

Optimal Disclosure and Fight for Attention*

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Abstract

In this paper, firm managers use their disclosure policy to direct speculators' scarce attention towards their firm. More attention implies greater outside information and strengthens the feedback effect from stock prices to firm investment. The model highlights a novel trade-off associated with disclosure. While more precise public information crowds-out the value of private information, it can also signal high firm quality to the financial market. If the spread between the (unknown) quality of firms is sufficiently high, there is a separating equilibrium with partial disclosure by the high-quality firm and no disclosure by the low-quality firm. Otherwise, there is a (more efficient) pooling equilibrium without disclosure. Surprisingly, an increase in the managers' short-run incentives encourages disclosure and renders equilibrium disclosure more efficient.

Keywords: attention allocation, disclosure, feedback effect, asymmetric information, real efficiency.

JEL Classification: D82, D83, G14.

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1 Introduction

Stock prices can play an important role for firms' real investment decisions by aggregating traders' private information. This "feedback effect" (Bond et al., 2012) improves the managers' information about the uncertain future and increases investment efficiency. As a result, benevolent firm managers have an incentive to shape the informational environment in a way that increases the informational content of their firm's price.

In this paper, I provide a theoretical framework in which firm managers compete for the scarce attention of privately informed speculators. The more attention speculators pay to a certain firm, the more outside information is reflected by the stock price and the more efficient is this firm's investment decision. In the model, firms can influence the informational environment through their commitment to a disclosure policy which requires each firm to reveal a pre-specified amount of information to the market. The main focus of the paper is to analyze what type of disclosure policy firms choose in equilibrium and what consequences this decision has on the feedback effect from prices to firm investment.

A key assumption of the model is that speculators have limited attention. In reality, financial market participants are overwhelmed with information like data releases, earnings reports or public announcements. Clearly, a rational trader with unlimited resources would like to thoroughly read and process all of this information to make the optimal trading decision. However, in actual markets even professional traders ("speculators") face capacity constraints. They have to decide how to allocate their limited time (or resources) between different tasks, firms or sectors. In this paper, I connect this attention allocation problem with two important firm decisions, information disclosure and real investment.

How firms should structure their disclosure policy is a controversial question. First, there is a substantial policy debate about the implementation of disclosure regulations or mandatory disclosure. Second, the existing academic literature has highlighted several costs and benefits associated with firm disclosure (see e.g. Leuz and Wysocki (2016) and Kanodia and Saprà (2016)). At the same time, there is ample evidence that investors regularly shift attention from one firm (or

industry) to another (see e.g. Barber and Odean (2008) and Da et al. (2011)).¹ Nevertheless, we still know very little about the way in which firms can actively control or influence this reallocation of attention. In this paper, the feedback effect from stock prices to firm investment encourages managers to compete for attention through their disclosure policy. This specific mechanism has received recent support from the empirical literature (Cunat and Groen-Xu (2016) and Edmans et al. (2017)).

In the model, a firm's commitment to a certain disclosure policy serves a dual purpose. On the one hand, committing to a high degree of transparency sends a powerful signal to the market because a firm of low quality (and therefore imprecise information) could never credibly promise to issue a precise signal in the future. On the other hand, giving all market participants an additional public signal crowds out the value of speculators' private information and lowers their incentive to pay attention to a certain firm. Hence, each firm's optimal disclosure decision has to balance these two forces.

I analyze a simple model with two types of firms that are run by benevolent managers who make two decisions to maximize their firm's expected value: committing to a disclosure policy and investing in an uncertain growth opportunity. Claims to each firm's terminal payoff are traded in a financial market by (potentially) informed speculators and liquidity traders. Each speculator can acquire private information about a firm if he pays attention to it. Thus, attention should be interpreted as time or effort devoted to fundamental research of a certain firm. More extensive research leads to a more precise signal that allows the speculator to invest more efficiently in the firm's stock.

A crucial feature of the model is that financial market participants are uncertain about each firm's type. They only know that a firm could be of low or high quality but cannot distinguish between the two types ex ante. I capture firm quality by the precision of the managers' private signals. The manager of a high-quality (*hq*) firm receives a very precise private signal about the return on the growth opportunity, and vice versa. Each manager knows his type and the only tool to communicate it to the financial market is through the firm's disclosure policy.

¹See also Dellavigna and Pollet (2009) and deHaan et al. (2015) for empirical evidence of investors' limited attention.

The model has four periods. In $t = 0$, the firm managers decide on their firm's disclosure policy, i.e. they decide how much of their private information to reveal to the public in $t = 2$. In $t = 1$, speculators choose how to allocate their scarce attention between the firms. This decision determines the precision of their private signals and the informational content of the respective stock price. In $t = 2$, the endogenously informed speculators trade in the risky assets against liquidity traders and a risk-neutral market maker sets the equilibrium prices competitively. In $t = 3$, the firm managers decide on their firm's investment in a growth opportunity. Importantly, they base this decision, in part, on the informative stock price which represents the "feedback effect" in the model.

When the firm managers decide on the investment decision at $t = 3$, they have access to two signals, a private signal and the stock price that aggregates the speculators' private information. In equilibrium, managers choose to invest if either signal reveals that their firm's fundamental value is high, $\theta_j = H$. Given that the project's ex ante NPV is negative, they choose not to invest if they are uncertain about θ_j or if they know that is low, $\theta_j = L$. At $t = 2$, a continuum of informed speculators with mass n_j trade the firm's stock in a financial market against a continuum of liquidity traders. The price of the asset is set by a market maker based on total order flow and the disclosed public signals with precisions determined at $t = 0$. Depending on the realized signal of the firm's fundamental and the value of liquidity trading, the stock price can take one of three values ("high", "intermediate", "low"). Importantly, if the price is in the high (low) state, both the market maker and the firm manager can infer that the speculators must have received a "high" ("low") signal. The intermediate stock price value is uninformative because liquidity trading clouds the speculators' information. Importantly, this state becomes less likely if more speculators pay attention to a given firm which is why managers have an incentive to maximize the speculators' aggregate attention towards their firm.

The mass of informed speculators n_j depends on their collective attention allocation decision. At $t = 1$, each speculator has to decide which firm to pay attention to. This decision, in turn, determines whether the speculator becomes privately informed about firm A or B . Importantly,

at $t = 1$ the only factor that differentiates the two firms is their disclosure decision, γ_j^* . Given that a higher mass of informed speculators leads to a more informative stock price, each firm tries to maximize this mass through its disclosure decision. I show that there are two mutually exclusive equilibria that depend on the possible quality levels of the low-quality and the high-quality firms. If this difference is relatively small, there is a pooling equilibrium in which both firm managers choose to withhold their private information. As a result, speculators cannot differentiate between the two firm types and each firm receives an equal share of attention. If the difference is high, however, there is a separating equilibrium. The low-quality firm still chooses to withhold its private information, but the high-quality firm partially discloses its information. In particular, it discloses just enough information to send a credible signal of its high quality to the financial market.

Next, I compare the two types of equilibria in terms of their efficiency, i.e. the implications on the ex ante expected firm values. Interestingly, the pooling equilibrium, in which financial market participants cannot discriminate between both firms, leads to a more efficient allocation of attention. Intuitively, from an ex ante perspective efficiency is maximized if the *low-quality* firm receives all attention because this firm relies more heavily on the aggregated private information in the stock price. Given that the equilibrium mass of speculators in this firm is higher in the pooling equilibrium, it follows that this outcome is more efficient.

Moreover, I discuss possible efficiency gains from two types of exogenous disclosure rules. First, if these rules can be conditioned on the firms' true types (e.g. because both firms are governed by the same board of directors), the most efficient disclosure rule is to force the high-quality firm to be fully transparent and the low-quality firm to be fully opaque. This choice ensures that the low-quality firm receives the speculators' entire attention. Second, if the disclosure rule cannot be conditioned on the true quality, it is most efficient to impose a "cap" on the maximum disclosure quality. Intuitively, this cap implies that the high-quality firm cannot deviate into the separating equilibrium, which would decrease the amount of attention allocated to the low-quality firm.

Lastly, I analyze a variation of the main model in which the firm managers are no longer

benevolent but are also endowed with short-run incentives through stock-based compensation. I show that this alternative setup changes the high-quality firm's disclosure policy because its manager directly benefits from the higher stock price that results from the market maker knowing the firm's true quality. As a result, short-run incentives increase the high-quality firm's willingness to disclose even if it results in a loss of attention. Most surprisingly, higher short-run incentives render the equilibrium more efficient as more attention is shifted to the low-quality firm that benefits more from it.

This paper is related to three literatures. First, the finance literature on the real effects of financial markets (reviewed in Bond et al. (2012)). Second, the accounting and finance literature on the real effects of disclosing information in financial markets (reviewed in Kanodia (2007) and Goldstein and Yang (2017)). Third, the economics literature on limited attention and endogenous information acquisition (reviewed in Veldkamp (2011)).

There is a substantial literature in accounting and finance on the real effects of disclosing information. Recent contributions are Gao (2008), Gao (2010) and Cheynel (2013).² These papers study the impact of corporate disclosure on the firm's cost of capital and price efficiency. One main difference with respect to this literature is the assumption that financial markets have spillover ("feedback") effects to the real economy. This feedback effect is modeled through the informational role of the two asset prices, as e.g. in Subrahmanyam and Titman (1999) and Goldstein et al. (2013)³. Other papers in the feedback literature that discuss firms' optimal disclosure are Goldstein and Yang (2018), Kurlat and Veldkamp (2015), Han et al. (2016), Bond and Goldstein (2015), and Edmans et al. (2016). Gao and Liang (2013) also discuss a feedback model with optimal disclosure. In their paper, disclosure crowds out private information production, reduces price informativeness and harms managerial learning. In Thakor (2015) firms may withhold information because disclosing it can actually lead to greater disagreement between managers and investors.

Closely related is an early paper by Fishman and Hagerty (1989). Their paper assumes that it is costly for traders to process the *disclosed* information. Therefore, firms compete for traders'

²See also Strobl (2013) for a setting in which greater (exogenous) disclosure increases the manager's incentive to engage in earnings manipulation.

³See also Boot and Thakor (1997), Foucault and Fresard (2014), and Foucault and Fresard (2016).

attention over the disclosed signal because traders have to choose which signal to pay attention to. Even though this mechanism might look similar to the one in my paper, it is quite different. In my paper, the disclosed public signal does not require any attention from the speculators. It does, however, have an important impact on the speculators' information acquisition decision regarding their *private* information. In particular, in my model the lack of disclosure is a useful tool to render a firm a more "attractive" target for speculators. As a result, the equilibrium in the two models are also fundamentally different. While in Fishman and Hagerty (1989), both firms have an incentive to disclose in equilibrium, I show that under limited attention *withholding* information or partial disclosure is the natural equilibrium outcome.

