Reach for Yield by U.S. Public Pension Funds

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Abstract

This paper studies whether U.S. public pension funds reach for yield by taking more investment risk in a low interest rate environment. To perform our analysis, we first present a simple theoretical model relating funds’ risk-taking behavior to the level of risk-free rates, the extent of underfunding, and the fiscal condition of their state sponsors. To test the model implications empirically, we create a new methodology for inferring funds’ risk from limited public information on their annual returns and portfolio weights for the interval 2002-2016. In order to better measure the extent of underfunding, we revalue funds’ liabilities using discount rates that better reflect their risk. In line with our model implications, we find that funds on average took more risk when risk-free rates were lower. This was the case especially for funds that were more underfunded or affiliated with state or municipal sponsors with weaker public finances. We estimate that up to one-third of the funds’ total risk was related to underfunding and low interest rates at the end of our sample period.

Keywords: U.S. public pension funds, reach for yield, Value at Risk, underfunding, duration-matched discount rates, state public debt.

JEL Classification: E43, G11, G23, G32, H74

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1. Introduction

How do low interest rates affect the investment behavior of institutional investors? How is this behavior influenced by and how does it influence investors’ financial condition? This paper studies these questions in the context of state and municipal U.S. public pension funds (PPFs or funds). Specifically, we investigate whether the PPFs reach for yield (RFY) by holding riskier investment portfolios to increase their expected returns when interest rates on relatively safe assets are low. In addition, we study how the extent of funds’ underfunding and the fund sponsors’ fiscal condition affect and are potentially affected by risk-taking and reach-for-yield by PPFs. To study these relationships, we first present a simple theoretical model relating funds’ risk-taking to their underfunding and to the fiscal condition of their state sponsors. To empirically test the theory, we create a new methodology for measuring funds’ risk, and then use it with a relatively new measure of plan underfunding (Rauh 2017), in a panel regression analysis of how funds’ asset-risk is related to underfunding, interest-rates, and to states’ fiscal condition. We then study the consequences of our results for state finances.

A public pension plan is underfunded if the value of its assets is less than the net present value of liability payments to its pension holders. When a PPF is underfunded or there is a change in its investment opportunities, state sponsors are limited in their ability to reduce promised pension benefits because in most states public employee retirement benefits are either guaranteed by state constitutions or constitute a contractual obligation between the sponsor and plan members.\(^2\) Political considerations are also likely to affect the PPFs’ investment behavior because governmental sponsors of PPFs have discretion regarding the level of contributions to the fund and the setting of funds’ target asset return.\(^3,4\) Sponsors do, however, have many other opportunities to close the funding gap: Their funds can reach for yield hoping that investments

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\(^2\) Munnell and Quinby (2012) provide an analysis of the restrictions on the reduction of pension benefits to public sector employees and retirees.

\(^3\) Government accounting standards require that plan sponsors develop a plan to fully fund public pension plans over a period no greater than thirty years. The plan requirements are not binding. Many plan sponsors do not adhere to the funding schedules specified in the plan. Moreover, standards governing public sector pension plans provide sponsors with considerable discretion in the choice of accounting assumptions. Naughton, Petacchi and Weber (2015) provide evidence that plan sponsors use this discretion to reduce reported levels of underfunding and contributions. Kelley (2014) finds that political factors have a significant influence on plan funding levels.

\(^4\) A large share of the public pension plan board members consists of political appointees and elected officials. Return objectives of state public sector pension plans are often set by state legislatures in the budgeting process. Similar processes are used by local government pension plan sponsors.
in assets with higher expected return—but also risk—will close the gap; they can also require
greater future contributions on the part of pension holders; or sponsors can provide higher
contributions to the plan which they would fund by current or future taxation, by borrowing, or
by cutting expenditures on other governmental programs and prerogatives.

