The Rise of Passive Funds Triggers Active Fund Consolidation

Jinyuan Zhang

ABSTRACT
Consolidation among asset managers surged in 2016. What triggered this new wave of consolidation? Through a noisy rational expectation model with imperfect competition, we show that the rise of passive funds could attribute to the new change. As the size of passive funds grows, the market becomes more liquid. As a result, the rents extracted from noise traders shrinks and the competition among active funds becomes severer. These effects squeeze down the expected profit margins of active funds, hence stimulating them to form a coalition to gain monopolistic trading power. The rise of passive funds affects market outcomes (liquidity, price informativeness and volatility) both directly and indirectly through fund consolidation. The aforementioned two mechanisms have competing effects. Hence it is essential to control for fund consolidation when examining the (casual) relationship between passive funds and market outcomes.
1 Introduction

The landscape of the asset management industry is changing. The recent batch of big mergers signals the start of a wave of consolidation among asset managers. In October 2016, Janus Capital Group merged with Henderson Group, creating one of the largest and most diversified active investment managers in the world, with total assets of over US$320 billion under management. In March 2017, Aberdeen Asset Management and Standard Life planned to form a combined group that would control 660bn pounds worth of assets, becoming the UK’s largest fund. This sudden surge of consolidation can be directly observed from Figure 1, which plots the average total net asset (TNA) of acquiring and acquired mutual fund shares from 2000 and 2016. Charles Morrison, the head of the groups $2.1tn asset management division, said this is just the start and the industry is going to look very different in five years when there will be fewer and larger asset managers.

![Figure 1. The average TNA of acquiring and acquired fund shares](image)

What have triggered this new wave of consolidation among asset managers?

It is tricky to rationalize consolidation in an industry where companies often have distinct organizational cultures but lack big fixed back-office costs that could be trimmed in a merger. Common reasons to consolidate, often mentioned by merged fund managers, are scale benefits from 1) the creation of wider investment options, strong brands, international clients, and 2) the reduction in

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1 More mergers can be found in Barron’s article “Mutual-Fund Mergers, Acquisitions Likely to Rise” ([http://www.barrons.com/articles/mutual-fund-mergers-acquisitions-likely-to-rise-1476506750](http://www.barrons.com/articles/mutual-fund-mergers-acquisitions-likely-to-rise-1476506750))

2 The details for data and sampling method can be found in the Appendix A.1. The results are robust for the average of the median at both fund share level and fund level. Also they are robust for acquired funds only and acquiring funds only.
regulation costs. Obviously, the creation of new opportunities can not account for the sudden surge in the consolidation since this demand has always existed. If it was the reduction in regulation costs that stimulated the new wave of consolidation, then we should have seen changes in policy that gave rise to more restricted rules, pushing up fund operating costs in 2016. This is not the case. An alternative explanation, provided by Jayaraman et al. (2002) and Ding (2006), was that consolidation helps eliminate inefficient funds that are small, underperforming, and lacked of growth opportunities. This explanation, however, does not apply to the new wave of consolidation as mergers happen between big and successful funds. In summary, existing explanations barely explain the sudden surge in the fund consolidation.

The rise of passive funds seems to be a more relevant cause. Figure 2\textsuperscript{3} shows that over time, investors have been moving out from traditional actively managed funds and shifting towards index funds and Exchange Traded Funds (ETFs), which simply replicate the market’s return and charge cheaper fees. In the first quarter of 2017, ETFs hit $197.3bn on record inflows. This puts lots of pressure on active managers to lower management fees as a fightback to passive funds if they can not beat the market and restore confidence from investors. This is not just an issue for active funds in the US. Cremers et al. (2016) analyze the mutual fund industry worldwide and document that if active funds face more competition from passive funds, they will charge lower fees and trade more actively. In a word, the margins of active funds have been eroded by the rise of passive funds.

\textbf{Figure 2.} Flows form active to passive funds in US equities

\textsuperscript{3}The figure is extracted from Financial Times article written by Wigglesworth 2017.
Faced with high competition from passive funds, consolidation seems to be a natural strategy for active funds to fight back. This strategy works for general corporations as merger increases market share, hence reduces competition and leads to higher profit margins. If this reasoning is applicable to asset management industry, a smooth increase in the number of consolidation should be observed as the size of passive funds grows over the years. However, Figure 1 shows a noticeable surge in consolidation only in 2016 despite a smooth trend in passive funds.

Do the unmatched patterns disapprove the causality relationship between two events? This paper theoretically shows that the consolidation among active funds can only be triggered when the size of passive funds exceeds a critical threshold. This is perfectly in line with the patterns depicted in Figures 1 and 2.

The model follows the seminal work of Kyle (1989). The market is imperfectly competitive. Three kinds of participants are involved, informed, passive funds and noise traders. Deviating from the original set-up, we allow active funds to form coalitions, a game theory term for consolidation. The investors in the same coalition share their information and profits, and trade against other coalitions and passive funds. In my model, the liquidity is measured as price impact due to one order, the same as Kyle’s $\lambda$.

Forming coalitions has two main effects. First, as competitors become allies, they achieve monopolistic power in trading (referred as monopolistic trading effect). Monopolistic power facilitates a better utilization of information. To understand the intuition, consider an extreme scenario with only one informed investor who has no price impact but absolute monopolistic power in information. As he has no competitors, he only tradeoffs the gain from the use of information and the cost from information leakage to passive funds via price. When other active funds jump into the market, he has one more dimension to consider — to preempt other competitors, which requires him to trade more aggressively on the information. Even if he still trades optimally, the excessive trading on the information over the absolute monopolistic scenario actually adds a marginal cost. Therefore, forming a coalition could reduce competition and excessive use of information.

Second, as fewer investors left in the market, the market becomes less liquid. Interestingly, illiquidity is a double-edged sword for the active funds. On the one hand, it restricts the use of information to take advantage of the passive funds, hammering their expected profits; on the other hand, it forces noise traders to pay a higher price for their inelastic demands, creating a bigger pie to
be split between informed and passive funds. In summary, the decision to form a coalition depends on the tradeoff of above effects.

How does an increase in the number of passive funds trigger the coalition among active funds? When the size of passive funds is small, the market is very illiquid. Forming coalitions worsens liquidity issue, and the gain at passive funds’ expense dominates the benefits from the monopolistic power effect and marginal gain from noise traders. Therefore, active funds prefer staying separated. When the size of passive funds grows, the market becomes more liquid. As a result, the rents extracted from noise traders shrinks and competing active funds trade more aggressively against each other. These effects squeeze down the expected profit margins of active funds, hence stimulating their incentives to form a coalition. This explains why active funds start to consolidate after passive funds grow for a while. Importantly, the liquidity increases arising from noise traders can not produce the same effect. Effectively, high demand from noise traders creates large rents for other market participants.

The above analysis shows that both imperfect competition and passive funds play important roles to generate the interesting phenomenon. Without imperfect competition, investors’ trading aggressiveness on information will not be affected by coalition formation or the change of number of the passive funds. The motivation to form a coalition purely comes from the power of an alliance to lower market liquidity and extract more from noise traders. Therefore, a coalition that includes all informed traders, a comprehensive coalition, will be formed. The case without passive funds is studied by Chen et al. (2015), who adopt imperfect competition framework by Kyle (1985). They show that when the cost to form a coalition is negligible, a comprehensive coalition is a dominant strategy for active funds. The mechanism is as follows. In Kyle (1985) model, the market maker breaks even and the trading is a zero sum game, so all the gains to active funds are extracted from noise traders. Therefore, active funds have strong motivations to form a comprehensive coalition to reduce liquidity in the market and rip off noise traders. If their model is correct, generating a sudden surge of mergers in mutual fund industry presented in Figure 1 requires a huge drop in the cost to form a coalition at 2016. This is not realistic.

In addition to explaining the increase in fund consolidation after 2016, my model generates some

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4Only the precision of signal and risk aversion have impacts on the trading strategy on information. The trading aggressiveness of an allied entity is the sum of trading aggressiveness of each individual when he trades alone, assuming all have the same risk aversion.
predictions on the effects of the rise of passive funds on the financial market outcomes, including liquidity, price informativeness and return volatility. As shown in Figure 3, two channels link passive funds to the market outcomes, a direct channel and an indirect channel through triggering coalition formation. A direct effect of the rise of passive funds is the improvement of the market liquidity. Consequently, active funds are less restricted on trading information and more information is impounded into the price. Moreover, this process stabilizes the price, leading to a decrease in the return volatility. The indirect channel has opposite effects on the market outcomes. Specifically, in my model, forming a coalition allows allied informed traders to gain monopolistic power. When they behave more strategically, the price becomes less informative, the market becomes less liquid, and the return becomes more volatile.