Overall, the main contribution of this paper is to highlight a novel trade-off associated with disclosure: (i) a crowding-out effect similar to that in Gao and Liang (2013) and (ii) less asymmetric information about the firm's true quality. The multi-firm setup allows me to study an alternative information acquisition technology, limited attention capacity as in Kacperczyk et al. (2016) or van Nieuwerburgh and Veldkamp (2010). Importantly, this novel setup leads to several new implications. For example, there can be different types of disclosure equilibria (pooling or separating) depending on the true quality of the two firms. Moreover, I show that in general the firm that requires less attention (from a social planner's perspective) generally receives more attention such that the equilibrium attention allocation and disclosure policy is inefficient, in general. Lastly, I highlight the role of different disclosure mandates and short-run managerial incentives on the speculators' attention allocation, price informativeness, and economic efficiency.

The remainder of this paper is organized as follows: Section 2 describes the basic model. Section 3 solves for the financial market and attention allocation equilibrium. I discuss the firms' fight for attention and disclosure policies in Section 4. Section 5 discusses efficiency and policy implications of the model mechanism and Section 6 discusses an extension with short-run incentives for the managers. Section 7 concludes.

2 The Model

The model has four dates, $t \in \{0, 1, 2, 3\}$. There are two firms $j \in \{A, B\}$ of unknown quality γ_j whose stock is traded in a financial market. Each firm's payoff depends on an unknown fundamental θ_j and the manager's investment decision, which is set to maximize the firm's expected value. At $t = 0$, each firm announces its disclosure policy and commits to send out a public signal with precision γ_j^* . Based on this decision, each speculator decides whether to pay attention to firm A or B in $t = 1$. This attention allocation decision determines the precision of the speculators' private signals which affects their trading behavior at $t = 2$. In addition to speculators, a continuum of liquidity traders and a market maker participate in the financial market. The latter sets the two stock prices equal to the expected firm value based on total order flow and the two disclosed signals. At $t = 3$, the firm managers potentially invest in a growth opportunity depending on their private signal and stock price, which partially reflects the speculators' aggregated private information. Figure 1 summarizes the sequence of events.

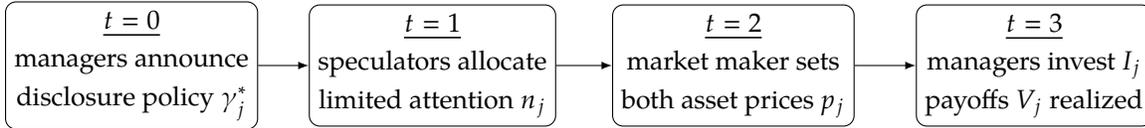


Figure 1: Timeline for the basic model.

2.1 The Firms

There are two unobservable states of the world, $\theta_j \in \Theta = \{L, H\}$ ("low" and "high") with equal probability that are independently drawn for each firm.⁴ Firm j 's future value is given by:

$$V_j = A + I_j x^{\theta_j} \quad (1)$$

where $A > 0$ is the constant return on the firm's assets in place, $I_j \in \{0, 1\}$ denotes the benevolent firm manager's investment decision and x^{θ_j} is the net present value (NPV) of investment in a risky

⁴See Foucault and Fresard (2014) for a model with correlated fundamentals. In their model firms have an incentive to learn from the stock price of the other firm as well.

growth opportunity.⁵ I assume $x^L < 0 < x^H$ and $x^L + x^H < 0$ such that the ex ante NPV of the growth opportunity is negative. Intuitively, the latter assumption implies that the firm managers only choose to invest if they know that $\theta_j = H$ but not if they remain uninformed about the firm's fundamental or know that $\theta_j = L$.

The firm manager's decision whether to invest or not is based on two signals. A private signal $\sigma_{mj} \in \{\theta_j, \emptyset\}$ and an endogenous feedback signal from the financial market based on the stock price p_j . Each firm's *quality* is captured by the precision of its private signal, i.e. the probability that it reveals the true fundamental, $\gamma_j \equiv \Pr(\sigma_{mj} = \theta_j)$.

At $t = 3$, firm manager j chooses firm investment to maximize the expected firm value:

$$\max_{I_j \in \{0,1\}} E[V_j | \sigma_{mj}, p_j, \gamma_j]. \quad (2)$$

Importantly, each firm's quality is private knowledge to the respective firm. All other financial market participants only know that there is one high-quality and one low-quality firm in the economy and that each firm is equally likely (ex ante) to be of either type.⁶ Moreover, it is common knowledge that the high-quality firm's manager receives a private signal with high precision $\gamma_{hq} = \bar{\gamma}$, while the other manager receives a low-precision signal ($\gamma_{lq} = \underline{\gamma}$) where $0 < \underline{\gamma} < \bar{\gamma} < 1$.

In addition to the investment decision in the growth opportunity, each firm manager also decides on his firm's disclosure policy. In particular, at $t = 0$ each firm commits to a disclosure policy that requires the firm to issue a public signal with precision γ_j^* to the financial market at $t = 2$. Importantly, I assume that the firms cannot promise a more informative signal than their own private signal σ_{mj} such that $\gamma_j^* \in [0, \gamma_j]$.⁷ An announced disclosure policy with precision γ_j^* therefore implies that the firm sends out a public signal σ_{mj}^* that reveals the fundamental θ_j with probability γ_j^* . Thus, each manager can release a signal with precision arbitrarily close to the true γ_j that can help to signal his identity to the financial market participants. He can, however, also decide to reveal a noisy version of his private signal that only reveals the true signal in some cases.⁸

⁵In Section 6, I analyze an alternative setup in which the firm managers are no longer fully benevolent but are also endowed with short-run incentives.

⁶An alternative, but tantamount, assumption would be to have a continuum of firms drawing their type (high or low quality) independently with equal probability. Thus, the two firms can be interpreted as representative firms for each type.

⁷The upper limit excludes γ_j for reasons of tractability but is not essential for any the main results.

⁸I assume that: $\Pr(\sigma_{m,j}^* = \emptyset | \theta = H) = \Pr(\sigma_{m,j}^* = \emptyset | \theta = L)$, i.e. the omitted states are chosen arbitrarily. An alternative interpretation

Formally, the disclosure policy is chosen to maximize the firm's expected value at $t = 0$:

$$\max_{\gamma_j^* \in [0, \gamma_j]} E [V_j | \gamma_j]. \quad (3)$$

As I show in greater detail below, the managers' disclosure policy is driven by the incentive to attract as much attention from informed speculators as possible. Intuitively, more attention leads to a more informative stock price and therefore to a more efficient investment decision and a higher firm value. In particular, the firm faces the following trade-off when choosing γ_j^* . On the one hand, it would like to provide as little information as possible to the market maker in order to increase the speculators' trading profits (and thus their attention towards the firm). On the other hand, the high-quality firm would like to signal its true quality to the financial market to show that the speculators' attention is well-spent at the firm because it generates a higher payoff, on average, due its more precise signal.

2.2 The Financial Market

There are three types of agents in the financial market: (i) a continuum (with unit mass) of risk-neutral speculators, (ii) liquidity traders who collectively trade $z_j \stackrel{iid}{\sim} U[-1, 1]$, and (iii) a risk-neutral market maker.

Speculator $i \in [0, 1]$ has to decide whether to pay attention to firm A (i.e. $n_{iA} = 1$ and $n_{iB} = 0$) or firm B (i.e. $n_{iA} = 0$ and $n_{iB} = 1$) at $t = 1$. This decision determines the nature of the speculator's private information in the next period, $t = 2$. In particular, if speculator i decides to pay attention to firm j , he receives a perfect signal $\sigma_{ij} = \theta_j$ about this firm's fundamental and no signal about the other firm's shock, $\sigma_{i,-j} = \emptyset$. I denote the endogenously determined number of informed speculators for firm j by $n_j \equiv \int_0^1 n_{ij} di \geq 0$, with $\sum_{j \in \{A, B\}} n_j = 1$. In addition to their private signals, all speculators receive the disclosed public signals $\sigma_{m,j}^*$ which reveal θ_j with probability γ_j^* . To keep the model simple, I assume that the speculators do not have to pay attention to the public signals in order to process them.⁹

is that the manager commits to reveal his signal (with precision γ_j) only with a certain probability (π_j). In this case, γ_j^* corresponds to the expected precision $\pi_j \times \gamma_j$.

⁹This assumption is in line with the empirical evidence in Engelberg et al. (2012). See Fishman and Hagerty (1989) for a model featuring costly interpretation of public information.

Each speculator can buy or sell up to one unit of each asset, i.e. $s_{ij} \left(\sigma_{ij}, \sigma_{mj}^*, \gamma_A^*, \gamma_B^* \right) \in [-1, 1]$.¹⁰ Since the speculators are atomistic, they ignore price impact and either trade the maximum amount ($s_{ij} = 1$ or $s_{ij} = -1$) or not at all ($s_{ij} = 0$). If a speculator is indifferent between trading and not trading, I assume that he chooses not to trade as in Edmans et al. (2015). Formally, each speculator thus solves:

$$\begin{aligned} \max_{n_{iA}, n_{iB} \in \{0,1\}} E[\Pi_i | \gamma_A^*, \gamma_B^*] \\ \text{s.t. } n_{iA} + n_{iB} = 1 \end{aligned} \quad (4)$$

at the attention allocation stage $t = 1$ and

$$\max_{s_{iA}, s_{iB} \in [-1,1]} E[\Pi_i | \sigma_{iA}, \sigma_{iB}, \sigma_{mA}^*, \sigma_{mB}^*, \gamma_A^*, \gamma_B^*] \quad (5)$$

at the trading stage $t = 2$. In these two expressions trader i 's total trading profit is denoted by:

$$\Pi_i = \Pi(s_{iA}, s_{iB}, \sigma_{iA}, \sigma_{iB}, \sigma_{mA}^*, \sigma_{mB}^*, \gamma_A^*, \gamma_B^*, n_A, n_B) = \sum_{j \in \{A,B\}} s_{ij}(V_j - p_j). \quad (6)$$

As in Kyle (1985), the market maker observes the total order flow $X_j = s_j + z_j$, where $s_j = \int_0^1 s_{ij} \left(\sigma_{ij}, \sigma_{mj}^*, \gamma_A^*, \gamma_B^* \right) di$, for each firm but not its individual components. Moreover, he also observes the publicly disclosed signals σ_{mj}^* . The market maker is competitive and sets each firm's equilibrium price equal to the expected payoff:

$$p_j = p \left(X_j, \sigma_{mj}^*, \gamma_A^*, \gamma_B^* \right) = E \left[V_j | X_j, \sigma_{mj}^*, \gamma_A^*, \gamma_B^* \right]. \quad (7)$$

In contrast to Kyle (1985), however, the asset price p_j not only *reflects* the future firm value V_j but also *affects* it through a learning mechanism.¹¹

A key factor for each manager's optimal disclosure decision is the impact on the speculators' *perceived* quality of the firm after observing $\{\gamma_A^*, \gamma_B^*\}$. I denote this updated quality assessment by $\gamma_j^{**} = E[\gamma_j | \gamma_A^*, \gamma_B^*] \in [\underline{\gamma}, \bar{\gamma}]$. As I show below, the firms' *perceived* quality plays an important role on the speculators attention allocation decision. Intuitively, a higher *perceived* quality increases the speculators' incentive to collect information about this firm because it increases the probability

¹⁰This position limit can be interpreted as a form of borrowing/short-selling constraint.

