Financial Networks and the Real Economy*

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Abstract

Financial intermediaries channel funds from less productive sectors of the economy to more productive sectors. This process links two otherwise unrelated sectors together through the intermediaries' balance sheet. The downside of this arrangement is that a negative shock to an individual sector can be transmitted to other sectors through connecting intermediaries. We characterize when spillovers can occur, and show that the size of the spillover declines with a sectors distance from the initial shock. Increased financial integration within the system can dampen spillovers. Our results shed new light on which industries are important from a systemic risk perspective.

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1 Introduction

Economists have long been interested in how small idiosyncratic shocks might lead to aggregate distortions. Classical mechanisms range from a lag in capital accumulation (Kydland and Prescott [1982]), to coordination failures (Diamond [1982], Kiyotaki [1988]), and credit market frictions (Bernanke and Gertler [1989], Kiyotaki and Moore [1997]). More recently, a nascent but expanding literature has proffered a more granular approach to analyzing the transmission of shocks by way of input-output linkages (Gabaix [2011], Acemoglu *et al* [2012], and Baqaee [2015]). However, by focusing only on production networks, this literature has ignored the role of the financial system as a potential transmitter.

Our paper takes a different approach to the production network literature and examines the role of a financial network between sectors in transmitting shocks across the real economy. We provide a novel characterization of connections between sectors, and show how and when shocks are transmitted. We show how integration can improve efficiency, and provide suggestive evidence on the importance of seemingly peripheral sectors. Our results have policy implications for shock containment and systemic risk.

To highlight how our channel differs from existing work, we assume sectors[1] are not connected - directly or indirectly - with each other via a production network. Instead, connections in our model arise through the financial system, where banks make loans to different sectors in the economy. In particular, an individual bank connects two different sectors by making separate loans to each sector. Moreover, although a given bank might only connect sectors which are similar on some characteristic (e.g. geography), unrelated sectors can become indirectly connected through a series of intermediaries.

In our model, each bank holds a portfolio of performing and non-performing loans. Banks are obligated to hold performing loans till maturity, whereas non-performing loans contain an option to be liquidated at an interim stage (*e.g.*, sale of collateral). Although loans held to maturity provide banks with a positive expected return, early liquidation generates immediate revenue which can be useful in covering banks' assumed liquidity needs. We model the liquidation market by a downward sloping demand curve, in which the liquidation price is determined endogenously.

[1] A sector in our model could be interpreted as a geographical region or an industry, among others.

Liquidation is socially costly. It imposes costs on firm's customers, workers, and suppliers. We do not explicitly derive welfare losses from liquidation in our model, instead we focus on the total amount of liquidation across different sectors, and assume liquidation negatively affects welfare. Such an approach has the advantage of allowing us to effectively characterize the transmission of shocks through the financial system in our economy. Moreover, by abstracting away from general equilibrium effects, we are able to better emphasize our channel as distinct from other mechanisms proposed in the extant literature.

Consider a negative shock to the expected returns of the non-performing loans in one sector. Because this shock reduces banks' opportunity cost of liquidation in the affected sector, it provides these banks with the incentive to increase their liquidation activity. These additional liquidations cause prices in the liquidation market to decline, and depending on the price elasticity of demand, banks' revenues from liquidation relative to pre-shock forecasts can be positively or negatively affected. If the liquidation market is inelastic, for example, total liquidation proceeds will be decreasing in the quantity of non-performing loans liquidated. Thus, following a large shock to an inelastic market, the ensuing drop in liquidation revenues will create a liquidity shortage if ex-ante banks were relying on liquidation revenue to meet their obligations. In this way, liquidity shortages arise as a result of banks' profit maximizing liquidation decisions, not because of an unexpected liquidity shock.

To cover these liquidity shortages, banks can obtain extra funds by liquidating additional non-performing loans in *other* sectors. However, in doing so, banks effectively act as transmitters of the shock to these sectors. Increased liquidation in sectors initially unaffected by the shock lowers prices in their respective liquidation markets. Consequently, a negative externality in the form of reduced liquidation revenues is imposed on banks whose activities were not directly affected by the initial shock. Should these banks respective liquidity constraints bind, they will in turn transmit the shock to other untouched sectors. In this way, the initial shock spreads across the economy to distant sectors.

Equipped with our theoretical foundation for shock transmission, we analyze how the transmission varies under different network structures. To do so, we first formally represent our economy as a network in which the nodes are sectors and the connections between nodes are banks. The transmission of the initial shock depends crucially on the distribution of banks across the economy. We provide conditions under which the shock will be transmitted from one sector to another. In particular, we show that if the size of a shock crosses a sector specific threshold, it will be transmitted to neighboring sectors.

Second, we provide conditions under which the shock will spread to the remainder of the economy (*i.e.* to non-neighboring sectors). Put differently, we show how large shocks might never be completely absorbed by a single sector. The impact of the shock dissipates with distance. Thus, intuitively, sectors located near the source are the most adversely effected. This result suggests that efforts to contain a shock which could potentially be passed through the banking sector should be concentrated at the source.

Third, we analyze the effect of integration on efficiency. For a fixed level of bank competition in each sector (*i.e.,* the number of banks operating in each sector), we say an economy is more integrated as the distribution of banks across sectors becomes more diversified. We show that increased integration improves efficiency by lowering the aggregate number of liquidations. That is, shocks become less disruptive as economy gets more integrated. The intuition for this result is that a shock to a particular sector can be better diversified across other sectors.

Finally, we provide evidence for the importance of seemingly peripheral sectors. We interpret peripheral sectors as those which are only sparsely connected to the rest of the economy through the banking system. We highlight how a shock to a peripheral sector can trigger greater liquidations in certain parts of the economy vis-a-vis a shock to a well-connected core sector. Underpinning this result is the benefits of sectoral diversification among banks. More specifically, the shock to the peripheral sector leaves liquidity constrained banks with few options to raise funds. Thus the initial shock transmission is significantly stronger than when the shock hits a core sector with well-diversified banks.

Identification of this new channel of shock propagation is important for policy. Our analysis suggests that industries located on the periphery of a production network should not be overlooked from a systemic risk perspective. That is, these peripheral production industries can be the source of a crisis if they hold an important position in the financial network.

1.1 Related Literature

Our paper is related to multiple strands of literature. First, recent work by Gabaix [2011], Acemoglu *et al* [2012], and Baqaee [2015] provide the theoretical underpinnings for the transmission of shocks in a production network. These papers highlight how the law of large numbers can fail to wash out idiosyncratic shocks when industries are connected in a production network. Motivated by theory, several recent empirical studies attempt to trace shocks through production networks (*e.g.,* Boehm, Flaaen, and Nayar [2015], Carvalho, Nirei, and Saito [2017]). As mentioned earlier, our paper differs from these production network based stories by analyzing how a financial network can transmit shocks across the real economy.