![Figure 3. Direct and indirect effects of the rise of passive funds on market outcomes](image)

Since the direct channel and the indirect channel provide opposite predictions regarding the relationship between the rise of passive funds and the market outcomes, the compounded effects depend on which effect dominates. This could potentially explain the mixed effects the literature finds on relations between the rise of ETFs and stock price informativeness, see Glosten et al. (2017) and Israeli et al. (2016), and liquidity, see Glosten et al. (2017) and Israeli et al. (2016). Therefore, one important takeaway is that we need to at least control for the merger of active funds or the concentration of active shares when studying the impacts of ETFs on market outcomes.

The last striking result is that the expected profits of passive funds increase after coalition formation even though they learn less information from the price. The reason is that the aggregate amount of private information stays unchanged and allied active funds strategically trade less aggressively on their signals, leaving some playfields for the passive funds. In other words, passive funds effectively get less adversely selected by the active funds. Therefore, it is not adverse selection that causes illiquidity and two effects actually go opposite directions after coalition formation. In empiric studies, however, both adverse selection and illiquidity are measured using bid-ask spreads.
We organize the rest of the paper as follows. Section 2 provides a brief overview of the literature. Section 3 presents the model with coalition among active funds and shows that the rise of passive funds could trigger the coalition among the investors. Section 4 discusses interesting predictions from the model on the impacts of the rise of passive funds on the market efficiency, price informativeness and return volatility. Finally, Section 5 concludes.

2 Literature Review

This paper links the mutual fund consolidations to the rise of passive funds through an imperfect competition on the asset price in the presence of exogenous information.

First, we contribute to limited studies on the rationale of the fund consolidation. Empirically, Jayaraman et al. (2002) first pointed out that the likelihood of a fund merger is high when the fund size is small for both within- and across-family mutual fund mergers. Ding (2006) provided further evidence that acquired funds are small, underperform persistently, and experience net outflows. However, these stories are inconsistent with the current situation.

Theoretically, Chen et al. (2015) is the only one studying coalition among competing investors, similar to my case. However, their model is based on Kyle (1985), where active funds submit orders based on signals only and one market maker sets price after observing aggregate order flows. As the market maker breaks even and the trading is zero sum game, all the gains for active funds are extracted from noise traders. The more illiquid the market is, the more they can take advantage on noise traders. Therefore, a comprehensive coalition is always the dominant strategy for competing investors when they make decisions on coalition formation ex-ante. To match the reality that not all the funds merge, they need to have exogenous coalition costs due to search, setup, and coordination efforts. If using this model to explain a surge in fund consolidation, then we need the exogenous coalition costs to decrease a lot in 2016, which is unrealistic. An easy way to have non-comprehensive coalition is to assume a decreasing return to scale, following Berk and Green (2004). But an immediate problem arises as there is no incentive to merge ex-ante under this assumption. My model, however, generates endogenous costs to push back comprehensive coalition in the beginning and creates motivations to consolidate after the passive funds exceed a certain
threshold. This could explain a surge in fund consolidation in 2016.

We also contribute to the empirical literature on EFTs’ impacts. The literature has found mixed impacts of the rise of ETFs on the market outcomes. For example, Glosten et al. (2017) document that ETF activity increases informational efficiency for stocks with imperfectly competitive markets, but not for those with perfectly competitive markets\(^5\), while Israeli et al. (2016) find that an increase in ETF ownership is associated with lower informativeness of underlying firms. De Winne et al. (2011) find index firm becomes more liquid, relative to those of non index stocks, after the introduction of the ETF, while Hamm (2014) find a positive association between the percentage of firm shares being held by ETFs and illiquidity\(^6\). Other evidence has shown that ETFs have distorted the capital markets as a whole, leading to increased volatility (Ben-David et al., 2017), co-movement (Da and Shive, 2016), systemic risk (Srichander, 2011). However, none of the studies controls for the fund mergers or concentration of active fund shares. From this paper, we see that without controlling indirect effects through fund mergers, the relation between ETFs and liquidity, price informativeness and return volatility could be ambiguous. Additionally, Cremers et al. (2016) document that actively managed funds are more active when they face more competition from passive funds, consistent with my predictions. But they do not recognize the active funds could mitigate this problem through consolidation.

Third, my work is closely related to the massive literature on imperfect competition, (see, e.g., Kyle 1989, Nezafat and Schroder 2016, Kacperczyk et al. 2017, among others). We bring in coalition formation into this literature. The decision to form a coalition can be deemed as a new way to obtain information, reduce competition and achieve higher expect returns.

3 A Strategic Model of Coalition

3.1 Model Set-up

The model is an extension of Kyle (1989). Consider a simple exchange economy with three dates, \(-1, 0,\) and \(1\). There exist a risk-free asset with a zero net return and a single risky asset with payoff \(v\) distributed normally with mean \(\bar{v}\) and standard deviation \(\tau_v^{-1}\), i.e, \(v \sim \mathcal{N}(\bar{v}, \tau_v^{-1})\). Without loss of

\(^5\)The effect in a competitive market is discussed before. With perfect competition, investors trading aggressiveness on information will not be affected by the change of the passive funds, so does the price informativeness.

\(^6\)The illiquidity is approximated by bid-ask spread in both cases.
generality, \( \bar{v} \) is set to be 0 in this paper. The risky asset is traded at a market price \( p \), which which is to be determined, and thus generates return \( r = v - p \).

Three kinds of risk neutral investors participant in the market: \( N \) active funds, \( M \) passive funds, and noise traders trading in an exogenous random quantity \( U \sim \mathcal{N}(0, \tau^{-1}_u) \). Each informed investor is endowed with a private signal about the asset payoff

\[ s_i = v + \epsilon_i, \]

where \( \epsilon_i \sim \mathcal{N}(0, \tau^{-1}_i) \) is independently distributed across investors and independent of \( v \).

At date \(-1\), before receiving signals, active funds meet up and decide whether to form coalitions. We assume that one investor can not appear in two different coalitions. In general, there are \( 2^N - N \) possible structures of coalition. For example, assuming there are three active funds, a, b, c, then the possible structures of coalition include: \{\{a\}, \{b\}, \{c\}\}, \{\{a, b\}, \{c\}\}, \{\{b, c\}, \{a\}\}, \{\{a, c\}, \{b\}\} and \{\{a, b, c\}\}. This game is called coalitional game in the game theory literature. The coalition formation will be discussed in the Section 3.4. Denote the structure of coalition as \( C_j = \{C_{j,1}, ..., C_{j,n_j}\} \in \mathcal{B}(\{1, ..., N\}) \), where \( j = 1, ..., 2^N - N \) and \( n_j \) is the number of coalitions in \( C_j \).

At date 0, each informed investor receives his signal. Investors in the same coalition share information and profit. Sharing profit guarantees that they coordinate trades perfectly; otherwise, active funds in the same coalition will compete fiercely against each other, potentially reducing their expected profits, see Colla and Mele (2010). We assume that investors within the coalition share information truthfully, then the information set for coalition \( C_{j,i} \) is \( \mathcal{F}_{C_{j,i}} = \{p, \{s_j\}_{j \in C_{j,i}}\} \). To simplify notation, we omit subscript \( j \) since coalition \( C_j \) is chosen at date 0. Each coalition submits the optimal demand schedule \( x_i \) as a function his information set to maximize the expected profit

\[ x_i = \arg \max_{x_i} \mathbb{E}\left[ x_i (v - p) \middle| \mathcal{F}_{C_i} \right]. \quad (1) \]

Similarly, each uninformed investor submits the optimal demand schedule \( x_u \) as a function of ob-
served price $p$ to maximize the expect profit

$$x_u = \arg \max_{x_u} \mathbb{E}\left[x_u(u - p)\bigg| p\right]. \quad (2)$$

The noise traders submit market orders, $U$. In equilibrium, the price $p$ satisfies the market clearing condition

$$\sum_{i=1}^{n} x_i + Mx_u + U = 0. \quad (3)$$

At date 1, the asset pays $v$, which is distributed to every investor according to his holdings.

### 3.2 The Link of Passive Funds Between Model and Reality

In the model presented above, the active funds are analogous to the informed investors and the passive funds are analogous to the uninformed investors. The first link is quite straightforward that active funds trade their private information. But the second link is arguably weak as passive funds usually hold index portfolio in reality, so one may not immediately see why they learn from the price. This section provides some justification for this analogy.

Consider an economy of $N$ assets and the risky asset discussed in the set-up is one of them, called asset 1. There could be many active funds in the economy but only part of them trade and only trade this asset due to, for example, their limited attentions. These active funds are active funds in my setting. All the passive funds hold the market portfolio, which is consist of all the $N$ assets based on their market capitalizations. The weight of each asset is

$$\left( \frac{\theta_1 p_1}{\sum_{i=1}^{N} \theta_i p_i}, \ldots, \frac{\theta_N p_N}{\sum_{i=1}^{N} \theta_i p_i} \right),$$

where $q_i$ is the total shares of asset $i$ and $p_i$ is price of each share. Then the demand of asset 1 for a passive fund with wealth capacity $W$ is

$$D_1 = \frac{\theta_1}{\theta_1 p_1 + \sum_{i=2}^{N} \theta_i p_i}.$$  

\(^7\)This can be interpreted as one portfolio as well.
which is a decreasing function in its price $p_1$. This implies that passive funds indeed look at price to decide demand and the demand goes up when the price is low. So one can treat passive funds as passive funds who only trade on the price.

