GLOBAL RISK AVERSION AND INTERNATIONAL RETURN COMOVEMENTS

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Abstract

This article addresses the ongoing debate about the relative importance of fundamental sources of risk that transmit across countries, and provides evidence for the role of “global” risk aversion. I first compare international equity and bond return comovements, and establish three new stylized facts: (1) bond return correlations are smaller in magnitude than equity return correlations, (2) equity returns have downside correlations that are significantly higher than upside correlations, while bond return correlations are symmetric, and (3) equity return correlations are countercyclical, while bond return correlations are weakly procyclical. I then interpret the stylized facts in the context of a linear dynamic factor model, which is motivated using a dynamic no-arbitrage asset pricing model. The theoretical model features time-varying global economic uncertainties (of output growth, inflation, and real interest rates) and time-varying risk aversion (of a global investor) and consistently prices international equities and Treasury bonds. I find that all three stylized facts above can be explained by the different sensitivities of equity returns (strongly negative) and bond returns (weakly positive or negative) to the global risk aversion shock. In addition, global risk aversion explains 90 percent of the fitted global equity conditional comovements and 40 percent of the fitted global bond conditional comovements, after controlling for a wide set of global economic uncertainties. Inflation upside uncertainty is the other key driver for global bond comovement.

Keywords: risk aversion, international return comovements, dynamic correlation models, asset pricing, dynamic factor model

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1 Introduction

Since the global financial crisis, there is renewed interest in understanding how asset returns comove across countries, for both risky and safe asset classes. A large empirical literature has focused on quantifying the evolution of international equity return comovements (see e.g. Bekaert, Hodrick, and Zhang (2009), and Christoffersen, Errunza, Jacobs, and Langlois (2012), among many others). However, given the important role of safe assets in both domestic and international markets, there is surprisingly little research on how government bond returns comove across countries (see Cappiello, Engle, and Sheppard (2006) for an exception). These correlations are important inputs when evaluating the benefits of international diversification for bond and equity investments. In addition, studying why the comovements among equities versus bonds are different contributes to the ongoing debate about the relative importance of fundamental sources of risk that transmit across borders. My paper formally contrasts global equity return comovements and global bond return comovements, both unconditionally and dynamically, and interprets the comovement differences in the context of a dynamic no-arbitrage asset pricing model with time-varying global economic uncertainties (of output growth, inflation, and the real interest rate) and risk aversion of a global investor.

In the first part of the paper, I formulate a new model of multi-dimensional dynamic dependence to estimate the global equity and bond comovements of 8 developed countries from March 1987 – December 2016. A parametric model helps reveal substantive time variation in global correlation, and provides testable empirical benchmarks in evaluating an asset pricing model (later). The model belongs to the class of Dynamic Equicorrelation models (Engle and Kelly, 2012) but with modifications to accommodate correlation asymmetry, and to ensure the simultaneous fit of domestic equity-bond comovement. In addition, I conduct three tests within the model to identify the differences between equity and bond comovements from three perspectives: magnitude, tail behavior, and cyclicality. Three new stylized facts are established and tested against unconditional moments:

1. Bond return correlations are smaller in magnitude than equity return correlations;
2. Equity returns have downside correlations that are significantly higher than upside correlations, while bond return correlations are symmetric;
3. Equity return correlations are countercyclical, while bond return correlations are weakly procyclical.

As prices are the sums of discounted cash flows, asset return innovations can be explained by cash flow (CF) shocks or discount rate (DR) shocks. Commonalities in asset returns across countries come from a collection of “common” shocks, CF or DR, that are priced in individual country assets. In particular, there has been growing interest regarding the role of global risk aversion as a DR source of international comovements. For instance, Miranda-Agrippino and Rey (2015) suggest that global risk aversion is a key transmission mechanism for exporting U.S. monetary policy to countries worldwide, thus driving the global financial cycle of risky assets. Jotikasthira, Le, and Lundblad (2015) suggest that around 70 percent of long-term bond yield
comovement derives from the commonality of term premia. While these studies have indicated potential economic determinants of global comovements within either risky or safe asset classes, no research has formally explained global equity and bond return comovements in a unified framework. Having a unified framework is helpful for identifying the relative importance of common shocks.

In the second part of the paper, I formulate and solve a U.S.-centric dynamic no-arbitrage asset pricing model, featuring time-varying global macroeconomic uncertainties and time-varying risk aversion of a global (U.S.) investor, that prices both international equities and government bonds. The state variable capturing risk aversion is motivated from a general HARA utility function; as in Bekäert, Engstrom, and Xu (2017; henceforth, BEX), it represents the non-fundamental variable (in contrast to the macroeconomic variables) in the real pricing kernel. In BEX, a risk aversion index is filtered using moments of risky assets, whereas in the current paper I also focus on government bond markets. Therefore, I consider a broader set of market-wide economic uncertainties in order to price nominal bonds: adding to the real output growth uncertainties as in BEX, inflation uncertainties and real short rate uncertainties. Furthermore, because the asset moment of interest in this paper is comovement, my model admits (1) heteroskedasticity and (2) conditional non-Gaussianity in the shock assumptions, which has the potential to capture (1) substantive time variation and (2) asymmetric properties as established in the first part of my paper using the parametric model. In particular, I acknowledge left- and right-tail sources of macroeconomic uncertainties, given the recent growing literature discovering their different asset pricing implications (see e.g. Bekäert and Engstrom (2017) in a habit formation framework). Hence, I adapt a realistic and tractable “Bad Environment–Good Environment” shock structure consisting of two centered heteroskedastic non-Gaussian shocks to model the dynamics of upside and downside uncertainties of the three macroeconomic variables. I estimate them using approximate maximum likelihood methods.

Despite the non-Gaussian shock structure, the theoretical model still fits in the affine class of asset pricing models with a closed-form solution. I use the model solution to motivate a dynamic factor model of asset returns where the factors are shocks of the aforementioned market-wide economic determinants (global risk aversion and economic uncertainties); I refer to them as “global shocks” in this paper. The dynamics of both global equity and bond return comovements are driven by the second moments of these global shocks, and the difference between the two comovements in my model is explained by the different sensitivities of asset returns to these shocks. In the last part of the paper, I estimate the dynamic factor model with asset returns of the 8 countries to interpret the three stylized facts.

The core finding is that different sensitivities of equity returns (strongly negative) and bond returns (weakly positive or negative) to the global risk aversion shock dominantly drive all three stylized facts. Regarding the first stylized fact, because not every country’s government bonds are considered safe (i.e., some bond prices increase with risk aversion while others decrease) but all equities are considered highly risky (i.e., asset prices decrease with risk aversion), bond return comovements are smaller in magnitude than equity return comovements. Second, the fact that bond returns do not respond to the global risk aversion shock with the same sign weakens the role of strongly positively-skewed risk aversion in driving global bond comove-
ments. Third, the finding that all bonds are safe in a good economic environment while only a few bonds remain safe in a bad economic environment results in higher comovement among international bond returns in these good environments, explaining a (weakly) procyclical global bond comovement.

In addition, factor models with time-varying betas spanned by economic uncertainties increase the correlation between the model-implied conditional correlations and conditional correlations implied by the flexible parametric model from the first part of the paper. The fit increases from 55% to 69% for global equity comovement, and from 0% to 17% for global bond comovement. Global risk aversion accounts for 90% (40%) of the fitted global equity (bond) comovement. As to the economic significance of economic uncertainties, real uncertainties (inflation uncertainties) explain 7% (47%) of the fitted global equity (bond) comovement.

The paper contributes to the finance and economics literature in three ways.

First, the core finding in the paper stresses the importance of the “non-cash flow channel” of international return comovements: risk aversion of the global investor. Risk aversion has featured as a source of capital flow waves (Forbes and Warnock, 2012), monetary policy shock transmission to foreign stock markets (Miranda-Agrippino and Rey, 2015), interest rate correlations (Jotikasthira, Le, and Lundblad, 2015). My contribution is to quantify the “price of risk” or “risk compensation” channel simultaneously with the “cash flow” channel for explaining global equity and bond return comovements.

Second, as documenting stylized facts of global equity and bond comovements, my empirical contribution is oriented toward global bond comovements. For example, the asymmetry of global equity return comovement is a widely-recognized fact and has been tested using different econometric approaches, such as exceedance correlation (Longin and Solnik, 2001), bivariate GARCH models (Cappiello, Engle and Sheppard, 2006), and asymmetric copula models (Christoffersen et al., 2012). The countercyclical behavior of global equity correlation is also well-known (see e.g. Longin and Solnik, 1995; De Santis and Gerard, 1997; Riberio and Veronesi, 2002). However, there is little formal research on these properties for global bond comovement. Most importantly, the first stylized fact is a new and surprising finding because the existing literature documents that 10-year Government bond yields are highly correlated at around 92% (Jotikasthira, Le, and Lundblad, 2015).

Third, from a modeling standpoint, the new econometric model contains two innovations. It allows for the possibility of asymmetric correlations in a multi-dimensional space. In addition, it offers a parsimonious way to ensure the simultaneous fit of time-varying domestic equity-bond comovement and within-asset comovements. Both innovations are shown to improve the statistical fit.

The remainder of the paper is organized as follows. Section 2 presents the econometric model and establishes the three stylized facts. Section 3 solves an asset pricing model and presents estimation results for the global economic determinants. A more detailed international model is relegated to the Appendix. Section 4 interprets the three stylized facts in a dynamic factor model with global factors. Section 5 provides additional evidence with a “Jackknife” exercise. Concluding remarks are presented in Section 6.
2 Stylized Facts of Global Comovements

In this section, I first exploit a high-dimensional dynamic dependence model to establish three new stylized facts of international return comovements from three perspectives: magnitude, tail behavior, and cyclicality. Then, I obtain unconditional, non-parametric data moments to evaluate the fit of the parametric model. Sections 2.1 and 2.2 present the econometric model, while estimation methodology, data description, and estimation results follow in Sections 2.3–2.5, respectively. Section 2.6 evaluates the model fit of unconditional moments.

2.1 Setup

Consider a world economy of $N$ countries. The log equity return of Country $i$ during period $t+1$ is modeled as follows, $r_{i,t+1}^{E} = \mu_{i}^{E} + \varepsilon_{i,t+1}^{E}$ where $\mu_{i}^{E}$ represents the constant mean, and $\varepsilon_{i,t+1}^{E}$ the return residual. The log bond return of Country $i$ during period $t+1$ is modeled similarly, $r_{i,t+1}^{B} = \mu_{i}^{B} + \varepsilon_{i,t+1}^{B}$. In the rest of the section, superscripts “$E$” and “$B$” denote equity and bond, respectively, and subscripts indicate country ID and time stamp.

Within each asset class, the conditional variance-covariance matrices of the residuals are defined as,

$$
H_{t}^{E} = E \left[ \varepsilon_{t+1}^{E} \varepsilon_{t+1}^{E'} | I_{t} \right],
$$

$$
H_{t}^{B} = E \left[ \varepsilon_{t+1}^{B} \varepsilon_{t+1}^{B'} | I_{t} \right],
$$

where $I_{t}$ denotes the information set at time $t$. I follow the dynamic dependence literature and express $H_{t}^{E}$ and $H_{t}^{B}$ in a quadratic form to estimate the conditional variances and the conditional correlation (off-diagonal elements) in separate steps,

$$
H_{t}^{E} = \Lambda_{t}^{E} \text{Corr}_{t}^{E} \Lambda_{t}^{E},
$$

$$
H_{t}^{B} = \Lambda_{t}^{B} \text{Corr}_{t}^{B} \Lambda_{t}^{B},
$$

where $\Lambda_{t}^{E}$ ($\Lambda_{t}^{B}$) contains square roots of the equity (bond) return conditional variances on the diagonal, and zeros elsewhere; $\text{Corr}_{t}^{E}$ ($\text{Corr}_{t}^{B}$) is the equity (bond) return conditional correlation matrix. $\Lambda_{t}^{E}$, $\Lambda_{t}^{B}$, $\text{Corr}_{t}^{E}$ and $\text{Corr}_{t}^{B}$ are $N \times N$ symmetric matrices. In the first step, return residuals are standardized using conditional variance estimates from univariate GARCH-class models; I relegate univariate conditional variance models and distributional assumptions to Appendices A and B. The second step takes standardized residuals as given and focuses on estimating the conditional correlation matrices, $\text{Corr}_{t}^{E}$ and $\text{Corr}_{t}^{B}$.

2.2 A New Econometric Model for Global Comovements

A well-known multi-dimensional dynamic dependence model is the Dynamic Equicorrelation (DECO) model introduced by Engle and Kelly (2012). They impose a strong assumption that all pairwise conditional correlations are the same for all country pairs. This assumption is suitable for my research because the goal is to obtain a series of global return comovements for each asset class.

My model highlights two new features. First, it accommodates asymmetry, which is
an improvement on the original DECO model. Second, a DECO model estimates global comovement within one asset class independently; my model also ensures a simultaneous fit of the time-varying domestic equity-bond comovement—which can be thought of as the flight-to-safety channel. In addition, my model is flexible enough to conduct three statistical tests within the model, leading to the three stylized facts.

2.2.A Global Bond Comovement and the Three Tests

Denote \( z_{t+1}^{B} \) \((N \times 1)\) as the standardized residuals of country bond returns during period \( t + 1 \). The conditional equicorrelation matrix of \( z_{t+1}^{B} \) is defined by,

\[
E_t[z_{t+1}^{B} z_{t+1}^{B}'] = Corr_t^{B} = (1 - \rho_t^{B}) I_N + \rho_t^{B} J_{N \times N},
\]

where \( I_N \) is an identity matrix and \( J_{N \times N} \) is a matrix of 1s. The equicorrelation by definition is an equally-weighted average of correlations of unique country pairs (i.e., total of \( N(N-1)/2 \) pairs) at time \( t \):

\[
\rho_t^{B} = \frac{2}{N(N-1)} \sum_{i>j} \frac{q_{i,j,t}^{B} q_{i,i,t}^{B} q_{j,j,t}^{B}}{q_{i,j,t}^{B} q_{j,j,t}^{B}},
\]

where \( q_{i,j,t}^{B} \) is the \((i, j)\)-th element of a symmetric matrix \( Q_t^{B} \) \((N \times N)\) which follows a generalized autoregressive heteroskedastic process. Therefore, the dynamic process of \( Q_t^{B} \) drives the time variation in \( Corr_t^{B} \). In a matrix representation, \( \rho_t^{B} \) is a \( N \times 1 \) vector of ones.

The original DECO framework, as in Engle and Kelly (2012), models the dynamic process of \( Q_t \) as follows, omitting superscript “B” for simplicity:

\[
Q_t = \bar{Q} + \beta_1 \left( \tilde{Q}_{t-1}^{\frac{1}{2}} z_t \tilde{z}_t' \tilde{Q}_{t-1}^{\frac{1}{2}} \bar{Q} - \bar{Q} \right) + \beta_2 \left( Q_{t-1} - \bar{Q} \right),
\]

where \( \bar{Q} \) (to be specific, \( \bar{Q}^B \)) is the pre-determined sample bond return correlation matrix \((N \times N)\); \( \beta_1 \) and \( \beta_2 \) are unknown parameters, to capture the relative importance of the cross products of shock realizations and persistence.

In this paper, I propose a more flexible dynamic process as follows:

\[
Q_t = \bar{Q} \circ \Phi_t + \beta_1 \left( \tilde{Q}_{t-1}^{\frac{1}{2}} z_t \tilde{z}_t' \tilde{Q}_{t-1}^{\frac{1}{2}} \bar{Q} \circ \Phi_{t-1} \right) + \beta_2 \left( Q_{t-1} - \bar{Q} \circ \Phi_{t-1} \right)
+ \gamma \left( \tilde{Q}_{t-1}^{\frac{1}{2}} n_t n_t' \tilde{Q}_{t-1}^{\frac{1}{2}} - \Xi \circ \bar{Q} \circ \Phi_{t-1} \right),
\]

where “\( \circ \)” denotes the Hadamard product operator (i.e., element-by-element multiplication). The first term “\( \bar{Q} \circ \Phi_t \)” represents the time-varying long-run conditional mean of the conditional covariance matrix. While \( \bar{Q} \) captures the unconditional component of the long-run mean as before, this model also has the capacity to capture cyclical behavior of global comovement.
through the new term $\Phi_t$ defined below,

$$
\Phi_t(N \times N) = \begin{bmatrix}
1 & 1 + \phi_t & 1 + \phi_t & \cdots \\
1 + \phi_t & 1 & 1 + \phi_t & \cdots \\
1 + \phi_t & 1 + \phi_t & 1 & \cdots \\
\vdots & \vdots & \vdots & \ddots
\end{bmatrix},
$$

(9)

where $\phi_t = \phi \tilde{\theta}_t^{world}$ and $\tilde{\theta}_t^{world}$ is the standardized world recession indicator. Therefore, by construction, the unconditional mean of $\Phi_t$ is a symmetric matrix of ones, and $\phi$ is an unknown constant parameter. As before, the second term captures the effect of news (scaled contemporaneous shock products) on the $Q_t$ process. The third term is an autoregressive term, capturing the persistence of the process.

To capture potential asymmetry in joint downside events, I introduce an asymmetric component to the conditional process. In the fourth term of Equation (8),

$$
n_t(N \times 1) = I_{z_t<0} \circ z_t,
$$

(10)

where $I_{z_t<0}$ ($N \times 1$) is assigned 1 if the standardized residual is less than 0, and assigned 0 otherwise. The constant symmetric matrix $\Xi = E[I_{z_t<0}I_{z_t<0}]$ ($N \times N$) represents the expected covariance of joint negative shocks. The $\gamma$-coefficient is not constrained to be positive. The sufficient conditions for a dynamic dependence model in the GARCH class to be positive definite for all possible realizations are that the intercept is positive semi-definite, and the initial covariance matrix is positive definite (see Ding and Engle (2001) for further details).

Three statistical tests are conducted within Equation (8), which leads to the three stylized facts:

(1) **Equality Test.** The magnitude of the long-run conditional mean of the $Q_t$ process has an unconditional component $\overline{Q}$, which in the equality test is modeled as below,

$$
\overline{Q}^E + \nu (J_{N \times N} - I_N)
$$

(11)

where $\overline{Q}^E$ is the pre-determined equity unconditional correlation matrix, and $\nu$ is a constant. The diagonal terms of the pre-determined equity unconditional correlation matrix are equal to 1; by construction, $\nu$ increases all off-diagonal correlations by an equal amount, which is a reasonable construct because the present research focuses on international return comovements at the global level. A positive (negative) $\nu$ suggests that global bond comovement is on average greater (smaller) than global equity comovement. The null hypothesis, $\nu = 0$, is that both asset comovements have the same unconditional magnitude.

(2) **Asymmetry Test.** A positive $\gamma$ indicates a higher left-tail (downside) comovement, whereas a negative $\gamma$ indicates a higher right-tail (upside) comovement. The closest paper to introducing asymmetry to dynamic dependence models in the GARCH class is Cappiello, Engle, and Sheppard (2006); however, their model is limited to a bivariate system, whereas the present model works with a multivariate system.

(3) **Cyclicality Test.** A positive $\phi$ indicates that the long-run conditional mean of dynamic comovement behaves countercyclically, because the long-run conditional mean comoves

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1The recession indicator is assigned 1 during recession periods, and 0 during non-recession periods; then, I standardize the indicator so that the unconditional mean (sample mean) of $\phi_t$ is 0.
positively with the countercyclical world recession indicator, whereas a negative \( \phi \) indicates procyclical behavior.

In principle, business cycle news can affect returns (and thus the “new” terms in my model, \( z_t z'_t \) and \( n_t n'_t \)) at relatively high frequency. However, my measurement of the business cycle involves the actual observed cycles and changes only at the lower frequency. Therefore, I argue that introducing cyclicality in the long-run conditional mean is more realistic because cycles are slow-moving, and thus they are more likely to influence the levels of global comovements rather than higher-frequency dynamics. This way, my model differentiates cyclical behavior (which is economic-based) from asymmetric tail behaviors (which is return-based). Similar instrument approaches with macroeconomic variables are used in many empirical studies, a few of which include Bekaert and Harvey (1995) on estimating the world price of risk, Duffee (2005) on testing the cyclicality of the amount of consumption risk, and more recently Xu (2017) on uncovering the procyclical comovement between dividend growth and consumption growth.

### 2.2.B Global Equity Comovement with Duo-DECO: Return Decomposition

To model global equity return comovements, I propose a “Duo-DECO” framework. The “Duo” part imposes salient feature of the time variation in domestic equity-bond return comovements. This time-variation is difficult to explain with economic factors (Baele et al., 2013; Ermolov, 2015) and sign-switches in the correlation are often associated with Flight-to-Safety (FTS; see Connolly, Stivers, and Sun, 2005). On a global scale, Baele et al. (2013) identify and characterize FTS episodes for 23 countries, and find that the majority of FTS events are country-specific rather than global. The correlation between country stock and bond returns is generally procyclical; that is, FTS episodes occur in a bad domestic economic environment. My model aims to accommodate the two empirical observations of domestic comovements—procyclical and lack of synchronization—while estimating global equity return comovements in a parsimonious fashion.

To be more specific, I impose a linear equity return decomposition. Denote \( z_{E,t+1}^i \) (\( z_{B,t+1}^i \)) as the standardized residuals of equity returns (bond returns) of Country \( i \) during period \( t+1 \). Define an unknown beta process for each country \( i \), \( b_{i,t} \), to capture the time-varying sensitivity of equity returns to bond returns.

\[
\begin{align*}
  z_{i,t+1}^E &= b_{i,t} z_{i,t+1}^B + \sqrt{1 - (b_{i,t})^2} z_{i,t+1}^E, \\
  b_{i,t} &= \frac{\exp(\delta_1 + \delta_2 x_{i,t})}{1 + \exp(\delta_1 + \delta_2 x_{i,t})} - 1,
\end{align*}
\]

where \( \delta_1 \) and \( \delta_2 \) are unknown constant parameters and \( x_{i,t} \) is a country recession indicator, assigned 1 during recession months and 0 during non-recession months.

Equations 12-13 have four immediate implications. First, because \( z_{i,t+1}^B \) and \( z_{i,t+1}^E \) are assumed to be mutually independent,

\[
Var_t (z_{i,t+1}^E) = b_{i,t}^2 Var_t (z_{i,t+1}^B) + (1 - b_{i,t}^2) Var_t (z_{i,t+1}^E).
\]

Given that the conditional variances of standardized residuals are 1 at all times, Equation 12 restricts the mean and conditional variance of \( z_{i,t+1}^E \) to 0 and 1, respectively.

Second, \( b_{i,t} \) ranges from -1 to 1 (exclusively). The two unknown parameters in \( b_{i,t} \) are...
the same across all country pairs; however, because recession periods in different countries are non-synchronized, the domestic equity-bond comovements are different across countries. Thus, \( \tilde{z}_{t+1}^{E} \) can be referred to as the “bond-purified” component of equity returns.

Third, this return decomposition conveniently implies a “correlation decomposition”. In a matrix representation, I denote \( \text{Corr}_t^E \) (\( N \times N \)) as the conditional correlation matrix\(^2\) of \( \tilde{z}_{t+1}^{E} \) (\( N \times 1 \)), or \( E_t[\tilde{z}_{t+1}^{E} \tilde{z}_{t+1}^{E}'] = \text{Corr}_t^E \). I denote \( b_t \) (\( N \times 1 \)) as a vector of domestic equity-bond comovement. Given the decomposition in Equation (12), the total conditional correlation of equity returns, \( \text{Corr}_t^E \), can be expressed as follows,

\[
\text{Corr}_t^E = \text{diag}(b_t) \text{Corr}_t^B \text{diag}(b_t) + \text{diag}\left( \sqrt{1 - (b_t)^{\odot 2}} \right) \text{Corr}_t^E \text{diag}\left( \sqrt{1 - (b_t)^{\odot 2}} \right),
\]

where “\( \text{diag}(\cdot) \)” is a matrix operator that generates a diagonal matrix with the vector on the diagonal and 0 elsewhere, “\( (\cdot)^{\odot 2} \)” indicates the Hadamard (element-by-element) squares, and \( 1 \) is a \( N \times 1 \) vector of ones; \( \text{Corr}_t^B \) is the conditional equicorrelation matrix of bond returns as formulated in Section 2.2.A.

