Electronic Trading in OTC Markets vs. Centralized Exchange*

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Abstract

We model a two-tiered market structure in which an investor can trade an asset on a trading platform with a set of dealers who in turn have access to an interdealer market. The investor's order is informative about the asset's payoff and dealers who were contacted by the investor use this information in the interdealer market. Increasing the number of contacted dealers lowers markups through competition but increases the dealers' costs of providing the asset through information leakage. We then compare a centralized market in which investors can trade among themselves in a central limit order book to a market in which investors have to use the electronic platform to trade the asset. With imperfect competition among dealers, investor welfare is higher in the centralized market if private values are strongly dispersed or if the mass of investors is large.

1 Introduction

Trading in over-the-counter (OTC) markets is traditionally done over the phone, i.e. an investor who wants to trade an asset has to call a dealer an negotiate the price bilaterally. A recent trend in OTC markets is the growing electronification. Instead of calling dealer by dealer separately, an investor can use electronic trading platforms to send a request-for-quote (RFQ) to many dealers at once to obtain quotes at which the dealers are willing to trade. Some estimates suggest that in 2015, more than 40% of OTC-traded credit default swaps and more than 60% of OTC-traded interest rate swaps were traded electronically.[1]

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[1]See for instance Stafford (2016) for a brief overview of recent developments in OTC markets.

Electronic trading platforms can potentially increase the connectedness between market participants and thereby make OTC markets more exchange-like. However, there remains a fundamental difference between centralized exchanges and electronic trading platforms in OTC markets. Whereas exchanges can be viewed as all-to-all platforms, electronic trading platforms in OTC markets are one-to-many platforms. On exchanges, each market participant can trade through a central limit order book with all other market participants. On electronic trading platforms, the RFQ trading protocol prescribes that only one investor can initiate a trade at a time and choose one dealer to trade with. Therefore, electronic trading still incorporates many of the features of traditional bilateral trading in OTC markets.

The contribution of this paper is twofold. First, we model the trading process on trading platforms via an RFQ protocol. In our model, an investor who has some information about the assets payoff can choose a quantity to trade on the platform. In equilibrium, this quantity is informative about the asset's payoff. Our model therefore provides a theoretical foundation of information leakage on electronic trading platforms that is examined in empirical studies such as Hendershott and Madhavan (2015) or Hagströmer and Menkveld (2016). Increasing the number of dealers who are contacted by an RFQ has three competing effects on trading costs: If an RFQ is sent to more dealers, (i) competition among dealers lowers the expected markup the investor has to pay, (ii) the investor is more likely to receive a quote in the first place, since each dealers' response is uncertain and (iii) information leakage about the asset's fundamental value increases the dealers' cost of providing the asset, which results in worse prices for the investor. If dealers respond very frequently to each RFQ, the cost of information leakage dominates the benefits of more competition and contacting only few dealers maximizes the investor's payoff. Only if the dealers' RFQ response rate is sufficiently low, an RFQ has to be sent to a certain minimum number of dealers in order for an equilibrium to exist in the first place. In an off-equilibrium analysis, we deal with the price impact an investor faces on the platform. The presence of adverse selection makes the permanent price impact on the trading platform larger than the permanent price impact in the interdealer market. This result is consistent with the findings of Collin-Dufresne et al. (2017).

Second, we determine conditions under which investors are better off trading on a centralized exchange among themselves and when they are better off in the two-tiered market structure with an electronic trading platform and an interdealer market. In our model, all investors are equally informed about the asset's fundamental value and benefits from trade in the centralized market only arise due to private values of obtaining the asset (e.g. hedging benefits). Since dealers are less informed about the asset's value, investors can also benefit from their information about the asset in the OTC market structure. The dealers are willing to trade with the more informed investor, because they expect to be able to partially offset the trade at a favorable price in the interdealer market. If private values of obtaining the asset are small, investors are better off in the OTC market structure where they can benefit from information asymmetries between them and the dealers. On the other hand, if the total mass of investors is large, information about the asset's fundamental value quickly leaks into the interdealer market. In this case, the price investors have to pay on the platform is approximately the sum of the fundamental value and a markup. Then, investors are better off in the centralized exchange where they can avoid the dealers' markups and uncertainty about transactions. Only if competition among dealers is very high, investors will prefer to trade in the OTC markets. In this case, markups are very low, a trade is very likely and dealers efficiently intermediate trades between their customers. Additionally, investors can benefit from their information advantage over dealers in the OTC market. These results extend previous research on the comparison between OTC markets and exchanges in terms of investor welfare (Babus and Kondor, 2016; Glode and Opp, 2017). In this strand of literature, our study is the first one to specifically look at electronic trading platforms.

The paper proceeds as follows. Section 2 relates our paper to previous research. In Section 3, we explain the basic setup that is studied in Section 4. In Section 5, we slightly modify this setup to accommodate a continuum of investors and compare the two-tiered market structure to a centralized market. Concluding remarks are presented in Section 6. All proofs are in Appendix A.

2 Related Literature

Collin-Dufresne et al. (2017) empirically study the two-tiered index CDS market in the US. In the market for the most liquid index CDSs, the Dodd-Frank Act required trading via swap execution facilities (SEFs). As a result, investors trade with dealers almost exclusively via RFQs on electronic trading platforms. Dealers, on the other hand, trade among themselves via a continuous limit order book.[2] This market structure very closely corresponds to the setup we assume in our paper. The results of Collin-Dufresne et al. (2017) suggest that the permanent price impact in the D2C segment, i.e. when the investor trades on the platform, is higher than the permanent price impact in the interdealer market. These results justify our assumption that investors have some information about the asset that dealers do not have and are consistent with our result that there is information leakage from the trading platform to the dealers. Hendershott and Madhavan (2015) empirically study what kind of bonds are traded over the phone and which bonds are traded on an

[2]Block trades are exempt from the requirement to be traded on SEFs. However, most trades in the interdealer market are executed in the continuous limit order book, which also allows for mid-market matching and workup.

electronic trading platform. Controlling for endogenous venue selection, they examine the trading costs on these two trading venues. Hagströmer and Menkveld (2016) estimate information flows between dealers and provide further empirical evidence for information leakage on trading platforms in OTC markets. **?** and **?** argue that dealers in the foreign exchange market learn from their clients' order flow and exploit this information in the interdealer market.

Babus and Parlatore (2017) and Glode and Opp (2017) theoretically study investor welfare in OTC markets and centralized markets. Our model is different from those studies, since we specifically assume an RFQ trading protocol in the OTC market. Moreover, the information structure in our model differs from that in Babus and Parlatore (2017), since we have a common value of the asset for both investors and dealers. Compared to Glode and Opp (2017) we allow the investor to trade continuous quantities of the asset in the OTC market. Malamud and Rostek (2014) show that decentralized exchange markets may be more efficient than centralized ones. Lester et al. (2017) show in a search-theoretic model that competition in fragmented markets may decrease welfare.

In modeling the information leakage on trading platforms, our paper relates to a large strand of literature that models how information is shared between economic agents. Notable papers in this strand of literature include Duffie and Manso (2007), Duffie et al. (2009, 2014), Andrei and Cujean (2017) and Babus and Kondor (2016). Traditionally, OTC markets are modeled as pure search markets as for instance in Duffie et al. (2005), Weill (2007), Lagos and Rocheteau (2009), Gârleanu (2009), Lagos et al. (2011), Feldhütter (2005), Pagnotta and Philippon (2011) or Lester et al. (2015). Zhu (2012) and Duffie et al. (2016) explicitly model dealer markets. Our paper differs from all of those those papers since we consider an electronic trading platform.

Our assumption that dealers' responses on trading platforms are uncertain has been used by Jovanovic and Menkveld (2015) and Yueshen (2017) to model the behavior of market makers in central limit order books to derive similar random-pricing strategies.[3]

We also draw on the techniques of noisy rational-expectations models of Grossman and Stiglitz (1980), Hellwig (1980) and Diamond and Verrecchia (1981). These models assume that agents behave competitively. Kyle (1989) showed that those models can be extended to allow for strategic traders that take their price impact into account. However, few closed-form solutions are available in this case. Since the competitive case is generally viewed as a reasonable approximation to the strategic case in large markets (Vives, 2010), we will model a competitive dealer market.

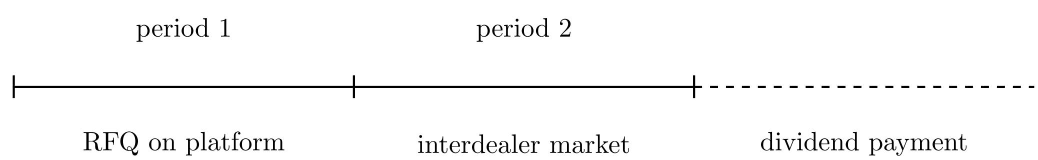
[3]Random-pricing strategies in turn have their origin in the consumer search literature. See for instance Varian (1980), Burdett and Judd (1983), Stahl (1989) and Janssen et al. (2005, 2011).

As Pagano and Röell (1996) argue, auction markets are in many ways more transparent than bilateral dealer markets. Naik et al. (1999) show that increased post-trade transparency has an ambiguous effect on dealers risk-sharing ability in two-tiered markets. Other papers who study the effects of transparency include De Frutos and Manzano (2002) and Yin (2005). In this paper, however, we do not consider any specific disclosure policies that are enforced by regulators. In our model, information is disseminated through the different trading mechanisms.

3 Model

There are two periods and two types of agents. In the first period, an investor can contact a number of dealers via an RFQ trading protocol on an electronic trading platform to buy or sell a quantity of an asset. In the second period, dealers trade with each other in a central limit order book. After period 2, the dividend is paid. This is illustrated in Figure 1.

Figure 1: Timeline



The asset pays an uncertain dividend $D = \theta + \varepsilon$ after the second period, where θ and ε are both independent and normally distributed random variables with zero mean and variances $\sigma_\varepsilon^2 > 0$ and $\sigma_\theta^2 > 0$, respectively. The informed investor knows the realization of θ already in the beginning of period 1. The investor also receives a private benefit $\delta \sim \mathcal{N}(0, \sigma_\delta^2)$, with $\sigma_\delta^2 \geq 0$ for holding one unit of the asset. This private benefit is realized in the beginning of period 1 and is independent from all other random variables. Only the investor can observe δ.

There are N, $\mathbb{N} \ni N \geq 2$ dealers. On the trading platform, the investor can specify the quantity x of the asset he wants to trade. The investor also selects M dealers, with $\mathbb{N} \ni M \leq N$ from which he wants to obtain prices at which they are willing to offer quantity x of the asset. The dealers respond independently with probability $q \in (0, 1]$ to the RFQ. That dealers do not necessarily respond may reflect the cost of paying attention. We will throughout this paper assume that the number of contacted dealers M is exogenously given, i.e. the trading protocol specifies that the investor has to contact exactly M dealers. This is a slight simplification of RFQ protocols in real-world markets where investors can often freely choose a number of dealers to contact.

The dealers are ex ante identical and hold zero initial inventory in the beginning of period 1. In period 2, the aggregate supply of the asset in the interdealer market is noisy. We denote the aggregate supply of the asset in the interdealer market by W. This aggregate supply is normally distributed: $W \sim \mathcal{N}(0, \sigma_W^2)$ and $\sigma_W^2 > 0$. A noisy aggregate supply is necessary in order to prevent uninformed dealers from observing the information of informed dealers. One can interpret noise in the aggregate supply as demand from noise traders or inventory shocks to dealers' portfolios, even though there is a slight difference between inventory shocks and noisy aggregate supply. Both dealers and the investor have mean-variance preferences. There is no discounting and each agent's utility is linear in the payments made when trading the asset. Let $\overline{\omega}_k$ denote dealer k's final inventory in the end of period 2 and let Z_k denote the sum of all payments made or received by dealer k from trading the asset. Then dealer k's utility in the end of period 2 with final inventory $\overline{\omega}_k$ is given by

$$U_d(\overline{\omega}_k, Z) = \overline{\omega}_k \cdot \mathbb{E}(D|\mathcal{I}_k) - \frac{\gamma_d}{2} \cdot \overline{\omega}_k^2 \cdot \mathbb{V}(D|\mathcal{I}_k) - Z_k, \tag{1}$$

where $\gamma_d > 0$ is the dealers' risk-aversion parameter. The expectation and the variance in equation (1) are taken with respect to each dealer k's specific information set $\mathcal{I}_k$, which will be determined later. Equation (1) says that dealers care linearly about the mean of their expected dividend payment in the end of period 1 and sum they have to pay in both period 1 and 2. They also have to pay an inventory cost which is increasing in the expected variance of the dividend payment. This inventory cost depends on the risk-aversion parameter $\gamma_d > 0$. It is clear that equation (1) can be derived from a first-order condition of an exponential utility function. We specifically do not assume exponential utility because an exponential utility function and the functional form specified in equation (1) have different implications for the equilibrium on the platform. On the platform, a dealer has to take into account the possibility of being undercut by another dealer when giving quotes to the investor. The model becomes more tractable, if the dealers' utility is linear in the payments made when trading. We will assume that the dealers follow symmetric strategies on the platform and symmetric and linear strategies in the interdealer market.

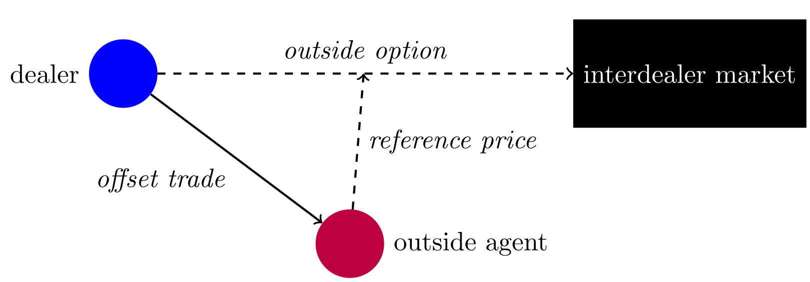
Similar to the dealers, the investor has mean-variance preferences. The investor, however, also receives the private benefit δ per unit of the asset held. If the investor buys a quantity $x_1 \in \mathbb{R}$ on the platform at price p_1, the investor's utility is given by

$$U_I(x_1, p_1) = x_1 \cdot (\theta + \delta) - \frac{\gamma_I}{2} \cdot x_1^2 \cdot \sigma_\varepsilon^2 - p_1 x_1, \tag{2}$$

where $\gamma_d > 0$ is the investor's risk-aversion parameter. Comparing (1) and (2), note that the investor's expectation of the dividend payment and its variance is given by θ and σ_ε, respectively. On the other hand, dealers potentially learn about the dividend from the other agents and thus have a less trivial information set $\mathcal{I}_k$ for each dealer k. Also, dealers trade with each other, which results in a more complex final inventory $\overline{\omega}_k$ and more complex total payments Z_k for each dealer k.

When dealing with the case of one investor in Section 4, we need to make a technical assumption in order to keep the model tractable. In Section 4, we will assume the presence of an "outside agent". If the investor contacts $M < N, M > 0$ dealers on the platform, these M dealers will learn from the investors about the the realization of θ. Thus, there will be informed and uninformed dealers in the interdealer market. In the interdealer market, the uninformed dealers may then make inferences about the dividend level from the observed market price. It will turn out that this price is affected by both the dealers' inventories and the informed dealers' expectation of the dividend payment. In order to keep this inference problem tractable, we make the dealers' inventory independent of the expected dividend level. To this end, we assume that a dealer who traded on the platform with the investor offsets this trade with the outside agent, who does not participate in the interdealer market. The outside agent does not behave strategically. The price at which the dealer offsets his trade with the investor is such that the dealer is indifferent between trading with the outside agent and going directly to the interdealer market. This way, we keep the dealers' inventories independent of the dividend level and still keep the key economic trade-offs that the dealers and the investor face in our model. This setup is summarized in Figure 2. After the trader has offset his trade with the outside agent, all dealers start to trade in the interdealer market. A version of our model without the outside agent will be studied in Section 5.

