce  $u_i = \sqrt{h_i} z_i = h_i(\hat{\mu}_{X,i} - P R_f)$ . Moreover, note that viewed from period 1 both— $\hat{\mu}_{X,i}$  and  $P R_f$ —are random variables. In particular, substituting  $Y_i$  by  $X + \varepsilon_i$  and  $\frac{P R_f - a + d \mu_Z}{b}$  by  $X - \frac{\rho}{q}(Z - \mu_Z)$  in the expression for the posterior mean (A3) yields:

$$\hat{\mu}_{X,i} = \frac{1}{h_i} \left( \left( \frac{\mu_X}{\sigma_X^2} + \frac{\mu_Z \bar{q}}{\rho \sigma_Z^2} \right) + q_i \varepsilon_i + \left( q_i + \frac{\bar{q}^2}{\rho^2 \sigma_Z^2} \right) X - \frac{\bar{q}}{\rho \sigma_Z^2} Z \right). \quad (\text{A8})$$

Consequently, after substituting (A8) for  $\hat{\mu}_{X,i}$  and (7) for  $P R_f$  in  $u_i$ , one gets:

$$\begin{aligned} u_i &= \left( \frac{\mu_X}{\sigma_X^2} + \frac{\bar{q} \mu_Z}{\rho \sigma_Z^2} \right) \left( 1 - \frac{h_i}{\bar{h}} \right) - \frac{h_i}{\bar{h}} \rho \bar{\gamma} + q_i \varepsilon_i + \left( \frac{1}{\sigma_X^2} \left( \frac{h_i}{\bar{h}} - 1 \right) \right) X \\ &\quad + \frac{\rho}{\bar{q}} \left( h_i - \frac{\bar{q}^2}{\rho^2 \sigma_Z^2} - \frac{h_i}{\bar{h} \sigma_X^2} \right) Z. \end{aligned}$$

Integrating over the distributions of  $P$  and  $Y_i$ , the time-1 expectation and variance are:

$$\begin{aligned} E_1[u_i] &= \frac{h_i \rho}{\bar{h}} (\mu_Z - \bar{\gamma}), \\ \text{Var}_1(u_i) &= \frac{h_i^2}{\bar{h}^2} \left( \frac{1}{\sigma_X^2} + \frac{\rho^2}{\bar{q}^2} \sigma_Z^2 \left( \bar{h} - \frac{1}{\sigma_X^2} \right)^2 \right) - h_i = \frac{h_i^2}{\bar{h}^2} (\bar{h} + \rho^2 \sigma_Z^2 + \bar{q}) - h_i. \end{aligned}$$

Using that  $E_1 [u_i^2] = \text{Var}_1(u_i) + E_1[u_i]^2$  (from the definition of variance), we get:

$$E_1 [u_i^2] = h_i^2 \frac{1}{\bar{h}^2} \left( \rho^2 \left( \sigma_Z^2 + (\mu_Z - \bar{\gamma})^2 \right) + \bar{h} + \bar{q} \right) - h_i = A h_i^2 - h_i.$$

Consequently, we get that  $E_1 [z_i^2] = \frac{1}{h_i} E_1 [u_i^2] = A(h_0 + q_i) - 1$ , such that the time-1 objective function (10) is given by:

$$E_1 \left[ E_2[C_i] - \frac{\rho}{2} \text{Var}_2(C_i) \right] = R_f \left( W_0 - \frac{\kappa(q_i)}{R_f} \right) + \frac{1}{2\rho} (A(h_0 + q_i) - 1). \quad (\text{A9})$$

Re-arranging the first-order condition of (A9) with respect to  $q_i$  (taking  $\bar{q}$  and, hence,  $\bar{h}$  and  $A$  as given), yields Theorem 3. In particular, while the right-hand side of the equation is constant, the left-hand is increasing in  $q_i$  (because  $\kappa$  is convex), so that an unique interior solution exists if  $2\kappa'(0) < A/\rho$ . Otherwise, one arrives at a corner solution:  $q_i = 0$ .