Our theoretical analysis models the asset portfolio choice of a public pension plan that is acting
on behalf of its sponsors and can invest in risky and risk-free assets. The model captures the
tradeoff that plan sponsors face when choosing between their constituents paying higher future
taxes to support pension beneficiaries, or by the plan taking more risk in the hopes that the risky
assets perform well. In the model, risk-free interest rates affect pension plans’ incentives to take
risk through two main channels. Through the first channel, interest rates affect plans’ funding
ratios, which is the present value of plans’ assets relative to their liabilities. If the present value
of a plan’s assets relative to liabilities drops with interest rates, the plan may choose to take more
risk in the hopes of meeting its obligations with a higher return on assets. This is the reach-for-
yield channel in the model. Through the second channel, the risk-premia on risky assets may
also be related to risk-free rates. This channel is separate from reach-for-yield, which allows
risk-free rates to affect funding ratios and hence risk-taking. Our empirical work distinguishes
between these two channels, as it examines funding ratios and risk-free rates as separate drivers
of risk-taking for pension funds. The model also captures the possibility that some sponsors may
choose to default on their non-pension debt if their required payments to pension beneficiaries
are too high. We examine how this possibility affects the risk-taking behavior of the pension
plan. In addition, within the model, we capture how risk-taking depends on the level of interest
rates, the funding ratio, and the amount of non-pension debt relative to state income.

Our empirical findings strongly support the model implications. First, we find that lower interest
rates and lower funding ratios caused PPFs to increase portfolio risk. Thus, lower interest rates
contributed to higher risk-taking through both a reach-for-yield effect and through the risk-
premium channel. Second, our findings show that there was an interaction between reach-for-
yield and low interest rates, i.e. the effect of a lower funding ratio on risk-taking was more
pronounced when interest rates were relatively low, such as during the last five years of our
sample (2012-2016). Third, PPFs affiliated with state or municipal sponsors with weaker public
finances—as reflected by their level of public debt or the state bond ratings—also took more
risk. In line with our model implications, there was a notable interaction between public finances and interest rates, as PPFs from states in worse fiscal shape took more risk especially during periods of low interest rates.

Our modeling of state finances suggests that if states can default on their non-pension debt with a penalty, then they may take higher risk in their pension funds if the state debt-to-income ratio is sufficiently high, because they can shift the risk of poor performance of the pension fund away from taxpayers and toward state debt holders. On the other hand, in our modeling, we also find that states may choose to take less risk if state debt increases but they cannot default on it. Our empirical analysis of state finances is mostly consistent with higher state debt-to-income ratios leading to higher risk in the pension plan, consistent with risk-shifting. Therefore, our analysis implies that, because PPFs in states with weaker financial conditions take more risk, they run the risk of further weakening state finances. We quantify that the potential loss to the states if a 1-in-20 years adverse return were to occur in 2016 would have been on average about 3% of state personal income, or about 36% of states’ debt.

A related theory of why PPFs’ risk-taking behavior has increased during the recent low yield environment is that sponsors may attempt to mask their PPFs’ extent of underfunding, and may do so by holding riskier assets with higher returns to reduce the reported value of their liabilities. Under-reporting of liability values can occur because GASB accounting rules allow U.S. PPFs to discount their liabilities based on the expected return on their assets. Andonov et al. (2017)’s cross-country study provides evidence consistent with this theory. However, Boubaker et al. (2017) find that given their asset holdings, PPFs tend to significantly exaggerate the expected returns on their assets; which means they do not necessarily have to hold riskier assets in order to mask a part of their underfunding. This phenomenon is partly illustrated in Figure 1 (panel a), which shows that the funds’ expected return targets have declined little while the Treasury yields have significantly declined.

This study makes several contributions relative to the existing literature on PPFs’ risk-taking behavior. First, we present a stylized theoretical model to help interpret our results. Importantly, the model highlights the role of risk-shifting as a determinant of PPFs’ risk-taking behavior. In addition, the theory highlights that risk-taking due to lower rates occurs through separate
channels that are captured by the funding ratios and interest rates, and illustrates the importance of distinguishing among the two channels.