The negative liquidation externalities within an industry stemming from an individual firm's bankruptcy have been well documented (*e.g.,* Lang and Stulz [1992], Jorion and Zhang [2007], and Benmelech and Bergman [2011]). Forced liquidation in these settings reduces collateral values, depletes balance sheets, and tightens the financial constraints of other firms. Banks respond by reducing their lending to the entire sector. Our channel differs in two ways. First, the shock in our model hits an entire sector. Naturally it follows that our focus is on spillovers across sectors, not within an individual sector. Second, in our model, banks respond to the shock by reconsidering their liquidation choices across sectors, not their lending decisions. Despite these differences, we view our work as complimentary to these papers, in that it explains a second level of transmission. That is, whereas this literature has focused on the spark which lights the fuse in a given industry, our paper analyzes the gasoline which is then poured on the fire.

Sectors in our model can represent different asset classes, industries, geographical regions or even countries. In line with our results, Van Rijckeghem and Weder [2001] present empirical evidence that banks which operated in a crisis country during the Mexican, Asian, and Russian crises reduced their lending in other countries in order to meet regulatory standards (*e.g.,* capital adequacy ratio). Their findings suggest a negative spillover which emanates from the crisis country and is transmitted elsewhere through banks.

In a related theory paper, Goldstein and Pauzner [2004] develop a model in which a financial crisis can be transmitted across countries if there exists a common pool of investors who suffer from a coordination problem. In their model, the financial crisis reduces investors' wealth, increases investors' effective risk aversion and ultimately exacerbates the coordination problem. The crisis is then transmitted through investors' self-fulling fear of pre-emptive withdrawals. Our model and underlying focus differ fundamentally from Goldstein and Pauzner [2004]. There, actions of investors exhibits complementarities, and contagion occurs because of investors' risk aversion. In contrast, banks' liquidation decisions across sectors are substitutes in our model, and contagion arises from liquidity shortages. Moreover, their model offers a channel of contagion between two asset classes whereas ours analyzes contagion across the entire economy.

Our paper is also related to the literature on financial contagion in bank networks. In this literature, banks are connected either directly through bilateral contracts as in Allen and Gale [2000] or indirectly by holding similar portfolios as in Cifuentes *et al* [2004]. In the former class of models, contagion occurs when a bank fails to meet its obligation to other banks, whereas in the latter, contagion is driven primarily by a decrease in asset prices which adversely affects the portfolio values of multiple banks concurrently. Among other things, our paper takes a different approach to this literature by modeling banks as the connections between nodes of a network, rather than the nodes themselves.

Finally, the banks' problem in our model also bares a resemblance to the portfolio rebalancing problem of asset managers in the asset pricing literature. In particular, much attention in this literature has centered on how contagion in asset prices can be a consequence of investors' strategic portfolio rebalancing decisions. For example, in Kodres and Pritsker [2002] and Calvo [2002], contagion between two assets can occur when an idiosyncratic shock to one asset is misinterpreted (due to informational asymmetries) for a common macroeconomic shock which affects both. Thus, the fall in the price of one asset leads to self-fulfilling expectations that the price of the other asset will also fall. Contagion, part of the inefficient equilibrium, would not arise absent common macroeconomic factors. In our model, however, there is no common factor which affects prices. Assets (*i.e.,* sectors) are linked indirectly and only by players (*i.e.,* banks) themselves.

2 Model

2.1 Setup

There are three dates $t = \{0, 1, 2\}$ and N banks. Banks operate in different sectors of the economy which we call islands. There are M islands, and each island represents a certain industry, area, or asset class. We use j to denote a bank and i to denote an island. Each bank is endowed with one unit of deposits per island on which they operate, and a fraction γ of each bank's depositors withdraw early at the intermediate date.

At the initial date, banks allocate their endowment between cash c_j and island specific investment opportunities (loans for short) y_{ij}. At the intermediate date, fraction p_i of the loans on island i are performing as expected. These loans cannot be liquidated and will pay bank j a gross return of R at the terminal date. The remaining loans on island i are considered to be non-performing. Non-performing loans can be liquidated early at the intermediate date[2] for a price $P_i = L_i(X_i)$, where $X_i = \sum_j x_{ij}$ is the aggregate quantity of liquidated loans on island i and x_{ij} is bank j's quantity of liquidated loans on island i. If a non-performing loan is not liquidated and instead held to maturity, it returns R with probability q_i at the terminal date, and zero otherwise. Thus the opportunity cost of liquidating a non-performing loan is $Z_i = q_i R$.

We assume a loan issued at the initial date and held to maturity has a strictly positive net return r, *i.e.*, $p_i R + (1 - p_i) Z_i - 1 = r > 0$. Essentially, this assumption implies the market for making loans on an island at the initial date is not perfectly competitive. We also assume a downward sloping linear demand curve for liquidated loans, where the maximum liquidation price is strictly greater than the opportunity cost of liquidation, *i.e.*, $L_i(X_i) = A_i - bX_i$ where $A_i > Z_i$ and $b_i > 0$.

Banks are heterogeneous in their size and locations. Among the N banks, there are N_1 large banks which operate on K islands and $N_2 = N - N_1$ small banks each operating on a solitary island. For convenience, we normalize the small banks to have a single unit of deposits. We summarize the islands on which bank j operates by the $M \times 1$ location vector $w_j = [w_{1j}, w_{2j}, ...w_{Mj}]$ which has $w_{ij} = 1$ if bank j operates on island i and $w_{ij} = 0$ otherwise. We can then stack these vectors column-wise into an $M \times N$ location matrix W_{ij} which summarizes all bank locations.

[2] The ability for banks to only liquidate non-performing loans may arise as a result of contracting frictions. A performing loan by definition is one where all payments have been made on time and thus banks have no rights to early cash flows.

For example, if the economy consists of three islands, one large bank which operates on island 1 and island 2, one large bank which operates on island 2 and island 3, and three small banks each operating on a different island, we can write the location matrix as:[3]

$$W = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Bank locations are fixed at the initial date. That is, any bank which is not initially located on island i is assumed to be both unable to make loans on island i at the initial date and unable to participate in island i's liquidation market at the intermediate date. Thus the location matrix summarizes the distribution of banks in our economy.

2.2 One Island Economy

To provide intuition, we first solve the model using backward induction when the economy consists of only one island. This assumption simplifies our problem considerably by ensuring each bank makes a single decision at both the initial and intermediate dates. Given c_j and y_j were chosen at the initial date, we can write each banks' problem at the intermediate date as follows:

$$\begin{aligned} \max_{x_j} \Lambda_j^1 &= (L(X) - Z)\, x_j && (1) \\ s.t\ x_j &\leq (1-p)y_j && (2) \\ \gamma &\leq L(X)x_j + c_j && (3) \end{aligned}$$

That is, banks engage in a simple Cournot liquidation game subject to capacity and liquidity constraints ((2) and (3)). Setting aside the liquidity constraint momentarily, we know a profit-maximizing bank will liquidate some non-performing loans because $L(0) > Z$. Assume for now that $y_j = y \ \ \forall j$. The exact quantity of liquidated loans depends on whether banks have sufficient

[3] It is worth noting that W will typically not be unique. However, it is easy to show that two location matrices W_1 and W_2 will represent the same distribution of banks if W_2 can be obtained from W_1 through a finite number of column switches.

non-performing loans in their portfolios. We can write banks' first order constraint as:

$$L'(X)x_j + L(X) - Z = \mu_j \tag{4}$$

where μ_j is the Lagrange multiplier on the capacity constraint. If $\mu_j^* = 0$, we have an interior solution in which banks do not liquidate their entire portfolio of non-performing loans. Moreover, if banks are not capacity constrained, it can be shown that the Nash equilibrium in pure strategies is both unique and symmetric, with each bank choosing $x_j^* = \frac{A-Z}{(N+1)b}$. If $\mu_j^* > 0$, banks liquidate their entire portfolio of non-performing loans, *i.e.*, $x_j^* = (1 - p_j)y_j$. Thus we can write bank *j*'s optimal liquidation decision as $x_j^* = x_j^*(y_j)$ indicating its dependence on bank *j*'s loan choice at the intermediate date.