¹¹See also Edmans et al. (2015), Foucault and Fresard (2014) or Gao and Liang (2013) for Kyle-type settings with a feedback effect from stock prices to firm investment.

that the informed speculator and the firm manager receive the same (perfect) signal about θ_j . Consequently, an informed speculator can more efficiently time the market and buy the stock when the shock is high and the firm chooses to invest (which leads to higher payoff), and vice versa.

Definition 1 *A Perfect Bayesian Equilibrium consists of (i) a trading and attention allocation strategy by each speculator, (ii) an investment and disclosure strategy by each firm manager, (iii) a price setting strategy by the market maker, such that:*

1. *the market maker breaks even for each asset*
2. *each speculator maximizes his expected profits*
3. *each manager maximizes his firm's expected firm value*
4. *all agents have rational expectations in that each player's belief about the other players' strategies is correct in equilibrium and out-of-equilibrium beliefs satisfy the conditions stated in the Appendix.*

2.3 Discussion

Before I proceed to the model solution, I discuss the two main assumptions of the model. First, speculators have to decide whether they want to pay attention towards firm A or firm B . Thus, in equilibrium, each speculator becomes a specialist and receives a perfect private signal about one firm and zero information about the other. This assumption is primarily made to keep the model tractable because it implies that the aggregate order flow from informed speculators of stock j is simply equal to the mass of speculator who pay attention to this firm (n_j). In the current setup with risk-neutral speculators, however, this assumption is rather innocuous because the speculators do not have an incentive to diversify their portfolio by splitting their fixed attention between the two firms, anyways. Therefore, the attention allocation problem in this paper is much simpler than that in other papers in the literature such as Mondria (2010), van Nieuwerburgh and Veldkamp (2010) or Kacperczyk et al. (2016). For the main results of the paper to go through, it is sufficient to allow traders to shift attention (or equivalently the precision of private signals) from one firm

to the other in response to the firms' disclosure decision, such that firms can actively compete for attention.

Second, I assume that at $t = 0$ both firms credibly commit to a policy that requires the firm to reveal the true fundamental with probability γ_j^* at the trading stage. In particular, I assume that this announced precision cannot exceed the firm's true signal precision (or quality) γ_j . Thus, firms cannot engage in "cheap talk" and overstate their true quality.¹² As a result, the type of information that is revealed should be interpreted as "hard" or quantitative information (e.g. regarding earnings) that can be verified ex post. One way to interpret this assumption is the following. All public firms have to meet a minimum disclosure requirement, which is equal to zero for simplicity, set by the regulator, such as the SEC. In reality, firms do, however, vary quite substantially in the amount of additional information they provide to the financial market. For example, firms have discretion about the informational content and the amount of details (such as segmental data) of their financial reports. A high degree of disclosure in the model therefore represents a firm's voluntary commitment to provide very detailed information to the public.

3 Model Solution

In this section, I solve for the pure-strategy equilibria in the different stages. I solve the model backwards and start with the financial market and investment equilibrium ($t = 2$ and $t = 3$, respectively). Then, I move on to the speculators' optimal attention allocation decisions ($t = 1$). The two firms' equilibrium disclosure decisions ($t = 0$) are characterized in Section 4.

3.1 Financial Market and Investment Equilibrium

First, I take the speculators' optimal attention allocation decision, which implies the mass of informed speculators n_j , and the firms' optimal disclosure decision γ_j^* as given and analyze the equilibrium at the last two stages.

Lemma 1 *Given a disclosure decision γ_j^* by each firm, an updated expectation of each firm's quality γ_j^{**} , and a mass n_j of informed speculators in firm j , there is a trading and investment equilibrium for each firm*

¹²See Almazan et al. (2008) for such a setting.

in which:

1. Each speculator i buys firm j 's stock if $\sigma_{mj}^* = \emptyset$ and $\sigma_{ij} = H$, sells if $\sigma_{mj}^* = \emptyset$ and $\sigma_{ij} = L$, and does not trade otherwise, i.e.:

$$s_{ij} = \begin{cases} 1 & \text{if } \sigma_{mj}^* = \emptyset \text{ and } \sigma_{ij} = H \\ -1 & \text{if } \sigma_{mj}^* = \emptyset \text{ and } \sigma_{ij} = L \\ 0 & \text{otherwise.} \end{cases}$$

which implies the following aggregate order flow by informed speculators for stock j :

$$s_j = \int_0^1 s_{ij} di = \begin{cases} n_j & \text{if } \sigma_{mj}^* = \emptyset \text{ and } \theta_j = H \\ -n_j & \text{if } \sigma_{mj}^* = \emptyset \text{ and } \theta_j = L \\ 0 & \text{otherwise.} \end{cases}$$

2. Firm j 's stock price satisfies:

$$p_j = \begin{cases} p^H = A + x^H & \text{if } \sigma_{mj}^* = H \text{ or } \sigma_{mj}^* = \emptyset \text{ and } X_j > 1 - n_j \\ p_j^M = A + \frac{1}{2} \frac{\gamma_j^* - \gamma_j}{1 - \gamma_j^*} x^H & \text{if } \sigma_{mj}^* = \emptyset \text{ and } -1 + n_j \leq X_j \leq 1 - n_j \\ p^L = A & \text{if } \sigma_{mj}^* = L \text{ or } \sigma_{mj}^* = \emptyset \text{ and } X_j < -1 + n_j. \end{cases}$$

3. Firm j 's investment decision satisfies:

$$I_j = \begin{cases} 1 & \text{if } \sigma_{mj} = H \text{ or } \sigma_{mj} = \emptyset \text{ and } p_j = p^H \\ 0 & \text{if } \sigma_{mj} = L \text{ or } \sigma_{mj} = \emptyset \text{ and } p_j \neq p^H. \end{cases}$$

Proof: See Appendix A.1.1.

Lemma 1 shows that informed speculators in firm j 's stock optimally choose to buy (sell) the asset, when they receive positive (negative) private information and the firm does not disclose the fundamental. Because a mass of n_j speculators receives a perfect signal about θ_j and trade size is restricted to be in $[-1, 1]$, the aggregate trading volume s_j is equal to n_j and $-n_j$, respectively. Importantly, informed speculators do not trade on their perfect private information, if the firm's fundamental is revealed by the disclosed public signal σ_{mj}^* . Intuitively, this information is already

accounted for in the equilibrium price set by the market maker in this case and the speculators' profit from trading on it is equal to zero. As I show in the next section, this crowding out effect of the firm's public signal plays an important role in the firm's disclosure decision at $t = 0$ which is designed to maximize the mass of informed traders n_j .

Furthermore, Lemma 1 also shows that the stock price of each firm can assume three values, $0 < p^L \leq p_j^M \leq p^H$. This price is set by the market maker and equals his conditional expectation of the firm's terminal value V_j . Thus, the price depends on the two signals that affect this conditional expectation, aggregate order flow X_j and the disclosed signal σ_{mj}^* . Clearly, if the firm manager announces that the true value is $\theta_j = H$, the market maker rationally infers that the manager chooses to invest in the growth opportunity, such that he sets the price equal to the firm value in this case, $p^H = A + x^H$. Vice versa, if the manager reveals a low signal ($\theta_j = L$), the market maker anticipates that the manager does not invest and sets the price equal to $p^L = A$. If the manager does not release a public signal ($\sigma_{mj}^* = \emptyset$), it can either indicate that he does not have any information about the fundamental or that he withholds some of this information (which happens with probability $\frac{\gamma_j^{**} - \gamma_j^*}{1 - \gamma_j^*}$). In these instances, a large enough order flow can push up the stock price to p^H and signal positive news about θ_j to the firm manager and encourages him to invest in the growth opportunity. The stochastic evolution of p_j and V_j is shown in Figure 5 and Figure 6, respectively.

3.2 Attention Allocation Equilibrium

Next, I solve the speculators' attention allocation problem at $t = 1$, taking the optimal disclosure decisions γ_A^* and γ_B^* as given. Each speculator has to decide whether to specialize in firm A or firm B . This decision, in turn, determines the speculator's information set at the trading stage. If speculator i specializes (i.e. pays attention to) firm j , he receives a perfect signal about firm j 's fundamental, $\sigma_{ij} = \theta_j$, and no information about the other firm's shock, $\sigma_{i,-j} = \emptyset$.

At $t = 1$, the only factor that differentiates the two firms is the released precision γ_A^* and γ_B^* , respectively. These two precisions serve a dual purpose for speculator i . First, γ_j^* represents the precision of the public signal that all financial market participants receive at $t = 2$. As argued above,

more precise public information decreases the speculator's informational advantage vis-a-vis the market maker and, thus, reduces the incentive to pay attention to this firm. Second, a given pair of precisions $\{\gamma_A^*, \gamma_B^*\}$ also serves as a signal about the firms' actual quality and allows the traders to update their belief about it from their prior expectation $\frac{\gamma + \bar{\gamma}}{2}$ to the posterior $\gamma_j^{**} = E[\gamma_j | \gamma_A^*, \gamma_B^*]$. For instance, if the disclosed signal is greater than the true quality of the low quality firm ($\gamma_j^* \geq \underline{\gamma}$), speculators rationally infer that this firm must be of high quality ($\gamma_j = \bar{\gamma}$) because the low-quality firm cannot credibly disclose such an informative signal.

In the attention allocation equilibrium each speculator has to be indifferent between paying attention to either firm. Thus, the equilibrium mass of informed speculators for both firms has to adjust such that the ex ante expected trading profits are equalized:

$$E_1[\Pi_i | \theta_A, \sigma_{m_A}^*, \sigma_{m_B}^*, \gamma_A^*, \gamma_B^*, n_A, n_B] = E_1[\Pi_i | \theta_B, \sigma_{m_A}^*, \sigma_{m_B}^*, \gamma_A^*, \gamma_B^*, n_A, n_B], \quad (8)$$

where $\Pi_i \equiv \sum_j s_{ij} (V_j - p_j)$ denotes speculator i 's trading profit and the left-hand (right-hand) side of equation (8) corresponds to the expected profit of a speculator who pays attention to firm A (B).

Next, I plug in the equilibrium values for s_{ij} , V_j and p_j derived in Lemma 1 to the indifference condition in equation (8) and solve for the optimal mass of speculators. Lemma 2 provides a closed form solution for the optimal share of informed speculators in each firm.

Lemma 2 *The equilibrium number of informed speculators is given by:*

$$\begin{aligned} n_A &= \frac{(1 - \gamma_B^*) \Delta_A}{(1 - \gamma_B^*) \Delta_A + (1 - \gamma_A^*) \Delta_B} \\ n_B &= 1 - n_A \end{aligned}$$

with $\Delta_j \equiv \gamma_j^{**} - \gamma_j^* > 0$ and $j \in \{A, B\}$.

Proof: See Appendix A.1.2.

