Abstract

How should governments choose tax rates when they face competition from other countries? To answer this question, I build a two country, dynamic general equilibrium, open economy model. An important aspect of the analysis is the endogenous determination of the real exchange rate arising from each country producing a distinct good. In the open economy setting, an endogenous real exchange rate has been shown to be important in the optimal taxation literature. As is typical in the optimal taxation literature, I rewrite the planners’ problems as directly choosing an allocation subject to feasibility, an implementability condition, an international solvency condition, and an international risk-sharing condition. Tax rates are the Nash equilibrium responses of the game played between the two Ramsey planners. In addition, each planner tries to manipulate the real exchange rate in favor of its own country. I find that the planners choose very high capital income tax rates in the short term, and zero rates in the long run (this last result being shown analytically) – results reminiscent of the closed economy optimal taxation literature. In other words, there is no “race to the bottom” with respect to capital income taxation. I compare the tax competition case to one with tax cooperation, I also analyze a model with a single world planner. When countries cooperate, they choose higher capital income taxation in the short run, but again set capital taxes to zero in the long term.
1 Introduction

Governments around the world compete with respect to taxes – chiefly capital income (corporate) taxation. Within Europe, Ireland is known for its low corporate income tax rate. In the U.S., the Tax Cuts and Job Act of 2017 lowered the corporate tax rate from 35% to 21%, and switched from a global to a territorial tax system – both changes arguably designed to bring U.S. corporate taxation policy into line with that of other countries. The question addressed in this paper is: How should benevolent governments choose tax rates in the face of tax competition from other countries?

To answer this tax competition question, I develop a two country open economy model featuring strategic interaction between Ramsey planners. Each country produces its own distinct good; since consumers in each country enjoy utility from both its own good as well as the foreign good, the real exchange rate is endogenously determined (the importance of this feature is discussed shortly). The government in each country levies taxes on labor and capital income as well as on consumption taxes. As is typical in the closed economy
Ramsey literature, at an initial date, each government fully commits to time paths for its tax rates given competitive equilibrium, and in addition the actions of the other government (its choice of taxes). In other words, paths for the tax rates in the two countries are the Nash equilibrium responses to a game played between the two Ramsey planners. The model is described in section 2 while details of the Ramsey tax competition are presented in section 3.

The usual practice in the Ramsey optimal taxation literature is to develop an implementability condition (Lucas and Stokey, 1983) that subsumes private sector equilibrium behavior; the planner then chooses an allocation directly (see Chamley (1986), Judd (1985), and the subsequent literature). The open economy introduces a pair of additional considerations. First, an international risk-sharing condition ensures a determinate international distribution of wealth. Second, the sequence of trade balances implied by the government’s actions must satisfy an international solvency condition (Auray, Eyquem, and Gomme, 2018). In the initial period, each planner chooses an allocation for its economy and a sequence for the real exchange rate subject to an implementability condition, international solvency, a sequence of international risk-sharing conditions, feasibility, and given the choices of the other Ramsey planner. Notice that, in addition to the strategic interaction arising from their choices of tax rates, the two governments also try to manipulate the real exchange rate in favor of their residents.

I show, analytically, that in the long run, capital income tax rates are zero, just as they are in closed economy Ramsey models. This result shows that there is no “race to the bottom” with respect to capital income taxation. Other results are obtained computationally. In the short term, capital income is taxed at very high rates – again, a result reminiscent of the closed economy literature. I find that in the short term, labor income is actually subsidized, while it is taxed at a positive rate in the long run. The open economy introduces an additional margin: differential taxation of imported versus locally produced goods.

To gain some more insight into the role played by dynamic competition, the results from the tax competition model are compared to a cooperative outcome. In this latter case, a world
Ramsey planner chooses tax rates for both countries to maximize population-weighted sum of the lifetime utilities of consumers in the two countries. The chief difference is that the cooperative outcome calls for even higher capital income tax rates in the short term than the tax competition model. In other words, tax competition leads to somewhat lower capital income tax rates in the short run, while both environments call for zero capital income taxation in the long run.

My paper is broadly related to the Ramsey optimal taxation literature initiated by Chamley (1986), Judd (1985). Correia (1996) was the first to study Ramsey taxation in a small open economy setting. She assumed that there was only one good which fixed the real exchange rate at one. Correia found that the capital income tax rate should immediately fall to zero (and stay there) – much different from the findings of the closed economy Ramsey literature. More recently, Auray, Eyquem, and Gomme (2018) looked at the more general setting with domestic- and foreign-produced goods – and so an endogenously determined real exchange rate – in a small open economy. They find that the dynamics of the capital income tax rate more closely resembles that of a closed economy model (initially very high, then zero) than Correia’s one good small open economy model (zero immediately). Auray, Eyquem, and Gomme attribute the difference to the endogenously-determined real exchange rate in their model which looses a no-arbitrage condition on the return to capital vis-à-vis net foreign assets.

My paper is more closely related to the dynamic tax competition literature. Mendoza and Tesar (2005) is an early and influential work in this area. They too solve for tax rates as the Nash equilibrium of a game between two governments in an open economy setting. They too find no race to the bottom. However, Mendoza and Tesar assume that tax rates are constant starting in the initial period. In choosing its (time invariant) capital income tax rate, the government adjusts either the labor income tax rate or the consumption tax rate to satisfy the present value government budget constraint. In brief, Mendoza and Tesar do not solve Ramsey optimal taxation problems.
More recently, Gross, Klein, and Makris (2017) have addressed this deficiency in Mendoza and Tesar (2005). Specifically, they solve for the Nash equilibrium of the tax competition game played between two Ramsey planners. Unlike Mendoza and Tesar, in Gross, Klein, and Makris the paths for tax rates vary over time. Interestingly, they find that the optimal paths for the capital income tax rate gradually and monotonically declines to zero. They too find that there is no “race to the bottom.” However, as in Mendoza and Tesar, Gross, Klein, and Makris analyze an environment in which both countries produce the same good which necessarily fixes the real exchange rate at one. In light of the differences in the path of capital income tax rates in small open economies between exogenously- (Correia, 1996) and endogenously-determined (Auray, Eyquem, and Gomme, 2018) real exchange rates, the one good assumption in Gross, Klein, and Makris may be too restrictive, and the time paths of capital income tax rates in their model may be an artefact of the exogenous real exchange rate.