At the initial date, banks allocate their deposits between cash and loans to maximize future profits. We can write banks' optimization problem at the initial date as follows:

$$\max_{c_j, y_j} \Lambda_j^0 = c_j + pRy_j + (1-p)Zy_j + (L(X^*) - Z)\, x_j^*(y_j) \tag{5}$$

$$s.t.\ 0 = 1 - c_j - y_j \tag{6}$$

$$\gamma \leq c_j + L(X^*)x_j^*(y_j) \tag{7}$$

Because loans have a higher expected return than cash, cash is held only with the purpose of covering liquidity shocks. Consequently, if $L(X^*)x^*(1) \geq \gamma$, banks hold zero cash. However, if the funds obtained from liquidation at the intermediate date will prove insufficient to cover the early withdrawing depositors, banks may indeed opt to hold some precautionary cash. For the remainder of the paper, we focus on the limiting case where $\lim r \to \underline{r} = 0$.[4] This assumption, together with the absence of uncertainty in our model, implies banks' cash holdings will be exactly equal to the difference between their intermediate date liquidity needs and the revenues generated from their optimal liquidation decision, *i.e.*, $c^* = \max(0, \gamma - L(X^*)x^*)$. It follows that the optimal number of loans in each banks' portfolio is $y^* = \min(1, 1 - \gamma + L(X^*)x^*)$.

Returning now to banks' problem at the intermediate date, we can rewrite the liquidity con-

[4]This assumption is purely for expositional purposes - our results are robust to non-limiting cases.

straint as:

$$L(X)x_j \geq \hat{\gamma}_j \tag{8}$$

where $\hat{\gamma}_j$ is bank j's intermediate date liquidity needs. Because banks set $c^* = \max\left(0, \gamma - L(X^*)x^*\right)$ at the initial date, the Lagrange multiplier on (8) will be zero, and thus banks will always liquidate their profit maximizing quantity of non-performing loans at the intermediate date in the absence of any unexpected shock. We now extend our analysis to the general economy.

2.3 Multi-Island Economy

Our one island economy can be generalized to an M island economy with some additional notation. Given our location matrix, we define $I = \{j \in I : W_{ij} = 1\}$ to be the set of banks which operate on island i. That is, bank j operates on island i, $j \in I$, if and only if $W_{ij} = 1$. In a similar fashion, we can define $J = \{i \in J : W_{ij} = 1\}$ to be the set of islands on which bank j operates. Let $D_j = \{1, K\}$ represent bank j's deposits, where $D_j = 1$ if bank j is a small bank and $D_j = K$ if bank j is a large bank. Finally, let n_i denote the number of banks operating on island i. We can write bank j's optimization problem at the intermediate date as follows:

$$\max_{x_{ij}} \Lambda_j^1 = \sum_{j \in I} (P_i - Z_i) x_{ij} \tag{9}$$

$$s.t \quad x_{ij} \leq (1 - p_i) y_{ij} \tag{10}$$

$$D_j \hat{\gamma}_j \leq \sum_{j \in I} L_i(X_i) x_{ij} \tag{11}$$

The solution is analogous to the one island economy. It is worth noting that provided large banks are not capacity constrained, their liquidation decisions will be made *independently* across islands. That is, if bank j operates on islands $i = \{1, 2\}$, its liquidation quantity on island 1, x_{1j}, will depend only on the liquidation decisions of its competitors on island 1, and not on any activity taking place on island 2. Similarly, bank j's liquidation quantity on island 2 depends only on its competitors behavior on island 2.

We can write bank j's optimization problem at the initial date as:

$$\max_{c_j, y_{ij}} \Lambda_j^0 = c_j + \sum_{j \in I} \left\{ p_i R y_{ij} + (1 - p_i) Z_i y_j + \left(L(X_i^*) - Z_i\right) x_{ij}^*(y_{ij}) \right\} \tag{12}$$

$$s.t.\ 0 = D_j - c_j - \sum_{j \in I} y_{ij} \tag{13}$$

$$D_j \gamma \leq c_j + \sum_{j \in I} \left\{ L(X_i^*) \cdot x_j^*(y_j) \right\} \tag{14}$$

The solution is also analogous to the one island economy. That is, banks hold cash only up until the point they have sufficient funds to cover their liquidity shock at the intermediate date and the remaining endowment is allocated to loans.

Having characterized the initial equilibrium allocation in our economy, our focus in the remaining sections transitions to analyzing the impact of unexpected shocks in our economy. Thus for the remainder of the paper we will focus on an initial equilibrium in which $\forall i, j$ such that $W_{ij} > 0$; $x_{ij}^* = \frac{A_i - Z_i}{(n_i+1)b}$, $c_j^* = D_j\gamma - \sum_{j \in I} L(X_i^*) x_{ij}^* > 0$, and $y_{ij}^* > \frac{x_{ij}^*}{1-p_i}$ with $\sum_{j \in I} y_{ij}^* + c_j^* = 1$. That is, we will restrict our attention to equilibria in which banks require some cash to cover their liquidity constraint and liquidate an interior quantity of non-performing loans.[5]

2.4 Connections Across Islands

Before we analyze the transmission of shocks through the economy, it will be useful to fix ideas with some additional definitions. Given (M, N, N_1), we define the set of all feasible location matrices as $\mathbf{W}_{M \times N} = \left\{ W \in \mathbf{W}_{M \times N} | \sum_i \sum_j W_{ij} = N_1 K + N_2, \sum_j W_{ij} \neq 0 \ \ \forall i \right\}$. That is, $\mathbf{W}_{M \times N}$ is the set of all possible distributions of banks across islands in the (M, N, N_1) economy. Although our location matrix provides a precise characterization of the islands on which each bank operate, it is difficult to directly compare two different financial systems, $W_1, W_2 \in \mathbf{W}$ using only their location matrices. To this end, we can transform the location matrix W into a *system matrix* S, which is defined below.

Definition 1. The *system matrix* is an $M \times M$ matrix S with diagonal entries $S_{ii} = \sum_j W_{ij}$ and off-diagonal entries $S_{ii'} = \sum_j W_{ij} W_{i'j}$ where $i \neq i'$.