### 2.2.C Global Equity Comovement with Duo-DECO: \( \text{Corr}_t^E \)

The conditional correlation matrix of the “bond-purified” equity returns is defined as a conditional equicorrelation matrix,

\[
\text{Corr}_t^E = (1 - \tilde{\rho}_t^{E}) I_N + \tilde{\rho}_t^{E} J_{N \times N},
\]

where the equicorrelation is similarly defined:

\[
\tilde{\rho}_t^{E} = \frac{2}{N(N - 1)} \sum_{i > j} \frac{\tilde{q}_{i,j,t}^{E}}{\tilde{q}_{i,t}^{E} \tilde{q}_{j,t}^{E}},
\]

where \( \tilde{q}_{i,j,t}^{E} \) is the (\( i, j \))-th element of a symmetric matrix \( \tilde{Q}_t^E \) (\( N \times N \)) which follows a generalized autoregressive heteroskedastic process, omitting superscript \( E \) for simplicity:

\[
\tilde{Q}_t = \overline{Q} \circ \Phi_t + \beta_1 \left( \tilde{Q}_{t-1}^{\frac{1}{2}} \tilde{z}_t \tilde{z}_t' \tilde{Q}_{t-1}^{\frac{1}{2}} - \overline{Q} \circ \Phi_{t-1} \right) + \beta_2 \left( \tilde{Q}_{t-1} - \overline{Q} \circ \Phi_{t-1} \right) + \gamma \left( \tilde{Q}_{t-1}^{\frac{1}{2}} \tilde{n}_t \tilde{n}_t' \tilde{Q}_{t-1}^{\frac{1}{2}} - \overline{Q} \circ \Phi_{t-1} \right),
\]

where \( \overline{Q} \) is the pre-determined equity return correlation matrix, \( \tilde{n}_t = I_{\tilde{z}_t < 0} \tilde{z}_t, \overline{\Xi} = E \left[ I_{\tilde{z}_t < 0} \tilde{z}_t < 0 \right] \), and similarly \( \Phi_t \) is the cyclical component of the long-run conditional mean.

### 2.3 Estimation Procedure

I follow the dynamic conditional correlation literature (e.g., Engle, 2002; Engle and Kelly, 2012) to pre-estimate the return conditional variances of each return series independently. I use the Maximum Likelihood Estimation (MLE) methodology to obtain the conditional variance estimates for each return series, and standardize return residuals with the best conditional volatility estimates given the Akaike information criterion (AIC) and Bayesian Information

\( ^2 \text{Corr}_t^E \) is a conditional correlation matrix in general; to be more accurate, I define it as a conditional equicorrelation matrix in Section 2.2.C.
Criteria (BIC). I consider four conditional variance models (see Appendix A) and four univariate
distributions (see Appendix B).

To estimate the global equity and bond correlations, I adopt a two-step procedure:

**Step 1, Bond Comovement.** Because there is no feedback from equity returns to bond
returns, I first estimate global bond correlations. According to Section 2.2.A, the bond model
has up to 4 (5) unknown parameters, \( \beta_1, \beta_2, (\nu, \phi) \). The sufficient conditions for \( Q_t^B \)
to be stationary are \( \beta_1 J N \times N + \beta_2 J N \times N + \gamma \Xi < J N \times N \) and \( \beta_1, \beta_2 > 0 \). The proof is relegated
to Appendix C. No other parameter restrictions are imposed. The global bond correlation
is estimated using the MLE methodology where I allow for two multivariate distributional
assumptions:

1. Multivariate Gaussian MLE. The log likelihood \( L^B \) is the sum of a constant and
   \[
   \frac{1}{2} \sum_{t=1}^{T} \left( \log |\text{Corr}_t^B| + z_t^B (\text{Corr}_t^B)^{-1} z_t^B \right).
   \]

2. Multivariate \( t \) MLE. The log likelihood \( L^B \) is the sum of a constant and
   \[
   \frac{1}{2} \sum_{t=1}^{T} \left( \log |\text{Corr}_t^B| + (df + N) \log \left( 1 + \frac{1}{df} z_t^B (\text{Corr}_t^B)^{-1} z_t^B \right) \right),
   \] where \( df \) is the degree
   of freedom of the \( N \)-variate \( t \) distribution (see Kotz and Nadarajah, 2004; Genz and Bretz,
   2009).

The best estimates of \( \text{Corr}_t^B \), according to the AIC and BIC, are used in the Step 2 estimation.

**Step 2, Equity Comovement.** As a key feature in the Duo construct, the return
decomposition implies a correlation decomposition as shown in Equation (15). That is, the total
equity correlation \( \text{Corr}_t^E \) is a “weighted” average of the estimated bond correlation (from Step
1) and the bond-purified equity correlation \( \text{Corr}_t^E \) (this step)—\( \text{diag}(b_t) \text{Corr}_t^B \text{diag}(b_t) +
\text{diag} \left( \sqrt{1 - (b_t)^2} \right) \text{Corr}_t^E \text{diag} \left( \sqrt{1 - (b_t)^2} \right) \)—where the time-varying weights (this step)
are strictly positive by design. Therefore, the equity model has up to six unknown parameters,
\( \{\delta_1, \delta_2, \beta_1, \beta_2, \gamma, \phi\} \). It is noteworthy that DECO is a special case of Duo-DECO; when
\( \delta_1 = \delta_2 = 0, b_t \) is 0 for all countries during all periods. The stationary conditions for \( Q_t^E \)
are similar to the ones for \( Q_t^B \). As in the first step, the total global equity correlation is estimated
using the MLE methodology and two multivariate distributional assumptions. Model selection
relies on the AIC and BIC.

**2.4 Data**

I use monthly USD-denominated log returns of eight developed countries: the United
States, USA; Canada, CAN; Germany, DEU; France, FRA; United Kingdom, GBR; Switzerland,
CHE; Japan, JPN; Australia, AUS. Log equity returns refer to changes in the log total
return index of the domestic stock market (United States: S&P500; Canada: S&P/TSX 60;
Germany: DAX 30; France: CAC 40; United Kingdom: FTSE 100; Switzerland: SMI; Japan:
NIKKEI 225; Australia: S&P/ASX 200); the CRSP value-weighted return (including dividend)

---

\(^3\)The equality test parameter \( \nu \) is not considered in the full model because estimation results with \( Q = Q^E + \nu (J N - J N) \) do not exploit the full cross-country information in the true unconditional correlation
matrix of bond return.
is used as the USA equity return; for other countries, the total return index can be obtained from DataStream. Log bond returns refer to changes in the log 10-year government bond index constructed by DataStream. The sample is from March 1987 to December 2016 (a total of 358 months). Country and world recession indicators are obtained from the OECD website.

Table 1 shows the summary statistics of log returns, with mean and standard deviations presented as annualized percentages. According to Panel A, the average of all pairwise unconditional correlations of raw log equity returns (before standardization) is 0.639, and that of raw log bond returns is 0.465. Using standardized returns, the average equity return correlation is 0.627, whereas the average bond return correlation is 0.461. In Panel B, U.S. equity and bond return volatilities are both the lowest among the eight countries, which is expected because returns for other countries are denominated in USD. In fact, when expressed in local currencies, the U.S. equity volatility remains the lowest. I comment on comovements in local currencies in the conclusions.

2.5 Estimation Results

In this section, I discuss the estimation results of the global equity and bond return comovements.

2.5.A Global Bond Comovement

The parameter estimates of global bond comovement models are reported in Table 2. Model “B(1)” is Engle and Kelly (2012)’s DECO model. Recall, DECO is a special case of the model in the current paper (Section 2.2.A); therefore, the DECO model is used as an informative null hypothesis to test whether there are improvements on the statistical fit by introducing a time-varying long-run conditional mean, and correlation asymmetry. According to the standard model selection criteria (i.e. AIC and BIC), models with the asymmetry term denoted as Model “B(2)” perform the best. Between the two multivariate distributional assumptions, fitting standardized bond returns with a multivariate \(t\) distribution consistently increases the statistical fit (in terms of likelihoods, AIC, and BIC), demonstrating that the data exhibits comovement non-Gaussianity.

The conditional equicorrelation process in the best model is highly persistent \((\beta_2 = 0.9017)\). The asymmetry parameter \(\gamma\) is borderline significant in the best model \((\gamma = 0.0263, t = 1.654, \text{one-sided } p\text{-value} = 0.0645)\), and remains borderline significant after controlling for the time-varying long-run conditional mean \((\phi = -0.0420)\) as in Model B(4). Therefore, I fail to reject the null that bond correlations are symmetric at the 5% significance level. Note that Model B(4) performs better than the best model in terms of the AIC. According to Model B(4), the cyclicality parameter \(\phi\) is estimated to be -0.0420 \((t = -1.76, \text{one-sided } p\text{-value} = 0.05)\), which indicates a weakly procyclical global bond comovement. Economically, the long-run conditional mean during recession periods is significantly lower than that during non-recession periods by an average of 0.086 \((i.e., -\frac{0.0420}{0.491} \text{ where } 0.491 \text{ is the standard deviation of the OECD world recession indicator})\). As a result, I find that global bond comovement is a persistent, weakly procyclical, and symmetric process.
Model B(5) conducts the equality test. By construction, the inequality parameter $\nu$ captures the average difference between the off-diagonal terms of equity and bond return correlation matrices. Using the multivariate Gaussian distributional assumption, $\nu$ is significant and negative (-0.275), suggesting that the bond correlations are on average smaller than equity correlations; a significant and negative $\nu$ is also found using the multivariate $t$ distributional assumption.

In Figure I, I graph the time variation in global bond comovement given the best model (dotted black line) together with the OECD world recessions (shaded regions). The correlation between the OECD world recession indicator and the global bond comovement is -0.0981 (two-sided p-value = 0.0635), which is consistent with the finding of weak procyclicality in the parameter estimation results above. Global bond comovement experienced a two-year drop beginning in 1992, and bounced back in 1994 for the next 17 years or so. The biggest monthly increase occurred during July 1987, which coincided with the Single European Act of 1987. The second biggest monthly increase occurred during January 1999 when the euro was formally introduced. In my sample, four out of eight countries are European. The formation of the monetary union in Europe certainly increased comovements among national assets. This increase in bond comovement around January 1999 is consistent with the pairwise evidence shown by Cappiello, Engle, and Sheppard (2006). The biggest monthly drop occurred during October 2008, the peak of financial crisis after the Lehman Brothers collapse; in two months, the global bond correlation dropped from 0.54 to 0.36. The Online Appendix shows that, during the same period, global bond return variances experienced the biggest increase since 1987—partly through an increase in currency volatilities—further contributes to the drop in correlation. However, during the full sample period, the dynamics of volatilities and correlations (given my estimations) are uncorrelated ($\rho = -0.0608$, p-value = 0.251).

2.5.B Global Equity Comovement

In Table 3, I report the estimation results of the global equity comovement model, given the best global bond comovement estimates. The multivariate $t$ distributional assumption improves the model fit, in terms of likelihood, the AIC, and the BIC, across all models. Model “E(1)” is the Duo system in which the purified equity return correlation is modeled with Engle and Kelly (2012)’s DECO model. The parameter estimates of $\gamma$ are significant and positive whether controlling for the time-varying long-run conditional mean or not, which rejects the null that global equity comovement is symmetric. The positive sign of $\gamma$ supports excessive downside comovement. This finding is consistent with the literature (Longin and Solnik, 2001; Ang and Chen, 2002; Cappiello, Engle, and Sheppard, 2006; among many others). Next, the cyclicality parameter $\phi$ is significant and positive—around 0.04—in all models. In particular, the $\phi$ estimate in the best model—Model “E(4)” with the multivariate $t$ distribution—is 0.0457 (p-value=0.022), which can be interpreted as follows: the long-run conditional mean during recession periods is significantly higher than that during non-recession periods by an average of 0.093 (i.e., $0.0457 / 0.391$).

---

4The equality test is discussed separately here because it does not improve statistical fit, but is applied as a way to test equality within my dynamic model.
The Duo part—consisting of the country-specific domestic comovement process \( b_{i,t} \) and the return decomposition/correlation decomposition—uses minimal assumptions to potentially capture the FTS channel. In Table 3, \( \delta_2 \) is estimated to be significant and negative in all models, suggesting a procyclical \( b_{i,t} \). The moment matching exercise in Appendix Table Aii enhances the estimation plausibility. In Appendix Table Aiii, I re-estimate the equity correlation models using a continuous business cycle variable: the standardized country industrial production growth, which is a procyclical business indicator. Although the standard model selection with the AIC and BIC rejects these models in Appendix Table Aiii, procyclical domestic comovement is still found with significant and negative \( \delta_2 \) estimates. Moreover, I show that models that include the Duo part perform better than those that do not. In Appendix Table Aiv, Model E(13) with multivariate \( t \) distribution retains all the features of the best equity model (Table 3) except for the restriction that \( \delta_1 = \delta_2 = 0 \), and performs worse than the best equity model in terms of likelihood, the AIC, and the BIC. Note that it is hard to make an exact comparison among multivariate \( t \) models because the shapes of the multivariate \( t \) distribution (governed by \( df \)) are estimated to be different; however, the multivariate Gaussian models without the Duo part are rejected by those with the Duo part, once introducing realistic features—such as asymmetry and cyclical conditional means—according to Appendix Table Aiv.

Figure 1 depicts the time variation in global equity comovement (solid red line), given the best model. Two interesting observations emerge. First, at 0.218, the equity correlation has a weak (but statistically significant) positive correlation with the bond correlation. However, their movements diverge during recession periods, for instance during the early 1990’s recessions (Gulf war), the 1994 Mexican economic crisis, and the 1998 Asian crisis. The recent 2007-08 global financial crisis and the 2012 European debt crisis witness the largest differences between global equity and bond comovements in my sample: 0.50 and 0.36, respectively. This observation confirms my findings of countercyclical global equity comovement and procyclical global bond comovement. Apart from recession periods, global equity comovements also peaked during the October 1987 global stock market crash.

Second, during the sample period, a significant and positive upward trend is found in global equity comovement. However, there is no evidence of a positive trend for global bond comovement (denominated in USD). Global equity and bond comovements both reached a low point after the euro debt crisis in January 2015, when Switzerland’s central bank stunned financial markets by abandoning a cap limiting the value of the Swiss franc against the euro and caused high currency volatility. Therefore, this drop in correlations during a non-recession period is volatility-induced.

Given the estimation results of the parametric model, this paper formally establishes the following:
Stylized Fact 1: Bond return correlations are smaller in magnitude than equity return correlations.

Stylized Fact 2: Equity return correlations are higher following joint negative shocks, while bond return correlations are symmetric.

Stylized Fact 3: Equity return correlations are strongly countercyclical, while bond correlations are weakly procyclical.

2.6 Model Fit

In this section, I document the stylized facts using unconditional, non-parametric data moments. Then, I compare these data moments with model moments implied from simulated datasets of the parametric models described in Sections 2.2.A–2.2.C in order to evaluate the model fit. Given the estimation results assuming multivariate $t$ or multivariate Gaussian distributions, country equity and bond returns are simulated 1,000 times with finite samples ($T=358$) and exogenous variables (country and world OECD recession indicators). Although the multivariate Gaussian model is rejected, I simulate it for the purpose of comparison, which serves as the null hypothesis when evaluating the fit of asymmetry.

2.6.A Model Fit: Equality

In Table 4, I test the equality between pairwise equity and bond unconditional correlations using the Jennrich (1970)'s $\chi^2$ test (see the description in Appendix D). According to Panel A of Table 4, the average pairwise correlation using standardized returns during the sample period is 0.6271 for equities and 0.4606 for bonds; the difference is significant ($\chi^2 = 227.087$ under the degree of freedom 28). The average global conditional correlations from the best models, denoted with “Conditional Model”, are 0.6568 for equities and 0.4926 for bonds, which are insignificantly different from the data moments. However, the unconditional correlation does not equal the average of conditional correlations. Thus, I calculate the average of the pairwise unconditional correlations using the simulated datasets. The best model is denoted with “Simulated Model (t)”, and I fail to reject that the model moments equal the data moments at the 5% significance test. In addition, I examine the fit in three 10-year subsamples and find: (1) global equity comovements are significantly higher than global bond comovements in all three subsamples; and (2) the simulation moments calculated using the best model are statistically close to the actual data moments. The subsample results also capture the widening equity-bond correlation difference after the 2007-08 financial crisis, as also found in the parametric model estimation results (see Section 2.5).

2.6.B Model Fit: Asymmetry

In order to replicate the second stylized fact non-parametrically, I follow Longin and Solnik (2001) and Ang and Chen (2002) and use exceedance correlations to demonstrate correlation asymmetry. The core objective is to quantify the comovement of return realizations that
are jointly in the lower or upper parts of their distributions. Figure 2 depicts exceedance correlations from the 20th to 80th quantiles. Apart from the data exceedance correlations, I compare exceedance correlations implied by three global comovement models from Tables 2 and 3: (1) Best models assuming the multivariate \( t \) distribution (“B (2)”, Table 2, “E (4)”, Table 3); (2) Models assuming the multivariate \( t \) distribution but with no asymmetry term (“B (1)”, Table 2, “E (3)”, Table 3); and (3) Best models assuming the multivariate Gaussian distribution (“B (2)”, Table 2, “E (4)”, Table 3). Simply put, I demonstrate the contribution of correlation asymmetry, and non-Gaussianity, by comparing the exceedance correlations implied by models in (1) and (2), and (1) and (3), respectively.

I discuss four relevant observations next. First, the data reveal significant asymmetry in equity return correlations. The equity plot in Figure 2 demonstrates a clear gap around the median, which is consistent with the literature. To be more specific, according to Table 5, the exceedance correlation jumps from 0.2619 at the 50th quantile to 0.3292 at the 49th quantile, and the gap is statistically significant. In contrast, symmetry in bond correlations is not rejected.

Second, according to the equity plot in Figure 2, the best model using the multivariate \( t \) distribution is able to match general patterns as seen in the data: equity exceedance correlations significantly increase around the median, whereas the bond exceedance correlation pattern is smooth. Quantitatively, bond exceedance correlations from the simulated datasets of the best model (see Row “Simulated Model (t)” of the bottom panel of Table 5) are within 95% confidence intervals of the data exceedance correlations across the spectrum of threshold quantiles; the Wald test that jointly test 4 quantiles-of-interest as indicated in this table fails to reject the null that the simulated best model is close to the data (\( \chi^2 = 3.07, \) p-value = 0.55). On the other hand, the best model for equity return comovements is able to match the general pattern and most key exceedance correlations (i.e., at the 25th, 49th, and 75th quantiles, according to the top panel of Table 5). However, the joint Wald test rejects the null that the model statistically matches the data. It is noteworthy that, for equities, an exact fit of asymmetry across threshold quantiles is not expected because of the slightly different asymmetry identification criteria. In my model, asymmetry is introduced when both returns are negative, whereas in the exceedance correlation literature, asymmetry arises when both returns are below the median. In my sample, the 43rd quantile, rather than the median, is where all equity returns turn negative; 14 out of 28 unique country pairs show joint negative returns at the 46th quantile.

Third, I simulate models without asymmetry to demonstrate that it is the asymmetry term in the new econometric model that explains the gap (around the median). In Table 5, with regard to equities, the model without asymmetry is strongly rejected (see Row “Simulated Model (t), No Asymmetry”). For bonds, the model-implied point estimates are mostly within 95% confidence interval, but the joint test is rejected at the 5% level. Interestingly, among the three models, the model with a multivariate \( t \) distribution but without asymmetry is closer to the data than the model with a multivariate Gaussian distribution and an asymmetry term. This supports the discussion in Campbell et al. (2008) about the importance of “fat” tails in understanding correlation asymmetry.

the value of a given percentile \( \tau \) for variable \( x \). Global exceedance correlations can be further defined as equally-weighted bivariate exceedance correlations across 28 unique country pairs. Exceedance correlations are typically found to be smaller than time-series correlations.
Lastly, Gaussian models do not produce enough asymmetry or capture fat tails for either equity or bond returns, and are thus rejected. This result is consistent with the estimation results (Tables 2 and 3). The tent-shaped exceedance correlation with Gaussian fundamentals is also typically found in the literature (see e.g. Ang and Bekaert, 2002; Campbell et al., 2008). The difference between equity downside comovements calculated using data, and equity downside comovements using the simulated dataset grows wider, as the threshold quantile grows smaller. The joint test regarding fitting data for all four exceedance correlations is rejected for both equity and bond Gaussian models of the 1% level.

2.6.C Model Fit: Cyclicality

In Table 6 I calculate the average pairwise correlations during OECD recession and non-recession periods within both asset classes. During non-recession periods, the pairwise equity return correlation averages 0.5952, which is significantly different from the 0.6571 average found during recession periods. The bond return correlation is slightly higher during non-recession periods, which is consistent with the weak procyclicality result from the model estimation. These unconditional patterns are replicated in the parametric model. For the simulated best model (“Simulated Model (t)”), all correlations are within 95% confidence interval of the data moments and the recession / non-recession pattern is matched for both equity and bond return correlations.

3 Economic Determinants

Asset return innovations can be explained by either cash flow or discount rate shocks. Therefore, the economic determinants of global comovements are the second moments of these shocks that are commonly priced in individual country assets. In Section 3.1, I formalize this intuition and identify the economic determinants using a dynamic no-arbitrage asset pricing model that consistently prices international equities and Treasury bonds. Then, I provide the estimation strategy and results of these economic determinants in Section 3.2. A dynamic factor model implied from the theoretical model solution is estimated using actual return data in Section 4.

3.1 An Asset Pricing Model

The reduced-form asset pricing model is defined by a global real pricing kernel and state variables. In this paper, I focus on the perspective of a U.S. (global) investor, which is consistent with the previous empirical section (i.e., USD-denominated returns). The U.S. state variables are proxies for global state variables. The global model has closed-form solutions, which immediately motivates the economic determinants of the global conditional comovements. Given that the asset moment of interest in this paper is a second moment, the shocks structure and the dynamics of the economic determinants are modeled carefully with realistic and sophisticated dynamic processes. The focus of the present research is to evaluate the ability of a global model to interpret all three stylized facts.
Note that under the assumption of market completeness, a reduced-form international asset pricing model assuming partial integration (i.e., the existence of different real pricing kernels for each country) can be shown to have similar model implications on the economic determinants of global comovements. A detailed solution is derived in closed form in Appendix F.

3.1.A The Global Real Pricing Kernel

The real pricing kernel in this paper is a variant of the Bekaert, Engstrom, and Xu (2017; BEX) kernel, but accommodates more economic state variables in order to price nominal bonds. The minus real pricing kernel is,

\[-m_{t+1} = x_t + J_t + \delta'_m \left[ \omega_{\theta u,t+1} \omega_{\theta d,t+1} \omega_{\pi u,t+1} \omega_{\pi d,t+1} \omega_{q,t+1} \right], \tag{19}\]

where \(x_t\) is the real short rate, \(J_t\) Jensen’s inequality term (see Appendix F for a full expression) and \(\delta_m\) a 5-by-1 constant vector, \(\left[ \delta_{\theta u} \delta_{\theta d} \delta_{\pi u} \delta_{\pi d} \delta_q \right] \). The five kernel shocks include real upside and downside uncertainty shocks, \(\omega_{\theta u,t+1}\) and \(\omega_{\theta d,t+1}\), inflation upside and downside uncertainty shocks, \(\omega_{\pi u,t+1}\) and \(\omega_{\pi d,t+1}\), and a preference shock \(\omega_{q,t+1}\). All shocks are non-Gaussian, heteroskedastic and mutually independent. I present each of the kernel state variables and shocks in more detail in Sections 3.1.B–3.1.C.