Lemma 2 shows that the mass of speculators in each firm depends on four variables, $\{\gamma_A^*, \gamma_B^*, \Delta_A, \Delta_B\}$. Intuitively, $\Delta_j = \gamma_j^{**} - \gamma_j^*$ measures firm j 's perceived excess quality because it captures the difference between the firm's *expected* (γ_j^{**}) and *disclosed* (γ_j^*) quality. If traders can perfectly infer the

firms' true quality from their disclosure policy, this difference is equal to $\gamma_j - \gamma_j^*$. However, there might also be a pooled outcome, in which the financial market cannot perfectly infer the firms' true type. In these cases, the best guess of the firms' quality is the average quality, $\gamma_j^{**} = \frac{\gamma + \bar{\gamma}}{2}$.

The equilibrium expressions for n_j in Lemma 2 show the inherent trade-off that both firms face when they decide on the optimal disclosure policy at $t = 0$. On the one hand, each firm would like to increase the traders' expected quality γ^{**} through a precise public signal σ_{mj}^* . On the other hand, disclosing a very precise signal also reduces the traders' incentive to pay attention to this firm because it improves the precision of the market makers information set such that this information is already priced in to the equilibrium stock price. Mathematically, the optimal mass of speculators for firm j depends positively on its own excess quality and negatively on that of the other firm, $\frac{\partial n_j}{\partial \Delta_j} > 0$ and $\frac{\partial n_j}{\partial \Delta_{-j}} < 0$. Furthermore, this mass depends negatively on its own disclosed quality and positively on that of the other firm, $\frac{\partial n_j}{\partial \gamma_j^*} < 0$ and $\frac{\partial n_j}{\partial \gamma_{-j}^*} > 0$.

4 Fight for Attention

In this section, I present the main results of the paper and characterize the firms' *fight for attention* through their disclosure policy at $t = 0$. I proceed in two steps. First, I solve for the optimal disclosure policy in a benchmark economy without uncertainty about the firms' quality. Then, I move on to the main model with uncertainty about γ_j . In an initial step, however, I formally establish the firms' motive to compete for attention.

Lemma 3 *Each firm's expected value (at $t = 0$) increases in the number of informed speculators n_j .*

Proof: *See Appendix A.1.3.*

Lemma 3 shows that each firm can increase its expected value by attracting the attention of a larger number of informed speculators. Intuitively, a larger mass of informed speculators renders the firm's stock price more informative about the fundamental θ_j . More specifically, a larger value of n_j reduces the likelihood of the intermediate stock price value $p_j = p_j^M$ in Lemma 1. Importantly, neither the market maker nor the firm manager can infer information about θ_j from this value. In the limit as $n_j \rightarrow 1$, i.e. all speculators only pay attention to firm j , this state vanishes and the

stock price is either high or low. In this extreme case, the market maker and the firm manager become perfectly informed about the firm's fundamental. Moreover, the incentive of the marginal speculator to pay attention to this firm would be zero because there is no informational advantage over the market maker and expected trading profits are equal to zero for this firm.

It is also important to note that each firm's ex ante value only depends on the respective firm's disclosure policy through the mass of informed speculators n_j . Therefore, if the speculators' attention was fixed, each firm would be indifferent between disclosing and not disclosing.

4.1 First-best Benchmark without Uncertainty about Firm Quality

In this benchmark economy without quality uncertainty, all market participants know each firm's true quality, such that their expected quality is equal to the firm's actual quality ($\gamma_j^{**} = \gamma_j$). As a result, the equilibrium mass of informed speculators for each firm given in Lemma 2 simplifies to:

$$n_{hq} = \frac{(1 - \gamma_{lq}^*) (\bar{\gamma} - \gamma_{hq}^*)}{(1 - \gamma_{lq}^*) (\bar{\gamma} - \gamma_{hq}^*) + (1 - \gamma_{hq}^*) (\underline{\gamma} - \gamma_{lq}^*)} \quad (9)$$

$$n_{lq} = 1 - n_{hq} \quad (10)$$

where the subscript hq (lq) denotes the high (low) quality firm with $\gamma_j = \bar{\gamma}$ ($\gamma_j = \underline{\gamma}$), respectively.

In this setting, the only channel through which firms can compete for the speculators' attention is by providing them more or less additional information about the firm's fundamental, i.e. through their choice of the released public signal γ_j^* . By simple differentiation of the attention shares above, it follows that the speculators pay more attention to a given firm if that firm releases a less precise signal. Intuitively, in this setting there is no upside associated with information disclosure but only a downside via the aforementioned crowding-out effect. A more precise public signal strengthens the comparative position of the market maker and reduces the marginal value of each speculator's private information about firm j .

Proposition 1 (No-Uncertainty Benchmark) *In the benchmark equilibrium without quality uncertainty, both firms choose to withhold their private signal and choose: $\gamma_A^* = \gamma_B^* = 0$.*

Proof: See Appendix A.1.4.

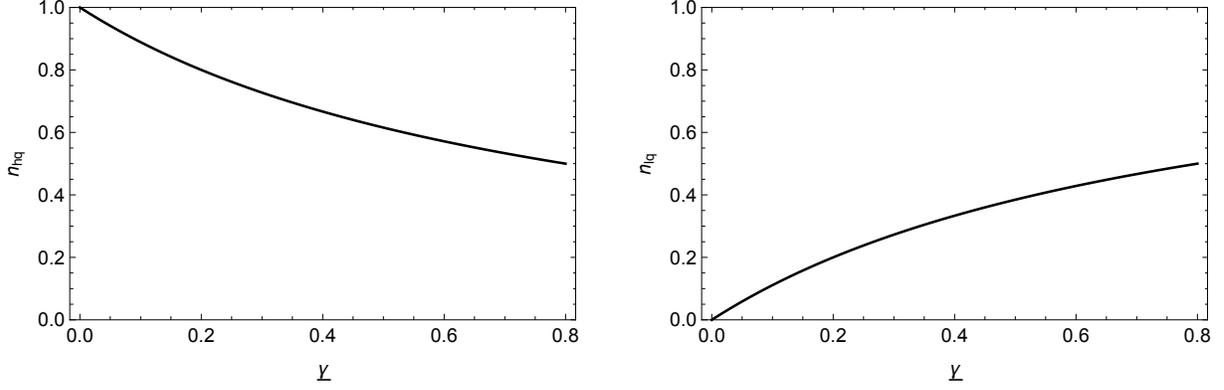


Figure 2: These two figures show the equilibrium mass of informed traders for both firms as a function of the quality of the low-quality firm in the benchmark economy without quality uncertainty. The quality of the high-quality firm is set to $\bar{\gamma} = 0.8$.

Proposition 1 confirms this intuition. In the benchmark equilibrium, both firms choose to withhold all of their private information and choose $\gamma_j^* = 0$. This choice, in turn, maximizes the speculators' informational advantage over the market maker who can only price the two assets based on the aggregate order flow X_j .

Corollary 1 *In the benchmark economy, the optimal mass of informed speculators for both firms is given by:*

$$\begin{aligned} n_{hq}^{bench} &= \frac{\bar{\gamma}}{\bar{\gamma} + \underline{\gamma}} \\ n_{lq}^{bench} &= 1 - n_{hq}^{bench} \end{aligned}$$

where the subscript hq (lq) denotes the high (low) quality firm with $\gamma_j = \bar{\gamma}$ ($\gamma_j = \underline{\gamma}$), respectively.

Proof: See Appendix A.1.5.

Corollary 1 provides a closed-form solution for the mass of informed speculators that pay attention to each firm. The respective mass only depends on the true quality of the two firms. More specifically, each firm's equilibrium attention is given by its relative quality $\frac{\bar{\gamma}}{\bar{\gamma} + \underline{\gamma}}$ and $\frac{\underline{\gamma}}{\bar{\gamma} + \underline{\gamma}}$, respectively. As a result, the attention share of the high-quality firm is always higher than that of the low-quality firm: $n_{hq}^{bench} > n_{lq}^{bench}$.

Figure 2 plots the equilibrium mass of informed speculators for the two firms as a function of $\underline{\gamma}$, the precision of the low quality firm. If the spread in firm quality is particularly large ($\underline{\gamma} = 0$), the high-quality firm receives all attention ($n_{hq} = 1$). As the two firms become more similar to each other ($\underline{\gamma}$ approaching $\bar{\gamma}$), the mass of informed speculators becomes equally split between the two

firms and converges to $\frac{1}{2}$.

4.2 Optimal Disclosure with Uncertainty about Firm Quality

Next, I solve for the two firms' optimal disclosure decision at $t = 0$ in the main model. A crucial friction in this setting is the uncertainty of all financial market participants regarding the firms' true quality, γ_j . Interestingly, this friction gives rise to an upside associated with the firms' disclosure policy. More specifically, the high-quality firm can now credibly signal its type to the market through a particularly transparent disclosure policy, i.e. a high value of γ_j^* . As shown before in Lemma 2, the mass of informed speculators increases in the firm's perceived quality such that the good firm can use the public signal to attract attention. However, when deciding on the optimal degree of disclosure, each firm also has to take into account how the additional public signal affects the speculators' marginal value of private information, which is generally "crowded-out" through more precise public information.

Proposition 2 (Disclosure Equilibrium) *There are two, mutually exclusive disclosure equilibria depending on the commonly known values of potential firm quality, $\bar{\gamma}$ and $\underline{\gamma}$.*

1. If $\underline{\gamma} \geq 1 - \sqrt{1 - \bar{\gamma}}$, there is a pooling equilibrium with:

- no disclosure, $\gamma_j^* = 0$
- expected quality equal to the unconditional expectation, $\gamma_j^{**} = \frac{\gamma + \bar{\gamma}}{2}$

for both firms $j \in \{A, B\}$.

2. If $\underline{\gamma} < 1 - \sqrt{1 - \bar{\gamma}}$, there is a separating equilibrium with:

- no disclosure by the low-quality firm $\gamma_{lq}^* = 0$ and partial disclosure by the high-quality firm $\gamma_{hq}^* = \underline{\gamma}$
- perfect revelation of the firms' true quality, $\gamma_{lq}^{**} = \underline{\gamma}$ and $\gamma_{hq}^{**} = \bar{\gamma}$

where the subscript lq (hq) indicates the low-quality (high-quality) firm.

Throughout, the quality levels satisfy $0 < \underline{\gamma} < \bar{\gamma} < 1$.

Proof: See Appendix A.1.6.

Proposition 1 characterizes the disclosure equilibrium in the main model. Depending on the relative size of $\bar{\gamma}$ to $\underline{\gamma}$, there is either a pooling or a separating equilibrium. If the low quality firm's precision is sufficiently high ($\underline{\gamma} \geq 1 - \sqrt{1 - \bar{\gamma}}$), there is a pooling equilibrium in which both firms take the same action at the disclosure stage. In particular, both firms choose to withhold all of their information, i.e. they both set $\gamma_j^* = 0$. As a consequence, the financial market participants cannot distinguish between both firms and assign the unconditional expectation of firm quality to each firm. If the difference between $\bar{\gamma}$ and $\underline{\gamma}$ is sufficiently high ($\underline{\gamma} < 1 - \sqrt{1 - \bar{\gamma}}$), however, there is a separating equilibrium. The low quality firm withholds all of its information, but the high quality firm issues a partially informative signal $\gamma_{hq}^* = \underline{\gamma}$. In this outcome, the two firms' type is revealed perfectly to all traders.