Figure 2: The outside agent



4 Equilibrium with one investor

The equilibrium is determined by backward induction. The first step is to establish the equilibrium in the interdealer market. We will assume and later verify that the investor reveals a noisy signal about the dividend level θ to the dealers he contacts. After an equilibrium in the interdealer market has been established, dealers on the platform can anticipate their expected final payoff conditional on the quantity they trade on the platform. This payoff will ultimately be a key determinant of the expected price for the asset on the platform which is derived by standard auction-theoretic arguments. Using the derived quoting strategies of the dealers and assuming that the quantity the investor wants to trade is linear in $\theta + \delta$, an equilibrium on the trading platform can be constructed.

4.1 The equilibrium in the interdealer market

The equilibrium in the interdealer market considered in this paper is a rational expectations equilibrium in linear demand schedules as first studied by Grossman and Stiglitz (1980). This means that dealers behave competitively. Even though not completely realistic, this assumption can be viewed as a rather good approximation in the case of large interdealer markets.

Let x_k denote the quantity of the asset that dealer k buys in the interdealer market. Since we assume that a dealer who trades on the platform offsets his trade with a outside agent, the final inventory $\overline{\omega}_k$ of dealer k is equal to the traded quantity in the interdealer market: $\overline{\omega}_k = q_k$ for all $k \in \{1, ..., N\}$.

Since $M \leq N$ dealers have been contacted on the platform, there will be M informed dealers, who observe $\theta + \delta$ from the investor's demand. The other $N - M$ dealers are uninformed and will use the market price to make inferences about the dividend level. In the following, we will represent all dealers by the set $\{1, ..., N\}$ and say that dealer k is informed if $k \leq M$. Conversely, we say that dealer k is uninformed if $k > M$.

Let p_2 denote the price for the asset in the interdealer market. Differentiating the dealer's utility (1) with respect to q_k and using $\frac{\partial Z_k}{\partial q_k} = p_2$ gives the first-order condition

$$\mathbb{E}(D|\mathcal{I}_k) - \gamma_d \overline{\omega}_k \mathbb{V}(D|\mathcal{I}_k) - p_2 = 0. \tag{3}$$

Since $\overline{\omega}_k = q_k$, the second order condition is $-\gamma_d \mathbb{V}(D|\mathcal{I}_k) < 0$. The second order condition always holds, since $\gamma_d > 0$ and $\mathbb{V}(D|\mathcal{I}_k) \geq \sigma_\varepsilon^2$. If dealer k receives the signal $s_d := \theta + \delta$, one obtains by standard Bayesian updating that

$$\xi := \mathbb{E}(D|s_d) = \frac{\sigma_\theta^2 s_d}{\sigma_\theta^2 + \sigma_\delta^2}. \tag{4}$$

Similarly, one obtains

$$\tau_\xi := \frac{1}{\sigma_\xi^2} := \frac{1}{\mathbb{V}(D|s_d)} = \frac{1}{\frac{\sigma_\theta^2 \sigma_\delta^2}{\sigma_\theta^2 + \sigma_\delta^2} + \sigma_\varepsilon^2}, \tag{5}$$

where we defined τ_ξ and σ_ξ^2 as the precision and the variance of the dividend payment based on the informed dealers' information that includes the signal s_d.

The first order condition (3) now implies the following demand schedule:

$$q_k = \frac{\tau_\xi(\xi - p_2)}{\gamma_d} \qquad \text{for } k \leq M. \tag{6}$$

If dealer k is uninformed, his demand is assumed to be of the form

$$q_k = \frac{\mathbb{E}(D|p_2) - p_2}{\gamma_d \mathbb{V}(D|p_2)} \qquad \text{for } k > M. \tag{7}$$

Equation (7) takes into account that uninformed dealers can only learn about the conditional distribution of D by observing the market price p_2. We will use the standard approach to conjecture a price that is linear in ξ and the aggregate supply of the asset W:

$$p_2 = a\xi + bW, \tag{8}$$

with $a, b \in \mathbb{R}$. Then, uninformed dealers can use the normal projection theorem to calculate $\mathbb{E}(D|p_2)$ and $\mathbb{V}(D|p_2)$.

In equilibrium, also the market clearing condition

$$\sum_{k=1}^{N} q_k = W \tag{9}$$

has to be satisfied. Using (6) and (7) in (9) determines the market clearing price. Matching of coefficients in the obtained expression for the market clearing price with the coefficients in the conjectured expression (8) then gives the rational expectations equilibrium price function. This price function in turn determines the uninformed dealers' equilibrium demand schedules.

The following Proposition confirms the existence of an equilibrium in the interdealer market and states the corresponding expressions for equilibrium price.

Proposition 1. *There is always a rational expectations equilibrium such that the market clearing price is given by (8). Define*

$$
\begin{aligned}
\rho : &= \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\delta^2}, && (10)\\
\tau_u : &= \frac{1}{Var(D|p_2)} = \frac{1}{\sigma_\theta^2 + \sigma_\epsilon^2 - \psi\rho\sigma_\theta^2}, && (11)\\
\psi : &= \frac{a^2\rho\sigma_\theta^2}{a^2\rho\sigma_\theta^2 + b^2\sigma_W^2} = \frac{\rho}{\rho + \frac{\gamma_d^2\sigma_W^2}{M^2\tau_\xi^2\sigma_\theta^2}} && (12)
\end{aligned}
$$

Then a and b are given by

$$a = \frac{M\tau_\xi + (N-M)\psi\tau_u}{M\tau_\xi + (N-M)\tau_u} \tag{13}$$

and

$$b = -\frac{\gamma_d}{M\tau_\xi}a, \tag{14}$$

One has $a > 0$ if $M > 0$. One also has $a \leq 1$ with strict inequality if $M < N$.

The fact that $a < 1$ for $M < N$ means tha the price in the interdealer market is inefficient in the sense that the price does not fully reflect the informed dealers' information. In the absence of private benefits for the investor ($\sigma_\delta^2 = 0$), dealers are only willing to trade with the investor because of this informational inefficiency in the interdealer market.

4.2 The equilibrium on the trading platform

The equilibrium on the trading platform is derived as follows. We will assume that dealers who are contacted by the investor can observe $s_d = \theta + \delta$ and therefore form a conditional expectation of θ given by ξ as defined in (4). We will then use Proposition 1 and the optimal demand schedules (6) to determine the lowest price at which a dealer is willing to sell (or the highest price at which he is willing to buy) a given quantity of the asset. The dealers then infer from the investor's utility function the maximum markup they can charge. In equilibrium, dealers will charge a random markup on the platform. The expectation of this price can be used to determine the investor's equilibrium strategy that reveals s_d.

Assume an investor submitted an RFQ to M dealers on the platform to buy x units of the asset (if $x < 0$, the investor wants to sell). If a dealer is contacted on the platform, but does not trade the asset, he will observe s_d and will therefore be informed in the interdealer market, expecting a dividend level of ξ. Let $V_{d,1}:\mathbb{R}^2\to\mathbb{R}$ denote the function that maps the expectation ξ and dealer k's traded quantity to dealer k's expected utility that he will get after period 2. The dealer will anticipate that the price p_2 is a linear function of ξ and W as stated in Proposition 1. Now, the optimal demand schedule (6) and the dealers utility function (1) imply the following payoff from not trading (i.e. from trading quantity 0):

$$V_{k,1}(\theta,0):=\mathbb{E}_k\left[q_k(D-p_2)-\frac{\gamma_d}{2}\sigma_\xi^2 q_k^2\right]=\frac{\xi^2(1-a)^2+b^2\sigma_W^2}{2\gamma_d\sigma_\xi^2}.$$

We now consider the case in which dealer k sells quantity x to the investor[4] and goes directly to the interdealer market, while other dealers think that dealer k already offset his trade with the outside agent. Now, dealer k has the initial inventory $-x$ in the beginning of period 2. However, only dealer k knows that.

The following result states the expected price in the interdealer market for dealer k and dealer k's optimal demand.

Lemma 1. *Assume dealer k traded quantity $x\neq 0$ with the investor on the platform and directly goes to the interdealer market. Let the other dealers believe, dealer k offset his trade before going to the interdealer market. Then, according to dealer k's information, the price in the interdealer market is given by*

$$p_2=a\xi-bx+bW$$

and his optimal demand schedule is given by

$$q_k=\frac{\xi-p_2}{\gamma_d\sigma_\xi^2}+x,$$

where a and b are defined as in Proposition 1.

Using Lemma 1, one can calculate dealer k's expected utility if he goes directly to the interdealer market holding a quantity $-x\neq 0$ and having expectation about the dividend payment ξ.

$$V_{k,1}(\xi,x):=\mathbb{E}_k\left[D(q_k-x)-p_2q_k-\frac{\gamma_d}{2}\sigma_\xi^2(q_k-x)^2\right]=\frac{\xi^2(1-a)^2+2(1-a)b\xi x+b^2\sigma_W^2+b^2x^2}{2\gamma_d\sigma_\xi^2}-(a\xi-bx)x.$$

[4] If $x<0$ the dealer is buying from the investor.

Comparing $V_{k,1}(\xi, x)$ and $V_{k,1}(\xi, 0)$ one can observe that the dealer expects a different return from holding a final inventory due to a different expected price. The second term in $V_{k,1}(\xi, x)$ represents the additional payment a dealer has to make to offset his inventory x in the interdealer market. We define

$$p_c(x) := \frac{V_{k,1}(\xi, 0) - V_{k,1}(\xi, x)}{x} \qquad k \leq M \tag{15}$$

as the break-even price for any contacted dealer k. A dealer who charges $p_c(x)$ per quantity of the asset and sells x units to the investor, does not change his final utility. The payment from the investor exactly matches the difference in utility due to different inventory holdings. Analogously, we define

$$p_v(x) := \theta + \delta - \frac{\gamma_I}{2} x\sigma_\varepsilon^2 = \frac{\xi(\sigma_\theta^2 + \sigma_\delta^2)}{\sigma_\theta^2} - \frac{\gamma_I}{2} x\sigma_\varepsilon^2 \tag{16}$$

as the price at which the investor is indifferent between trading and not trading the asset. As one can immediately verify, equation (2) implies $U_I(x, p_v(x)) = 0$. One can interpret $p_c(x)$ as the cost for each contacted dealer of supplying x units of the asset. Analogously, $p_v(x)$ is the investor's value of acquiring x units of the asset. The investor can only trade a certain quantity $x > 0$ with a dealer if $p_v(x) \geq p_c(x)$. Analogously, it has to hold that $p_c(x) \geq p_v(x)$ if $x < 0$.

In the following, we assume that dealers follow symmetric strategies when giving a quote to the investor. This approach is standard, since dealers are ex-ante identical. In the appendix we show that standard search-theoretic arguments imply that the price a dealer quotes on the platform for a certain quantity x has to be a continuous random variable if $p_v(x) \neq p_c(x)$. Let $F_x : \mathbb{R} \to [0, 1]$ denote the distribution of the price a dealer quotes on the platform conditional on the quantity x that the investor wants to trade. If $x > 0$ and $p_v(x) > p_c(x)$, then $p_v(x)$ will turn out to be the supremum of the support of F_x. That quoting a higher price than $p_v(x)$ cannot be optimal follows from $U_I(x, p) < 0$ for $p > p_v(x)$ and $x > 0$. The investor would not be willing to buy the asset at such a price since doing so would make him worse off. Analogously, $p_v(x)$ is the infimum of the support of the distribution of quoted prices if $x < 0$. The investor would not be willing to sell the asset at a lower price.

Dealers are only willing to quote random prices if the expected profit they make is the same for any price in the support of F_x. If $p_v(x)$ is in the support of F_x, this indifference condition means that

$$x(p - p_c(x)) \sum_{j=0}^{M-1} \binom{M-1}{j} (1-q)^{M-1-j} q^j (1 - F_x(p))^j = (1-q)^{M-1}(p_v(x) - p_c(x))x \tag{17}$$

has to hold for all $p \in \text{supp}(F_x)$. The left-hand side of equation (17) describes the expected profit a dealer makes by quoting any $p \in \text{supp}(F_x)$. The payment $x(p - p_c(x))$ in excess of the indifference level $xp_c(x)$ is weighted by the probability that the dealer has the best quote among all dealers that respond to the RFQ. Since the response of a dealer is uncertain and occurs with probability $q < 1$, one has to consider the cases in which $j = 0, ..., M-1$ other dealers respond. The right hand side describes the expected profit for a dealer that quotes $p_v(x)$. Since F_x is continuous, this dealer will only sell the asset if no other dealer responds to the RFQ. This happens with probability $(1-q)^{M-1}$. In this case, the dealer's utility will increase by $x(p_v(x) - p_c(x)) > 0$.

The following result gives the closed-form expression for the distribution function F_x that solves (17) for any x with $x(p_v(x) - p_c(x)) > 0$. The last inequality is a necessary condition for the existence of strictly positive benefits of trade between dealers on the platform and the investor. In the statement of Lemma 2, we will leave implicit that p_c and p_v depend on x. In Section 5, we will study a version of the model in which the dealers' cost p_c does not depend on x. Since Lemma 2 holds irrespective of what variables p_v and p_v depend on, we will state it without reference to any of those variables.

Lemma 2. *Let p_c be the dealers' cost of providing a certain quantity $x \in \mathbb{R} \setminus \{0\}$ of the asset and let p_v denote the investor's value of acquiring x units of the asset. Let the investor submit an RFQ to $M \geq 2$ dealers on the platform to trade quantity x with $x(p_v - p_c) > 0$. Let $q < 1$. Assume that dealers who get contacted know θ.*

If a dealer responds to an RFQ, he will charge a random price that is distributed according to the distribution function F_x. This function is defined by

$$F_x(p) := \frac{1}{q} - \frac{1-q}{q} \left(\frac{p_v - p_c}{p - p_c} \right)^{1/(M-1)}. \tag{18}$$

If $x > 0$, the support of F_x is given by $[\overline{p}_x, p_v]$, where $\overline{p}_x$ is determined by $F_x(\overline{p}_x) = 0$ and satisfies $\overline{p}_x > p_c$.

If $x < 0$, the support of F_x is given by $[p_v, \overline{p}_x]$, where $\overline{p}_x$ is again determined by $F_x(\overline{p}_x) = 0$ and satisfies $\overline{p}_x < p_c$.

In this case, the expected price the investor has to pay for the asset conditional on at least one response to the RFQ is given by

$$P(x) := \mathbb{E}(p_1 \mid x, \textit{at least one response}) = \int_{\text{supp}(F_x)} p \mathrm{d}G_x(p) = p_c + \kappa(p_v - p_c), \tag{19}$$

where the distribution $G_x : \mathbb{R} \to [0,1]$ *is defined by*

$$G_x(p) := \frac{1 - (1 - qF_x(p))^M}{1 - (1-q)^M}$$

and

$$\kappa := \frac{Mq(1-q)^{M-1}}{1-(1-q)^M} < 1 \in [0,1) \tag{20}$$

If $q = 1$*, Bertrand competition implies that dealers have to set a price that equal to their cost* p_c*. Thus, the above expression for* $P(x)$ *holds for all* $q \in (0,1]$*.*

Equation (19) states that the expected price the investor receives on the platform is equal to the dealers cost p_c plus a fraction of the total gains from trade $p_v - p_c$. The fraction of this surplus that the investor has to pay is equal to κ, defined as in (20). Thus, κ can be viewed as the endogenously determined bargaining power of the dealers. By taking derivatives, it can be shown that κ is decreasing in M and q, which is consistent with economic intuition. As M becomes larger, competition among the dealers for the business of the investor increases. This competition is also higher, if the presence of other dealers on the platform becomes more likely.

Note that the results in Lemma 2 required the assumptions that $x(p_v - p_c) > 0$ and that contacted dealers observe $\theta + \delta$. In the remaining part of this section we will derive an optimal strategy of the investor that allows both assumptions to hold in equilibrium. We will restrict the possible strategies of the investor to strategies that are linear in the sum $\theta + \delta$. This means that the quantity the investor wants to trade is a (positive) multiple of $\theta + \delta$. It is obvious that dealers then can infer $\theta + \delta$ from the quantity the investor wants to trade. However, it is a nontrivial result that the investor finds it indeed optimal to reveal $\theta + \delta$ and the associated information about θ through his choice of the quantity x. The reason why such an equilibrium is possible, even as the private value δ becomes negligible, lies in the fact that the parameter a as defined in 1 is generally less than one. If the investor reveals a given value of $\theta + \delta$ to the dealers, the dealers expect a dividend payment equal to ξ as defined in (4). The price for the asset in the interdealer market will be $\xi a < \xi$ in expectation. This price in the interdealer market determines the cost for dealers of providing the asset, which according to Lemma 2 determines the expected price the investor receives on the platform. If $a < 1$, the quotes the investor gets on the platform are less sensitive to θ than the investor's utility. This makes an equilibrium possible in which the investor partially reveals his information θ to the dealers.