Note that the first four shocks—filtered from two macro fundamental processes, real output growth and inflation—are referred to as “uncertainty” shocks. The non-Gaussian property of these macro shocks allows them to govern the shape of one tail of the fundamental distribution at a time, e.g. upside and downside output growth shocks. Take the real uncertainty shocks for example. Decomposed from the output growth innovation, the two real shocks also drive the dynamics of the upside and downside uncertainties, respectively, in order to capture this simple intuition: increases in upside (“good”) or downside (“bad”) uncertainty are likely to coincide with increases or decreases in the fundamentals, e.g. output growth is likely to increase when good volatility increases. Therefore, with no restrictions on \(\delta_m\), Equation (19) implicitly assumes that both level shocks and uncertainty shocks drive the real pricing kernel. Formal mathematical expressions are presented in Section 3.1.B.

Regarding the economic motivation, consumption-based models (as in Campbell and Cochrane, 1999) allow real fundamental shocks to span the real pricing kernel. BEX additionally consider in an orthogonal preference shock. The presence of inflation uncertainty shocks may induce an inflation risk premium, which is an important component of the nominal term structure.

Assets are risky to the extent that they have negative returns when the macroeconomic environment is in a “bad” state defined by realizations of macro shocks, and when risk aversion increases, as captured by a positive preference shock. The quantity of economic risk is measured by the second moments of the real and inflation shocks. Both the quantity and price of preference

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6The BEX pricing kernel is derived from the general HARA utility function \(U(C) = \frac{(C/Q)^{1-\gamma}}{1-\gamma}\); when \(Q\) increases, consumption delivers less utility, and marginal utility increases. A special case can be found in Campbell and Cochrane, where \(Q = \frac{C}{H}\) is the inverse surplus consumption ratio and \(H\) represents the habit stock. In this case, the relative risk aversion is \(\gamma Q\). Therefore, there are two drivers for log real pricing kernel \(m_{t+1}\): consumption growth \(\Delta c\) and changes in log relative risk aversion \(\Delta q\). In BEX, \(\Delta c\) is spanned by real economic fundamental shocks (filtered from industrial production growth) and cash flow shocks; \(\Delta q\) is spanned by fundamental shocks and an orthogonal preference shock.
risk are determined by the second moment of the preference shock.

3.1.B The Global Macro Environment

Macro shocks are likely to be non-Gaussian and asymmetric with time-varying volatilities (see e.g. Hamilton, 1990; Fagiolo, Napoletano, and Roventini, 2008; Gambetti, Pappa, and Canova, 2008). Therefore, I adopt the “Bad Environment-Good Environment” (BEGE) framework of Bekaert and Engstrom (2017), to construct the innovations of industrial production growth and inflation. BEGE is particularly suitable in this paper because it admits heteroskedasticity and non-Gaussianity in shocks, while the theoretical model remains in the affine asset pricing class, which demonstrates simplicity.

Real-Side Shocks and Uncertainties I follow BEX in modeling the change in the log industrial production index, \( \theta_{t+1} \). The innovation is decomposed into two independent shocks each period, one governing the upside skewness (\( \omega_{\theta u,t+1} \)) and the other one governing the downside skewness (\( \omega_{\theta d,t+1} \)):

\[
\theta_{t+1} = m_{\theta,t} + \delta_{\theta u} \omega_{\theta u,t+1} - \delta_{\theta d} \omega_{\theta d,t+1},
\]

where the conditional mean \( m_{\theta,t} \) is a persistent process to accommodate a time-varying long-run mean of output growth\(^7\) and the two shocks follow centered gamma distributions with time-varying shape parameters:

\[
\omega_{\theta u,t+1} \sim \tilde{\Gamma}(\theta_{u,t}, 1), \quad \omega_{\theta d,t+1} \sim \tilde{\Gamma}(\theta_{d,t}, 1),
\]

where \( \tilde{\Gamma}(x, 1) \) denotes a centered gamma distribution with shape parameter \( x \) and a unit scale parameter. The shape factors, \( \theta_{u,t} \) and \( \theta_{d,t} \), follow autoregressive processes,

\[
\theta_{u,t+1} = \bar{\theta}_u + \rho_{\theta u}(\theta_{u,t} - \bar{\theta}_u) + \delta_{\theta u} \omega_{\theta u,t+1},
\]

\[
\theta_{d,t+1} = \bar{\theta}_d + \rho_{\theta d}(\theta_{d,t} - \bar{\theta}_d) + \delta_{\theta d} \omega_{\theta d,t+1}.
\]

In this paper, \( \theta_{u,t} \) and \( \theta_{d,t} \) govern the higher conditional moments of the real upside and downside shocks, respectively (see Appendix E for properties of a gamma-distributed shock). More specifically, because the two shocks are mutually independent, the conditional variance and the conditional unscaled skewness of output growth are as follows,

\[
\text{Conditional Variance: } \delta_{\theta u}^2 \theta_{u,t} + \delta_{\theta d}^2 \theta_{d,t},
\]

\[
\text{Conditional Unscaled Skewness: } 2\delta_{\theta u} \theta_{u,t} - 2\delta_{\theta d} \theta_{d,t}.
\]

This reveals why \( \theta_{u,t} \) represents “good” and \( \theta_{d,t} \) represents “bad” volatility. \( \theta_{u,t} \) (\( \theta_{d,t} \)) increases (decreases) the skewness of industrial production growth, and thus represents the real upside (downside) uncertainty at time \( t \). Shocks \( \theta_{u,t+1} \) and \( \theta_{d,t+1} \) represent real upside and downside uncertainty shocks, respectively.

Nominal-Side Shocks and Uncertainties Regarding modeling the inflation process, I allow inflation to respond to output shocks, which is approximately in line with a standard New Keynesian AS curve relating inflation to the output gap. To be specific, inflation is assumed to

\[
\tilde{m}_{\theta,t} = \bar{\theta} + \rho_{\theta}(\theta_t - \bar{\theta}) + m_{\theta_u}(\theta_{u,t} - \bar{\theta}_u) + m_{\theta_d}(\theta_{d,t} - \bar{\theta}_d).
\]
have constant exposures to the two real shocks, and the residual is decomposed into two nominal-
side shocks governing the behavior of left- or right-tail, respectively. Denote \( \pi_t \) as the change in
the log consumer price index for all urban consumers, \( \pi u_t \) the nominal upside uncertainty, and
\( \pi d_t \) the nominal downside uncertainty. Inflation system follows these reduced-form dynamics,
\[
\pi_{t+1} = m_{\pi,t} + (\delta_{\pi u}\omega_{\pi u,t+1} + \delta_{\pi d}\omega_{\pi d,t+1}) + \delta_{\pi u}\omega_{\pi u,t+1} - \delta_{\pi d}\omega_{\pi d,t+1},
\]
where the conditional mean is a persistent process\(^8\), and the two inflation shocks follow centered
gamma distributions with time-varying shape parameters,
\[
\omega_{\pi u,t+1} \sim \Gamma (\pi u_t, 1), \quad \omega_{\pi d,t+1} \sim \Gamma (\pi d_t, 1).
\]
The time-varying shape parameters follow autoregressive processes,
\[
\pi u_{t+1} = \pi u_t + \rho_{\pi u}\pi u_t - \pi u_t + \delta_{\pi u}\omega_{\pi u,t+1}
\]
\[
\pi d_{t+1} = \pi d_t + \rho_{\pi d}\pi d_t - \pi d_t + \delta_{\pi d}\omega_{\pi d,t+1},
\]
Similarly, \( \pi u_t \) and \( \pi d_t \) (\( \omega_{\pi u,t+1} \) and \( \omega_{\pi d,t+1} \)) represent nominal upside and downside uncertainties
(uncertainty shocks), respectively.

The four macro shocks, both real and nominal, are by design mutually independent, centered gamma shocks with heteroskedastic shape parameters. Thus, the conditional variance and the conditional unscaled skewness of inflation are,

\[
\text{Conditional Variance: } (\delta_{\pi u}^2 \theta_{u,t} + \delta_{\pi d}^2 \theta_{d,t}) + (\delta_{\pi u}^2 \pi u_t + \delta_{\pi d}^2 \pi d_t),
\]
\[
\text{Conditional Unscaled Skewness: } (2\delta_{\pi u}^3 \theta_{u,t} - 2\delta_{\pi d}^3 \theta_{d,t}) + (2\delta_{\pi u}^3 \pi u_t - 2\delta_{\pi d}^3 \pi d_t).
\]

### 3.1.3 Global Risk Aversion

The state variable capturing risk aversion, \( q_t \), according to BEX is motivated using a
representative agent economy with a HARA utility. In the present research, the stochastic risk
aversion process follows a reduced-form process:
\[
q_{t+1} = m_{q,t} + \delta_{q u} \omega_{q u,t+1} + \delta_{q d} \omega_{q d,t+1} + \delta_{q u} \omega_{q u,t+1} + \delta_{q d} \omega_{q d,t+1} + \delta_{q u} \omega_{q u,t+1},
\]
where the conditional mean \( m_{q,t} \) is a linear function of \( q_t, \theta_t, \theta u_t, \theta d_t, \pi_t, \pi u_t \) and
\( \pi d_t \), and the preference innovation receives macro shocks and an orthogonal preference shock which follows
a centered heteroskedastic gamma distribution,
\[
\omega_{q,t+1} \sim \Gamma (q_t, 1).
\]
Note that \( q_t \) does not have a feedback effect on the macro variables, which enables \( \omega_{q,t+1} \) to represent a non-fundamental preference shock. The variance and unscaled skewness of \( \omega_{q,t+1} \) are proportional to its level: While controlling for current business conditions, when current
risk aversion is higher, there is a greater chance that the future preference shock will see a large
and positive realization.

The risk aversion disturbance contains three parts: exposure to real shocks as motivated by
consumption-based habit models, exposure to nominal shocks as suggested by Brandt and Wang
(2003), and the preference shock as appeared in BEX. Given the distributional assumptions
\[
\sum_{m_{q,t}} = \pi + \rho_{\pi u}(\theta_{u,t} - \pi) + \rho_{\pi u} \theta_{u,t} - \pi_u) + \rho_{\pi d}(\theta_{d,t} - \pi d) + \rho_{\pi u}(\pi_u - \pi) + \rho_{\pi u}(\pi u_t - \pi u) + \rho_{\pi u}(\pi d_t - \pi d).
\]
19
regarding these shocks, the model-implied conditional variance of risk aversion in the current paper is 
\[
\delta^2 \theta_u \theta_{ut} + \delta^2 \theta_d \theta_{dt} + \delta^2 \pi_u \pi_{ut} + \delta^2 \pi_d \pi_{dt} + \delta^2 \theta_t, \quad \text{and the conditional unscaled skewness}
\]
\[
(2\delta^3 \theta_u \theta_{ut} + 2\delta^3 \theta_d \theta_{dt}) + (2\delta^3 \pi_u \pi_{ut} + 2\delta^3 \pi_d \pi_{dt}) + 2\delta^3 \theta_t.
\]
In other words, the higher moments of risk aversion are perfectly spanned by macroeconomic uncertainties on one hand and pure sentiment on the other.

As mentioned before, the idea that fundamental shocks span the risk aversion process is motivated by Campbell and Cochrane (1999) where the real pricing kernel is spanned by one shock. At first glance, the CC model looks very different as non-linearities in the pricing kernel are introduced through time-varying sensitivity function of the surplus consumption ratio \(-q_t\) to consumption shocks, whereas in my model the kernel shock exposures appear time-invariant. However, the framework here, as well as in BEX, does not explicitly require a time-varying sensitivity function to induce non-linearity in the real pricing kernel. To be more specific, the conditional variance of \(m_{t+1}\) in Campbell and Cochrane (1999) is \(\gamma^2(1 + \lambda_t)^2\sigma^2\), where \(\sigma^2\) is the constant variance of consumption growth, \(\gamma\) the constant curvature parameter, and \(\lambda_t\) the time-varying sensitivity function that causes non-linearity in the pricing kernel. In my paper, the conditional variance of \(m_{t+1}\) is \(\delta^2 \theta_u \theta_{ut} + \delta^2 \theta_d \theta_{dt} + \delta^2 \pi_u \pi_{ut} + \delta^2 \pi_d \pi_{dt} + \delta^2 \theta_t\). The presence of \(q_t\) alone suffices to make it non-linear as it has an asymmetric non-Gaussian shock.

### 3.1.D Real Short Rate

The reduced-form process of the real short rate is as follows,
\[
x_{t+1} = m_{x,t} + f_x (\omega_{u,t+1}, \omega_{d,t+1}, \omega_{\pi_u,t+1}, \omega_{\pi_d,t+1}, \omega_{q,t+1}) + \delta_x \omega_{xu,t+1} - \delta_x \omega_{xd,t+1},
\]  
(28)
Fundamental & Preference shock exposures

where the conditional mean is a linear function of \(x_t\), \(x_{u_t}\) (later), \(x_{d_t}\) (later), \(q_t\), \(\theta_t\), \(\theta_{ut}\), \(\theta_{dt}\), \(\pi_t\), \(\pi_{ut}\) and \(\pi_{dt}\). The real short rate innovation has a systematic component which not only comprises the fundamental shocks but also the risk aversion shock. Note that this is potentially consistent with the Campbell and Cochrane model reflecting the effect of risk aversion on the real short rate (through precautionary savings and intertemporal substitution effects). The residual is then decomposed into two centered gamma distributions with autoregressive shape parameters,
\[
\omega_{xu,t+1} \sim \Gamma(x_{u_t}, 1), \quad \omega_{xu,t+1} = \pi_{u_t} + \rho_{xu}(x_{u_t} - \pi_{u_t}) + \delta_{xu}\omega_{xu,t+1},
\]  
(29)
\[
\omega_{xd,t+1} \sim \Gamma(x_{d_t}, 1), \quad \omega_{xd,t+1} = \pi_{d_t} + \rho_{xd}(x_{d_t} - \pi_{d_t}) + \delta_{xd}\omega_{xd,t+1}.
\]  
(30)
The two real short rate shocks can be interpreted as discretionary monetary policy shocks in my framework. There is no feedback from real short rate shocks to the risk aversion process.

### 3.1.E Closed-Form Solution

To price U.S. equities, the cash flow growth processes are modeled to receive global real shocks \((\omega_{u_t} \text{ and } \omega_{d_t})\), and the residual is referred to as the global cash flow shock (a homoskedastic shock denoted \(\omega_g\)). In order to price individual country equities, cash flow processes are projected on global shocks \((\omega_{u_t}, \omega_{d_t} \text{ and } \omega_g)\), and the idiosyncratic residuals are assumed to be homoskedastic and mutually independent (across countries). In order to price nominal assets,
individual inflation processes are subject to global real and nominal shocks and an idiosyncratic shock.

This global model, from the perspective of a global investor, has a closed-form solution that fits in the affine class of asset pricing models because the moment generating function of a gamma shock is exponentially affine. The government bond prices can be expressed as an exact affine function of the state variables. The equity price-dividend ratios can be expressed as a quasi-affine function of the state variables. For either asset of country $i$, equity or long-term government bond, the log asset return can be written as the following general process,

$$r_{i,t+1} = E_t(\tilde{r}_{i,t+1}) + Global\ Shock\ Exposure + Orthogonal\ Idiosyncratic\ Residual$$

where the global shocks are: the six heteroskedastic economic uncertainty shocks, the heteroskedastic preference shock ($\omega_q$), and the homoskedastic cash flow shock ($\omega_g$). The approximation error is primarily introduced to account for the quasi-affine solutions of equity price-dividend ratios; for simplicity, the error is assumed homoskedastic and Gaussian. Because a linear combination of Gaussian shocks still follows a Gaussian distribution, the country return processes can be further written as a global dynamic factor model with seven heteroskedastic gamma shocks,

$$\{\omega_{\theta u}, \omega_{\theta d}, \omega_{xu}, \omega_{xd}, \omega_{\pi u}, \omega_{\pi d}, \omega_q\},$$

and a single homoskedastic Gaussian residual that is orthogonal to the seven heteroskedastic global factors and allowed to have non-zero covariances with other country residuals due to its exposure to the homoskedastic global cash flow shock.

As a result, the model solution indicates a reduced-form dynamic factor model, implying that the time variation in the global return comovement is driven by the time variation in the second moments of relevant global shocks. Therefore, the second moments of global shocks are motivated as the economic determinants of global comovements. It immediately follows that the difference in global equity and bond comovements can be explained by the different sensitivities of equity and bond returns to (some of) these global shocks.

### 3.2 The Identification of the Economic Determinants

In what follows, I describe the estimation strategy as well as estimation results of these economic determinants described in Section 3.1.E.

#### 3.2.A Procedure

**Real and Inflation Uncertainties:** I pre-filter the real uncertainties $\theta u_t$ and $\theta d_t$ using the monthly changes in the log industrial production index (source: FRED), to ensure these variables are identified from macroeconomic information alone, and not contaminated by asset prices. I use Bates (2006)’s Approximate Maximum Likelihood estimation methodology, which allows filtering non-Gaussian shocks and exploits exponential affine characteristic functions.

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9In data, after controlling for heteroskedastic fundamental shocks, the cash flow growth is not rejected from Gaussianity nor homoskedasticity.
Therefore, the estimation provides time series of the latent uncertainty state variables and their shocks, given the observed growth series. Next, given the filtered real uncertainty shocks, and the assumption that the real shocks are orthogonal to inflation shocks, I project inflation (changes in log Consumer Price Index; source: FRED) onto output growth, real uncertainties and real shocks. Then, I filter the nominal uncertainty shocks from the inflation residual using Bates (2006). I use the longest period of data available for the estimations of these four economic uncertainties, \( \{\theta_u, \theta_d, \pi_u, \pi_d\} \); the sample period is January 1947 to December 2016.

**Risk Aversion:** Bekaert, Engstrom, and Xu (2017) estimate a utility-based risk aversion index from a no-arbitrage framework using a wide set of macro and financial information. I argue that their risk aversion estimate is still consistent with my framework. First, as discussed earlier, my framework is a modified version of theirs to accommodate short rate shocks and uncertainties that do not feed back on risk aversion. In addition, although BEX’s risk aversion process does not control for inflation uncertainty shocks as my model here does, according to my estimation, filtered inflation shocks (after controlling for real shocks) are insignificantly correlated with the risk aversion shock in the overlapping sample \( \rho(\omega_q, \omega_{\pi u}) = 0.038, \rho(\omega_q, \omega_{\pi d}) = -0.037 \). I therefore use the BEX risk aversion process in this article. The sample period spans June 1986 to February 2015.

**Real Short Rate Uncertainties:** The estimation procedure of latent real short rate uncertainties exploits the no-arbitrage condition and the assumed pricing kernel shock structure. Given the closed-form solution (Appendix F), observed nominal 30-day T-bill rate (source: CRSP), and macro and preference shocks, the real short rate as well as its shocks can be filtered using Bates (2006)’s methodology. The general estimation strategy—i.e., using the no-arbitrage condition and a pricing kernel to estimate the real term structure—is commonly used in the literature (e.g., Chen and Scott, 1993; Ang, Bekaert, and Wei, 2008). However, my model is more complicated because I also filter two shocks from the real short rate innovation, namely upside and downside short rate uncertainties.

### 3.2.B Results

In Figure 3, I show the time variation in the seven economic determinants of global comovements. Full sample plots are shown in the Online Appendix.

First, the weak countercyclicality of the BEX risk aversion (calculated as \( \gamma \exp(q_t) \) where \( \gamma = 2 \) represents the utility curvature) is immediately apparent, with risk aversion spiking in all three recessions, but also showing distinct peaks in other periods. The highest risk aversion of 11.58 is reached at the end of January in 2009, at the height of the Great Recession. But the risk aversion process also peaks during the October 1987 crash, the August 1998 crisis at the time of Russia default and LTCM collapse, after the TMT bull market ended in August 2002, and in August 2011 during the Euro area debt crisis.

Second, the real downside uncertainty is strongly countercyclical with a correlation coefficient of 0.71 with a NBER recession dummy (in the long sample). The correlation coefficient between the real upside uncertainty and NBER recessions is positive as well, but not statisti-
cally significant using the full sample. However, using the short sample (1987.03–2015.02), real upside uncertainty is weakly procyclical with a correlation coefficient of -0.38 with a NBER recession dummy. The two real uncertainty state variables are negatively correlated at -0.31 (statistically different from zero).

While downside uncertainty delivers the countercyclicality in real uncertainty, among inflation uncertainties, the inflation upside uncertainty exhibits more cyclical behavior than the inflation downside uncertainty. In the long sample (beginning in 1947), the time variation is less spiky, in the sense that high inflation upside uncertainty seems to appear in clusters, for example, the 1973 recession, the 1980s recession, and the recent financial crisis. The full sample plot is located in the Online Appendix. The source of the countercyclicality of inflation variation is consistent with Ball (1992). When actual and expected inflation are low, there is a consensus that the monetary authority will try to keep them low. However, when inflation is high, the public does not know whether the policy maker will disinflate or keep inflation high with the fear that disinflation could result in a recession. This dispersion in beliefs potentially results in fluctuations in inflation upside uncertainty. This “high inflation-high upside uncertainty” theory is consistent with the modeling of inflation shocks appearing in both the inflation process and the inflation uncertainty processes, as in Section 3.1.B. Empirically, the estimated inflation upside uncertainty is significantly countercyclical (i.e., a 0.52 correlation with a NBER recession dummy) which may reflect the wider dispersion in general beliefs in a bad economic environment. (The two nominal uncertainty state variables are weakly positively correlated with a coefficient of 0.16.) Because macro shocks are independent by design, it is now possible to quantify the relative importance of the four shocks in explaining total inflation uncertainty. According to my estimation, inflation upside and downside uncertainties account for 47.63% and 50.25% of the total inflation variance, respectively. Surprisingly, real upside and downside uncertainties together explain less than 3% of total variance within the short sample period.

Lastly, the real short rate innovation process depends on the risk aversion shock, real-side shocks filtered from industrial production growth, and nominal-side shocks filtered from inflation; the residual is then decomposed into two shocks. Thus, these two shocks are “cleansed” from all systematic monetary policy determinants; therefore in my framework, the residuals can potentially be interpreted as discretionary monetary policy shocks. In this sample, inflation shocks explain around 64% of the total real short rate variability to reflect the close relationship between the two; for example, to disinflate, monetary authorities are likely to increase the interest rate. Risk aversion explains 17% of the variability; when risk aversion increases, investors save more, driving down the interest rate (i.e., the precautionary savings channel dominates). In the full sample, of the total variance explained by \( x_u \) and \( x_d \), the downside uncertainty has a share of 73%. In the past 10 years, during which the nominal short rate is close to the zero boundary, expected inflation is positive, and real short rate is negative, the share increases to 88%. According to the last two plots of Figure 3, the real short rate upside uncertainty \( (x_u) \) during 1986 – 1989 was high when the Federal Reserve responded to high inflation by raising interest rates; in the early 1990s, however, the downside uncertainty starts to become relatively more elevated. The Online Appendix provides detailed estimation results.
4 A Theory-Motivated Factor Model

In this section, I evaluate the ability of the asset pricing model in Section 3 to interpret the three stylized facts established in Section 2. Sections 4.1 and 4.2 present the factor model and estimation strategy. Section 4.3 evaluates the model fit. Section 4.4 conducts a global conditional comovement decomposition for both covariance and correlation, followed by a discussion of the economic significance of the factors in interpreting the three stylized facts in Section 4.5.