Intuitively, the non-disclosure equilibrium in the first part of Proposition 1 is stable for the following two reasons. First, the low quality firm does not want to deviate by disclosing more information because it would lose its share of speculators' attention as (i) the market maker has more public information about its fundamental and (ii) its perceived quality (γ^{**}) would not change. Second, the high quality firm could raise its perceived quality by deviating (sufficiently strongly) from this pooling equilibrium which would increase the speculators' attention. However, this benefit is not strong enough to outweigh the loss in attention due to more public information for the market maker (and lower trading profits for speculators) because $\bar{\gamma}$ is not sufficiently higher than $\underline{\gamma}$. Also note that the non-disclosure equilibrium ($\gamma_j^* = 0$) is the only possible pooling equilibrium because both firms would always be better off by releasing less information. By doing so, the firm could (i) keep its perceived quality unchanged and (ii) release less information which would render the speculators' private information more valuable.

In the separating equilibrium, the high quality firm releases just enough information to the financial market to credibly signal its true quality $\bar{\gamma}$ because the low quality firm could never release such an informative signal. The best response for the low quality firm is to withhold all of

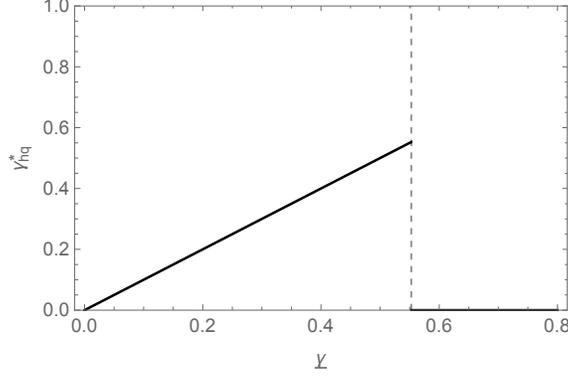


Figure 3: This figure shows the equilibrium disclosure precision of the high-quality firm against the quality of the low-quality firm. The quality of the high quality firm is set to $\bar{\gamma} = 0.8$.

its information because it cannot compete with this signal anyways (i.e. its true type is revealed) and it can minimize its loss in attention by diluting the market makers information set as much as possible.

Figure 3 plots the disclosure decision of the high-quality firm against the quality of the low-quality firm. As long as $\underline{\gamma}$ is sufficiently small there is a separating equilibrium and γ_{hq}^* is equal to $\underline{\gamma}$. If $\underline{\gamma}$ crosses the threshold $1 - \sqrt{1 - \bar{\gamma}} \approx 0.55$, however, the disclosure of the high-quality firm decreases to zero.

Corollary 2 *The equilibrium mass of informed speculators in each firm's stock is given by:*

1. If $\underline{\gamma} \geq 1 - \sqrt{1 - \bar{\gamma}}$ (pooling equilibrium):

$$n_{hq}^P = n_{lq}^P = \frac{1}{2}$$

2. If $\underline{\gamma} < 1 - \sqrt{1 - \bar{\gamma}}$ (separating equilibrium):

$$n_{hq}^S = \frac{\bar{\gamma} - \underline{\gamma}}{\bar{\gamma} - \underline{\gamma}^2} = 1 - n_{lq}^S$$

which implies that $n_{hq}^S > \frac{1}{2} > n_{lq}^S$.

The subscript lq (hq) indicates the low (high) quality firm and the commonly known quality levels satisfy

$$0 < \underline{\gamma} < \bar{\gamma} < 1.$$

Proof: See Appendix A.1.7.

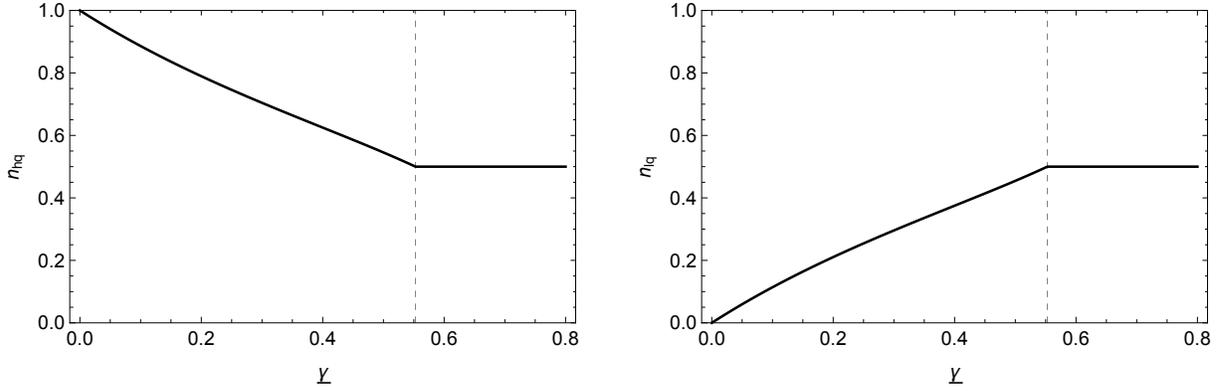


Figure 4: These two figures show the equilibrium mass of informed traders for both firms as a function of the quality of the low quality firm. The quality of the high quality firm is set to $\bar{\gamma} = 0.8$.

Corollary 2 provides the equilibrium values of n_j implied by the firm's optimal disclosure decisions. In the pooling equilibrium, both firms choose to withhold their information and the traders cannot distinguish between the high-quality and the low-quality firm. Since these two firms only differ in terms of their unknown true quality, they are equally attractive for speculators in this equilibrium and receive the same amount of attention. Consequently, half of the speculators become informed about firm A and the other half about firm B . In the separating equilibrium, traders can perfectly infer the type of both firms. In this outcome, speculators realize that they can make a larger trading profit in the stock of the high quality firm such that its attention share increases. Initially, speculators realize that their private signal is more valuable for them when trading in the high quality firm's stock such that its attention share increases up to the point when the marginal speculator is indifferent between both firms. Intuitively, the high quality firm is more likely to invest in the growth opportunity on its own, i.e. without a feedback signal from the stock market and thus the market maker's knowledge, which increases the expected profit of a privately informed speculator paying attention to this firm. Of course, this intuition only applies when spread between both firms' quality is sufficiently high ($\underline{\gamma} < 1 - \sqrt{1 - \bar{\gamma}}$).

Figure 4 shows the equilibrium mass of informed speculators for both firms as a function of $\underline{\gamma}$ and for a fixed value of $\bar{\gamma} = 0.8$. The plots show that the total amount of attention allocated to the *high* quality firm is decreasing in the quality of the *low* quality firm as long as $\underline{\gamma} < 1 - \sqrt{1 - \bar{\gamma}}$ (which is approximately 0.55) such that the economy is in the separating equilibrium. A higher

value for the low quality firm's precision $\underline{\gamma}$ implies that the high quality firm issues a more precise public signal to credibly display its quality to the financial market. Because more public information decreases the speculators relative information advantage vis-a-vis the market maker, less speculators become informed about this firm, i.e. n_{hq} decreases. In the pooling equilibrium ($\underline{\gamma} \geq 1 - \sqrt{1 - \bar{\gamma}}$), the traders cannot distinguish between both firms such that the low and the high quality firm receive an equal share of the speculators' overall attention.

Corollary 3 *Asymmetric information about firm quality reduces equilibrium attention for the high-quality firm and increases attention for the low-quality firm:*

$$n_{hq}^{bench} > n_{hq}^S > \frac{1}{2} \quad \text{and} \quad n_{lq}^{bench} < n_{lq}^S < \frac{1}{2}$$

Proof: See Appendix A.1.8.

Corollary 3 compares the mass of informed speculators in the benchmark economy to that in the main equilibrium. It can be seen that the high-quality (low-quality) firm always receives more (less) attention in the benchmark economy. Therefore, asymmetric information about firm quality affects the two firms asymmetrically. Intuitively, the high-quality firm is negatively affected by this type of uncertainty through two channels. It could either be mistaken as a low-quality firm or it could be identified as a high-quality through signaling. Both outcomes are undesirable for the high-quality firm because they reduce the speculators' incentive to pay attention.

4.3 Empirical Implications

The main model provides several empirical implications. First, the model shows that firms of higher quality might be willing to voluntarily commit to a more transparent disclosure policy. Importantly, both the decision to do so and the degree of disclosure depend on the quality of other firms in the market. In particular, a high quality firm is more likely to disclose more information if the difference to the low quality firm is small. However, the difference has to be large enough such that it pays off to separate itself from this firm. Second, the model presents a new insight to the relationship between corporate disclosure and speculators' attention. Generally, the model

predicts that the firm which discloses more information receives a larger share of attention from the financial market which provides a theoretical explanation for the empirical finding in Lang and Lundholm (1996) that firms with more informative disclosure policies have higher analyst coverage. In the model, the positive relationship between disclosure and attention is due to the fact that disclosure is used as a signal of quality. Per se, more disclosure crowds out private information production and reduces attention. Interestingly, this theoretical result is in line with recent empirical evidence in Francis et al. (2008) who show that the positive association between disclosure and a firm's cost of capital vanishes after controlling for earnings quality.

Another potential application of the model mechanism is related to the firms' decision to go public. During the initial public offering (IPO) process, information disclosure is particularly relevant for firms because it offers support for the initial offer price or range.¹³ Importantly, firms have a significant amount of leeway regarding the degree of information disclosure in the premarket. The model mechanism in this paper implies that higher information disclosure should be followed by a larger share of attention post-IPO, especially for firms that rely heavily on outside information in their investment decision.

Moreover, the model implies that the amount of "outside information" in the stock price can actually increase even though a firm increases the amount of "inside information" through its commitment to a transparent disclosure policy. Therefore, the firm is able to infer more novel information from its stock price such that its investment-q ratio should increase. This result is in contrast to much of the existing literature on the relationship between inside and outside information (e.g. Edmans et al. (2017)) where these two sources of information are usually viewed as substitutes.

5 Efficiency and Disclosure Rules

The previous section shows the firms' optimal disclosure policies in equilibrium. A key driver of these decisions is the firm's competitive behavior regarding the speculators' limited attention capacity. In this section, I analyze whether this "fight for attention" leads to any inefficiencies. As

¹³See e.g. Hanley and Hoberg (2010).

a natural measure of efficiency, I use the expected average firm value of both firms at $t = 0$. Thus, if a given disclosure equilibrium leads to higher efficiency, it leads to a higher average expected firm value in the economy. In particular, note that at $t = 0$ both firms appear exactly identical. Therefore the efficiency measure can only take into account commonly known parameters of the economy $\{A, x^H, \underline{\gamma}, \bar{\gamma}\}$, as well as the probability distributions of θ_j and z_j . Next, I formally define the underlying efficiency measure.