By construction, every location matrix W has a corresponding system matrix S. Moreover, analogous to our definition of $\mathbf{W}_{M \times N}$, we can also define the set of all possible system matrices as $\mathbf{S}_{M \times M} = \{S \in \mathbf{S}_{M \times M} | \exists W \in \mathbf{W}_{M \times N} \ \ s.t.\ \ S = \Psi(W)\}$, in which the mapping $\Psi(W) : \mathbf{W} \rightarrow \mathbf{S}$

[5] We believe this equilibrium to be the most realistic in our economy. In particular, banks generally hold some cash to cover their liquidity needs, and typically do not plan on liquidating their entire portfolio of non-performing loans.

translates $W \in \mathbf{W}_{M \times N}$ to an $S \in \mathbf{S}_{M \times M}$ as per our definition of the system matrix.

The properties of the system matrix are most easily illustrated by way of example. Consider the location matrix from section 2.1. We can compute its corresponding system matrix as:

$$S = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

The system matrix provides a useful description of our economy's financial system. In particular, the diagonal entries of the system matrix are the number of banks on a given island. For any given island i, because each bank's marginal liquidations have an identical impact on the liquidation price, we say S_{ii} provides a measure of the level of *competition* on island i. In our example, we observe from S that there are two banks operating on island 1 and island 3, and three banks operating on island 2. Thus we say island 2 is more competitive than islands 1 and 3.

Similarly, the off-diagonal entries of the system matrix are the number of large banks which operate on two particular islands. We argue S_{ij} provides information about the level of *financial integration* in our economy. To be concrete, we say for two economies with M islands and system matrices S and S', the economy with system matrix S is more *integrated* if $\forall i \neq i'$, $S_{ii'} \geq S'_{ii'}$ and $S_{ii'} > S'_{ii'}$ for some $i \neq i'$.

The system matrix also informs us about where shocks might potentially be transmitted. In particular, we can use S to determine whether islands are *connected.*

Definition 2. Given a system matrix S, we say island i and island i' are *directly connected* if and only if $S_{ii'} > 0$. We say island i and island i' are *indirectly connected* if and only if there exists a sequence of "intermediary" islands $i_l = \{i_1, ...i_m\}$ such that $S_{ii_1} \left(\Pi_{l=1}^{m-1} S_{i_l i_{l+1}} \right) S_{i_m i'} > 0$ and $S_{ii'} = 0$.

In our example, islands 1 and 2, and islands 2 and 3 are directly connected because $S_{12} > 0$ and $S_{23} > 0$ respectively. Similarly, islands 1 and 3 are not directly connected ($S_{13} = 0$), but they are indirectly connected because $S_{12}S_{23} > 0$. Graphically, one might represent the connections in our example economy as follows:

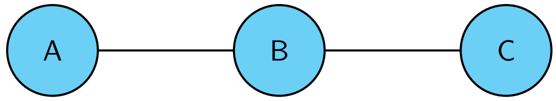

Figure 1: Example of three island economy

However, unlike the location matrix, the system matrix does not tell us *which* banks connect two islands. Moreover, because the system matrix only provides information about pairwise connections, if large banks operate on $K > 2$ islands, one might be worried about losing information when mapping a location matrix into a system matrix. Put differently, without further restrictions, a given S could correspond to many different financial systems, and if so, we may be unable to use the system matrix to effectively characterize the transmission of shocks across islands.

To see this more clearly, it is instructive to momentarily think of w_j, the islands bank j operates on, as bank j's type. Two banks j and j' have the same type if and only if $\forall i \; w_{ij} = w_{ij'}$. Thus we can think of the inverse mapping $\Psi^{-1}(S) : \mathbf{S} \to \mathbf{W}$, if it exists, as finding the distribution of bank types across islands. Fixing M and K, we have up to ${}^{M}C_K + M$ possible bank types (unknowns).[6] In contrast, our system matrix provides us with only $((M+1) \times M)/2$ equations.[7] Thus in many instances, the inverse mapping will not provide us with a unique solution. Despite this dimensionality problem, the following lemma provides a condition under which information is not lost.

Lemma 1. *If $K \leq 2$ or $K \geq M - 2$, then the mapping $\psi^{-1}(S) : \mathbf{S} \to \mathbf{W}$ has a unique solution up to a finite series of column switches.*

Thus for any (M, N, N_1), and providing $K \leq 2$ or $K \geq M - 2$, the system matrix is a sufficient statistic for the location matrix, and thus the financial system.[8] To see why Lemma 1 must be true, notice that if $K \leq 2$ or $K \geq M - 2$, the maximum number of possible bank types becomes $\frac{M \times (M+1)}{2}$. Therefore providing S is a feasible system matrix ($S \in \mathbf{S}$), we will be able to back out its

[6] There are M distinct islands and thus there are M unique small bank types. Large banks are on K islands and thus there are potentially ${}^{M}C_K$ unique large banks types.

[7] Because S is symmetric, $S_{ij} = S_{ji}$ and thus we only have the M diagonal elements and the $(M-1) \times M/2$ off-diagonal elements.

[8] If W_1 can be obtained from W_2 via a series of finite column switches we say W_1 and W_2 are equivalent financial systems.

corresponding W by applying $\Psi^{-1}(S)$.

2.5 Transmission of Shocks

We now perturb the model by introducing a state of the world which was assigned zero probability at the initial date. In particular, upon realization of this state, the success probability of non-performing loans on one island, island k, falls to $q'_k < q_k$. In a manner similar to Allen and Gale [2000], we interpret this state as the occurrence of an unexpected shock to the economy. This unexpected shock reduces the opportunity cost of liquidation for banks on island k by an amount $\delta_k = \left(q_k - q'_k\right) R$. We define δ_k to be the size of the shock, set $Z'_k = Z_k - \delta_k$, and restrict $\delta_k \in (0, Z_k]$. To further ease our exposition, we assume the initial equilibrium which was described in section 2.3 has aggregate island liquidation prices and quantities P_i^*, X_i^*, which imply liquidation markets on all islands i are elastic. The remainder of the section analyzes how the financial system transmits this shock to other islands k' in our economy.

The transmission of the shock depends on the responses of the banks operating on island k. Initially, the shock impacts the liquidation decisions of banks on island k through a pure profit-maximizing channel. More precisely, as is standard in a Cournot game, the reduction in the opportunity cost of liquidation on island k causes each bank operating on k to liquidate additional non-performing loans. However, of central interest in this paper, is how and when a shock to one island spreads to other parts of the economy.

Definition 3. A shock to island k is *transmitted* to island k' if the liquidation decisions of banks on island k' differ from the pre-shock equilibrium.

Given the above definition, we now provide a condition under which our shock is transmitted to other islands.

