# A Theory of Collateral for the Lender of Last Resort

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December 14, 2017

*The views stated herein are those of the authors and are not necessarily the views of the Federal Reserve Bank of New York or the Federal Reserve System. We would like to thank Patrick Bolton, Charles Calomiris, Elena Carletti, Jean-Edouard Colliard, Douglas Gale (discussant), Itay Goldstein, Denis Gromb, Barney Hartman-Glaser (discussant), Charles Kahn, Francois Koulischer, David Martinez Miera, Emanuel Moench, Martin Oehmke, Rafael Repullo, Javier Suarez, Sergio Vicente (discussant), and Ansgar Walther, as well as audiences at Warwick, Bank of Spain, CEMFI, Deutsche Bundesbank, ECB, University of Amsterdam, New York Fed/NYU Stern Financial Intermediation Conference, 2017 FIRS, and 2017 EFA for useful comments.

# A Theory of Collateral for the Lender of Last Resort

We take a macroprudential approach to analyze optimal lending policies for the central bank, focusing on externalities such policies impose on private markets. Lending against high-quality collateral protects central banks against losses but could adversely affect the collateral pool in markets, impairing their efficient functioning. Lending against low-quality collateral creates counterparty risk but could improve the collateral pool and the functioning of markets. We characterize optimal central bank policy incorporating these trade-offs. We show that, contrary to what is generally accepted, lending against high-quality collateral can have a negative effect, whereas it may be optimal to lend against low-quality collateral and restore markets.

**Keywords:** Central bank, liquidity, macroprudential policy, externality, interbank market, lending facilities

# 1 Introduction

In his famous 1873 book *Lombard Street*, Walter Bagehot advocates four principles for central banks when they act as the lender of last resort (LoLR): lend only to illiquid but solvent banks, at a penalty rate, and against good collateral valued at pre-panic prices, and make clear in advance the readiness to lend any amount to any institution that meets the conditions for solvency and collateral. Despite having shaped central bank policies around the world for more than a hundred years, these principles continue to be the subject of intense debate, in part because they still lack a rigorous theoretical foundation. Moreover, the institutional environment has significantly changed since Bagehot's time. For one thing, unlike today's central banks, which are public institutions, the Bank of England was then privately held (with some privileges from the government). Also, the financial system today is much more connected and complex than ever before, making financial stability a bigger concern for authorities and, in some countries, an explicit goal of central banks.

In this paper, we investigate the optimal lending policy for a central bank. In contrast to the existing literature on the LoLR, which typically takes a microprudential approach focusing individual institutions, we take a macroprudential approach focusing on externalities central bank policies can impose on private markets by affecting the pool of collateral and liquidity creation in such markets. Charles Calomiris captures this idea in an article featured in Forbes: "[...] the Fed's purchases of treasury bonds and AAA MBS has withdrawn collateral from the market that otherwise would support repo contracting. Repos fund lending and other financial transactions, both within and outside the regulated banking system, which promote economic activity. According to that point of view, therefore, Fed QE policies may have been contractionary."[1]

Lending against high-quality collateral protects the central bank from potential losses, but it can adversely affect the pool of collateral in private markets and impair their efficient functioning because high-quality collateral gets tied up with the central bank rather than

[1] Available at https://www.forbes.com/sites/charlescalomiris/2016/01/05/federalreserveqelessons/#4e90a0c274e2.

circulating in private markets and facilitating liquidity creation. In contrast, lending against low-quality collateral exposes the central bank to counterparty risk, but it improves the pool of collateral in money markets, which, in turn, promotes the functioning of markets and can even unlock frozen markets. We characterize the optimal policy for the central bank by taking into account these trade-offs. We show that, contrary to what is generally accepted, lending against high-quality collateral can have a negative effect on markets; whereas it can be optimal for the central bank to lend against low-quality collateral since it improves the functioning of private markets.

We build a model that involves maturity transformation and collateral circulation. The model has three dates, two different groups of banks, outsiders, and a central bank. The first group of banks, denoted as "borrowers," have access to long-term projects at the initial date. These projects are risky but they have a positive net present value (NPV). However, projects are illiquid; that is, they suffer a loss when liquidated at the interim date. Borrowers do not have funds at the initial date, but they have collateral, which can be of high or low quality, and can use their collateral to raise funding from our second set of banks, denoted as "lenders." Lenders have assets in place but do not have access to new projects. They also have funds they can lend against borrowers' collateral at the initial date. Lending and borrowing are short term, creating a maturity mismatch.

At the interim date, lenders experience a liquidity shock and their existing assets generate long-term returns at the final date only if they meet these liquidity shocks. After observing their liquidity shock, lenders search for other banks that can provide them liquidity. If they find another bank, they can borrow from that bank and their long-term returns from existing assets are not disrupted. However, with some probability, the search fails and they need to borrow from outsiders, pledging the collateral they received from the borrowers at the initial date. Outsiders are unable to identify the quality of individual collateral; they only know the composition of high- and low-quality collateral in the market. Hence, they lend only up to the average value of collateral in the market.

If the average quality of collateral is high enough compared with their liquidity needs, lenders can raise enough funding from outsiders. However, when the overall quality of collateral is low, it becomes more likely that the lenders will not be able to borrow enough from outsiders. In that case, lenders call back the loans they provided to borrowers at the initial date, which leads to a costly liquidation of the borrowers' projects. In other words, lenders' demand for liquidity at the interim date creates the risk of early liquidation for borrowers. Borrowers will invest at the initial date only when the expected return from investing, net of any liquidation cost, is positive, that is, borrowers forgo valuable investment when the expected cost of liquidation outweighs the return from the project.

Note that it is socially optimal for banks to invest at the initial date, since all projects have a positive NPV. By imposing losses on banks, which could be mere private losses, early liquidations can lead banks to bypass valuable projects, resulting in real effects. This creates a role for the LoLR to insure banks against liquidity risk, which facilitates valuable investment ex ante. However, because the central bank lends against collateral, it affects the pool of collateral in private markets. This, in turn, imposes externalities on markets, which can strengthen or offset the direct effects associated with central bank lending. A key innovation of our paper is its macroprudential approach, which takes into account the externalities induced by the central bank's lending policy. This allows us to derive results that would have been overlooked had we relied on a microprudential approach.

Next, we discuss our main results. Consider a situation where private markets are functioning so that banks finance themselves and take their valuable investments ex ante. However, banks with high-quality collateral may find that liquidity risk is too high since they are pooled with banks with low-quality collateral, and prefer to borrow from the central bank instead, pledging their collateral. This would have two effects that go in opposite directions. First, it would have the direct effect of insuring banks that borrow from the central bank against liquidity risk, which would spur investment and output without imposing any losses on the central bank. However, it would decrease the quality of collateral in private markets, thereby imposing a negative externality on the banks that are left in the market by increasing their liquidity risk. We show that the negative indirect effect on private markets can more than offset the positive direct effects on banks borrowing from the central bank. In addition, if central banks lend in large quantities against high-quality collateral, this could lead to a deterioration in the quality of collateral in private markets to the point where these markets stop functioning. In all of these cases, welfare would be enhanced if high-quality collateral remained in the private market, where it could circulate and generate liquidity, rather than being locked up with the central bank.

However, this does not imply that it's never optimal for the central bank to lend against high-quality collateral. Consider the case where the liquidity risk is so high that all banks refrain from investing in the first place; that is, private markets are frozen. In this case, when the central bank lends against high-quality collateral, it can facilitate the investment of banks borrowing from the central bank and protect itself from potential losses. This would be optimal when the central bank is highly concerned about its losses. The optimal policy in that case is to lend *freely* against high-quality collateral, similar to what Bagehot proposed.

Alternatively, the central bank can lend against low-quality collateral. Although this would expose the central bank to potential losses, it would improve the quality of collateral in private markets, where this positive effect can outweigh such losses. An interesting case arises when the central bank faces a frozen market. By lending against low-quality collateral at a large enough scale, the central bank can improve the quality of the remaining collateral in the market sufficiently so that it can revive a frozen market resulting in a discontinuous effect on output. Furthermore, it would be optimal for the central bank to continue lending even when the marginal cost from additional lending is greater than the marginal benefit, at which point a central bank that takes a microprudential approach would stop lending, if such an action eventually jump starts a frozen market and the resulting discrete increase in output more than offsets aggregate losses. When the costs from potential losses are not too taxing, this, in turn, can improve welfare more than the policy of lending against high-quality collateral, in which case only the banks that borrow from the central bank invest.

In sum, the optimal policy can be summarized as follows. As long as private markets are functioning, it is never optimal to lend against high-quality collateral, and lending against low-quality collateral can be optimal provided that the potential losses from counterparty risk are not too costly. On the other hand, when the markets are frozen during a deep crisis, the optimal policy could be to lend *freely* against high-quality collateral if the central bank is highly concerned about potential losses from counterparty risk. However, when potential losses from counterparty risk are not too costly, it is optimal to lend against low-quality collateral and jump start the frozen markets. This, in turn, may require the central bank to lend beyond the point where the marginal cost of lending exceeds the marginal benefit. All of these effects would be overlooked if one used a microprudential approach.

We present several extensions of our baseline setup. We first extend the intermediation chain, so that collateral can be circulated multiple times. We show that the effects from the baseline model become amplified as the intermediation chain gets longer; that is, a small intervention by the central bank can have a significant effect on markets when the chain is longer. In another extension, we introduce cash-in-the-market pricing so that the liquidation value changes with the scale of liquidations. We show that the central bank lending, by decreasing the scale of liquidations, can have an extra kick arising from improved liquidation values in private markets as in Allen and Gale (1998) and Choi, Eisenbach, and Yorulmazer (2016). In a third extension, we allow for the possibility of costly information generation, as in Gorton and Ordoñez (2014, 2017), where the owners of the collateral can credibly convey information about the quality of their collateral to outsiders.

Our paper is related to the vast literature on LoLR, which dates back to Thornton (1802) and Bagehot (1873).[2] Goodfriend and King (1988) argue that with efficient interbank markets, central banks should not lend to individual banks but instead provide sufficient liquidity via open market operations, which the interbank market would then allocate efficiently

[2]For surveys, see Bordo (1990), Santos (2006), and Freixas et al. (1999). For some recent empirical studies, see Drechsler et al. (2016), Acharya et al. (2017), and Garcia-de-Andoain et al. (2016).

among banks. Others, however, argue that interbank markets may fail to allocate liquidity efficiently owing to frictions such as asymmetric information about banks' assets (Flannery (1996), Freixas and Jorge (2007)); the market power of banks in the interbank market (Acharya, Gromb, and Yorulmazer (2012)); banks' free-riding on each other's liquidity (Bhattacharya and Gale (1987)); banks' free-riding on the central bank's liquidity (Repullo (2005)); or banks' incentives to hoard liquidity for strategic and precautionary motives (Diamond and Rajan (2011); Gale and Yorulmazer (2013)).

While there is a large literature in corporate finance and banking on the importance of collateral, this issue has not been fully explored in the context of central bank lending with the exception of some recent papers such as Koulischer and Struyven (2014) and Bindseil (2013), who, like us, examine the role of central bank collateral policies. Koulischer and Struyven (2014) show that, during systemic crises, central banks can improve welfare by also lending against low-quality collateral when all high-quality collateral has been exhausted. Bindseil (2013) suggests widening the set of collateral that the central bank would accept during a crisis. However, we show that lending against low-quality collateral even when high-quality collateral is still available can improve welfare, a result similar to the effect of purchasing certain types of assets as in Tirole (2012) to jump start markets. Finally, Gorton and Ordoñez (2017) analyze the role of an information externality that an LoLR generates, as we do in this paper. They show that "opaque" lending facilities that do not reveal the identities of borrowing banks can improve the average quality of assets in the banking sector and mitigate the risk of runs by depositors. They examine how externalities differ depending on whether the LoLR maintains "secrecy", while we focus on externalities by different lending policies when the central bank transparently communicates their rules.

Different from most of the literature, the main feature of our paper is its macroprudential approach, whereby we characterize the optimal central bank policy taking into account externalities these policies can generate on private markets.

The rest of the paper is organized as follows. Section 2 introduces the model. Section

3 solves for bank behavior, both in the absence and in the presence of an LoLR. Section 4 investigates the effect of central bank lending on output and liquidity in markets. Section 5 discusses the optimal LoLR policy taking into account potential losses. Section 6 presents extensions of our model, and Section 7 provides a discussion of various important issues. Section 8 concludes.

# 2 Model setup

In this section, we present our model setup. We introduce the agents, illustrate the trading of liquidity in markets and lending by the central bank.