We now conjecture that the investor's demand for the asset on the platform is given by

$$x = \alpha(\theta + \delta), \tag{21}$$

for some $\alpha \in \mathbb{R}$. In the appendix we show that the expected price $P(x)$ from Proposition 2 is linear in x and ξ:

$$P(x) = \beta_1 \xi + \beta_2 x, \tag{22}$$

with $\beta_1, \beta_2 \in \mathbb{R}$. From the investor's conjectured strategy (21), the contacted dealers infer $\theta + \delta = \frac{x}{\alpha}$. Using (2), (22) and (4), the investor's problem therefore becomes

$$\max_{x \in \mathbb{R}} \left[(\theta + \delta)x - x^2 \frac{\gamma_I}{2} \sigma_\varepsilon^2 - x \left(\beta_1 \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\delta^2} \frac{x}{\alpha} + \beta_2 x \right) \right]. \tag{23}$$

Note that (23) considers the investor's expected payoff conditional on at least one response to the RFQ. Since the probability of this event is exogenous and always gives a zero payoff, it can be neglected. The first-order condition for (23) implies the investor's optimal demand schedule

$$x = (\theta + \delta) \frac{\alpha(\sigma_\theta^2 + \sigma_\delta^2)}{2\alpha\beta_2(\sigma_\theta^2 + \sigma_\delta^2) + \alpha\gamma_I\sigma_\varepsilon^2(\sigma_\theta^2 + \sigma_\delta^2) + 2\beta_1\sigma_\theta^2}. \tag{24}$$

Therefore, the investor's optimal demand is indeed linear in $\theta + \delta$. Matching the coefficient in (24) with the conjectured strategy (21) gives

$$\alpha = \frac{\sigma_\theta^2 + \sigma_\delta^2 - 2\beta_1\sigma_\theta^2}{(2\beta_2 + \gamma_I\sigma_\varepsilon^2)(\sigma_\theta^2 + \sigma_\delta^2)}. \tag{25}$$

The following proposition summarizes these results and states formal conditions under which the equilibrium exists.

Proposition 2. *The expected price on the platform $P(x)$ from Lemma 2 is linear in ξ and x, as stated in (22). Let $M \geq 2$. If*

$$\kappa < \frac{1}{2}, \tag{26}$$

with κ as in (20), there is a threshold $\overline{a} > 0$, such that the equilibrium on the platform described below exists if and only if $a < \overline{a}$. The last condition holds as $N \to \infty$ and $\sigma_W^2 \to \infty$ or as $\sigma_\delta \to \infty$. The inequality in (26) will always hold for all $M \geq 2, \sigma_\theta^2, \sigma_\delta^2 > 0$ if $q \to 1$. If (26) does not hold, the equilibrium does not exist.

The equilibrium is characterized as follows. The investor submits a demand x as determined in equations

([21](#)) and ([25](#)). The dealers quote independently with probability q according to the distribution function F_x in ([18](#)).

One furthermore has $0 < \beta_1 < \frac{1}{2}\frac{\sigma_\theta^2+\sigma_\delta^2}{\sigma_\theta^2}$, $\beta_2 > -\frac{\gamma_I}{2}\sigma_\varepsilon^2$ and $\alpha > 0$ in each such equilibrium.

With the RFQ trading protocol, an equilibrium with linear strategies described in Proposition 2 is possible even though a linear equilibrium in double auctions and two strategic traders would not exist due to correlated values as Du and Zhu (2017) show. With the RFQ trading protocol, only the investor has the option to avoid price impact by reducing his demand. The dealers have to take the traded quantity as given and can merely charge a markup in addition to their cost of providing the asset.

We will illustrate the results derived so far with an example.

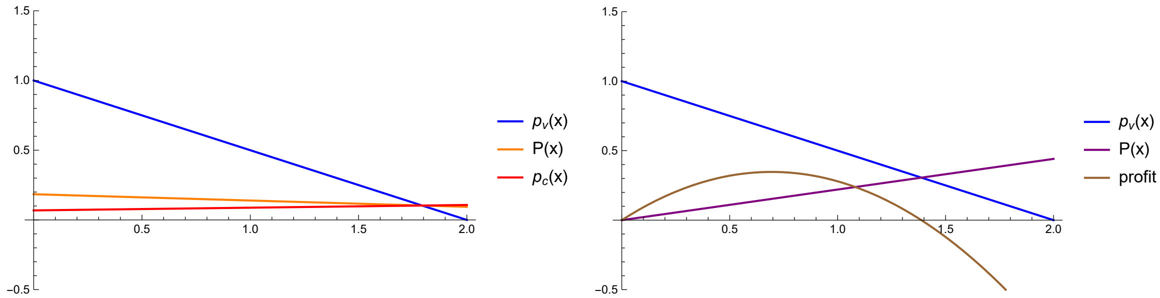
4.3 A brief example

For illustrative purposes we fix the exogenous parameters as follows: $N = 100$, $M = 10$, $\sigma_\varepsilon = 1$, $\sigma_W = N$, $\sigma_\theta = 1$, $\gamma_d = 1$, $\gamma_I = 1$, $q = 0.3$. To illustrate the economic mechanism of our model, we first consider the case in which $\theta + \delta = 1$, which corresponds to a realization one standard deviation above the mean. Afterwards we consider the case when $\theta + \delta = -1$. It is sufficient to only consider the sum of the common value and the investor's private, since both the investor's demand and the dealers' inferences depend only on this sum.

In Figure 3, $\theta + \delta$ has the high realization. In Panel (a) we plot the price $p_v(x)$ that the investor is willing to pay for x units of the asset. If the absolute value of x is small, this price is approximately equal to $\theta + \delta$, since the cost of bearing risk is small. The price $p_v(x)$ is linearly decreasing in x because of the quadratic cost of bearing risk. We also plot the dealer's cost $p_c(x)$ of providing x units of the asset, if they believe the dividend payment is normally distributed with mean ξ and precision σ_ξ^2, as defined in (4) and (5). One can see that this cost is slightly increasing in x, which represents the difficulty of offsetting the trade in the interdealer market or with the outside agent, respectively. The average price the investor can expect conditional on at least one response to the RFQ, $P(x)$, is between the other two curves.

In Panel (b) of Figure 3 we keep the investor's reservation price $p_v(x)$, but now look at the average price dealers quote when they the expected dividend level ξ from the investors demand x one can see that this price increases faster in x than the cost $p_c(x)$ in Panel (a). We also plot the profit the investor investor gets for demanding a certain quantity x. This profit is the solution to problem (23) weighted by the probability of at least one response to the RFQ. We see that the optimum is approximately at $x = 0.69$. This also turns out to be the value of α. Thus, Panel (b) illustrates, that the investor has indeed no incentive to deviate from the equilibrium strategy determined in the last section.

Figure 3: High realization of $\theta + \delta$

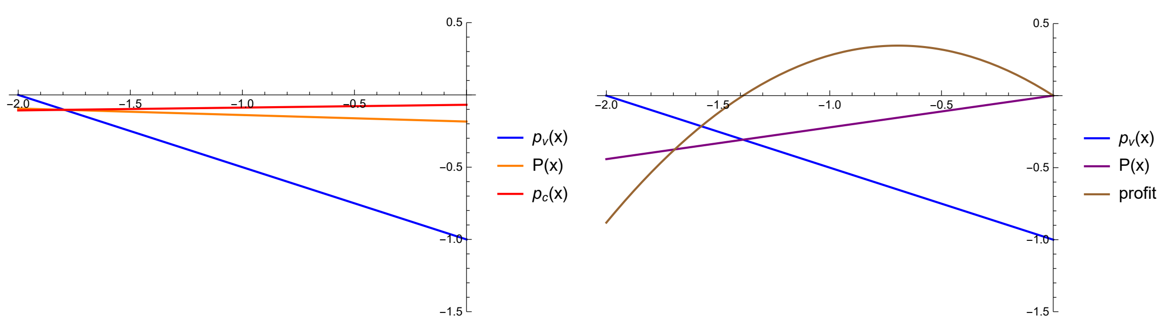


(a) Expected prices and reservation prices if ξ is held fixed.

(b) Dealers infer ξ from the investor's demand.

In Figure 4, we consider the low realization of $\theta + \delta$. Comparing panel (a) to Panel (a) in Figure 3, we observe that all curves have been shifted downwards by a constant. The curve of $p_v(x)$ has been shifted downwards more than the curve of $p_c(x)$. This has two reasons. First, the dealers expectation ξ is a weighted average between $\theta + \delta$ and zero, as (4) shows. Second, the dealers do not find it as costly to hold a bad asset as the investor does. The dealers expect to be able to resell the asset again at a favourable price, since there are many uninformed dealers in the interdealer market. Panel (b) of Figure 4 shows a similar picture as Panel (b) of Figure 3. In Figure 4, however, the investor sells the asset at a negative expected price. The investor finds it profitable to do so, since $p_v(x)$ indicates that he would be willing to sell the asset at an even lower price due to the negative expected dividend. The equilibrium strategies have not changed in Figure 3 and Figure 4. Therefore, the optimal demand in Figure 4 is the negative of the optimal demand in Figure 3, since the respective realizations of $\theta + \delta$ have the same absolute value in both cases.

Figure 4: Low realization of $\theta + \delta$



(a) Expected prices and reservation prices if ξ is held fixed.

(b) Dealers infer ξ from the investor's demand.

4.4 Competition vs. information leakage

In this section we take a closer look at the equilibrium described in Proposition 2. Specifically, we take a look how the investor's profits from trading on the platform are affected by varying the number of dealers who are contacted on the trading platform.

We define π_I as the investor's ex-ante expected payoff in the equilibrium described in Proposition 2. By the investor's utility function (2), his equilibrium strategy (25) and (22), one has

$$\pi_I = \mathbb{E}\left[(1-(1-q)^M)(\theta+\delta)^2\frac{1}{2}\alpha\right]. \tag{27}$$

Equation (27) takes into account that the investor does not receive any quote with probability $(1-q)^M$ and that dealers infer ξ from the investor's demand.

Our first goal is to study the role of M, the number of recipients of each RFQ. Increasing M has three major effects that determine the investor's profit:

- As is evident from (27) a higher M increases the probability of a trade $1-(1-q)^M$, whenever $q<1$. Holding everything else equal, this increases expected profits.
- A higher M increases the fraction of informed dealers in the interdealer market. One can verify that a as defined in (13) is strictly increasing in M for $M<N$. This makes prices in the interdealer market more informative and it therefore becomes more difficult to offset any inventory that was acquired on the platform.
- A higher M decreases κ, as mentioned in the discussion after Lemma 2. Therefore, the bargaining power of the investor increases, which has a positive effect on his profit.

Considering these three bullet points, the investor's profit should be maximal for $M=2$, if $q=1$. If $q=1$, one has $\kappa=0$ and $1-(1-q)^M=1$, i.e. the investor's bargaining power is maximal and a trade happens with probability 1. Then the first and third bullet point above become irrelevant and increasing M is only associated with the cost of information leakage, discussed in the second bullet point. The following proposition formally confirms that the conclusion of the above heuristic reasoning is indeed true. Since we focus on the cost of information leakage we assume for better algebraic tractability that there are no private benefits, i.e. $\sigma_\delta=0$.

Proposition 3. *Let $2\leq M$ and $q=1$ and $\sigma_\delta=0$. The equilibrium described in Proposition 2 exists if and only if a is below a certain threshold $\overline{a}$, with $\overline{a}<\frac{1}{2}$. In this equilibrium, one has $\beta_1,\beta_2,\alpha>0$.*

Furthermore, the equilibrium exists for any other choice of the number M' of dealers to contact with $2 \leq M' < M$. If $M = 2$, the payoff for the investor is higher than in any other possible equilibrium with $M > 2$.

When $q < 1$, the investor has incentive to contact more dealers, i.e. $M \geq 2$. Because when $q \neq 1$, the first and third effects turn out to be relevant: increasing M will improve the probability of trading , as well as the bargaining power of the investor. But at the same time, the cost of information leakage is also increased (second bullet point).

The following proposition states that M sometimes has to be larger than a certain threshold in order for an equilibrium to exist in the first place. If q is relatively small, the bargaining power κ of dealers may be so high that investors do not want to incur any price impact they have on the trading platform. Increasing M lowers this bargaining power. Under the condition that prices in the interdealer market remain sufficiently uninformative, an equilibrium exists for a sufficiently large M. On the other hand, there is a clear upper bound on the possible number of dealers that are contacted on the platform for which an equilibrium exists. In particular, if more than half of the dealers are contacted and there is strong asymmetric information about the asset's payoff ($\sigma_\delta = 0$), an equilibrium cannot exist, because information leakage on the platform is too strong.

Proposition 4. *Let $\sigma_\delta^2 = 0$. If $M > \frac{1}{2}N$, there is no equilibrium on the trading platform as described in Proposition 2.*

If $q < \frac{1+\sqrt{1-\frac{2\left(a(2)^2-5a(2)+2\right)}{a(2)^2-5a(2)+4}}}{2}$, there is no such equilibrium with $M = 2$. If furthermore $a < \bar{a}$, for an $\bar{a} \in (0, \frac{1}{2})$, then there is such an equilibrium with $M \geq 3$.

4.5 Price impact

In this section we want to relate our theoretical results to the empirical findings of Collin-Dufresne et al. (2017). In particular, we want to study the price impact that an investor faces on the trading platfrom and the price impact that dealers face in the interdealer market. The total price impact a trader faces can be decomposed as

$$\text{price impact} = \text{permanent impact} + \text{transitory impact}.$$

Collin-Dufresne et al. (2017) find that price impact in the D2C segment is higher than in the D2D segment. This difference is largely due to a differene in the permanent price impact.

We now want to derive the price impact and find analogues in our model that correspond to a permanent component and a transitory component. As commonly argued in theoretical studies (Sannikov and Skrzypacz, 2016; Kyle et al., 2017), the study of price impact is an off-equilibrium analysis. We will therefore assume an equilibrium as described in Proposition 2 and examine how the price a trader faces changes if the demanded quantity changes.

Equation (19) in Lemma 2 directly provides an expression of the expected price an investor receives on the platform. If the investor changes his demanded quantity x, then p_v and p_c in (19) and consequently the expected price for this quantity will change. Since the model presented in this paper is static, we have to find a decomposition of this price impact that would correspond to a decomposition into a permanent and a transitory component in a dynamic model. In empirical studies in Market Microstructure, it is generally assumed that the transitory component reflects a markup of the dealers, whereas the permanent component reflects the cost of the dealers of providing the asset due to future price changes. In our following analysis, we adopt this interpretation. We say that the price impact is permanent, if it was caused by a change in the dealers' cost of providing the asset.[5] Therefore, we define

$$PI := \frac{\partial}{\partial x} p_c(x) \tag{28}$$

as the permanent price impact of the investor, because (28) reflects the change in the price that is due to an increase in the dealers' cost of trading the asset.

In the following, we will consider p_c as defined in (15). Due to adverse selection, we also need to take into account that the dealers form their expectation ξ about the dividend payment based on (4) and (21). The following proposition contains some statements about the price impact on the platform and in the interdealer market.