4.1 Dynamic Equity and Bond Return Factor Model

Suppose there are \( N \) (8) asset return series for each asset class and \( P \) (7) global factors. Denote log asset returns (raw, not standardized) of two asset classes during month \( t + 1 \) with \( r_{t+1} \) \((2N \times 1)\), which are assumed to follow:

\[
\begin{bmatrix}
\mathbf{r}_{t+1} \\
\end{bmatrix}_{2N \times 1} = E_t \begin{bmatrix}
\mathbf{r}_{t+1} \\
\end{bmatrix}_{2N \times 1} + \begin{bmatrix}
\Omega_{t+1} & 0 & \cdots & 0 \\
0 & \Omega_{t+1} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \Omega_{t+1} \\
\end{bmatrix}_{2N \times 2NP} \begin{bmatrix}
\beta_{1,t} \\
\beta_{2,t} \\
\vdots \\
\beta_{2N,t} \\
\end{bmatrix}_{2NP \times 1} + \varepsilon_{t+1},
\]

where \( E_t \begin{bmatrix}
\mathbf{r}_{t+1} \\
\end{bmatrix}_{2N \times 1} \) denotes a vector of expected returns which, in my model, is a linear function of the state variables. \( \Omega_{t+1} \) denotes a row vector of the factors (shocks) introduced in Section 3:

\[
\Omega_{t+1} = \begin{bmatrix}
\omega_{q,t+1} & \omega_{\theta u,t+1} & \omega_{\theta d,t+1} & \omega_{\eta u,t+1} & \omega_{\eta d,t+1} & \omega_{\pi u,t+1} & \omega_{\pi d,t+1} & \omega_{x u,t+1} & \omega_{x d,t+1} \\
\end{bmatrix}
\]

where each shock follows a centered gamma distribution with time-varying shape parameters as discussed before.

Given the empirical focus of the paper, I also allow the possibility that betas of global factors are time-varying. For each country-asset class, the return sensitivity to each shock is defined as follows:

\[
\beta_t = \beta_0 + \beta_1 s_t,
\]

where \( s_t \) denotes a standardized shape parameter.

Time-varying betas, or conditional betas, can be motivated both empirically and economically. First, empirical studies have found that the response of volatility to macroeconomic news depends on the conditional states of the business cycle (Andersen, Bollerslev, Diebold, and Vega, 2007). Second, time-varying betas can also arise in economic models with various debits motivated by various departures of rational expectations. I discuss two plausible mechanisms below: the Bayesian Learning theory by David and Veronesi (2013), and the Confidence Risk theory by Bansal and Shaliastovich (2010). According to the Bayesian Learning theory, investors learn about (unobserved) shifts in economic states by observing signals in fundamentals and asset prices. In times of precise prior beliefs, large news is not necessary to move posterior probabilities (that is, betas are small); but when there is large uncertainty, which may be correlated with economic uncertainty measures, even small news moves posterior distributions (that is, betas are large). According to the Confidence Risk theory, a widening cross-section
of variance in economic signals lowers investor’s confidence placed in future growth forecasts, leading to Through this channel, large moves in the confidence measure lead to large declines (negative jumps) in asset prices, though there are no large moves in consumption.

Both empirical evidence and the economic mechanisms discussed above have been documented in the equity markets. Given that inflation upside uncertainty reflects the widening dispersion in beliefs (Ball, 1992), I consider standardized $\pi_u$ as $s_t$ in modeling the log equity return betas; in addition, this instrument is shown to improve the empirical fit the most, among the six economic uncertainties in this paper. On the other hand, there is little research on directly examining the time-varying betas in bond returns; therefore, the beta instruments for bond returns are selected based on the best empirical fit. As a result, betas in bond returns are spanned by real uncertainties. As both real uncertainties filtered from industrial production growth exhibit strong business cycle behavior, the modeling of bond betas is potentially consistent with the modeling of the domestic comovement channel in Section 2 where I use the OECD recession indicator that is also identified from industrial production growth.

The residuals in Equation (31) are mean zero, and assumed to be correlated:

$$E[\varepsilon_{t+1}|\Omega_{t+1}] = 0,$$

$$E[\varepsilon_{t+1}\varepsilon'_{t+1}|\Omega_{t+1}] = \Sigma.\quad (33)$$

The model-implied pairwise conditional covariance between country-asset $i$ and $j$ of the same asset class that is explained through the heteroskedastic global shocks is,

$$\beta'_{i,t} Var(\Omega_{t+1})\beta_{j,t},\quad (35)$$

where $\beta_{i,t}$ ($P \times 1$) and $\beta_{j,t}$ ($P \times 1$) are return sensitivities to $\Omega_{t+1}$, and $Var(\Omega_{t+1})$ ($P \times P$) a conditional covariance-variance matrix of $\Omega_{t+1}$ with zeros in all off-diagonal terms because all these common shocks are by design mutually independent.

Then, the model-implied pairwise conditional correlation through the heteroskedastic global shocks is,

$$\frac{\beta'_{i,t} Var(\Omega_{t+1})\beta_{j,t}}{\sqrt{\beta'_{i,t} Var(\Omega_{t+1})\beta_{i,t} + Var(\varepsilon_{i,t+1})\sqrt{\beta'_{j,t} Var(\Omega_{t+1})\beta_{j,t} + Var(\varepsilon_{j,t+1})}}}.$$

The global correlation is the equal-weighted average of the pairwise correlations, to be consistent with the parametric model in Section 2; this is referred to as the factor model-implied global correlation.

### 4.2 Estimation

The dynamic factor model is a system of regression equations with correlated residuals, which is in the class of Zellner (1962)’s Seemingly Unrelated Regression (SUR). I use feasible Generalized Least Squares (FGLS) estimators for betas and the residual covariance matrix (see Zellner, 1962; Zellner and Huang, 1962), and estimate them jointly with MLE.\footnote{The reason is as follows: With correlated residuals, the Ordinary Least Squares (OLS) estimators are no longer Best Linear Unbiased Estimators (BLUE), whereas Generalized Least Squares (GLS) estimators are, by construction (Greene, 2003). Both OLS and GLS estimators are unbiased and consistent; however, the variance of the OLS estimator is biased and inefficient. Then again, GLS assumes a known residual covariance matrix,} The
theoretical model (see Section 3.1.E) implies a multivariate Gaussian residual structure with non-zero correlations is indicated. However, the residuals could be heteroskedastic empirically; there are two potential remedies: (1) model the time series variation in $\Sigma$, and (2) change the estimation methodology to Generalized Method of Moments (GMM). The concern with (1) is misspecification error. The concern with (2) is that the estimation speed with GMM is significantly slower than MLE in this case, mainly because of the large number of moment conditions involved.\footnote{There are 16 asset returns, and 7 main factors; with time-varying betas, the total number of unknown parameters are 224 ($16 \times 7 \times 2$); with (at least) an exact identification, a convergence for a 3-step GMM takes 30 minutes, whereas the MLE procedure takes less than 1 minute.}

Due to the large number of parameters involved in the system, I relegate detailed equation-by-equation estimation results to Appendix Tables $\text{Av}$ (with constant betas) and $\text{Avi}$ (with time-varying betas).

### 4.3 Model Fit

Figures 4 and 5 compare the factor model-implied global conditional correlations with the best global correlation estimates implied from the DECO class in Section 2 (henceforth, the empirical benchmarks). In both figures, the top plots assume constant betas, and the bottom plots assume time-varying betas.

The constant beta model generates a 0.55 correlation with the empirical benchmark. According to Figure 4, it matches the October 1987 spike, the correlation decrease during the early 1990s, and the drop during the expansion between the dot-com bubble and the 2007 financial crisis. However, the constant beta model underestimates the global comovement level during the peak of the 2007-2008 financial crisis by 0.1, and overestimates by as much as 0.2 during the 1990s. This is because constant betas do not introduce enough non-linearity, and most economic determinants considered here do not have trends, while the empirical benchmark is tested with significant and positive time trend during the sample period (see Figure 1).

According to the equity return estimation, the time-varying beta instrument is the time-varying inflation upside uncertainty; a high inflation upside uncertainty potentially captures a wide dispersion in beliefs (Ball, 1992). Linking to Bansal and Shaliastovich (2010)’s confidence risk theory for example, this wide dispersion in beliefs could result in large negative jumps in investor confidence and thus large declines in asset prices given a unit of negative fundamental shock. Given the estimation results, the time-varying beta model improves the constant beta model fit (i.e., the correlation with the empirical benchmark) from 55% to 69%. In particular, the time-varying beta model is able to match the global equity comovement during the 2007-2008 financial crisis and generate a positive albeit weak trend.

The constant beta model generates a global bond comovement that is uncorrelated with the empirical benchmark comovement from Section 2. However, the time-varying beta model, at a 17% correlation with the empirical benchmark, shows improvements during certain periods. Between the two real uncertainties serving as beta instruments, real upside uncertainty improves the model fit more. For example, when the Single European Act is introduced during June/July...
1987, the empirical benchmark reflects this major monetary union integration event with a large spike. In Figure 5, while the constant beta model generates a negative jump, the time-varying beta model generates a positive jump. The economic reason behind this divergent behavior is that, in the time-varying beta model, the sensitivity of almost all foreign bond returns in this sample to the global risk aversion shock becomes positive when real upside uncertainty increases substantively (see Appendix Table A6), which drives up the comovement among bonds. Because beta instruments are standardized, a one standard deviation increase in real upside uncertainty would change the signs of bond sensitivities to the risk aversion shock from negative to positive for all four European countries (Germany, France, United Kingdom, Switzerland), which drives up global comovement. With a 2.79 standard deviation increase, all bonds are “safe”. The July 1987 integration event comprises a 1.67 standard deviation increase in real upside uncertainty. As another improvement, the time-varying beta model matches the increase in global bond comovement from 1993 to the introduction of the euro in 1999. This improvement is again due to the beta instrument reflecting good and bad states of the global economy in terms of the output growth uncertainty; the peak corresponds to a 2.8 standard deviation increase in the instrument.

The main goal of this section is to explain the three stylized facts established in the first part of the paper. Therefore, I formally evaluate the ability of this theory-motivated dynamic factor model—featuring changes in global risk aversion and fundamental economic uncertainties as factors—in fitting the three stylized facts. To do so, I first summarize and represent the stylized facts in terms of six numerical moments, two for each fact; they are calculated from the empirical benchmarks (i.e., conditional correlations implied from the parametric model), rather than the simulated ones. Then, I obtain the factor model-implied moments to confront the benchmark moments. Both moments and their closeness tests are presented in Table 7. Columns with “Full” indicates that models are implied from a dynamic factor model with the full set of seven heteroskedastic global shocks. Here are several observations. The time-varying beta model is able to match all three stylized facts, whereas the constant beta model fails to match Fact 2 because the upside comovement difference 0.2245 is insignificant from the downside comovement difference 0.2368. The time-varying model is statistically closer to the empirical benchmarks according to the higher p-values in brackets, which is consistent with the dynamic fit shown in Figures 4 and 5. I conclude that, despite that fitting the true values in correlations is imperfect, the dynamic factor model with only global state variables suffices to match the three stylized fact motivated in the article.

4.4 Global Conditional Comovement Decomposition

I now examine the contributions of each factor to the fit by conducting global conditional covariance and correlation decompositions that are discussed in Sections 4.4.A and 4.4.B respectively.
4.4.A Covariance

Given the total pairwise conditional covariance shown in Equation (35), I calculate the share of conditional covariance explained by factor $\omega_\kappa$ (e.g.):

$$\frac{\beta_{i,t,\kappa} Var_t(\omega_{t+1}) \beta_{j,t,\kappa}}{\beta_{i,t} Var_t(\Omega_{t+1}) \beta_{j,t}}$$

(37)

where $\beta_{i,t} = \beta_{i,0} + \beta_{i,1} s_t$ and $\beta_{j,t} = \beta_{j,0} + \beta_{j,1} s_t$ are scalars, indicating the beta for that particular factor; the values of $\beta_{i,0}, \beta_{i,1}, \beta_{j,0}$ and $\beta_{j,1}$ are given by the estimation results; the conditional variance of that factor, $Var_t(\omega_{t+1})$, is a scalar.

Table 8 presents the average conditional covariance decomposition (across all months and across 28 unique country pairs). Three observations stand out. First, the risk aversion factor explains around 90% of the equity return covariance both in the constant beta and time-varying beta models. This quantitative result formally contributes to the ongoing debate about the relative importance of fundamental sources of risk that transmit across countries (see Miranda-Agrippino and Rey, 2015; Jotikasthira, Le, and Lundblad, 2015), and supports a potentially stronger role for the risk compensation channel in explaining international return comovement as opposed to the cash flow variables (industrial production and inflation) or the interest rate. Adding to the recent, growing literature on the strong predictive power of the variance risk premium for equity returns (see Bollerslev, Tauchen, and Zhou, 2009; Bekaert and Hoerova, 2014; Bollerslev et al., 2014; among many others), the current work establishes an important role of risk aversion for second moment dynamics.12 Besides the dominant role of risk aversion in explaining international equity return covariances, real economic uncertainties also explain an amount (5.2% in constant beta models and 7.4% for time-varying beta models). Figure 6 shows the conditional covariance decomposition over time. While the share of total equity return conditional covariance explained by risk aversion dominates during my sample period, the relative weights of real and inflation uncertainties occasionally spike but their magnitudes always remain below 40%. Note that, even though the relative weight of risk aversion decreases during the financial crisis, both the total conditional covariance and the conditional covariance accounted for by risk aversion spike. It is more of a surprising finding on the strictly dominant role of risk aversion relative to a wide range of economic uncertainties in explaining the dynamics of conditional covariance during good times, which is not suggested by extant theories.

Second, inflation upside uncertainty—the part of inflation uncertainty that comoves strongly with business cycles (see the discussion in Section 3.2)—explains 48.6% of the fitted bond return conditional covariance in the factor model with time-varying betas. The constant part $\beta_{i,0} Var_t(\omega_{t+1}) \beta_{j,0}$ already accounts for 43.1%. In the time-varying beta model, risk aversion is a less dominant factor (40% of the total explained covariance instead of 78% in the constant beta model), whereas inflation uncertainty (47% instead of 34%) and real short rate upside uncertainty (22% instead of a negative contribution) now play a larger role. My model reveals non-linearities in the effects of $q_t$ and $x_{u_t}$ on global bond comovements. As mentioned in Section 4.3, the risk characteristics of bond returns might be different during good and bad times. The fact that all bond returns are positively correlated with risk aversion (i.e., safe)

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12Bekaert, Engstrom, and Xu (2017) show that their risk aversion index is highly correlated with the variance risk premium.
during good times drives up global bond comovement; when the world is entering a bad economic environment, several foreign bonds become risky, resulting in lower global bond return comovement. As a result, this non-linear effect of risk aversion contributes positively to the third stylized fact, a weakly procyclical global bond comovement. On the other hand, in a bad economic environment, when the real short rate upside uncertainty increases (i.e., there is a higher chance of an increase in interest rate), bond prices drop for all countries except for the USA in this sample, also indicating higher global bond comovement. Therefore, the non-linear effect of $xu$ contributes negatively to the third stylized fact, which renders the effect of risk aversion the dominant force in fitting the stylized facts.

Third, global factor model with constant betas explains 49.4% of the total equity return covariance. Allowing for time-varying betas, the factor model explains covariances slightly better, at 54.6%. On the other hand, fitting bond return covariances with a constant-beta factor model is not successful, with only 0.9% of the covariance explained. The fit improves with a time-varying beta model to a 15.4% explained fraction.

4.4.B Correlation

While covariances can be easily decomposed, a correlation decomposition is not as straightforward. Here, I propose a new correlation decomposition test. I denote $\text{CORR}_{0,t}$ as the factor model-implied global conditional correlation using all 7 factors, $\text{CORR}_{\kappa,t}$ as the factor model-implied global conditional correlation using all factors except for factor $\omega$ (where the parameters are re-estimated), and $BM_t$ as the empirical benchmark. Then, under the null of the factor having zero contribution to the model fit, $\rho(\text{CORR}_{0,t}, BM_t) - \rho(\text{CORR}_{\kappa,t}, BM_t)$ should not be indifferent from zero. I show the statistics in Table 9.

The first line reports the correlations between the empirical benchmark for the case of comparison. Clearly, risk aversion has the largest marginal contribution to the correlation fit. Let’s focus on the time-varying beta models which exhibit better statistical fit than the constant beta models given the previous analysis. A dynamic factor model without the risk aversion shock has -21% (-12%) correlation with the empirical benchmark for global equity comovement (global bond comovement). This indicates that risk aversion has the largest marginal contribution to the fit of global equity and bond conditional correlations, according to the second line. Moreover, I report the marginal contribution of economic uncertainties in the dynamic fit. By adding any of the 6 uncertainty shocks to the equity model, the factor model-implied global equity comovement fits the empirical benchmark better by less than 0.1—which are weakly significant, albeit small and significantly smaller than the 0.9 marginal contribution by risk aversion. In the case of the bond model, the inflation upside uncertainty shock improves the model fit the most among all 6 uncertainty shocks by around 0.14, leaving the risk aversion shock still the largest contributor in terms of statistical fit.

---

13 The share of total explained covariance is calculated by dividing the time-series average of pairwise conditional covariance by the unconditional pairwise covariance matrix, and then taking the equal-weight cross-sectional average.
4.5 Economic Significance of Risk Aversion

The global comovement decompositions establish risk aversion as a critical economic determinant of global comovements. In this section, I formally quantify the economic significance of risk aversion (and other factors for comparison), and explain why $\omega_q$ helps interpret all three stylized facts. Because the constant beta model fits bond comovement poorly to begin with, I focus on the models with time-varying betas in this section.

In the last two columns of Table 7, I report the fit of a factor model omitting the risk aversion shock ($\omega_q$). The time-varying beta model without $\omega_q$ fails to jointly fit the facts. The top right plot of Figure 7 suggests that the biggest misfit (in terms of both magnitude and dynamics) in global equity-bond correlation difference occurs during and after the 2007 financial crisis, when global equity and bond comovement decoupled. The decoupling is an interesting phenomenon, which requires more detailed examination. In my framework, the decoupling is likely an FTS effect (equity to bond) coupled with a large positive risk aversion shock. Table 10 evaluates the fit of factor models omitting economic uncertainty shocks is at a time. These models are still able to generate reasonable comovement differences between international equities and government bonds markets, as the three stylized facts are not rejected. That is, they are not the core factors delivering the comovement stylized facts established in the current paper.

Next, I revisit the three stylized facts and provide details on why risk aversion helps interpreting the three stylized facts:

First, equity return sensitivities to the risk aversion shock are significant and negative (i.e. risky), whereas bond return sensitivities to the risk aversion shock are not only much smaller in magnitude but have different signs in different countries. Therefore, equity comovements are higher in magnitude than bond comovements.

Second, the second moment of the risk aversion shock is positively-skewed, $skew(q_t) > 0$. With bond returns displaying relatively weaker return sensitivities to the risk aversion shock, asymmetry in bond return comovements is naturally less strong than in equity return comovements.

Third, the U.S. (global) investor demands higher risk compensation from risky assets when her risk aversion is high; moreover, the risk compensation channel is strengthened during periods when dispersion in beliefs is wider (captured by high inflation upside uncertainty in this model). Therefore, stock prices drop across all countries simultaneously and global equity comovement increases in a bad economic environment.

In a good economic environment when real upside uncertainty (or good volatility associated with positive skewness of future output growth) is higher than average, almost all bond prices increase with the risk aversion shock, rendering them safe assets. In a bad economic environment with low real upside uncertainty, six foreign bonds turn risky while the USA and JPN bonds remain safe assets. Therefore the commonality in bond returns is higher during good times, implying a procyclical bond comovement.

\[14\] The constant beta model without $\omega_q$ is also immediately rejected.
Countercyclical Divergence of Bond Risk Characteristics

In this paper, I interpret the procyclical global bond comovement with divergence in country bond risk characteristics during global economic turmoil. The sensitivities of country bond returns to the global risk aversion shock depend on different states of the world economy. In a good environment with high good real-side volatility, all country bonds are identified as safe assets, meaning their prices increase in response to increases in risk aversion. In a bad environment, some country bond prices soar when global risk aversion is higher, indicating safe heaven behaviors, whereas other country bond prices drop, constituting the higher-risk segment of the global bond market; hence, there exhibits a divergence in the risk characteristics of country government bonds. An intuition explanation is a Flight-To-Quality effect within the international bond market. In this country sample, I identify United States and Japanese government bonds as safe assets; other foreign country bonds become risky during recessions.

To further demonstrate the significant role of countercyclical divergence of bond risk characteristics in resulting in the substantive time variation in global bond correlations as observed in Figure 1 in this final section, I conduct a “Jackknife” exercise. I re-estimate the global bond correlations with subsets of the full country set using the parametric model in Section 2 and compare them with the empirical benchmark (with a full country set).

In the first three plots of Figure 8, I omit one country at a time in the re-estimations. Clearly by omitting either USA or JPN bonds, global bond correlations (depicted by thin black lines) become significantly higher than the empirical benchmark (depicted by wide red lines). It is because the strongly comoving risky bonds now have more weights in the aggregate bond return comovement of this subsample. Note that the global bond comovement omitting Japanese government bond is higher and exhibits less time variation than the one omitting United States government bond, which suggests that the JPN bond might play a more significant role of a “safe” asset. Moreover, according to the second plot of Figure 8 omitting any of the European government bonds does not influence the level or cyclicality of the global bond comovement significantly.

The final plot depicts the time variation in the global bond comovement without USA and JPN government bonds. The global comovement of risky bonds is higher than the empirical benchmark value, which is expected; most importantly, it is uncorrelated with the NBER recession indicator. Major declines in global bond comovements during economic turmoil as in the empirical benchmark (e.g., 2007-08 global financial crisis, 2012 European debt crisis) do not appear in this plot. In fact, the risky bond comovement is extremely persistent (AR(1)=0.98) than the empirical benchmark using a full country set (AR(1)=0.90). Thus, this jackknife exercise further demonstrates that the divergence of bond risk characteristics is an important driver of the dynamics of global bond comovement documented in this paper. However, I do not explain why bond risk characteristics diverge in this paper; but eventually, it is closely related to the monetary policy regime and/or the exchange rate regime, which deserves more scrutiny.
6 Conclusion

In this paper, I formally establish three new stylized facts contrasting global equity and bond comovements, both conditionally and unconditionally. The three facts are as follows: (1) bond return correlations are smaller than equity return correlations, (2) equity returns have higher downside than upside correlations, while bond return correlations are symmetric, and (3) equity return correlations are countercyclical while bond return correlations are weakly procyclical. The stylized facts regarding bond comovements are new contributions. The new global dynamic comovement model accommodating asymmetry and domestic comovement (Duo-DECO) is potentially a methodological contribution. Next, I motivate and identify economic determinants of global comovements in a dynamic no-arbitrage asset pricing model with time-varying global economic uncertainties (of output growth, inflation, and the real short rate) and the risk aversion (of a global investor). Finally, I interpret the three stylized facts in the context of a dynamic factor model motivated by the theoretical model. I find that different sensitivities of equity returns (strongly negative) and bond returns (weakly positive or negative) to the global risk aversion shock dominantly drive all three stylized facts. While Miranda-Agrippino and Rey (2015) suggest that global risk aversion drives the global risky-asset cycle, my paper documents global risk aversion as a major source of asset return comovements across countries, for both equities and government bonds, which contributes to the ongoing debate about the relative importance of fundamental sources of risk in affecting global comovements.