## 2.1 Agents and liquidity shocks

The model has three dates: $t = 0, 1, 2$. There are three groups of banks—each with measure 1, denoted by $i = A, B_0, B_1$—plus outsiders, and a central bank that can act as the lender of last resort. Note that $B$ banks are divided into two subgroups, $B_0$ and $B_1$, as we explain below. We assume that all agents have access to a storage technology, with a rate of return equal to 1, and consume only at $t = 2$. All agents are risk neutral and have a discount factor of 1.

Banks in group $A$ ("borrowers") are endowed with an investment opportunity that requires one unit of cash at $t = 0$. The investment generates a random return at $t = 2$: with probability $p$ it generates $\bar{R} > 1$, and with probability $(1 - p)$ it generates $\underline{R}$, where we assume $\underline{R} = 0$ for simplicity. We denote the expected return from the project by $R = p\bar{R}$ and assume that the investment has a positive NPV; that is, $R > 1$. The project is illiquid and can be liquidated at $t = 1$ in a lump-sum fashion, in which case it only generates $r < 1$.[3] $A$ banks start out with no cash but are endowed with one unit of collateral that we explain in detail below.

[3]In the benchmark model, we assume that the liquidation value $r$ is fixed. However, the liquidation value may depend on the number of projects being liquidated and the liquidity available within the buyers. We analyze this case in Section 6.2, where we introduce cash-in-the-market pricing.

Since $A$ banks do not have any cash at the outset, they need to borrow (from $B_0$ banks) to finance the investment at $t = 0$. We assume that lending is short term and needs to be rolled over at $t = 1$, which is typical of models of financial intermediation that feature maturity transformation.[4]

All $B$ banks ($B_0$ and $B_1$) have assets in place that produce an output worth $R^B > 1$ at $t = 2$, but do not have any new investment opportunity at $t = 0$ or $t = 1$. $B_0$ banks ("lenders") are endowed with one unit of cash at $t = 0$, which they can lend to $A$ banks. At $t = 1$, $B_0$ banks receive a liquidity shock, which requires a cash injection $\ell$, with $\ell \sim U[0, 1]$.[5] $B_0$ banks' long-term output $R^B$ is realized only if they are able to fully satisfy their liquidity need $\ell$ at $t = 1$; otherwise long-term output is zero. The $B_0$ bank that needs funding can borrow from a $B_1$ bank or from outsiders, pledging the collateral it received from $A$ at $t = 0$ when necessary. We assume that $B_1$ banks can verify the return of a $B_0$ bank's project at $t = 2$ so that collateral is not needed when $B_0$ banks borrow from $B_1$ banks.[6] However, outsiders cannot verify the return of a $B_0$ bank's project so that a $B_0$ bank can borrow only up to the value of its collateral from outsiders (e.g., Hart and Moore 1994, 1998).

$B_1$ banks have no cash at $t = 0$ but can receive a cash endowment of one unit at $t = 1$. Finally, outsiders have no investment opportunity but they do receive one unit of cash at $t = 1$, which they can lend in the money market.

As we noted above, $A$ banks have one unit of collateral at $t = 0$, which can be of two types. A fraction $\alpha$ of the $A$ banks have "high-quality" collateral, denoted by $j = H$, which

[4]As we show below, by lending short term $B_0$ banks can completely eliminate any liquidity or credit risk, which could be the rationale for lending short-term. The literature features models that show the disciplining effect of short term debt on banks, see Calomiris and Kahn (1991) and Diamond and Rajan (2001). For papers that analyze the maturity structure, see, for instance, Flannery (1986, 1994), Diamond (1991), and Brunnermeier and Oehmke (2013).

[5]For simplicity, in the benchmark model, we use a uniform distribution for liquidity shocks. Our results go through with general p.d.f.s (see Section 6.4). We further assume that all $B_0$ banks experience the same liquidity shock $\ell$, which simplifies the analysis and allows us to rule out interbank lending among $B_0$ banks, without affecting our main results. See the Appendix for the case with idiosyncratic liquidity shocks for $B_0$ banks.

[6]This could be due to, for instance, the relationships among banks. When maintaining the established relationships is valuable, banks would have incentives to repay the loans whenever possible. See Section 7.3 for a discussion of relationships in the interbank market.

has the high value $c_H \geq 1$. The remaining $A$ banks have "low-quality" collateral, denoted by $j = L$, which has the low value $c_L < 1$.

We assume that banks $A$ and $B$ as well as the central bank are "insiders" and can identify the type of collateral, whereas outsiders cannot do so.[7] Note that we assume that the central bank has an informational advantage over outsiders, which can arise through its supervisory and regulatory duties, but not over other banks.[8] The only information outsiders have is the fractions $\alpha$ of high- and $(1-\alpha)$ of low-quality collateral at $t = 0$. Hence, from the outsider's point of view, the expected value of the collateral is

$$c_1 = \alpha c_H + (1-\alpha)c_L,$$

which is the maximum amount $B_0$ banks can borrow from outsiders at $t = 1$, as outsiders are unable to distinguish the type of collateral. We assume that $c_1 < 1$ as otherwise the model is trivial, since $B_0$ banks can always borrow the cash needed to satisfy their liquidity need $\ell$ at $t = 1$.

For simplicity, we make a set of assumptions about the parameters of our model. To start with, we assume that $c_H$ is high enough ($c_H \geq 1$), while $c_L$ is low enough ($c_L < 1$). Since $c_H \geq 1$, lending against high-quality collateral does not expose the lender to credit losses when the investment matures at $t = 2$, while lending against low-quality collateral exposes the lender to credit losses at $t = 2$, since $c_L < 1$ and with probability $1-p$ the bank's project has the low return 0. We also assume $c_L + r > 1$, which implies that a lender who grants a short-term loan to $A$ banks at time $t = 0$ is not exposed to any credit or liquidity risk, since it can always recover one unit by calling back the loan at $t = 1$. We summarize these conditions in the following assumption:

[7] For models of endogenous choice of becoming an "insider" versus an "outsider," see, for instance, Allen and Gale (1994) and Acharya, Shin and Yorulmazer (2013).

[8] The central bank can examine the collateral ex post and impose a penalty on banks that do not report truthfully, which would induce banks to report the true quality ex ante. In addition, central banks can have a longer horizon than private agents so that it can lend longer term, and can have a larger financial capacity so that it can lend and acquire assets at larger scales. For a discussion of the features that put central banks in a special position to act as the lender of last resort, see Section 7.4.

**Assumption:** $1 - r < c_L < c_1 < 1 \leqslant c_H$.

## 2.2 Market for liquidity

Money markets are competitive such that lenders anticipate to receive an expected rate of return equal to 1. At $t = 0$, an $A$ bank can take out a short-term loan of one unit from a $B_0$ bank, pledging its collateral. Recall that since $c_L + r > 1$, granting a short-term loan to an $A$ bank does not expose the lender to any credit or liquidity risk.

At $t = 1$, the $B_0$ bank can either roll over the loan or refuse to renew it. If the $B_0$ bank refuses to renew the loan, $A$ bank needs to liquidate its project early. However, if the $B_0$ bank agrees to roll over the loan, it needs to borrow at $t = 1$ since it will need to meet its own liquidity need $\ell \sim U[0, 1]$, either from $B_1$ banks or outsiders. Recall that collateral is not needed when $B_0$ borrows from $B_1$ but $B_0$ can borrow only up to the average value of its collateral from outsiders. Furthermore, there is a search friction such that with probability $\rho$, $B_0$ banks cannot be matched with $B_1$ banks in the interbank market so that these $B_0$ banks have to go to outsiders for funding. One possible interpretation is that $B_1$ banks' cash flows at $t = 1$ get delayed with probability $\rho$ so that $B_1$ banks do not have any cash they can lend to $B_0$ banks at $t = 1$, which can be interpreted as a state of "systemic illiquidity" in the interbank market.

For simplicity, we assume that $A$ banks cannot directly borrow from outsiders at $t = 1$, when their collateral is already pledged with $B_0$ banks. Also, note that outsiders do not have any cash at $t = 0$, since they receive their one unit of cash at $t = 1$. As a result, $A$ banks cannot borrow from outsiders at $t = 0$, either.

## 2.3 Lender of last resort

The central bank can lend to $A$ banks at $t = 0$. We focus on central bank lending at $t = 0$ because we want to understand how the central bank's collateral policy can affect liquidity in the money market by changing the pool of collateral available in the market at $t = 1$.[9] Note that the central bank, just like $A$ and $B$ banks, can verify the quality of collateral that banks may post for loans from the central bank. Hence, the central bank does not have an informational advantage over $A$ and $B$ banks, but only over outsiders. This advantage could come from the central bank's supervisory activity—through supervision the central bank can acquire information on banks' assets that is not publicly available to all market participants.

At $t = 0$, the central bank announces its policy, which may include a requirement—$j \in \{H, L\}$ (i.e., high-quality, low-quality, or both)—about the collateral it will accept against the loan, the penalty rate $\gamma_j$ ($\geq 0$) it demands on its loans, and finally the maximum amount $x_j$ it is willing to lend. Note that the banks that borrow from the central bank would pay at least the "fair" rate; that is, they pay at least one unit in expectation, which allows the central bank to recover the funds it lends in expectation. We call $\gamma_j$ the penalty rate, since this is what banks pay over and above the fair rate. We assume that $\gamma_j$ is a mere transfer.

Note that both $x_j$ and $\gamma_j$ can depend on the borrower's collateral.[10] When the central bank pre-specifies the amount $x_j$ it is willing to lend, and there is excess demand, we assume that it allocates $x_j$ randomly among banks seeking loans.[11]

[9] We discuss the effect of ex-post lending at $t = 1$ by the central bank in the Appendix and show that our main results go through. Furthermore, in a dynamic setting, the distinction between ex-ante and ex-post lending becomes blurred, since any action by the central bank will have an impact in future periods. See our multi-period extension in Section 6.1.

[10] Note that banks in our model are identical in terms of their solvency, and the only difference is the type of collateral they have. See Choi (2014) for the macroprudential effect of policy interventions across banks with heterogeneous solvency.

[11] We discuss in Section 7.2 alternative arrangements, including auctions, that the central bank can use to address banks' excess demand for liquidity.

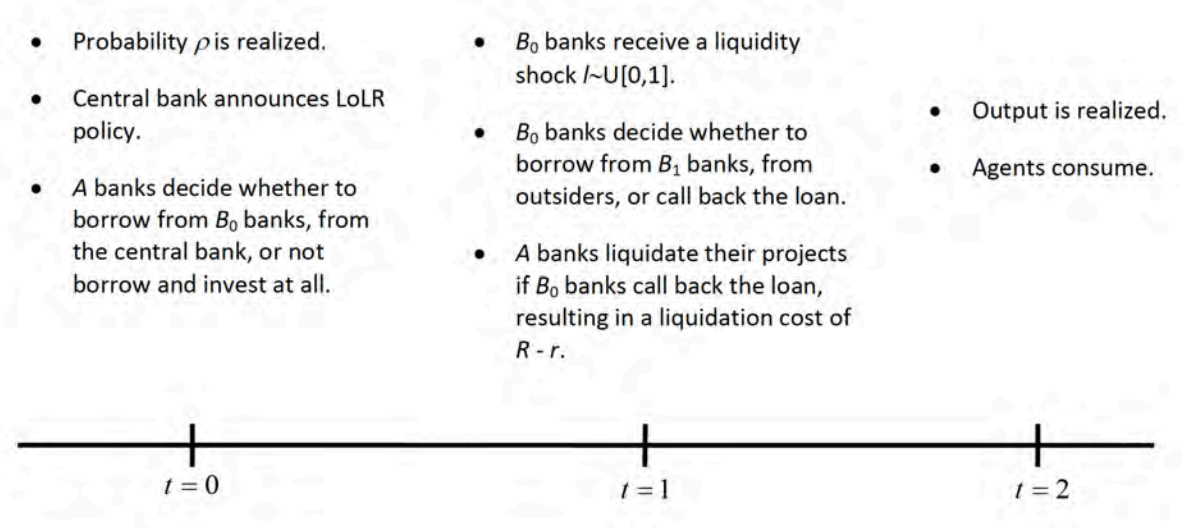


Figure 1: Timeline

## 2.4 Timeline

Figure 1 summarizes the timeline of our model. At $t = 0$, the probability $\rho$ is realized and the central bank announces its lending policy. $A$ banks choose to (i) borrow from the central bank; (ii) borrow from $B_0$ banks in the private market; or (iii) not borrow (and thus not invest). At $t = 1$, the liquidity shock $\ell$ is realized. After they learn about the liquidity shock $\ell$, $B_0$ banks borrow from $B_1$ banks if they can; otherwise, they borrow from outsiders or call back their loan to $A$ banks.[12] Output is realized at $t = 2$, and agents consume. Figure 2 summarizes the flows between agents at $t = 0$ and $t = 1$.