Proposition 1. *Assume there is at least one large bank located on island k. An unexpected shock to island k of size δ_k will trigger contagious liquidations on at least one other island if and only if $\delta_k > \delta_k^*$ where $\delta_k^* = 2Z_i - \frac{(n_i-1)}{n_i} A_i$. If $\delta_k \leq \delta_k^*$ there will be no change in banks' liquidation decisions on other islands.*

Intuitively, the presence of a large bank j on island k creates a channel through which the shock might be transmitted. In particular, given that our initial shock affects bank j's liquidation decisions on island k, it might also affect bank j's liquidation decision on the other islands it is located. The exact channel is via bank j's balance sheet. If the shock is sufficiently large, $\delta_k > \delta_k^*$, then bank j's post-shock profit maximizing level of liquidation on island k provides *less* revenue than its pre-shock equilibrium liquidation quantity. As a direct consequence, bank j's liquidity constraint will now bind. Thus bank j will have to raise additional revenue by increasing its liquidation quantities on the other islands where it is located.[9]

Put differently, we observe the shock causes profit-maximizing banks to expand their liquidation quantities, which in turn lowers the price in the liquidation market. If the shock is sufficiently large, the quantity of loans liquidated on island i moves banks sufficiently down the liquidation demand curve into the inelastic region where revenues decline. Conversely, if the shock is small, bank revenues do not decrease, and the shock does not propagate.

Before continuing, we provide the following definition of an equilibrium at the intermediate date in the post-shock economy.

Definition 4. An *equilibrium in the post-shock economy* consists of a set of liquidation decisions by banks j operating on islands $j \in I$, x_{ij}^*, and a price vector $P = [P_1, ..., P_M]$ such that:

1. Each bank's liquidation decisions x_{ij}^* are the solutions to its optimization problem characterized in equations (9), (10), and (11) with $Z_k = Z_k'$.
2. Banks' liquidity and capacity constraints are satisfied.
3. Each element of the price vector P_i satisfies $P_i = L_i(X_i) = A_i - bX_i$.

Put simply, banks maximize profits subject to their constraints and the liquidation market clears. Our next proposition characterizes the islands to which our shock is transmitted.

Proposition 2. *If all islands have liquidation markets which are elastic, $\delta_k > \delta_k^*$, and island k' is connected to island k - directly or indirectly - liquidation on island k' in the post-shock equilibrium will increase following the shock to island k. If all islands are connected, the shock is transmitted to the entire economy.*

Proposition 2 states that any island which is connected to island k, the source of our shock, will experience an increase in liquidations, even if its connection is only indirect. We view this result

[9] If we instead assumed some islands had inelastic liquidation markets, some banks may opt to curtail their liquidation activities on those particular islands. However, in this instance the shock will still be transmitted.

as important for two reasons. First, it suggests that two islands which are completely unrelated on the production side of the economy, are in fact linked together through the financial sector. Although this result can be shown in reduced form with a representative financial sector, such models cannot speak to *which* islands are (particularly) connected. To our current knowledge ours is the first result showing how a series of banks can transmit shocks across the economy to seemingly distant islands.

Second, and perhaps more importantly, from a policy perspective this result suggests that efforts to contain a shock should be concentrated at the source. Put differently, Proposition 2 states that *if* the shock is transmitted from its source, it touches *every* part of the economy. Thus intuitively, simply stopping the shock on an individual island which is not the source, will prove ineffective unless the island itself represents a bottleneck in the financial system. In contrast, preventing the shock from triggering the initial series of contagious liquidations is likely to prove more fruitful. There is a natural analogy which can be made to oil spills, where the most effective method to stop the spread of oil is usually putting a lid on the exposed oil well.

Next, we seek to characterize which islands are more affected by the transmission of our shock. To simplify the remaining analysis we assume investment opportunities are identical across islands. That is, we assume $\{p_i, q_i, A_i, b_i\} = \{p, q, A, b\}$. The next two definitions will also prove to be useful.

Definition 5. We say there is a *path* from island i to island i' if i and i' are connected - either directly or indirectly. The *length* of a particular path between i and i' is equal to $1 + m$ where m is the number of intermediary islands in $i_l = \{i_1, ..., i_m\}$. The *shortest path* between two connected islands i and i' consists of the fewest intermediary islands $i_l = \{i_1, ..., i_m\}$ required to connect islands i and i'.[10]

In any network model, the notion of distance is usually central to understanding the variable of interest. Our paper is no different. However, unlike many network models, connections between our nodes are implicit. That is, two nodes are connected by the *decisions* of the parties operating on those nodes. Such a structure makes obtaining analytically tractable results challenging. Thus we shift our focus to fully symmetric networks, which we define below.

[10]If i and i' are directly connected, then the shortest path has $i_l = \emptyset$.

We view these restrictions as reasonable for two reasons. First, our focus to fully symmetric networks is in fact consistent with much of the financial networks literature. For example, Allen and Gale [2000] compare two fully symmetric networks; the ring network and the complete network. Second, doing so allows us to provide analytical results which do not rely on numerical simulations. Finally, we should note that in Section 2.7, we will consider some networks which are not fully symmetric.

Definition 6. A *symmetric financial network* is a financial system in which banks are identically distributed across islands. A *fully symmetric network* is a symmetric network in which all the off-diagonal elements of the system matrix, $S_{ii'}$, $i \neq i'$, equal either zero or a positive constant, c.[11]

To help fix ideas, we provide the following two intuitive examples of fully symmetric networks. Depicted below are the complete and ring financial networks. The symmetric aesthetic "nature" of each network is easily observed. However, in each case, these depictions are fully symmetric networks only if there is the same number of banks connecting each island. In fact, these may not even represent symmetric networks by our definition if the majority of the banks are concentrated on a single island.

For the sake of this example, assume there are no small banks in the economy. Then if every line drawn in the ring network represents one bank, the ring network is indeed fully symmetric. However, if this is not the case, say for example the line between island D and island E in the ring network instead represents three banks, then the final part of definition 6 is violated and our ring network would not constitute a fully symmetric network.

[11]I.e. For any two pairs of directly connected islands in a fully symmetric network, the intensity of both connections (the number of banks which connect each pair of islands) is identical.

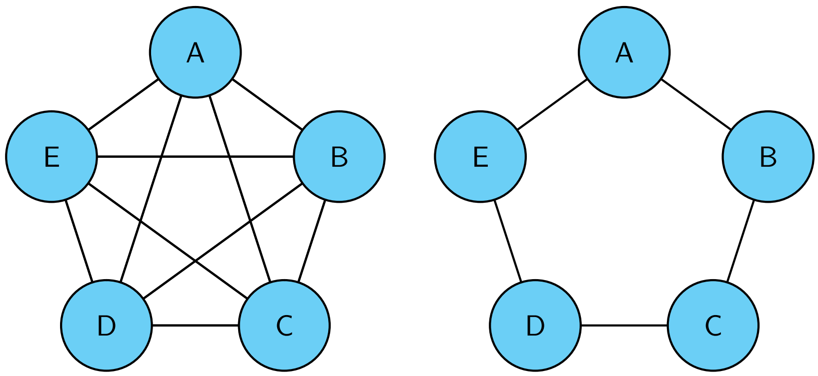

Figure 2: The complete (left) and ring (right) financial networks

Having restricted our attention to fully symmetric networks, we can now characterize which islands in our economy will be more affected by the transmission of our shock. Our next proposition focuses on the two extreme cases of fully symmetric networks which we provided above; the ring network and the complete network.