Finally, using the definition of  $A$  in (11), we get Lemma 1:

$$\frac{\partial A}{\partial \bar{\gamma}} = -\frac{2\rho^2}{\bar{h}} (\mu_Z - \bar{\gamma}) < 0, \quad \text{if } \mu_Z - \bar{\gamma} > 0. \quad (\text{A10})$$

One can also show that the value of information is declining in average aggregate precision  $\bar{h}$ :

$$\frac{\partial A}{\partial \bar{h}} = -\frac{2}{\bar{h}^3} \left( \rho^2 (\sigma_Z^2 + \mu_Z^2) + \bar{h} + \bar{q} \right) + \frac{1}{\bar{h}^2} = \underbrace{-\frac{1}{\bar{h}^3}}_{<0} \underbrace{\left\{ 2(\rho^2 (\sigma_Z^2 + \mu_Z^2) + \bar{q}) + \bar{h} \right\}}_{>0} < 0, \quad (\text{A11})$$

and, consequently, also in the average precision of private information  $\bar{q}$ :

$$\frac{\partial A}{\partial \bar{q}} = \frac{\partial A}{\partial \bar{h}} \frac{\partial \bar{h}}{\partial \bar{q}} = \underbrace{\frac{\partial A}{\partial \bar{h}}}_{<0} \left( \frac{2\bar{q}}{\rho^2 \sigma_Z^2} + 1 \right) < 0. \quad (\text{A12})$$

## Proof of Theorem 4

In equilibrium, precision choices are *mutual* best response functions. That is, each investor's information choice  $q_i$  affects  $\bar{q}$  and  $\bar{q}$  affects  $q_i$  through  $A$ . Therefore, the equilibrium value

of the precision of the private signal of the average investor,  $\bar{q}$ , is determined by plugging  $q_i$  from (11) into the definition (9) which yields (12).

To prove that the equilibrium is unique (within the class of linear equilibrium), it suffices to show that  $\bar{q}$  is uniquely defined. Let  $\Sigma(\bar{q}) = \frac{1}{\bar{q}} \int_0^1 q_i(\bar{q}) di \geq 0$ . Then  $\bar{q}$  is defined as the solution of  $\Sigma(\bar{q}) = 1$ . Differentiating  $\Sigma(\bar{q})$  yields:

$$\Sigma'(\bar{q}) = \underbrace{\frac{-1}{\bar{q}^2} \int_0^1 q_i(\bar{q}) di}_{=-\frac{1}{\bar{q}} \Sigma(\bar{q}) < 0} + \underbrace{\frac{1}{\bar{q}} \int_0^1 \frac{\partial q_i}{\partial \bar{q}} di}_{> 0}. \quad (\text{A13})$$

Differentiating (11) with respect to  $\bar{q}$  yields:

$$2 \kappa''(q_i) \frac{\partial q_i}{\partial \bar{q}} = \frac{1}{\rho} \frac{\partial A}{\partial \bar{q}} \Leftrightarrow \frac{\partial q_i}{\partial \bar{q}} = \underbrace{\frac{1}{\rho 2 \kappa''(q_i)}}_{> 0} \underbrace{\frac{\partial A}{\partial \bar{q}}}_{< 0 \text{ (see (A12))}} < 0$$

where we used that  $\kappa$  is strictly convex. Consequently,  $\Sigma'(\bar{q})$  is negative and  $\Sigma(\bar{q})$  is decreasing over the real positive line. Together with  $\Sigma(0) = +\infty$  and  $\Sigma(\infty) = 0$ , this implies that  $\Sigma(\bar{q})$  “crosses” each real point once. Hence, there is a unique  $\bar{q}$  satisfying  $\Sigma(\bar{q}) = 1$  and, hence, (12).

## Proof of Corollary 1

Taking the derivatives of (12) with respect to  $\bar{\gamma}$ , yields:

$$\frac{d\bar{q}}{d\bar{\gamma}} = \int_0^1 \frac{\partial q_i}{\partial \bar{\gamma}} di. \quad (\text{A14})$$