[12]We assume that, when indifferent, $B_0$ banks borrow from outsiders instead of calling back the loan.

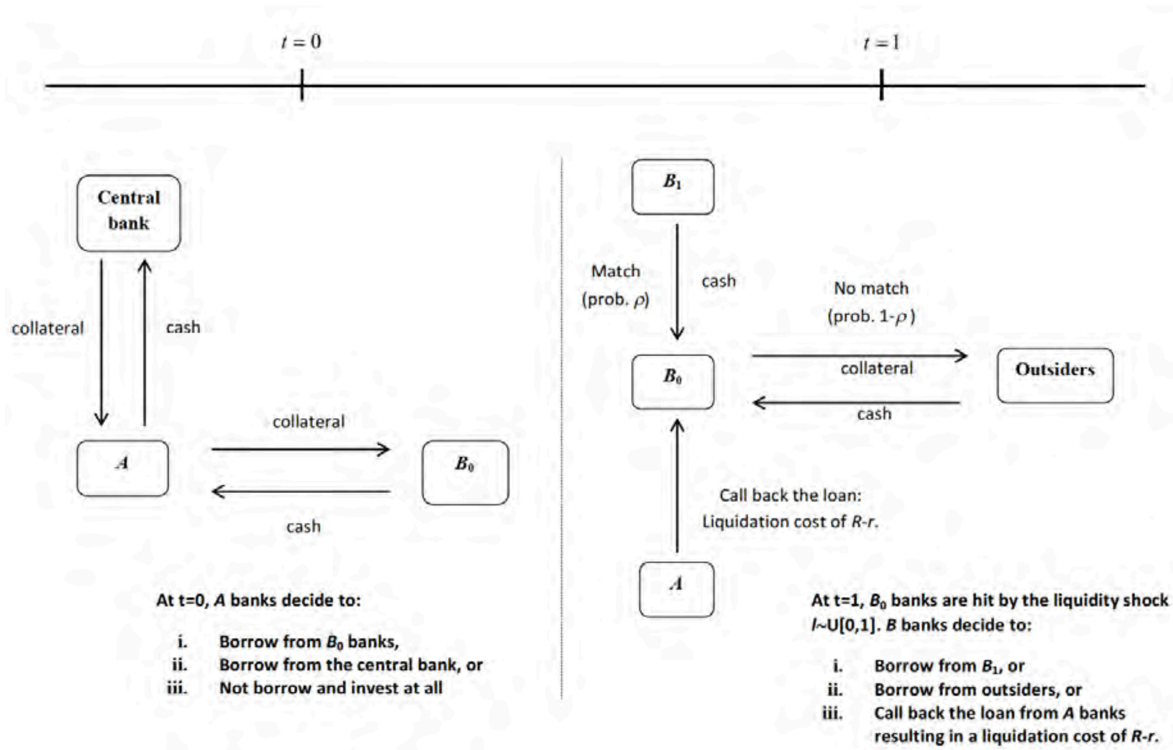


Figure 2: Flows between agents at $t = 0$ and $t = 1$

# 3 Individual bank behavior

In this section, we analyze optimal bank decisions in the absence, and then in the presence, of the LoLR. On the basis of these results, we will compare the implications of different LoLR policies for output generated by banks in Section 4, and discuss optimal policies, taking into account the potential losses from the central bank's LoLR activities, in Section 5.

We start by investigating banks' decisions when there is no LoLR. We focus on $A$ banks' decisions, since $B_0$ banks' problem is trivial as they can always avoid any credit or liquidity risk by calling back the loan at $t = 1$. In the absence of an LoLR, at $t = 0$ after $\rho$ is realized, $A$ banks decide whether to borrow from $B_0$ banks and invest, or to forgo their investment opportunity.

As discussed, $B_0$ banks are willing to lend to $A$ banks at $t = 0$, since $c_L + r > 1$ and they can always recover their loan by refusing to renew it at $t = 1$. However, when a $B_0$ bank does not renew its loan, this will lead to the early liquidation of $A$ banks' investment, resulting in a loss of $R - r$ to $A$ banks.

Note that with probability $1 - \rho$, $B_0$ banks are matched with $B_1$ banks and can always meet their liquidity need at $t = 1$. However, with probability $\rho$, they are not matched with $B_1$ banks and need to borrow from outsiders. Recall that outsiders cannot distinguish the quality of the collateral so that they have the average valuation of $c_1$ for all the collateral. Since $B_0$ banks can only borrow up to the average collateral value $c_1$ from outsiders, they can meet their liquidity needs only if $c_1 \geq \ell$. When the average quality of collateral $c_1$ is low, and/or when the liquidity shock $\ell$ is high, $B_0$ banks will not be able to meet their liquidity needs in the market and, in turn, will call back their loans. This occurs with probability $1 - c_1$ since $\ell \sim U[0, 1]$.

Hence, the net expected payoff for an $A$ bank from investing can be written as:

$$\begin{aligned} \Pi &= (1 - \rho)R + \rho\left[c_1 R + (1 - c_1)\, r\right] - 1 \\ &= R - 1 - \underbrace{\rho\,(1 - c_1)\,(R - r)}_{\text{liquidity risk}}, \end{aligned}$$

since with probability $1 - \rho$ lenders are matched with a $B_1$ bank and there is no liquidity risk; with probability $\rho(1 - c_1)$, lenders cannot meet their liquidity needs and call back the loan, forcing $A$ banks to liquidate their projects, resulting in a loss of $(R - r)$ at $t = 1$. Since lending is competitive with the risk-free rate equal to 1, $A$ banks will pay the expected funding cost equal to 1.

$A$ banks will not invest if their net expected payoff is negative, that is, when $R - 1 < \rho\,(1 - c_1)\,(R - r)$. Hence, $A$ banks will not invest when

$$\rho > \rho^* = \frac{R - 1}{(1 - c_1)(R - r)}, \tag{1}$$

which gives the following proposition.

**Proposition 1 (No investment):** *A banks invest if and only if* $\rho \leqslant \rho^{*}$.

Note that $\rho^{*}$ is increasing in $c_1$ so that loans are rolled over more often for higher average values of the collateral, which in turn induces more investment ex ante. Hence, higher quality collateral reduces liquidity risk for both lenders and borrowers and facilitates investment by generating more liquidity.

We now expand our analysis to consider the implications of the LoLR.[13] As we will see, banks' incentives to access the LoLR will depend on the central bank's lending policy, including its collateral, interest rate, and penalty rate policies, and on banks' benefits from access to the LoLR, that is, the resulting reduction in their liquidity risk exposures.

In the presence of an LoLR, banks could, in principle, eliminate liquidity risk entirely by borrowing from the central bank since the central bank can afford to lend long term because it would have a longer horizon than the market participants that may have their own liquidity needs in the short run. Banks will do so, however, only if the liquidity risk is high enough to offset the cost of borrowing from the central bank.

Note that for $R-1 < \gamma_j$, $A$ banks never borrow from the central bank since the penalty rate $\gamma_j$ is higher than the highest net expected return from the project. However, for $R-1 \geqslant \gamma_j$, $A$ banks with collateral type $j \in \{H, L\}$ choose to borrow from the central bank if the benefit from lowering liquidity risk is greater than the penalty rate $\gamma_j$. We distinguish two cases. For $\rho \leqslant \rho^{*}$, where $A$ banks would borrow in the private market and invest, $A$ banks with collateral type $j$ will borrow from the central bank only when $\rho\left(1-c_1\right)\left(R-r\right) \geq \gamma_j$, which becomes

$$\rho \geqslant \rho_j^{CB} = \frac{\gamma_j}{\left(1-c_1\right)\left(R-r\right)}. \tag{2}$$

For $\rho > \rho^{*}$, where the liquidity risk is sufficiently high and $A$ banks do not borrow in the market, they would borrow from the central bank when $R-1 \geq \gamma_j$. We summarize these results in the following proposition.

[13] Another alternative for banks with high-quality collateral would be to invest in generating and credibly conveying information about the quality of their collateral. We analyze this case in Section 6.3.

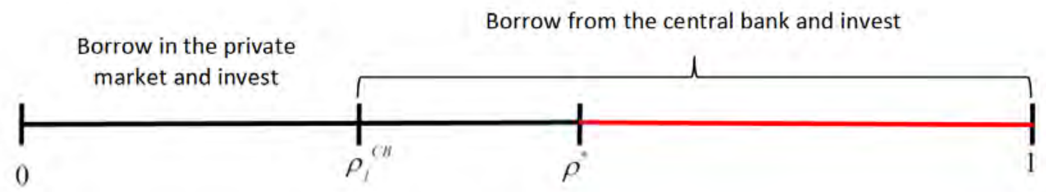


Figure 3: $A$ banks borrow from the central bank and invest when $\rho \geq \rho_j^{CB}$. In the red region ($\rho \geq \rho^*$), $A$ banks would have bypassed investment if they could not borrow from the central bank.

**Proposition 2 (LoLR):** *For $R-1 < \gamma_j$, A banks do not borrow from the central bank. For $R-1 \geqslant \gamma_j$, we have two cases: (i) For $\rho > \rho^*$, an A bank with collateral type $j \in \{H, L\}$ borrows from the LoLR only if $\gamma_j \leq R-1$; (ii) for $\rho \leqslant \rho^*$, it borrows from the central bank only if $\rho \geqslant \rho_j^{CB}$, where, $\rho_j^{CB}$, given in (2), is increasing in $\gamma_j$.*

Figure 3 illustrates the result. Note that in the red region, where $\rho_j^{CB} \leqslant \rho^* < \rho$, banks would borrow from the central bank and invest, whereas they would have bypassed investment in the absence of LoLR funding. In sum, given that access to the central bank insures banks against liquidity risk, the maximum rate they are willing to pay for a loan from the central bank will depend on the expected loss they incur due to liquidity risk. For example, when they can borrow in private markets, that is, for $\rho \leqslant \rho^*$, the expected loss from early liquidation is increasing in the loss induced by early liquidation, $R - r$, and the likelihood $\rho(1 - c_1)$ that $B_0$ banks call back the loan. This probability, in turn, is higher when the banking sector is adversely affected by a negative shock, represented by high $\rho$ or high $\ell$, or a low average collateral quality $c_1$. While the liquidity shock $\ell$ is exogenous, the average quality of collateral in private markets will be affected by the central bank's lending and collateral policy. In the next section, we investigate the effect of LoLR policies on liquidity creation and output in this economy.

# 4 LoLR and impact on output

In this section, we investigate how central bank policies affect aggregate liquidity and thereby output in the economy. While lending to banks can expose the central bank to credit risk and potential losses, to clearly show the effect of central bank policies on the functioning of private markets and output generated by banks, we postpone the analysis of such losses to Section 5.

As described in Section 2, the central bank lends to eligible $A$ banks at $t = 0$ after the probability $\rho$ of not finding a match is realized. Also, recall that $A$ banks will lose $R - r$ on their investment if it is liquidated early at $t = 1$. We assume that out of the loss $(R - r)$, a fraction $\Delta \in [0, 1]$ is attributed to an actual *real output loss*; that is, $\Delta(R - r)$ represents output losses from liquidations, and the rest $(1 - \Delta)(R - r)$ is a mere transfer within the economy.[14]

Note that when a lender is matched with a $B_1$ bank, there is no liquidation. However, liquidations can occur when lenders need to borrow from outsiders, as we discussed earlier. Hence, output in our economy, denoted by $Y$, with no LoLR, can be written as

$$Y = [\{(R - 1) - \rho(1 - c_1)(R - r)\Delta\} \times \mathbb{1}_{\rho \leqslant \rho^*}] + 2R^B,$$

where $\mathbb{1}_{\rho \leqslant \rho^*}$ is an indicator function that takes the value 1 when $\rho \leqslant \rho^*$ and 0 otherwise. Note that for $\rho \leqslant \rho^*$, $A$ banks invest and are exposed to early liquidation risk that occurs with probability $\rho(1 - c_1)$, while for $\rho > \rho^*$ they do not invest at all. The long-term return of $B$ banks' investments $2R^B$ is trivial—$B_0$ banks can always meet their liquidity needs, either by borrowing from outsiders or by calling back the loan to $A$ banks; $B_1$ banks always generate $R^B$, since they do not experience any interim liquidity shock requiring them to abandon their existing investments.

Recall that all $A$ banks invest when $\rho \leqslant \rho^*$, while no $A$ bank invests when $\rho > \rho^*$. In each

[14] For instance, the transfer could include legal costs, brokers' fees, or a lower price paid by the buyers in the secondary market due to their opportunistic behavior or limited cash holdings.

of these cases, we first analyze the "marginal" effect of central bank policy on output, without entailing a regime switch from investment to no-investment and vice versa; we then analyze the effects of the central bank policy when it does trigger a regime switch—that is, when it restores a frozen market or causes a well-functioning market to freeze.