Proposition 5. *The (permanent) price impact an informed dealer faces in the interdealer market is given by $-b$, where b is defined as in Proposition 1. Without adverse selection (dealers do not update their belief ξ), one has*

$$PI < -b,$$

i.e. the permanent impact on the trading platform is smaller then the permanent impact the dealers face in

[5] This assumes that changes in the dealers' cost have no transitory component. Transitory changes in the dealers' cost may arise due to inventory holding costs or order processing costs. In our model, dealers can immediately offset their inventory and the interdealer market is competitive. Order processing is costless. Therefore, such transitory components of dealers' costs are not present in our model.

the interdealer market. In the presence of adverse selection and $\rho = \frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\delta^2} > 1/4$*, one has*

$$PI > -b.$$

The dealers' permanent price impact $-b$ derived in Proposition 5 is due to a change in the uninformed dealers' belief about the dividend payment and a permanent change in the aggregate inventory held by other dealers in the interdealer market. Proposition 5 shows that the permanent price impact on the trading platform higher than the permanent impact in the interdealer market if and only if investors know more about the asset than dealers do. If there is no adverse selection and dealers do not update their belief about the asset's payoff, the dealers' cost of providing the asset changes by a lower rate than the price in the interdealer market would when trading the same quantity. This result is due to the dealers' optimal portfolio choice in period 2. A dealer could always offset the investor's demand in the interdealer market with price impact $-b$. If the investor however changed his demanded quantity, the dealer would, due to risk sharing considerations, in general not offset the total amount of this quantity in the interdealer market. Due to optimality of the dealer's portfolio choice, the dealer must be able to provide the quantity at a lower price than the one he would pay for this quantity in the interdealer market.

In the presence of adverse selection, however, the permanent price impact on the trading platform is higher than the permanent price impact the dealers face in the interdealer market. This makes our model (which assumes information asymmetries) consistent with the findings of Collin-Dufresne et al. (2017) that the permanent price impact is higher on the trading platform than in the interdealer market.

5 Centralized trading vs. electronic trading via RFQs

This section develops the market-design implications of our model. Our final goal is to characterize situations in which investors are better off trading in a centralized market and when an OTC market can improve their utility. In order to do this, we extend our previous model from to the case in which there is a is a continuum of investors of measure μ[6] who all know the realization of θ in the beginning of period 1. The investors' utility function is still give by (2). The risk-aversion parameter γ_I is the same for all investors. The investors receive a private benefit δ_i, where as before $\delta_i \sim \mathcal{N}(0, \sigma_\delta^2)$. The private benefits for different investors are essentially pairwise independent for different investors. This assumptions lets us apply the exact law of large

[6]Formally, let $(\Omega, \mathcal{F})$ denote the measurable space of investors. Then there is a bijective measurable map $\Phi : \Omega \to [0, \mu]$ and the measure of any set of investors $F \in \mathcal{F}$ is equal to the Lebesgue measure of the set $\Phi(F)$.

numbers of Sun (2006). The model assumptions about the dealers are as in Section 3, except that we do not assume the presence of an outside agent in this section. Before we establish an equilibrium in the OTC market, we quickly describe how the investors would trade in a centralized market.

5.1 The centralized-market benchmark

Investors trade through double auctions in the centralized market. In these double auctions, each investor specifies a demand schedule, i.e. conditional on each price $p \in \mathbb{R}$ the investor specifies a quantity he wants to trade. The equilibrium price in the centralized market will be the market-clearing price. The market clearing price will be the unique price for which the investors' aggregate demand is equal to the aggregate supply of the asset (zero). The specification of the investor's utility function (2) gives the following maximization problem for each investor for each $p \in \mathbb{R}$:

$$\max_{x_i \in \mathbb{R}} \left[x_i(\theta + \delta_i - p) - \frac{\gamma_I \sigma_\epsilon^2}{2} x_i^2 \right],$$

where x_i denotes the quantity the investor demands given the price p on the exchange.

The sufficient first-order condition for the above optimization problem gives

$$x_i \ = \ \frac{\theta + \delta_i - p}{\gamma_I \sigma_\epsilon^2},$$

To determine the market-clearing price, we substitute each investor's demand schedule x_i into the market clearing condition, $\int x_i \mathrm{d}i = 0$.[7] We get

$$0 \ = \ \mu \frac{\theta}{\gamma_I \sigma_\epsilon^2} - \mu \frac{p}{\gamma_I \sigma_\epsilon^2} \Leftrightarrow p = \theta\,,$$

where we have used the fact that $\int \delta_i \mathrm{d}i = 0$ almost surely by the exact law of large numbers.

Using each investor's optimal demand schedule, the utility function (2) and the fact that the market clearing price is given by θ, we can define each investor's ex-ante payoff:

$$\pi_i^c \ := \mathbb{E}\left(\frac{1}{2} \frac{(\theta + \delta_i - p)^2}{\gamma_I \sigma_\epsilon^2} \right) = \frac{1}{2} \frac{\sigma_\delta^2}{\gamma_I \sigma_\epsilon^2}\,. \tag{29}$$

Equation (29) states that the centralized market realizes all the gains from trade that arise due to dispersed private values. When all investors have the same valuation of the asset ($\sigma_\delta^2 = 0$), no trade happens

[7]The notation di means that we integrate with respect to the measure on set of investors defined in Footnote 6.

and the investor's profits become zero. Each investor's profit decreases if the cost of bearing risk increases.

5.2 Electronic trading with a continuum of investors

The model with a continuum of investors is very similar to the model with one investor. It will turn out that a continuum of investors allows us to derive an equilibrium without the assumption of an outside agent. We will let the mass of investors have a measure $\mu \in (0, \infty)$. In period 1, all investors submit RFQs to M dealers. Afterwards, dealers trade in the interdealer market. All investors contact the same M dealers at the same time. The dealers then independently respond with a probability q to each RFQ. As before, we will determine the equilibrium in this model by backward induction.

Since there is no outside agent anymore in this section, uninformed dealers in the interdealer market take into account that the aggregate supply of the asset is correlated with the investors' information about the dividend level θ. We will conjecture that each investor demands a quantity x_i on the trading platform, where

$$x_i = \alpha_1 \theta + \alpha_2 \delta_i, \tag{30}$$

for some $\alpha_1, \alpha_2 \in \mathbb{R}$. As in Section 4, it will turn out that an investor always trades the asset if he receives a quote on the trading platform. Since each dealer responds independently with probability q to each RFQ, the an investor is able to trade the asset with probability $\mathbb{P}(trade) = 1 - (1-q)^M$. By the exact law of large numbers and (30), the investors' aggregate demand traded on the platform given by

$$X^{agg} := \int \mathbb{P}(trade)(\alpha_1 \theta + \alpha_2 \delta_i) \mathrm{d}i = (1 - (1-q)^M) \int (\alpha_1 \theta + \alpha_2 \delta_i) \mathrm{d}i = (1 - (1-q)^M) \mu \alpha_1 \theta, \tag{31}$$

where the last equality holds almost surely. By symmetry, each dealer gets an equal fraction of this aggregate demand. We define $X_k := -\frac{X^{agg}}{M}$ as the inventory of each dealer $k \leq M$ who gets contacted on the trading platform. From the dealers' utility function (1), one obtains the optimal demand schedule q_k for each dealer $k \leq M$:

$$q_k = \frac{\theta - p_2}{\gamma_d \sigma_\varepsilon^2} + X_K. \tag{32}$$

Notice that X_k is a multiple of θ, this will simplify the inference problem that the uninformed dealers face in the interdealer market. Analogously to Section 4, we conjecture that the market-clearing price in the interdealer market is given by

$$p_2 = a\theta + bW, \tag{33}$$

where W is the noise in the aggregate supply of the asset. The uninformed dealers use the normal projection theorem obtain the distribution of the dividend payment conditional on the market-clearing price p_2. The dealers' utility function (1) now gives the optimal demand

$$q_k = \frac{\mathbb{E}(D|p_2) - p_2}{\mathbb{V}(D|p_2)} \tag{34}$$

for the uniformed dealers who do not get contacted on the trading platform. Analogously to Proposition 1, we now state the equilibrium in the interdealer market, conditional on the investors' trading strategy (30).

Proposition 6. *For any given α_1, there is a rational expectations equilibrium such that the market clearing price is given by (33). Define*

$$\varphi : \; = \; (1-(1-q)^M)\mu, \tag{35}$$

$$\tau_u : \; = \; \frac{1}{Var(D|p_2)} = \frac{1}{\sigma_\theta^2 + \sigma_\epsilon^2 - \psi\sigma_\theta^2}, \tag{36}$$

$$\psi : \; = \; \frac{a^2\sigma_\theta^2}{a^2\sigma_\theta^2 + b^2\sigma_W^2} = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \left(\frac{\gamma_d}{M\tau_\epsilon + \gamma_d\varphi\alpha_1}\right)^2 \sigma_W^2} \tag{37}$$

Then a and b are given by

$$a = \frac{M\tau_\epsilon + \gamma_d\alpha_1\varphi + (N-M)\psi\tau_u}{M\tau_\epsilon + (N-M)\tau_u} \tag{38}$$

and

$$b = -\frac{\gamma_d}{M\tau_\epsilon + \gamma_d\varphi\alpha_1} a, \tag{39}$$

One has $a > 0$. One also has $a \leq 1$ with a strict inequality if $M < N$.

Lemma 2 gives the dealers optimal quoting strategy for any aggregate quantity X_k that dealer k trades with the investors and any demand x_i they face from an individual investor i. There is only a slight difference between the case in Section 4 and the setup considered here. Whereas the aggregate demand a dealer faced was equal to the demand by the single investor in Section 4, the quantities x_i and X_k are different here. Using Lemma 2 and taking account of this difference gives the expected price $P(x_i)$, investor i gets for his demand x_i conditional on at least one response to the RFQ:

$$P(x_i) = p_c(X_k) + (p_v(x_i) - p_c(X_k)\frac{Mq(1-q)^{M-1}}{1-(1-q)^M}. \tag{40}$$

We already determined in (30) which form each investor's demand x_i takes. We also know the quantity X_k given these individual demand schedules. We now determine the values of $p_v(x_i)$ and $p_c(X_k)$, so that we can use (40) to determine the expected price that each investor faces for his demand.

We define this dealer's value function that maps his inventory after period 1 X_k to expected utility as

$$V_{k,1}(\theta, X_k) := \mathbb{E}_k\left[D(q_k - X_k) - p_2 q_k - \frac{\gamma_d}{2}\sigma_\varepsilon^2(q_k - X_k)^2\right], \tag{41}$$

where we used the dealer's utility function (1).

Having obtained the dealer's utility $V_{k,1}(\theta, X_k)$ when holding X_k units of the asset, we define the dealer's break even price $p_c(X)$ such the payment compensates for the marginal cost of holding an additional marginal unit of the asset. The resulting expression is stated in the following Lemma.

Lemma 3. *Conditional on the equilibrium inventory of dealer $k \leq M$, the dealer's equilibrium break-even price for for the asset is given by*

$$p_c(X_k) := -\frac{\partial}{\partial X_k}V_{k,1}(\xi, X) = a\theta - b\frac{(1-a)\theta}{\gamma_d\sigma_\varepsilon^2} + b\frac{\varphi}{M}\alpha_1\theta, \tag{42}$$

where a, b, φ are defined as in Proposition 6.

Given that a dealer inferred the realization of θ from the investors' demand, a dealer can infer the private value δ_i of investor i from this investor's individual demand and (30). Given the investor's demand for x_i units of the asset, a dealer can infer the maximum price the investor is willing to pay for these x_i units by using (2):

$$p_v(x_i) := \theta + \delta_i - \frac{\gamma_I}{2}x\sigma_\varepsilon^2. \tag{43}$$

Using (31), (42) and (42), one can rewrite (40) as

$$P(x_i) = \beta_1\theta + \beta_2 x_i, \tag{44}$$

for some $\beta_1, \beta_2 \in \mathbb{R}$ stated in the appendix. We now determine the optimal amount x_i that an investor wants to demand given that the expected price he faces on the platform is given by (44). The maximization problem of investor i is given by

$$\max_{x\in\mathbb{R}} \left[(\theta + \delta_i)x_i - x^2\frac{\gamma_I}{2}\sigma_\varepsilon^2 - x_i\left(\beta_1\theta + \beta_2 x_i\right)\right]. \tag{45}$$

The expression in (45) considers the investor's expected payoff conditional on at least one response to the RFQ, since the investor's payoff is maximized when his payoff conditional on at least one response is maximized. The first-order condition to the problem in (45) gives the investor's optimal demand schedule

$$x = \theta\frac{1-\beta_1}{2\beta_2 + \gamma_I\sigma_\varepsilon^2} + \delta\frac{1}{2\beta_2 + \gamma_I\sigma_\varepsilon^2}. \tag{46}$$

One can immediately determine α_1 and α_2 from (30) by looking at (46):

$$\alpha_1 = \frac{1-\beta_1}{2\beta_2 + \gamma_I\sigma_\varepsilon^2}, \tag{47}$$

$$\alpha_2 = \frac{1}{2\beta_2 + \gamma_I\sigma_\varepsilon^2}. \tag{48}$$

We are now ready to establish the existence of an equilibrium.

Proposition 7. *The expected price on the platform $P(x_i)$ that an investor gets on the platform for his demand x_i is given by (44) for some $\beta_1, \beta_2 \in \mathbb{R}$. Let $M \geq 2$. There is an equilibrium on the platform described below if and only if $M < N$ and*

$$\kappa = \frac{Mq(1-q)^{M-1}}{1-(1-q)^M} < \frac{1}{2}.$$

The equilibrium is characterized as follows. The investor submits a demand x_i as determined in equations (30) with $\alpha_1, \alpha_2 \in \mathbb{R}$, with $0 < \alpha_1 < 1$ and $\alpha_1 \leq \alpha_2$. The dealers quote independently with probability q according to the distribution function F in (18) with $p_c(X_k)$ and $p_v(x_i)$ given by (42) and (43).

5.3 Market design

In this section we will use the results derived in Section 5.1 and Section 5.2 and study when investors prefer the centralized market and when they prefer the OTC market with an electronic trading platform. Proposition 7 states that there cannot be an equilibrium on the electronic trading platform if $\kappa \geq \frac{1}{2}$ or $N = M$. In this case, there is only an equilibrium in the centralized market. Therefore, we restrict our further discussion to the case in which $\kappa < \frac{1}{2}$ and $M < N$. The following claim follows from (29) and Proposition 7.

Proposition 8. *Let $0 < \kappa < \frac{1}{2}$ and $2 \leq M < N$. As $\sigma_\delta^2 \to 0$, investors prefer to trade in on the trading platform in the OTC market. As $\sigma_\delta^2 \to \infty$, investors prefer to trade in the centralized market.*

If $\sigma_\delta^2 \to 0$, equation (29) implies that investors' gains from trading in the centralized market go to zero. However, due to information asymmetries between dealers and investors, investors can still benefit from trading in the OTC market.

Suppose on the other hand, that $\sigma_\delta^2 > 0$ and the mass of investors μ becomes very large. Then holding everything else constant, the investors' demand will be very sensitive to variations in θ. In this case, an equilibrium is only possible if α_1, the coefficient in the investors' demand on θ is very small and investors will mainly trade based on their private value of holding the asset. If markups in the interdealer market are positive, investors will therefore prefer to trade in the centralized market instead. The following proposition proofs this statement formally.

Proposition 9. *Let $0 < \kappa < \frac{1}{2}$, $2 \leq M < N$ and $\sigma_\delta^2 > 0$. As $\mu \to \infty$, investors prefer to trade in the centralized market.*

The proof of Proposition 9 shows that $\alpha_1 \to 0$ as $\mu \to 0$. According to (47), this is equivalent to $\beta_1 \to 1$, holding everything else equal and noting that by (75), β_2 is unaffected by μ. Thus, (44) implies that the expected price an investor receives on the platform when $\mu \to \infty$ is approximately the sum of the common value θ of the dividend payment and a markup. In this case, the investors' gains from trade are derived mostly from their private values.

So far, we assumed that $\kappa > 0$, which lead to positive expected markups for the dealers when quoting on the trading platform. In the following we consider the case in which $q \to 1$, which leads to $\kappa \to 0$. If $\kappa \to 0$, these markups become negligible and dealers efficiently intermediate trades between their customers as if these customers were trading in a centralized market. Furthermore, the probability of not receiving a quote goes to zero as $q \to 1$. Thus, all the gains from trade that could be realized in the centralized market would also be realized in the OTC market. However, investors can still benefit from information asymmetries between them and the dealers in the OTC market. As $q \to 1$ investors therefore prefer to trade in the OTC market. This claim is formally proved in the next proposition.