The two country open economy model is described in section 2. The Ramsey problem is developed in section 3, including the implementability conditions and international solvency. The model is calibrated in section 4, and numerical results for the Ramsey plan presented in section 5. Some final remarks are in section 6.

2 Economic Environment

The “world” consists of two large open economies. The home country is of size $n$ while the foreign country is of size $1 - n$. Each country is populated by a continuum of infinitely lived households, a continuum of firms, and a benevolent government. Except where noted, the two countries are symmetric and so attention is focused on the home country; foreign variables are distinguished by an asterisk superscript.
2.1 Households

At the start of a period, the household holds physical (home) capital, $k_{t-1}$, home government debt, $d_{t-1}$, and net foreign assets, $a_{t-1}$. It is well known that stability of open economy models requires a closing assumption. Here, complete international asset markets do the trick. As a result, net foreign assets are, conceptually, state contingent bonds, and the restriction that only home households hold home capital and home government debt is innocuous.

The representative household in the home country receives utility from consumption of its own good, $c_{ht}$, consumption of the foreign good, $c_{ft}$, and disutility from working, $h_t$. The household’s Lagrangian is

$$
L = \max_{c_t, h_t, k_t, d_t, a_t} \sum_{t=0}^{\infty} \beta^t \left\{ U(c_{ht}, c_{ft}, h_t) + \xi_t \left[ (1 - \tau_{wt}) w_t h_t + R_{kt} k_{t-1} + d_{t-1} + e_t a_{t-1} - (1 + \tau_{ct}) c_{ht} - (1 + \tau_{mt}) e_t c_{ft} - k_t - \frac{d_t}{R_{dt}} - \frac{a_t}{R_{at}} \right] \right\}
$$

(1)

with $R_{kt} \equiv 1 - \delta+(1-\tau_{kt})r_t$, $\tau_{wt}$, $\tau_{kt}$ are factor income tax rates; and $\tau_{ct}$ and $\tau_{mt}$ are tax rates on the locally produced consumption goods and imports, respectively; $w_t$ and $r_t$ are factor prices; $e_t$ is the real exchange rate, expressed as the number of units of domestic output per unit of foreign output; $R_{dt}$ and $R_{at}$ are, respectively, the gross real return to (domestic) government debt and net foreign assets. In (1), net foreign assets are denominated in units of foreign output, hence the real exchange rate terms; in the foreign household budget constraint, the real exchange rate terms associated with net foreign assets do not appear.

The household’s first-order conditions are:

$$
c_{ht} : \quad U_1(c_{ht}, c_{ft}, h_t) = \xi_t(1 + \tau_{ct})
$$

(2)

$$
c_{ft} : \quad U_2(c_{ht}, c_{ft}, h_t) = \xi_t(1 + \tau_{mt})e_t
$$

(3)

$$
h_t : \quad U_3(c_{ht}, c_{ft}, h_t) + \xi_t(1 - \tau_{wt})w_t = 0
$$

(4)

$$
k_t : \quad \xi_t = \beta \xi_{t+1} R_{k,t+1}
$$

(5)
\[ d_t : \frac{\xi_t}{R_{dt}} = \beta \xi_{t+1} \]  
\[ a_t : \frac{e_t \xi_t}{R_{at}} = \beta e_{t+1} \xi_{t+1} \] 

Later, the set of no-arbitrage conditions on returns implied by (5)–(7) will prove useful:

\[ R_{k,t+1} = R_{dt} = R_{at}. \] 

The problem of the foreign household is presented in full in Appendix A.1.

2.2 Domestic Firms

Goods producing firms are perfectly competitive and face a sequence of static profit maximization problems:

\[ \max_{k_{t-1}, h_t} \{ F(k_{t-1}, h_t) - r_t k_{t-1} - w_t h_t \}. \] 

The associated first-order conditions are

\[ r_t = F_1(k_{t-1}, h_t) \text{ and } w_t = F_2(k_{t-1}, h_t). \] 

2.3 Government

The Ramsey problem will be considered shortly. For now, it suffices to note that the government faces a sequence of budget constraints,

\[ \frac{d_t}{R_{dt}} - d_{t-1} = \tau c_t c_{ht} + \tau m_t e_t c_{ft} + \tau w_t w_t h_t + \tau r_t r_t k_{t-1} - g \] 

where the term on the right-hand side is the government primary deficit. The government is also subject to the usual transversality condition on its debt.

Alternatively, (11) can be solved forward; applying the transversality condition then delivers the present value form on the government budget constraint,

\[ d_{t-1} + \sum_{t=0}^{\infty} \left( \prod_{j<0} \frac{1}{R_{dj}} \right) \PrDef_t = 0. \]
A feasible fiscal policy consists of a path for the tax rates that satisfies the present value constraint, (12).