In the first case, where $\rho \leqslant \rho^*$, all $A$ banks invest. Suppose that the central bank lends to a measure $x_H$ of $A$ banks with high-quality collateral. Suppose also, for now, that $x_H$ is small enough so that we can focus on the marginal effect. Note that the central bank's actions will affect the pool of collateral in the private market, where the average quality of collateral will now be

$$c_H^{CB} = \frac{(\alpha - x_H)c_H + (1-\alpha)c_L}{1 - x_H}.$$

In that case, output can be written as

$$Y_H = x_H\,(R-1) + (1-x_H)\left[(R-1) - \rho(1-c_H^{CB})(R-r)\Delta\right] + 2R^B, \tag{3}$$

since banks receiving funding from the central bank are not exposed to any liquidity risk, while those remaining in the private market, a fraction $1 - x_H$, experience changes in their liquidity risk exposure from $\rho\,(1-c_1)$ to $\rho\left(1 - c_H^{CB}\right)$. Note that central bank loans have two effects that go in opposite directions. On the one hand, they eliminate the liquidity risk for $x_H$ banks, increasing output as captured by the first term in equation (3). On the other hand, they impair the quality of collateral in the market since $c_H^{CB} < c_1$, which increases liquidity risk for the banks remaining in the market and indirectly decreases output, as captured by the second term in (3).

We have

$$\begin{aligned}
\frac{\partial Y_H}{\partial x_H} &= \rho(R-r)\Delta\left[(1-c_H^{CB}) + (1-x_H)\frac{\partial c_H^{CB}}{\partial x_H}\right] \\
&= \rho(R-r)\Delta(1-x_H)(1-c_H).
\end{aligned}$$

Note that $\frac{\partial Y_H}{\partial x_H} \leq 0$ since $c_H \geq 1$ — even though lending against high–quality collateral directly increases output, the negative externality on the rest of the banks remaining in the market more than offsets the direct effect. In other words, high-quality collateral would generate more liquidity and output if it remained and circulated in the private market than being locked up with the central bank.

However, when the central bank lends against low-quality collateral to a measure $x_L$ of banks, the average quality of collateral in the market improves to

$$c_L^{CB} = \frac{\alpha c_H + (1 - \alpha - x_L)c_L}{1 - x_L}.$$

Output can be written as

$$Y_L = x_L(R-1) + (1-x_L)\left[(R-1) - \rho(1-c_L^{CB})(R-r)\Delta\right] + 2R^B, \tag{4}$$

so that

$$\frac{\partial Y_L}{\partial x_L} = \rho(R-r)\Delta(1-x_L)(1-c_L) > 0.$$

Hence, when $\rho \leqslant \rho^*$, all banks invest and lending against high-quality collateral decreases output, whereas lending against low-quality collateral has a positive effect on output.

We now analyze the second case, where $\rho > \rho^*$. In this case, $A$ banks do not invest and the central bank stimulates only the investment of banks to which it directly lends, that is, $\frac{\partial Y_H}{\partial x_H} = \frac{\partial Y_L}{\partial x_L} = R-1$. Assuming that central bank lending is small enough and does not lead to a regime switch, we can summarize our results in the following proposition, which is illustrated in Figure 4.

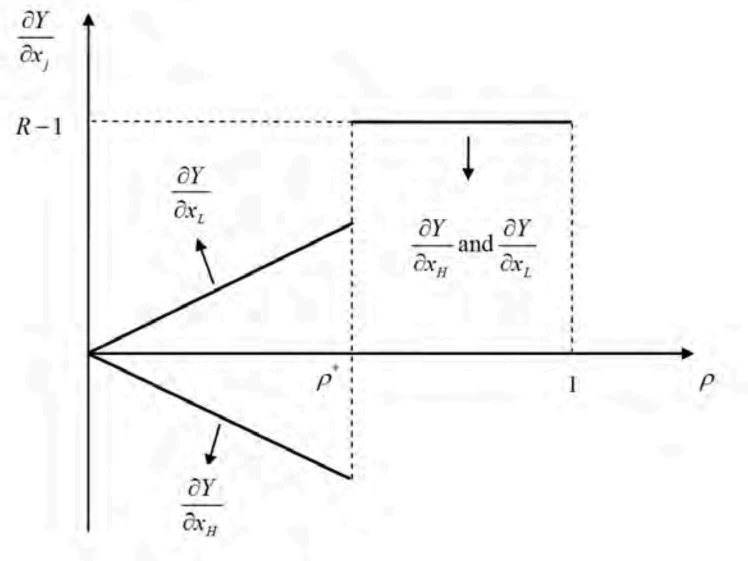


Figure 4: The effect of LoLR on output, by collateral type. When $\rho > \rho^*$, no $A$ bank invests, and different central bank policies have the same positive effect on output. When $\rho \leqslant \rho^*$, output increases (decreases) when the central bank lends against low-quality (high-quality) collateral.

**Proposition 3** *For $\rho > \rho^*$, central bank lending against high- and low-quality collateral has the same effect on output, $\frac{\partial Y}{\partial x_H} = \frac{\partial Y}{\partial x_L} = R - 1$. For $\rho \leqslant \rho^*$, central bank lending against high- and low-quality collateral has the following effects on output: $\frac{\partial Y}{\partial x_H} = \rho(1 - c_H)(R - r)\Delta \leq 0$ and $\frac{\partial Y}{\partial x_L} = \rho(1 - c_L)(R - r)\Delta > 0$, respectively.*

So far, we have focused on the effect of "marginal" lending, where central bank lending does not lead to regime switches. Next, we analyze how it can lead to such a shift. First, we show that improvements in the average quality of collateral can restore an impaired interbank market. Suppose $\rho > \rho^*$, so that no $A$ bank invests. If the average quality of collateral in the private market increases sufficiently, liquidity risk for $A$ banks is mitigated, possibly leading (some of) these banks to start investing.

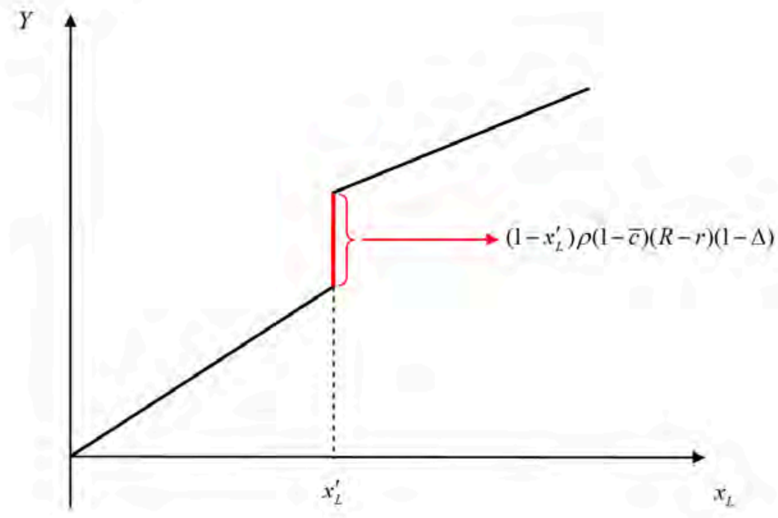


Figure 5: Market jump-start. All $A$ banks start to borrow and invest if the central bank lends to $x'_L$ or more banks with low-quality collateral. Output jumps upward.

In particular, let $c_1$ be the average quality of collateral at the outset. Liquidity risk is sufficiently mitigated and all banks will invest when the average quality of collateral increases to $c'$ so that $\rho(1-c')(R-r) \leqslant R-1$, that is, when

$$c' \geqslant \bar{c} \equiv 1 - \frac{R-1}{\rho(R-r)}, \tag{5}$$

where $\bar{c}$ represents the threshold level of the average quality of collateral above which $A$ banks borrow and invest. The central bank can achieve this by lending to, and taking out of the market, a minimum measure $x'_L$ of $A$ banks with low-quality collateral, where

$$x'_L = \frac{(R-1) - \rho(1-c_1)(R-r)}{(R-1) - \rho(1-c_L)(R-r)}. \tag{6}$$

Hence, when the central bank lends to a sufficiently large proportion of banks with low-quality collateral, the economy switches from a no-investment regime to one where banks take on investment by borrowing in restored interbank markets.

Figure 5 illustrates this result. As the central bank increases its lending to banks with low-quality collateral, it increases output by inducing the investment of the direct borrowers, but at the same time, the quality of the collateral in the private market improves. If the central bank takes out $x'_L$ of the low-quality collateral, the average quality of the collateral in the market improves sufficiently to trigger the investment of all the remaining $A$ banks in the private market. At that point, output jumps upward by $(1 - x'_L)\rho(1 - \bar{c})(R - r)(1 - \Delta)$.

Next, we show that a central bank policy that adversely affects the quality of collateral in private markets can lead to the breakdown of an otherwise functioning interbank market. For $\rho \leqslant \rho^*$, all banks borrow and invest. When the average quality of collateral falls below $c' < \bar{c}$, the economy switches to a state in which banks stop borrowing and investing. Central bank policy can lead to such a freeze when it lends to a measure $x'_H$ (or more) of $A$ banks with high-quality collateral, where

$$x'_H = \frac{(R-1) - \rho(1 - c_1)(R - r)}{(R-1) - \rho(1 - c_H)(R - r)}. \tag{7}$$

Figure 6 presents this result. As the central bank takes out the high-quality collateral, output decreases owing to the negative externality imposed on borrowers remaining in the market by increasing their liquidity risk. At the threshold $x'_H$, the liquidity risk becomes significant enough such that the remaining $A$ banks simply opt out from investing, and the output of remaining $A$ banks drops to 0. We summarize these results in the following proposition.

**Proposition 4** *Central bank policies can have the following effects on banks' investment decision: (i) when the central bank lends against low-quality collateral, which increases the average quality of collateral to a level above $\bar{c}$, given in (5), it can jump-start investment; and (ii) when the central bank lends against high-quality collateral, which decreases the average quality of collateral to a level below $\bar{c}$, banks stop borrowing in private markets and stop investing.*

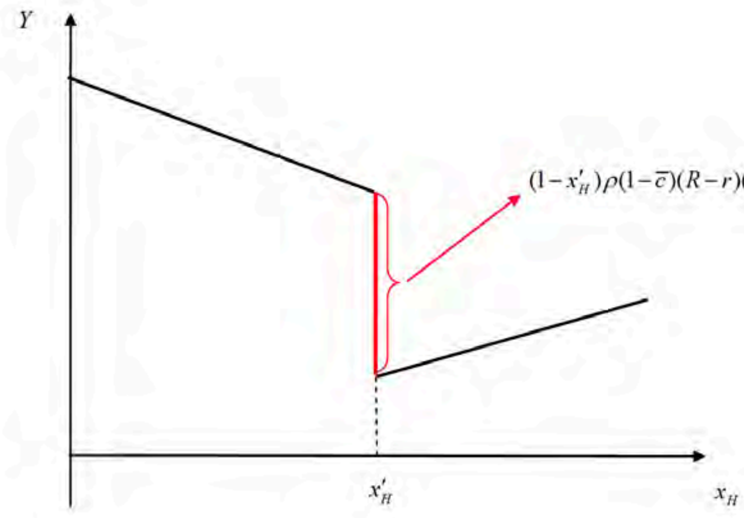


Figure 6: Market freeze. All remaining $A$ banks stop investing if the central bank lends to $x'_H$ or more banks with high-quality collateral. $A$ banks' output drops to $x'_H(R-1)$.

# 5 Potential losses and optimal LoLR policy

We now analyze the optimal LoLR policy taking into account the potential losses the central bank can experience from lending. We first define the objective function of the central bank taking into account the potential cost of such losses. Banks that borrow from the central bank pay at least the "fair" rate for the loan; that is, they pay at least 1 in expectation since $\gamma_j \geq 0$. However, the central bank can suffer losses when the projects of the banks it lends to have a low return and the collateral it received has a low value. This may be costly to the central bank, because it may affect its reputation or impair its independence.[15]

The central bank does not face any counterparty risk when it lends against high-quality collateral, since $c_H \geqslant 1$. However, the central bank suffers a loss of $1 - c_L$ when it lends against low-quality collateral and the bank's project has the low return 0 at $t = 2$. Hence, when the

[15]These costs can be different for different central banks, which can manifest itself in different collateral and lending policies for different central banks. For a more detailed discussion of these costs, see Section 7.4.

central bank lends to a measure $x_L$ of banks against low-quality collateral, the resulting counterparty risk exposes the central bank to expected losses of $z = x_L\rho(1-p)(1-c_L)$. We denote the cost to the central bank from incurring these losses by $f(z)$ and assume this function is convex and increasing in $z$ with $f(0) = 0$.