Second, we use a new and more flexible approach for measuring funds’ risk based on the limited data that are publicly available. To do so, we assume that funds’ returns in each asset category (e.g., equities, fixed income, alternatives, etc.) consist of the return on an unknown category return index that is common across funds plus a fund-specific component. We estimate the category return indices and their constituents, as well as the funds’ residual risks econometrically. Then we use the estimated series and the funds’ portfolio weights to estimate funds’ risk. Unlike our approach, some papers in the literature measure funds’ risk while assuming that a particular index (such as the S&P 500 for equities) is representative of funds’ returns in each asset category. Other papers measure risk using less comprehensive measures, such as the share of equities or risky assets in the funds’ investment portfolios, without accounting for all asset classes. In contrast, our approach to risk measurement is more comprehensive because it accounts for all asset classes (where data is reported) and their return correlations.

Our third contribution is that we use improved measures of underfunding based on the methodology in Rauh (2017), which discounts the value of funds’ liabilities at rates that better reflect the riskiness of liabilities than the rates associated with the GASB reporting standards. The use of more appropriate (in our view) discount rates reduces error in the measurement of funds’ underfunding. Consistent with our measurement error interpretation, our improved measures of underfunding enhance the goodness of fit for regressions relating PPFs’ risk to underfunding.

Our paper is related to three strands of the literature. The first is the literature on PPFs’ risk taking. Boubaker et al. (2017) and Mohan and Zhang (2014) measure funds’ risk using aggregate market beta coefficients, with different betas assumed for each asset category. Andonov et al. (2017) measure funds’ risk as the share of risky assets in the portfolio. Pennacchi and Rastad (2011) measure funds’ risk based on the tracking error volatility between the value of assets and liabilities. The volatility of assets is measured under the assumption that returns in each asset category are determined by specific return indices. Relative to these papers, our
approach improves the measurement of risk in two ways. First, measures of risk based on covariation with the market, such as beta, or on the share of risky assets held do not measure the time series aspect of risk. For example, high covariation with the market or large holdings of risky assets in the portfolio increases a PPF’ riskiness when the market is expected to be more volatile, but these measures do not account for this time variation. In contrast, our approach based on funds’ Value-at-Risk accounts for it, as described in our risk measurement section. We use an approach to measuring the variance of asset portfolios that is similar to Pennacchi and Rastad (2011), but instead of assuming that returns in each category are driven by a particular index, our approach is more flexible. We believe our method of measuring the riskiness of pension funds’ portfolios has potential to improve on other methods used when the data are limited.

The second strand of related literature concerns the risk taking by financial intermediaries as a function of changes in macroeconomic conditions. As is the case with our study, these papers rely on cross-sectional differences between institutions in their response to changes in macroeconomic conditions to identify reach-for-yield behavior. Becker and Ivashina (2015) examine reach-for-yield behavior among life insurance companies. They find that life insurers tend to assume greater levels of investment risk during economic expansions and that this effect is more pronounced among more poorly capitalized firms. DiMaggio and Kacperczyk (2017) find evidence of greater risk-taking by money funds when interest rates are low. This effect is stronger for independent funds than for funds affiliated with insurance companies, commercial or investment banks. They argue that reputational considerations tend to moderate reach for yield behavior by affiliated funds. Studies of commercial banks also find evidence of increased risk taking in low rate environments but the effect of financial condition on risk taking is mixed. Jiménez et al. (2014) examine lending activity by Spanish banks. They find that lower overnight rates induce banks to do more risky lending. This effect is stronger among more poorly capitalized institutions. Dell’Ariccia et al. (2017) examine commercial banks in the United States. They also find evidence of greater risk taking in a low rate environment; however the increase in risk taking is more prevalent among well-capitalized institutions. Closer to this study, Boubaker et al. (2017) study reach-for-yield by PPFs in a framework that models the evolution of PPFs’ asset category risk-exposures and monetary policy innovations in a Bayesian VaR framework with Markov regime switching. Consistent with our results, they find that
monetary easing is consistent with greater portfolio risk. Because of differences in how risks are measured, how reach-for-yield is measured, and our very different modeling methodologies, we view our two approaches as complementary.