2.4 Balance of Payments

The home country balance of payments equation is

\[ \frac{1 - n}{n} c_{ht} - e_t c_{ft} + e_t a_{t-1} - \frac{e_t a_t}{R_{at}} = 0 \]  

where the first two terms constitute the trade balance. Solving (13) forward and applying the transversality on home net foreign assets delivers an international solvency condition,

\[ a_{-1} + \sum_{t=0}^{\infty} \left( \prod_{j<t} \frac{1}{R_{aj}} \right) \frac{TB_t}{e_t} = 0. \]  

2.5 Competitive Equilibrium

Given a feasible home fiscal policy, \( \{\tau_{ct}, \tau_{mt}, \tau_{wt}, \tau_{kt}\}_{t=0}^{\infty} \), and a feasible foreign fiscal policy, \( \{\tau_{ct}^*, \tau_{mt}^*, \tau_{wt}^*, \tau_{kt}^*\}_{t=0}^{\infty} \), a competitive equilibrium is given by a set of home quantities, \( \{c_{ht}, c_{ft}, h_t, k_t, d_t, a_t\}_{t=0}^{\infty} \), a set of foreign quantities, \( \{c_{ht}^*, c_{ft}^*, h_t^*, k_t^*, d_t^*, a_t^*\}_{t=0}^{\infty} \), prices, \( \{r_t, w_t, r_t^*, w_t^*, e_t\}_{t=0}^{\infty} \), and interest rates \( \{R_{dt}, R_{dt}^*, R_{at}\}_{t=0}^{\infty} \) such that

1. The quantities for the home households solve their problem given prices and government policy; the same for foreign households.

2. The quantities for the home firms solve their problems given prices; the same for foreign firms.

3. The international risk-sharing condition holds:

\[ e_t \xi_t = \varphi \xi_t^*. \]  

\footnote{The international risk-sharing condition is obtained by combining the home first-order condition for net foreign assets, (7), with the corresponding equation for foreign households, then iterating back in time. \( \varphi \) is a constant given by initial conditions, and typically involves a ratio of the marginal utility of consumption for home households to that of foreign households.}
4. Markets clear: goods markets (recall that $n$ is the size of the home country, and so $1 - n$ the size of the foreign country)

\[ c_{ht} + \frac{1 - n}{n} c^*_t + k_t + g = F(k_{t-1}, h_t) + (1 - \delta)k_{t-1} \quad (16) \]

\[ c^*_{ft} + \frac{n}{1 - n} c_{ft} + k^*_t + g^* = F(k^*_{t-1}, h^*_t) + (1 - \delta)k^*_{t-1} \quad (17) \]

standard factor market clearing conditions, adn the balance of payments condition, (14). Given the present value constraints, (12) and (14) (and corresponding constraints for the foreign economy), there is no need to include bond market clearing conditions.

3 The Ramsey Problem

Relative to the closed economy setting, the international dimension introduces a number of additional considerations. The remainder of this section works through these issues.

As in Mendoza and Tesar (2005) and Gross, Klein, and Makris (2017) both planners are assumed to fully and credibly commit to tax policies starting from $t = 0$; the interaction between the benevolent government planners is modeled as a once-and-for-all game at time $t = 0$; and attention is focused on the Nash equilibrium of this game. So that the government’s taxation problem to be “interesting”, the initial capital income tax rate is fixed by history; absent this restriction, the government would levy a sufficiently large capital income tax rate that it would never need to raise revenue in the future. Following the usual in the Ramsey literature, it is convenient to think of each planner as choosing not only a feasible tax policy, but also the relevant quantities and prices subject to private sector optimization. More specifically, given the actions of the foreign planner, the home planner chooses: a feasible fiscal plan and \{c_{ht}, c_{ft}, h_t, k_t, r_t, w_t, e_t\}_{t=0}^{\infty} subject to (2)–(5), (10), (12) and (14)–(16) with $R_{kt} = (1 - \tau_{kt})F_1(k_{t-1}, h_t) + 1 - \delta$, and $R_{dt}$ and $R_{st}$ satisfying the return arbitrage conditions in (8). The foreign planner solves an analogous problem. Notice that both planners try to manipulate the real exchange rate, $e_t$, and that both are subject an international
risk-sharing condition ((14) in the case of the home government).

The chief difference between the Ramsey problem described above and the problem studied in Mendoza and Tesar (2005) is that they assume that tax rates are constant starting at $t = 0$, the beginning of the planning horizon, while the Ramsey problem above allows for a time-varying path for tax rates. In short, Mendoza and Tesar (2005) do not solve a Ramsey problem.

While Gross, Klein, and Makris (2017) solve a Ramsey problem, there are conceptual differences relative to the Ramsey problem solved here. In particular, they assume that the home planner also chooses private sector foreign variables (subject, of course, to a set of equations describing optimizing behavior by foreign households and firms). In the end, this is probably more a difference in style rather than substance. After all, the home planner in Gross, Klein, and Makris (2017) takes as given the actions of the foreign planner, and vice versa. In the end, both planners are choosing best responses, and the only question is whether private sector behavior in the other country should be internalized by the local planner (as in Gross, Klein, and Makris (2017)), or left external to the the local planner (as above).