Proposition 3. *Suppose S represents either the fully symmetric ring network or the fully symmetric complete network. Consider a shock to island k of size $\delta_k = Z > \delta_k^*$. For any two islands k' and k'' which are connected to island k, the increase in liquidations on k' will be strictly greater than the increase in liquidations on k'' if and only if the shortest path from k to k' is strictly less than the shortest path from k to k''. If the shortest path between k and k' equals the shortest path between k and k'', then k' and k'' will experience an identical increase in the level of liquidations.*

Proposition 3 states that the islands closest to the initial shock, as measured by the shortest path, will be the most affected. Thus Proposition 3 states that the impact of our shock dissipates with distance. If we view the islands in our model as geographical regions, Proposition 3 implies a shock to Florida will affect nearby regions more than distant regions. However, what constitutes a "nearby region" in our model is open to interpretation. If our large banks are those which operate across state lines but within particular Fed jurisdictions (*e.g.* a Florida bank might operate in Georgia as both are regulated by the Atlanta Fed, yet not in Texas which is regulated by the Dallas Fed), then a shock to Florida will have the greatest impact on Georgia. However, if our large banks represent billion dollar financial institutions such as Bank of America, then a shock to Florida might have greater consequences for New York as opposed to Florida's geographical neighbor Georgia.

2.6 Financial Integration

The level of financial integration in the economy has received increased attention since the 2008 financial crisis. In this section, we examine how financial integration in our model can improve efficiency. To do so, we must first formally define financial integration.

Definition 7. Consider two (M, N, N_1) economies, each with fully symmetric financial systems represented by the system matrices S, S'. We say the economy with system matrix S exhibits a higher level of financial integration than the economy with system matrix S' if and only if $c < c'$.

We return now to our examples of the ring and complete financial networks from section 1.6. First, we start by fixing the number of banks on each island in both network to be four. By our definition of the fully symmetric network, it follows that the complete network requires each connection depicted in Figure 2(a) to represent *one* bank (*i.e.* $c = 1$). In contrast, the ring network requires each connection depicted in Figure 2(b) to represent *two* banks (*i.e.* $c = 2$). Thus we say the complete network exhibits a higher level of financial integration than the ring network. Our next proposition compares shock transmission in the ring and complete networks.

Proposition 4. *Consider an (M, N, N_1) economy with a fully symmetric complete network and an (M, N, N_1) economy with a fully symmetric ring network. For any shock satisfying $\delta_k = Z > \delta_k^*$, the aggregate increase in liquidations across all islands j will be greater under the ring network than the complete network.*

Proposition 4 states that the same shock to a fully symmetric ring network, which is otherwise identical to its complete network counterpart, causes greater liquidations in the ring network. The intuition behind Proposition 4 is a simple diversification story. The banks which operate in the complete network are better diversified across the remainder of the economy than their counterparts in the ring network. Thus a shock to an island in the complete network which creates "pressure" in the liquidation markets has more immediate release valves. More specifically, banks which operate on island k who ultimately increase their liquidations elsewhere, face less competition from other affected banks when liquidating on other islands k'. In contrast, a shock to an island in the ring network leaves its banks with few options to cover their resulting liquidity shortages.

As argued earlier, liquidation harms welfare through its impact on firm's customers, workers, and suppliers. Thus we view this result as evidence financial integration can improve efficiency. That is, efficiency is higher in the relatively well integrated complete network as opposed to the relatively poorly integrated ring network. Our result contrasts with some of the work in the literature examining how shocks transmit through bank's liabilities (*e.g.* Acemoglu et al [2015]). Here, large shocks to more integrated networks can cause greater spillovers. However, we do not consider our result to directly contradict these papers, rather we see this result as helping unify our understanding of how shocks propagate through networks of intermediaries.

2.7 Heterogeneity

Finally, we are interested in how heterogeneity might affect the propagation of shocks in our economy. Our model provides many natural candidates through which we can introduce heterogeneity. For example, we could consider different liquidation markets characterized by different $L_i(X_i)$, different levels of competition across islands, or perhaps asymmetric network structures such as the well known star network. Although these are all good candidates, we instead opt to focus on the "butterfly" network depicted below, and leave these other candidates to future research.

Although the butterfly network with an identical number of banks located on each island appears to be symmetric, it does not satisfy our definition of a symmetric network. To see why, notice that the position of island A in our butterfly network is very different to the position of all other islands in this economy. Thus, conveniently, heterogeneity in the butterfly network arises purely in a post-shock sense. A shock to island A will have a very different transmission path than a shock to island B.

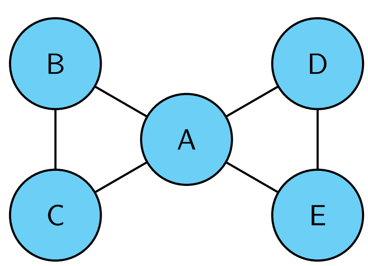

Figure 3: The Butterfly Financial System

Consider the following parameterization of the butterfly network; $M = 5$, $N = 20$, $N_1 = 20$, $A = 1.75$, $b = 1$, $Z = 1$, $\gamma = 0.345$, $\delta = 1$. Plotted below are the liquidation responses on each island. The left panel displays the responses when the shock hits island A, whereas the right panel displays the liquidation responses when the shock hits island B. The numbers in each plot are scaled to highlight the differences across islands.

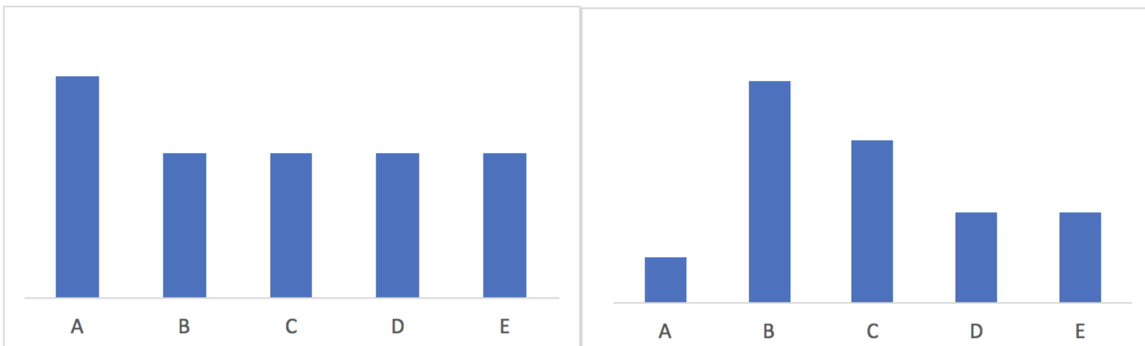

Figure 4: Liquidation Patterns in the Butterfly Network

Aggregating across islands we find total liquidations in panel (a) are marginally greater than total liquidations in panel (b). However, the distribution of shocks across islands are very different. Abstracting away from the liquidations on the shocked islands, we find that unsurprisingly, the shock to central island affects all peripheral islands symmetrically. However, in contrast, a shock to a peripheral island produces a distribution of shocks which is at first glance counter-intuitive. In particular, we observe the distant islands D and E are *more* affected by a shock to island B than the centrally located island A. Underpinning this result is the benefit of diversification for banks on island A. Essentially, although island A is located in closer proximity to the shock on island B, each of its banks relieves stress from the shock by increasing liquidations on different islands. Conversely, the majority of the banks operating on more distant islands D and E, have fewer options to relieve pressure.