Moreover, assuming each passive fund has a limited capacity, so when more capital flows into the passive fund industry, more passive funds are open to accommodate extra capital flows. In this way, the number of passive funds can approximate the size of the passive fund industry.

In what follows, we solve for the equilibrium using backward induction. In Section 3.3, we focus on the trading stage at date 0. Suppose that the coalition $C_j$ has already been formed and we solve for the optimal demand schedule for each coalition $C_{j,i}$. In Section 3.4, we focus on coalition formation at date $-1$. In Section 3.5, we discuss how the rise of passive funds affect the coalition formation at date $-1$.

3.3 Trading at Date 0

At date 0, the coalition structure is decided, so the signal precision for each coalition is determined. As in Kyle (1989), the equilibrium concept is Bayesian Nash equilibrium. It is characterized by demand schedules $x_i$ and $x_u$ satisfying Equations (1) and (2), and well-defined market clearing price $p$ satisfying Equation (3). Specifically, a symmetric linear equilibrium is defined as an equilibrium where the demand schedules $x_u$ and $x_i$, $i = 1, ..., n$, are identical linear functions in observed price, $p$ and aggregate signal in coalition $C_i$, $s_{C_i}$:

$$x_u = -\beta_u p, \quad x_i = \alpha_i s_{C_i} - \beta_i p,$$

where $\alpha_i$, $\beta_i$ and $\beta_u$ are endogenous parameters. Specifically, $s_{C_i}$ can be written as

$$s_{C_i} = \frac{\sum_{j \in C_i} s_j \tau_j}{\sum_{j \in C_i} \tau_j} = v + \epsilon_{C_i},$$

where $\epsilon_{C_i} \sim N(0, \tau_{C_i}^{-1})$ and $\tau_{C_i} = \sum_{j \in C_i} \tau_j$. As $s_{C_i}$ is sufficient statistic of $\{s_j\}_{j \in C_i}$, $s_{C_i}$ provides the same amount of information as $\{s_j\}_{j \in C_i}$.

With the imperfect competition, each investor’s trade affects the price, in other words, each informed trader acts as a monopolistic with respect to his residual supply curve. Following Kyle
(1989), we develop equilibrium solution in standard four steps using non-regret pricing approach: each trade 1) solves for his residual supply curve, 2) updates his belief about the payoff $v$ based on price and private information, 3) submits the optimal demand schedule to trade, which is 4) the same as other traders’ conjectured demand schedules. We only show derivation steps for the informed coalitions.

**Residual Supply Curve.** The market clearing condition (3) implies that the residual supply curve for each consolidated investor is given as

$$ p = \lambda_i x_i + p_i, \quad (5) $$

where $\lambda_i = (\sum_{j \neq i} \beta_j + \beta_u M)^{-1}$ is the price impact, measuring the change of price in response to one unit share purchased by each informed coalition, and $p_i = \lambda_i \left( \sum_{j \neq i} \alpha_j s_{Cj} + U \right)$ is the intercept, aggregating the information of other investors and random demands from noise traders.

**Learning.** The posterior precision of payoff after observing the aggregate signal $s_{C_i}$ and price $p$ is

$$ \bar{\tau}_i = \text{var}[v|s_{C_i}, p]^{-1} = \tau_v + \tau_{C_i} + \varphi_i \quad (6) $$

where

$$ \varphi_i = \frac{(\sum_{j \neq i} \alpha_j)^2}{\sum_{j \neq i} \alpha_j^2 + \frac{1}{\tau_u}}. \quad (7) $$

Here, $\varphi_i$ measures the total information inferred from the price. This term is increasing in the competitors trading aggressiveness $\alpha_j$ and signal precision $\tau_{C_j}$.

The conditional expectation of payoff is given by

$$ \mathbb{E}[v|s_{C_i}, p] = \frac{1}{\bar{\tau}_i} \left( \tau_{C_j} - \alpha_i \frac{\varphi_i}{\sum_{j \neq i} \alpha_j} s_{Cj} + \frac{1}{\lambda} \frac{\varphi_i}{\sum_{j \neq i} \alpha_j} p \right). \quad (8) $$

**Optimal Demand Schedule.** The first-order condition of Equation (1) gives

$$ x_i = \frac{\mathbb{E}[v|s_{C_i}, p] - p}{\lambda_i} = \frac{\mathbb{E}[v|s_{C_i}, p_i] - p_i}{2\lambda_i}, \quad (9) $$
The second order condition is

\[ 2\lambda_i > 0. \]

Substituting Equation (8) into (9) yields the solution for \( \alpha_i \) and \( \beta_i \),

\[
\alpha_i = \frac{\tau C_i}{\lambda_i \bar{r}_i + \sum_{j \neq i} \varphi_i}, \quad \beta_i = \frac{-\frac{1}{\lambda} \sum_{j \neq i} \varphi_i + \bar{r}_i}{\lambda_i \bar{r}_i},
\]

(10)

where \( \lambda = (\sum_{i=1}^{n} \beta_i + \beta_u M)^{-1} \) measures the increase in price when the noise traders increase their demand by one unit. To understand the trading strategy, it is better to re-write demand schedule on the private information and the information inferred from the price, that is \( x_i = \tilde{\alpha}_i sC_i + \tilde{\beta}_i p_i \). The formula for \( \tilde{\alpha}_i \) and \( \tilde{\beta}_i \) is given in the following lemma.

**Lemma 1.**

\[
\tilde{\alpha}_i = \frac{\tau C_i}{2\lambda_i \bar{r}_i}, \quad \tilde{\beta}_i = \frac{-\frac{1}{\lambda} \sum_{j \neq i} \varphi_i + \bar{r}_i}{2\lambda_i \bar{r}_i}.
\]

(11)

\[
\tilde{\alpha}_i = (1 - \lambda \beta_i) \alpha_i, \quad \tilde{\beta}_i = (1 - \lambda \beta_i) \beta_i.
\]

**Proof.** See the Appendix.

Straightforwardly, \( \tilde{\alpha}_i \) measures the investor’s *aggressiveness* to use his own information without considering information revealed from others. The information accuracy can affect aggressiveness from two channels. An increase in information provides more confident to trade, encouraging him to trade more aggressively. Meanwhile, it pushes up the endogenous market noncompetitiveness \( 2\lambda_i \bar{r}_i \), deterring trading on the information. To understand why \( 2\lambda_i \bar{r}_i \) measures the market noncompetitiveness, we need to re-derive formula for \( \tilde{\alpha}_i \) under CARA utility function, which has a form as \( \tilde{\alpha}_i = \frac{\tau C_i}{\gamma_i + 2\lambda_i \bar{r}_i} \), where \( \gamma_i \) is risk aversion. In the scenario that the investor ignores his price impact or market is perfectly competitive, the trading aggressiveness is reduced to \( \tilde{\alpha}_i = \frac{\tau C_i}{\gamma_i} \). Therefore, the term \( 2\lambda_i \bar{r}_i \) only appears in the imperfect competition scenario, so it can be interpreted as market noncompetitiveness.
Following Kyle (1989), we define
\[
\xi_i := \lambda \beta_i = \lambda_i \bar{\beta}_i
\]
to measure the marginal share demanded by the investor when there is an extra unit demanded by other investors\(^9\). There are two components included. Define
\[
\kappa_i := \frac{\varphi_t}{\sum_{j \neq i, j \neq i, j}}
\]
which can be interpreted as the information precision per price, measured as the average information revealed per aggressive trade from competitors. Then the first component \(-\frac{\kappa_i}{\lambda_i \bar{\tau}_i}\) measures the investor’s aggressiveness to trade information revealed from the price\(^{10}\), similar to \(\tilde{\alpha}_i\). This term increases when the competitors reveal more information or the investor has low market power. The second component \(\frac{\bar{\tau}_i \lambda_i}{2\bar{\tau}_i \lambda_i} = \frac{1}{2}\) measures how aggressively the investor would correct price movement caused by noise traders. To see this, consider the case that the investor has absolute monopolistic power in information, that is \(\kappa_i = 0\) and \(\xi_i = \frac{1}{2}\), then for every order submitted by noise traders, the investor only trades one-half share against it. It is clear that \(0 \leq \xi_i \leq \frac{1}{2}\).

**Match Conjectured Demand Schedules** The above analysis can be applied to obtain the optimal demand schedules for the passive funds. Then the equilibrium can be characterized by the following theory for an exogenous set of private signals in the informed coalitions.