The third strand of related literature concerns the effect of public pension funds’ obligations on state and local finances. Increased risk taking and reach-for-yield behavior increases the exposure of plan sponsors to large declines in asset values, and hence increases the volatility of contributions necessary to fund pension promises. A growing literature considers the impact of pension costs, underfunding, and investment losses on state and local government borrowing costs (Novy-Marx and Rauh (2012), Boyer (2018)). Several academic and policy studies have examined the effect of a decline in asset prices on the required contributions of plan sponsors (Novy-Marx and Rauh (2014), Boyd and Ying (2017) and Mennis et. al. (2018)). Measures of PPFs’ risk taking presented herein should be useful in future work concerning the vulnerability of public pension plans and plan sponsors to adverse shocks in asset prices.

The remainder of the paper proceeds as follows. Section 2 presents a stylized theoretical model. Section 3 present our data and our methodology for measuring PPFs’ asset portfolio risk and plan underfunding. Section 4 contains our empirical analysis of how PPFs’ risk has changed over time and in the cross section. It examine the relationship between asset portfolio risk, the interest rate environment, plan underfunding, and the financial condition of fund sponsors. Section 5 discusses the implications of our results for states’ public finances. Section 6 concludes.

### 2. Model

To guide our thinking about the determinants of risk-taking by public pension plans, and to provide some intuition for how to interpret the findings from our econometric analysis, here we present a very simple “benchmark” two-period model of risky portfolio choice for a state or municipal pension plan. We refer to the plan sponsor as the state throughout.

For our modeling, we assume there is no conflict of interest between the state and the manager of its pension investments, and that therefore the pension plan is managed in accordance with the
wishes of plan sponsors. Therefore we model the state as controlling the amount of assets managed by the pension fund, and how those assets are invested. Multi-period treatments of the pension fund’s portfolio choice problem are contained in Pennacchi and Rahstad (2011) and D’Arcy et al (1999); following their approach, the state is assumed to choose its pension assets to maximize a utility function that is based on the preferences of its citizens, denoted as the representative citizen RC, hereafter. We interpret the representative citizen as the median voter within the state or municipality associated with a pension plan, but we acknowledge the utility function has rich interpretations. In particular, it may also depend on how the plans and sponsors actions affect conflicting special interests such as plan beneficiaries, state taxpayers, and the holders of state debt. Because of these potential conflicts and other potential imperfections, we don’t interpret maximization of the utility function as maximization of social welfare, but we regard it as a useful modeling device for our positive analysis. In our modeling below, we rely primarily on a median voter interpretation but we allow for the possibility that the interests of the median voter and state debt holders may be in conflict and examine the consequences of that conflict for plan actions in our theory and our empirical analysis.

There are two dates in the model. Date 0, which represents today, and date $t$, a date $t$ years in the future. The RC is endowed with income $Y_0$ and $Y_t$. The income $Y_t$ is assumed to be net of all tax payments other than those that may need to be made to support pension beneficiaries, or to payoff state debt. To simplify the analysis $Y_t$ and $Y_0$ are assumed to be known at date 0. In addition, the state has zero coupon debt with face value $D_t$ that must paid at date $t$ and it has a pension liability of $L_t$ that must be paid to its workers at date $t$. We model the debt $D_t$ in two different cases. In the first the debt is risk-free and the state will pay it out of state income $Y_t$. In the second the state can choose to default on its debt and will do so if the taxes needed to support its pension plan and state debt are too high. We first focus on the risk-free debt case and later turn to the case with risky debt. To partially fund the pension plan the state sets aside an amount $A_0$ to invest on behalf of its workers. At date $t$, the portfolio grows to value $A_0 R_{p,t}$, where $R_{p,t}$

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7 The preferences could reflect median voters, vocal interest groups or a blend of the median voter and the median voter as in Kelley (2014); or they could represent the preferences of median voters or fund managers as in Pennacchi and Rahstad (2011). Or they could represent the preferences of the median voter in a setting where there are information asymmetries among voters regarding pay structure as in Glaeser et al (2014).