Using the central bank's cost function $f(.)$, we can define the central bank's objective function ("net output") as:

$$W = Y - f(z), \tag{8}$$

which is the output $Y$ net of any cost to the central bank from its LoLR actions. Note that we assumed that the penalty rate $\gamma_j$ is a mere transfer. Next, we discuss the central bank policies that would maximize its objective function.

We start with the case $\rho \leqslant \rho^*$, where all $A$ banks invest. In that case, while lending against high-quality collateral does not expose the central bank to any counterparty risk, it is never optimal because it will decrease output and possibly lead to a freeze in the private market (Propositions 3 and 4). Hence, the central bank would be better off refraining from lending against high-quality collateral and rather leave it in the market, where it generates more liquidity and output.

When lending against low-quality collateral, the central bank should weigh the benefit of increased output against the costs arising from counterparty risk. Hence, the central bank lends to a measure $x_L$ of $A$ banks with low-quality collateral, where the first order condition

$$\frac{\partial Y}{\partial x_L} = f'(z)\rho(1-p)(1-c_L), \tag{9}$$

which can also be written as $\frac{(R-r)\Delta}{1-p} = f'(z)$, is satisfied for $x_L < 1-\alpha$; or to all (no) $A$ banks with low-quality collateral, that is, $x_L = 1-\alpha$ (0), if

$$\frac{(R-r)\Delta}{1-p} \geq (<) f'(z) \text{ for } x_L = 1-\alpha \ (0). \tag{10}$$

Next, we consider the case where $\rho > \rho^*$, such that none of the $A$ banks invests. Lending against high-quality collateral increases output by $R - 1$ per unit lent (Proposition 3) and does not expose the central bank to counterparty risk; hence, the net increase in output is $R-1$ per unit lent. Lending against low-quality collateral also increases output by $R-1$ per unit lent but exposes the central bank to counterparty risk entailing the cost $f(z)$. However, lending against low-quality collateral has the benefit of improving the quality of collateral in the market, which can lead to a switch from a no-investment regime to one where investment is restored (Proposition 4). When analyzing the optimal central bank policy, we need to take into account such discontinuous effects.

A feasible policy for the central bank, denoted as Policy $H$, is to (i) lend to all $A$ banks with high-quality collateral (i.e., $x_H = \alpha$), which would increase net output by $\alpha(R-1)$, and then (ii) lend to the $A$ banks with low-quality collateral according to the marginal cost-benefit analysis characterized by the first-order conditions given in (9) and (10). Note that when the central bank lends to all $A$ banks with high-quality collateral, only $A$ banks with low-quality collateral are left in the market, and the latter will invest only if they can borrow from the central bank; that is, the private market is completely replaced by the central bank. This results in a net output of

$$W_H = (\alpha + x_L)(R-1) + 2R^B - f(z),$$

since only the banks that borrow from the central bank, a measure $(\alpha + x_L)$, invest. Under this policy, similar to the policy proposed by Bagehot, the central bank lends "freely" to all banks with high-quality collateral.

Under an alternative policy, denoted by Policy $L$, the central bank lends against low-quality collateral. This would be optimal only if it leads to a regime switch by restoring investment in the interbank market, since, otherwise, incremental lending against high-quality collateral would generate the same increase in output ($R-1$ per unit lent) without exposing the central bank to any counterparty risk. Hence, Policy $L$ with no regime switch would be dominated by Policy $H$. For the central bank to induce a regime shift, it should lend at least to a measure $x'_L$, given in equation (6), of banks with low-quality collateral. When the economy switches to the investment regime, output jumps upward by $(1 - x'_L)\rho(1 - \bar{c})(R - r)(1 - \Delta)$, and further lending to $A$ banks with low-quality collateral will generate a continuous effect, as in Figure 5.

The optimal capacity of this lending facility depends on the cost to the central bank. Suppose that the central bank just managed to jump start investment with $x_L = x'_L$. If additional lending to $A$ banks with low-quality collateral at that point is too costly, that is,

$$\frac{(R-r)\Delta}{1-p} < f'(z) \tag{11}$$

for $x_L = x'_L$, then the central bank should just restore investment by lending to $x'_L$ of $A$ banks with low-quality collateral but should not lend beyond $x'_L$. Otherwise, the central bank will lend to a measure $x_L > x'_L$ of $A$ banks with low-quality collateral, where the first order condition

$$\frac{(R-r)\Delta}{1-p} = f'(z), \tag{12}$$

is satisfied for $x_L < 1 - \alpha$; or it will lend to all $A$ banks with low-quality collateral, that is, $x_L = 1 - \alpha$ if

$$\frac{(R-r)\Delta}{1-p} \geqslant f'(z) \tag{13}$$

for $x_L = 1-\alpha$.[16] Policy $L$ results in a net output of

$$W_L = (R-1) - (1-x_L)\rho(1-c_L^{CB})(R-r)\Delta + 2R^B - f(z),$$

where the first term is potential output and the second term represents the expected output loss, due to liquidity risk, for banks that borrow in the interbank market.

For $W_L > W_H$, the optimal policy would be Policy $L$, that is, to lend against low-quality collateral and induce a shift to an investment regime. For $W_L \leqslant W_H$, the optimal policy would be Policy $H$, that is, lending to all $A$ banks with high-quality collateral, and then to (some of the) $A$ banks with low-quality collateral.

Let $x_L^*$ and $x_L^{**}$ be the optimal levels of lending to $A$ banks with low-quality collateral under Policy $H$ and Policy $L$, respectively; and let $f(z^*)$ and $f(z^{**})$ represent the resulting costs from counterparty risk under Policy $H$ and Policy $L$, respectively. We have $W_L > W_H$ when

$$(1-\alpha-x_L^*)(R-1) > (1-x_L^{**})\rho(1-c_L^{CB})(R-r)\Delta + \left(f(z^{**}) - f(z^*)\right), \tag{14}$$

where the LHS is the loss in output arising from the inability of banks that cannot borrow from the central bank to invest under Policy $H$; the first term on the RHS represents the output loss from liquidity risk for banks that borrow in the market under Policy $L$; and the second term represents the difference between the costs of counterparty risk under Policy $L$ and Policy $H$. We summarize these results in the following proposition.

**Proposition 5** *The optimal central bank policy can be characterized as follows:*

*i) For $\rho \leqslant \rho^*$, it is never optimal to lend against high-quality collateral. The optimal level of lending against low-quality collateral $x_L$ satisfies $\frac{\partial Y}{\partial x_L} = f'(z)\frac{\partial z}{\partial x_L}$; or $x_L = 1-\alpha$ (0)*

[16] Recall from Proposition 4 that the high-quality collateral is better left in the private market than kept by the central bank, unless the interbank market is frozen. Hence, once the economy has switched to an investment regime, any lending against high-quality collateral is suboptimal.

*for* $\frac{\partial Y}{\partial x_L} > (<) f'(z) \frac{\partial z}{\partial x_L}$.

*ii) For* $\rho > \rho^*$, *when* $W_L > W_H$, *that is, when (14) holds, the central bank optimally lends against low-quality collateral and induces a switch to an investment regime. Otherwise, the central bank lends to all A banks with high-quality collateral, that is,* $x_H = \alpha$, *and to (some of) the A banks with low-quality collateral characterized by the FOCs in (12) and (13).*

A unique case is depicted in Figure 7, which corresponds to Policy $L$ satisfying (11). In this case, the central bank should increase its lending to $A$ banks with low-quality collateral even if the marginal benefit $\frac{\partial Y}{\partial x_L}$ is lower than the marginal cost from the counterparty risk exposures $f'(z)\frac{dz}{dx_L}$. This is because, by doing so, it can eventually lead to a regime switch with a discrete increase in net output. In Figure 7, the first order condition $\frac{\partial Y}{\partial x_L} = f'(z)\frac{\partial z}{\partial x_L}$ holds at $x_L = x_L^{FOC}$, and beyond that point, net output decreases with greater $x_L$. That is, the objective function attains its local maximum at $x_L = x_L^{FOC}$. However, if the central bank expands its lending to $x_L = x_L^{'}$, investment is restored and output jumps upward by $(1 - x'_L)\rho(1 - \bar{c})(R - r)(1 - \Delta)$. If net output $W_L$ when investment has been restored is greater than the local maximum $W^{FOC}$, then the central bank should expose itself to more counterparty risk than the marginal cost-benefit analysis would suggest—an approach for a central bank with a microprudential view—in order to restore private markets.

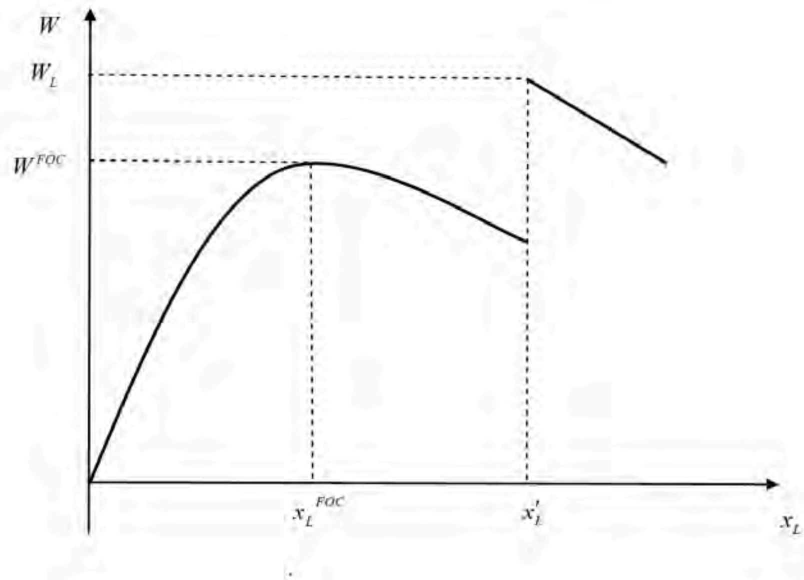


Figure 7: Optimal regime shift. The marginal increase in output is less than the marginal cost to the central bank beyond $x_L = x_L^{FOC}$. However, a regime switch from no-investment to investment arises when $x_L = x'_L$, at which point output and net output increase in a discrete fashion. If $W_L > W^{FOC}$, the central bank should bear greater counterparty risk to restore private markets than the marginal cost-benefit analysis suggests.

# 6 Extensions

In this section, we provide several extensions of the benchmark model to tackle important issues such as the role of a longer intermediation chain in amplifying the results from the benchmark model (Section 6.1); the role of cash-in-the-market pricing and the positive price effects that central bank lending can generate on the liquidation value in the secondary markets (Section 6.2); endogenous generation of information on collateral quality by banks and how central bank lending can affect information generation (Section 6.3); and illustration of our results using a general p.d.f. for the liquidity shocks (Section 6.4).

## 6.1 Intermediation chain and collateral circulation

In the benchmark model, we kept the intermediation chain short to illustrate the main results in a simple framework. In this section, we extend the intermediation chain, where we allow for the collateral to circulate among $B$ banks multiple times and show that the effects we obtain in our benchmark setup become amplified.

We extend our baseline setup and divide the intermediate period $t = 1$ into $N$ subperiods denoted by $n = 0, 1, \ldots N - 1$. Furthermore, we have more than $N$ subgroups of $B$ banks, where we denote the $B$ banks that lent to $A$ banks at $t = 0$ as $B_0$.

At $n = 0$, as in the benchmark case, $B_0$ banks get hit by a liquidity shock $\ell_0$, where $\ell_0$ is uniformly distributed over the unit interval $[0, 1]$. Furthermore, at subperiod $n = 0$, $B_1$ banks receive cash that they can lend to $B_0$ banks. $B_0$ banks search for a $B_1$ bank to borrow from in the market and they are matched with a $B_1$ bank with probability $1 - \rho$. However, with probability $\rho$ they do not find a match and need to borrow from outsiders. They can borrow up to $c_1$ from an outsider so that with probability $\rho(1 - c_1)$ they cannot satisfy their liquidity need and call back the loan, which leads to an early liquidation of $A$ banks' projects.

The model continues in the same fashion, where $B_n$ banks, who lent to $B_{n-1}$ banks in subperiod $n - 1$ if there were successful matches so far, are hit by a liquidity shock $\ell_n$ in subperiod $n$. They search for a $B_{n+1}$ bank in the market, but with probability $\rho$, they are not matched with one and need to borrow from an outsider.[17] If they cannot satisfy their liquidity needs, $B_n$ banks call back the loan they gave to $B_{n-1}$ banks in subperiod $n-1$, which starts a chain of loans being called back so that, ultimately, $A$ banks' projects get liquidated. Note that if a $B_n$ bank is not matched with a $B_{n+1}$ bank and needs to go to an outsider, the intermediation chain breaks at subperiod $n$ and, from then on, all the $B$ banks in subgroups $k > n$ are self sufficient since they receive enough cash at subperiod $k$ to satisfy their own liquidity needs $\ell_k$. Figure 8 summarizes the flows between agents.