**Theorem 1.** Assume all investors are risk neutral. Suppose that there are \(n\) informed coalitions having signals with different precision \(\tau_{C_i}\) and \(M\) passive funds, where \(n + N \geq 3\). Then investors’ demands satisfy the following system of equations:
\[
\begin{align*}
\alpha_i &= \frac{\tau_{C_i}}{\lambda_i \bar{\tau}_i + \kappa_i}, \\
\beta_i &= -\frac{-\kappa_i / \lambda + \bar{\tau}_i}{\lambda_i \bar{\tau}_i}, \\
\beta_u &= -\frac{-\kappa_u / \lambda + \bar{\tau}_u}{\lambda_u \bar{\tau}_u}
\end{align*}
\]

\(^9\)When other investors demand one less unit of share, the price decreases by \(\lambda\) unit, then the investor would increase the quantity traded by \(\lambda \beta_i\) unit.

\(^\text{10}\)Remind that the sign before \(\bar{\beta}_i\) is negative.
where

\[
\lambda = \left( \sum_{i=1}^{n} \beta_i + \beta_u M \right)^{-1}, \quad \lambda_i = \left( \sum_{j \neq i} \beta_j + \beta_u M \right)^{-1}, \quad \lambda_u = \left( \sum_{i=1}^{n} \beta_i + \beta_u (M - 1) \right)^{-1}
\]

\[
\kappa_i = \frac{\sum_{j \neq i} \alpha_j}{\sum_{j \neq i} \alpha_j^2 + \frac{1}{\tau_v}}, \quad \kappa_u = \frac{\sum_{i=1}^{n} \alpha_i}{\sum_{i=1}^{n} \alpha_i^2 + \frac{1}{\tau_v}}
\]

\[
\bar{\tau}_i = \text{var}[v|s_{Ci}, p]^{-1} = \tau_v + \tau_{Ci} + \varphi_i, \quad \text{where} \quad \varphi_i = \frac{\left( \sum_{j \neq i} \alpha_j \right)^2}{\sum_{j \neq i} \alpha_j^2 + \frac{1}{\tau_v}}
\]

\[
\bar{\tau}_u = \text{var}[v|p]^{-1} = \tau_v + \varphi_u, \quad \text{where} \quad \varphi_u = \frac{\left( \sum_{i=1}^{n} \alpha_i \right)^2}{\sum_{i=1}^{n} \alpha_i^2 + \frac{1}{\tau_v}}
\]

The equilibrium price is

\[
p = \lambda \left( \sum_{i=1}^{n} \alpha_i s_{Ci} + U \right).
\]

The variance of return is

\[
\text{var}[v - p] = (1 - \lambda \sum_{i=1}^{n} \alpha_i^2 \tau_v^{-1} + \lambda^2 \sum_{i=1}^{n} \alpha_i^2 \tau_i^{-1}) + \lambda \tau_u^{-1}
\]

\[
= \frac{(\xi_u)^2}{(1 - 2\xi_u)(1 - \xi_u)} \sum_{i=1}^{n} \alpha_i \lambda \frac{1}{\tau_v} + \bar{\tau}_u^{-1}.
\]

The ex-ante expected profits for the active fund, the passive funds and noise traders are

\[
\Pi_i = \mathbb{E}\left[ \max_{x_i} \mathbb{E}\left[ (x_i(v - p)) | s_{Ci}, p \right] \right] = \frac{(\xi_i)^2}{1 - 2\xi_i} \frac{\sum_{j \neq i} \alpha_j}{\tau_v} + \frac{\alpha_i}{2\tau_v} \left( 1 - \xi_i - \lambda \frac{\sum_{j \neq i} \alpha_j}{1 - 2\xi_i} \right), \quad (12)
\]

\[
\Pi_u = \mathbb{E}\left[ \max_{x_u} \mathbb{E}\left[ (x_u(v - p)) | p \right] \right] = \frac{(\xi_u)^2}{1 - 2\xi_u} \frac{\sum_i \alpha_i}{\tau_v} \quad (13)
\]

\[
\Pi_n = -\frac{\lambda}{\tau_u}. \quad (14)
\]

Since trading is an zero-sum game, the sum of all participants’ expected profits is zero.

There is no close form solution for the general case described in the Theorem 1. If assuming the active funds have symmetric information, that is \( \tau_{C_1} = \ldots = \tau_{C_n} = \tau_c \), and \( M \geq 1 \). The trading aggressiveness of each coalition \( \alpha_1 = \ldots = \alpha_n \) is the unique positive
solution to the following equation

\[
\frac{(n-1)}{(n-1)\alpha_1^2/\tau_e + 1/\tau_u} = \frac{1 - \frac{1}{1 + \frac{(n-1)\alpha_1^2}{\tau_e}}}{1 - \frac{2 - \frac{2}{\tau_e} - \frac{(n-1)\alpha_1^2}{\tau_e}}{\tau_u + \frac{(n-1)^2\alpha_1^2}{(n-1)\alpha_1^2/\tau_e + 1/\tau_u}}}
\]

Equilibrium \( \xi_1 = \ldots = \xi_n \) and \( \xi_u \) are

\[
\xi_1 = \ldots = \xi_n = \frac{1}{2} \left( 1 - \frac{(n-1)\tau_u}{\tau_e} \alpha_1^2 \right)
\]

\[
\xi_u = \frac{1}{M} \left( 1 - \frac{n}{2} + \frac{n(n-1)\tau_u}{2\tau_e} \alpha_1^2 \right).
\]

After obtaining \( \alpha_1 \) and \( \xi_u \), the price impact can be derived easily as

\[
\lambda = \frac{\lambda_u}{\lambda_u} = \frac{\kappa_u(1 - \xi_u)}{\tau_u(1 - 2\xi_u)}.
\]

\[N = 2 \text{ informed investors, } M \text{ uninformed investors}\]

\[x_i = \frac{E[v|S] - p}{\lambda_i} = \alpha S - \beta p, \quad x_u = \beta_u p\]

\[\beta = \frac{1}{\lambda_i} = \beta + M\beta_u\]

No solution

The case with a signal active funds is characterized by Nezafat and Schroder (2016). The details can be find in Proposition 2.

3.4 Coalition Formation

3.4.1 Equilibrium Solution

At date \(-1\), active funds play a coalitional game with partition form, introduced by Thrall and Lucas (1963). Specifically, the coalition structure is defined as \( C = \{C_{j,1}, \ldots, C_{j,n_j}\} \in \mathcal{B}\{1, \ldots, N\}\), where \( j = 1, \ldots, 2^N - N \), such that \( \forall i \neq k, C_{j,i} \cap C_{j,k} = \emptyset \) and \( \cup_{i=1}^{n_j} C_{j,i} = \{1, \ldots, N\} \). Denote the expected profit of partition \( k \) of coalition \( C_{j} \) as \( \Pi_{j,k} \), and the expected profit of investor \( i \) in partition
$k$ of coalition $C_j$ as $\Pi_{j,k}^i = \frac{\Pi_{j,k}}{|C_{j,k}|}$, where $|C_{j,k}|$ is the number of investors in partition $k$. Then a coalition game is defined by $(C_j, \Pi_j)$, where $\Pi_j$ is a vector of expected profit of partitions in $C$. The Pareto equilibrium is defined as follows.

**Definition 1** (Pareto Equilibrium). Coalition $C_{j_1}$ is preferred over coalition $C_{j_2}$ by Pareto order if at least one investor improves in $C_{j_1}$ without hurting the other investors, i.e., $\Pi_{j_1} \geq \Pi_{j_2}$ with at least one investor $i$ having $\Pi_{j_1,k_1}^i > \Pi_{j_2,k_2}^i$.

To find a Pareto equilibrium, a centralized approach can be used. For example, a social planner distributes the location to each informed investor. However, such an approach is a NP-hard problem. The reason is that finding an optimal partition requires iterating over all the partitions, with total possibilities as $2^N - N$.

Luckily, it is unnecessary to solve the general coalition game to show that an increase in the number of passive funds could trigger coalition among active funds. Once we can find a Pareto equilibrium without comprehensive coalition and prove that the change of passive funds will alter the desires of active funds to form a comprehensive coalition, we can make my point. Therefore, it is enough to consider a simple case with only two active funds ($N = 2$) in the market. In this case, there exist merely two types of coalitions $C_1 = \{\{1\}, \{2\}\}$ and $C_2 = \{\{1, 2\}\}$. A comprehensive coalition $C_2$ pareto dominates $C_1$ if $\Pi_{2,\{1\}} + \Pi_{2,\{2\}} \geq \Pi_{1,\{1, 2\}}$, and visa versa. Proposition 1 and 2 characterize the equilibrium solutions for these two coalitions.