8 If instead $Y_t$ is stochastic, then the investment choices of the pension fund would be used to hedge against the risk of $Y_t$ as pointed out by Lucas and Zeldes (2009).
is the gross return on the portfolio, then the portfolio is liquidated and the full proceeds from liquidation are turned over to workers.\(^9\) If the workers' liabilities exceed the proceeds, the difference is paid as a transfer from taxpayers to the pension beneficiaries. The taxes to support the pension plan at date \(t\) are given by \(T_t = \text{Max}(L_t - A_0 R_{p,t}, 0)\). The consumption of the RC at date \(t\) is then given by income at date \(t\) less debt payments and taxes. \(C_t = Y_t - D_t - T_t\). Similarly, consumption at date 0 is \(C_0 = Y_0 - A_0\).

The state chooses \(A_0\) and \(\omega\) to maximize the discounted present value of the consumption of the RC subject to the constraint that the pension liabilities and debt are paid off:

\[
\text{Max}_{A_0, \omega} U_0(C_0) + E_0 \delta^t U_t(C_t)
\]

where \(\delta\) is instantaneous rate at which the RC discounts the future, and \(U_0\) and \(U_t\) are strictly increasing concave functions of utility over consumption in each period. The optimization is also equivalent to minimizing the expected utility loss due to tax payments at period \(t\).\(^{10}\) In this view of the problem, the utility functions can be interpreted as incorporating the costs of distortionary taxes.\(^{11}\)

In our theoretical analysis the pension fund can invest in only two assets.\(^{12}\) There is a risk-free asset with instantaneous net return \(r_f\) which we treat as fixed between dates 0 and \(t\). A dollar invested in the risk-free asset at date 0 grows to \(e^{r_f t}\) at date \(t\). In addition, the pension fund can invest in a risky asset, which we think of as an equity index, whose log-return between dates 0 and \(t\) is normally distributed:

\[
\ln(R_t) \sim N(\left[r_f + \lambda - .5 \sigma^2\right]t, \sigma^2 t)
\]

where \(\lambda\) is the market price of risk, which is the reward for exposure to stock-market risk, and \(\sigma\) is the instantaneous standard deviation of the return on the risky asset. This assumption implies the gross return on \(R_t\) has the functional form,

\(^9\) The workers are assumed to get all of the cashflows from their asset portfolio even if it exceeds the liabilities. This assumption follows the practice that only workers have access to the cashflows that have been set aside in their pension funds.

\(^{10}\) Utility at time \(t\) is decreasing and convex in taxes \(T_t\).

\(^{11}\) Distortionary tax representations of the problem in two period and multiperiod settings are contained in Lucas and Zeldes (2009) and Epple and Schipper (1981).

\(^{12}\) We consider a larger number of investment types in our empirical analysis.
\[ R_t = \text{Exp}(r_f + \lambda - 0.5\sigma^2)t + \sigma \sqrt{t}\epsilon \]  \hspace{2cm} (2)

where \(\epsilon \sim N(0,1)\).

The pension fund invests \(\omega\) percent of its wealth in the risky asset and \(1-\omega\) percent in the risk-free asset subject to the constraint that \(0 \leq \omega \leq 1\). This constraint rules out the use of leverage by the fund as well as the use of short-sales. The resulting return on the asset portfolio is given by

\[ R_{p,t} = (1-\omega)e^{r_ft} + \omega e^{(r_f + \lambda - 0.5\sigma^2)t + \sigma \sqrt{t}\epsilon} \]

Substituting \(R_{p,t}\) into the expression for utility, given a choice of \(A_0\) at time 0, the maximization for the choice of \(\omega\), the share of risky assets in the portfolio, after simplification reduces to:

\[ \text{Max}_{\omega} E_0 U_t \left( Y_t - D_t - \text{Max} \left( L_t - A_0 \left[ (1-\omega)e^{r_ft} + \omega e^{(r_f + \lambda - 0.5\sigma^2)t + \sigma \sqrt{t}\epsilon} \right], 0 \right) \right) \]  \hspace{2cm} (3)