Moreover, the derivative of the best information response (11) with respect to  $\bar{\gamma}$  is:

$$2 \underbrace{\frac{\partial \kappa'}{\partial q_i}}_{=\kappa''(q_i)} \frac{\partial q_i}{\partial \bar{\gamma}} \frac{d\bar{\gamma}}{d\bar{\gamma}} = \frac{1}{\rho} \underbrace{\left( \frac{\partial A}{\partial \bar{\gamma}} \frac{d\bar{\gamma}}{d\bar{\gamma}} + \frac{\partial A}{\partial \bar{q}} \frac{d\bar{q}}{\bar{q} d\bar{\gamma}} \right)}_{\frac{dA}{d\bar{\gamma}}} \Leftrightarrow \frac{\partial q_i}{\partial \bar{\gamma}} = \frac{1}{2 \rho \kappa''(q_i)} \left( \frac{\partial A}{\partial \bar{\gamma}} \frac{d\bar{\gamma}}{d\bar{\gamma}} + \frac{\partial A}{\partial \bar{q}} \frac{d\bar{q}}{\bar{q} d\bar{\gamma}} \right).$$

Substituting this expression into (A14) and simplifying, yields Corollary 1:

$$\begin{aligned} \frac{d\bar{q}}{d\bar{\gamma}} &= \int_0^1 \frac{1}{2\rho\kappa''(q_i)} \left( \frac{\partial A}{\partial \bar{\gamma}} \frac{d\bar{\gamma}}{d\bar{\gamma}} + \frac{\partial A}{\partial \bar{q}} \frac{d\bar{q}}{d\bar{q}} \right) di = \underbrace{\int_0^1 \frac{1}{2\rho\kappa''(q_i)} di}_{\equiv H} \left( \frac{\partial A}{\partial \bar{\gamma}} \frac{d\bar{\gamma}}{d\bar{\gamma}} + \frac{\partial A}{\partial \bar{q}} \frac{d\bar{q}}{d\bar{q}} \right) \\ \Leftrightarrow \left( 1 - \frac{\partial A}{\partial \bar{q}} H \right) \frac{d\bar{q}}{d\bar{\gamma}} &= H \frac{\partial A}{\partial \bar{\gamma}} \Leftrightarrow \frac{d\bar{q}}{d\bar{\gamma}} = \underbrace{\left( 1 - \frac{\partial A}{\partial \bar{q}} H \right)^{-1}}_{>0 \text{ because } \frac{\partial A}{\partial \bar{q}} < 0 \text{ (see (A12))}} H \underbrace{\frac{\partial A}{\partial \bar{\gamma}}}_{< 0 \text{ (see (A10))}} < 0. \end{aligned}$$

## Proof of Theorem 5

In the information choice period, the objective of an investor with CARA expected utility is given by:

$$\begin{aligned} E_1 [-E_2 [\exp(-\rho C_i)]] &= E_1 \left[ -\exp \left( E_2 [-\rho C_i] + \frac{1}{2} \text{Var}_2(-\rho C_i) \right) \right] \\ &= -\exp \left( \rho R_f \left( \frac{\kappa(q_i)}{R_f} - W_0 \right) \right) E_1 \left[ \exp \left( -\frac{1}{2} z_i^2 \right) \right], \end{aligned} \quad (\text{A15})$$

where we used (A6) and (A7) for the mean and variance of compensation  $C_i$ .

To compute the expectation of the exponential of a squared normal variable ( $z_i^2$ ), we can use Brunnermeier (2001, page 64):

$$\begin{aligned} E_1 \left[ \exp \left( -\frac{1}{2} z_i^2 \right) \right] &= \left| 1 - 2 \text{Var}_1(z_i) \left( -\frac{1}{2} \right) \right|^{-\frac{1}{2}} \times \\ &\quad \exp \left( \underbrace{\frac{1}{2} (-E_1[z_i])^2 \left( 1 - 2 \text{Var}_1(z_i) \left( -\frac{1}{2} \right) \right)^{-1} \text{Var}_1(z_i) - \frac{1}{2} E_1[z_i]^2}_{= \frac{1}{2} E_1[z_i]^2 \left( \frac{\text{Var}_1(z_i)}{1 + \text{Var}_1(z_i)} - 1 \right) = \frac{1}{2} E_1[z_i]^2 \left( \frac{\text{Var}_1(z_i) - 1 - \text{Var}_1(z_i)}{1 + \text{Var}_1(z_i)} \right)} \right) \\ &= \left( (1 + \text{Var}_1(z_i)) \exp \left( \frac{E_1[z_i]^2}{1 + \text{Var}_1(z_i)} \right) \right)^{-\frac{1}{2}}. \end{aligned}$$