**Proposition 1.** Suppose there are two informed investor $N = 2$ and $M \geq 2$ passive funds. The signal prevision of the each informed investor is $\tau_\epsilon$. Then,
1) \[
\alpha_1^2 = \alpha_2^2 = -\frac{(M + 2)\tau_v \tau_\epsilon + \tau_\epsilon \sqrt{(3M + 2)^2 \tau_v^2 + 16\tau_v \tau_\epsilon (M^2 + M)}}{4(M + 1)(2\tau_\epsilon + \tau_v)\tau_u}
\]
\[
\xi_1 = \xi_2 = \frac{1}{2}(1 - M\xi_u), \quad \xi_u = \frac{1}{M} \tau_u \alpha_1^2 = -\frac{(M + 2)\tau_v + \sqrt{(3M + 2)^2 \tau_v^2 + 16\tau_v \tau_\epsilon (M^2 + M)}}{4M(M + 1)(2\tau_\epsilon + \tau_v)}
\]
\[
\lambda = \frac{2\tau_u \sqrt{M\xi_u \tau_v}}{4M\xi_u \tau_\epsilon + (2M\xi_u + 1)\tau_v} \frac{1 - \xi_u}{1 - 2\xi_u}, \quad \lambda_u = \frac{\lambda}{1 - \xi_u}, \quad \lambda_1 = \lambda_2 = \frac{2\lambda}{1 + M\xi_u}
\]
\[
\bar{\tau}_u = \tau_v + \frac{4M\xi_u \tau_\epsilon}{2M\xi_u + 1}, \quad \bar{\tau}_1 = \bar{\tau}_2 = \frac{4M\xi_u \tau_\epsilon + (2M\xi_u + 1)\tau_v}{4M\xi_u} \frac{1 - 2\xi_u}{1 - \xi_u}
\]
\[
\Pi_n = -\frac{2\sqrt{M\xi_u \tau_v}}{4M\xi_u \tau_\epsilon + (2M\xi_u + 1)\tau_v} \frac{1 - \xi_u}{1 - 2\xi_u}, \quad \Pi_u = \frac{2\sqrt{M\xi_u \tau_v}}{\tau_v} \frac{\xi_u^2}{1 - 2\xi_u}, \quad \Pi_{2,(1)} + \Pi_{2,(2)} = (-\Pi_n - M\Pi_u)
\]
\[
\var[v - p] = \frac{(\xi_u)^2 \tau_\epsilon}{(1 - 2\xi_u)^2 \tau_v} \frac{4M\xi_u}{4M\xi_u \tau_\epsilon + (2M\xi_u + 1)\tau_v} + \frac{2M\xi_u + 1}{4M\xi_u \tau_\epsilon + (2M\xi_u + 1)\tau_v}
\]

2) \[
\frac{\partial \alpha_1}{\partial M} > 0, \quad \frac{\partial \lambda}{\partial M} < 0, \quad \frac{\partial \bar{\tau}_u}{\partial M} > 0.
\]

**Proposition 2.** Nezafat and Schroder (2016) When two active funds form one coalition, denoted using superscript $C$, then the signal precision of the coalition is $2\tau_\epsilon$. Then,

1) \[
\alpha_1^C = \sqrt{\frac{M - 1}{M} \frac{2\tau_\epsilon}{\tau_v + 2\tau_\epsilon \tau_u}}, \quad \beta_u^C = \frac{1}{M} \sqrt{\frac{M - 1}{M} \frac{2\tau_\epsilon}{\tau_v + 2\tau_\epsilon \tau_u}}, \quad \beta_1^C = \frac{M\beta_u^C}{\tau_v + 2\tau_\epsilon \tau_u}
\]
\[
\xi_1^C = \frac{1}{2}, \quad \xi_u^C = \frac{1}{2M}
\]
\[
\lambda^C = \frac{1}{2\beta_u^C M}, \quad \lambda_1^C = \frac{1}{\beta_u^C M}, \quad \lambda_u^C = \frac{1}{(2M - 1)\beta_u^C}
\]
\[
\bar{\tau}_1^C = \tau_v + 2\tau_\epsilon, \quad \bar{\tau}_u^C = \tau_v + \frac{2(M - 1)\tau_\epsilon \tau_v}{(M - 1)\tau_v + M(\tau_\epsilon + 2\tau_\epsilon)}
\]
\[
\Pi_n^C = -\frac{1}{2M\beta_u^C \tau_u}, \quad \Pi_u^C = \frac{1}{4M^3 \beta_u^C \tau_u}, \quad \Pi_{1,(1,2)}^C = \frac{2M - 1}{4M^2 \beta_u^C \tau_u}
\]
\[
\var[v - p]^C = \frac{1}{4M^3 \beta_u^C \tau_u (2M - 1)\beta_u^C} + \left(\tau_v + \frac{2(M - 1)\tau_\epsilon \tau_v}{(M - 1)\tau_v + M(\tau_\epsilon + 2\tau_\epsilon)}\right)^{-1}
\]

2) \[
\frac{\partial \alpha_1^C}{\partial M} > 0, \quad \frac{\partial \lambda^C}{\partial M} < 0, \quad \frac{\partial \bar{\tau}_u^C}{\partial M} > 0.
\]
Both propositions show that, as expected, the demand sensitivities to the private signals, $\alpha_1^C$ and $\alpha_1$, are both increasing in information precision. By making the price more informative, the price impact $\lambda^C$ and $\lambda$ increases as well. This can be thought of as an adverse selection effect because informed coalition can profit at passive funds’ expense due to a large information gap. A decrease in the $\tau_u$ reduces adverse selection problem, therefore, reduces the price impact for all traders.

An increase in the number of passive funds also reduces price impact for all traders, hence encouraging informative trading. This result, however, does not come from the adverse selection channel but purely arises since more participants in the market dilute the price impact of each individual. The key difference between liquidity arising from two channels is that a reduction of $\tau_u$ does not lead to a change in allocation the pie between active funds and passive funds\(^{11}\), while a reduction in $M$ does. This helps understand why noise traders could not trigger coalition, which will be discussed more detailedly in Section 3.5.

### 3.4.2 The Impacts of Coalition Formation

Forming one coalition has two main effects, increasing monopolistic power in trading and reduce liquidity in the market. In this session, we detailedly discuss these two effects.

First, as competitors become allies, they not only have a bigger information advantage over other investors but also achieve monopolistic power in trading, facilitating a better utilization of information. As a result, allied active funds trade less aggressively, compared to the case that they stay separated, stated in Lemma 2.

**Lemma 2.** *Informed investors have higher monopolistic power in trading after coalition formation, that is, $\alpha_1^C < \alpha_1 + \alpha_2$.*

To understand what the monopolistic power is, consider an extreme case of only one informed investor who has no price impact but absolute monopolistic power in information\(^{12}\). As he has no competitors, he only tradeoffs the gain from the use of information and the cost from information leakage to passive funds via price, the same mechanism as in Grossman and Stiglitz (1980). When other competitors jump into the market, he needs to trade more aggressively to preempt other active

---

\(^{11}\)All investors adjust their demand by the same scale when $\tau_u$ changes, so that the price informativeness does not change. In fact, the price informativeness does not depend on $\tau_u$ at all.

\(^{12}\)In this case, one needs to assume the investor is risk averse; otherwise, his trading on information is unlimited.
funds. However, the excessive use of information is undesirable, otherwise, he should trade to reach the same level in the absolute monopolistic case. Therefore, active funds have incentives to form a coalition to reduce competition and excessive use of information. A direct consequence is that the price reveals less information, summarized in Lemma 3.

**Lemma 3.** *Less information is revealed from price after coalition formation, that is, \( \bar{\tau}_u^C < \bar{\tau}_u \).*

Second, the market becomes less liquid as fewer investors get involved. A side effect of an illiquid market is the rise of return volatility, see Lemma 4. Liquidity is measured by the inverse of price impact, same to Kyle (1985).

**Lemma 4.** *The market becomes less liquid and asset return becomes more volatile after coalition formation, that is, \( \lambda^C > \lambda \), and \( \text{var}[v - p]^C > \text{var}[v - p] \).*

Interestingly, illiquidity is a double-edged sword for the active funds. On the one hand, it restricts the use of information to take advantage of the passive funds, hammering their expected profits; on the other hand, it forces noise traders to pay a higher price for their inelastic demands, creating a bigger pie to be split between informed and passive funds, see Lemma 5.

**Lemma 5.** *The market noncompetitiveness for active funds increases after coalition formation, that is, \( \lambda^C \bar{\tau}_1^C > \lambda_1 \bar{\tau}_1 \). Moreover, the total expected cost for noise traders increases, that is, \( \Pi_n^C > \Pi_n \).*

It is worth mentioning that worse illiquidity market does not result from worse adverse selection problem between active funds and passive funds\(^{13}\). The reason is that, after forming a coalition, the aggregate amount of private information stays unchanged and allied active funds strategically trade less aggressively on their signals (see Lemma 2), leaving some playfields for the passive funds. In other words, passive funds effectively get less adverse selected by the active funds. This leads to another interesting outcome that passive funds get better off after coalition formation even if they learn less information from the price, summarized in Lemma 6. Therefore, it is not the adverse selection that causes illiquidity and two effects actually go opposite directions after coalition formation.

**Lemma 6.** *Unactive funds get better off after coalition formation, that is, \( \Pi_u^C > \Pi_u \).*

\(^{13}\)In Chen et al. (2015), the market maker sets the price. The price impact is set higher when the adverse selection problem is more severe.
Summarily, the decision to form a coalition depends on the tradeoff of monopolistic power effect, restricted trading effect and free-ride noise traders effect. Note that the first two effects do not appear in a perfectly competitive world, where the trading aggressiveness is decided by the precision of signal and risk aversion. In this case, neither coalition formation nor the change of the passive funds could affect the aggregate use of total information\textsuperscript{14}. The motivation to form a coalition purely comes from the power of an alliance to lower market liquidity and extract more from noise traders. Hereby, a comprehensive coalition will always be formed in a perfectly competitive market.