Definition 2 *Real efficiency is defined as the average unconditional expected firm value at $t = 0$.*

$$RE \equiv \frac{1}{2}E_0[V_A + V_B] \quad (11)$$

In Appendix A.1.9, I show that RE depends positively on the parameters A and x^H , i.e. the return on the firms' assets in place and the growth opportunity. Moreover, a higher average value for firm quality $\mu_\gamma \equiv \frac{\bar{\gamma} + \underline{\gamma}}{2}$ also increases real efficiency because it implies that, on average, the firms are able to invest more efficiently in the growth opportunity. More interestingly, however, RE is also (negatively) affected by the allocation of attention which is captured by the terms $n_{hq}\bar{\gamma} + n_{lq}\underline{\gamma}$. In the following analysis, I want to keep all other parameters in equation (11) fixed and focus on this *allocational efficiency of attention*. Then it follows from Definition 2 that real efficiency is higher if low values of γ_j are matched with high values of n_j , and vice versa. Therefore, it is more efficient if the low quality firm receives relatively more attention.

Lemma 4 *For a fixed average quality $\mu_\gamma = \frac{\gamma + \bar{\gamma}}{2}$ and parameters $\{A, x^H\}$, the most efficient (as defined in equation (11)) allocation of attention is: $n_{lq} = 1$ and $n_{hq} = 0$.*

Proof: See Appendix A.1.9.

Lemma 4 shows that real efficiency is maximized if the high-quality firm receives zero attention and all speculators pay attention to the low-quality firm instead. Intuitively, the speculators' scarce attention yields a higher "return" when paid to the low quality firm because this firm relies more heavily on the price signal in its investment decision and both firm types have access to an equally profitable growth opportunity.

Of course, a key problem with the result in Lemma 4, is that in reality it is impossible to implement the efficient allocation outcome directly by dictating values for n_j to speculators. Therefore, I focus on the efficiency of the equilibrium attention allocation decisions that are implied by the firms' optimal disclosure policies next. Subsequently, I analyze the impact of ex ante disclosure rules on real efficiency.

Proposition 3 (Efficiency of Disclosure Equilibria) *For a fixed average quality $\mu_\gamma = \frac{\gamma + \bar{\gamma}}{2}$ and parameters $\{A, x^H\}$, real efficiency, as defined in equation (11), in the pooling equilibrium is higher than in the separating equilibrium.*

Proof: See Appendix A.1.10.

Proposition 3 ranks the disclosure equilibria according to their (ex ante) real efficiency, keeping the average quality of firms constant. As shown in Corollary 2, the high-quality firm is able to attract a larger mass of informed speculators in the separating equilibrium. Therefore, the implied attention allocation in the pooling equilibrium is closer to the efficient allocation stated in Lemma 4 such that, everything else equal, the pooling equilibrium leads to a more efficient outcome.

Next, I focus on mandated disclosure policies and their impact on efficiency. In particular, I consider two different cases. First, a setting in which the optimal disclosure rule can be conditioned on firm quality and second one in which a "social planner" does not know firm quality and, thus, dictates a uniform rule to both firms.

Proposition 4 (Optimal Disclosure Rules) *The following set of mandatory disclosure rules maximizes efficiency:*

1. *If the rules can be conditioned on firm quality:*

- *Full disclosure by the high-quality firm and zero disclosure by the low-quality firm.*

$$\gamma_{hq}^* = \bar{\gamma} \quad \text{and} \quad \gamma_{lq}^* = 0$$

2. *If the rules cannot be conditioned on firm quality:*

- A cap $\gamma^{*,cap} < \underline{\gamma}$ on disclosure quality such that

$$\gamma_{hq}^* \leq \gamma^{*,cap} \quad \text{and} \quad \gamma_{lq}^* \leq \gamma^{*,cap}$$

Proof: See Appendix A.1.11.

Proposition 4 distinguishes between two cases: whether the disclosure rule can be conditioned on firm quality or not. In the first case, the most efficient rule differentiates between the high-quality and the low-quality firm. In particular, the high-quality firm is forced to reveal its true quality to the financial market by disclosing as much information as possible, $\gamma_{hq}^* = \bar{\gamma}$.¹⁴ The low-quality firm, however, is forced to withhold all of its private information. Of course, the financial market participants are still able to infer its type because they know that one firm has to be of lower quality. Intuitively, this disclosure rule is efficient because it implements the most efficient allocation of attention as outlined in Lemma 4. The speculators' entire attention flows to the low-quality firm which has the highest use for the speculators' aggregated private information.

If the disclosure rule cannot be conditioned on firm quality, the most efficient outcome is achieved by constraining the maximum disclosure amount by a cap $\gamma^{*,cap}$. Importantly, this cap has to be below the low-quality firm's quality $\underline{\gamma}$. Intuitively, $\gamma^{*,cap}$ prevents the high-quality firm from deviating into the separating equilibrium and thus implements a pooling equilibrium for all possible values of $\underline{\gamma}$ and $\bar{\gamma}$. The implied attention allocation share for each firm is thus $\frac{1}{2}$ which means that the low-quality firm receives a (weakly) higher share of attention than in the main equilibrium.¹⁵

6 Extension: Stock-Based Compensation

In this section, I study a variation of the main model in which the firm managers are partially compensated through their firm's stock. Therefore, firm manager j 's payoff is given by a combination of the firm's long-run value V_j and the short-run stock price p_j ,

$$U_j = V_j + \omega_j p_j \tag{12}$$

¹⁴For convenience, I ignore the physical constraint from before that the disclosed quality has to be strictly smaller than the true quality.

All the results in Proposition 4 go through if γ_{hq}^* takes on the highest possible value.

¹⁵This result is similar to the optimal rule in Edmans et al. (2016).

where $\omega_j \geq 0$ determines the importance of the short-run stock price. Thus, this variation collapses to the main model for $\omega_j = 0$.

As the stock price is determined at the time managers choose firm investment, the financial market and investment equilibrium are still given by Lemma 1. Similarly, the equilibrium number of informed speculators is unaffected by ω_j and given in Lemma 2. Thus, the strength of the short-run incentives, only impacts the firms' disclosure decision in the first period.

Proposition 5 (Disclosure Equilibrium with Stock-Based Compensation) *There are two, mutually exclusive disclosure equilibria depending on the commonly known values of potential firm quality, $\bar{\gamma}$ and $\underline{\gamma}$, and the strength of the managers' short-run incentives ω_j .*

1. If $\bar{\gamma} \leq g(\underline{\gamma}, \omega_{hq})$, there is a pooling equilibrium with:

- no disclosure, $\gamma_j^* = 0$
- expected quality equal to the unconditional expectation, $\gamma_j^{**} = \frac{\gamma + \bar{\gamma}}{2}$

for both firms $j \in \{A, B\}$.

2. If $\bar{\gamma} > g(\underline{\gamma}, \omega_{hq})$, there is a separating equilibrium with:

- no disclosure by the low-quality firm $\gamma_{lq}^* = 0$ and partial disclosure by the high-quality firm $\gamma_{hq}^* = \underline{\gamma}$
- perfect revelation of the firms' true quality, $\gamma_{lq}^{**} = \underline{\gamma}$ and $\gamma_{hq}^{**} = \bar{\gamma}$

where the subscript lq (hq) indicates the low-quality (high-quality) firm.

The exact expression for the cutoff value $g(\underline{\gamma}, \omega_{hq})$ is given in the Appendix. Moreover, the cutoff increases in $\underline{\gamma}$, $\frac{\partial g(\underline{\gamma}, \omega_{hq})}{\partial \underline{\gamma}} > 0$.

Proof: See Appendix A.1.12.

Proposition 5 shows that there are two mutually exclusive disclosure equilibria as before. If the high-quality firm's quality is sufficiently high, there is a separating equilibrium, otherwise both firms choose to not disclose anything. Importantly, the cutoff $g(\underline{\gamma}, \omega_{hq})$ that determines

the disclosure equilibrium, depends on the low-quality firm's quality and the high-quality firm manager's short-run incentives. Intuitively, it is desirable for the high-quality firm to separate itself through disclosure if this decision leads to a significant increase in its perceived quality and therefore attention. Moreover, disclosure also leads to an increase in the high-quality firm's stock price because it increases the market makers perceived quality of the firm. If the firm manager has short-run incentives ($\omega > 0$), this channel provides an additional incentive for the manager to disclose.

Corollary 4 *Increasing the short-run incentives of the high-quality manager leads to:*

- a decrease in the cutoff $g(\underline{\gamma}, \omega_{hq})$, $\frac{\partial g(\underline{\gamma}, \omega_{hq})}{\partial \omega_{hq}} < 0$
- a decrease in the share of attention for the high quality firm, $\frac{\partial n_{hq}}{\partial \omega_{hq}} \leq 0$
- an increase in real efficiency.

Changes in the short-run incentives of the low-quality manager do not change these variables.

Proof: See Appendix A.1.13.

Corollary 4 analyzes the impact of changes in the managers' short-run incentives ω_j . An increase in this parameter gives the manager a direct incentive to structure the disclosure policy in a way that maximizes the expected stock price p_j . The corollary shows that this strategy only works for the high-quality firm because the low-quality firm cannot credibly signal its quality through its disclosure policy. For the high-quality firm, however, an increase in short-run incentives renders the firm more likely to disclose information. As a result, this firm might even be willing to disclose information if it leads to a drop in the attention share because the implied drop in the informational content of the price is compensated for by a higher price level. As a consequence, an increase in the high-quality manager's short-run incentives shifts attention from his firm to the low-quality firm and moves the attention allocation equilibrium closer to the efficiency benchmark.

7 Conclusion

This paper shows that corporate disclosure can have important and nuanced consequences for speculators' allocation of attention and the informational role of stock prices. The main model highlights a novel trade-off associated with disclosure. On the one hand, disclosure by one firm crowds-out attention allocation because it reduces the speculators' informational advantage. On the other hand, disclosure can also attract attention if it (credibly) conveys information about the firm's type. The optimal disclosure policy trades off these two forces. If the difference between the high-quality and the low-quality firm is large enough, there exists a separating equilibrium with partial disclosure by the high-quality firm and zero disclosure by the low-quality firm. Otherwise, both firms choose to withhold their information.

Interestingly, the pooling equilibrium with zero disclosure by both firms is more efficient because it implies a relatively larger attention share for the low-quality firm. From a "social-planner perspective" this firm is the more efficient user of information because its own private signal is less precise such that the feedback effect is stronger. For the same reason, the socially optimal disclosure rule forces the high-quality firm to disclose all of its information, while the low-quality firm withholds its information.

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A Appendix

A.1 Proofs

A.1.1 Proof of Lemma 1

Speculator trading: Let $\Pi(s_{ij}, \sigma_{ij}, \sigma_{mj}^*)$ denote the expected profit of a speculator who trades $s_{ij} \in \{-1, 0, 1\}$, observes the private signal $\sigma_{ij} \in \{L, H, \emptyset\}$ and the public signal $\sigma_{mj}^* \in \{L, H, \emptyset\}$. Consider first, $\sigma_{ij} = H$ and $\sigma_{mj}^* = \emptyset$. The overall demand of informed speculators is then given by $s_j = n_j$. The order flow is given by $X_j = n_j + z_j \sim U[n - 1, n + 1]$. Hence, it follows that:

$$\begin{aligned} \Pi(1, H, \emptyset) &= \frac{\gamma_j^{**} - \gamma_j^*}{1 - \gamma_j^*} \left(A + x^H - n_j p^H - (1 - n_j) p_j^M \right) + \frac{1 - \gamma_j^{**}}{1 - \gamma_j^*} \left(n_j (A + x^H - p^H) + (1 - n_j) (A - p_j^M) \right) \\ &= \frac{1}{2} \frac{\gamma_j^{**} - \gamma_j^*}{1 - \gamma_j^*} (1 - n_j) x^H > 0, \end{aligned}$$

and $\Pi(-1, H, \emptyset) = -\Pi(1, H, \emptyset) < 0$ and $\Pi(0, H, \emptyset) = 0$. Hence, it is optimal for speculator i to buy if $s_{ij} = H$ and $\sigma_{mj}^* = \emptyset$.