As discussed in the Introduction, the model in Gross, Klein, and Makris (2017) is similar to that of Mendoza and Tesar (2005) in that the home and foreign economies produce the same good which necessarily fixes the real exchange rate at one. An important difference in my analysis is that each economy produces a unique good which leads to an endogenously determined real exchange rate. Auray, Eyquem, and Gomme (2018) point to the importance of an endogenous real exchange rate in an small open economy Ramsey problem; again, see the discussion in the Introduction.
3.1 The Implementability Condition

Substituting the home household’s first-order conditions, (2)–(7) into the second term in the household’s Lagrangian, namely,
\[
\sum_{t=0}^{\infty} \beta^t \xi_t \left[ (1 - \tau_{wt}) w_t h_t + R_k k_{t-1} + d_{t-1} + e_t a_{t-1} - (1 + \tau_{et}) p_t c_t - k_t - \frac{d_t}{R_{dt}} - e_t a_t \right]
\]
yields the familiar implementability condition,
\[
\sum_{t=0}^{\infty} \beta^t \left[ U_1(c_{ht}, c_{ft}, h_t) c_{ht} + U_2(c_{ht}, c_{ft}, h_t) c_{ft} + U_3(c_{ht}, c_{ft}, h_t) h_t \right] = \xi_0 [R_k k_{-1} + d_{-1} + e_0 a_{-1}]
\]
(18)

3.2 International Solvency

Solving the Ramsey problem is easier if the product of the interest rate terms in the earlier international solvency condition, (14), is eliminated. This task is accomplished by using the household’s first-order condition with respect to net foreign assets, (7):
\[
e_0 \xi_0 a_{-1} + \sum_{t=0}^{\infty} \beta^t \xi_t T_{B_t} = 0
\]
(19)

where \( \xi_t \) is the Lagrange multiplier associated with the home household’s budget constraint.

3.3 The Home Planner’s Problem

Finally, the home Ramsey planner’s problem can be cast: taking as given the actions of the foreign Ramsey planner, choose \( \{ c_{ht}, c_{ft}, h_t, k_t, e_t \}_{t=0}^{\infty} \) to maximize lifetime utility of the representative home household subject to the international risk-sharing condition, (15), home feasibility, (16), the home implementability condition, (18) and the international solvency condition, (19). The government has an excess of tax instruments at its disposal. In particular, eliminating the common Lagrange multiplier from the home household’s first-order conditions, (2)–(5), leaves three equations that can be used to determine three of the four tax rates. One of the four taxes must be set exogenously; to this end, assume that the
consumption tax \((\tau_{ct})\) is constant. In this case, use (2) to solve out for the multiplier \(\xi_t\); the home planner’s Lagrangian is

\[
\max_{\{c_{ht}, c_{ft}, h_t, k_t, e_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left\{ W(c_{ht}, c_{ft}, h_t) \right. \\
+ \lambda_{1t} \left[ F(k_{t-1}, h_t) + (1 - \delta)k_{t-1} - c_{ht} - \frac{1 - n}{n}c_{ht}^* - k_t - g \right] \\
+ \lambda_{2t} \left[ U_1(c_{ht}, c_{ft}, h_t) - \frac{U_2^*(c_{ht}^*, c_{ft}^*, h_t^*)}{1 + \tau_c^*} \right] \\
+ \Omega U_1(c_{ht}, c_{ft}, h_t) \left[ \frac{1 - n}{n}c_{ht}^* - e_t c_{ft} \right] \left\} \right.
\]

\[+ \Omega e_0 \frac{U_1(c_{ht}, c_{ft}, h_t)}{1 + \tau_c} \frac{a_{-1}}{1 + \tau_c} \]

\[= \Lambda \left[ (1 - \delta + (1 - \tau_{k0})F_1(k_{-1}, h_0))k_{-1} + d_{-1} + e_0 a_{-1} \right] \]

with

\[W(c_{ht}, c_{ft}, h_t) \equiv U(c_{ht}, c_{ft}, h_t) + \Lambda [U_1(c_{ht}, c_{ft}, h_t)c_{ht} + U_2(c_{ht}, c_{ft}, h_t)c_{ft} + U_3(c_{ht}, c_{ft}, h_t)h_t] \]

where \(\lambda_{1t}\) is the multiplier associated with the home goods market condition, \(\lambda_{2t}\) the multiplier on the international risk-sharing condition, \(\Omega\) is the multiplier for the international solvency condition, and \(\Lambda\) is the multiplier associated with the implementability condition. The foreign Ramsey planner solves a similar problem; see Appendix A.3 for its problem in full.

The first-order conditions for \(t > 0\)

\[c_{ht} : \quad W_1(c_{ht}, c_{ft}, h_t) - \lambda_{1t} + \lambda_{2t} e_t \frac{U_1(c_{ht}, c_{ft}, h_t)}{1 + \tau_c} + \Omega \frac{U_1(c_{ht}, c_{ft}, h_t)}{1 + \tau_c} \text{TB}_t = 0 \]

\[c_{ft} : \quad W_2(c_{ht}, c_{ft}, h_t) + \lambda_{2t} e_t \frac{U_2(c_{ht}, c_{ft}, h_t)}{1 + \tau_c} + \frac{U_1(c_{ht}, c_{ft}, h_t)}{1 + \tau_c} \text{TB}_t = 0 \]

\[\frac{\Omega U_1(c_{ht}, c_{ft}, h_t)}{1 + \tau_c} = 0 \]
and the first-order conditions for the initial period \((t = 0)\)

\[ c_{h0} : \quad W_1(c_{h0}, c_{f0}, h_0) - \lambda_{10} + \lambda_{20}c_0\frac{U_{11}(c_{h0}, c_{f0}, h_0)}{1 + \tau_c} \]
\[ + \Omega \frac{U_{13}(c_{h0}, c_{f0}, h_0)}{1 + \tau_c} TB_t = 0 \]  

\[ k_0 : \quad - \lambda_{11} + \beta\lambda_{11} [F_1(k_0, h_1) + 1 - \delta] = 0 \]  