3.5 The Impact of the Passive Investing on Coalition Formation

As shown in Figure 4, when the number of passive funds is small, the market is very illiquid. Forming a coalition worsens liquidity issue, and the gain at passive funds’ expense dominates the benefits from the monopolistic power effect and marginal gain from noise traders. Therefore, active funds prefer to stay separated. As the number of passive funds increases, the expected profits of active funds decrease in both cases. Nevertheless, the profit margin drops quicker in the no-coalition case than in the coalition case, so active funds choose to form a coalition after a certain threshold. In the following, I will explain why there are decreasing trends of expected profits and then why decreasing speed is higher in the no-coalition case.

![Figure 4](image)

**Figure 4.** The expected profit of all active funds as the number of passive funds increase

When the size of passive funds grows, as shown in Proposition 1, the market becomes more

\textsuperscript{14}The trading aggressiveness of an allied entity is the sum of trading aggressiveness of each individual when he trades alone, assuming all have the same risk aversion.
liquid, leading to two competing outcomes. First, the total rents one can extract from noise traders decrease, see the size of pie decreases after the number of passive funds increase in Figure 5. Second, active funds can better use their information and take more advantage of passive funds, obtaining a bigger share of the pie in Figure 5. The negative effect dominates the positive effect as noisy traders have perfectly inelastic demand curves, but passive funds do not. This explains the decreasing trends in expected profits as the number of passive funds increases.

For the coalition case, however, the positive effect from the better use of information is stronger for the following reasons. First, the coalition sees all the private signals. Second, the coalition does not worry about being taken advantage by other informed competitors as in the no-coalition case, so its trading aggressiveness increase more marginally, the same as the Grossman and Stiglitz (1980) effect. Consequently, the proposition of pie obtained by active funds in the coalition grows quicker than the no-coalition case, and, hence, the net expected profit decreases slower.

![Diagram](image)

**Figure 5. Illustration**

Importantly, the liquidity increases arising from noise traders can not produce the same effect. There are two ways to understand this. First, consider the case that two active funds stay separated initially. As varying $\tau_u$ scale up or down all the demands from investors at the same scale, it does not alter the informativeness of price and affects the allocation of the pie between the passive and the active. So the only effect of a decrease in $\tau_u$ (an increase in liquidity) is the creation of a big
pie for market participants, which will not trigger the coalition among active funds. An alternative way is to compare Proposition 1 and Proposition 2. Essentially, the expected profits for active funds are a constant multiplied by $\frac{1}{\sqrt{\tau_u}}$ in both cases and the levels of constants are not determined by $\tau_u$. The intuition is similar as before. The level of $\tau_u$ only decides the size of pie provided by noise traders but not the allocation of the pie. So the structure of the active funds will not be affected by the change of $\tau_u$ at all.

4 Empirical Predictions

Above analysis shows that an increase in passive investing can trigger fund consolidation. In this section, we will discuss some predictions generated from the model. The key message is that the rise of passive funds can affect market outcomes through two competing channels: a direct channel and an indirect channel via fund consolidation, as shown in Figure 3. Since two channels provide opposite predictions regarding the relationship between the rise of passive funds and the market outcomes, the compounded effects depend on which effect dominates.

Illiquidity  A direct effect of the rise of passive funds is the improvement of the market liquidity, as proved in Proposition 1 and Proposition 2. However, it can trigger coalition among active funds, drying up the market liquidity, see Lemma 4. Two effects are shown in Figure 6.

![Graph](image)

**Figure 6.** The illiquidity of market $\lambda$ as the number of passive funds increase
**Price Informativeness**  A liquid market relaxes restrictions on active funds to trade on information and more information is impounded into the price. So price becomes more informative as the number of passive funds increases, see Proposition 1 and Proposition 2. However, the growing of passive funds could trigger consolidation, allowing allied informed traders to gain monopolistic power. When they behave more strategically, the price becomes less informative, see Lemma 2. Two effects are shown in Figure 7.

**Figure 7.** The price informativeness of market \( \lambda \) as the number of passive funds increase

**Return Volatility**  Lastly, liquid market stabilizes the price, leading to a decrease in the return volatility, while consolidation formation reduces market liquidity, so does return volatility, see Figure 8. As summarized in the literature review, the empirical literature finds mixed evidence on the

**Figure 8.** The return volatility of market \( \lambda \) as the number of passive funds increase
impacts of ETFs on different market outcomes. None of the studies, however, control for the fund mergers in their analysis. So their studies can only capture the compounded effects. Additionally, Ben-David et al. (2017) adopt exogenous changes in index membership as an instrument variable to study the impact of ETFs on stock volatility. However, it is possible that active investors will also jump in to trade the stocks which are added to the index since those stocks’ liquidities are enhanced, and active trading affects return volatility directly. So the validity of the instrument variable is questionable.

One may argue that above studies do not cover 2016 and 2017 and fund consolidation should not be an issue for them. This argument is loose. Remind that the key mechanism is that the rise of passive funds squeezes the profit margins of active funds. So, expect for fund consolidation, dropout of some active funds can be another potential outcome. In either case, the consequence is that the shares held by active funds become more concentrated, meaning fewer active funds holding a large proportion of shares. Then all the above intuitions stay and all the analysis goes through. Therefore, old papers are subject to the issue discussed above.

Therefore, one important takeaway is that we need to at least control for the merger of active funds or the concentration of active shares when studying the impacts of ETFs on market outcomes. A new study conducted by Kacperczyk et al. (2017) already provide some evidence that an increase in the concentration of institutional ownership leads to lower informativeness, in line with my prediction. To complete my story, we need to formally test whether active fund consolidation or the change of concentration of active shares is mediation between passive funds and market outcomes in the next step.

The last point is about distinguishment of illiquidity and adverse selection. Illiquidity is the price impact from one trade, while the adverse selection is the effective information gap between active funds and passive funds. In this paper, high illiquidity does not come with a high adverse selection. In fact, two effects can go opposite directions. In empiric studies, however, both adverse selection and illiquidity are measured using bid-ask spreads. It is unclear which effect is captured by the spreads.
5 Conclusion

The recent wave of consolidation among asset managers is striking. This paper provides an explanation for this phenomenon. We show that fund consolidation is an endogenous outcome of the rise of passive investing. This endogenous change leads to the jumps in illiquidity, price informativeness and return volatility. Hence it is essential to control for fund consolidation when examining the (casual) relationship between passive funds and market outcomes.
A Appendix

A.1 Data and Sampling Method for Figure 1

The sample of mutual fund mergers adopted in this study is collected from the 1999 CRSP Survivor-Bias Free Mutual Fund Data Base ("CRSP"). Thus CRSP is free of survivor-bias. This study greatly benefits from this feature since we are investigating the funds involved in mergers.

In the funds list table, CRSP records whether a fund share class is merged out of business and its corresponding acquiring fund and merger date. As pointed out by Elton et. al (2001), the merger dates reported by CRSP may be inaccurate. But we ignore it because we are only interested in the year level. We also exclude funds with total net assets (TNA) less than $15 million to eliminate the upward bias in their reported returns.

For the fund level analysis, we group shares if they satisfy the following criteria: (1) acquired share classes have the same fund portfolio name; (2) acquiring share classes have the same fund portfolio name; (3) same merger dates.

A.2 Proofs

A.2.1 Proof of Lemma 1

\[ x_i = \alpha_i sC_i - \beta_i p = \alpha_i sC_i - \beta_i (p_i + \lambda_i x_i) \]
\[ \implies x_i = \frac{\alpha_i}{1 + \beta_i \lambda_i} sC_i + \frac{\beta_i}{1 + \beta_i \lambda_i} p_i \]
\[ \implies \hat{\alpha}_i = \frac{\alpha_i}{1 + \beta_i \lambda_i}, \quad \hat{\beta}_i = \frac{\beta_i}{1 + \beta_i \lambda_i} \]

A simple transformation gives

\[ \frac{1}{1 + \beta_i \lambda_i} = \frac{\lambda_i}{\lambda} = 1 - \beta_i \lambda. \]

Moreover, \( \beta_i \lambda = \hat{\beta}_i \lambda_i. \) \( \square \)
A.2.2 Proof of Theorem 1

Proof. Plug the optimal demand schedule in (9) in the expected profit function 13 yields

\[
\Pi_i = \mathbb{E}\left[\frac{(\mathbb{E}[v|F_i] - p)^2}{\lambda_i}\right] = \frac{\text{var}\left[\mathbb{E}[v|F_i] - p\right]}{\lambda_i} = \frac{\text{var}\left[\mathbb{E}[v|F_i] - p_i\right]}{4\lambda_i} = \text{var}[x_i] \lambda_i
\]