Consider next the case $\sigma_{ij} = L$ and $\sigma_{mj}^* = \emptyset$.

$$\Pi(-1, L, \emptyset) = (1 - n_j) p_j^M + n_j p^L - A = \frac{1}{2} \frac{\gamma_j^{**} - \gamma_j^*}{1 - \gamma_j^*} (1 - n_j) x^H > 0$$

and $\Pi(1, L, \emptyset) = -\Pi(-1, L, \emptyset) < 0$ and $\Pi(0, L, \emptyset) = 0$. Hence, it is optimal for speculator i to sell if $s_{ij} = L$ and $\sigma_{mj}^* = \emptyset$.

Consider next the case $\sigma_{ij} = \emptyset$ and $\sigma_{mj}^* = \emptyset$.

$$\Pi(1, \emptyset, \emptyset) = E[V_j - p_j | \sigma_{ij} = \emptyset, \sigma_{mj}^* = \emptyset] = 0$$

which follows from the law of iterated expectations because $p_j = E[V_j | X_j, \sigma_{mj}^*]$. Moreover, $\Pi(0, \emptyset, \emptyset) = \Pi(-1, \emptyset, \emptyset) = 0$.

Lastly, consider the case if the manager sends out a perfectly informative signal, $\sigma_{mj}^* = \theta_j$. Then, $p_j = E[V_j | \theta_j]$ such that:

$$\Pi(1, \emptyset, \emptyset) = E[V_j - p_j | \theta_j] = 0$$

and $\Pi(0, \emptyset, \theta_j) = \Pi(-1, \emptyset, \theta_j) = 0$.

Optimal investment policy: If manager j receives the signal $\sigma_{mj} = H$, he knows that $\theta_j = H$.

Hence, the expected firm value is given by

$$E[V_j|\theta_j = H] = I_j x^H$$

and the optimal investment decision is $I_j = 1$ because $x^H > 0$. Similarly, if the manager receives the signal $\sigma_{mj} = L$, he knows that $\theta_j = L$. The expected firm value is given by:

$$E[V_j|\theta_j = L] = I_j x^L$$

and the optimal investment decision is $I_j = 0$ because $x^L < 0$.

If manager j receives no information ($\sigma_{mj} = \emptyset$), he bases his investment decision on the stock price p_j . If $p_j = p^H$, everybody knows that $\theta_j = H$ and the manager sets $I_j = 1$ (again because $x^H > 0$). If $p_j = p_j^M$, then the manager's posterior about θ_j is uniform and the expected firm value is given by:

$$E[V_j|p_j = p_j^M] = I_j \frac{x^H + x^L}{2}$$

and the optimal decision is $I_j = 0$ because $x^H + x^L < 0$ by assumption. Lastly, if $p_j = p^L$ everybody knows that $\theta_j = L$ and the manager chooses $I_j = 0$ because $x^L < 0$.

Equilibrium price: For the market maker to break even, the equilibrium price has to satisfy:

$p_j = E[V_j|X_j, \sigma_{mj}^*]$. If $\sigma_{mj}^* = H$, the market maker knows that $\theta_j = H$ and sets the price equal to:

$$E[V_j|\sigma_{mj}^* = H] = A + x^H.$$

Similarly, if the manager announces a negative signal the price equals:

$$E[V_j|\sigma_{mj}^* = L] = A.$$

If the manager does not disclose an informative signal ($\sigma_{mj}^* = \emptyset$), the equilibrium price depends on order flow. If $X_j > 1 - n_j$, the market maker knows that $\theta_j = H$ and sets the price equal to:

$$E[V_j|X_j > 1 - n_j, \sigma_{mj}^* = \emptyset] = A + x^H.$$

Similarly, if $X_j < n_j - 1$, he knows that $\theta_j = L$ and sets the price equal to:

$$E[V_j|X_j < n_j - 1, \sigma_{mj}^* = \emptyset] = A.$$

For intermediate values $1 - n_j \geq X_j \geq n_j - 1$, the market maker cannot infer the state of the world. He knows that $\theta \in \{L, H\}$ with equal probability. Furthermore, if $\theta = H$ the manager could have received a positive signal (which occurs with probability γ_j^{**}) and did not disclose it (which occurs with probability γ_j^*). Thus the equilibrium price is equal to:

$$E[V_j | 1 - n_j \geq X_j \geq n_j - 1, \sigma_{mj}^* = \emptyset] = A + \frac{1}{2} \frac{\gamma_j^{**} - \gamma_j^*}{1 - \gamma_j^*} x^H.$$

Hence, the strategies form an equilibrium.

A.1.2 Proof of Lemma 2

At $t = 1$, speculator i 's expected profit from paying attention to firm A is equal to:

$$E_1[\Pi_i | \gamma_A^*, \gamma_A^{**}, n_A, \sigma_{iA} = \theta_A, \sigma_{iB} = \emptyset] = \frac{\gamma_A^{**} - \gamma_A^*}{1 - \gamma_A^*} (1 - n_A) x^H$$

which follows from the definition $\Pi = \sum_j s_{ij}(V_j - p_j)$ and the equilibrium expressions provided in Lemma 1. In equilibrium n_A and n_B have to make each speculator indifferent between both firms:

$$E_1[\Pi_i | \gamma_A^*, \gamma_A^{**}, n_A, \sigma_{iA} = \theta_A, \sigma_{iB} = \emptyset] = E_1[\Pi_i | \gamma_B^*, \gamma_B^{**}, n_B, \sigma_{iB} = \theta_B, \sigma_{iA} = \emptyset]$$

Thus, solving the following equation for n_A (or n_B) leads to the expressions given in Lemma 2.

A.1.3 Proof of Lemma 3

To compute each firm's expected value at $t = 0$, I can use the event tree given in Figure 6. As a consequence,

$$E_0[V_j | \gamma_j] = A + \frac{1}{2} x^H (\gamma_j + (1 - \gamma_j) n_j)$$

which implies that each firm's value is increasing in the mass of informed traders:

$$\frac{\partial E_0[V_j | \gamma_j]}{\partial n_j} = \frac{1}{2} x^H (1 - \gamma_j) > 0$$

because $x^H > 0$ and $\gamma_j \in (0, 1)$.

A.1.4 Proof of Proposition 1

This result follows directly from Lemma 3 and the expressions for n_{hq} and n_{lq} given in the text. In particular, $\frac{\partial n_{hq}}{\partial \gamma_{hq}^*} < 0$ such that a more precise public signal leads to a smaller mass of informed speculators for the high quality firm in this case. In equilibrium, both firms release as little information as possible such that $\gamma_A^* = \gamma_B^* = \emptyset$.

A.1.5 Proof of Corollary 1

The implied equilibrium values for n_{hq} and n_{lq} directly follow from the result of Proposition 1 that firms do not disclose and the expressions for n_j given in the text (as a function of γ_j^*).

A.1.6 Proof of Proposition 2

First I specify the traders' off-equilibrium beliefs. If the disclosed precision is less than the quality of the low-quality firm ($\gamma_j^* < \underline{\gamma}$), traders assign the unconditional expectation $\mu_\gamma = \frac{\gamma + \bar{\gamma}}{2}$ to γ_j^{**} because the signal could have been sent by both types. If $\gamma_j^* \geq \underline{\gamma}$ only the high-quality firm could have sent the signal such that $\gamma_j^{**} = \bar{\gamma}$ in this case.

I start with the Pooling equilibrium in which both firms withhold their information. This equilibrium is stable if neither the good nor the bad firm has an incentive to deviate. First, the low-quality firm: for this firm increasing its disclosure precision is not beneficial because it strictly decreases the mass of informed traders: the inferred quality is unchanged (due to the off-equilibrium belief stated above) and so higher precision crowds out informed trading. For the high-quality firm deviating is only profitable if the mass of informed speculators after deviating is greater than $\frac{1}{2}$ (the mass in the pooling equilibrium). Simple algebra shows that this is true as long as $\underline{\gamma} \geq 1 - \sqrt{1 - \bar{\gamma}}$.

If $\underline{\gamma} < 1 - \sqrt{1 - \bar{\gamma}}$, there is a separating equilibrium in which the low-quality does not disclose and the high-quality firm discloses $\underline{\gamma}$. The low-quality firm does not want to deviate from $\gamma_{lq}^* = 0$ because this would decrease the mass of informed speculators and not change its inferred quality. The high-quality would not deviate upwards for the same reason. It would also not deviate downwards into the pooling equilibrium as long as the initially stated inequality holds.

A.1.7 Proof of Corollary 2

The expressions for n_{hq} and n_{lq} follow from plugging in the expressions for γ_j^* and γ_j^{**} provided in Proposition 2 into the expression for n_j provided in the text.

A.1.8 Proof of Corollary 3

The result simply follows from taking the differential between n_{hq}^{bench} and n_{hq}^S together with the previous result that $n_{hq}^S > \frac{1}{2}$ and the assumption that $n_{lq} + n_{hq} = 1$.

A.1.9 Proof of Lemma 4

First note, that by plugging in the expected firm values according to the event tree in Figure 6, it follows that:

$$RE = A + \frac{1}{2}x^H \left(\mu_\gamma + \frac{1}{2} - \frac{1}{2}\gamma_A n_A - \frac{1}{2}\gamma_B n_B \right)$$

where $\mu_\gamma \equiv E_0[\gamma_j] = \frac{\gamma + \bar{\gamma}}{2}$ denotes the average quality of the two firms.

Then we know that either of the two following events is true, $\{\gamma_A = \bar{\gamma}, \gamma_B = \underline{\gamma}\}$ or $\{\gamma_B = \bar{\gamma}, \gamma_A = \underline{\gamma}\}$. Because $\bar{\gamma} > \underline{\gamma}$, it follows that a corner solution maximizes RE , in which $n_{lq} = 1$ and $n_{hq} = 0$ (given that $n_A, n_B \geq 0$ and $n_A + n_B = 1$).

A.1.10 Proof of Proposition 3

From the definition of real efficiency in equation (11) and the result in Corollary 2 that $n_j = \frac{1}{2}$ in the pooling equilibrium it follows that:

$$RE^{pool} = A + \frac{1}{2}x^H \left(\mu_\gamma + \frac{1}{2} - \left(\frac{1}{4}\bar{\gamma} + \frac{1}{4}\underline{\gamma} \right) \right)$$

In the separating equilibrium:

$$RE^{sep} = A + \frac{1}{2}x^H \left(\mu_\gamma + \frac{1}{2} - \left(\frac{1}{2}\bar{\gamma}n_{hq} + \frac{1}{2}\underline{\gamma}n_{lq} \right) \right).$$

Because $n_{hq} > \frac{1}{2} > n_{lq}$ (as shown in Corollary 2) and $\bar{\gamma} > \underline{\gamma}$ (by assumption), it follows that $RE^{pool} > RE^{sep}$.