Proposition 10. *Let $2 \leq M < N$. As $q \to 1$, investors prefer to trade on the trading platform.*

6 Discussion and concluding remarks

Electronic trading platforms play a central role in today's OTC markets. The implications of our model are consistent with recent empirical research that studies OTC markets with electronic trading platforms. One important feature of our model is information leakage which is studied in Hendershott and Madhavan (2015) and Hagströmer and Menkveld (2016). We also showed that information asymmetries between dealers and investors are a sufficient and necessary condition to generate the price impact patterns observed in Collin-Dufresne et al. (2017). Therefore, the first part of this paper can be viewed as a theoretical foundation of several empirical findings in recent research. The model can also be used to evaluate the impact of recent financial regulation on investors' trading profits. The Dodd-Frank Act mandates that the most liquid index CDS in the US are trades on electronic platforms. An RFQ furthermore should be sent to at least three dealers.[8] We show that increasing the number of contacted dealers may decrease investor's profits if the cost of information-leakage is high. On the other hand, the number of contacted dealers has to be sufficiently high in order for an equilibrium to exist, if competition among dealers on the platform (in terms of response rates) is low.

In the second part of the paper, we considered a hypothetical scenario in which there is either a centralized exchange or an OTC market and studied the respective implications on investor welfare. Some of our results are consistent with the recent theoretical literature in the area of market design. That investor welfare is generally higher on exchanges if the investors associate strong private values with holding the asset, can be viewed as an analogue to the result of Babus and Parlatore (2017) that there is only a centralized-market equilibrium if the investors' values of holding the asset are sufficiently independent. We also emphasize the role of information asymmetries that becomes important in OTC markets. In this respect our paper is related to Glode and Opp (2017). However, the specific trading protocol on electronic trading platforms features some aspects that are not present in other models of OTC markets. As the RFQ response rate q of dealers becomes high, our model shows that electronic trading platforms indeed become similar to exchanges,

[8] See Collin-Dufresne et al. (2017) for an overview of the regulatory changes in the US CDS market.

in the sense that dealers efficiently intermediate the demand from their customers. This result justifies the common opinion that electronic trading platforms represent a natural compromise between exchanges and OTC markets.[9]

To conclude, we want to make some general remarks on our model assumptions. As every theoretical model, also the one presented in this paper is build on some simplifying assumptions trading-off analytical tractability against appropriate representation of the real world. The fact that all investors are equally informed about the asset's payoff is certainly not completely realistic, but should capture the general information asymmetry between investors and dealers that in many markets seem to exist. To justify the way we model trading in the interdealer market, we want to refer to the event that made both the academic world and international regulatory authorities focus so much on OTC markets in the first place: the recent financial crisis. Arguably, demand for certain credit derivatives originated from informed hedge funds who wanted to bet against a credit bubble in the US credit market. Some investment banks may have learned about the value of certain securities from this informed demand and may have tried to use this knowledge against other less informed investment banks or other clients (which may be represented by noise traders in our model).

This example also suggests to interpret the welfare results derived from our model with a slight grain of salt. In this paper, we exclusively focused on investor welfare. While this approach may be viewed as standard in market design, it does not take into account financial stability considerations that may be important when determining the optimal level of transparency in the market. If losses to dealers or noise traders are large, the financial system may very well be affected in ways that cannot be captured in the model presented here. While the trade-off between the efficient allocation of assets and financial stability is a common theme in banking, examining the trade-off between investor welfare and financial stability in OTC markets may be a theme for future research.

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[9]See Stafford (2016).

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Appendix A

This appendix contains all proofs.

Proof of Proposition 1. By the conjecture (8), the market clearing price p_2 is jointly normally distributed with θ. By the definition of ξ in (4), one has

$$\mathrm{Cov}(D,p_2) = \mathrm{Cov}\left(\theta+\varepsilon, a\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\delta^2}(\theta+\delta)\right) = a\rho\sigma_\theta^2. \tag{49}$$

Furthermore, one has

$$\mathbb{V}(\xi) = \mathbb{V}\left(\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\delta^2}(\theta+\delta)\right) = \rho\sigma_\theta^2. \tag{50}$$

Now (49), (50) and the normal projection theorem give

$$\mathbb{E}(D|p_2) = \frac{a\rho\sigma_\theta^2}{a^2\rho\sigma_\theta^2+b^2\sigma_W^2}p_2 = \frac{\psi}{a}(a\xi+bW), \tag{51}$$

$$\mathbb{V}(D|p_2) = \frac{1}{\tau_u} = \sigma_\theta^2+\sigma_\varepsilon^2 - \frac{a^2\rho^2\sigma_\theta^4}{a^2\rho\sigma_\theta^2+b^2\sigma_W^2} = \sigma_\theta^2+\sigma_\varepsilon^2-\psi\rho\sigma_\theta^2. \tag{52}$$

Plugging (51) and (52) into (7), using the result with (6) in the market-clearing condition (9):

$$\frac{M\tau_\xi(\xi-p_2)}{\gamma_d} + \frac{(N-M)\tau_u(\psi\xi+\frac{\psi b}{a}W-p_2)}{\gamma_d} = W$$

solving for p_2 and matching coefficients with (8) yields

$$\begin{aligned} M\tau_\xi + (N-M)\psi\tau_u &= [M\tau_\xi+(N-M)\tau_u]a & (53)\\ (N-M)\tau_u\frac{\psi b}{a} - \gamma_d &= [M\tau_\xi+(N-M)\tau_u]b. & (54) \end{aligned}$$

Substituting $\psi = \frac{a^2\rho\sigma_\theta^2}{a^2\rho\sigma_\theta^2+b^2\sigma_W^2}$ into equations (53) and (54) and solve for a and b gives the expressions in (13) and (14).

It is immediately clear from (13) that $a>0$ if $M>0$, since both numerator and denominator are always positive in this case. Since $\psi>0$ it follows also that $a\leq 1$, with an equality only if $N=M$. □

Proof of Lemma 1. The dealer's optimal demand schedule follows directly from the first-order condition (3) by substituting $\overline{\omega}_k = q_k - x$. The demand schedules of other informed dealers do not change, since they do not make inferences from the price in the interdealer market. The dealers who have not been contacted by the investor perform inferences as described in Proposition 1. One can now conjecture $p_2 = a\xi + b(W-x)$. Thus using dealer k's demand schedule and demand schedules (6) and (7) for the other dealers in the market clearing condition and following the exact procedure described in the proof of Proposition 1 determines a and b as in 1.

$\square$

Proof of Lemma 2. Let F_x denote the dealers' optimal quoting strategy. This means dealers quote a price p_0 that is a random variable with the distribution function F_x.

Let $x > 0$. Then $x(p_v - p_c) > 0$ implies $p_v > p_c$. If the dealers' optimal strategy were such that there is a $p^* \in (p_v, p_c)$ such that dealers quote a price $p \leq p^*$ with a probability of 1, then a dealer could profitably deviate from this strategy by quoting p_v. This would contradict optimality. On the other hand, quoting a prices greater than p_v with any positive probability cannot be optimal, since the investor would not buy the asset at that price. Thus, one obtains $\sup \operatorname{supp}(F_x) = p_v$.

Now we show that F_x must be continuous, i.e. there cannot be any atoms in the distribution of p_0. Clearly, quoting a price less than or equal to p_c with any positive probability cannot be optimal, since a dealer would not make any positive profit by doing so, whereas he would make a positive expected profit by quoting p_v. Now, suppose there is a price p' with $p_v \geq p' > p_c$ that is quoted with probability $\rho > 0$ by all dealers.

Then a single dealer could again profitably deviate from this strategy which contradicts optimality. The profitable deviation is constructed as follows. Since the number of prices charged with positive probability must be countable, one can find for each $\delta > 0$ an ε_δ, such that $\delta \geq \varepsilon_\delta > 0$ and the price $p' - \varepsilon_\delta$ is charged with probability zero by all dealers. The deviating dealer can now charge price $p' - \varepsilon_\delta$ with probability ρ and charge price p' with probability zero. Using the fact that $\lim_{\delta \to 0} F_x(p' - \varepsilon_\delta) = F_x(p') - \rho$, one can express the difference Δ in profits between the original strategy and the proposed deviation as follows. A dealer quoting p' only makes a positive profit if no other dealer on the platform quotes a lower price. If no other dealer quotes a lower price, there might be $j = 0, 1, ..., M - 1$ dealers who quote p' as well. In the latter case, each of the $j + 1$ is equally likely to be chosen by the investor for trading the asset. The calculation below considers the cases in which j dealers quote price p on the platform separately.

$$
\begin{aligned}
\Delta \;=\;& (1 - qF_x(p' - \varepsilon_\delta) - q\rho)^{M-1}(p' - \varepsilon_\delta - p_c)x \\
& -(1 - qF_x(p'))^{M-1}(p - c)x \\
& + \sum_{j=1}^{M-1} \binom{M-1}{j} (1 - qF_x(p' - \varepsilon_\delta) - q\rho)^{M-1-j}(q\rho)^j (p' - \varepsilon_\delta - p_c)x \\
& - \sum_{j=1}^{M-1} \binom{M-1}{j} (1 - qF_x(p'))^{M-1-j}(q\rho)^j (p' - p_c)\frac{x}{j+1}.
\end{aligned}
$$

The first two lines in the above expression compare expected profits from quoting $p' - \varepsilon_\delta$ and expected profits from quoting p' in the event that all other dealers quote a price above p'. Since $\lim_{\delta \to 0} F_x(p' - \varepsilon_\delta) = F_x(p') - \rho$, the difference in these two lines goes to zero as δ goes to zero. The last two lines compare the respective profits in the cases in which $j > 0$ other dealers quote p'. Since $M \geq 2$, the deviating dealer can get a jump in expected trading volume in this case, since he can avoid ties with other dealers. Therefore one obtains

$$\Delta \to \sum_{j=1}^{M-1} \binom{M-1}{j} (p' - p_c) \frac{jx}{j+1} (1 - qF_x(p'))^{M-1-j} (q\rho)^j > 0 \quad \text{as } \delta \to 0.$$

Thus, the proposed deviation is profitable for a small δ. In equilibrium, F_x cannot have any atoms.

If $x < 0$, one verifies analogously to the case of $x > 0$, that $\inf \operatorname{supp}(F_x) = p_v$ must hold for any optimal strategy. That the distribution cannot have any atoms follows analogously as well.

The dealers are only willing to randomize over prices if they earn the same profit in expectation with each price in the support of F_x. This profit must be equal to the profit in which the dealer quotes $p_v(x)$. This gives the indifference condition expressed in (17).

Using the binomial formula $(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$, (17) simplifies to

$$(p - p_c)x(1 - qF_x(p))^{M-1} = (1-q)^{M-1}(p_v(x) - p_c)x,$$

which can be solved for F_x. The solution is given by (18).

Using (18) and solving $F_x(\overline{p}_x) = 0$ for $\overline{p}_x$ gives

$$\overline{p}_x = p_c + (p_v - p_c)(1-q)^{M-1}.$$

Since $x(p_v - p_c) > 0$, one obtains $\overline{p}_x > p_c$ for $x > 0$ and $\overline{p}_x < p_c$ for $x < 0$.

The event that at least one dealer is on the platform happens with probability $1 - (1-q)^M$, since all dealers respond independently with probability q. The unconditional probability that no dealer quotes above $p \in \operatorname{supp}(F_x)$ can be expressed by $(1 - qF_s(p))^M$. Therefore, the conditional distribution G_x has to satisfy $G(p)(1 - (1-q)^M) = 1 - (1 - qF_x(p))^M$. Performing a change of variables $p = p_c + \frac{(p_v - p_c)(1-q)^{M-1}}{(1-(1-(1-q)^M)u)^{(M-1)/M}}$, one can calculate

$$\int_{\mathrm{supp}(F_x)} p \mathrm{d}G_x(p) = \int_0^1 \left[p_c + \frac{(p_v - p_c)(1-q)^{M-1}}{(1-(1-(1-q)^M)u)^{(M-1)/M}} \right] \mathrm{d}u = p_c + \frac{(p_v - p_c)(1-q)^{M-1}}{1-(1-q)^M} Mq.$$

The claim that $0 \le \kappa < 1$, can be shown as follows. That $0 \le \kappa$ is immediately clear from the definition (20). The other inequality can be seen as follows.

- κ as a function of q is strictly decreasing in q for all $q \in (0,1]$, since

$$\frac{\partial \kappa}{\partial q} = \frac{-M(1-q)^{M-2}\left[(1-q)^M + Mq - 1\right]}{(1-(1-q)^M)^2} < 0$$

  for $q \in (0,1]$.

- By L'Hospital's rule, one has

$$\lim_{q \to 0} \kappa = \frac{\lim_{q\to 0}(M(1-q)^{M-1} - M(M-1)q(1-q)^{M-2})}{\lim_{q \to 0} M(1-q)^M} = \frac{M}{M} = 1.$$

The last two bullet points imply $\kappa < 1$ for all $q \in (0,1]$.

This proves all statements in the lemma.

□

Proof of Proposition 2. Claim 1: The expected price on the platform is linear in ξ and x. Define

$$\beta_1 := \left[1 - \frac{Mq(1-q)^{M-1}}{1-(1-q)^M}\right]\left[a - \frac{(1-a)b}{\gamma_d \sigma_\xi^2}\right] + \frac{Mq(1-q)^{M-1}}{1-(1-q)^M}\left(1 + \frac{\sigma_\delta^2}{\sigma_\theta^2}\right) \tag{55}$$

and

$$\beta_2 := \left[1 - \frac{Mq(1-q)^{M-1}}{1-(1-q)^M}\right]\left[-b - \frac{b^2}{2\gamma_d \sigma_\xi^2}\right] + \frac{Mq(1-q)^{M-1}}{1-(1-q)^M}\left(-\frac{\gamma_I \sigma_\epsilon^2}{2}\right). \tag{56}$$

Using the definitions of $p_c(x)$ and $p_v(x)$, it follows by direct computation that $P(x)$ as defined in Lemma 2 is given by (22).

Claim 2: Let $\frac{M(1-q)^{M-1}}{1-(1-q)^M} < \frac{1}{2}$. An equilibrium exists if and only if $a < \overline{a}$ for some $\overline{a} \in \mathbb{R}$.

To verify the existence of the described equilibrium, there are several things to check. The strategy (25) is well-defined if

$$2\beta_2 + \gamma_I \sigma_\varepsilon^2 \neq 0. \tag{57}$$

Furthermore, the investor's second-order condition from the maximization problem (23) requires

$$-\gamma_I\sigma_\varepsilon^2 - \left(2\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\delta^2}\frac{\beta_1}{\alpha} + 2\beta_2\right) < 0. \tag{58}$$

In order to apply Lemma 2 we also need to verify that

$$x(p_v(x) - p_c(x)) > 0 \tag{59}$$

for holds for any $x \neq 0$ demanded by the investor in the proposed equilibrium.

If (57), (58) and (59) hold, one can use Lemma 2 to see that there exist optimal strategies for the dealers that yield $P(x)$ as the expected price on the platform conditional on at least one response. As demonstrated in the text, the stated strategy for the investor (25) indeed solves the first order condition (24), given that dealers rationally infer $\theta+\delta$ from the investor's demand. Thus, both dealers and the investor behave optimally given the strategies of the others and an equilibrium is established.

The strategy of the proof of this claim is as follows. We will assume that the average price is given by the expression in Lemma 2. We then show that the investor's strategy is well-defined so that the first-order and second-order conditions of the maximization problem (23) are satisfied. We that verify that in this case

In order to prove our claim, we first note that (59) is satisfied in this case, so that dealers indeed find it optimal to quote as described in Lemma 2.

For the following proof, it is worth noting that Lemma 2 states that

$$0 \leq \kappa < 1 \tag{60}$$

for all $q \in (0,1]$ and $M \geq 2$.

„$\Rightarrow$": *Proof that equilibrium exists under the stated conditions.*

Let now $\kappa < \frac{1}{2}$.