The paper also leaves three puzzles for future research. First, bond comovement remains poorly explained. Second, although this paper focuses on return comovements from the perspective of a global investor, comovements denominated in local currencies are also potentially interesting given the increasing demand for currency-hedged bonds (see Viceira, Wang, and Zhou, 2017). In fact, I also examine properties of global bond comovement denominated in local currencies, and find four facts: it is smaller than global equity comovement, it is symmetric, it is acyclical, and it has a clear upward trend that disappears in the global bond comovement denominated in USD. Lastly, the three stylized facts suggest that international bond investments is more attractive for a U.S. investor from a diversification perspective. However, world bond home bias is significantly higher than world equity home bias (Coeurdacier and Rey, 2013).
Appendices

A Univariate Conditional Variance Models

The univariate variance model for each return series is selected using the Bayesian information criterion (BIC) from a class of models capable of capturing the common features of financial asset return variance: persistent, clustering, and (sometimes) asymmetric. Although commonly-acknowledged, these features do not appear in conditional variances of all asset returns. For example, as asymmetry in both domestic stock returns and international stock returns is widely documented (see, e.g., French, Schwert, and Stambaugh (1987), Hentschel (1995), Wu (2001), Li et al. (2005); Kenourgios, Samitas, and Faltalidis (2011) among many others), little evidence of asymmetry is found in bond returns, both domestically or internationally (see a thorough examination in Cappiello, Engle, and Sheppard (2006) for instance). As a result, in this paper, I consider four conditional variance models in the GARCH class with four residual distributional assumptions; thus, 16 models are included in the model selection.

Suppose the residual follows a conditional distribution, \( \varepsilon_{t+1} \sim D(0, h_t) \) where \( h_t \) denotes the conditional variance. The first conditional variance model follows an autoregressive conditional heteroskedastic process with one lag of the innovation and one lag of volatility, or “GARCH” as in Bollerslev (1986):

\[
    h_t = \alpha_0 + \alpha_1 \varepsilon^2_t + \alpha_2 h_{t-1}
\]

where \( \alpha_1 \) denotes the sensitivity of conditional variance to news and \( \alpha_2 \) the autoregressive coefficient. Then, I use three widely-used asymmetric GARCH models that introduce non-linearity to different transformations of the conditional variance \( h_t \). The second model is the exponential GARCH, or “EGARCH” as in Nelson (1991), which includes a signed standardized residual term to capture the (potential) higher downside risk variance. The third model is the threshold GARCH, or “TARCH” as in Zakoian (1994), which introduces asymmetry to conditional volatility, whereas the fourth model, Glosten, Jagannathan, and Runkle (1993)’s “GJR-GARCH”, introduces asymmetry to conditional variance. The model specifications are shown below:

\[
    \ln(h_t) = \alpha_0 + \alpha_1 \frac{\varepsilon_t}{\sqrt{h_{t-1}}} + \alpha_2 \ln(h_{t-1}) + \alpha_3 \frac{\varepsilon_t}{\sqrt{h_{t-1}}}, \quad (A2)
\]

\[
    \sqrt{h_t} = \alpha_0 + \alpha_1 \varepsilon_t + \alpha_2 \sqrt{h_{t-1}} + \alpha_3 I_{\varepsilon_{t-1}<0} |\varepsilon_t|, \quad (A3)
\]

\[
    h_t = \alpha_0 + \alpha_1 \varepsilon^2_t + \alpha_2 h_{t-1} + \alpha_3 I_{\varepsilon_{t-1}<0} \varepsilon^2_t, \quad (A4)
\]

where \( \alpha_3 \) is the asymmetry term. If the downside uncertainty is higher than the upside uncertainty, then \( \alpha_3 \) in Equation (A3) is expected to be negative because downside risk in these models is identified when residuals are negative, whereas \( \alpha_3 \) in Equations (A3) and (A4) are expected to be positive because the asymmetry terms in last two models are sign-independent.

The standardized residuals, \( \tilde{\varepsilon}_{t+1} \), are defined to be \( \frac{\varepsilon_{t+1}}{\sqrt{h_t}} \).

B Four distributional assumptions in estimating the conditional variances of return series in Section A

I consider four distributions. First, Gaussian distribution; \( \varepsilon_{t+1} \sim N(0, h_t) \) with conditional probability density function equal to \( \frac{1}{\sqrt{2\pi h_t}} \exp \left( \frac{-\varepsilon^2_{t+1}}{2h_t} \right) \). Second, Student’s t distribution; \( \varepsilon_{t+1} \sim STD(0, h_t, \zeta_t) \) with conditional probability density function equal to

\[
    \frac{\Gamma\left(\frac{1}{2} (1+\zeta_t)\right)}{\sqrt{\pi} \Gamma\left(\frac{1}{2}\right) h_t^{\frac{1}{2} (1+\zeta_t)}} \left(1 + \frac{\varepsilon^2_{t+1}}{(1+\zeta_t)h_t} \right)^{-\frac{1}{2} (1+\zeta_t)}
\]

where \( \zeta_t > 2 \) denotes the degree of freedom capturing the thickness of both tails and \( \Gamma \) denotes the gamma distribution. A higher \( \zeta_t \) indicates a thinner tail. Third, Generalized error distribution; \( \varepsilon_{t+1} \sim GED(0, h_t, \zeta_t) \) with conditional probability density function equal to

\[
    \frac{\zeta_t}{2\sqrt{\pi} \Gamma\left(\frac{1}{2}\right) h_t^{\frac{1}{2} \zeta_t}} \exp \left( \frac{-\varepsilon^2_{t+1}}{2h_t} \right)^{\zeta_t}.
\]

Platykurtic densities (with tails lighter than Gaussian) are defined if \( \zeta_t > 2 \); on the other hand, leptokurtic densities (with tails heavier than Gaussian) are defined if \( 1 < \zeta_t < 2 \). Fourth, Skewed student t distribution; \( \varepsilon_{t+1} \sim SEWT(0, h_t, \zeta_t, \psi_t) \) where conditional probability density function (according to Hansen, 1994) equals

\[
    f(\varepsilon_{t+1} \mid h_t, \zeta_t, \psi_t) = \begin{cases} 
    bc \left[1 + \frac{\varepsilon^2_{t+1}}{h_t} \left(\frac{b_{t+1}}{h_t} - \frac{b^2_{t+1}}{h_t^2}\right)\right]^{\frac{\zeta_t+1}{2}} & \varepsilon_{t+1} < -\frac{\psi_t}{\sqrt{h_t}}, \\
    bc \left[1 + \frac{\varepsilon^2_{t+1}}{h_t} \left(\frac{b_{t+1}}{h_t} - \frac{b^2_{t+1}}{h_t^2}\right)\right]^{\frac{\zeta_t+1}{2}} & \varepsilon_{t+1} \geq -\frac{\psi_t}{\sqrt{h_t}}, 
    \end{cases}
\]

(A1)
where $2 < \zeta_1 < \infty$, $-1 < \zeta_2 < 1$, constants $a = 4\zeta_2 c \left(\frac{\zeta_2 - 2}{\zeta_2}\right)$, $b^2 = 1 + 3\zeta_2^2 - a^2$, and $c = \frac{1}{\sqrt{\pi(\zeta_2 - 1)}} \left((\zeta_2 - 2)b\right)^{-\frac{1}{2}}$.

The density function is continuous, and has a single mode at $-\frac{a}{b}$, which is of opposite sign with the parameter $\zeta_2$. Thus if $\zeta_2 > 0$, the mode of the density is to the left of zero and the distribution is right-skewed, and vice-versa when $\zeta_2 < 0$. To summarize, all distributions except for the first distribution allow for thick tails; in addition, the last distribution also captures the skewness.

C  Prove Covariance Stationarity of the Global Dynamic Comovement Model in Equation (8).

In this section, I prove that $Q_t$ ($N \times N$) is a stationary process. As introduced in Section 2.2.A, $Q_t$ follows a generalized autoregressive heteroskedastic process,

$$Q_t = \bar{Q}^* \circ \Phi_t + \beta_1 \left(\hat{Q}_{t-1}^2 z_t - \hat{Q}_{t-1}^2 - \bar{Q}^* \circ \Phi_{t-1}\right) + \beta_2 \left(Q_{t-1} - \bar{Q}^* \circ \Phi_{t-1}\right)$$

$$+ \gamma \left(\hat{Q}_{t-1}^2 n_t n_t' \hat{Q}_{t-1}^2 - \Xi \circ \bar{Q}^* \circ \Phi_{t-1}\right).$$

(A1)

where “$\circ$” denotes the Hadamard product operator (element-by-element); $\bar{Q}^*$ is the unconditional component of the long-run conditional mean; $\Phi_t$ is $Q_t$, with off-diagonal terms being zeros, which is a modification to Engle (2002) proposed by Aielli (2013); $n_t (N \times 1) = I_{z_t < 0} \circ z_t$, where $I_{z_t < 0} (N \times 1)$ is assigned 1 if the residual is less than 0, and assigned 0 otherwise; $\Xi = E[I_{z_t < 0} I_{z_t < 0}']$; $\Phi_t (N \times N) = \begin{bmatrix} 1 & 1 & \phi_1 & \phi_1 & \cdots \\ 1 & 1 + \phi_1 & 1 & \phi_1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$.

$\phi_t = \phi \bar{\theta}^{\text{world}}, \bar{\theta}^{\text{world}}$ is the standardized $\theta^{\text{world}}$ and $\phi$ is an unknown constant parameter.

C.1 Time-Invariant Mean

First, given that $E_{t-1} (z_t z_t') = \text{Corr}_{t-1} = \hat{Q}_{t-1}^2 \hat{Q}_{t-1}$, one-period conditional mean has the following process,

$$E_{t-1} (Q_t) = \bar{Q}^* \circ E_{t-1} (\Phi_t) + \beta_1 \left(\hat{Q}_{t-1}^2 E_{t-1} (z_t z_t') \hat{Q}_{t-1}^2 - \bar{Q}^* \circ \Phi_{t-1}\right) + \beta_2 \left(Q_{t-1} - \bar{Q}^* \circ \Phi_{t-1}\right)$$

$$+ \gamma \left(E_{t-1} (I_{z_t < 0} I_{z_t < 0}') \hat{Q}_{t-1}^2 - \Xi \circ \bar{Q}^* \circ \Phi_{t-1}\right).$$

(A2)

$$E_{t-1} (I_{z_t < 0} I_{z_t < 0}' \hat{Q}_{t-1}^2) \circ \hat{Q}_{t-1}^2 \circ \Phi_{t-1} + \beta_1 \left(Q_{t-1} - \bar{Q}^* \circ \Phi_{t-1}\right) + \beta_2 \left(Q_{t-1} - \bar{Q}^* \circ \Phi_{t-1}\right)$$

$$+ \gamma \left(E_{t-1} (I_{z_t < 0} I_{z_t < 0}') \hat{Q}_{t-1}^2 \circ \Phi_{t-1} - \Xi \circ \bar{Q}^* \circ \Phi_{t-1}\right).$$

(A3)

Given the law of iterated expectation and $E [E_{t-1} (I_{z_t < 0} I_{z_t < 0}')] = \Xi$, the unconditional mean of $Q_t$ can be shown to be time-invariant as below,

$$E [E_{t-1} (Q_t)] = \bar{Q}^* \circ E [\Phi_t] + (\beta_1 \bar{\theta} + \beta_2 \bar{\theta} + \gamma \Xi) \circ \left(E [Q_{t-1}] - \bar{Q}^* \circ E [\Phi_{t-1}]\right);$$

(A5)

$$E [Q_{t}] \left(\gamma - \beta_1 \bar{\theta} - \beta_2 \bar{\theta} + \gamma \Xi\right) = \bar{Q}^* \circ E [\Phi_t] \left(\gamma - \beta_1 \bar{\theta} - \beta_2 \bar{\theta} - \gamma \Xi\right).$$

(A7)

$$E [Q_t] = \bar{Q}^*.$$

(A8)

C.2 Time-Invariant Variance

$$\text{Var} (Q_t) = \bar{Q}^* \circ \text{Var} (\Phi_t) \circ \bar{Q}^* + \beta_1 \left(\text{Var} \left(\hat{Q}_{t-1}^2 z_t z_t' \hat{Q}_{t-1}^2\right) - \bar{Q}^* \circ \text{Var} (\Phi_{t-1}) \circ \bar{Q}^* \right)$$

$$+ \beta_2 \left(\text{Var} (Q_{t-1}) - \bar{Q}^* \circ \text{Var} (\Phi_{t-1}) \circ \bar{Q}^* \right).$$
\[ + \gamma \left( \text{Var} \left( \tilde{Q}_{t-1} n_t n_t' \tilde{Q}_{t-1} \right) - \Xi \otimes \mathbb{E} \circ \text{Var} (\Phi_t) \otimes \mathbb{E} \right), \]

where
\[ [A] = \text{Var} \left( \tilde{Q}_{t-1} n_t z_t \tilde{Q}_{t-1} \right) = E \left[ \text{Var}_{t-1} \left( \tilde{Q}_{t-1} n_t z_t \tilde{Q}_{t-1} \right) \right] + \text{Var} \left[ E_{t-1} \left( \tilde{Q}_{t-1} n_t z_t \tilde{Q}_{t-1} \right) \right], \]

and \[ [B] = \text{Var} \left( \tilde{Q}_{t-1} n_t n_t' \tilde{Q}_{t-1} \right) = E \left[ \text{Var}_{t-1} \left( \tilde{Q}_{t-1} n_t n_t' \tilde{Q}_{t-1} \right) \right] + \text{Var} \left[ E_{t-1} \left( \tilde{Q}_{t-1} n_t n_t' \tilde{Q}_{t-1} \right) \right]. \]

Given that \( z_t \) is assumed to be a stationary vector, higher moments of \( z_t \) are time-invariant; it immediately suggests that the unconditional means of Components \([C1]\) and \([C2]\) in the equation above are time-invariant. Given the stationary \( \tilde{Q}_{t-1} \) process as shown earlier, the unconditional mean of products of stationary processes in \([A]\) are time-invariant. Similar arguments can be applied to \([B]\).

**D** The Jennrich (1970) Correlation Test

Suppose two \( N \)-variate sample correlation matrices, \( R_1 \) \((N \times N)\) and \( R_2 \) \((N \times N)\) with sample sizes \( t_1 \) and \( t_2 \) (per variate), the test statistics is, \( \chi^2 = \frac{1}{2} \text{tr}(Z'Z - \text{diag}(Z)S^{-1}\text{diag}(Z)) \) where \( \text{"tr"} \) calculates the matrix trace and \( \text{"diag"} \) the diagonal terms; \( Z(N \times N) = c^{1/2} \bar{R}^{-1}(R_1 - R_2) \) where \( c = \frac{t_1 t_2}{t_1 + t_2} \) and \( \bar{R} = (t_1 R_1 + t_2 R_2)/(t_1 + t_2) \); \( S(N \times N) = I_N + \bar{R} \circ \bar{R}^{-1} \) where \( I_N \) is the identity matrix and \( \circ \) denotes the Hadamard product operator (element-by-element). The test statistics (see further details in Jennrich, 1970) has an asymptotic \( \chi^2 \) distribution with degrees of freedom \( N(N-1)/2 \).

**E** Review on the Statistical Properties of a Gamma Distribution

For a Gamma random variable, \( y \sim \Gamma(s, \theta) \) where \( s \) denotes the shape parameter and \( \theta \) the scale parameter, it has the following PDF,
\[ f_{Y_{\text{Gamma}}}^{\text{Gamma}}(y; s, \theta) = \frac{1}{\Gamma(s)\theta^s} y^{s-1} \exp \left( -\frac{y}{\theta} \right), \]

where \( \Gamma(v) \) is a complete Gamma function.

The moment generating function is,
\[ M_{Y_{\text{Gamma}}}^{\text{Gamma}}(t; s, \theta) = (1 - \theta t)^{-s}, \forall t < \frac{1}{\theta}. \]

The mean is \( \theta s \); the variance is \( \theta^2 s \); the unscaled skewness is \( 2\theta^3 s \).

**F** Solving an International Asset Pricing Model

In the main text (Section 3), for simplicity, I assume that there exists a global investor who prices both U.S. and foreign country assets (equities and Treasury bonds), and thus the asset prices are solved from the perspective of this global investor. The advantage of that parsimonious framework is to motivate a global dynamic factor model examined in Section 3.

In this appendix section, I acknowledge the exchange rates dynamics and different real pricing kernel of each country. For each country, its domestic investor prices domestic assets where (1) the domestic macro environment and investor risk aversion receive global state variable exposures, and (2) the domestic investor’s pricing kernel reflects partial integration. Section F.1 introduces the U.S. state variables and real pricing kernel and solves the U.S. asset prices; Section F.2 discusses the individual country real pricing kernels and state variables as well as model solutions.
The main take-away is that a global dynamic factor model still holds.

F.1 The U.S. Asset Market

F.1.1 U.S. State Variable Dynamics

F.1.1.a Matrix representation In a matrix representation, the U.S. state vector at time \( t \) is denoted as \( X_{t+1} \) (11 x 1),

\[
\begin{bmatrix}
\theta_{t+1} \\
\theta_{ut, t+1} \\
\theta_{dt, t+1} \\
\pi_{t+1} \\
\pi_{ut, t+1} \\
\pi_{dt, t+1} \\
x_{t+1} \\
x_{ut, t+1} \\
x_{dt, t+1} \\
g_{t+1} \\
q_{t+1}
\end{bmatrix}
\]

which follows this general dynamics:

\[ X_{t+1} = \xi_{X,t} + \text{Jensen's } (\delta_X, S_t) + \delta_X \omega_{t+1}, \quad (A1) \]

\[ \omega_{t+1} \sim \Gamma(S_t, 1) - \sigma^2, \quad (A2) \]

where \( \xi_{X,t} \) (11 x 1) denotes the conditional mean vector; \( \omega_{t+1} \) (8 x 1) denotes the global state variable shock matrix \( \Omega_{\omega,u, t+1} \omega_{\theta,u, t+1} \omega_{\theta,d, t+1} \). The shocks are mutually independent; \( \delta_X \) (11 x 8) denotes the constant coefficient matrix to the state variable shocks \( S_t \) (8 x 1) is the vector of the shock shape parameters [\( \theta_{ut}, \theta_{dt}, \pi_{ut}, \pi_{dt}, x_{ut}, x_{dt}, g_t, q_t \]; Jensen’s (\delta_X, S_t) denotes the Jensen’s inequality term from Gamma distributions; \( \Gamma(s, 1) \) denotes the Gamma random variable with a shape parameter \( s \) and a scale parameter \( 1 \).

The six uncertainty state variables and their shocks are denoted as:

\[
\omega_{t, t+1} = \left[ \omega_{\theta_{ut, t+1}} \omega_{\theta_{dt, t+1}} \omega_{\pi_{ut, t+1}} \omega_{\pi_{dt, t+1}} \omega_{x_{ut, t+1}} \omega_{x_{dt, t+1}} \omega_{g_{t+1}} \omega_{q_{t+1}} \right].
\]

F.1.1.b Output growth and uncertainties I follow Bekaert, Engstrom, and Xu (2017) to model industrial production growth innovation with two centered independent gamma shocks where each shock has a time-varying shape parameter that governs the higher moments of the shock. I name the shape parameter that governs the right-tail (left-tail) skewness the real upside (downside) uncertainty, \( \theta_{ut} \) (\( \theta_{dt} \))\footnote{Note that Bekaert, Engstrom, and Xu (2017) name them “good” and “bad” uncertainties to assign economic meanings of real uncertainties, whereas my notation here is more general and consistent as, for example, inflation upside uncertainty (later) is not typically considered as “good” uncertainty.}. Formally, \( \theta_{t+1} \) has the following process:

\[ \theta_{t+1} = \bar{\theta} + \mu_{\theta} (\theta - \bar{\theta}) + \rho_{\theta u} (\theta_{ut} - \bar{\theta}_{ut}) + \rho_{\theta d} (\theta_{dt} - \bar{\theta}_{dt}) + u_{\theta, t+1}, \quad (A3) \]

where the growth shock is decomposed into two independent shocks,

\[ u_{\theta, t+1} = \delta_{\theta u} \omega_{\theta_{ut, t+1}} - \delta_{\theta d} \omega_{\theta_{dt, t+1}}, \quad (A4) \]

The shocks follow centered Gamma distributions with time-varying shape parameters,

\[
\omega_{\theta_{ut, t+1}} \sim \tilde{\Gamma}(\theta_{ut}, 1),
\]

\[
\omega_{\theta_{dt, t+1}} \sim \tilde{\Gamma}(\theta_{dt}, 1),
\]

where \( \tilde{\Gamma}(y, 1) \) denotes a centered Gamma-distributed random variable with shape parameter \( y \) and a unit scale parameter. The shape factors, \( \theta_{ut} \) and \( \theta_{dt} \), follow autoregressive processes,

\[
\theta_{ut, t+1} = \bar{\theta}_{ut} + \rho_{\theta u} (\theta_{ut} - \bar{\theta}_{ut}) + \delta_{\theta u} \omega_{\theta_{ut, t+1}},
\]

\[
\theta_{dt, t+1} = \bar{\theta}_{dt} + \rho_{\theta d} (\theta_{dt} - \bar{\theta}_{dt}) + \delta_{\theta d} \omega_{\theta_{dt, t+1}},
\]

where \( \rho_y \) denotes the autoregressive term of process \( y_{t+1} \), \( \delta_u \) the sensitivity to \( \omega_{\theta_{ut, t+1}} \), and \( \bar{\theta} \) the constant long-run mean. Given that Gamma distributions are right-skewed by design, the growth shock with a negative loading on \( \omega_{\theta_{dt, t+1}} \) models the left-tail events; hence, \( \omega_{\theta_{dt, t+1}} \) is interpreted as the downside uncertainty shocks, and \( \theta_{dt} \) the real downside uncertainty.

State variables: \( \{ \theta, \theta_{ut}, \theta_{dt} \} \).

State variable shocks: \( \{ \omega_{\theta_{ut}}, \omega_{\theta_{dt}} \} \).

F.1.1.c Inflation and uncertainties Inflation process receives contemporaneous shocks from the real side. Denote \( \pi_{t+1} \) as the change in the log consumer price index for all urban consumers, \( \pi_{ut} \) the nominal
State variable shocks:
Denote upside uncertainty and $\pi_d$ the nominal downside uncertainty. The inflation system follows this reduced-form dynamics,

$$
\pi_{t+1} = \pi + \rho_d \theta (\pi_t - \tilde{\pi}) + \rho_{\pi \theta} (\theta u_t - \bar{\theta} u) + \rho_{\theta d} (\theta d_t - \tilde{\theta} d)
$$

where the inflation disturbance is sensitive to the two real uncertainty shocks, and the residual is decomposed into two nominal uncertainty shocks that are mutually independent of one another,

$$
u_{t+1}^\pi = (\delta \theta g \omega u_{t+1} + \delta g \omega \bar{d}_{t+1}) + (\delta \pi \pi u_{t+1} - \delta \pi d \omega d_{t+1}).
$$

The shocks follow centered Gamma distributions with time-varying shape parameters,

$$
\omega_{u_{t+1}} \sim \tilde{\Gamma} (\pi u_{t+1}),
\omega_{d_{t+1}} \sim \tilde{\Gamma} (\pi d_{t+1})
$$

Importantly, the theoretical structural representation of the inflation dynamics above is,

$$
\pi_{t+1} = \xi_{\pi, t} + [\delta_\pi - \ln (1 + \delta_\pi)] S_t + \delta_\pi \omega_{t+1},
$$

where $\delta_\pi = [\delta_{\pi u} \quad \delta_{\pi d} \quad \pi \pi - \delta_{\pi d} 0 0 0] \pi_{t+1}$ and the relevant shocks are $\omega_{u_{t+1}}, \omega_{d_{t+1}}, \omega_{\pi_{t+1}}, \omega_{\theta d_{t+1}}$, and $\omega_{\pi_{t+1}}$. The signs of the innovation loadings on the two real uncertainty shocks, $\omega_{u_{t+1}}$ and $\omega_{\pi_{t+1}}$, are not restricted in the model, whereas $\delta_{\pi u}$ and $\delta_{\pi d}$ are assumed to be positive.