3 Discussion

The relevance of the transmission channel described in this paper depends on several aspects relating to the health and the structure of the financial system. First, the propagation of an idiosyncratic shock becomes less of a concern when the liquidity in the financial system is abundant. Banks do not need to engage in additional liquidations to make up for their missing revenue when they hold more liquidity. Hence, a financial system with significant holdings of liquid assets is less prone to transmit shocks. Liquidity of the financial system is more likely to be low at times of bank distress. This suggests that bank distress increases the likelihood of the propagation of shocks in the real economy. Such an argument provides a novel rationale as to why the health of the financial system is vital for the real economy.

Competition also plays an important role in our transmission of shocks. If banks act as price takers, they fail to internalize the price impact of their decisions in the liquidation market. Less competitive and thus more concentrated banking systems reduce the probability of transmission by lowering banks' incentives to sell in the liquidation market. Moreover, more integrated banking systems in contrast to more specialized ones reduce the impact of the shock by diversifying its effect across sectors.

Finally, banks in our model do not default. This feature of our setup, despite its tractability, is not realistic. However, ignoring this aspect works against us from the perspective of producing quantatatively meaningful spillovers. Thus, more realistic setups seeking to produce quantative implications might seek to incorporate the possibility of defaults, mixed strategies, and amplification. We leave these possibilities to future research.

4 Conclusion

It has long been recognized that banks are not neutral players in the economy. Their decisions affect real variables. In this paper, we developed a model in which banks played a central role in transmitting negative shocks to different sectors of the economy. In our model, banks increased their liquidations in response to a shock to an individual sector in our economy. If the shock was sufficiently large, it spilled over to other sectors of the economy. These spillovers dissipated with distance. We showed how financial integration can reduce spillovers, and how heterogeneity has non-trivial implications. Our findings have policy implications for systemic risk mitigation and shock containment.

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Appendix

Proof of Proposition 1

Using equilibrium quantities derived in section 2.3, the total revenue generated on island k before the shock can be calculated as

$$\left[A - bn_k \frac{A_k - Z_k}{(n_k+1)b}\right] n_k \frac{A_k - Z_k}{(n_k+1)b}. \tag{15}$$

If all banks remain unconstrained after the shock, the revenue generated will be

$$\left[A - bn_k \frac{A_k - Z_k'}{(n_k+1)b}\right] n_k \frac{A_k - Z_k'}{(n_k+1)b} \tag{16}$$

where $Z_k' = Z_k - \delta_k$. Given banks set $c_j^* + \sum_i L(X_i^*)x_{ij}^* = \gamma$, the shock will spread if and only if the second expression is less than the first. That is, the condition for the shock to spread is

$$\left[A - bn_k \frac{A_k - Z_k}{(n_k+1)b}\right] n_k \frac{A_k - Z_k}{(n_k+1)b} > \left[A - bn_k \frac{A_k - Z_k'}{(n_k+1)b}\right] n_k \frac{A_k - Z_k'}{(n_k+1)b} \tag{17}$$

which implies

$$\delta_k^* = 2Z_k - \frac{(n_k-1)A_k}{n_k}. \tag{18}$$

Proof of Proposition 2

It will be convenient to introduce some additional notation. Define $\alpha_j = \frac{1}{1+\lambda_j}$, where λ_j is the Lagrange multiplier associated with bank j's liquidity constraint in expression (14). Before the shock, these Lagrange multipliers are zero for every bank (*i.e.*, $\lambda_j = 0$). Define $\hat{\gamma}_j = \gamma - c_j^*$ as bank j's planned pre-shock liquidation revenue. Using first order conditions, bank j's optimal liquidation amount on island i and the liquidation price can be obtained as

$$x_{ij} = \frac{A_i + Z_i(\sum \alpha_r - (n_i+1)\alpha_j)}{(n_i+1)b} \tag{19}$$

and

$$P_i = \frac{A_i + Z_i \sum \alpha_r}{n_i + 1} \tag{20}$$

where the summations are over all banks operating on island j.

First, we show $\alpha_j < 1$ for all banks (*i.e.*, all banks are constrained). By the assumption $\delta_k > \delta_k^*$, we know $\alpha_j < 1$ for all banks operating on island k. Consider island k', where bank j and bank j' both operate. Without loss of generality, assume bank j also operates on island k, whereas bank j' operates on some other island(s) $k'' \neq k$. Suppose bank j' is not constrained ($\alpha_{j'} = 1$). Because $\alpha_j < 1$, all else equal, by (19) and (20) both $x_{ij'}$ and $P_{k'}$ will decrease following the shock. Thus, bank j''s liquidation revenue on island k' will decrease following the shock. Moreover, because bank j' has no incentive to change its liquidation decisions on any other island (bank j' is unconstrained by our supposition), it follows that its post-shock liquidation revenues will be strictly less than $\hat{\gamma}_{j'}$:

$$\sum_{i \in J'} P_i x_{ij'} < \hat{\gamma}_{j'}. \tag{21}$$

However, we have a contradiction, because (21) implies bank j' is indeed constrained. Thus it follows bank j' cannot be unconstrained in equilibrium, *i.e.* bank j' must be constrained. A similar argument can be constructed to prove any bank which is competing with a constrained bank on a particular island must also be constrained. It follows that all banks which are connected - directly or indirectly - to island k will become constrained after the shock occurs.

Given all connected banks are constrained post-shock, we now show liquidations increase on all islands. For any island i, total liquidations are given by:

$$\sum_{j \in I} x_{ij} = \frac{n_i A_i - Z_i \sum \alpha_r}{(n_i + 1)b} \tag{22}$$

The derivative of this expression with respect to $\sum \alpha_r$ is negative. Because post-shock all α's decrease, $\sum \alpha_r$ also decreases, and thus total liquidations on every island must increase. That is, the initial shock - if sufficiently large - leads to increased liquidations on all connected islands.

Proof of Proposition 3

Consider a ring network with M islands and two banks operating on each island. The price on island i, $\frac{A+Z(\alpha_j+\alpha_{j'})}{3}$, is a sufficient statistic for the total liquidation on island i, where $j, j' \in I$. It is useful to define

$$f(\alpha_j, \alpha_j') = \frac{\left[A + Z(\alpha_j + \alpha_{j'})\right]\left[A + Z(\alpha_{j'} - 2\alpha_j)\right]}{9b}. \tag{23}$$

The function $f(\cdot, \cdot)$ is the revenue of bank j on an island where it competes only with bank j'. This function's derivatives with respect to its first and the second arguments are negative and positive respectively.