We rewrite (55) and (56) as using $b = -\frac{\gamma_d\sigma_\xi^2}{M}a$:

$$\beta_1 = (1-\kappa)\left(a + \frac{(1-a)a}{M}\right) + \kappa\frac{\sigma_\theta^2+\sigma_\delta^2}{\sigma_\theta^2} > 0, \tag{61}$$

$$\beta_2 = (1-\kappa)\left(\frac{\gamma_d\sigma_\xi^2}{M}a - \frac{a^2\gamma_d\sigma_\xi^2}{2M^2}\right) - \kappa\left(\frac{\gamma_I\sigma_\epsilon^2}{2}\right) > -\frac{\gamma_I\sigma_\epsilon^2}{2}, \tag{62}$$

where the inequalities follow from (60) and $1 \leq a > 0$.

Define

$$\Psi := \frac{\sigma_\theta^2 + \sigma_\delta^2}{\sigma_\theta^2} \frac{\frac{1}{2} - \kappa}{1 - \kappa} > 0$$

and define $\overline{a}$ as the smaller solution to the quadratic equation

$$a + \frac{(1-a)a}{M} = \Psi,$$

if there is a real solution to the equation. Set $\overline{a} = 1$ otherwise. Then it follows from (61) that

$$\beta_1 < \frac{1}{2} \frac{\sigma_\theta^2 + \sigma_\delta^2}{\sigma_\theta^2},$$

if $a < \overline{a}$.

Thus, all that remains to show is that there is an equilibrium if the last inequality involving β_1 holds. As described above it is sufficient to check that (57), (58) and (59) hold. It is immediatly clear from (62) that (57) always holds for any set of parameters.

Regarding (58), note that using (25), (62) and the assumption on β_1 imply $\alpha > 0$. Using (61), one therefore obtains

$$-\gamma_I \sigma_\varepsilon^2 - \left(2 \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\delta^2} \frac{\beta_1}{\alpha} + 2\beta_2 \right) < 2 \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\delta^2} \frac{\beta_1}{\alpha} + 2\beta_2 < 0.$$

Thus, the investor's second-order condition holds if $\beta_1 < \frac{1}{2} \frac{\sigma_\theta^2 + \sigma_\delta^2}{\sigma_\theta^2}$.

Lastly, we check that (59) holds which justifies the use of Lemma 2 for determining the expected price on the platform. Note that by optimality of the investor's choice of x and $\alpha > 0$, it follows that the investor makes a positive profit if $x \neq 0$ This can be seen, since the investor could always make a zero profit by not trading, but instead chooses a different x. By the convexity of the maximization problem (23), the optimal quantity is uniquely determined and therefore must give a positive profit. This implies

$$x(p_v(x) - p_c(x)) \geq x(p_v(x) - p_c(x))(1 - \kappa) = x(p_v(x) - P(x)) > 0.$$

Therefore (59) indeed holds and Lemma 2 can be used to determine the dealer's quoting strategies on the platform.

Since (57), (58) and (59) indeed hold, the equilibrium exists.

,,$\Leftarrow$": *Proof that equilibrium does not exist if* $a \geq \overline{a}$.

The definition of $\overline{a}$ and β_1 imply that $\beta_1 \geq \frac{1}{2}\frac{\sigma_\theta^2+\sigma_\delta^2}{\sigma_\theta^2}$ if $a \geq \overline{a}$. If the last inequality is an equlity, it follows that $\alpha = 0$. This means, the investor does not trade and the quoting strategies of the dealers are not defined. Let the inequality be strict. Note that by (61), $\kappa \in [0,1]$ and $a \in [0,1]$, one has $\beta_1 \leq a + (1-a) = 1$. This in turn implies

$$\frac{1}{1 - 2\beta_1 \frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\delta^2}} > -1.$$

One now obtains

$$\begin{aligned}
& -\gamma_I \sigma_\varepsilon^2 - \left(2\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\delta^2}\frac{\beta_1}{\alpha} + 2\beta_2 \right) \\
= \quad & -\gamma_I \sigma_\varepsilon^2 + 2\beta_2 - 2\frac{2\beta_2 + \gamma_I \sigma_\varepsilon^2}{1-2\beta_1}\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\delta^2}\beta_1 \\
> \quad & -\gamma_I \sigma_\varepsilon^2 + 2\beta_2 + 2(2\beta_2 + \gamma_I \sigma_\varepsilon^2)\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\delta^2}\beta_1 \\
> \quad & -\gamma_I \sigma_\varepsilon^2 + 2\beta_2 + (2\beta_2 + \gamma_I \sigma_\varepsilon^2) = 0.
\end{aligned}$$

Therefore, the second-order condition for the investor's maximization problem (23) is not satisfied. Thus, the investor's strategy is clearly not optimal and the described equilibrium does not exist.

Claim 3: The equilibrium does not exist if $\kappa \geq \frac{1}{2}$

In this case, $a \geq 0$ and (61) imply $\beta_1 \geq \frac{1}{2}\frac{\sigma_\theta^2+\sigma_\delta^2}{\sigma_\theta^2}$. The prove that the equilibrium doe not exist is identical to the proof in Claim 2.

Claim 4: $a \to 0$ as $N \to \infty$ and $\sigma_W \to \infty$.

By equation (13), one can see that

$$\begin{aligned}
\lim_{\sigma_W \to \infty} \lim_{N \to \infty} a \quad &= \quad \lim_{\sigma_W \to \infty} \lim_{N \to \infty} \frac{\frac{M\tau_\xi}{N-M} + \psi\tau_u}{\frac{M\tau_\xi}{N-M} + \tau_u} \\
&= \quad \lim_{\sigma_W \to \infty} \psi \\
&= \quad 0.
\end{aligned}$$

Claim 5: An equilibrium exist if $\kappa < \frac{1}{2}$ and $\sigma_\delta \to \infty$. As $\sigma_\delta \to \infty$, one has $\Psi \to \infty$. This means $a + \frac{(1-a)a}{2M} < \Psi$ for all $a \in \mathbb{R}$ and in particular for all $a \in (0,1]$ As shown in the proof of Claim 2, this implies

$\beta_1 < \frac{1}{2}\frac{\sigma_\theta^2+\sigma_\delta^2}{\sigma_\theta^2}$ and the equilibrium exists. $\square$

Proof of Proposition 3. We divided this proof into several steps. The first step is an auxiliary result that will be used later in the proof.

Step 1: $\frac{\partial a}{\partial M} \geq \frac{1}{M}a(1-a)$.

Since $\sigma_\delta = 0$, we get $\sigma_\xi = \sigma_\varepsilon$ and $\rho = \sigma_\theta^2$. We now rewrite a as defined in (13) as

$$a = 1 - \frac{\gamma_d^2\sigma_\varepsilon^4\sigma_W^2(N-M)}{M^2N\sigma_\theta^2 + \gamma_d^2M\sigma_\varepsilon^2\sigma_\theta^2\sigma_W^2 + \gamma_d^2N\sigma_\varepsilon^4\sigma_W^2} \tag{63}$$

Using the expression in (63), one obtains by direct calculation and simplifying terms that

$$\begin{aligned}\frac{\partial a}{\partial M} - \frac{1}{M}a(1-a) &= \frac{\gamma_d^2M\sigma_\varepsilon^4\sigma_W^2\left(\gamma_d^2\sigma_\varepsilon^2\sigma_W^2\left(\sigma_\varepsilon^2+\sigma_\theta^2\right)+N^2\sigma_\theta^2\right)}{\left(M^2N\sigma_\theta^2 + \gamma_d^2M\sigma_\varepsilon^2\sigma_\theta^2\sigma_W^2 + \gamma_d^2N\sigma_\varepsilon^4\sigma_W^2\right)^2} \\ &\geq 0.\end{aligned}$$

This proves the first step

Step 2: The equilibrium exists if and only if a is below a certain threshold.

This result follows directly from Proposition 2 by noting that κ as defined (60) is equal to zero if $q = 1$. Furthermore, since $q = 1$, one has $\Psi = \frac{1}{2}$, where Ψ is defined in the proof of Proposition 1. Defining $\overline{a}$ as in the proof of Proposition 1, one gets that $\overline{a}$ is the smaller real solution to

$$a + \frac{(1-a)a}{M} = \frac{1}{2},$$

which is always greater than zero and less than $\frac{1}{2}$.

Step 3: There is an equilibrium for all $M' < M$.

In the proof of Proposition 2 it was established that the described equilibrium exists if and only if $\beta_1 < \frac{1}{2}\frac{\sigma_\theta^2+\sigma_\delta^2}{\sigma_\theta^2}$. If an equilibrium exists when M dealers get contacted, it consequently must be the case that $\beta_1 < \frac{1}{2}$. If furthermore, $\beta_1 < \frac{1}{2}$ for all $M' < M$, the result follows. The last claim will be shown next. If $q = 1$, one has

$$\begin{aligned}
\frac{\partial \beta_1}{\partial M} &= \frac{(1-2a)\frac{\partial a}{\partial M}}{M} + \frac{\partial a}{\partial M} - \frac{(1-a)a}{M^2} \\
&= \frac{M+1-2a}{M}\frac{\partial a}{\partial M} - \frac{(1-a)a}{M^2} \\
&\geq \frac{M+1-2a}{M}\frac{1}{M}a(1-a) - \frac{(1-a)a}{M^2} \\
&> (3(1-a)+a^2)\frac{(1-a)a}{M^2} \\
&> 0.
\end{aligned}$$

The third line follows from Step 1. Therefore, one has $0 \leq \beta_1 < \frac{1}{2}\frac{\sigma_\theta^2+\sigma_\delta^2}{\sigma_\theta^2}$ for all $M' < M$. Note that event though M represents an integer in the model, β_1 can be interpreted as a function in $C^1(\mathbb{R})$.

Step 4: The investor's payoff is highest if $M' = 2$ compared to all other $M'' \leq M$.

Since we know that a nonzero-trade equilibrium exists for all $M' < M$, we can calculate the investor's equilibrium payoff as defined by (27).

Using the expressions for β_1 and β_2 stated in Step 2 and using the definition of α from (25), one gets

$$\pi_I = \frac{\sigma_\theta^2 M\left(2a^2 - 2a(M+1) + M\right)}{2\sigma_\varepsilon^2\left(2a\gamma_d M + \gamma_I M^2 - a^2\gamma_d\right)}.$$

In equilibrium, one has $\pi_I > 0$. Since $a < \overline{a}$, the numerator in the above expression for π_I is positive. Therefore, the denominator must be positive as well. Interpreting π_I as a function in $C^1(\mathbb{R})$, one can show that π_I is strictly decreasing in M by showing that $\ln(\pi_I)$ is strictly decreasing in M. It then follows that the lowest possible M', i.e. $M' = 2$ is profit maximizing among all possible values less than M.

$$\begin{aligned}
\frac{\partial}{\partial M}\ln \pi_I &= \frac{M\left(4a\frac{\partial a}{\partial M} - 2(M+1)\frac{\partial a}{\partial M} - 2a + 1\right) + \left(2a^2 - 2(M+1)a + M\right)}{M\left(2a^2 - 2(M+1)a + M\right)} \\
&\quad - \frac{2\gamma_d M\frac{\partial a}{\partial M} - 2\gamma_d a\frac{\partial a}{\partial M} + 2\gamma_d a + 2\gamma_I M}{2\gamma_d M a - \gamma_d a^2 + \gamma_I M^2}
\end{aligned}$$

Collecting terms gives

$$
\begin{aligned}
\frac{\partial}{\partial M}\ln \pi_I \quad = \quad & \frac{1}{M}+\frac{1}{2a^2-2(M+1)a+M}-\frac{2a}{2a^2-2(M+1)a+M} \\
& -\frac{2\gamma_I M}{2\gamma_d Ma-\gamma_d a^2+\gamma_I M^2}-\frac{2\gamma_d a}{2\gamma_d Ma-\gamma_d a^2+\gamma_I M^2}+ \\
& +\Big(\frac{4a}{2a^2-2(M+1)a+M}-\frac{2(M+1)}{2a^2-2(M+1)a+M} \\
& \frac{2\gamma_d a}{2\gamma_d Ma-\gamma_d a^2+\gamma_I M^2}-\frac{2\gamma_d M}{2\gamma_d Ma-\gamma_d a^2+\gamma_I M^2}\Big)\frac{\partial a}{\partial M}
\end{aligned}
$$

Since $a<\overline{a}$ one has $a^2-a(M+1)+M/2>0$. One can now see that the term in front of $\frac{\partial a}{\partial M}$ is negative. Therefore, one can obtain an upper bound for the $\frac{\partial a}{\partial M}\ln\pi_I$ by plugging in the result from Step 1 for $\frac{\partial a}{\partial M}$. Simplifying gives

$$
\frac{\partial}{\partial M}\leq\frac{-2a\left(-2a^2(\gamma_d-\gamma_I M)+Ma(\gamma_d-\gamma_I(M+2))+\gamma_d a^3+\gamma_I M^2\right)}{\left(2a^2-2(M+1)a+M\right)\left(2\gamma_d Ma-\gamma_d a^2+\gamma_I M^2\right)}.
$$

The denominator is positive due to $a<\overline{a}$. Simplifying the numerator gives

$$
-2\gamma_I a\left(2Ma^2-(M+2)Ma+M^2\right)-2\gamma_d a\left(a^3-2a^2+Ma\right)<0.
$$

Therefore one has $\frac{\partial}{\partial M}\ln\pi_I<0$ and the claim follows.

Step 5: $M=2$ is profit-maximizing among all possible values.

Assume there would be an $M'>2$ such that $M=M'$ gives a higher profit than $M=2$ in equilibrium. By Step 3, it must be the case that $a<\overline{a}$ for $M=M'$. Now it follows by Step 3 that having $M=2$ gives a higher profit for the investor than having $M=M'$. Thus, contacting only 2 dealers is indeed profit-maximizing.

□

Proof of Proposition 4. It is shown in proposition 3 that equilibrium exists when $a<\bar{a}<\frac{1}{2}$. We now replace a as defined in equation (13): $\frac{M^2N\sigma_\theta^2+\gamma_d^2\sigma_\epsilon^2\sigma_\theta^2\sigma_W^2M+\gamma_d^2\sigma_\epsilon^4\sigma_W^2M}{M^2N\sigma_\theta^2+\gamma_d^2\sigma_\epsilon^2\sigma_\theta^2\sigma_W^2M+\gamma_d^2\sigma_\epsilon^4\sigma_W^2N}<\frac{1}{2}$. Equivalently,

$$
M<\frac{1}{2}\frac{M^2N\sigma_\theta^2+\gamma_d^2\sigma_\epsilon^2\sigma_\theta^2\sigma_W^2M+\gamma_d^2\sigma_\epsilon^4\sigma_W^2N}{MN\sigma_\theta^2+\gamma_d^2\sigma_\epsilon^2\sigma_\theta^2\sigma_W^2+\gamma_d^2\sigma_\epsilon^4\sigma_W^2}<\frac{1}{2}N.
$$

Therefore, if $M>\frac{1}{2}N$, one has $a\geq\bar{a}$, which implies that the equilibrium does not exist from proposition 3.

In the following, we show that the equilibrium existence condition $a<\bar{a}$ is equivalent to $M\in(\bar{M}_1,\bar{M}_2)$, where $\bar{M}_1$ and $\bar{M}_2$ are roots to the equation $a(M)=\bar{a}(M)$.

First, $a(M)$ is an increasing function of M and $\lim_{M\to N} a(M) = 1$. In terms of $\bar{a}(M,q)$, one can calculate the two derivatives

$$\frac{\partial \bar{a}}{\partial M} = 1 - \frac{2M + \frac{\kappa(1-\kappa) + \frac{M\partial\kappa}{\partial M}}{(1-\kappa)^2}}{\sqrt{4M^2 + 1 + 4M(1-2\Psi)}},$$

where $\frac{\partial\kappa}{\partial M} = \kappa\left[\frac{1}{M} + \frac{\ln(1-q)}{1-(1-q)^M}\right] < 0$, so $\frac{\partial\bar{a}}{\partial M} > 0$.