State variables: $[\pi, \pi u, \pi d]$.
State variable shocks: $\{\omega_{\pi_{t+1}}, \omega_{d_{t+1}}\}$.

**F.1.1.d Risk aversion** Denote $q_t$ as the time-varying risk aversion variable.

$$
q_{t+1} = q + \rho_q (\theta_t - \tilde{\theta}) + \rho_{q \theta} (\theta u_t - \bar{\theta} u) + \rho_{q d} (\theta d_t - \tilde{\theta} d)
$$

where the risk aversion shock is sensitive to the real and nominal uncertainty shocks, the short rate shock and a risk aversion-specific heteroskedastic shock,

$$
u_q^{xq} = (\delta_{\pi u} q u_{t+1} + \delta_{\pi d} q \bar{d}_{t+1}) + (\delta_{\pi \pi u} q u_{t+1} + \delta_{\pi \pi d} q \bar{d}_{t+1}) + \delta_{q \pi} q_{t+1},
$$

where the risk aversion-specific shock follows a centered heteroskedastic Gamma distribution,

$$
q_{t+1} \sim \tilde{\Gamma} (q u_{t+1}).
$$

State variables: $\{q\}$.
State variable shocks: $\{\omega_q\}$.

**F.1.1.e Real short rate and uncertainties** Denote $x_t$ as the latent real short rate,

$$
x_{t+1} = x + \rho_x (\theta_t - \tilde{\theta}) + \rho_{x u} (\theta u_t - \bar{\theta} u) + \rho_{x d} (\theta d_t - \tilde{\theta} d)
$$

where the short rate shock is sensitive to the real and nominal uncertainty shocks as well as a short rate-specific homoskedastic shock,

$$
u_x^{xq} = (\delta g \theta u_{u_{t+1}} + \delta g \pi \bar{d}_{t+1}) + (\delta g \pi u_{t+1} + \delta g \pi d \bar{d}_{t+1}) + \delta g q_{t+1} x_{t+1},
$$

where the (exogenous) short rate shocks follow centered Gamma distributions with time-varying shape parameters,

$$
\omega_{x u_{t+1}} \sim \tilde{\Gamma} (x u_{t+1}), \omega_{x d_{t+1}} \sim \tilde{\Gamma} (x d_{t+1})
$$

State variables: $\{x, x u, x d\}$.
State variable shocks: $\{\omega_{x u}, \omega_{x d}\}$.

**F.1.1.f Real dividend growth** Denote $g_t$ as the change in log real dividend per share,

$$
g_{t+1} = g + \rho_g (\theta_t - \tilde{\theta}) + \rho_{g u} (\theta u_t - \bar{\theta} u) + \rho_{g d} (\theta d_t - \tilde{\theta} d)
$$

where the dividend growth shock is sensitive to the real and nominal uncertainty shocks as well as a dividend-specific homoskedastic shock,

$$
u_g^{xq} = (\delta g \theta u_{u_{t+1}} + \delta g \pi \bar{d}_{t+1}) + \delta g q_{t+1},
$$

where the sign of $\delta_g$ is not restricted, and the dividend-specific shock is assumed to follow a homoskedastic Gamma distribution,

$$
g_{t+1} \sim \tilde{\Gamma} (g u_{t+1}).
$$

Importantly, the theoretical structural representation of the real growth dynamics above is,

$$
g_{t+1} = \xi_{g, t} + (\delta g + \ln (1 - \delta_g)) S_t + \delta g \omega_{t+1},
$$

It is a risk aversion variable, because the exact definition is risk aversion (motivated from a HARA utility is $\gamma \exp (q u)$).
where \( \delta \in [\delta_{\rho u}, \delta_{\rho d}, 0, 0, 0, \delta_{y}] \) so that the relevant shocks are \( \omega_{\rho u, t+1}, \omega_{\rho d, t+1} \), and \( \omega_{y, t+1} \).

State variables: \( \{q\} \).

State variable shocks: \( \{\omega\} \).

**F.1.2 U.S. Real Pricing Kernel**

I specify the (minus) logarithm of the real global pricing kernel to be affine to the global state variable levels and shocks,

\[
-m_{t+1} = x_t + [\delta_m - \ln (1 + \delta_m)] S_t + \delta_m \omega_{t+1},
\]

where the drift \( x_t \) is the real short rate, \( \delta_m (1 \times 8) \) prices of risks, \( \omega_{t+1} (8 \times 1) \) the state variable shock matrix defined earlier, and \( [\delta_m - \ln (1 + \delta_m)] S_t \) the Jensen’s inequality term given the Gamma distributional assumptions.

The real global pricing kernel is spanned by five global shocks: the real upside and downside uncertainty shocks (\( \omega_{\rho u} \) and \( \omega_{\rho d} \)), the inflation upside and downside uncertainty shocks (\( \omega_{\rho u} \) and \( \omega_{\rho d} \)), and the risk aversion shock (\( \omega_{y} \)). First, the real two-side uncertainty shock and the risk aversion shock span the pricing kernel, which can be motivated in Campbell and Cochrane (1999) and Bekker, Engstrom, and Xu (2017). Second, the two inflation uncertainty shocks span the real pricing kernel, which is to induce the inflation risk premium.

**F.1.3 U.S. Asset Prices and Risk Premiums**

**F.1.3.a Nominal Treasury Bonds** The real global short rate (\( y_{t,1} = -\ln(\mathbb{E}[\exp(m_{t+1})]) \)) and the nominal global short rate (\( \tilde{y}_{t,1} = -\ln(\mathbb{E}[\exp(m_{t+1} + \pi_{t+1})]) \)) are solved in closed forms,

\[
y_{t,1} = x_t, \quad \tilde{y}_{t,1} = x_t + \zeta_{x,t} + \ln [(1 + \delta_m + \delta_a) \circ (1 + \delta_m) \circ (1 + \delta_a) - 1] S_t.
\]

inflation compensation

where “\(\ln(.)\)” is the element-wise logarithm operator, “\(\circ\)” the Hadamard product of two identically sized matrices (or element-by-element matrix multiplication), and “\(\circ (.)^{-1}\)” the Hadamard inverse. The three components in nominal short rate are the real short rate (\( x_t \)), the expected inflation rate (\( \zeta_{x,t} \)), and the inflation risk premium to compensate investors for bearing the inflation risk associated with the nominal bonds. It is noteworthy that the linear approximation of the inflation risk premium, \( \mathbb{E}[(1 + \delta_m + \delta_a) \circ (1 + \delta_m) \circ (1 + \delta_a) - 1] \), is \( - (\delta_m + \delta_a) S_t, \) or \( \text{Cov}_t(m_{t+1}, \pi_{t+1}) \) as derived in the Gaussian-augmented nominal term structure literature (see, e.g., Campbell, Sunderam, and Viceira, 2017).

The price of the \( n \)-period zero-coupon nominal bond (\( \tilde{P}_{t,n}^m \)) can be then solved recursively in exact closed forms, and is an exponential affine function of a set of time-varying state variables.

\[
\tilde{P}_{t,n}^m = E_t \exp \left( \tilde{y}_{t+1,n-1} + m_{t+1} - \pi_{t+1} \right)
\]

\[
= E_t \exp \left( x_{t+1} + \zeta_{x,t+1} + \ln [(1 + \delta_m + \delta_a) \circ (1 + \delta_m) \circ (1 + \delta_a) - 1] S_{t+1} + m_{t+1} - \pi_{t+1} \right)
\]

where \( A_{0,n} \), \( A_{1,n} \), and \( X_t \) are constant scalars or matrices.

The log return of the global nominal \( n \)-period zero-coupon bonds from \( t \) to \( t+1 \) can be expressed as follows,

\[
\tilde{r}_{t+1,n}^b = \ln \left( \frac{\tilde{P}_{t+1,n-1}}{\tilde{P}_{t,n}^m} \right),
\]

where \( \tilde{r}_{t+1,n}^b \) is a homoskedastic Gaussian shock to potentially capture approximation error.

**F.1.3.b Bond Risk Premium** Given the no-arbitrage condition, \( E_t[\exp(m_{t+1} + \tilde{r}_{t+1,n}^b)] = 1 \), the global bond risk premium (ignoring the Jensen’s inequality terms) has a closed-form solution,

\[
E_t[\tilde{y}_{t+1,n} + \frac{1}{2} \sigma_y^2] = \ln (1 + \delta_m + \delta_a - \mathbb{E}_t[\delta_m] \circ (1 + \delta_m + \delta_a) \circ (1 - \mathbb{E}_t[\delta_m] - 1) S_t.
\]

which in a quadratic Gaussian approximation has the following expression,

\[
\approx (\delta_m + \delta_a) \circ \mathbb{E}_t[\delta_m] \circ S_t = - \text{Cov}_t(\tilde{m}_{t+1}, \tilde{r}_{t+1,n}^b).
\]

where \( \delta_m \) is the SDF loading on the four global uncertainty shocks subject to the time-varying global risk aversion as discussed in Section F.1.2 and \( \delta_a \) is the inflation rate loading on the four global uncertainty shocks as discussed in Section F.1.2.
F.1.3.c Equities

Bekaert, Engstrom, and Xu (2017) show that log equity returns are quasi-affine to the state variable levels and shocks as below:

\[ \tilde{r}_{t+1}' = \ln \left( \frac{P_{D_{t+1}} + 1}{P_{D_{t}}} \cdot \frac{	ilde{D}_{t+1}}{D_{t}} \right), \] (A36)

\[ = \Omega^0_2 + \Omega^1_2 X_t + \Omega^2_2 \omega_{t+1} + [\Omega^3_2 + \ln (1 - \Omega^4_2) S_t] + \epsilon^r_{t+1}, \] (A37)

where \( \epsilon^r_{t+1} \sim N(0, \sigma^r_2) \) is a homoskedastic Gaussian shock to potentially capture approximation error.

F.1.3.d Equity Risk Premium

Given the no-arbitrage condition, \( E_t[\exp(\tilde{m}_{t+1} + \tilde{r}_{t+1}')] = 1 \), the global equity risk premium has a closed-form solution using the return process,

\[ E_t[\tilde{r}_{t+1}'] = \tilde{y}_t + \frac{1}{2} \tilde{\sigma}^2 = \ln \left[ (1 + \delta_m + \delta_v - \Omega^0_2) \circ (1 + \delta_m + \delta_v)^{-1} \circ (1 - \Omega^4_2)^{-1} \right] S_t, \] (A38)

\[ \approx [(\delta_m + \delta_v) \circ \Omega^2_2] S_t = -Cov_t(\tilde{m}_{t+1}, \tilde{r}_{t+1}'). \] (A39)

F.1.3.e Variances

The physical variance for Asset \( a \in \{b, e\} \),

\[ V_t^{a,P} = E_t\left[ (\tilde{r}_{t+1}' - E_t(\tilde{r}_{t+1}'))^2 \right], \] (A40)

\[ = \Omega^2_2 S_t \Omega^2_2' + \sigma^2_a, \] (A41)

where the parameter matrices are discussed in Equations (A33) and (A36).

The risk-neutral variance for Asset \( a \in \{b, e\} \),

\[ V_t^{a,Q} = E_t^n\left[ (\tilde{r}_{t+1}' - E_t^n(\tilde{r}_{t+1}'))^2 \right], \] (A42)

\[ = \frac{\partial^2 mgf^n(\tilde{r}_{t+1}'; \nu)}{\partial \nu^2}|_{\nu=0} = \frac{\partial^2 mgf^n(\tilde{r}_{t+1}'; \nu)}{\partial \nu^2}|_{\nu=0} \] (A43)

\[ = [\Omega^2_2 \circ (1 + \delta_m + \delta_v)] S_t [\Omega^2_2 \circ (1 + \delta_m + \delta_v)^{-1}] + \sigma^2_a, \] (A44)

where the moment generating function is \( mgf^n(\tilde{r}_{t+1}'; \nu) = E_t^n[\exp(\tilde{m}_{t+1} + \nu \tilde{r}_{t+1}')] \). "\( \circ \)" is the Hadamard product of two identically sized matrices (or element-by-element matrix multiplication), and "\( (\cdot)^{-1} \)" is the Hadamard inverse. \( \Omega^2_2 \) is the asset return loading vector on the common shocks, or an “amount-of-risk” loading vector; \( (\delta_m + \delta_v) \) represents the nominal pricing kernel loading vector on the common shocks, or a “price-of-risk” loading vector. Intuitively, a positive downside uncertainty shock is perceived as bad news, driving up the intertemporal variance for \( n \theta m, d, l \) in the minus \( m_t, t \) expression is smaller than 0 and less than \( \delta_m n_\theta m, d, l \).

F.1.3.f Variances as Assets: Variance Risk Premium

Hence, the solutions of variances in closed form imply a premium of \( V_t^{a,Q} \) over \( V_t^{a,P} \). For asset \( a \in \{b, e\} \),

\[ VRP^n_{t+1} = V_t^{a,Q} - V_t^{a,P}. \] (A45)

F.2 Other Asset Markets

This world economy is partially integrated. Each market is complete. Each country-level state variable has a global component with constant exposures to the global levels and shocks and an idiosyncratic component. Idiosyncratic shocks are uncorrelated across countries. Under the no-arbitrage assumption, there exists closed-form solutions for country equity and bond prices.

F.2.1 Local State Variables: Matrix representation

In a matrix representation, the regional state vector denoted as \( X_{t+1}' (11 \times 1) \),

\[ \begin{bmatrix} \theta_{t+1}' & \theta d_{t+1}' & \pi_{t+1}' & \pi u_{t+1}' & \pi d_{t+1}' & x_{t+1}' & x u_{t+1}' & x d_{t+1}' & \bar{y}_{t+1}' & \bar{q}_{t+1}' \end{bmatrix}', \]

follows this general dynamics:

\[ X_{t+1}' = \alpha_X' \circ \xi_{X,t} + (1 - \alpha_X') \circ \xi_{X,t} + \text{Jensen’s terms} \left( \alpha_X' \circ \delta_X' \circ \xi_{X,t} + \text{Jensen’s terms} \left( (1 - \alpha_X') \circ X_{t}' \circ S_t \right) \right) \]

Jensen’s inequality terms

\[ + \left( \alpha_X' \circ \delta_X' \circ X_{t}' \circ S_t \right) \omega_{t+1}' \] (A46)

\[ \omega_{t+1}' \sim \Gamma(S_t^4, 1) - S_t. \] (A47)
where $\xi_{X,t}$ (11 $\times$ 1) denotes the conditional mean vector of the global state variables $X_{t+1}$ in Section F.1.1. $\omega_{t+1}$ (9 $\times$ 1) the global state variable shock matrix, $\delta_X$ (9 $\times$ 1) the constant local coefficient vector to the global state variable shock $\omega_{t+1}$ (which are not constraint to be the same with global state variable loadings on global shocks $\delta_X$), $S_t$ (9 $\times$ 1) the time-varying shape parameters of local shocks, and $Y (\alpha_X^* \circ \delta_X, S_t)$ is the Jensen’s inequality term from Gamma distributions. The local counterparts are defined as follows. $\xi_{X,t}$ (11 $\times$ 1) denotes the local component of the conditional mean vector of the regional state variables, $\omega_{t+1}$ (9 $\times$ 1) the local state variable shock matrix, $\xi_{X,t+1}$ (9 $\times$ 1) the constant coefficient vector to the local state variable shocks $\omega_{t+1}$, $S_t$ (9 $\times$ 1) the time-varying shape parameters of local shocks, $\xi_{X,t}$ (11 $\times$ 1) captures the constant global exposures.

The shock structures of each local state variables follow the global counterparts to ensure local shocks are also mutually independent from each other.

F.2.2 Local Real Pricing Kernel

I specify the logarithm of the local real local pricing kernel to be affine to the global and local state variable levels and shocks,

$$-m^1_{t+1} = \alpha^i_m \left( x_t + \delta^i_m \omega_{t+1} \right) + (1 - \alpha^i_m) \left( x_t^i + m^i_\omega \omega_{t+1}^i \right)$$

$$\left[ \begin{array}{cccc} \omega_{b,t+1} & \omega_{b,t+1}^i & \omega_{b,t+1}^\ell & \omega_{b,t+1} \end{array} \right]$$

$\xi_{X,t+1}$ (11 $\times$ 9) the constant coefficient vector to the local state variable shocks $\omega_{t+1}$, $S_t$ (9 $\times$ 1) the time-varying shape parameters of local shocks, $\omega_{b,t+1}^i$ (1 $\times$ 9) denotes a vector of constant sensitivities to global shocks. Similarly, $m^i_\omega$ (1 $\times$ 7) denotes a vector of constant sensitivities to local shocks.

The drift term, $\alpha^i_m x_t + (1 - \alpha^i_m) x_t^i$, is the real regional short rate.

F.2.3 Local Asset Prices and Risk Premiums

F.2.3.a Nominal Treasury Bonds The real local short rate ($y^i_{t,1} = -\ln (E_t [\exp (m^i_{t+1})])$) and the nominal regional short rate ($\bar{y}^i_{t,1} = -\ln (E_t [\exp (m^i_{t+1} - \pi^i_{t+1})])$) are solved in closed forms,

$$y^i_{t,1} = \alpha^i_m x_t + \alpha^i_\xi \xi_{t,i} + (1 - \alpha^i_m) x_t^i + (1 - \alpha^i_\xi) \xi_{t,i}^i$$

$$\bar{y}^i_{t,1} = \alpha^i_m x_t + \alpha^i_\xi \xi_{t,i} + (1 - \alpha^i_m) x_t^i + (1 - \alpha^i_\xi) \xi_{t,i}^i + \ln \left( 1 + (1 - \alpha^i_m) \delta^i_m \right) \circ \left( 1 + (1 - \alpha^i_\xi) \delta^i_\xi \right)^{-1} \circ \left( 1 + (1 - \alpha^i_m) m^i_\omega \right) \circ \left( 1 + (1 - \alpha^i_\xi) \pi^i_\omega \right) S_t$$

(49)

(50)

The price of n-period zero-coupon nominal bond ($P^i_{t,n}$) can be then solved recursively in exact closed forms, given the shock specifications. The nominal local bond return from t to t + 1 can be approximately expressed as follows,

$$\bar{y}^i_{t,1} = \ln \left( \frac{P^i_{t+1,n+1}}{P^i_{t,1,n}} \right)$$

$$= \Omega^i_{t,n} + \Omega^i_{t,n} X_t + \Omega^i_{2,n} \omega_{t+1} + \left[ \Omega^i_{2,n} + \ln \left( 1 - \Omega^i_{2,n} \right) S_t \right]$$

$$+ \Omega^i_{b,n} X_t^i + \Omega^i_{b,n} \omega_{t+1} + \left[ \Omega^i_{2,n} + \ln \left( 1 - \Omega^i_{2,n} \right) S_t \right] + \bar{y}^i_{t+1}.$$

(51)

(52)

F.2.3.b Bond Risk Premium Given the no-arbitrage condition, $E_t [\exp \left( \bar{m}^i_{t+1} + \bar{y}^i_{t+1} \right)] = 1$ where $\bar{y}^i_{t+1}$ is the nominal bond return, the regional bond risk premium has a closed-form solution,

$$E_t [\bar{y}^i_{t+1}] = \ln \left( 1 + \alpha^i_m \delta_m + \alpha^i_\xi \delta_\xi - \Omega^i_{2,n} \right) \circ \left( 1 + \alpha^i_m \delta_m + \alpha^i_\xi \delta_\xi \right)^{-1} \circ \left( 1 - \Omega^i_{2,n} \right) S_t$$

(1) compensation for global risk exposure

40
\[ + \ln \left[ (1 + (1 - \alpha^i_m) m^i + (1 - \alpha^i_e) \pi^i_e - \Omega^b_{4,m} \otimes \left( (1 + (1 - \alpha^i_m) m^i + (1 - \alpha^i_e) \pi^i_e \right)_{\alpha^i_m}^{-1} \otimes (1 - \Omega^b_{4,m} \otimes)_{\alpha^i_m}^{-1} S_t^i. \right] \]

(2) compensation for regional risk exposure

(A53)

which in a quadratic Gaussian approximation has the following expression,
\[ \approx \left( \alpha^i_m \delta_m + \alpha^i_e \delta_e^2 \right) \otimes \Omega^b_{2,m} S_t + \left( (1 - \alpha^i_m) m^i + (1 - \alpha^i_e) \pi^i_e \right) \otimes \Omega^b_{4,m} S_t^i = -\text{Cov}_t(\tilde{m}^i_{t+1}, \tilde{p}^b_{i+1}). \]

(A54)

\[ \approx (1) \approx (2) \]

\textbf{F.2.3.c Equities} The nominal local equity return from \( t \) to \( t + 1 \) can be approximately expressed as follows,
\[ \tilde{r}^e_{t+1} = \ln (\tilde{P}^e_{t+1,n-1} / \tilde{P}^e_{t+1}) \]
\[ = \Omega^e_0 + \Omega^e_{t} X_t + \Omega^e_{t+1} \delta_{\omega_t+1} + \left[ \Omega^e_{t+1} + \ln \left(1 - \Omega^e_{t} \right) S_t \right] \]
\[ + \Omega^e_{t+1} X_t + \Omega^e_{t+1} \delta_{\omega_t+1} + \left[ \Omega^e_{t+1} + \ln \left(1 - \Omega^e_{t} \right) S_t \right] + \varepsilon^e_{t+1}, \]
\[ \text{where } \varepsilon^e_{t+1} \text{ is a homoskedastic Gaussian shock with volatility } \sigma^e_t \text{ to capture approximation error.} \]

\[ \approx (1) \approx (2) \]

\textbf{F.2.3.d Equity Risk Premium} Given the no-arbitrage condition, \( E_t[\exp(\tilde{m}^e_{t+1} + \tilde{r}^e_{t+1})] = 1 \) where \( \tilde{r}^e_{t+1} \) is the nominal local equity return from \( t \) to \( t + 1 \) can be approximately expressed as follows,
\[ E_t[\tilde{r}^e_{t+1}] = \ln \left[ (1 + (1 - \alpha^e_m) m^e + (1 - \alpha^e_e) \pi^e_e - \Omega^b_{4,m} \otimes \left( (1 + (1 - \alpha^e_m) m^e + (1 - \alpha^e_e) \pi^e_e \right)_{\alpha^e_m}^{-1} \otimes (1 - \Omega^b_{4,m} \otimes)_{\alpha^e_m}^{-1} S_t \right] \]
\[ + \ln \left[ (1 - \alpha^e_m) m^e + (1 - \alpha^e_e) \pi^e_e \right] \otimes \Omega^b_{4,m} S_t^i = -\text{Cov}_t(\tilde{m}^e_{t+1}, \tilde{p}^e_{t+1}). \]

(A55)

\[ \approx (1) \approx (2) \]

\textbf{F.2.3.e Variances} The physical variance for Asset \( a \in \{b, e\}, \)
\[ V^{a,i}_{t+1} \equiv E_t \left[ \left( \tilde{P}^a_{t+1} - E_t(\tilde{P}^a_{t+1})^2 \right) \right], \]
\[ = \Omega^a_{t} \otimes S_t \omega^a_{t} + \sigma^2_a, \]
\[ \text{where the parameter matrices are discussed in Equations (A51) and (A55).} \]

The risk-neutral variance for Asset \( a \in \{b, e\}, \)
\[ V^{a,i,Q}_{t+1} \equiv E_t \left[ \left( \tilde{P}^{a,i}_{t+1} - E_t(\tilde{P}^{a,i}_{t+1})^2 \right) \right], \]
\[ = \left[ \Omega^a_{t} \otimes (1 + \delta_m + \delta_e)^{\alpha^i_m} \right] S_t \left[ \Omega^a_{t} \otimes (1 + \delta_m + \delta_e)^{\alpha^i_e} \right] \]
\[ + \left[ \Omega^a_{t} \otimes (1 + m^e + \pi^e_e)^{\alpha^i_e} \right] S_t^i \left[ \Omega^a_{t} \otimes (1 + m^e + \pi^e_e)^{\alpha^i_e} \right] + \sigma^2_a. \]

(A59)

(A60)

(A61)

(A62)

\textbf{F.2.3.f Variances as Assets: Variance Risk Premium} The present tripartite model derives closed-form solutions for VRP which show potentials to capture its empirical time variation characteristics. For

\footnote{Note that, the non-linearity is due to the non-linearities in the moment generating function of Gamma shocks.}
\footnote{The quadratic Taylor approximation for \( "y - \ln(1 + y)" \) is \( \frac{1}{2} y^2 \).}
\footnote{Note that, the non-linearity is due to the non-linearities in the moment generating function of Gamma shocks.}
asset \ a \in \{b,e\},
\begin{align*}
VP_{t}^{P} &= V_{t}^{a,Q} - V_{t}^{a,P} \\
&= \Omega_{2}^{a} \circ (1 + \delta_{m} + \delta_{\pi})^{\omega-1} S_{t} \left[ \Omega_{2}^{a} \circ (1 + \delta_{m} + \delta_{\pi})^{\omega-1} \right] - \Omega_{2}^{a} S_{t} \Omega_{2}^{a'} \\
&+ \left[ \Omega_{4}^{a} \circ (1 + m_{w}^{i} + \pi_{w}^{i})^{\omega-1} \right] S_{t}^{\epsilon} \left[ \Omega_{4}^{a} \circ (1 + m_{w}^{i} + \pi_{w}^{i})^{\omega-1} \right] - \Omega_{4}^{a} S_{t}^{\epsilon} \Omega_{4}^{a'},
\end{align*}
(A63)
where \( \Omega_{2}^{a} \) and \( \Omega_{4}^{a,i} \) are the “amount-of-risk” coefficients, and \( \delta_{m} \) and \( m_{w}^{i} \) are the “price-of-risk” coefficients that are linear to the global and regional risk aversions respectively. In the tripartite framework, the variance risk premium can be decomposed into a global component and a regional component.