\[ e_0 : \quad \lambda_{20}c_0\frac{U_1(c_{h0}, c_{f0}, h_0)}{1 + \tau_c} - \Omega \frac{U_1(c_{h0}, c_{f0}, h_0)}{1 + \tau_c} c_{f0} + \Omega \frac{U_1(c_{h0}, c_{f0}, h_0)}{1 + \tau_c} a_{-1} \]
\[ - \Lambda \frac{U_1(c_{h0}, c_{f0}, h_0)}{1 + \tau_c} a_{-1} = 0 \]

where

\[ A_0 \equiv [1 - \delta + (1 - \tau_{k0})F_1(k_{-1}, h_{-1})]k_{-1} + d_{-1} + e_0a_{-1} \]

is the initial value of household wealth and

\[ TB_t = \frac{1 - n}{n} e_{ht} - \varepsilon_t c_{ft} \]

is, again, the trade balance.
Given the allocation that solves the above Ramsey problem, the paths of taxes, \( \{\tau_{mt}, \tau_{wt}, \tau_{k,t+1}\} \), are obtained from

\[
\frac{U_1(c_{ht}, c_{ft}, h_t)}{U_2(c_{ht}, c_{ft}, h_t)} = \frac{1 + \tau_{mt}}{1 + \tau_c} e_t
\]  

\[
U_3(c_{ht}, c_{ft}, h_t) + \frac{1 - \tau_{wt}}{1 + \tau_c} U_1(c_{ht}, c_{ft}, h_t) F_2(k_{t-1}, h_t)
\]

\[
U_1(c_{ht}, c_{ft}, h_t) = \beta U_1(c_{ht+1}, c_{ft+1}, h_{t+1}) [1 - \delta + (1 - \tau_{k,t+1}) F_1(k_t, h_{t+1})]
\]

which, in turn, are obtained by combining the households’ first-order conditions with those of the firms.

Assuming that the economy eventually settles into a steady state, (35) reads

\[
1 = \beta [(1 - \tau_k) F_1(k, h) + 1 - \delta].
\]  

(36)

For the planner, the relevant steady state condition comes from the planner’s first-order condition with respect to capital, (25):

\[
1 = \beta [F_1(k, h) + 1 - \delta].
\]  

(37)

Comparing (36) and (37) delivers the familiar Chamley (1986)–Judd (1985) prescription: in the long run, capital income should not be taxed.

### 3.4 International Tax Cooperation

Below, the tax competition model is compared to an environment with international tax cooperation. In this latter case, a world Ramsey planner maximizes a population-weighted sum of the utility of representative households in the two countries subject to feasibility, international risk sharing, the two international solvency conditions, and the two implementability
conditions. The full problem is:

\[
\max_{\{c_{ht}, c_{ft}, h_t, k_t, c_{ht}^*, c_{ft}^*, h_t^*\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left\{ nW(c_{ht}, c_{ft}, h_t) + (1 - n)W^*(c_{ht}^*, c_{ft}^*, h_t^*) 
\right. \\
+ \lambda_{1t} \left[ F(k_{t-1}, h_t) + (1 - \delta)k_{t-1} - c_{ht} - \frac{1 - n}{n}c_{ht} - k_t - g \right] \\
+ \lambda_{2t} \left[ F(k_{t-1}^*, h_t^*) + (1 - \delta)k_{t-1}^* - c_{ft}^* - \frac{n}{1 - n}c_{ft} - k_t^* - g^* \right] \\
+ \lambda_{2t} \left[ e_t \frac{U_1(c_{ht}, c_{ft}, h_t)}{1 + \tau_c} - \varphi \frac{U_2^*(c_{ht}^*, c_{ft}, h_t^*)}{1 + \tau_c^*} \right] \\
+ \Omega \left[ \frac{U_1(c_{ht}, c_{ft}, h_t)}{1 + \tau_c} \right] \left[ \frac{1 - n}{n}c_{ht} - e_t c_{ft} \right] \\
+ \Omega^* \left[ \frac{U_2^*(c_{ht}^*, c_{ft}, h_t^*)}{1 + \tau_c^*} \right] \left[ \frac{n}{1 - n}c_{ft} - \frac{c_{ht}^*}{e_t} \right] \right\} \\
+ \Omega \epsilon_0 \frac{U_1(c_{h0}, c_{f0}, h_0)}{1 + \tau_c} a_{-1} \\
+ \Omega^* \frac{U_2^*(c_{h0}^*, c_{f0}^*, h_0^*)}{1 + \tau_c^*} a_{-1}^* \\
- \Lambda \frac{U_1(c_{h0}, c_{f0}, h_0)}{1 + \tau_c} \left[ (1 - \delta + (1 - \tau_{k0})F_1(k_{-1}, h_0))k_{-1} + d_{-1} + \epsilon_0 a_{-1} \right] \\
- \Lambda^* \frac{U_2^*(c_{h0}^*, c_{f0}^*, h_0^*)}{1 + \tau_c^*} \left[ (1 - \delta + (1 - \tau_{k0})F_1(k_{-1}^*, h_0^*))k_{-1}^* + d_{-1}^* + a_{-1}^* \right]
\]

where, recall, \( n \) is the size of the home economy. In other words, the world planner max

\section{Parameterization and Calibration}

Constant relative risk aversion utility

\[
U(c_h, c_f, h) = \frac{[C(c_h, c_f)(1 - h)^{\omega}]^{1-\sigma}}{1 - \sigma}
\]

with consumption aggregator

\[
C(c_h, c_f) = \left[ \varphi \frac{\mu - 1}{\mu} c_h^{\frac{\mu}{\mu - 1}} + (1 - \varphi) \frac{\mu - 1}{\mu} c_f^{\frac{\mu}{\mu - 1}} \right]^{\frac{\mu}{\mu - 1}}
\]
Cobb-Douglas production function

\[ F(k, h) = k^\alpha h^{1-\alpha} \]