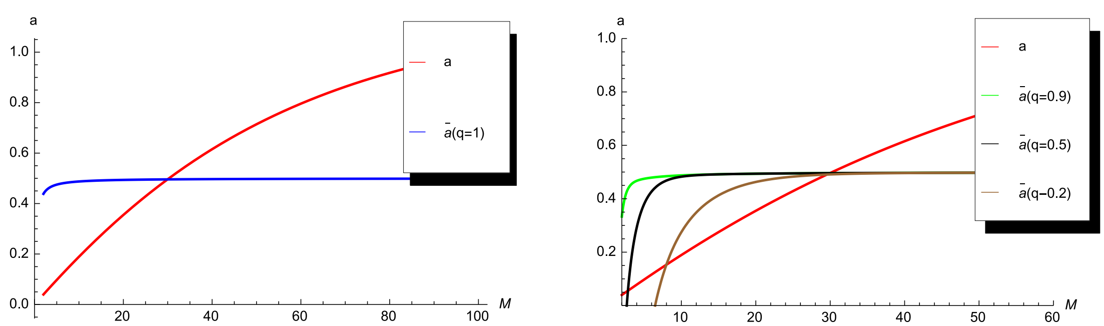
And

$$\frac{\partial \bar{a}}{\partial q} = -\frac{\frac{M}{(1-\kappa)^2}\frac{\partial\kappa}{\partial q}}{\sqrt{4M^2 + 1 + 4M[\frac{\kappa}{1-\kappa} - \frac{\sigma_\delta^2}{\sigma_\theta^2}]}},$$

where $\frac{\partial\kappa}{\partial q} = \frac{M(1-q)^{M-2}(1-Mq-(1-q)^M)}{(1-(1-q)^M)^2} < 0$, so $\frac{\partial\bar{a}}{\partial q} > 0$. Thus, $\bar{a}$ is an increasing function of both M and q.

Moreover, comparing the the value of $a(M)$ and $\bar{a}(M,q)$ at the limits, one gets

$$\begin{aligned}
\lim_{M\to N} a(M) = 1 \quad &> \quad \frac{1}{2} > \lim_{M\to N} \bar{a}(M,q), \\
\lim_{M\to 2} a(M) \quad &< \quad \lim_{M\to 2, q\to 1} \bar{a}(M,q), \\
\lim_{M\to 2} a(M) > 0 \quad &> \quad \lim_{M\to 2, q\to 0} \bar{a}(M,q).
\end{aligned}$$



(a) $a(M)$ and $\bar{a}(M, q = 1)$ (b) $a(M)$ and $\bar{a}(M, q = 0.9, 0.5, 0.2)$

Figure 5: $a(M)$ and $\bar{a}(M,q)$

So there are maximum two roots to the equation $a(M) = \bar{a}(M,q)$ for $M \in [2, N]$. As has been shown and demonstrated by figure (5) that there exists at least one root when $q = 1$, since $\bar{a}(M,q)$ decreases when q decreases, the larger root $\bar{M}_2$ also decreases. Note that $\frac{\partial^2\bar{a}(M,q)}{\partial q^2} < 0$, implies that the concavity of $\bar{a}(M,q)$ becomes larger , so the smaller root $\bar{M}_1$ increases when q decreases. More specifically,

(1) When $q = 1$, $\bar{M}_1 < 0$ and $\bar{M}_2 > 2$.

(2) When $q \in (\frac{1+\sqrt{1-\frac{2\left(a(2)^2-5a(2)+2\right)}{a(2)^2-5a(2)+4}}}{2}, 1]$, $\bar{M}_1 < 2$ and $\bar{M}_2 > 2$.

(3) When $q = \frac{1+\sqrt{1-\frac{2\left(a(2)^2-5a(2)+2\right)}{a(2)^2-5a(2)+4}}}{2}$, $\bar{M}_1 = 2$ and $\bar{M}_2 > 2$.

(4) When $q \in (\underline{q}, \frac{1+\sqrt{1-\frac{2\left(a(2)^2-5a(2)+2\right)}{a(2)^2-5a(2)+4}}}{2})$, $\bar{M}_1 > 2$ and $\bar{M}_2 > \bar{M}_1 > 2$.

(5) When $q = \underline{q}$, $\bar{M}_1 = N$, where $\underline{q}$ is the solution to the equation $\frac{Nq(1-q)^{N-1}}{1-(1-q)^N} = \frac{1}{2}$.

(6) When $q \in [0, \underline{q})$, there is no solution to $a(M) = \bar{a}(M)$ and $a(M) > \bar{a}(M)$.

The existence of equilibrium is summarized in figure (6).

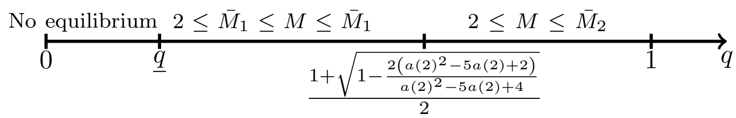


Figure 6: The existence of equilibrium

The above results show that when $q < \frac{1+\sqrt{1-\frac{2\left(a(2)^2-5a(2)+2\right)}{a(2)^2-5a(2)+4}}}{2}$, the equilibrium exists when $\bar{M}_1 < M < \bar{M}_2$. But $\bar{M}_1 \geq 2$, so there is no equilibrium when $M = 2$. Moreover, the minimum value of M such that the equilibrium exist is 3. Overall, we can conclude that when $q < \frac{1+\sqrt{1-\frac{2\left(a(2)^2-5a(2)+2\right)}{a(2)^2-5a(2)+4}}}{2}$, there exists an equilibrium with $M \geq 3$.

□

Proof of Proposition 5. Using the optimal demand schedule (6) for $M-1$ informed dealers and demand schedule (7) for the uninformed dealers, where the the conditional beliefs of uninformed dealers are formed as described in the proof of Proposition 1, the market clearing condition

$$q_k + \sum_{l \leq M, l \neq k} q_l + \sum_{lM+1}^{N} = W$$

can be rearranged as

$$p_2 = b\left(W - \frac{(M-1)\xi}{\gamma_d \sigma_\xi^2} - q_k\right).$$

therefore, one has $\frac{\partial}{\partial q_k} p_2 = -b$, whenever dealer k is informed.

If dealers do not update their belief about ξ when the investor changes his demanded quantity x, taking the derivative of (15) w.r.t. x gives the following permanent price impact

$$\frac{\partial}{\partial x}p_c = -\frac{b^2}{2\gamma_d\sigma_\xi^2} - b < -b.$$

Using (15) and taking into account that the dealers form their expectation ξ about the dividend payment based on (4) and (21), one obtains

$$\begin{aligned}\frac{\partial}{\partial x}p_c &= \frac{a\rho}{\alpha} - b - \frac{2(1-a)b\frac{\rho}{\alpha} + b^2}{2\gamma_d\sigma_\xi^2} \\ &= \frac{\rho\tilde{\beta}_1}{\alpha} + \tilde{\beta}_2 \\ &\geq \frac{\rho\tilde{\beta}_1}{\tilde{\alpha}} + \tilde{\beta}_2,\end{aligned}$$

where $\tilde{\alpha}$, $\tilde{\beta}_1$ and $\tilde{\beta}_2$ denote the value of α, β_1 and β_2 when $q = 1$, respectively. The inequality holds since $\alpha = \tilde{\alpha} - \frac{\kappa}{1-\kappa}\frac{1}{2\tilde{\beta}_2+\gamma_I\sigma_\epsilon^2}$. Replacing $\tilde{\alpha}$ by (25), one has

$$\begin{aligned}\frac{\partial}{\partial x}p_c &\geq \frac{\rho\tilde{\beta}_1}{1-2\rho\tilde{\beta}_1}(2\tilde{\beta}_2 + \gamma_I\sigma_\epsilon^2) + \tilde{\beta}_2 \\ &> \frac{\rho\tilde{\beta}_1}{1-2\rho\tilde{\beta}_1}2\tilde{\beta}_2 + \tilde{\beta}_2 \\ &= \frac{\tilde{\beta}_2}{1-2\rho\tilde{\beta}_1} \\ &= \frac{-b - \frac{b^2}{2\gamma_d\sigma_\xi^2}}{1-2\rho\left[a - \frac{(1-a)b}{\gamma_d\sigma_\xi^2}\right]} \\ &= \frac{-b\left(1-\frac{a}{2M}\right)}{1-2\rho a\left(1-\frac{1-a}{M}\right)} \\ &> -b.\end{aligned}$$

The last inequality holds since $1 - \frac{a}{2M} > 1 - 2\rho a\left(1 - \frac{1-a}{M}\right)$, that is equivalent to $a > 0 > 1 - M + \frac{1}{4\rho}$. $\square$

Proof of Proposition 6. By the conjecture (33), the market clearing price p_2 is jointly normally distributed with θ. One has

$$\mathrm{Cov}(D, p_2) = \mathrm{Cov}\left(\theta + \varepsilon, a\theta\right) = a\sigma_\theta^2. \tag{64}$$

Now (64), (37) and the normal projection theorem give

$$\mathbb{E}(D|p_2) = \frac{a\sigma_\theta^2}{a^2\sigma_\theta^2 + b^2\sigma_W^2} p_2 = \frac{\psi}{a}(a\theta + bW), \tag{65}$$

$$\mathbb{V}(D|p_2) = \frac{1}{\tau_u} = \sigma_\theta^2 + \sigma_\varepsilon^2 - \frac{a^2\sigma_\theta^4}{a^2\sigma_\theta^2 + b^2\sigma_W^2} = \sigma_\theta^2 + \sigma_\varepsilon^2 - \psi\sigma_\theta^2. \tag{66}$$

Plugging (65) and (66) into (34), using the result with (32) in the market-clearing condition (9):

$$\frac{M\tau_\epsilon(\theta - p_2)}{\gamma_d} + MX_k + \frac{(N-M)\tau_u(\psi\theta + \frac{\psi b}{a}W - p_2)}{\gamma_d} = W$$

solving for p_2 and matching coefficients with (33) yields

$$\begin{aligned} M\tau_\epsilon + (N-M)\psi\tau_u + \gamma_d\varphi\alpha_1 &= [M\tau_\xi + (N-M)\tau_u]a & (67)\\ (N-M)\tau_u\frac{\psi b}{a} - \gamma_d &= [M\tau_\xi + (N-M)\tau_u]b. & (68)\end{aligned}$$

Solving for a and b gives the expressions in (38) and (39).

It is immediately clear from (38) that $a > 0$, both numerator and denominator are always positive. Since $\psi > 0$ it follows also that $a \leq 1$ with a strict inequality only if $N = M$. □

Proof of Lemma 3. To show the second equality in (42), we note that the equilibrium price in the interdealer market depends on the aggregate inventory by market clearing. If market clearing holds, then

$$\sum_{l=1}^{M} q_l + \sum_{k=M+1}^{N} q_l + W,$$

where the demand schedules are defined as in (32) and (34). Using these definitions, the normal projection theorem to determine the conditional expectations gives and solving the previous equation for p_2 gives

$$p_2 = \frac{W - \sum_{l=1}^{M} X_l - \frac{\theta M}{\gamma_d\sigma_\varepsilon^2}}{\frac{a\sigma_\theta^2(N-M)}{\gamma_d\left(a^2\sigma_\theta^2+b^2\sigma_W^2\right)\left(-\frac{a^2\sigma_\theta^4}{a^2\sigma_\theta^2+b^2\sigma_W^2}+\sigma_\varepsilon^2+\sigma_\theta^2\right)} - \frac{N-M}{\gamma_d\left(-\frac{a^2\sigma_\theta^4}{a^2\sigma_\theta^2+b^2\sigma_W^2}+\sigma_\varepsilon^2+\sigma_\theta^2\right)} - \frac{M}{\gamma_d\sigma_\varepsilon^2}}.$$

Using the definition of a and b in Proposition 6, some algebra yields that the denominator on the right-hand side of the previous equation is equal to $\frac{1}{b}$. Therefore, it follows that

$$\frac{\partial}{\partial X_k} p_2 = -b. \tag{69}$$

Using $\mathbb{E}(p_2) = a\theta$, one can now calculate

$$\begin{aligned}
&\frac{\partial}{\partial X_k}\mathbb{E}_k\left[D(q_k - X_k) - p_2 q_k - \frac{\gamma_d}{2}\sigma_\varepsilon^2(q_k - X_k)^2\right] \\
= \quad & a\theta - b\frac{(1-a)\theta}{\gamma_d\sigma_\varepsilon^2} + bX_k.
\end{aligned}$$

Since in equilibrium, one has $X_k = \frac{\varphi}{M}\alpha_1\theta$, the result follows. □

Proof of Proposition 7. Step 1: expressions of β_1 and β_2

Substituting equation $p_c(x_i)$ and $p_v(x)$ into the price $P(x)$ formula gives

$$\beta_1 = \kappa\left(1 - \frac{\alpha_1}{\alpha_2}\right) + (1-\kappa)\left[a + \frac{b(1-a)}{\gamma_d\sigma_\epsilon^2} - \frac{b\varphi\alpha_1}{M}\right] \tag{70}$$

$$\beta_2 = \kappa\left(\frac{1}{\alpha_2} - \frac{\gamma_I\sigma_\epsilon^2}{2}\right). \tag{71}$$

Step 2: Solving α_1, α_2, β_1 and β_2

Combing the equations (47), (48), (70) and (71) and solving α_1, α_2, β_1 and β_2 leads to the following:

$$\alpha_1 = \frac{1 - a - \frac{b(1-a)}{\gamma_d\sigma_\epsilon^2}}{1 - \frac{b\varphi}{M}\frac{1-2\kappa}{(1-\kappa)\gamma_I\sigma_\epsilon^2}}\frac{1-2\kappa}{(1-\kappa)\gamma_I\sigma_\epsilon^2}, \tag{72}$$

$$\alpha_2 = \frac{1-2\kappa}{(1-\kappa)\gamma_I\sigma_\epsilon^2}, \tag{73}$$

$$\beta_1 = \frac{a + \frac{b(1-a)}{\gamma_d\sigma_\epsilon^2} - \frac{b\varphi}{M}\frac{1-2\kappa}{(1-\kappa)\gamma_I\sigma_\epsilon^2}}{1 - \frac{b\varphi}{M}\frac{1-2\kappa}{(1-\kappa)\gamma_I\sigma_\epsilon^2}}, \tag{74}$$

$$\beta_2 = \frac{\kappa}{2(1-2\kappa)}\gamma_I\sigma_\epsilon^2. \tag{75}$$

Note that when $\kappa < \frac{1}{2}$, one gets $\alpha_2 > 0$ and $\beta_2 > 0$.

Step 3: Show the existence of $\alpha_1 \in (0, \alpha_2]$ and $a \in (0, 1]$

We first show that α_1 is a decreasing function of a. Secondly, show that a is an increasing function of α_1, then prove that the two curves insect at $\{\alpha_1 \times a : (0, \alpha_2] \times (0, 1]\}$. Replacing b by equation (39) into formula (72) and derive the expression of α_1 as a function of a:

$$\alpha_1 = \frac{1}{2\gamma_d\varphi}\left[-M\tau_\epsilon - \gamma_d\varphi\alpha_2\left(\frac{a}{M} + a - 1\right) + \sqrt{[M\tau_\epsilon + \gamma_d\varphi\alpha_2\left(\frac{a}{M} + a - 1\right)]^2 + 4\gamma_d\varphi\alpha_2 M\tau_\epsilon(1-a)\left(\frac{a}{M} + 1\right)}\right] \tag{76}$$

Once $a \leq 1$, one could derive that $\alpha_1 > 0$ and further α_1 is monotonically decreasing on a. Since

$$
\begin{aligned}
\frac{\partial \alpha_1}{\partial a} &= \frac{\frac{\alpha_2}{2}(\frac{1}{M}+1)\left[M\tau_\epsilon + \gamma_d\varphi\alpha_2(\frac{a}{M}+a-1) - \sqrt{[M\tau_\epsilon + \gamma_d\varphi\alpha_2\left(\frac{a}{M}+a-1\right)]^2 + 4\gamma_d\varphi\alpha_2 M\tau_\epsilon(1-a)\left(\frac{a}{M}+1\right)}\right]}{\sqrt{[M\tau_\epsilon + \gamma_d\varphi\alpha_2\left(\frac{a}{M}+a-1\right)]^2 + 4\gamma_d\varphi\alpha_2 M\tau_\epsilon(1-a)\left(\frac{a}{M}+1\right)}} \\
&+ \frac{\alpha_2 M\tau_\epsilon(\frac{-2a}{M}+\frac{1}{M}-1)}{\sqrt{[M\tau_\epsilon + \gamma_d\varphi\alpha_2\left(\frac{a}{M}+a-1\right)]^2 + 4\gamma_d\varphi\alpha_2 M\tau_\epsilon(1-a)\left(\frac{a}{M}+1\right)}} \\
&< 0.
\end{aligned}
$$

Moreover, one has

$$
\begin{aligned}
\lim_{a\to 0}\alpha_1 &= \frac{1}{2\gamma_d\varphi}\left[-M\tau_\epsilon + \gamma_d\varphi\alpha_2 + \sqrt{(M\tau_\epsilon - \gamma_d\varphi\alpha_2)^2 + 4\gamma_d\varphi\alpha_2 M\tau_\epsilon}\right] = \alpha_2, \\
\lim_{a\to 1}\alpha_1 &= \frac{1}{2\gamma_d\varphi}\left[-M\tau_\epsilon - \gamma_d\varphi\alpha_2 + \sqrt{(M\tau_\epsilon + \gamma_d\varphi\alpha_2)^2}\right] = 0.
\end{aligned}
$$

Since α_1 is monotonically decreasing on a, one gets $\alpha_1 \in [0, \alpha_2)$.