One can further simplify the two components within the exponential function as follows:

$$\begin{aligned} 1 + \text{Var}_1(z_i) &= 1 + \frac{\text{Var}_1(u_i)}{h_i} = h_i \frac{1}{\bar{h}^2} (\bar{h} + \rho^2 \sigma_Z^2 + \bar{q}) \equiv A_1 h_i, \\ \frac{E_1[z_i]^2}{1 + \text{Var}_1(z_i)} &= \frac{h_i}{h_i + \text{Var}_1(u_i)} \frac{1}{h_i} E_1[u_i]^2 = (\bar{h} + \rho^2 \sigma_Z^2 + \bar{q})^{-1} \rho^2 (\mu_Z - \bar{\gamma})^2 \equiv A_2. \end{aligned}$$

Consequently, the CARA objective function (A15) can be written as:

$$\max_{q_i} E_1 [-\exp(-\rho C_i)] = -\exp\left(\rho R_f \left(\frac{\kappa(q_i)}{R_f} - W_0\right)\right) (A_1 (h_0 + q_i) \exp(A_2))^{-\frac{1}{2}}$$

Note that neither  $A_1(\bar{q})$  nor  $A_2(\bar{q}, \bar{\gamma})$  depend on  $q_i$ . Hence, taking the first-order condition with respect to  $q_i$  and re-arranging yields Theorem 5.

### Proof of Theorem 6

Taking the time-1 expectation of the equilibrium price (7) yields:

$$S = \frac{1}{\bar{h}} \left( \frac{\mu_X}{\sigma_X^2} + \rho \bar{\gamma} + \frac{\mu_Z \bar{q}}{\sigma_Z^2 \rho} \right) + \frac{1}{\bar{h}} \left( \bar{h} - \frac{1}{\sigma_X^2} \right) \mu_X - \frac{1}{\bar{h}} \left( \frac{\bar{q}}{\rho \sigma_Z^2} + \rho \right) \mu_Z = \mu_X - \frac{1}{\bar{h}} \rho (\mu_Z - \bar{\gamma}).$$

Hence, the total derivative with respect to  $\bar{\gamma}$  is given by:

$$\frac{dS}{d\bar{\gamma}} = \frac{\partial S}{\partial \bar{\gamma}} \frac{d\bar{\gamma}}{d\bar{\gamma}} + \frac{\partial S}{\partial \bar{h}} \frac{d\bar{h}}{d\bar{\gamma}} = -\frac{1}{\bar{h}} \rho (-1) - \frac{-1}{\bar{h}^2} \rho (\mu_Z - \bar{\gamma}) \frac{d\bar{h}}{d\bar{\gamma}} = \frac{1}{\bar{h}} \rho + \frac{1}{\bar{h}^2} \rho (\mu_Z - \bar{\gamma}) \frac{d\bar{h}}{d\bar{\gamma}}$$

Accordingly, the time-1 expectation of the excess return  $M \equiv E_1[X - PR_f]$  is given by:

$$M = E_1[X] - E_1[PR_f] = \mu_X - \left( \mu_X - \frac{1}{\bar{h}} \rho (\mu_Z - \bar{\gamma}) \right) = \frac{\rho}{\bar{h}} (\mu_Z - \bar{\gamma}), \quad (\text{A16})$$

and, thus, its total derivative with respect to  $\bar{\gamma}$  is:

$$\frac{dM}{d\bar{\gamma}} = \frac{\partial M}{\partial \bar{\gamma}} \frac{d\bar{\gamma}}{d\bar{\gamma}} + \frac{\partial M}{\partial \bar{h}} \frac{d\bar{h}}{d\bar{\gamma}} = \frac{-\rho}{\bar{h}} + \frac{-\rho}{\bar{h}^2} (\mu_Z - \bar{\gamma}) \frac{d\bar{h}}{d\bar{\gamma}}.$$