Steady state tax rates \((\tau_c, \tau_m, \tau_w, \tau_k)\) are set exogenously as is the relative size of the home economy \((n)\), risk aversion \((\sigma)\), and the trade-related parameters \((\gamma, \mu)\). There is a one-to-one relationship between the targets for capital’s share of income, the capital depreciation rate, and the discount factor and the parameters \(\alpha, \delta, \beta\). The preference parameter \(\omega\) is chosen so that in steady state the model matches the target for the average fraction of time spent working. In addition, government’s share of output is to be matched; government debt is determined by the steady state version of the government present value budget constraint. The tax rates are taken from Mendoza and Tesar (2005) and correspond to the U.K. (home) and continental Europe (foreign). The calibrated parameter values are summarized in Table 1 (along with the auxiliary parameters, \(\varphi, \varphi^*, \text{ and } \vartheta\)), and the resulting steady state reported in Table 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Home</th>
<th>Foreign</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption tax, (\tau_c)</td>
<td>0.140</td>
<td>0.166</td>
</tr>
<tr>
<td>Import tax, (\tau_m)</td>
<td>0.140</td>
<td>0.166</td>
</tr>
<tr>
<td>Labor income tax, (\tau_w)</td>
<td>0.250</td>
<td>0.374</td>
</tr>
<tr>
<td>Capital income tax, (\tau_k)</td>
<td>0.530</td>
<td>0.265</td>
</tr>
<tr>
<td>Discount factor, (\beta)</td>
<td>0.962</td>
<td>0.962</td>
</tr>
<tr>
<td>Risk aversion, (\sigma)</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Leisure weight, (\omega)</td>
<td>1.549</td>
<td>1.297</td>
</tr>
<tr>
<td>Trade openness, (\gamma)</td>
<td>0.300</td>
<td>0.300</td>
</tr>
<tr>
<td>Trade elasticity, (\mu)</td>
<td>1.500</td>
<td>1.500</td>
</tr>
<tr>
<td>Capital share, (\alpha)</td>
<td>0.300</td>
<td>0.300</td>
</tr>
<tr>
<td>Capital depreciation, (\delta)</td>
<td>0.075</td>
<td>0.075</td>
</tr>
<tr>
<td>(\varphi)</td>
<td>0.850</td>
<td>0.150</td>
</tr>
<tr>
<td>(\vartheta)</td>
<td>1.181</td>
<td>1.181</td>
</tr>
<tr>
<td>size ((n \text{ or } 1-n))</td>
<td>0.500</td>
<td>0.500</td>
</tr>
</tbody>
</table>
Table 2: Initial Steady State

<table>
<thead>
<tr>
<th>Variable</th>
<th>Home</th>
<th>Foreign</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>0.327</td>
<td>0.397</td>
</tr>
<tr>
<td>Consumption: Home</td>
<td>0.193</td>
<td>0.040</td>
</tr>
<tr>
<td>Consumption: Foreign</td>
<td>0.034</td>
<td>0.228</td>
</tr>
<tr>
<td>Consumption: Aggregated</td>
<td>0.227</td>
<td>0.268</td>
</tr>
<tr>
<td>Hours worked</td>
<td>0.300</td>
<td>0.300</td>
</tr>
<tr>
<td>Capital stock</td>
<td>0.401</td>
<td>0.760</td>
</tr>
<tr>
<td>Capital-output ratio</td>
<td>1.226</td>
<td>1.917</td>
</tr>
<tr>
<td>Consumption tax</td>
<td>0.140</td>
<td>0.166</td>
</tr>
<tr>
<td>Gov. share</td>
<td>0.196</td>
<td>0.196</td>
</tr>
<tr>
<td>Tax Revenues</td>
<td>0.141</td>
<td>0.180</td>
</tr>
<tr>
<td>Primary deficit</td>
<td>−0.077</td>
<td>−0.102</td>
</tr>
<tr>
<td>Public debt</td>
<td>2.006</td>
<td>2.660</td>
</tr>
<tr>
<td>Debt-output ratio</td>
<td>6.126</td>
<td>6.709</td>
</tr>
<tr>
<td>Exchange rate</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Net Exports</td>
<td>0.006</td>
<td>−0.006</td>
</tr>
<tr>
<td>Net foreign assets</td>
<td>−0.160</td>
<td>0.160</td>
</tr>
</tbody>
</table>

5 The Ramsey Plan

Fig. 1 presents the tax competition case.

Fig. 2 presents the cooperative case.

6 Conclusion
Figure 1: Ramsey Tax Competition

(a) Output

(b) Consumption

(c) Hours

(d) Capital Income Tax

(e) Labor Income Tax

(f) Government Debt

(g) Capital

(h) Wage

(i) Interest Rate

(j) Net Foreign Assets

(k) Trade Balance

(l) Exchange Rate

(m) Labor Income Tax

(n) Capital Income Tax

(o) Primary Deficit (%)
Figure 2: Ramsey Tax Cooperation

(a) Output

(b) Consumption

(c) Hours

(d) Capital Income Tax

(e) Labor Income Tax

(f) Government Debt

(g) Capital

(h) Wage

(i) Interest Rate

(j) Net Foreign Assets

(k) Trade Balance

(l) Exchange Rate

(m) Labor Income Tax

(n) Capital Income Tax

(o) Primary Deficit (%)
A The Foreign Economy

A.1 Foreign households

\[ L^* = \max_{(c^*_{ht}, c^*_{ft}, h^*_t, k^*_t, d^*_t, a^*_t)} \sum_{t=0}^{\infty} \beta^t \left\{ U^*(c^*_{ht}, c^*_{ft}, h^*_t) 
+ \xi^*_t \left[ (1 - \tau^*_w) w^*_t h^*_t + R^*_k k^*_{t-1} + d^*_{t-1} + a^*_{t-1} - (1 + \tau^*_m) \frac{c^*_{ht}}{e_t} - (1 + \tau^*_c) c^*_{ft} - k^*_t \frac{d^*_t}{R^*_d} - \frac{a^*_t}{R^*_a} \right] \right\} \]