[17] We assume that the $B_{n+1}$ banks lend $\ell_n$ to $B_n$ banks and consume any liquidity left with them. This helps us keep the analysis simple, since otherwise we would have to track unused liquidity, which would complicate the analysis without adding much to the results.

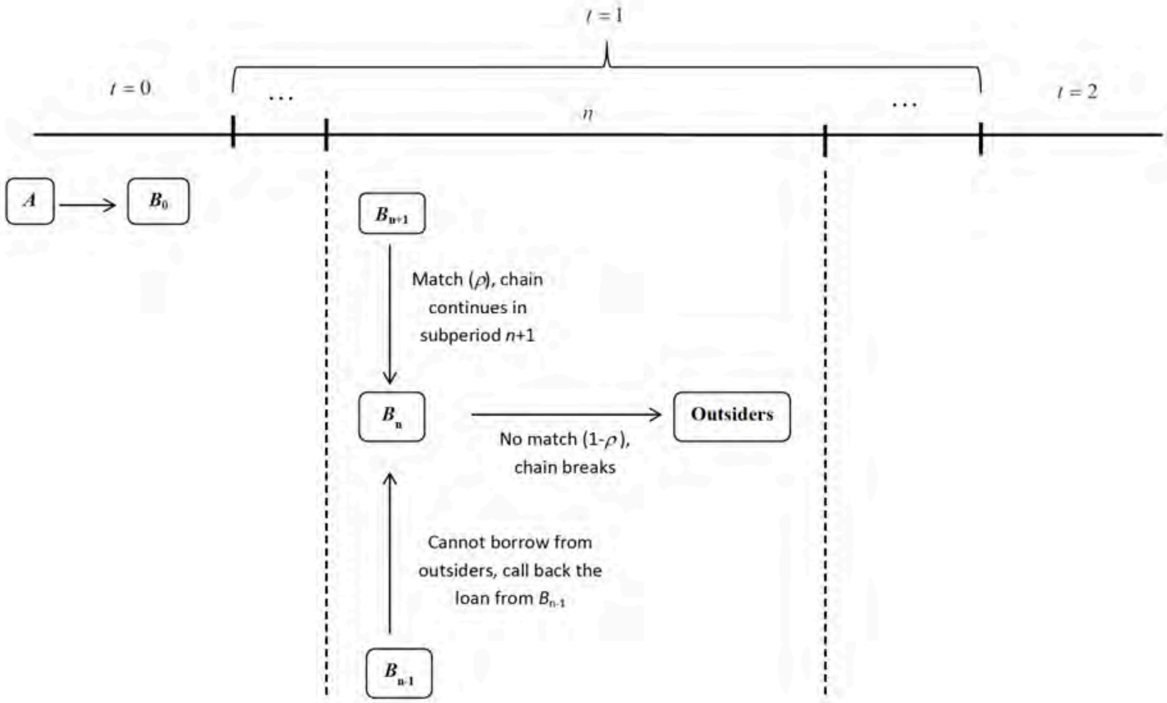


Figure 8: Flows between agents at subperiod $n$ of $t = 1$.

With this extended intermediation chain, we can derive the probability of $A$ banks' projects being liquidated as follows. At $n = 0$, the probability that $B_0$ banks cannot satisfy their liquidity need is $\rho(1 - c_1)$. The probability that $B_0$ borrows from $B_1$ but $B_1$ cannot satisfy its liquidity need at $n = 1$ is $(1 - \rho)\rho(1 - c_1)$. Similarly, we can easily show that the ex-ante likelihood of (i) $B_k$ banks' successfully borrowing from $B_{k+1}$ banks for all $k \in \{0, ..., n-1\}$; but (ii) $B_n$ banks' failure to satisfy their liquidity need, is equal to $(1-\rho)^n \rho(1-c_1)$.[18] Hence, as of $t = 0$, the probability of an early liquidation of $A$ banks' project can be written as

$$\rho(1 - c_1) \sum_{n=0}^{N-1} (1 - \rho)^n = (1 - c_1) \left(1 - (1 - \rho)^N\right).$$

Note that as the intermediation chain gets longer, the probability of early liquidation increases,

[18]Note that there are more subgroups of $B$ banks than $N$ so that the $B_{N-1}$ banks have other $B$ banks they can be matched with in subperiod $N - 1$. Since the $B$ banks receive the long-term return $R^B$ from their existing projects at $t = 2$, the $B_N$ banks that lent to $B_{N-1}$ banks always have enough cash to satisfy their own liquidity needs and loans are never called back after subperiod $N - 1$.

and this probability converges to $1 - c_1$ as $N \to \infty$.

$A$ banks will not invest when $R - 1 < (1 - c_1)\left(1 - (1-\rho)^N\right)(R - r)$, that is, when

$$\rho > \rho_N^* = 1 - \left(1 - \frac{R-1}{(1-c_1)(R-r)}\right)^{1/N}.$$

Note that as the chain gets longer, the threshold $\rho_N^*$ decreases, that is, $\frac{d\rho_N^*}{dN} < 0$. Hence, longer chains result in higher expected liquidation costs if $A$ banks invest and, hence, $A$ banks bypass investment for a larger set of parameter values.

Next, we show that as the chain gets longer, the effect of central bank actions on output is amplified. Suppose that the central bank lends to a measure $x_H$ of $A$ banks with high-quality collateral so that the average quality of collateral in the private market will be $c_H^{CB}$. We focus on the marginal effect.[19] In that case, output can be written as

$$Y_H = (R-1) - (1 - x_H)\left[\left(1 - (1-\rho)^N\right)(1 - c_H^{CB})(R-r)\Delta\right] + NR^B. \tag{15}$$

We thus have

$$\frac{\partial Y_H}{\partial x_H} = \left(1 - (1-\rho)^N\right)(R-r)\Delta(1-x_H)(1-c_H).$$

Note that $\frac{\partial Y_H}{\partial x_H} \leq 0$, since $c_H \geq 1$ as in the benchmark case. Furthermore, the negative effect $\frac{\partial Y_H}{\partial x_H}$ is stronger for longer chains. We can show a similar effect when the central bank lends against low-quality collateral to a measure $x_L$ of banks, where we obtain

$$\frac{\partial Y_L}{\partial x_L} = \left(1 - (1-\rho)^N\right)(R-r)\Delta(1-x_L)(1-c_L) > 0.$$

Again, the effect $\frac{\partial Y_L}{\partial x_L}$ gets larger as the chain gets longer.

When the intermediation chain gets longer, collateral can circulate multiple times so that

[19] We can show similar amplification effects when we analyze discontinuous effects such as a jump start or a market freeze.

its capacity to generate liquidity increases. Hence, the central bank's actions that affect the average quality of collateral in markets have bigger effects on the functioning of markets. In particular, as the intermediation chain gets longer, a smaller intervention by the central bank can have a bigger impact on markets.

## 6.2 Cash-in-the-market pricing

So far, we have assumed that the liquidation value $r$ is fixed. However, the liquidation value can be a function of the number of projects being liquidated. In particular, a large proportion of projects being liquidated can lead to fire sales in secondary markets when buyers are financially constrained (Shleifer and Vishny (1992)) and the liquidation value can be determined by the available cash in the market, resulting in cash-in-the-market pricing (Allen and Gale (1994, 1998)).

Next, we analyze the effects of cash-in-the-market pricing. Suppose that the liquidation value decreases as more projects are liquidated. Hence, in turn, the liquidation value is increasing in the measure $x_j$ of banks to which the central bank lends, that is, $\partial r/\partial x_j > 0$, since banks that borrow from the central bank do not experience any liquidity risk and their projects can be held until maturity at $t = 2$.

Suppose that $\rho \leqslant \rho^*$ so that $A$ banks invest and the central bank lends to a measure $x_L$ of $A$ banks with low-quality collateral. In that case, using the output given in equation (4), we obtain

$$\frac{\partial Y_L}{\partial x_L} = \rho\Delta(R-r)\left((1-c_L^{CB}) + (1-x_L)\frac{\partial c_L^{CB}}{\partial x_L}\right) + \underbrace{\rho\Delta(1-x_L)(1-c_L^{CB})\frac{\partial r}{\partial x_L}}_{\text{price effect } (>0)}.$$

Note that the second term, which has a positive sign, results from the effect central bank lending has on the liquidation value $r$, since fewer projects need to be liquidated in the secondary market now.[20] Hence, central bank lending has an additional boosting effect on

[20]We can show that when the central bank lends against high-quality collateral, the price effect on output

output through improved prices in the secondary market.

Furthermore, central bank lending can decrease the investment threshold $\rho_L^*$ through improved prices in the secondary market. In particular, we have $\rho_L^* = \frac{R-1}{\left(1-c_L^{CB}\right)(R-r(x_L))}$, so that

$$\frac{\partial \rho_L^*}{\partial x_L} = \frac{1}{\left(1-c_L^{CB}\right)(R-r)} \left[ \frac{\partial c_L^{CB}/\partial x_L}{1-c_L^{CB}} + \underbrace{\frac{\partial r/\partial x_L}{R-r}}_{\text{price effect } (>0)} \right] > 0.$$

Note that the second term in the parentheses has a positive sign. Hence, central bank lending can have an additional effect on output through improved prices in the secondary market even when $A$ banks were not investing in the first place, that is, in addition to the output $R-1$ generated by the $A$ banks with low-quality collateral that borrow from the central bank, the threshold for the probability of the liquidity shock $\rho_L^*$ below which $A$ banks invest increases, which makes investment more likely.

## 6.3 Information on collateral quality

In this section, we provide an extension of our model that allows collateral holders to endogenous generate information. Suppose that the true quality of collateral would be revealed to the outsiders only if information that can be credibly conveyed to them is generated, as in Gorton and Ordoñez (2014, 2017); otherwise, outsiders would simply attribute the average quality $c_1$ to any collateral at $t=1$ as in our original setup. In particular, at $t=0$, $A$ banks can generate information about their collateral type by incurring a cost of $\delta$.[21] Note that banks will never generate information when $\delta > R-1$. Next, we analyze the case with $\delta \leqslant R-1$.

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and $\rho^*$ would be equal to $\rho\Delta(1-x_H)(1-c_H^{CB})\frac{\partial r}{\partial x_H}$ and $\frac{\partial r/\partial x_H}{\left(1-c_H^{CB}\right)(R-r)^2}$ , respectively.

[21]Note that information generation can take time and cannot be done immediately when facing a run at $t=1$. Furthermore, $B_0$ banks would not have any incentive to generate information, since they can always satisfy their liquidity need by calling back the loan from $A$ banks.

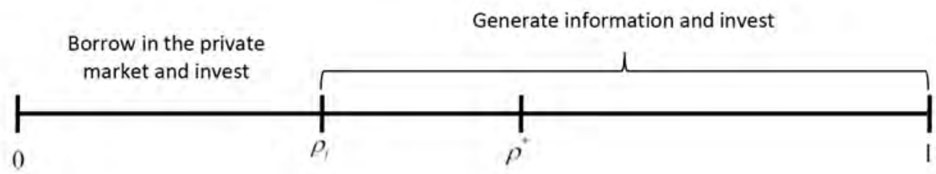


Figure 9: $A$ banks with high-quality collateral generate information and invest when $\rho \geq \rho_i$.

An $A$ bank with low-quality collateral will never produce information since it would only be worse off by doing so, and an $A$ bank with high-quality collateral will incur the cost of $\delta$ to separate itself from the $A$ banks with low-quality collateral if the benefit from lowering liquidity risk is greater than the cost of generating information. Once perceived as a borrower with high-quality collateral, the $A$ bank can eliminate liquidity risk altogether since $c_H \geq 1$. Therefore, $A$ banks with high-quality collateral will choose to generate information when $\rho(1-c_1)(R-r) \geq \delta$, which can also be written as

$$\rho > \frac{\delta}{(1-c_1)(R-r)} \equiv \rho_i. \tag{16}$$

Note that $\rho_i < \rho^*$ with $R-1 > \delta$, which gives us the following proposition (see Figure 9).

**Proposition 6 (Information generation):** *For $R-1 < \delta$, $A$ banks will not generate information. For $R-1 \geqslant \delta$, we have two cases: For $\rho < \rho_i$, no information is generated and $A$ banks with both types of collateral are pooled and invest. For $\rho > \rho_i$, $A$ banks with high-quality collateral will generate information to separate themselves.*

Hence, information generation arises endogenously as the probability $\rho$ gets higher or the average quality $c_1$ of collateral deteriorates. Note that when information is generated, the liquidity risk for the $A$ banks with low-quality collateral *increases*, since their collateral is no longer pooled with high-quality collateral, and these banks might not invest at all if their liquidity risk becomes sufficiently high.