### Proof of Theorem 7

The unconditional variance  $V^2$  is given by:

$$V^2 \equiv \text{Var}_1(X - PR_f) = E_1 \left[ (X - PR_f)^2 \right] - E_1 [(X - PR_f)]^2 = A - M^2,$$

where we used the law of iterated expectations, to re-write the first part as:

$$E_1 \left[ E_2 \left[ (X - PR_f)^2 \right] \right] = \underbrace{E_1 [\text{Var}_2(X)]}_{=\text{Var}_2(X)=\frac{1}{h_i}} + \underbrace{E_1 [(\hat{\mu}_{X,i} - PR_f)^2]}_{=E_1[z_i^2](1/h_i)} = \frac{1}{h_i} + \frac{1}{h_i} (A h_i - 1) = A.$$

Accordingly, the unconditional variance is given by:

$$V^2 = \frac{1}{\bar{h}^2} (\rho^2 (\sigma_Z^2 + \mu_Z^2) + \bar{h} + \bar{q}) - \frac{\rho^2}{\bar{h}^2} (\mu_Z - \bar{\gamma})^2 = \frac{1}{\bar{h}^2} (\rho^2 \sigma_Z^2 + \bar{h} + \bar{q}),$$

which does not explicitly depend on  $\bar{\gamma}$ , but implicitly (through  $\bar{q}$  in  $\bar{h}$ ):

$$\frac{dV^2}{d\bar{\gamma}} = \frac{\partial V^2}{\partial \bar{\gamma}} \frac{d\bar{\gamma}}{d\bar{\gamma}} + \frac{\partial V^2}{\partial \bar{h}} \frac{d\bar{h}}{d\bar{\gamma}} = \frac{-1}{\bar{h}^3} \frac{d\bar{h}}{d\bar{\gamma}}.$$

## B Proof and Derivations for Section 3

### Proof of Lemma 2

The first- and second-order derivatives of the CRRA-utility function (13) with respect to wealth  $W_i$  are given by:

$$\frac{\partial C_i^{1-\rho}/(1-\rho)}{\partial W_i} = C_i^{-\rho}, \quad \frac{\partial^2 C_i^{1-\rho}/(1-\rho)}{\partial W_i^2} = -\rho C_i^{-\rho-1}.$$

As a result, the local coefficient of relative risk-aversion, denoted by  $\hat{\rho}_i$ , is given by:

$$\hat{\rho}_i = \frac{(-W_i)(-\rho)C_i^{-\rho-1}}{C_i^{-\rho}} = \rho \frac{W_i}{C_i} = \rho \frac{W_i}{W_i - \gamma_i R_B - \chi_i},$$

which yields Lemma 2.

### Derivations for the Approximate Portfolio Holdings

Substituting wealth  $W_i$  into compensation  $C_i$ , we get:

$$C_i = W_{0,i} R_f + \phi_i r^e - \gamma R_B - \chi_i = (1-\gamma) \underbrace{\left( \frac{1}{1-\gamma} (W_{0,i} R_f + \phi_i r^e) - \frac{\gamma}{1-\gamma} R_B - \frac{\kappa(q_i)}{1-\gamma} \right)}_{\equiv \tilde{C}_i}.$$

Importantly, with CRRA preferences, maximizing utility over  $C_i$  is equivalent to maximizing utility over  $\tilde{C}_i$  because any multiplicative constant drops out of the optimization problem.

With the mean of  $\tilde{C}_i$  being around 1, one can approximate  $\tilde{C}_i$  using a log-normal distribution:

$$\tilde{C}_i \approx \exp\left(-1 + E_1[\tilde{C}_i] - \frac{1}{2}Var_1(\tilde{C}_i) + \sqrt{Var_1(\tilde{C}_i)}\nu\right), \quad \nu \sim \mathcal{N}(0, 1).$$

Because of log-normality of the approximated  $\tilde{C}_i$ , the time-2 portfolio choice problem with CRRA preferences can then be written as:

$$\begin{aligned} & \max_{\phi_1} \frac{1}{1-\rho} \exp\left((1-\rho)\left(-1 + E_1[\tilde{C}_i] - \frac{1}{2}Var_1(\tilde{C}_i)\right) + \frac{1}{2}(1-\rho)^2 Var_1(\tilde{C}_i)\right) \\ \Leftrightarrow & \max_{\phi_1} E_1[\tilde{C}_i] - \frac{\rho}{2}Var_1(\tilde{C}_i). \end{aligned}$$

Taking the first-order condition with respect to  $\phi_i$  and re-arranging, yields (15).