F.2.3.g Foreign Exchange Returns

Denote \( s_{i}/i \) as the log of the spot exchange rate in units of dollars per foreign currency \( i \) at region \( i \). As stated in the Proposition 1 of Backus, Foresi, and Telmer (2011), the change in the nominal exchange rate, \( \Delta s_{i+1}/i = s_{i+1}/i - s_{i}/i \), in a frictionless world is equivalent to the nominal pricing kernel difference,
\( \Delta \hat{\delta}_{i+1}/i = m_{i+1}/i - m_{i+1}/i + \pi_{i+1}/i - \pi_{i+1}/i. \) (A64)

The change in the nominal exchange rate means a depreciation in dollars (and an appreciation in region \( i \) currency). In this model, a hypothetical world with perfect integration (i.e., \( \alpha_{m}^{i} = 1 \)) still obtains a time-varying spot rate to address the inflation risk amid the real macroeconomic risks. The regional currency excess return is the log return to U.S. investors of borrowing in dollars to hold foreign investment currency \( i \), can be expressed as an exact dynamic factor model,
\begin{align*}
\hat{r}_{t+1}^{f,i} &= \hat{\Delta} s_{t+1}/i + \tilde{y}_{t+1}, \\
&= \Omega_{0}^{f,i} + \Omega_{1}^{f,i} \epsilon_{t+1}/i + \Omega_{2}^{f,i} \omega_{t+1} + \Omega_{3}^{f,i} X_{t} + \Omega_{4}^{f,i} \omega_{t+1} + \epsilon_{t+1}/i + Jensen's + \epsilon_{t+1}/i,
\end{align*}
(A65)
where \( \Omega_{0}^{f,i} \), \( \Omega_{1}^{f,i} \), \( \Omega_{2}^{f,i} \), \( \Omega_{3}^{f,i} \), and \( \Omega_{4}^{f,i} \) are constant matrices; \( \epsilon_{t+1}/i \) is the approximation error term that follows a homoskedastic Gaussian distribution with volatility \( \sigma_{f,i}^{2} \).

F.2.3.h Foreign Exchange Risk Premium

Given the no-arbitrage condition, \( E_{t}[\exp(\hat{m}_{t+1}/i + r_{t+1}^{f,i})] = 1 \) where \( r_{t+1}^{f,i} \) is the nominal foreign exchange return (from the U.S. investor’s view point), the foreign exchange risk premium has a closed-form solution,
\begin{align*}
E_{t}[\hat{r}_{t+1}^{f,i} - \hat{y}_{t+1}/i + \frac{1}{2} \sigma_{f,i}^{2}] &= \ln \left[ \left( 1 + \delta_{m} + \delta_{\pi} - \Omega_{2}^{f,i} \right) \circ (1 + \delta_{m} + \delta_{\pi})^{\omega-1} \circ (1 - \Omega_{2}^{f,i})^{\omega-1} \right] S_{t},
\end{align*}
(A67)
which in a quadratic Gaussian approximation has the following expression,
\(\approx (\delta_{m} + \delta_{\pi}) \circ \Omega_{2}^{f,i} S_{t} = -\text{Cov}(\hat{m}_{t+1}/i, \hat{r}_{t+1}^{f,i})\). (A68)

\footnote{Note that, the non-linearity is due to the non-linearities in the moment generating function of Gamma shocks.}
References


Table 1: Summary Statistics.

This table presents the unconditional correlation matrices of USD-denominated log returns of 8 developed countries (United States, USA; Canada, CAN; Germany, DEU; France, FRA; United Kingdom, GBR; Switzerland, CHE; Japan, JPN; Australia, AUS) in Panel A and unconditional univariate moments (with bootstrapped standard errors in parentheses) in Panel B. Mean and standard deviations are presented in annualized percentages. “Equity” return refers to the change in log total return index of domestic country stock market (United States: S&P500; Canada: S&P/TSX 60; Germany: DAX 30; France: CAC 40; United Kingdom: FTSE 100; Switzerland: SMI; Japan: NIKKEI 225; Australia: S&P/ASX 200); CRSP value-weighted return is used to obtain the USA equity return; other return series are obtained from DataStream. “Bond” (“Gov-Bond”) return refers to the change in log 10-year government bond index constructed by DataStream. Data is at monthly frequency. The sample is from March 1987 to December 2016 (T=358). Bold (italics) values indicate <5% (10%) significance level.

Panel A. Unconditional Correlation Matrices, 8 countries, 1987/03 - 2016/12

<table>
<thead>
<tr>
<th></th>
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<th>Australasia</th>
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</thead>
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<td>CAN</td>
<td>DEU</td>
</tr>
<tr>
<td>USA</td>
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<td>0.782</td>
<td>0.725</td>
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<td>0.872</td>
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<tr>
<td>FRA</td>
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<td>GBR</td>
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<td>CHE</td>
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<td>JPN</td>
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<tr>
<td>AUS</td>
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Panel B. Unconditional Univariate Moments (annualized percentages)

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<td>DEU</td>
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<td>(4.220)</td>
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<td>(1.018)</td>
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<td>-0.961</td>
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<tr>
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<td>(0.386)</td>
<td>(0.252)</td>
<td>(0.255)</td>
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<table>
<thead>
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<th>North America</th>
<th>Europe</th>
<th>Australasia</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>USA</td>
<td>CAN</td>
<td>DEU</td>
</tr>
<tr>
<td>(0.984)</td>
<td>(1.652)</td>
<td>(1.879)</td>
<td>(1.861)</td>
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<tr>
<td>(0.330)</td>
<td>(0.500)</td>
<td>(0.520)</td>
<td>(0.509)</td>
</tr>
<tr>
<td>Skewness</td>
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<td>-0.416</td>
<td>-0.006</td>
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<tr>
<td>(0.230)</td>
<td>(0.261)</td>
<td>(0.186)</td>
<td>(0.188)</td>
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</table>
Table 2: Estimation Results of Global Bond Comovement.

This table presents the estimation results of the global bond return comovement models as described in Section 2. The full model builds on Engle and Kelly (2012)'s Dynamic Equicorrelation (DECO) model and features three new tests: (1) whether global equity and bond comovements are equal, (2) whether global bond comovement is symmetric and (3) whether it is cyclical. **Model details:** Denote $\bar{z}_{i,t}^{B}$ ($N \times 1$) as the standardized residuals of country bond returns during period $t+1$. The conditional equicorrelation matrix of $\bar{z}_{i,t}^{B}$ is defined by $E_{i}[\bar{z}_{i,t}^{B} \bar{z}_{j,t+1}^{B}] = Corr^{B}(N \times N)$,

$$Corr^{B} = (1 - \rho_{B}^{B})I_{N} + \rho_{B}^{B} J_{N \times N},$$

where $I_{N}$ is an identity matrix; $J_{N \times N}$ is a matrix of ones. The equicorrelation (by definition) is an equal-weighted average of correlations of unique country pairs (i.e., total of $N(N-1)/2$ pairs) conditional given information set at $t$:

$$\rho_{B}^{B} = \frac{2}{N(N-1)} \sum_{i>j} \frac{q_{i,j,t}^{B} q_{j,i,t}^{B},}{\sqrt{q_{i,i,t}^{B} q_{j,j,t}^{B}}},$$

where $q_{i,j,t}^{B}$ is the $(i,j)$-th element of a symmetric matrix $Q^{B}$ ($N \times N$) which follows a generalized autoregressive heteroskedastic process, (omitting superscript “B” below for simplicity)

$$Q_{t} = Q^{*} \odot \Phi_{t} + \beta_{1} \left( \frac{Q_{t-1}^{2} - z_{t}^{2} \hat{Q}_{t-1}^{2}}{\gamma} \right) + \beta_{2} \left( Q_{t-1}^{2} - Q^{*} \odot \Phi_{t-1} \right) + \gamma \left( \frac{Q_{t-1}^{2} - \hat{Q}_{t-1}^{2}}{\gamma} \right),$$

where “$\odot$” denotes the Hadamard product operator (element-by-element): $\hat{Q}_{t}$ is $Q_{t}$ with off-diagonal terms being zeros, which is a modification to Engle (2002) proposed by Aielli (2013). **Tests:** [1. Equality] The constant part of the long-run conditional mean ($Q^{*} \odot \Phi_{t}$), $Q^{*}$, can be defined as $Q^{*} + \nu (J_{N \times N} - I_{N})$ (to test) or $Q^{B}$, where $Q^{B}$ ($Q^{*}$) is the pre-determined unconditional correlation matrix of equity (bond) returns respectively. [2. Asymmetry] $n_{i}(N \times 1) = I_{z_{t} < 0} \odot z_{t}$, where $I_{z_{t} < 0} (N \times 1)$ is assigned 1 if the residual is less than 0, and assigned 0 otherwise; $\hat{z} = [I_{z_{t} \leq 0}]$. [3. Cyclicality] $\Phi_{t} (N \times N) = J_{N \times N} + \Phi_{t} (J_{N \times N} - I_{N})$, where $\Phi_{t} = \Phi_{t}^{\text{world}}$ is the standardized world recession indicator (source: OECD). **Estimation:** The unknown parameters are $(\beta_{1}, \beta_{2}, \nu, \gamma, \phi)$, where $\nu$ is estimated separately. Sufficient stationarity conditions for $Q^{B}$ are $\beta_{1} J_{N \times N} + \beta_{2} J_{N \times N} + \gamma \hat{z} < J_{N \times N}$ and $\beta_{1}, \beta_{2} > 0$. Two distributions are considered to obtain log likelihood function: (1) multivariate Gaussian:

$$L^{B}[n] = \frac{1}{2} \sum \left[ \log Corr^{B} + z_{t+1}^{B}(Corr^{B})^{-1} z_{t+1}^{B} \right];$$

(2) multivariate $t$:

$$L^{t}[n] = \frac{1}{2} \sum \left[ \log Corr^{B} + (df + N) \log \left( 1 + \frac{1}{2} z_{t+1}^{B}(Corr^{B})^{-1} z_{t+1}^{B} \right) \right],$$

where $df$ is the degree of freedom of the $N$-variate $t$ distribution. Best estimates of $Corr^{B}$ according to AIC and BIC are used in the second step estimation. Model estimation uses MLE at monthly frequency covering period from March 1987 to December 2016 (T=358), and model selection follows BIC. Bold (italics) values indicate <5% (10%) significance level.

<table>
<thead>
<tr>
<th>Multivariate Gaussian</th>
<th>Multivariate $t$</th>
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<td>$\beta_{2}$</td>
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<tr>
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<td>0.0705</td>
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<tr>
<td>0.0701</td>
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<td>0.0423</td>
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<tr>
<td>0.0304</td>
<td>0.0325</td>
</tr>
<tr>
<td>-0.2746 (0.0551)</td>
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<tr>
<td>$\nu$</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>-0.2746 (0.0551)</td>
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<tr>
<td>-0.0170 (0.0166)</td>
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<tr>
<td>0.0263 (0.0158)</td>
<td>0.0195</td>
</tr>
<tr>
<td>$\phi$</td>
<td>$\phi$</td>
</tr>
<tr>
<td>-0.0375 (0.0253)</td>
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<td>-0.0572 (0.0466)</td>
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<td>0.0431 (0.0238)</td>
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<tr>
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<td>11.1770 (2.0198)</td>
<td>9.8033 (2.3917)</td>
</tr>
<tr>
<td>10.0123 (2.1231)</td>
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**Log-Likelihoods:** LL: -3599.93, -3596.17, -3597.87, -3594.27, -3598.88; AIC: 7023.87, 7018.34, 7021.74, 7016.53, 7023.75; BIC: 7031.63, 7029.99, 7033.38, 7032.06, 7035.40.
Table 3: Estimation Results of Global Equity Comovement.

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<td>6083.74</td>
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</table>
Table 4: Model Fit: (In)Equality between Global Equity Comovement and Global Bond Comovement.

This table replicates the test results on the inequality between global equity comovement and global bond comovement with data. **Data Moments & Test Statistics:** Rows “Data” report the equally-weighted unconditional pairwise correlations (excluding diagonal terms) of returns denominated in USD. Bootstrapped standard errors are reported in parentheses. “Jennrich’s $\chi^2$ (E-B)” is a statistical test adapted from Jennrich (1970) to test the equality of two sample correlation matrices (in this paper, equity and bond). Test details are relegated to Appendix D. “***” denotes <1% significance level, or the equality hypothesis is rejected. **Model Moments:** Three model moments are considered. (1) Rows “Conditional Model” reports time-series averages of the model-implied global equity and bond comovements obtained from the best model according to Tables 2 and 3. (2 and 3) Given the estimation results assuming multivariate $t$ or multivariate Gaussian distributions, Rows “Simulated Model (t)” (“Simulated Model (n)”) report averages of 1000 unconditional global correlations obtained from finite-sample simulations assuming multivariate $t$ distribution (multivariate Gaussian distribution). Bold (italics) values indicate the model point estimates are within 95% (99%) confidence intervals of the corresponding data moments. **Panels:** Full sample and three subsample periods are considered.

<table>
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<th>Bond</th>
<th>Equity</th>
<th>Bond</th>
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</thead>
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<td><strong>Panel A. Full Sample</strong></td>
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<td></td>
<td></td>
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<tr>
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<td>0.6271</td>
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<td>S.E.</td>
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<td>Jennrich’s $\chi^2$</td>
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<td>91.701(***</td>
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<td>Conditional Model</td>
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<td>Data</td>
<td>0.6401</td>
<td>0.5469</td>
<td>0.7538</td>
<td>0.5021</td>
</tr>
<tr>
<td>S.E.</td>
<td>(0.0270)</td>
<td>(0.0223)</td>
<td>(0.0268)</td>
<td>(0.0225)</td>
</tr>
<tr>
<td>Jennrich’s $\chi^2$</td>
<td>116.729(***</td>
<td>124.005(***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Conditional Model</td>
<td>0.6614</td>
<td>0.5391</td>
<td>0.7319</td>
<td>0.5384</td>
</tr>
<tr>
<td>Simulated Model (t)</td>
<td>0.6409</td>
<td>0.4637</td>
<td>0.7578</td>
<td>0.4336</td>
</tr>
<tr>
<td>Simulated Model (n)</td>
<td>0.5997</td>
<td>0.5088</td>
<td>0.5993</td>
<td>0.3896</td>
</tr>
</tbody>
</table>
Table 5: Model Fit: (A)symmetry in Global Comovements.

This table evaluates the fit of global dynamic comovement model(s) to the non-parametric estimates of global upside and downside comovements. **Data Moments:** Following Longin and Solnik (2001) and Ang and Chen (2002), the exceedance correlation of standardized daily returns ($\tilde{x}$ and $\tilde{y}$) at a certain threshold quantile $\tau$ is $\rho(\tilde{x}, \tilde{y}|\tilde{x} < \Phi^{-1}_x(\tau), \tilde{y} < \Phi^{-1}_y(\tau))$ if $\tau < 0.5$ or $\rho(\tilde{x}, \tilde{y}|\tilde{x} > \Phi^{-1}_x(\tau), \tilde{y} > \Phi^{-1}_y(\tau))$ if $\tau \geq 0.5$. Global exceedance correlations are equally-weighted bivariate exceedance correlations across 28 unique country pairs. Daily returns (from 1987/03 to 2016/12) are standardized using the best GARCH-class conditional volatility estimates. Standard errors for “bivariate” exceedance correlations are obtained using Cohen and Cohen (2003); then, standard errors for global exceedance correlations are obtained using Delta’s Method. **Model Moments:** Three global comovement models are considered: (1) Best models assuming multivariate t (“B (2)”, Table 2; “E (4)”, Table 3). (2) Models assuming multivariate t but with no asymmetry term (“B (1)”, Table 2; “E (3)”, Table 3). (3) Best models assuming multivariate Gaussian (“B (2)”, Table 2; “E (4)”, Table 3). Column “Distance” reports the sum of squared standardized distance between model and data moments (of the four quantiles); p-value of the distance is reported in the last column. Bold (italics) values indicate the model point estimates are within 95% (99%) confidence intervals of the corresponding data moments. More quantile choices can be found in Figure 2.

<table>
<thead>
<tr>
<th></th>
<th>Equity Distance, p-value</th>
<th>25%</th>
<th>49%</th>
<th>51%</th>
<th>75%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td></td>
<td>0.3682</td>
<td>0.3292</td>
<td>0.2619</td>
<td>0.2469</td>
</tr>
<tr>
<td>S.E.</td>
<td></td>
<td>0.0199</td>
<td>0.0147</td>
<td>0.0153</td>
<td>0.0216</td>
</tr>
<tr>
<td>Simulated Model (t)</td>
<td></td>
<td>0.3229</td>
<td>0.3326</td>
<td>0.3024</td>
<td>0.2872</td>
</tr>
<tr>
<td>Simulated Model (n)</td>
<td></td>
<td>0.2316</td>
<td>0.2763</td>
<td>0.2631</td>
<td>0.2161</td>
</tr>
<tr>
<td>Simulated Model (t), No Asymmetry</td>
<td></td>
<td>0.3094</td>
<td>0.3133</td>
<td>0.3115</td>
<td>0.3197</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Bond Distance, p-value</th>
<th>25%</th>
<th>49%</th>
<th>51%</th>
<th>75%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td></td>
<td>0.3029</td>
<td>0.3024</td>
<td>0.3079</td>
<td>0.3245</td>
</tr>
<tr>
<td>S.E.</td>
<td></td>
<td>0.0209</td>
<td>0.0149</td>
<td>0.0149</td>
<td>0.0206</td>
</tr>
<tr>
<td>Simulated Model (t)</td>
<td></td>
<td>0.3365</td>
<td>0.3098</td>
<td>0.3114</td>
<td>0.3337</td>
</tr>
<tr>
<td>Simulated Model (n)</td>
<td></td>
<td>0.2165</td>
<td>0.2767</td>
<td>0.2774</td>
<td>0.2394</td>
</tr>
<tr>
<td>Simulated Model (t), No Asymmetry</td>
<td></td>
<td>0.2718</td>
<td>0.2865</td>
<td>0.2888</td>
<td>0.2723</td>
</tr>
</tbody>
</table>

Table 6: Model Fit: Cyclicalities of Global Equity Comovement and Global Bond Comovement.

This table evaluates the fit of best global models for cyclical moments in data. **Data Moments:** “Non-recession” (“Recession”) periods are identified when the OECD world recession indicator is 0 (1). Then the average pairwise unconditional correlations are calculated within each subsample. **Model Moments:** Three model moments are considered. (1) Rows “Conditional Model” reports time-series averages of the model-implied global equity and bond comovements as shown in Tables 2 and 3. (2 and 3) Given the estimation results assuming multivariate t or multivariate Gaussian distribution, Rows “Simulated Model (t)” (“Simulated Model (n)”): report averages of 1000 unconditional global correlations obtained from finite-sample simulations assuming multivariate t distribution (multivariate Gaussian distribution). Bold (italics): point estimates within 95% (99%) confidence intervals of the data moments.

<table>
<thead>
<tr>
<th></th>
<th>Equity Bond</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Non-Recession</td>
<td>Recession</td>
<td>Non-Recession</td>
<td>Recession</td>
<td></td>
</tr>
<tr>
<td>Data</td>
<td>0.5952</td>
<td>0.6571</td>
<td>0.4705</td>
<td>0.4520</td>
<td></td>
</tr>
<tr>
<td>S.E.</td>
<td>(0.0334)</td>
<td>(0.0161)</td>
<td>(0.0302)</td>
<td>(0.0480)</td>
<td></td>
</tr>
<tr>
<td>t Statistics</td>
<td></td>
<td>1.67 (*)</td>
<td>-0.33</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Conditional Model</td>
<td></td>
<td>0.6479</td>
<td>0.6740</td>
<td>0.5285</td>
<td>0.4437</td>
</tr>
<tr>
<td>Simulated Model (t)</td>
<td></td>
<td>0.6364</td>
<td>0.6761</td>
<td>0.4188</td>
<td>0.4169</td>
</tr>
<tr>
<td>Simulated Model (n)</td>
<td></td>
<td>0.5777</td>
<td>0.5885</td>
<td>0.4112</td>
<td>0.4256</td>
</tr>
</tbody>
</table>
Table 7: Dynamic Factor Model Fit & Economic Significance of Risk Aversion

This table evaluates the fit of the full models and demonstrates the economic significance of risk aversion in explaining global return correlations. **Models:** “Empirical BM” denotes the empirical benchmarks obtained from the first part of the paper. Four dynamic factor models are considered: (a full set of factors, a subset of factors excluding the risk aversion shock \( \omega_q \)) \( \hat{\omega} \) (constant betas, time-varying betas). Details of estimation results of the return loadings are relegated to Tables A6 (constant beta) and A7 (time-varying beta). **Fact Checks:** Each stylized fact is summarized by 2 moments in this table to be matched. On Fact 1, average conditional global correlations are calculated over the sample. On Fact 2, periods are considered “downside” (“upside”) when the world return (equal average of all 8 countries) is less than or equal to 0. On Fact 3, the model-implied conditional correlations are regressed on the OECD world recession indicator. The p-values in brackets correspond to the t-test results of the closeness between the empirical benchmark moments and the factor model-implied moments; standard errors of the empirical benchmark moments are calculated using Delta’s method. Bold (italics) values indicate the point estimates are within 95% (99%) confidence intervals of the corresponding data moments. “Yes” (“No”) indicates that the model fits (fails to fit) the stylized facts.