In terms of a, one can rewrite a as a function of α_1 by substituting ψ and τ_u by equations (37) and (36), and rearranging:

$$
a = \frac{(M\tau_\epsilon + \gamma_d\varphi\alpha_1)^2(N\tau_\epsilon + \gamma_d\varphi\alpha_1) + \gamma_d^2\sigma_W^2(\tau_\theta + \tau_\epsilon)(M\tau_\epsilon + \gamma_d\varphi\alpha_1)}{(M\tau_\epsilon + \gamma_d\varphi\alpha_1)^2 N\tau_\epsilon + \gamma_d^2\sigma_W^2\tau_\epsilon(M\tau_\epsilon + N\tau_\theta)}
$$

Next, one can compute the derivatives of a in terms of α_1 as

$$
\begin{aligned}
\frac{\partial a}{\partial \alpha_1} &= \frac{(M\tau_\epsilon + \gamma_d\varphi\alpha_1)^4 N\tau_\epsilon + 2\gamma_d^2\sigma_W^2\tau_\epsilon(M\tau_\epsilon + \gamma_d\varphi\alpha_1)^2(M\tau_\epsilon + N\tau_\theta) + \gamma_d^4\sigma_W^4\tau_\epsilon(\tau_\theta + \tau_\epsilon)(M\tau_\epsilon + N\tau_\theta)}{\left[(M\tau_\epsilon + \gamma_d\varphi\alpha_1)^2 N\tau_\epsilon + \gamma_d^2\sigma_W^2\tau_\epsilon(M\tau_\epsilon + N\tau_\theta)\right]^2} \\
&+ \frac{\gamma_d^2\sigma_W^2(N-M)\tau_\epsilon^2(M\tau_\epsilon + \gamma_d\varphi\alpha_1)(M\tau_\epsilon + 2N\tau_\theta - \gamma_d\varphi\alpha_1)}{\left[(M\tau_\epsilon + \gamma_d\varphi\alpha_1)^2 N\tau_\epsilon + \gamma_d^2\sigma_W^2\tau_\epsilon(M\tau_\epsilon + N\tau_\theta)\right]^2} \\
&> 0.
\end{aligned}
$$

Moreover, the values at the two bounds:

$$
\begin{aligned}
\lim_{\alpha_1\to 0} a &= \frac{M^2N\tau_\epsilon^3 + M\gamma_d^2\sigma_W^2\tau_\epsilon(\tau_\theta + \tau_\epsilon)}{M^2N\tau_\epsilon^3 + \gamma_d^2\sigma_W^2\tau_\epsilon(M\tau_\epsilon + N\tau_\theta)} < 1, \\
\lim_{\alpha_1\to +\infty} a &= +\infty,
\end{aligned}
$$

where the inequality above inequality follows from $M < N$. So, one gets that a is a monotonically increasing function of α_1 and $a \in (\frac{M^2N\tau_\epsilon^3 + M\gamma_d^2\sigma_W^2\tau_\epsilon(\tau_\theta+\tau_\epsilon)}{M^2N\tau_\epsilon^3 + \gamma_d^2\sigma_W^2\tau_\epsilon(M\tau_\epsilon + N\tau_\theta)}, +\infty)$

Since $\alpha_1(a)$ is monotonically decreasing on a and $\alpha_1 \in [0, \alpha_2)$, $a(\alpha_1)$ is monotonically increasing on α_1 and $a \in (\frac{M^2 N\tau_\epsilon^3 + M\gamma_d^2\sigma_W^2\tau_\epsilon(\tau_\theta+\tau_\epsilon)}{M^2 N\tau_\epsilon^3 + \gamma_d^2\sigma_W^2\tau_\epsilon(M\tau_\epsilon + N\tau_\theta)}, +\infty)$, by the fixed point theorem, there exists one unique solution (α_1^*, a^*) to the problem

$$\begin{cases} a(\alpha_1) = a \\ \alpha_1(a) = \alpha_1 \end{cases}$$

as demonstrated in figure(7).

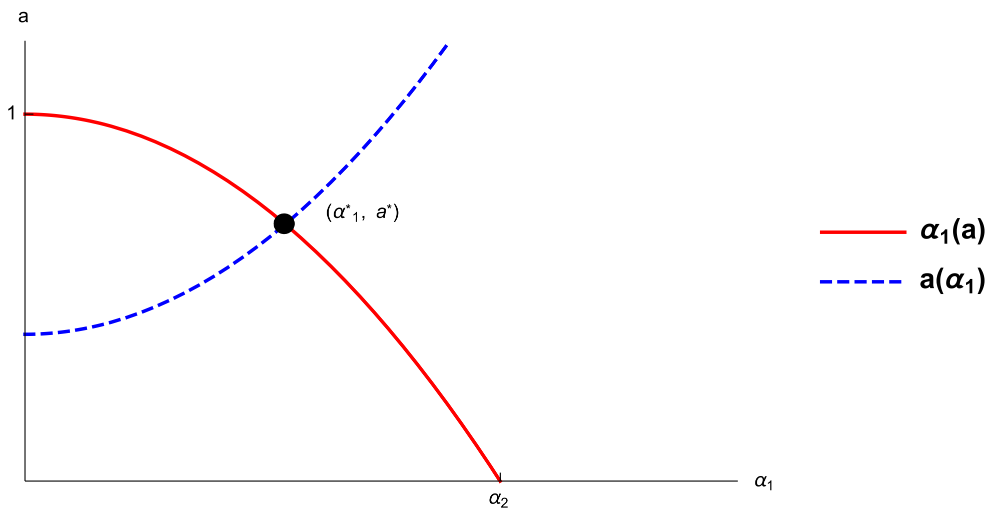


Figure 7: The curves of $\alpha_1(a)$ and $a(\alpha_1)$

Step 4: prove that $0 < \alpha_1^* \leq \alpha_2$ *and* $0 < a^* < 1$

First, it's obvious that $\alpha_1^* > 0$, we only need to prove that $\alpha_1^* \leq \alpha_2$. Suppose that $\alpha_1^* > \alpha_2$, then $a(\alpha_1^*) > a(\alpha_2)$, since a is an increasing function of α_1. Note that when $\alpha_1 = \alpha_2$, we have $\beta_1 = 1$ by equation(47), which implies that $a + \frac{b(1-a)}{\gamma_d\sigma_\epsilon^2} = 1$. Further, we get that either $a = 1$, or $a \neq 1$ and $b = \gamma_d\sigma_\epsilon^2$. But neither of the solution is consistent with the properties of α_1 and a function. On one hand, if $a = 1$, then $a(\alpha_1^*) > a(\alpha_2) = 1$, which is contrary to $a^* \leq 1$. On the other hand, if $b = \gamma_d\sigma_\epsilon^2$, then $a < 0$ by the equation (39), which is also contrary to $a > 0$. So $\alpha_1^* \leq \alpha_2$.

Next, we prove that $a^* < 1$. Assuming $a^* \geq 1$, then we should have $\alpha_1(a^*) \leq \alpha_1(1) = 0$, which is contrary to $\alpha_1^* > 0$. So $a^* < 1$. As in the equilibrium, $0 < \alpha_1 < \alpha_2$, one could get that $0 < \beta_1 < 1$ since $\alpha_1 = (1-\beta_1)\alpha_2$.

Last, we verify that the second order condition of the maximization problem (45) is satisfied when $\kappa < \frac{1}{2}$:

$$-(2\beta_2 + \gamma_I \sigma_\epsilon^2) = -\left(\frac{\kappa}{1-2\kappa}\gamma_I \sigma_\epsilon^2 + \gamma_I \sigma_\epsilon^2\right) = -\frac{1-\kappa}{1-2\kappa}\gamma_I \sigma_\epsilon^2 < 0$$

The fact that Lemma 2 is applicable in order to derive the dealers' quoting strategies is proved as in the proof of Proposition 2.

Step 5: Show the equilibrium does not exist when $\kappa \geq \frac{1}{2}$

First, when $\kappa = \frac{1}{2}$, one has $\alpha_1 = 0$, $\alpha_2 = 0$ and $\beta_2 = +\infty$ from equation (72), (73) and (71). That is, the investor does not trade, and the price is not defined since $P(x_i) = \infty$. Thus, the equilibrium does not exist.

Second, when $\kappa > \frac{1}{2}$, one has $\beta_2 < 0$, and the second order condition of the optimization problem (45):

$$-(2\beta_2 + \gamma_I \sigma_\epsilon^2) = -\left(\frac{\kappa}{1-2\kappa}\gamma_I \sigma_\epsilon^2 + \gamma_I \sigma_\epsilon^2\right) = -\frac{1-\kappa}{1-2\kappa}\gamma_I \sigma_\epsilon^2 > 0.$$

This means that the investor's maximization problem does not have a solution and the equilibrium does not exist. □

Proof of Proposition 8. Claim 1: investors prefer to trade on the platform as $\sigma_\delta^2 \to 0$.

Equation (29) implies that each investor's ex-ante profits go to zero in the centralized market if $\sigma_\delta \to 0$. Proposition 7 states that an equilibrium exists if $\kappa < \frac{1}{2}$. All that is left to show is that expected profits for each investor remain strictly positive as $\sigma_\delta^2 \to 0$. This can be seen as follows. From the definition of the investors' utility (2), the dealers expected quotes conditional on a response on the platform (44) and the investors' equilibrium strategy (30) one obtains the following expression for the expected profit π_i of an investor trying to trade on the platform:

$$\pi_i = (1-(1-q)^M)\mathbb{E}\left[(\alpha_1\theta + \alpha_2\delta_i)\left(\theta + \delta - \beta_1\theta - (\alpha_1\theta + \alpha_2\delta_i)\left(\frac{\gamma_I \sigma_\varepsilon^2}{2} + \beta_2\right)\right)\right]. \tag{77}$$

As $\sigma_\delta^2 \to 0$, one obtains from (77) that

$$
\begin{aligned}
\pi_i &\rightarrow (1-(1-q)^M)\alpha_1\sigma_\theta^2\left[(1-\beta_1)-\alpha_1\left(\frac{\gamma_i\sigma_\varepsilon^2}{2}+\beta_2\right)\right] \\
&= (1-(1-q)^M)\alpha_1\sigma_\theta^2\left[(1-\beta_1)-\alpha_1\frac{1}{2\alpha_2}\right] \\
&= (1-(1-q)^M)\alpha_1\sigma_\theta^2\left[(1-\beta_1)-\frac{1-\beta_1}{2}\right] \\
&> 0,
\end{aligned}
$$

where the second line follows from the expressions for β_1 and α_2 in (72) and (75). The third line follows from the expressions for α_1, α_2 in (47) and (48). The inequality follows from $\alpha_1 > 0$ and (74), which implies $\beta_1 < 1$, since $b < 0$ and $a < 1$ by the proof of Proposition 7. This proves the first claim.

Claim 2: investors prefer to trade in the centralized market as $\sigma_\delta^2 \rightarrow \infty$.

Computing the expectation in (77) gives

$$
\pi_i = (1-(1-q)^M)\left\{\underbrace{\alpha_1\sigma_\theta^2\left[(1-\beta_1)-\alpha_1\left(\frac{\gamma_i\sigma_\varepsilon^2}{2}+\beta_2\right)\right]}_{A}+\underbrace{\alpha_2\sigma_\delta^2\left[1-\alpha_2\left(\frac{\gamma_i\sigma_\varepsilon^2}{2}+\beta_2\right)\right]}_{B}\right\} \tag{78}
$$

In the(78), A is not affected by σ_δ^2. Using the expressions for β_1 and α_2 in (72) and (75), one gets

$$
B = \frac{1-2\kappa}{1-\kappa}\frac{\sigma_\delta^2}{2\gamma_I\sigma_\varepsilon^2} = \frac{1-2\kappa}{1-\kappa}\pi_i^c, \tag{79}
$$

where π_i^c is the expected profit of the investor in the centralized market as defined in (29). It trivially follows that $\pi_i^c \rightarrow \infty$ as $\sigma_\delta^2 \rightarrow \infty$. Therefore, it follows that

$$
\lim_{\sigma_\delta^2\rightarrow\infty}\frac{\pi_i}{\pi_i^c} = \lim_{\sigma_\delta^2\rightarrow\infty}(1-(1-q)^M)\frac{A+\frac{1-2\kappa}{1-\kappa}\pi_i^c}{\pi_i^c} = (1-(1-q)^M)\frac{1-2\kappa}{1-\kappa} < 1,
$$

because of our assumption $\kappa > 0$. Therefore, investors will have a higher expected payoff in the centralized market as $\sigma_\delta^2 \rightarrow \infty$.

□

Proof of Proposition 9. We will show that the term denoted by A in (78) goes to zero as $\mu \rightarrow \infty$. Then it follows from (79) and $\kappa > 0$ that $\pi_i < \pi_i^c$ as $\mu \rightarrow \infty$, with $\pi_i < \pi_i^c$ defined as in (77) and (29).

In order to show $A \to 0$ as $mu \to \infty$, it is sufficient to show that $\alpha_1 \to 0$ as $mu \to \infty$, since β_2 is by (75) unaffected by μ and β_1 is by (74) between zero and one.

We show In order to show $A \to 0$ as $mu \to \infty$ as follows. Define the function $a(\cdot)$ as in the proof of Proposition 7. It has been shown in the proof of Proposition 7 that $\alpha_1 > 0$ for any $\mu > 0$ must hold in equilibrium. For any fixed $\alpha_1 > 0$, one has $a(\alpha_1) \to \infty$ for $\mu \to \infty$. The equilibrium condition $a(\alpha_1) = a < 1$ can only hold if $\alpha_1 \to 0$ for $\mu \to \infty$ (since $a(\cdot)$ is monotone increasing with $\lim \alpha_1 a(\alpha_1) \in (0, 1)$). This proves the claim.

□

Proof of Proposition 10. Using (78), (79) and (29), one gets

$$\lim_{q\to 1}(\pi_i - \pi_i^c) = \lim_{q\to 1} A,$$

where A is defined as in (78). We proceed as in the proof of Proposition 8:

$$\begin{aligned}
\lim_{q\to 1} A &= \lim_{q\to 1} \alpha_1 \sigma_\theta^2 \left[(1-\beta_1) - \alpha_1 \left(\frac{\gamma_i \sigma_\varepsilon^2}{2} + \beta_2 \right) \right] \\
&= \lim_{q\to 1} \alpha_1 \sigma_\theta^2 \left[(1-\beta_1) - \alpha_1 \frac{1}{2\alpha_2} \right] \\
&= \lim_{q\to 1} \alpha_1 \sigma_\theta^2 \left[(1-\beta_1) - \frac{1-\beta_1}{2} \right] \\
&> 0,
\end{aligned}$$

where the second line follows from the expressions for β_1 and α_2 in (72) and (75). The third line follows from the expressions for α_1, α_2 in (47) and (48). The inequality follows from $\alpha_1 > 0$ and (74), which implies $\beta_1 < 1$ as $q \to 1$, since $b < 0$ and $a < 1$ by the proof of Proposition 7.

□