## Posterior Beliefs

Because each investor is small, the equilibrium stock price is a function of a stock's payoff and its supply (both unobservable) only:  $P(X, Z)$ . Consequently, if the payoff  $X$  is binomially distributed, each investor can back out the two combinations of payoff and noise, denoted by  $\{(X_L, Z_L), (X_H, Z_H)\}$ , which are consistent with a given price  $P$ .<sup>34</sup> Using the distribution of the noise, an investor can then compute the *posterior probability* of the payoff  $X$ .

Formally, investor  $i$ 's posterior probability of realization  $X_H$ , conditional on price  $P$  and signal  $Y_i \in \{Y_L, Y_H\}$ , is given by

$$\hat{\pi}_i = \mathbb{P}(X_H | P, Y_i) = \frac{f_z(z_H) \mathbb{P}(X_H | Y_i)}{\sum_o f_z(z_o) \mathbb{P}(X_o | Y_i)}, \quad (\text{B17})$$

with  $o \in \{L, H\}$  and  $f_z(\cdot)$  denoting the density function of the stochastic supply  $Z$ .  $\mathbb{P}(X_o | Y_i)$  can be computed directly as:

$$\mathbb{P}(X_o | Y_i) = \frac{\mathbb{P}(X_o, Y_i)}{\sum_{o'} \mathbb{P}(X_{o'}, Y_i)}, \quad o' \in \{L, H\};$$

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<sup>34</sup>The corresponding supply  $Z_o$ ,  $o \in \{L, H\}$ , is simply given by the aggregate demand in the economy at price  $P$ —conditional on  $X_o$ . See also the descriptions and derivations in [Breugem \(2016\)](#) for learning from price in a dynamic setting.

using the (chosen) correlation  $\mathbb{P}[X_o | Y_i = Y_o]$  between investor  $i$ 's private signal and the payoff:

$$\mathbb{P}(X_{o'}, Y_{m'}) = \mathbb{P}[X_{o'} | Y_i = Y_{m'}]/2, \quad \text{with } o', m' \in \{L, H\}.$$

## Numerical Solution Method

In the case of CRRA preferences, that is, in the economic setting described in Section 3.1, the equilibrium price function is nonlinear, and its specific functional form is unknown. Accordingly, we rely on a novel numerical solution method to solve for the equilibrium. In the following, we provide some details of the algorithm for the single-stock economy.

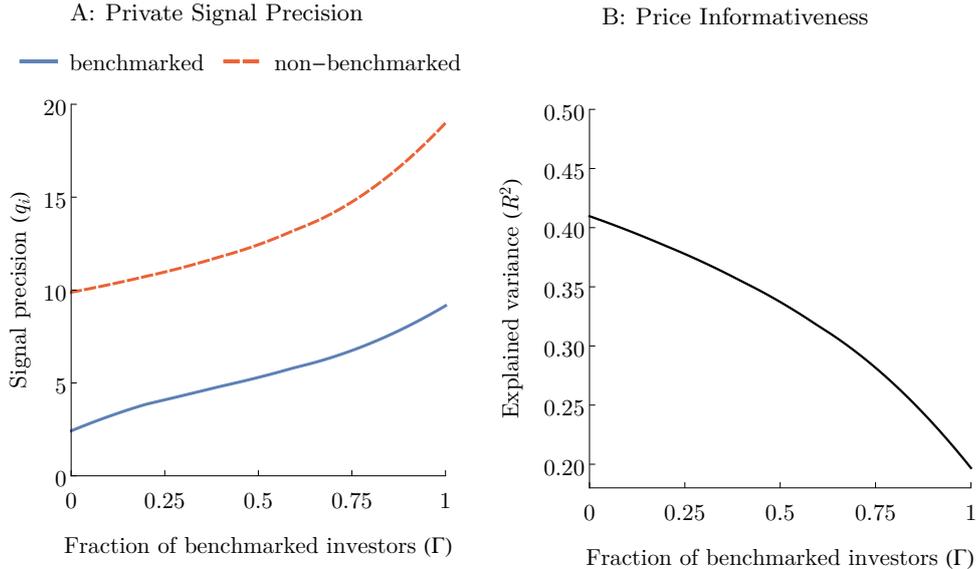
The algorithm relies on solving a large-scale fixed-point problem globally.<sup>35</sup> In particular, we discretize the state space for the stochastic supply  $Z$ , using  $N_Z$  grid points.<sup>36</sup> The full equation system then consists of the following set of equations: First,  $2 \times 2 \times 2 \times N_Z$  ‘‘posterior equations’’ (B17), describing the posterior beliefs of the two groups of investors for the 2 (realized) underlying payoff realizations, the 2 (conjectured) possible realizations of the payoff, 2 possible signals, and the  $N_Z$  potential random supply realizations; second,  $2 \times 2 \times 2 \times N_Z$  first-order conditions resulting from the optimal portfolio choice (3), again for the two groups of investors, the 2 (realized) underlying payoff realizations, 2 possible signals, and the  $N_Z$  grid points of the supply; third,  $2 \times N_Z$  market clearing condition for the 2 underlying payoff realizations, and the  $N_Z$  grid points of the supply; and fourth, two first-order conditions resulting from the optimal information choice (4) for the two groups of investors. That is, in total, we arrive at  $26 \times N_Z + 2$  equations.