<table>
<thead>
<tr>
<th></th>
<th>Empirical BM:</th>
<th>Dynamic Factor Models:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>constant</td>
<td>time-varying</td>
</tr>
<tr>
<td></td>
<td>Full</td>
<td>Full</td>
</tr>
<tr>
<td><strong>Test Fact 1:</strong> Equity Correlation &gt; Bond Correlation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>{Moment 1: Average Conditional Global Correlations}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Global Equity Correlation</td>
<td>0.6568</td>
<td><strong>0.6919</strong></td>
</tr>
<tr>
<td>Closeness to BM [p-value]</td>
<td>[41.5%]</td>
<td>[64.9%]</td>
</tr>
<tr>
<td>Global Bond Correlation</td>
<td>0.4926</td>
<td><strong>0.4628</strong></td>
</tr>
<tr>
<td>Closeness to BM [p-value]</td>
<td>[42.1%]</td>
<td>[28.4%]</td>
</tr>
<tr>
<td>Fit Fact 1?</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td><strong>Test Fact 2:</strong> Excessive Left-Tail Global Correlation in Equities</td>
<td></td>
<td></td>
</tr>
<tr>
<td>{Moment 2: Global Equity Correlation – Global Bond Correlation}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( r_{\text{World}} &gt; 0 )</td>
<td>0.1579</td>
<td><strong>0.2245</strong></td>
</tr>
<tr>
<td>( r_{\text{World}} \leq 0 )</td>
<td>0.1725</td>
<td><strong>0.2368</strong></td>
</tr>
<tr>
<td>Fit Fact 2?</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td><strong>Test Fact 3:</strong> Countercyclical Equity Correlation, Weakly Procyclical Bond Correlation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>{Moment 3: Sensitivity to OECD World Output Growth}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Global Equity Correlation</td>
<td>-1.2511</td>
<td><strong>-0.6688</strong></td>
</tr>
<tr>
<td>Global Bond Correlation</td>
<td>0.5880</td>
<td><strong>0.5114</strong></td>
</tr>
<tr>
<td>Fit Fact 3?</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>
Table 8: Global Return Covariance Decomposition.

This table calculates the extent to which each state variable contributes to the global equity conditional covariance and global bond conditional covariance. For Country \( i \) and Country \( j \) \((i \neq j)\), the covariance share explained by factor \( \omega \) is,

\[
\frac{\beta_{i,t,\omega} \text{Var}(\omega_{t+1}) \beta_{j,t,\omega}}{\beta_{i,t} \text{Var}(\Omega_{t+1}) \beta_{j,t}},
\]

where \( \beta_{i,t,\omega} \text{Var}(\omega_{t+1}) \beta_{j,t,\omega} \) can be further decomposed into a constant-beta part, \( \beta_{i,0,\omega} \text{Var}(\omega_{t+1}) \beta_{j,0,\omega} \), and a time-varying beta part, \( \beta_{i,1,\omega} \text{Var}(\omega_{t+1}) \beta_{j,1,\omega} s^2_t \) \((s_t \text{ is the standardized instrument})\). Lines in bold indicate the four types of factors; \([\beta_0]\) \((\tilde{\beta}_1)\) indicates the constant \((\text{time-varying})\) part of the explained covariance. The share of total explained comovement is obtained by dividing the time-series average of the pairwise conditional covariance by the unconditional pairwise covariance matrix.

<table>
<thead>
<tr>
<th></th>
<th>(\omega_q)</th>
<th>(\omega_{\theta u})</th>
<th>(\omega_{\pi u})</th>
<th>(\omega_{x u})</th>
<th>(\omega_{\theta d})</th>
<th>(\omega_{\pi d})</th>
<th>(\omega_{x d})</th>
<th>Total Equity</th>
<th>Total Bond</th>
<th>Equity</th>
<th>Bond</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk Aversion:</td>
<td>90.3%</td>
<td>78.2%</td>
<td>90.5%</td>
<td>40.0%</td>
<td>-1.9%</td>
<td>5.1%</td>
<td>0.2%</td>
<td>1.1%</td>
<td>-1.5%</td>
<td>1.3%</td>
<td></td>
</tr>
<tr>
<td>(\omega_{\theta u})</td>
<td>5.5%</td>
<td>-0.4%</td>
<td>5.1%</td>
<td>3.2%</td>
<td>4.9%</td>
<td>0.2%</td>
<td>0.9%</td>
<td>1.4%</td>
<td>7.0%</td>
<td>3.2%</td>
<td></td>
</tr>
<tr>
<td>(\omega_{\pi u})</td>
<td>1.8%</td>
<td>10.8%</td>
<td>1.3%</td>
<td>48.6%</td>
<td>1.2%</td>
<td>0.3%</td>
<td>-0.5%</td>
<td>1.1%</td>
<td>-1.8%</td>
<td>0.7%</td>
<td>-2.5%</td>
</tr>
<tr>
<td>(\omega_{x u})</td>
<td>-0.2%</td>
<td>-10.1%</td>
<td>-0.1%</td>
<td>21.9%</td>
<td>-0.2%</td>
<td>-0.5%</td>
<td>1.9%</td>
<td>1.1%</td>
<td>1.0%</td>
<td>17.0%</td>
<td></td>
</tr>
<tr>
<td>Real Uncertainties:</td>
<td>Total</td>
<td>2.8%</td>
<td>33.6%</td>
<td>1.1%</td>
<td>1.0%</td>
<td>-9.9%</td>
<td>-0.9%</td>
<td>1.7%</td>
<td>-4.9%</td>
<td>54.6%</td>
<td>15.4%</td>
</tr>
<tr>
<td>(\omega_{\pi d})</td>
<td>1.0%</td>
<td>22.8%</td>
<td>-0.2%</td>
<td>-1.8%</td>
<td>3.0%</td>
<td>0.3%</td>
<td>1.1%</td>
<td>-0.1%</td>
<td>1.0%</td>
<td>-4.9%</td>
<td>1.7%</td>
</tr>
<tr>
<td>(\omega_{x d})</td>
<td>1.9%</td>
<td>0.1%</td>
<td>1.1%</td>
<td>21.9%</td>
<td>-0.3%</td>
<td>0.2%</td>
<td>1.1%</td>
<td>-0.1%</td>
<td>1.0%</td>
<td>14.7%</td>
<td>7.2%</td>
</tr>
<tr>
<td>Share of Explained Comovement</td>
<td>49.4%</td>
<td>9.9%</td>
<td>54.6%</td>
<td>15.4%</td>
<td>4.8%</td>
<td>0.2%</td>
<td>5.2%</td>
<td>9.2%</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 9: Global Return Correlation Decomposition.

This table presents the extent to which each factor contributes to fitting the global correlations (empirical benchmarks). Row “All Shocks” shows the correlation between factor model-implied correlation and the empirical benchmarks where the factor model includes all shocks, denoted as \( \rho(Corr_{0,t}, BM_t) \); asymptotic standard errors are shown in the parentheses. Then, denote the correlation between factor model-implied correlation and the empirical benchmarks where the factor model excludes shock \( \omega_k \) as \( \rho(Corr_{k,t}, BM_t) \). In rest of the rows, \( \rho(Corr_{0,t}, BM_t) - \rho(Corr_{k,t}, BM_t) \) are reported with the corresponding shock name in the first column.

<table>
<thead>
<tr>
<th></th>
<th>Constant Beta</th>
<th>Time-Varying Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Equity</td>
<td>Bond</td>
</tr>
<tr>
<td>All Shocks</td>
<td>0.549 (0.046)</td>
<td>0.004 (0.055)</td>
</tr>
<tr>
<td>Risk Aversion</td>
<td>0.332</td>
<td>0.029</td>
</tr>
<tr>
<td>Real Upside Uncertainty</td>
<td>0.028</td>
<td>0.063</td>
</tr>
<tr>
<td>Real Downside Uncertainty</td>
<td>0.024</td>
<td>0.063</td>
</tr>
<tr>
<td>Inflation Upside Uncertainty</td>
<td>0.032</td>
<td>0.069</td>
</tr>
<tr>
<td>Inflation Downside Uncertainty</td>
<td>0.027</td>
<td>0.062</td>
</tr>
<tr>
<td>Real Short Rate Upside Uncertainty</td>
<td>0.037</td>
<td>0.062</td>
</tr>
<tr>
<td>Real Short Rate Downside Uncertainty</td>
<td>0.030</td>
<td>0.060</td>
</tr>
</tbody>
</table>
Table 10: Dynamic Factor Model Fit & Economic Significance of Other State Variables

This table evaluates the fit of the models excluding a specific factor shock and illustrate the economic significance of that shock in explaining global asset comovements. Panel A considers constant beta models, and Panel B considers time-varying beta models. Details on the model fit are described in Table 7. Bold (italics) values indicate the point estimates are within 95% (99%) confidence intervals of the corresponding data moments.

### A. Dynamic Factor Model with Constant Betas:

#### Excluding: \( \omega_u \theta_u \omega_d \theta_d \omega_{\pi_u} \omega_{\pi_d} \omega_{x_u} \omega_{x_d} \)

**Test Fact 1: Equity Correlation > Bond Correlation**
- \( \{ \text{Moment 1: Average Conditional Global Correlations} \} \)
  - Global Equity Correlation: 0.7732, 0.7733, 0.7751, 0.7737, 0.7729, 0.7747
  - Closeness to BM [p-value]: 1.8%, 1.8%, 1.7%, 1.8%, 1.8%, 1.7%
  - Fit Fact 1?: Yes, Yes, Yes, Yes, Yes, Yes

**Test Fact 2: Excessive Left-Tail Global Correlation in Equities**
- \( \{ \text{Moment 2: Global Equity Correlation - Global Bond Correlation} \} \)
  - Global Equity Correlation: 0.2852, 0.2937, 0.2912, 0.2955, 0.2935, 0.2971
  - Closeness to BM [p-value]: 1.1%, 0.8%, 0.7%, 0.9%, 0.8%, 0.7%
  - Fit Fact 2?: Yes, Yes, Yes, Yes, Yes, Yes

**Test Fact 3: Countercyclical Equity Correlation, Weakly Procyclical Bond Correlation**
- \( \{ \text{Moment 3: Sensitivity to OECD World Output Growth} \} \)
  - Global Equity Correlation: -0.2929, -0.3227, -0.2849, -0.2868, -0.2898, -0.2854
  - Closeness to BM [p-value]: 4.6%, 5.2%, 4.5%, 4.5%, 4.6%, 4.5%
  - Fit Fact 3?: Yes, Yes, Yes, Yes, Yes, Yes

### B. Dynamic Factor Model with Time-Varying Betas

#### Excluding: \( \omega_u \theta_u \omega_d \theta_d \omega_{\pi_u} \omega_{\pi_d} \omega_{x_u} \omega_{x_d} \)

**Test Fact 1: Equity Correlation > Bond Correlation**
- \( \{ \text{Moment 1: Average Conditional Global Correlations} \} \)
  - Global Equity Correlation: 0.7601, 0.7600, 0.7645, 0.7622, 0.7592, 0.7618
  - Closeness to BM [p-value]: 3.1%, 3.1%, 2.6%, 2.9%, 3.2%, 2.9%
  - Fit Fact 1?: Yes, Yes, Yes, Yes, Yes, Yes

**Test Fact 2: Excessive Left-Tail Global Correlation in Equities**
- \( \{ \text{Moment 2: Global Equity Correlation - Global Bond Correlation} \} \)
  - Global Equity Correlation: 0.5518, 0.5408, 0.5819, 0.5464, 0.5463, 0.5547
  - Closeness to BM [p-value]: 86.8%, 82.8%, 97.8%, 84.8%, 84.8%, 87.8%
  - Fit Fact 2?: Yes, Yes, Yes, Yes, Yes, Yes

**Test Fact 3: Countercyclical Equity Correlation, Weakly Procyclical Bond Correlation**
- \( \{ \text{Moment 3: Sensitivity to OECD World Output Growth} \} \)
  - Global Equity Correlation: -0.4693, -0.5162, -0.4105, -0.4639, -0.4943, -0.4624
  - Closeness to BM [p-value]: 9.3%, 11.2%, 7.4%, 9.1%, 10.3%, 9.1%
  - Fit Fact 3?: Yes, Yes, Yes, Yes, Yes, Yes
Table 11: Conditional Variance Decomposition.

### Panel A. Constant Beta

<table>
<thead>
<tr>
<th></th>
<th>$\omega_q$</th>
<th>$\omega_{\theta u}$</th>
<th>$\omega_{\theta d}$</th>
<th>$\omega_{\pi u}$</th>
<th>$\omega_{\pi d}$</th>
<th>$\omega_{x u}$</th>
<th>$\omega_{x d}$</th>
<th>Explained</th>
</tr>
</thead>
<tbody>
<tr>
<td>USA Equity</td>
<td>93.6%</td>
<td>4.0%</td>
<td>0.0%</td>
<td>1.2%</td>
<td>0.2%</td>
<td>0.3%</td>
<td>0.6%</td>
<td>56.7%</td>
</tr>
<tr>
<td>CAN Equity</td>
<td>71.3%</td>
<td>7.2%</td>
<td>0.2%</td>
<td>2.4%</td>
<td>17.1%</td>
<td>0.0%</td>
<td>1.7%</td>
<td>47.4%</td>
</tr>
<tr>
<td>DEU Equity</td>
<td>95.4%</td>
<td>0.5%</td>
<td>0.1%</td>
<td>1.6%</td>
<td>0.9%</td>
<td>1.0%</td>
<td>0.6%</td>
<td>39.6%</td>
</tr>
<tr>
<td>FRA Equity</td>
<td>88.1%</td>
<td>3.9%</td>
<td>0.2%</td>
<td>2.9%</td>
<td>0.2%</td>
<td>3.2%</td>
<td>1.5%</td>
<td>37.3%</td>
</tr>
<tr>
<td>GBR Equity</td>
<td>84.6%</td>
<td>8.8%</td>
<td>2.0%</td>
<td>1.0%</td>
<td>0.6%</td>
<td>2.6%</td>
<td>0.4%</td>
<td>33.9%</td>
</tr>
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</tr>
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</tr>
<tr>
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</tr>
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### Panel B. Time-Varying Beta

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<th>$\omega_{\theta d}$</th>
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</tr>
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<td>0.3%</td>
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<tr>
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<tr>
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<td>0.5%</td>
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<td>24.1%</td>
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<td>0.5%</td>
<td>0.4%</td>
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<td>1.7%</td>
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<td>JPN Gov-Bond</td>
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<td>1.0%</td>
<td>2.6%</td>
<td>1.0%</td>
<td>2.4%</td>
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<td>3.5%</td>
<td>5.4%</td>
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<tr>
<td>AUS Gov-Bond</td>
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<td>7.6%</td>
<td>9.8%</td>
<td>0.0%</td>
<td>4.0%</td>
<td>1.5%</td>
<td>4.8%</td>
<td>7.0%</td>
<td>5.6%</td>
</tr>
</tbody>
</table>
Figure 1: Global Dynamic Comovement Estimates.

The red line depicts the global equity correlations and the black dashed line the global bond correlations. The shaded regions are OECD world recession months from the OECD website. Model details are presented in Tables 2 and 3.
Figure 2: Global Exceedance Correlations of Asset Returns Denominated in USD.

This plot compares empirical global exceedance correlations calculated using standardized returns with model exceedance correlations calculated using simulated datasets, at full spectrum of the distribution. Table 5 provides more details on data moment and model moments (from simulated data). **Lines:** In this plot, black dashed lines and the yellow bandwidth depict the empirical global exceedance correlations and their 95% confidence intervals. Three global models from Tables 2 and 3 are considered. (1) Best models assuming multivariate t ("B (2)", Table 2; "E (4)", Table 3). (2) Models assuming multivariate t but with no asymmetry term ("B (1)", Table 2; "E (3)", Table 3). (3) Best models assuming multivariate Gaussian ("B (2)", Table 2; "E (4)", Table 3).
Figure 3: Dynamics of the Seven Economic Determinants.

The U.S. state variables used as global proxies. They are estimated at monthly frequency. For economic uncertainties, each state variable is estimated using the longest sample available: real and inflation upside and downside uncertainties, 1947/01–2016/12; real short rate, 1987/03–2015/02. The shaded regions are NBER world recession month from the NBER website.
Figure 4: Data-Implied (Empirical Benchmark) and Model-Implied (Dynamic Factor Model) Global Equity Return Comovements.

The shaded regions are OECD world recession months from the OECD website.
Figure 5: Data-Implied (Empirical Benchmark) and Model-Implied (Dynamic Factor Model) Global Bond Return Comovements.

The shaded regions are OECD world recession months from the OECD website.
Figure 6: Time Variation in Shares of Economic Determinants in Explaining the Fitted Conditional Covariance Decomposition.

The share is first calculated within each country pair (excluding self pairs), and then obtain the cross-section average across all country pairs. The shaded regions are OECD world recession months from the OECD website.
Figure 7: Fit of Equity and Bond Comovement Differences. Global Return Comovements When Omitting One Factor.

The data-implied differences are depicted in dashed blue lines, and the model-implied differences in solid black lines. The shaded regions are OECD world recession months from the OECD website.
The thick red lines in all four plots depict the empirical benchmark of global bond return correlations (from Figure [1]), In the top three plots, the rest of the lines depict the (re-estimated) time-varying global bond return correlations omitting one country at a time. The three plots correspond to the three regions in my sample. In the fourth plot, the black line shows the (re-estimated) time-varying global bond return correlations omitting both USA and JPN bonds; both are identified as safe assets in this sample. The shaded regions are OECD world recession months from the OECD website.

Figure 8: Global Dynamic Comovement Estimates, Omitting Certain Countries
Table A1: Conditional Volatility Models for Asset Returns.

This table presents best GARCH-class models and distributional assumptions for asset return conditional volatility. The four GARCH-class models are GARCH ("GARCH"), exponential GARCH ("EGARCH"), Threshold GARCH ("TARCH"), and Glosten-Jagannathan-Runkle GARCH ("GJRGARCH"). The four distributions-of-interest are Gaussian ("\(\mathcal{N}\)"), Student t ("\(t\)" characterized by a tail parameter \(\xi_1\)), GED ("GED" characterized by a tail parameter \(\xi_1\)), and Skewed t ("Skewt" characterized by a tail parameter \(\xi_1\) and an asymmetry parameter \(\xi_2\)) distributions. Suppose \(r_{t+1} = \mu + \varepsilon_{t+1}\), where \(\varepsilon_{t+1} \sim D(0, h_t)\).

(1) GARCH, Bollerslev (1986) : 
\[
h_t = \alpha_0 + \alpha_1 \varepsilon_t^2 + \alpha_2 h_{t-1}
\]

(2) EGARCH, Nelson (1991) : 
\[
\log(h_t) = \alpha_0 + \alpha_1 \frac{|\varepsilon_t|}{\sqrt{h_{t-1}}} + \alpha_2 \log(h_{t-1}) + \alpha_3 \frac{\varepsilon_t}{\sqrt{h_{t-1}}}
\]

(3) TARCH, Zakoian (1994) : 
\[
\sqrt{h_t} = \alpha_0 + \alpha_1 |\varepsilon_t| + \alpha_2 \sqrt{h_{t-1}} + \alpha_3 I_{\varepsilon_t<0}|\varepsilon_t|
\]

(4) GJRGARCH, Glosten, Jagannathan, and Runkle (1993) : 
\[
h_t = \alpha_0 + \alpha_1 \varepsilon_t^2 + \alpha_2 h_{t-1} + \alpha_3 I_{\varepsilon_t<0}\varepsilon_t^2.
\]

Model estimation uses MLE at monthly frequency covering period from March 1987 to December 2016 (T=358), and model selection follows BIC. Bold values indicate <5% significance level.

<table>
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<th>Asset</th>
<th>Best Model</th>
<th>Variance Equation Parameters</th>
<th>Distribution Parameters</th>
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<td>(\alpha_1)</td>
<td>(\alpha_2)</td>
<td>(\alpha_3)</td>
</tr>
<tr>
<td>USA Equity</td>
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<td>0.2652</td>
<td>0.8694</td>
</tr>
<tr>
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<td>GARCH-Skewt</td>
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<td>0.8079</td>
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<td>EGARCH-Skewt</td>
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<td>0.8325</td>
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<td>CHE Equity</td>
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</table>
Table Aii: Model Fit: Flight-to-Safety Channel, Given Best Model in Table 3

The model implicitly include a FTS channel. To provide the right empirical moments to be compared with, “Empirical” reports the average of time-varying correlation (estimated using a parsimonious dynamic conditional correlation model as in Engle, 2002) between standardized monthly equity returns and bond returns—both denominated in USD as consistently used in this paper. Three model moments are reported. (1) “Conditional Model” reports the time-series averages of the model-implied equity beta Table 3. (2 and 3) Because simulations do not effect the beta realizations, “Simulated Model (t)” and “Simulated Model (n)” report averages of model-implied beta averages given parameter estimates in Table 3. Because best conditional model is also the best model assuming multivariate t distribution, (1) and (2) report the same numbers.

**States:** Good (Bad) states, when country recession indicator = 0 (1). Bold (italics) values indicate the model point estimates are within 95% (99%) confidence intervals of the corresponding data moments.

<table>
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<th>Bad</th>
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<td><strong>0.3051</strong></td>
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<td>0.2299</td>
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Table Aiii: Estimation Results of Global Equity Comovement: \( x_{i,t} \) = Standardized Country Output Growth.

This table provides one of the robustness checks of the global equity correlation estimates involving the FTS channel (as in Table 3). Here, I use standardized country output growth (industrial production growth) as \( x_{i,t} \) in the FTS process. Model estimation uses MLE at monthly frequency covering period from March 1987 to December 2016 (T=358), and model selection follows BIC. Bold (italics) values indicate <5% (10%) significance level.

<table>
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<th>( \text{Multivariate t} )</th>
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<td>( \gamma )</td>
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<td>(0.0183)</td>
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<td>-0.0364</td>
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<tr>
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<td>(0.0190)</td>
<td>(0.0195)</td>
</tr>
<tr>
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A.ii
Table Aiv: Estimation Results of Global Equity Comovement: DECO Estimates, No Domestic Comovement Part.

This table provides one of the robustness checks of the global equity correlation estimates involving the FTS channel (as in Table 3). Here, I directly estimate the DECO model with tests. Model estimation uses MLE at monthly frequency covering period from March 1987 to December 2016 (T=358), and model selection follows BIC. Bold (italics) values indicate <5% (10%) significance level.

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A.iii
Table Av: Factor Exposures of Global Asset Returns in a Seemingly Unrelated Regression (SUR) Framework; Constant Beta.

In this table, I jointly estimate the constant exposures of global equity and bond returns to global factor shocks in a SUR framework. The error terms may have cross-equation contemporaneous correlations. SUR models are estimated with MLE. The sample period covers from March 1987 to February 2015; February 2015 is the last month given the availability of the risk aversion estimate from Bekaert, Engstrom, and Xu, 2017. Standard errors are shown in the parentheses. Bold (italics) values indicate <5% (10%) significance level.

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<th>FRA Equity</th>
<th>GBR Equity</th>
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<th>JPN Equity</th>
<th>AUS Equity</th>
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Table Avi: Factor Exposures of Global Asset Returns in Seemingly Unrelated Regression (SUR) Framework; USD; Time-Varying Beta.

In this table, I jointly estimate the time-varying exposures of global equity and bond returns (in USD) to global factor shocks in a SUR framework. The error terms may have cross-equation contemporaneous correlations. SUR models are estimated with MLE. The sample period covers from March 1987 to February 2015. Standard errors are shown in the parentheses. Bold (italics) values indicate <5% (<10%) significance level.

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