This result implies that a (partial) market breakdown can arise even when information is generated endogenously. Unless information is generated, all of the previous results on marginal central bank lending (Proposition 3) and on regime switch (Proposition 4) go through in a similar way. Note that the information generation threshold $\rho_i$ is increasing in $c_1$; hence, when the central bank lends against high-quality collateral, which would result in $c_H^{CB} < c_1$ and, therefore, lower $\rho_i$, could force banks with high-quality collateral to generate and convey information about their collateral. Hence, central bank lending can trigger information generation by banks with high-quality collateral, which, in turn, can result in a freeze in an otherwise functioning interbank market, with no investment by banks with low-quality collateral. On the other hand, lending against low-quality collateral can improve the overall quality of collateral in the market and can reduce the incentives for information generation.

## 6.4 Probability distribution for liquidity shocks

In the benchmark model, we assumed that the liquidity shock $\ell$ has a uniform distribution. In this section, we show that this is not necessary for our results. Suppose that the liquidity shock $\ell$ has a probability density function $g(.)$, and the c.d.f. $G(.)$. In that case, the probability of an early liquidation will be $\rho(1 - G(c_1))$, as opposed to $\rho(1 - c_1)$ for the case with the uniform distribution and the results in Propositions 1 and 2 follow immediately with this new modification.

Next, we analyze the "marginal" effect of central bank lending on output as characterized in Proposition 3. In this new setup, when the central bank lends against low-quality collateral to a measure $x_L$ of banks, we have

$$Y_L = (R - 1) - (1 - x_L)\left[\rho(1 - G(c_L^{CB}))(R - r)\Delta\right] + R^B,$$

so that

$$\frac{\partial Y_L}{\partial x_L} = \rho(R-r)\Delta\left[\left(1-G\left(c_L^{CB}\right)\right)+g\left(c_L^{CB}\right)\frac{\alpha\left(c_H-c_L\right)}{1-x_L}\right] > 0.$$

Note that when the central bank lends against low-quality collateral, it eliminates the liquidity risk for the $x_L$ banks it lends to and, furthermore, the quality of collateral in the market improves so that the liquidity risk for the remaining banks are mitigated. These two effects go in the same direction and increase output. Note that this result is independent of the probability distribution $g(.)$.

Similarly, when the central bank lends against high-quality collateral to a measure $x_H$ of banks, we have

$$\frac{\partial Y_H}{\partial x_H} = \rho(R-r)\Delta\left[\left(1-G\left(c_H^{CB}\right)\right)-g\left(c_H^{CB}\right)\frac{(1-\alpha)\left(c_H-c_L\right)}{1-x_H}\right].$$

We can show that $\frac{\partial Y_H}{\partial x_H} \leqslant 0$ when

$$\frac{g\left(c_H^{CB}\right)}{1-G\left(c_H^{CB}\right)} \geqslant \frac{1-x_H}{(1-\alpha)\left(c_H-c_L\right)}. \tag{17}$$

When the central bank lends against high-quality collateral, it eliminates the liquidity risk for the $x_H$ banks it lends to. However, the quality of collateral in the market deteriorates so that the liquidity risk for the remaining banks increases. The negative effect dominates the positive effect when the condition in (17) is satisfied, that is, when the value of the probability density $g\left(c_H^{CB}\right)$ is sufficiently large compared with the probability of early liquidation $\left(1-G\left(c_H^{CB}\right)\right)$, in which case a small change in the overall quality of collateral significantly affects liquidity risk.

# 7 Discussion

In this section, we provide a discussion of some of the features of our model and various issues such as how central banks can use the penalty rates $\gamma_j$ to prevent moral hazard and create incentives for banks to hold high-quality collateral (Section 7.1) and limit the size of their lending facilities (Section 7.2); relationships in the interbank market (Section 7.3); and various features of central banks that put them in a special position to act as the lender of last resort and the costs associated with their LoLR activities (Section 7.4).

## 7.1 Moral hazard

One potential downside of the central bank's lending against low-quality collateral is that it can create incentives for banks to generate and hold such collateral when it is more costly for them to hold high-quality collateral (Nyborg 2017). This, in turn, can expose the central bank to counterparty risk, especially during systemic crises, when the central bank is compelled to lend to banks.[22] A central bank policy that could prevent such moral hazard without curbing valuable investments by banks would be to charge a high enough penalty rate when it lends against low-quality collateral.

Consider the following addition to our existing setup. Suppose that the $A$ banks choose the collateral they hold at an earlier stage, say, at date $t = -1$. Let $\beta_j$ denote the cost of holding collateral of quality $j = H, L$ with $\beta_H > \beta_L$. Recall that $\gamma_j$ is the penalty rate the central bank charges when it lends against collateral type $j = H, L$. For simplicity, let's assume that $\gamma_H = 0$ so that the central bank does not charge a penalty rate when it lends against high-quality collateral. We denote by $\bar{\gamma}_L$ the expected penalty rate the bank pays when it borrows from the central bank against low-quality collateral that takes into account the probability of borrowing from the central bank, which, in turn, depends on the distribution of $\rho$ as of $t = -1$. Furthermore, let $E(\Pi_H)$ and $E(\Pi_L)$ be the expected profit for the bank when it

[22] Mailath and Mester (1994) and Acharya and Yorulmazer (2007, 2008) analyze time-inconsistency associated with regulatory actions. Goodfriend and Lacker (1999) and Freixas (1999) investigate the importance of "constructive ambiguity" when central banks are unable to commit to limiting lending to troubled institutions.

holds high- and low-quality collateral, respectively, excluding the cost of holding collateral and the cost of borrowing from the central bank. Again, the expectation is defined over the distribution of $\rho$ as of $t = -1$. Hence, the central bank can use the rate $\gamma_L$ to induce banks to hold high-quality collateral, where, for

$$E(\Pi_L) - \bar{\gamma}_L - \beta_L < E(\Pi_H) - \beta_H,$$

the bank would have a higher expected profit when it holds high-quality collateral. Thus, the central bank can facilitate valuable investment by insuring banks against liquidity risk, while at the same time preventing moral hazard by using an appropriate penalty rate when it lends against low-quality collateral.

## 7.2 LoLR capacity

In the benchmark model, we assume that the central bank could arbitrarily limit the capacity of LoLR lending to $x_j$. Note that the $A$ banks that have the same type of collateral are homogeneous in our setup; thus given the penalty rate $\gamma_j$, the demand for LoLR funding should always be a corner solution. In the case of excess demand, we assumed that central bank funds would be assigned randomly or on a first-come-first-served basis. Nonetheless, the central bank can achieve the desired allocation through a market clearing rate or via an auction mechanism if banks are heterogeneous.[23]

Suppose that the long-term return of the $A$ banks, denoted by $R$, is heterogeneous, such that $R \sim [R - \epsilon, R + \epsilon]$ for all $A$ banks with high- and low-quality collateral. Since there is no adverse selection with respect to LoLR lending,[24] the marginal borrower from the central bank is the bank with the return $R^*$ so that its benefit $\rho(1 - c_1)(R^* - r)$ from LoLR access is

[23]Central banks frequently use an auction mechanism for liquidity allocation. In the US, the term auction facility (TAF) allocated a set amount of funds to banks against collateral through auctions. The ECB controls liquidity via refinancing operations, which allocate the specified amount of liquidity through an auction mechanism.

[24]Note that all of the borrowers would need to pay $1 + \gamma_j$ in expectation, unlike in the credit rationing model in Stiglitz and Weiss (1981).

just equal to the penalty rate $\gamma_j$. Any bank with $R > R^*$ will borrow from the central bank since the benefit is greater for more productive banks.

This implies that starting from a sufficiently high penalty rate, the LoLR will attract only the most productive banks, and, by lowering the penalty rate slightly, it would attract the next set of productive banks. Suppose that the central bank can maximize net output by lending to $x_j$ of most productive $A$ banks with collateral type $j = H, L$. Hence, the central bank would like to limit its lending capacity to $x_j$, and would also aim to clear the demand for its funds by setting $\gamma_j$ so as to make the demand equal $x_j$. This is possible by choosing the right penalty rate, since the demand for central bank funds decreases monotonically in $\gamma_j$. In other words, as the cost of accessing the central bank funds increases, only the productive banks whose benefits from liquidity insurance are large enough will borrow from the central bank; and the central bank can achieve the desired allocation by choosing the penalty rate to clear the market.

Note that with the heterogeneous productivity of $A$ banks, the same allocation could be achieved through an auction even without the adjustment of the penalty rate. Again, the more productive banks would bid more for central bank funds, and given the limit $x_j$ of the facility, the marginal bank should bid the same rate $\gamma_j$, as we discussed in the penalty rate setup.

## 7.3 Relations in the interbank market

In our model, we assume that outsiders ask for collateral when lending to $B_0$; and $A$ banks cannot borrow directly from outsiders at $t = 1$ if their collateral is already pledged with $B_0$ banks. This could arise, for instance, if relations in the interbank market matter and outsiders do not have a relationship with banks. In this section, we discuss relationships in the interbank market and why banks have relationships only with a limited number of other banks.

Relationships in the interbank market can arise from "tiering," whereby some banks ("tier-1" banks) access central bank liquidity and act as clearing banks for other banks ("tier-2" banks). Even in large interbank markets, such tiering exists, and hence, issues of market power remain important. For example, in the U.S. federal funds market, JPMC and Bank of America are much bigger borrowers than others, and State Street and JPMC are much bigger lenders. Furthermore, many banks are connected with only one or two banks, and the average number of connections is between three and four (Bech and Atalay 2010). The UK also has a tiered system, and the volatility induced in interbank lending rates due to the cornering of collateral and liquidity by some of the large settlement banks during 2001–2005 was one of the main rationales for the Sterling Money Market Reform in 2006. Post-reform, the Bank of England increased the number of banks allowed to participate in open market operations from 10 to more than 35 (Bank of England 2005 and Tucker 2004).

Another important feature is that lending and borrowing in the interbank market make peer monitoring among banks important (see Rochet and Tirole 1996 and Freixas and Holthausen 2005). Such monitoring can create information monopolies in interbank markets. Cocco, Gomes, and Martins (2009) report evidence of strong relationships in the Portuguese interbank market, suggesting that some banks are more important lenders and are pivotal, even in normal times. Furthermore, smaller banks, with limited access to foreign interbank markets, concentrate all of their borrowing in only a few large banks in the domestic interbank markets. They also highlight the bilateral nature of interbank lending: most of the lending volume is accounted for by "direct" loans in which loan amount and interest rate are agreed to on a one-to-one basis between the borrower and the lender, whereas other banks do not necessarily have access to the same terms. Furthermore, such relationships become more important during stress periods, which is the main focus of our paper.

## 7.4 Central bank as LoLR

Central banks have certain features that put them in a special position to act as lenders of last resort such as acquiring information through their supervisory activities, being able to lend at a longer horizon than market participants, and their financial capacity to lend and acquire assets on large scales that private agents may not be able to do owing to financial constraints.[25]

Through its supervisory activities, the central bank can acquire information on banks' assets that is not publicly available to all market participants. In his testimony at the U.S. House of Representatives,[26] Stephen Cecchetti talks about this issue: "Operating as the lender of last resort requires two pieces of information: (1) a determination of an institution's solvency, and (2) a valuation of the collateral that is being posted to back the loan. Supervisors, with their intimate knowledge of the bank's operations, are the officials expected to have both of these." William Buiter makes a similar point in a Treasury Committee hearing, pointing to the separation of responsibilities between the UK Treasury, the Bank of England, and the FSA during the crisis: "The notion that the institution that has the knowledge of the individual banks that may or may not be in trouble would be a different institution from the one that has the money, the resources, to act upon the observation that a particular bank needs lender of last resort support is risky. It is possible, if you are lucky, to manage it, but it is an invitation to disaster, to delay, and to wrong decisions."[27] Furthermore, even if the central bank could not evaluate the quality of collateral that banks pledge exactly, it could examine it ex post as a regulator/supervisor and impose a penalty on banks that do not report truthfully. This would induce banks to report the true quality ex ante.

Another advantage of central banks would be that they can afford to lend long term, since they would have a longer horizon than private agents that may experience liquidity shocks in the short run. Furthermore, central banks can conduct asset purchases and lending on a large scale that private agents may not be able to do due to financial constraints. Such large scale lending can have a significant impact on prices, as we show in Section 6.2. Furthermore, they can eliminate various market failures that can arise owing to adverse selection and moral

[25]See Repullo (2000), Kahn and Santos (2005, 2006), and Ponce (2010) for papers that analyze who should act as the LoLR in the presence of asymmetric information and differences in policy objectives among regulators.