The unknowns of the equation system are given by the following variables: First,  $2 \times 4 \times 2 \times N_Z$  posterior beliefs  $\pi$ . for the two groups of investors, the 2 (realized) underlying payoff realizations, the 2 (conjectured) possible realizations of the payoff, the 2 possible signals, and the  $N_Z$  grid points of the supply; second,  $2 \times 2 \times 2 \times N_Z$  portfolio shares of the stock  $\phi_i$  for the two groups of investors, the 2 underlying payoff realizations, 2 possible signals, and the  $N_Z$  grid points of the supply; third,  $2 \times N_Z$  stock prices  $P$  for the 2 underlying payoff realizations and the  $N_Z$  grid points of the supply; and fourth, 2 signal precisions  $q_i$  for the two groups of investors, which makes in total  $26 \times N_Z + 2$  variables.

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<sup>35</sup>The equation system cannot be solved recursively because the period-1 choice of the signal precision  $x_j$  affects the period-2 posterior beliefs and, in turn, the portfolio choices and market clearing.

<sup>36</sup>In the case of a log-normally distributed payoff and log-normally distributed signals, as discussed in the succeeding section, we furthermore discretize the payoff and signal space by  $N_X$  and  $N_Y$  gridpoints, respectively. As a result, the total number of equations (and unknowns) is equal to  $N_X \times N_Z \times (2N_X \times N_Y + 2N_Y + 1) + 2$ .



**Figure A1: Equilibrium information choice with log-normal distributions.** The figure illustrates the equilibrium signal precision, as a function of the fraction of benchmarked investors in the economy,  $\Gamma$ . Panel A shows the optimal signal precision of the two groups of investors and Panel B depicts the resulting price informativeness, that is, the precision of the public price signal. Precision is measured as  $R^2$ , that is, the fraction of the variance of the payoff  $X$  that is explained by the investors' private information and the stock price, respectively. The graphs are based on the CRRA framework described in Section 3.1, but with log-normally distributed payoff and signals. The following parameter values are used:  $\mu_X = 1.05$ ,  $\sigma_X^2 = 0.25$ ,  $\mu_Z = 1.0$ ,  $\sigma_Z^2 = 0.2$ ,  $\rho = 3$ ,  $\gamma_i = \gamma = 1/3 \forall i \in \mathcal{BI}$ , and an information cost function  $\kappa(q_i) = \omega q_i^2$ , with  $\omega = 0.015$ .

## C Lognormally Distributed Payoff and Signals

The binomial distributions for the asset's payoff and the investors' private signals, as described in Section 3.1, are chosen for numerical convenience only. The results also hold for the case of a log-normally distributed payoff and log-normally distributed signals.

In particular, Figure A1 depicts the investors' optimal signal precisions and the resulting price informativeness for this setting. Similar to the case with a binomially distributed payoff and signals, the non-benchmarked investors endogenously chose a higher signal precision and price informativeness declines as the fraction of benchmarked investors increases. The results for the other quantities (omitted for brevity) are directly comparably as well.

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