(15)

Now, focus on the variance part.

\[
\text{var}\left[\mathbb{E}[v - p_i|\tau_i, p_i]\right] = \text{var}\left[\tau_i s_i - (\chi_i - k_i)(\sum_{j \neq i} \alpha_j s_j + U)\right] \bar{\tau}_i^{-1}
\]

\[
= \left(\left(\tau_i - \left(\sum_{j \neq i} \alpha_j (\chi_i - k_i)\right)\right)^2 \frac{1}{\bar{\tau}_i} + \tau_i + (\chi_i - k_i)^2 \frac{\sum_{j \neq i} \alpha_j}{k_i}\right)
\]

\[
= \left(\tau_i^2 \bar{\tau}_i^{-1} + \left(\sum_{j \neq i} \alpha_j\right)^2 (\chi_i - k_i)^2 \bar{\tau}_i^{-1} + 2 \tau_i \left(\sum_{j \neq i} \alpha_j\right) (\chi_i - k_i) \bar{\tau}_i^{-1} + \tau_i + (\chi_i - k_i)^2 \frac{\sum_{j \neq i} \alpha_j}{k_i}\right)
\]

\[
+ (\chi_i - k_i)^2 \frac{\sum_{j \neq i} \alpha_j}{k_i} \bar{\tau}_i^{-1} - (\chi_i - k_i)^2 \frac{\sum_{j \neq i} \alpha_j}{k_i} \bar{\tau}_i^{-1}
\]

\[
= \left(\chi_i - k_i\right)^2 \bar{\tau}_i^{-1} \left(\sum_{j \neq i} \alpha_j\right)^2 + (\tau_i + \xi_i) \frac{\sum_{j \neq i} \alpha_j}{k_i} + \frac{\tau_i}{\bar{\tau}_i} \left(\tau_i - \xi_i^2 \frac{\sum_{j \neq i} \alpha_j}{k_i}\right) \bar{\tau}_i^{-1}
\]

\[
= (\kappa_i - \chi_i)^2 \frac{\sum_{j \neq i} \alpha_j}{k_i} \bar{\tau}_i^{-1} + \frac{\tau_i}{\tau_v} \left(1 - \tau_i^{-1} \lambda \frac{\sum_{j \neq i} \alpha_j}{k_i}\right)
\]

\[
\Rightarrow \Pi_i = \frac{(\xi_i)^2}{1 - 2\xi_i} \frac{\sum_{j \neq i} \alpha_j}{\tau_v} + \frac{\alpha_i}{2\tau_v} \left(1 - \xi_i - \lambda \frac{\sum_{j \neq i} \alpha_j}{1 - 2\xi_i}\right).
\]

For passive funds, set \(\alpha_i = 0\), then

\[
\Pi_a = \frac{(\xi_a)^2}{1 - 2\xi_a} \frac{\sum_i \alpha_i}{\tau_v}.
\]

(16)

Denote \(y = \mathbb{E}[v|s_{C_i}, p] - p\), then \(\mathbb{E}[y] = 0\) and \(\text{var}[y] = \text{var}[\mathbb{E}[v - p|s_{C_i}, p]] = \text{var}[v - p] - \mathbb{E}[\text{var}[v - p|s_{C_i}, p]] = \text{var}[v - p] - \text{var}[v|s_{C_i}, p]\). Then above equation can be written as

\[
\Pi_i = \frac{\text{var}[v - p] - \text{var}[v|s_{C_i}, p]}{\lambda_i}.
\]

The expected profit for liquidity trader is

\[
\mathbb{E}[u(v - p)] = -\frac{\lambda}{\tau_u}.
\]

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The variance of return can be written as

$$\text{var}[v - p] = \Pi_u \lambda_u + \text{var}[v|p] = \frac{(\xi_u)^2}{1 - 2\xi_u} \sum_i \frac{\alpha_i}{\bar{\tau}_v} \lambda_u + \bar{\tau}_u^{-1}.$$ 

\[\square\]

### A.2.3 Proof of Theorem 2

**Proof.** Essentially, there are three unknowns \(\alpha_1 = \ldots = \alpha_n\), \(\xi_1 = \ldots = \xi_n\) and \(\xi_u\) and three equations

\[
\alpha_1 = \frac{\tau_\epsilon}{\lambda_1 \bar{\tau}_1 + \kappa_1}, \quad \xi_1 = \frac{\lambda \bar{\tau}_1 - \kappa_1}{\lambda_1 \bar{\tau}_1} = \frac{\lambda_1 \bar{\tau}_1 - \kappa_1}{2\lambda_1 \bar{\tau}_1}, \quad \xi_u = \frac{\lambda \bar{\tau}_u - \kappa_u}{\lambda_u \bar{\tau}_u} = \frac{\lambda_u \bar{\tau}_u - \kappa_u}{2\lambda_u \bar{\tau}_u}.
\]

We will express \(\xi_1\) and \(\xi_u\) in terms of \(\alpha_1\), then take ratio of \(\xi_1\) and \(\xi_u\) to pin down \(\alpha_1\).

Reorganize formula for \(\xi_1\) yields

\[
\kappa_1 = (2\lambda - \lambda_1)\bar{\tau}_1 \Rightarrow 2\lambda \bar{\tau}_1 = \kappa_1 + \lambda_1 \bar{\tau}_1
\]

\[
2\lambda_1 \bar{\tau}_1 \xi_1 = \lambda_1 \bar{\tau}_1 - \kappa_1 \Rightarrow \kappa_1 = (1 - 2\xi_1)\lambda_1 \bar{\tau}_1
\]

Plug the first equation into the formula of \(\alpha_1\) gives

\[
\alpha_1 = \frac{\tau_\epsilon}{2\lambda \bar{\tau}_1}.
\]

Plug this equation into the formula of \(\xi_1\) gives

\[
\xi_1 = \frac{1}{2} - \frac{\kappa_1}{2\lambda_1 \bar{\tau}_1} = \frac{1}{2} - \frac{\kappa_1 \alpha_1}{\lambda_1} = \frac{1}{2} - \frac{\kappa_1 \alpha_1}{\tau_\epsilon} (1 - \xi_1)
\]

\[
\Rightarrow \frac{2(1 - \xi_1)}{1 - 2\xi_1} = \frac{\tau_\epsilon}{\kappa_1 \alpha_1}
\]

\[
1 + \frac{1}{1 - 2\xi_1} = \frac{(n - 1)\alpha_1^2 + \tau_\epsilon}{(n - 1)\alpha_1^2} \Rightarrow 1 + \frac{\tau_\epsilon}{(n - 1)\alpha_1 \bar{\tau}_u}
\]

\[
\xi_1 = \frac{1}{2}(1 - \frac{(n - 1)\tau_u \alpha_1^2}{\tau_\epsilon}).
\]

As \(0 \leq \xi_1 \leq \frac{1}{2}\), we have \(0 \leq \alpha_1^2 \leq \frac{\tau_u}{(n - 1)\tau_\epsilon}\) immediately.

The equation \(n\xi_1 + M\xi_u = 1\) holds for certain, so the relationship between \(\xi_u\) and \(\xi_u\) can be
written as,

$$\xi_u = \frac{1}{M} \left( 1 - \frac{n}{2} + \frac{n(n-1)\tau_u}{2\tau_e} \alpha_1^2 \right).$$

As $0 \leq \xi_u \leq \frac{1}{2}$, we have $(1 - \frac{2}{n}) \frac{\tau_e}{(n-1)\tau_u} \leq \alpha_1^2 \leq \frac{\tau_e}{(n-1)\tau_u} \frac{n+M-2}{n}$ immediately. Therefore, the range of $\alpha_1^2$

$$a_1^2 \in \left[ (1 - \frac{2}{n}) \frac{\tau_e}{(n-1)\tau_u}, \min \left( \frac{\tau_e}{(n-1)\tau_u}, \frac{\tau_e}{(n-1)\tau_u} \frac{n+M-2}{n} \right) \right]. \quad (17)$$

Similar to $\xi_1$, we have $\kappa_u = (2\lambda - \lambda_u)\bar{\tau}_u$. The ratio of $\kappa_1$ and $\kappa_u$ gives

$$\frac{\kappa_1}{\kappa_u} = \frac{(2\lambda - \lambda_1) \bar{\tau}_1}{(2\lambda - \lambda_u) \bar{\tau}_u} = \frac{2 - \frac{1}{1-\xi_1}}{2 - \frac{1}{1-\xi_u}} \frac{\bar{\tau}_1}{\bar{\tau}_u} \frac{(n-1)\alpha_1^2/\tau_u + 1/\tau_u}{\sqrt{2\alpha_1^2 + n\alpha_1^2/\tau_u + 1/\tau_u}}$$

The last equation gives close form solution to $\alpha_1^2$.