A.1.11 Proof of Proposition 4

First, for the case when the disclosure rules can be condition on true quality. Note that Lemma 4 shows that in this case the most efficient attention allocation is $n_{lq} = 1$ and $n_{hq} = 0$. Then it directly follows from Lemma 2 that this outcome is achieved if $\Delta_{hq} \rightarrow 0$, i.e. if $\gamma_{hq}^* \rightarrow \bar{\gamma}$ and if Δ_{lq} is as large as possible, such that $\gamma_{lq}^* = 0$.

Second, when the disclosure rule cannot be conditioned on the firms' true quality, both firms are subject to the same regulation. As shown in Proposition 3, the most (constrained) efficient allocation of attention is to maximize (minimize) n_{lq} (n_{hq}). Furthermore, it follows from Corollary 2 that attention for the low-quality firm is highest in the pooling equilibrium. Thus, the most efficient outcome is obtained if both firms are forced into a disclosure equilibrium with $\gamma_{lq}^* = \gamma_{hq}^*$. Therefore, a maximum amount of disclosure $\gamma^{*,cap} < \bar{\gamma}$ prevents the high-quality firm from deviating into the separating equilibrium if $\underline{\gamma} < 1 - \sqrt{1 - \bar{\gamma}}$ and leads to the most efficient allocation of attention: $n_{hq} = n_{lq} = \frac{1}{2}$.

A.1.12 Proof of Proposition 5

First note that firm manager j 's expected utility at the disclosure stage can be written as:

$$E[U_j|\gamma_j] = A(1 + \omega_j) + \frac{1}{2}x^H(\gamma_j + n_j(1 - \gamma_j + \omega_j(1 - \gamma_j^{**})) + \gamma_j^{**}\omega_j).$$

Therefore, manager j chooses γ_j^* to maximize $E[U_j|\gamma_j]$.

First, assume both firms withhold their information and choose $\gamma_j^* = 0$. The low-quality firm does not have an incentive to deviate from this equilibrium because disclosing information would reduce n_{lq} and not change γ_j^{**} , thus that the manager's objective function would be lowered. For the high-quality firm, there might exist an incentive to deviate from this outcome because this firm can credibly signal its quality to the market which would increase γ_j^{**} and/or n_j . In particular, it is straightforward to show that:

$$E[U_{hq}|\gamma_{hq}, \gamma_{lq}^* = \underline{\gamma}, \gamma_{lq}^* = 0] > E[U_{hq}|\gamma_{hq}, \gamma_{hq}^* = 0, \gamma_{lq}^* = 0] \Leftrightarrow \gamma_{hq} > g(\underline{\gamma}, \omega_{hq})$$

with $g(\underline{\gamma}, \omega_{hq}) = \frac{-3\underline{\gamma}^2\omega_{hq} + (\underline{\gamma}-1)\sqrt{\underline{\gamma}^2(3\omega_{hq}+2)^2 + 4\underline{\gamma}(\omega_{hq}-2)(\omega_{hq}+1) + 4(\omega_{hq}+1)^2 - 2\underline{\gamma}^2 + 3\underline{\gamma}\omega_{hq} + 4\underline{\gamma} + 2\omega_{hq} + 2}}{2(\omega_{hq}+2)}$. As before, any

further disclosure beyond $\gamma_j^* = \underline{\gamma}$ would harm the high-quality firm because it would decrease n_j . Lastly, the low-quality firm's best response to $\gamma_{hq}^* = \underline{\gamma}$ is $\gamma_{lq}^* = 0$ because any amount of disclosed information would only reduce the firm's attention share. Hence, both firms chose not to disclose if $\gamma_{hq} \leq g(\underline{\gamma}, \omega_{hq})$ and only the high-quality firm discloses ($\underline{\gamma}$) if $\gamma_{hq} > g(\underline{\gamma}, \omega_{hq})$.

The first derivative of the cutoff value with respect to $\underline{\gamma}$ is given by:

$$\begin{aligned} \frac{\partial g}{\partial \underline{\gamma}} &= \frac{\sqrt{\underline{\gamma}^2(3\omega_{hq} + 2)^2 + 4\underline{\gamma}(\omega_{hq} - 2)(\omega_{hq} + 1) + 4(\omega_{hq} + 1)^2} + \frac{(\underline{\gamma} - 1)\left(\underline{\gamma}(3\omega_{hq} + 2)^2 + 2(\omega_{hq} - 2)(\omega_{hq} + 1)\right)}{\sqrt{\underline{\gamma}^2(3\omega_{hq} + 2)^2 + 4\underline{\gamma}(\omega_{hq} - 2)(\omega_{hq} + 1) + 4(\omega_{hq} + 1)^2}}}{2(\omega_{hq} + 2)} \\ &+ \frac{-6\underline{\gamma}\omega_{hq} - 4\underline{\gamma} + 3\omega_{hq} + 4}{2(\omega_{hq} + 2)} \end{aligned}$$

it is straightforward to confirm (with a statistical software package) that this expression is positive for $\underline{\gamma} \in (0, 1)$ and $\omega_{hq} \geq 0$.

A.1.13 Proof of Corollary 4

The first derivative of the cutoff g with respect to ω_{hq} is given by:

$$\begin{aligned} \frac{\partial g}{\partial \omega_{hq}} &= \frac{(1 - \underline{\gamma})}{(\omega_{hq} + 2)^2} + \\ &+ \frac{(\underline{\gamma} - 1)\left(\underline{\gamma}\left(-2\sqrt{\underline{\gamma}^2(3\omega_{hq} + 2)^2 + 4\underline{\gamma}(\omega_{hq} - 2)(\omega_{hq} + 1) + 4(\omega_{hq} + 1)^2} + \underline{\gamma}(6\omega_{hq} + 4) + 5\omega_{hq} + 2\right) + 2\omega_{hq} + 2\right)}{(\omega_{hq} + 2)^2\sqrt{\underline{\gamma}^2(3\omega_{hq} + 2)^2 + 4\underline{\gamma}(\omega_{hq} - 2)(\omega_{hq} + 1) + 4(\omega_{hq} + 1)^2}} \end{aligned}$$

it is straightforward to confirm (with a statistical software package) that this expression is negative for $\underline{\gamma} \in (0, 1)$ and $\omega_{hq} \geq 0$.

This result together with Proposition 5 implies that an increase in ω_{hq} decreases the cutoff for the two equilibria. In particular, an increase in ω_{hq} increases the range of parameters that lead to a separating equilibrium. Note that an increase in ω_{hq} only changes the firm's attention share if it changes the firm's disclosure policy (from pooling to separating). Thus, for a high-quality firm that moves from pooling to separating, the attention share changes from $n_{hq}^P = \frac{1}{2}$ to $n_{hq}^S = \frac{\bar{\gamma} - \underline{\gamma}}{\bar{\gamma} - \underline{\gamma}^2}$. Since $n_{hq} = \frac{1}{2}$ at $g(\underline{\gamma}, 0)$, and $\frac{\partial n_{hq}^P}{\partial \bar{\gamma}} < 0$, $n_{hq} < \frac{1}{2}$ for values of $\bar{\gamma}$ that are $\leq g(\underline{\gamma}, 0)$ but $> g(\underline{\gamma}, \omega_{hq})$ with $\omega_{hq} > 0$.

Together with Lemma 4, this result implies that real efficiency is increasing in ω_{hq} because

the disclosure outcome leads to an attention share closer to the efficiency maximizing allocation ($n_{lq} = 1, n_{hq} = 0$).

A.2 Additional Figures and Tables

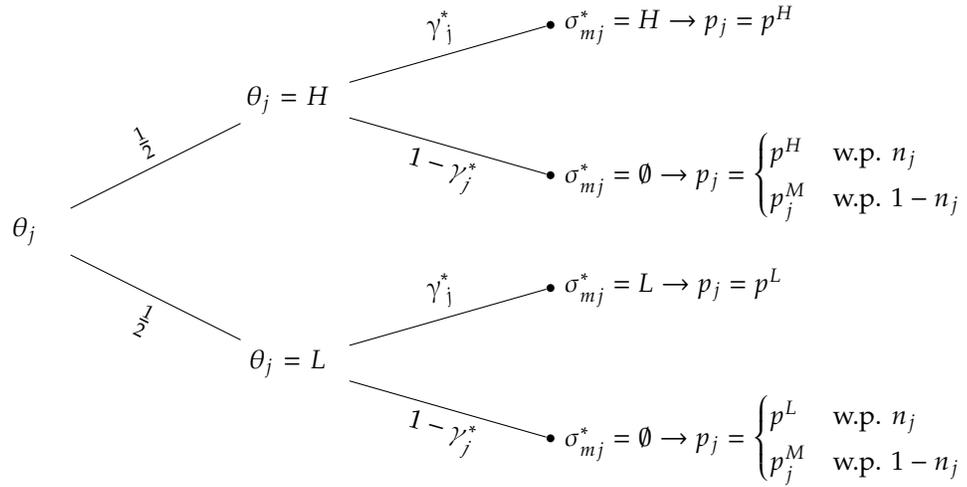


Figure 5: Event tree of firm j 's stock price.

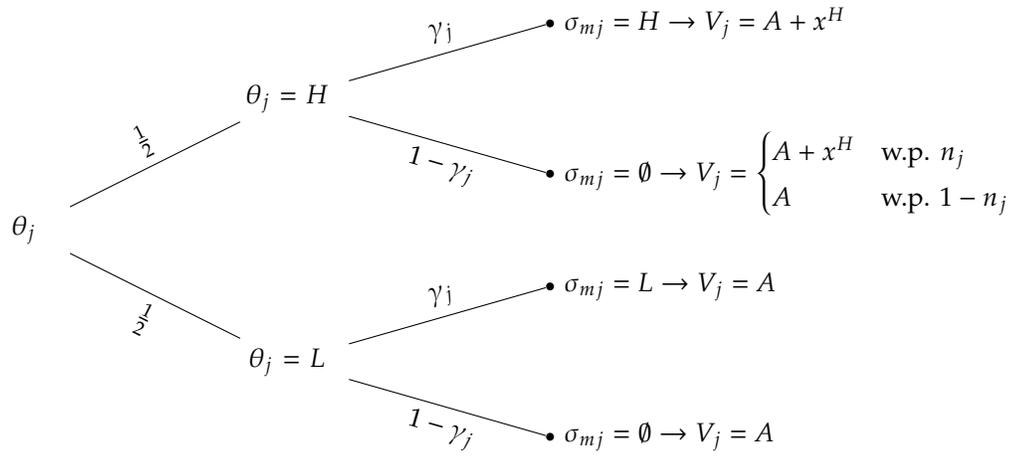


Figure 6: Event tree of firm j 's firm value.