(39)

First-order conditions:

\[ c^*_h : \quad U_1^*(c^*_{ht}, c^*_{ft}, h^*_t) = \frac{\xi^*_t (1 + \tau^*_m)}{e_t} \]

\[ c^*_f : \quad U_2^*(c^*_{ht}, c^*_{ft}, h^*_t) = \xi^*_t (1 + \tau^*_c) \]

\[ h^*_t : \quad U_3^*(c^*_{ht}, c^*_{ft}, h^*_t) + \xi^*_t (1 - \tau^*_w) w^*_t = 0 \]

\[ k^*_t : \quad \xi^*_t = \beta \xi^*_{t+1} R^*_k k^*_{t+1} \]

\[ d^*_t : \quad \frac{\xi^*_t}{R^*_d} = \beta \xi^*_{t+1} \]

\[ a^*_t : \quad \frac{\xi^*_t}{R^*_a} = \beta \xi^*_{t+1} \]

Notes:

1. The real exchange rate, \( e_t \), is expressed as the number of units of the home good per foreign good.

2. International bonds are priced in units of the foreign good. Hence, for foreign households, government debt \( (d^*_t) \) and net foreign assets \( (a^*_t) \) are treated as equivalent.

A.2 Other Constraints

The foreign implementability condition:

\[ \sum_{t=0}^{\infty} \beta^t \left[ U_1^*(c^*_{ht}, c^*_{ft}, h^*_t) c_{ht} + U_2^*(c^*_{ht}, c^*_{ft}, h^*_t) c_{ft} + U_3^*(c^*_{ht}, c^*_{ft}, h^*_t) h_t \right] = \xi^*_{10} \left[ R^*_k k^*_{-1} + d^*_{-1} + a^*_{-1} \right] \]

(40)
where \( R_{kt}^* = (1 - \tau_{kt}^*) F_1(k_{t-1}^*, h_t^*) + 1 - \delta \).

The present value government budget constraint:

\[
d_{t-1}^* + \sum_{t=0}^\infty \left( \prod_{j<t} \frac{1}{R_{dj}^*} \right) \underbrace{\tau_{ft}^* c_{ft}^* + \tau_{mt}^* \frac{c_{ht}^*}{e_t} + \tau_{wt}^* w_t^* h_t^* + \tau_{kt}^* r_t^* k_{t-1}^* - g^*}_{\text{PnDeF}_t^*}.
\]

The foreign economy international solvency condition:

\[
a_{t-1}^* + \sum_{t=0}^\infty \left( \prod_{j<t} \frac{1}{R_{aj}^*} \right) \left( \frac{n}{1-n} c_{ft}^* - \frac{c_{ht}^*}{e_t} \right). \tag{42}
\]

A.3 The Foreign Planner’s Ramsey Problem

As for the home government, assume that the consumption tax, \( \tau_{ct}^* \), is fixed; recall that otherwise there is a surplus of tax rates to determine.

Again using the household’s first-order conditions for bond holdings simplifies the expressions for the two present value constraints. The foreign government’s Ramsey problem is:

\[
\max \left\{ \{c_{ht}^*, c_{ft}^*, h_t^* \} \right\} \sum_{t=0}^\infty \beta^t \left\{ W^*(c_{ht}^*, c_{ft}^*, h_t^*) \right. \\
+ \lambda_{1t}^* \left[ F(k_{t-1}^*, h_t^*) + (1 - \delta) k_{t-1}^* - c_{ft}^* - \frac{n}{1-n} c_{ft}^* - k_t^* - g^* \right] \\
+ \lambda_{2t}^* \left[ \varphi \frac{U_2^*(c_{ht}^*, c_{ft}^*, h_t^*)}{1 + \tau_c^*} - e_t \frac{U_1(c_{ht}^*, c_{ft}^*, h_t^*)}{1 + \tau_c^*} \right] \\
+ \Omega^* \frac{U_2^*(c_{ht}^*, c_{ft}^*, h_t^*)}{1 + \tau_c^*} \left[ \frac{n}{1-n} c_{ft}^* - \frac{c_{ht}^*}{e_t} \right] \right\} \\
+ \Omega^* \frac{U_2^*(c_{ht0}^*, c_{ft0}^*, h_{0t}^*)}{1 + \tau_c^*} a_{t-1}^* \\
- \Lambda^* \frac{U_2^*(c_{ht0}^*, c_{ft0}^*, h_{0t}^*)}{1 + \tau_c^*} \left[ (1 - \delta + (1 - \tau_{k0}^*) F_1(k_{t-1}^*, h_{0t}^*)) k_{t-1}^* + d_{t-1}^* + a_{t-1}^* \right]
\]

\[ \tag{43} \]
The first-order conditions \((t > 0):\)