[26]Available at https://financialservices.house.gov/uploadedfiles/hhrg-115-ba15-wstate-scecchetti-20170912.pdf

[27]Source: "The Run on the Rock," report for the House of Commons Treasury Committee, page 105.

hazard, as Flannery (1996) shows. For example, even though private agents may know the overall quality of a portfolio, they may not be able to know the exact quality of each and every asset in the portfolio. Hence, when they cannot acquire or lend against the entire portfolio but only a fraction of it, they worry that the borrower will pledge the assets with lower value. As a result, they may shy away from acquiring assets or lending against them, whereas an agent with sufficient financial capacity, like the central bank, can lend against the entire portfolio.

While central banks are uniquely positioned to act as lenders of last resort, this function can expose them to counterparty risks and corresponding losses. A central bank can suffer losses when the banks it lends to have a low return and the collateral it received turns out to have a low value. These losses can be costly for the central bank. For example, they may result in a loss of reputation for the central bank that can impair its independence. Furthermore, the provision of immediate funds can entail fiscal costs, which can be linked to a variety of sources such as (i) the distortionary effects of tax increases required to fund losses, and (ii) the likely effect of government deficits on the country's exchange rate, manifested in the fact that banking crises and currency crises have often occurred as twin crises in many countries. In addition, while government expenditures and inflows during the regular course of events are smooth, we observe a rapid growth of off-balance-sheet contingent liabilities such as deposit insurance funds, and the costs of bank bailouts during crisis periods, where governments need to come up with funds in a short period of time.

Also, the costs associated with LoLR activities can vary among central banks. For example, the costs can be higher for more conservative central banks and central banks with more restrictions on their ability to provide liquidity to banks. These differences can manifest themselves in the central banks' collateral and lending policies. The Bank for International Settlements (2013) examines how collateral frameworks differ across central banks and how they have changed after the crisis. First, the recent crisis prompted central banks to modify their collateral policies, whereas central bank collateral frameworks today tend to be broader, accepting more asset types. Furthermore, the frameworks differ not only in terms of eligible asset types but also along other dimensions such as eligibility across lending facilities and collateral management.

## 8 Conclusion

In this paper, we provided a macroprudential approach to the analysis of optimal lending and collateral policy for the lender of last resort, a policy issue that dates back to Thornton (1802) and Bagehot (1873). The main idea in our approach is that the central bank's lending policy imposes externalities on private markets by affecting the pool of collateral and liquidity creation in these markets. By lending to individual banks, the central bank insures them against liquidity risk and spurs investment by those banks. In addition, lending against high-quality collateral has the advantage of protecting the central bank against potential losses. However, this has an adverse impact on the pool of collateral left in private markets, since high-quality collateral stays on the central bank's balance sheet rather than circulating and creating liquidity in private markets. This, in turn, can offset the positive effect generated by the investment of banks that directly borrow from the central bank. Furthermore, when the pool of collateral in private markets deteriorates sufficiently, liquidity risk increases to the point where banks refrain from investment in the first place. Hence, the central bank, by extracting high-quality collateral from private markets, can trigger a freeze in the markets that it tries to ensure their functionality.

On the other hand, lending against low-quality collateral can expose the central bank to potential losses, but it improves the pool of collateral in private markets. This reduces liquidity risk for banks that borrow in private markets and has a positive effect on output. Furthermore, when markets are frozen, by lending to a sufficiently large number of banks against low-quality collateral, the central bank can jump start these markets. This would require the central bank to go beyond the marginal cost-benefit analysis and take into consideration the discontinuous effects resulting from restoring private markets.

Hence, to get a better understanding of central banks' lender of last resort policy, we should go beyond the marginal cost-benefit analysis of a microprudential approach and consider a macroprudential approach, taking into account both the externalities that central bank lending imposes on markets and the discontinuous effects that this lending may engender. In this paper, we limit our focus to central banks' lender of last resort policy, but the framework we develop could be used more generally to consider, for example, the implications for the implementation of monetary policy.

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# Appendix: Ex-post central bank lending

Throughout the paper, we assumed that the central bank lends at $t = 0$ after the probability $\rho$ is realized, instead of lending at $t = 1$ after the liquidity shock $\ell$. We chose this timing so that we could examine the effects of central bank lending on private market functionalities going forward. Furthermore, as is clear from our multi-period extension of Section 6.1, in a dynamic setting, the distinction between ex-ante and ex-post lending becomes blurred, since any action by the central bank will have an effect in future periods. Nevertheless, we now discuss lending by the central bank at $t = 1$ and show that our main results would go through.

Suppose that the central bank announces its lending policy at the beginning of $t = 0$, but lends at $t = 1$ after $B_0$ banks' liquidity shock $\ell$ realize. Note that $B_0$ banks do not have any incentives to borrow from the central bank because of the penalty rate $\gamma_j$. Rather, $B_0$ banks call back the loan from $A$ banks, since, that way, $B_0$ banks are guaranteed to be paid in full. In that case, $A$ banks get back the collateral they pledged to $B_0$ banks at $t = 0$ and use it to borrow from the central bank.

We first start with the case where all $B_0$ banks are hit by the same liquidity shock $\ell$ as in the benchmark case. This way, we do not need to consider interbank lending among $B_0$ banks at $t = 1$. We analyze the case with idiosyncratic liquidity shocks later on.

We focus on the case where $A$ banks invest at $t = 0$, since, otherwise, there are no projects to be liquidated at $t = 1$ and central bank lending would be irrelevant. We focus on the marginal effect first. We show that when the central bank lends against low-quality collateral, the effect on output is the same as in the benchmark case. However, when the central bank lends against high-quality collateral, banks borrow from the central banks only when markets are frozen. Therefore, even though central bank lending lowers the quality of collateral in private markets, it does not lead to any actual effect on output, since the market is already frozen and only banks that borrow from the central bank invest.

We begin with the case where the central bank lends to a measure $x_L$ of banks against low-quality collateral. In this case, the average quality of collateral in the market is $c_L^{CB}$ as in the benchmark case of Section 4.

Note that with probability $1 - \rho$, $B_0$ banks are matched with $B_1$ banks so that they can borrow to meet their liquidity needs and there is no early liquidation. However, with probability $\rho$, there is no matching with $B_1$ banks and $B_0$ banks go to outsiders for funds. For $\ell < c_1$, $B_0$ banks can borrow from outsiders so that there is no early liquidation. For $c_1 < \ell < c_L^{CB}$, a measure $x_L$ of banks borrow from the central bank, and the rest can borrow from outsiders. However, for $\ell > c_L^{CB}$, while a measure $x_L$ of banks borrow from the central bank, the rest cannot borrow from outsiders so that $A$ banks' projects are liquidated.

Hence, the expected output can be written as

$$Y_L = x_L(R-1) + (1-x_L)\left[(R-1) - \rho(1-c_L^{CB})(R-r)\Delta\right] + 2R^B,$$

the same as in equation (4) in our benchmark case.

Next, we analyze the case when the central bank lends against high-quality collateral. When the central bank lends to a measure $x_H$ of banks against high-quality collateral, the average quality of collateral in the market becomes $c_H^{CB}$ as in the benchmark case of Section 4. Note that $c_H^{CB} < c_1$, since the quality of collateral in the market deteriorates.

Note that, with probability $1-\rho$, $B_0$ banks are matched with $B_1$ banks and there is no early liquidation. However, with probability $\rho$, there is no matching with $B_1$ banks. In that case, for $\ell < c_1$, $B_0$ banks can borrow from outsiders and there is no early liquidation. For $\ell > c_1$, a measure $x_H$ of banks with high-quality collateral borrow from the central bank but the rest cannot borrow from outsiders, since $c_H^{CB} < c_1$ as central bank lending leads to a deterioration in the quality of collateral in the market.

In other words, for $\ell < c_1$, $B_0$ banks can borrow from outsiders, they do not call back the loan, and no one needs to borrow from the central bank. However, for $\ell > c_1$, $B_0$ banks cannot borrow from outsiders; that is, the private market is frozen already and the central bank, by lending against high-quality collateral, only lowers the quality of collateral in the market without having any actual effect on output.

Hence, in this case, the central bank lends against high-quality collateral only when markets are frozen and we would only observe the direct effect this has on banks that borrow from the central bank, similar to the policy suggested by Bagehot as in our benchmark case.

**Idiosyncratic liquidity shocks**

Suppose that $B_0$ banks experience idiosyncratic liquidity shocks at $t = 1$.[28] We analyze the marginal effects when all $A$ banks are investing at $t = 0$. We first analyze the case where the central bank lends against low-quality collateral and then the case where it lends against high-quality collateral.

When the central bank lends to a measure $x_L$ of banks with low-quality collateral, the average quality of collateral in the market becomes $c_L^{CB}$ as before. Now, suppose that the central bank lends to the banks that have difficulty borrowing in the private market, that is, the banks with a liquidity shock $\ell > c_L^{CB}$. Again, with probability $1-\rho$, $B_0$ banks are matched with $B_1$ banks and there is no early liquidation. However, with probability $\rho$, there

[28]Note that $B_0$ banks with low liquidity shocks have spare debt capacity. They can borrow from outsiders and lend the excess liquidity they have to $B_0$ banks with high liquidity shocks. This would not change our results qualitatively. To keep the analysis simple, we do not focus on this case.

is no matching with $B_1$ banks. In that case, $B_0$ banks with a liquidity shock $\ell \leqslant c_L^{CB}$ can borrow from outsiders. However, $B_0$ banks with high liquidity shocks $\ell > c_L^{CB}$ cannot borrow from outsiders and a measure $x_L$ of them borrow from the central bank. Hence, a measure $(1 - c_L^{CB} - x_L)$ cannot borrow from the central bank or from outsiders, and they call back the loan, leading to an early liquidation of $A$ banks' projects. In this case, output can be written as

$$Y_L = (R-1) - \rho\left[1 - c_L^{CB} - x_L\right](R-r)\Delta + 2R^B.$$

Hence, the effect of central bank lending on output can be written as:

$$\begin{aligned}\frac{\partial Y_L}{\partial x_L} &= \rho(R-r)\Delta\left[1 + \frac{\partial c_L^{CB}}{\partial x_L}\right] \\ &= \rho(R-r)\Delta\left[1 + \frac{(1-\alpha)(c_H - c_L)}{(1-x_L)^2}\right].\end{aligned}$$

Note that both the direct effect through the banks that borrow from the central bank and the indirect effect, which works through improved collateral quality in markets, are positive, and are stronger in this case, compared to $\frac{\partial Y_L}{\partial x_L}$ for the benchmark case. In this case, the central bank lends only to those banks whose projects would otherwise be liquidated, whereas in the benchmark case where it lends at $t = 0$, the central bank ends up lending to banks that might have had a low liquidity shock $\ell \leqslant c_L^{CB}$ and might have been able to borrow in private markets at $t = 1$. Hence, the positive effect of central bank lending is stronger in this case.

When the central bank lends to a measure $x_H$ of banks with high-quality collateral, the average quality of collateral in the market is $c_H^{CB}$ as before. Now, suppose that the central bank lends to the banks with a liquidity shock $\ell > c_H^{CB}$. Again, with probability $1 - \rho$, there is no early liquidation. However, with probability $\rho$, there is no matching with $B_1$ banks. In that case, $B_0$ banks with a liquidity shock $\ell < c_H^{CB}$ can borrow from outsiders. However, $B_0$ banks with high liquidity shocks $\ell > c_H^{CB}$ cannot borrow from outsiders and a measure $x_H$ of them borrow from the central bank. Hence, a measure $(1 - c_H^{CB} - x_H)$ of loans are called back, leading to an early liquidation of $A$ banks' projects. In this case, output can be written as

$$Y_H = (R-1) - \rho\left[1 - c_H^{CB} - x_H\right](R-r)\Delta + 2R^B.$$

Hence, the effect of central bank lending on output can be written as:

$$\frac{\partial Y_H}{\partial x_H} = \rho(R-r)\Delta\left[1 - \frac{(1-\alpha)(c_H - c_L)}{(1-x_H)^2}\right].$$

Note that both the direct effect, which is positive, and the indirect effect, which is negative, are stronger in this case, compared to $\frac{\partial Y_H}{\partial x_H}$ for the benchmark case. Hence, the overall negative effect we received in the benchmark case is not guaranteed. However, a sufficient condition for $\frac{\partial Y_H}{\partial x_H} < 0$ is $c_H - c_L > \frac{1}{\alpha}$; that is, when the difference between the values of high- and low-quality collateral is sufficiently large, lending against high-quality collateral would have a negative effect on output, as in the benchmark case. Again, in this case, the central bank lends only to those banks whose projects would otherwise have been liquidated so that the positive effect of central bank lending is stronger.