**A.3 Proof of Proposition 1**

Proof. So the ratio of $\beta_1$ and $\beta_u$ is

$$\frac{\beta_1}{\beta_u} = \frac{\xi_1}{\xi_u} = \frac{M}{2} \left( \frac{\tau_e}{\tau_u \alpha_1^2} - 1 \right)$$

In the next step, take the ratio between $\kappa_1$ and $\kappa_u$

$$\frac{\bar{\tau}_u \kappa_1}{\bar{\tau}_1 \kappa_u} = \frac{M\beta_u}{\beta_1 + M\beta_u} \frac{2\beta_1 + (M - 1)\beta_u}{2\beta_1 + (M - 2)\beta_u} = \frac{M \frac{\tau_e}{\tau_u \alpha_1^2} - 1}{M \frac{\tau_e}{\tau_u \alpha_1^2} - 2 \alpha_1^2/\tau_e + 1/\tau_u} \frac{2\alpha_1^2/\tau_e}{2 \alpha_1^2/\tau_e + 1/\tau_u}$$

$$\Rightarrow \alpha_1^2 = -\frac{(M + 2)\tau_v \tau_e + \tau_e \sqrt{(3M + 2)^2 \tau_v^2} + 16\tau_e \tau_v (M^2 + M)}{4(M + 1)(2\tau_e + \tau_v)\tau_u}$$

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Another solution is less than zero, so omit it. Then the endogenous variables \( \alpha_1, \xi_1 \) and \( \xi_u \) can be written as

\[
\xi_u = \beta_u \lambda = \frac{1}{M} -(M + 2)\tau_v + \sqrt{(3M + 2)^2 \tau_v^2 + 16\tau_v \tau_u (M^2 + M)} \quad \frac{4(M+1)(2\tau_v + \tau_u)}{4(M+1)(2\tau_v + \tau_u)}
\]

\[
\alpha_1 = \sqrt{M \xi_u \frac{\tau_v}{\tau_u}}, \quad \xi_1 = \beta_1 \lambda = \frac{1}{2} (1 - M \xi_u)
\]

Then we will derive other statistics as a function of \( \xi_u \).

\[
\bar{\tau}_u = \tau_v + \frac{4\alpha_1^2}{2\alpha_1^2/\tau_v + 1/\tau_u} = \tau_v + \frac{4M \xi_u \tau_v}{2M \xi_u + 1}
\]

\[
\lambda = \frac{\lambda}{\lambda_u} = \frac{\kappa_u(1 - \xi_u)}{\tau_u(1 - 2\xi_u)} = \frac{2\tau_u \sqrt{M \xi_u \frac{\tau_v}{\tau_u}}}{4M \xi_u \tau_v + (2M \xi_u + 1) \tau_v} \frac{1 - \xi_u}{1 - 2\xi_u}
\]

\[
\bar{\tau}_1 = \bar{\tau}_2 = \frac{\tau_v}{2\alpha_1 \lambda} = \frac{4M \xi_u \tau_v + (2M \xi_u + 1) \tau_v}{4M \xi_u} \frac{1 - 2\xi_u}{1 - \xi_u}
\]

Then other statistics are easy to derive:

\[
\lambda_u = \frac{\lambda}{1 - \xi_u}, \quad \lambda_1 = \lambda_2 = \frac{2\lambda}{1 + M \xi_u}
\]

Lastly, we will derive expected profit for investors. For noise traders, it is easy to get

\[
\Pi_n = -\frac{\lambda}{\tau_u} = -\frac{2 \sqrt{M \xi_u \frac{\tau_v}{\tau_u}}}{4M \xi_u \tau_v + (2M \xi_u + 1) \tau_v} \frac{1 - \xi_u}{1 - 2\xi_u}.
\]

For passive funds, plug in \( \alpha_1 \) in to Equation 16 yields

\[
\Pi_u = \frac{(\xi_u)^2}{1 - 2\xi_u} \frac{\alpha_1 + \alpha_2}{\tau_v} = \frac{2 \sqrt{M \xi_u \frac{\tau_v}{\tau_u}}}{\tau_v} \frac{\xi_u^2}{1 - 2\xi_u}
\]

Then the active funds equally spit the remaining profit,

\[
\Pi_1 = \Pi_2 = \frac{1}{2} (\Pi_n - M \Pi_u).
\]
A.3.1 Proof of Proposition 2

Proof. The derivation for $\alpha_1, \beta_1, \beta_u, \lambda_1, \lambda_u, l$ can be found in Nezafat and Schroder (2016). Here, we only derive the expressions for the expected profits. First, plug in $\lambda$ in to Equation (14) yields

$$\Pi_n = -\frac{1}{2M\beta_u \bar{\tau}_u}$$

Then,

$$\Pi_u = \text{var}(x_u)\lambda_u = \text{var}[\xi_u(\alpha_1s_1 + U)]\lambda_u$$

$$= \frac{1}{(4M)^2} \left( \frac{M - 1}{M} \frac{1}{\tau_u} + \frac{1}{\tau_u} \right) \frac{1}{(2M - 1)c}$$

$$= \frac{1}{4M^3\beta_u \tau_u}$$

Since the trading is zero sum game, so the expected profit for the only informed investor is

$$\Pi_i = -\Pi_n - M\Pi_u = \frac{2M - 1}{4M^2\beta_u \tau_u}.$$ 

The variance of return is

$$\text{var}[v - p] = \Pi_u \lambda_u + \text{var}[v|p] = \frac{1}{4M^3\beta_u \tau_u} \frac{1}{(2M - 1)\beta_u} + \left( \tau_v + \frac{(M - 1)\tau_1 \tau_v}{(M - 1)\tau_v + M(\tau_v + 1)} \right)^{-1}$$
A.3.2 Proof of Lemma 2

Proof. 

\[ \alpha_1^C < \alpha_1 + \alpha_2 \iff (\alpha_1^C)^2 < (2\alpha_1)^2 \]

\[ (\alpha_1^C)^2 = \frac{M-1}{M} \frac{2\tau_v}{\tau_v + 2\tau_u} < 4(\alpha_1)^2 = \frac{-(M+2)\tau_v + \tau_v \sqrt{(3M+2)^2\tau^2_v + 16\tau_u \tau_v (M^2 + M)}}{(M+1)(2\tau_v + \tau_u)\tau_u} \]

\[ \frac{2\tau_v}{M-1} < \frac{-(M+2)\tau_v + \sqrt{(3M+2)^2\tau^2_v + 16\tau_u \tau_v (M^2 + M)}}{(M+1)} \]

\[ 2(M^2-1)\tau_v < -(M^2 + 2M)\tau_v + M\sqrt{(3M+2)^2\tau^2_v + 16\tau_u \tau_v (M^2 + M)} \]

\[ 9M^2\tau^2_v < (3M + 2)^2\tau^2_v + 16\tau_u \tau_v (M^2 + M) \]

\[ 0 < (12M + 4)\tau^2_v + 16\tau_u \tau_v (M^2 + M) \]

The last inequality holds for sure, so \( \alpha_1^C < \alpha_1 + \alpha_2 \).

A.3.3 Proof of Lemma 3

Proof. The precision of information revealed from price for coalition and non-coalition is

\[ \bar{\tau}_u^C = \tau_v + \frac{1}{2\tau_v} + \frac{1}{\alpha_1^C \tau_u}, \quad \bar{\tau}_u = \tau_v + \frac{1}{2\tau_v} + \frac{1}{4(\alpha_1)^2\tau_u}. \]

As \( (\alpha_1^C)^2 < 4(\alpha_1)^2 \) holds, \( \bar{\tau}_u^C < \bar{\tau}_u \) holds.

A.3.4 Proof of Lemma 5

Proof. From the proof of Theorem 2, we have

\[ \lambda_1^C \tau_1^C = \frac{2\tau_v}{\alpha_1^C}, \quad \lambda_1 \tau_1 = \frac{2\tau_v}{2\alpha_1} - \frac{\alpha_1}{\alpha_1^2 / \tau_v + 1 / \tau_u}. \]

As \( \alpha_1^C < 2\alpha_1 \), \( \lambda_1^C \tau_1^C = \frac{2\tau_v}{\alpha_1^C} > \frac{2\tau_v}{2\alpha_1} > \lambda_1 \tau_1 \).

\[ 33 \]
A.3.5 Proof of Lemma 4

Proof. The precision of information revealed from price for coalition and non-coalition is

\[ \lambda^C = \frac{2\tau_v}{2\alpha_1^C \tau_1}, \quad \lambda = \frac{\tau_v}{2\alpha_1 \tau_1}. \]

To prove \( \lambda^C = \frac{2\tau_v}{2\alpha_1^C \tau_1} > \lambda = \frac{\tau_v}{2\alpha_1 \tau_1} \) is equivalent to prove \( \alpha_1^C \tau_1^C < 2\alpha_1 \tau_1 \).

\[ \alpha_1^C (\tau_v + 2\tau_e) < 2\alpha_1 (\tau_v + 2\tau_e) < 2\alpha_1 (\tau_v + \tau_e + \frac{\alpha_1^2}{\tau_e} + \frac{1}{\tau_a}) \]

The last inequality holds for sure, so \( \lambda^C > \lambda \). \qed
References


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