\[
c_h^* : \quad W_1^*(c_{h_t}^*, c_{f_t}^*, h_t^*) + \lambda_{2t}^* \frac{U_{12}^*(c_{h_t}^*, c_{f_t}^*, h_t^*)}{1 + \tau_c^*} + \Omega^* \frac{U_{12}^*(c_{h_t}^*, c_{f_t}^*, h_t^*)}{1 + \tau_c^*} \text{TB}_t^* - \frac{\Omega^* U_2^*(c_{h_t}^*, c_{f_t}^*, h_t^*)}{e_t} + \frac{\Omega^* U_2^*(c_{h_t}^*, c_{f_t}^*, h_t^*)}{1 + \tau_c^*} \text{TB}_t^* = 0 \quad (44)
\]

\[
c_f^* : \quad W_2^*(c_{h_t}^*, c_{f_t}^*, h_t^*) - \lambda_{1t}^* + \lambda_{2t}^* \frac{U_{22}^*(c_{h_t}^*, c_{f_t}^*, h_t^*)}{1 + \tau_c^*} + \Omega^* \frac{U_{22}^*(c_{h_t}^*, c_{f_t}^*, h_t^*)}{1 + \tau_c^*} \text{TB}_t^* = 0 \quad (45)
\]

\[
h_t^* : \quad W_3^*(c_{h_t}^*, c_{f_t}^*, h_t^*) + \lambda_{1t}^* F_2(k_{t-1}^*, h_{t-1}^*) + \lambda_{2t}^* \frac{U_{23}^*(c_{h_t}^*, c_{f_t}^*, h_t^*)}{1 + \tau_c^*} + \Omega^* \frac{U_{23}^*(c_{h_t}^*, c_{f_t}^*, h_t^*)}{1 + \tau_c^*} \text{TB}_t^* = 0 \quad (46)
\]

\[
k_t^* : \quad -\lambda_{1t}^* + \beta \lambda_{1,t+1}^* [F_1(k_{t}^*, h_{t+1}^*) + 1 - \delta] = 0 \quad (47)
\]

\[
e_t : \quad -\lambda_{2t}^* \frac{U_1(c_{h_t}^*, c_{f_t}^*, h_t^*)}{1 + \tau_c^*} + \Omega^* \frac{U_2^*(c_{h_t}^*, c_{f_t}^*, h_t^*)}{1 + \tau_c^*} \frac{c_{h_t}^*}{e_t^2} = 0 \quad (48)
\]

The first-order conditions \((t = 0):\)

\[
c_{h_0}^* : \quad W_1^*(c_{h_0}^*, c_{f_0}^*, h_0^*) + \lambda_{20}^* \frac{U_{12}^*(c_{h_0}^*, c_{f_0}^*, h_0^*)}{1 + \tau_c^*} + \Omega^* \frac{U_{12}^*(c_{h_0}^*, c_{f_0}^*, h_0^*)}{1 + \tau_c^*} \text{TB}_t^* - \frac{\Omega^* U_2^*(c_{h_0}^*, c_{f_0}^*, h_0^*)}{\epsilon_0} + \frac{\Omega^* U_2^*(c_{h_0}^*, c_{f_0}^*, h_0^*)}{1 + \tau_c^*} A_0^* = 0 \quad (49)
\]

\[
c_{f_0}^* : \quad W_2^*(c_{h_0}^*, c_{f_0}^*, h_0^*) - \lambda_{10}^* + \lambda_{20}^* \frac{U_{22}^*(c_{h_0}^*, c_{f_0}^*, h_0^*)}{1 + \tau_c^*} + \Omega^* \frac{U_{22}^*(c_{h_0}^*, c_{f_0}^*, h_0^*)}{1 + \tau_c^*} \text{TB}_t^* + \Omega^* \frac{U_{22}^*(c_{h_0}^*, c_{f_0}^*, h_0^*)}{1 + \tau_c^*} A_{t-1}^* - \Lambda^* \frac{U_{22}^*(c_{h_0}^*, c_{f_0}^*, h_0^*)}{1 + \tau_c^*} A_0^* = 0 \quad (50)
\]

\[
h_0^* : \quad W_3^*(c_{h_0}^*, c_{f_0}^*, h_0^*) + \lambda_{10}^* F_2(k_{-1}^*, h_{-1}^*) + \lambda_{20}^* \frac{U_{23}^*(c_{h_0}^*, c_{f_0}^*, h_0^*)}{1 + \tau_c^*} + \Omega^* \frac{U_{23}^*(c_{h_0}^*, c_{f_0}^*, h_0^*)}{1 + \tau_c^*} \text{TB}_t^* + \Omega^* \frac{U_{23}^*(c_{h_0}^*, c_{f_0}^*, h_0^*)}{1 + \tau_c^*} A_{t-1}^* - \Lambda^* \frac{U_{23}^*(c_{h_0}^*, c_{f_0}^*, h_0^*)}{1 + \tau_c^*} A_0^* = 0 \quad (51)
\]

\[
k_0^* : \quad -\lambda_{10}^* + \beta \lambda_{11}^* [F_1(k_{0}^*, h_{0}^*) + 1 - \delta] = 0 \quad (52)
\]

\[
e_0 : \quad -\lambda_{20}^* \frac{U_1(c_{h_0}^*, c_{f_0}^*, h_0^*)}{1 + \tau_c^*} + \Omega^* \frac{U_2^*(c_{h_0}^*, c_{f_0}^*, h_0^*)}{1 + \tau_c^*} \frac{c_{h_0}^*}{e_0^2} = 0 \quad (53)
\]

where

\[
A_0^* = [1 - \delta + (1 - \tau_{k_0}) F_1(k_{-1}^*, h_{0}^*)] k_{-1}^* + d_{-1}^* + a_{-1}^*
\]
and

\[ TB_t^* = \left[ \frac{n}{1 - n} c_{ft} - \frac{e_{ht}^*}{e_t} \right] \]

References