We prove the case where M is an even number. A similar argument can be constructed when M is an odd number. Due to the symmetric nature of the ring, there are $t = \frac{N}{2}$ different types of banks. Without loss of generality, assume the shock hits island $k = 1$. Moreover, we label bank types by their shortest distance to the island which received the shock (*e.g.* banks denoted $j = 1$ are those which operate on the island which received the shock, whereas banks denoted $j = 2$ are those which are one island removed from the initial shock). Because each bank operating on island 1 receives liquidation revenue of $P_1 x_{11} = \frac{A^2}{9b}$ after the shock, their budget constraints can be written as

$$\frac{A^2}{9b} + f(\alpha_1, \alpha_2) = \hat{\gamma} \tag{24}$$

where $\hat{\gamma}$ is equal to each banks' pre-shock expected revenue from its liquidation activities. The budget constraint of a bank j operating on islands j and $j+1$ is

$$f(\alpha_j, \alpha_{j-1}) + f(\alpha_j, \alpha_{j+1}) = \hat{\gamma} \tag{25}$$

where $1 < j < t$. Finally budget constraints of the banks operating on island $t+1$ are

$$f(\alpha_t, \alpha_{t-1}) + f(\alpha_t, \alpha_t) = \hat{\gamma}. \tag{26}$$

We first show that prices on any two connected islands can not be the same. Suppose the opposite; $P_j = P_{j'}$. Consider banks $j-1$, j, and $j+1$ which operate on islands $(j-1, j)$, $(j, j+1)$ and $(j+1, j+2)$ respectively. Because prices are identical it follows that,

$$\frac{A + Z(\alpha_j + \alpha_{j-1})}{3} = \frac{A + Z(\alpha_{j+1} + \alpha_j)}{3} \tag{27}$$

or $\alpha_{j-1} = \alpha_{j+1}$. Budget constraints of banks $j-1$ and $j+1$ together imply:

$$\alpha_{j-2} = \alpha_{j-1} = \alpha_j = \alpha_{j+1} = \alpha_{j+2}. \tag{28}$$

Repeating this argument using the budget constraints of banks $j-2$ and $j+2$, and so on, we obtain $\alpha_j = \alpha \ \forall j$. Moreover, to satisfy the budget constraints of banks $j \geq 2$, we require $f(\alpha, \alpha) = \hat{\gamma}/2$. However, we have a contradiction because the first budget constraint requires $f(\alpha_1, \alpha_2) > \hat{\gamma}/2$. Therefore, prices on any two connected islands cannot be the same in equilibrium.

Next, we refine the equilibrium concept to rule out potential unstable candidates. In particular, we ignore any equilibrium candidate in which two banks j, j' operating on island i both fail to produce half of their required liquidation revenue on island i; *i.e.* we rule out $f(\alpha_j, \alpha_{j'}), f(\alpha_{j'}, \alpha_j) < \hat{\gamma}/2$. This set of strategies is not stable. If there is a small deviation in either bank's liquidation quantity on island i, both banks - via their best response functions - will be pushed back to the pre-shock equilibrium in which each will liquidate $x^* = \frac{A-Z}{3b}$.

This refinement also rules out situations where the price on island j is smaller then prices on both islands $j-1$ and $j+1$. To see why, first note that among the two islands a bank operates, it liquidates more on the island where the price is higher. Now suppose $P_j < P_{j-1}$ and $P_j < P_{j+1}$. Consider bank j operating on islands j and $j+1$ and bank $j-1$ on islands $j-1$ and j. Bank j liquidates $\frac{A+Z(\alpha_{j+1}-2\alpha_j)}{3b}$ on island $j+1$ and $\frac{A+Z(\alpha_{j-1}-2\alpha_j)}{3b}$ on island j. Because the price on island j is smaller than the price on island $j+1$ by the supposition, $\alpha_{j+1} > \alpha_{j-1}$. Then, bank j's budget constraint implies bank j's revenue on island j is less than $\hat{\gamma}/2$. Likewise, bank $j-1$ liquidates $\frac{A+Z(\alpha_j-2\alpha_{j-1})}{3b}$ on island j and $\frac{A+Z(\alpha_{j-2}-2\alpha_{j-1})}{3b}$ on island $j-1$. Because the price on island j is smaller than the price on island $j-1$, $\alpha_{j-2} > \alpha_j$. Consequently, bank $j-1$'s revenue on island j is also less than $\hat{\gamma}$. Our refinement rules out this possibility.

Now, suppose $f(\alpha_t, \alpha_t) < \hat{\gamma}/2$. The last budget constraint implies $f(\alpha_t, \alpha_{t-1})$ should be greater than $\hat{\gamma}/2$. However, this cannot be an equilibrium due to our refinement. Banks operating on island t can improve their revenues and profits by chosing their pre-shock liquidation amounts.

Next, suppose $f(\alpha_t, \alpha_t) = \hat{\gamma}/2$. However this implies $\alpha_j = \alpha \ \forall j$, which we know from earlier produces a contradiction. Thus, $f(\alpha_t, \alpha_t)$ is greater than $\hat{\gamma}$. Thus the last budget constraint implies

$P_t > P_{t-1}$:

$$\frac{A + Z(\alpha_t + \alpha_{t-1})}{3} < \frac{A + Z(\alpha_t + \alpha_t)}{3}. \tag{29}$$

Moreover, P_{t-1} must be greater than P_{t-2} according to our refinement. Similarly, P_{t-2} must be greater than P_{t-3}. This argument can be iterated until the first island to give:

$$P_1 < P_2 < ... < P_t. \tag{30}$$

That is, prices increase monotonically with an island's distance from the shock. Then, because the liquidation price on island i is a decreasing function of the total liquidations on island i, total liquidations on each island decrease with distance.

Proof of Proposition 4

Suppose there are M islands and $N = (M-1)M/2$ banks. Consider the ring and the complete networks, in which each has $M-1$ banks operating on all islands. The revenue on island $k = 1$ prior to the shock is the same in both networks. It is

$$(M-1)P_1 x_{11} = \frac{(M-1)A^2}{M^2 b}. \tag{31}$$

Define the total revenue on island i in the ring and the complete networks as $P_i(X_i^r)X_i^r$ and $P_i(X_i^c)X_i^c$. The total proceeds from liquidation on all islands in both networks after the shock must be the same:

$$\sum_{i=1}^{M} P_i(X_i^r)X_i^r = \sum_{i=1}^{M} P_i(X_i^c)X_i^c \tag{32}$$

and

$$\sum_{i \neq k}^{M} P_i(X_i^r)X_i^r = \sum_{i \neq k}^{M} P_i(X_i^c)X_i^c = (M-1)F \tag{33}$$

where $F = P_i(X_i^c)X_i^c \ \forall \ i \neq k$. Now, observe the revenue function is increasing and concave on all islands $i \neq k$. That is, the second derivative of $P_i(X_i)X_i$ is negative. For any strictly increasing concave function $g(x)$ it is true that

$$g(x_1) + g(x_2) + ... + g(x_n) = ng(x^*) \tag{34}$$

if and only if

$$x_1 + x_2 + ... + x_n > nx^*. \tag{35}$$

Thus, total liquidations in the ring should be higher than total liquidations in the complete network:

$$\sum_{i=1}^{M} X_i^r > \sum_{i=1}^{M} X_i^c. \tag{36}$$