Our analysis of reach for yield studies how the riskiness of the portfolio depends on risk-free interest rates, plan funding, and state finances. We measure pension underfunding by its funding ratio which is the present value of the fund’s assets divided by the present value of its liabilities. When the funding ratio is 1 or over, a plan is fully funded, when it is less than one, then the plan is underfunded. Because, as discussed further below, payments to beneficiaries are very likely to be paid in full, we treat them as risk-free and discount their value using risk-free rates. Furthermore, we can proxy for the debt burden of state finances to the representative citizen by the ratio \(D_t/Y_t\), the state debt to income ratio, \(SDI_t\); and \(L_t/Y_t\), which is the pension debt to income ratio, \(PDI_t\). With these transformations, the portfolio choice problem in equation (3) can be rewritten as

\[ \text{max}_{\omega} E_0 U_t [Y_t \times \left( 1 - \frac{D_t}{Y_t} - \frac{L_t}{Y_t} \text{Max} \left( 1 - \frac{A_0}{e^{r_ft}} \left[ (1-\omega) + \omega e^{(\lambda - 0.5\sigma^2)t + \sigma \sqrt{t}\epsilon} \right], 0 \right) \right)] \]

\[ = \text{max}_{\omega} E_0 U_t [Y_t \times \left( 1 - SDI_t - PDI_t \times \text{Max} \left( 1 - FR_0(\eta), \left( 1 - \omega \right) + \omega e^{(\lambda - 0.5\sigma^2)t + \sigma \sqrt{t}\epsilon} \right), 0 \right) \]  \hspace{2cm} (4)
where \( FR_0(\tau_f) = \frac{A_0}{e^{\tau_f T}} \) is the funding ratio of the pension fund.

Several results are immediately apparent from equation (4):

1. The short-term risk-free interest rate \( \tau_f \) only affects risk-taking in the simple setting of equation (4) through its effect on the funding ratio since this is the only place that risk-free rates appear in the maximization.

2. Whether a pension fund chooses to “reach for yield” by taking more risk when rates are low is captured by how risk-taking depends on the funding ratio.

For example, if the funding ratio is greater than or equal to 1, corresponding to a fully funded pension plan, equation (4) shows that by investing only in the risk-free asset (by setting \( \omega = 0 \)), the proceeds from the asset portfolio are sufficient to pay off the pension liability. If instead the funding ratio is less than one because risk-free rates have decreased or for any other reason, then the equation shows the plan’s obligations cannot be met by investing in risk-free assets alone. Instead the plan must meet its obligations by taking on more risk, i.e. reaching for yield, as described in Rajan (2005) or Yellen (2011), and/or plan sponsors must pay more to pension beneficiaries through taxes at time \( t \). Our empirical assessment of reach for yield measures how pension funds’ underfunding affects the amount of investment risk that they choose to take.

In a more general setting interest-rates could affect risk-taking through other channels, such as through the risk-premium \( \lambda \). We refer to this as a risk-premium channel. A series of papers tracing back to Campbell (1987) study whether equity risk premia are time varying and predictable from interest rates or other variables. Our reading of the recent literature is that in univariate return predictability regressions, the evidence for predictability is weak, and the regression coefficients are time varying and unstable [Welch and Goyal (2008) and Paye and Timmermann (2006)]. However when univariate forecasts are combined, forecastability

13 Rajan (2009) discusses the need for private insurance companies to increase portfolio risk when rates fall: “Insurance companies may have entered into fixed rate commitments. When interest rates fall, they may have no alternative but to seek out riskier investments – if they stay with low return but safe investments, they are likely to default for sure on their commitments, while if they take riskier but higher return investments, they have some chance of survival.” Similarly, Yellen (2011) states “[I]mportant classes of generally unlevered investors (for example, pension funds) are reportedly finding it difficult in the present low rate environment to meet nominal return targets and may be reaching for yield by assuming greater interest-rate and credit risk in their portfolios.”

14 For reviews of this literature see Rapach et al (2013), and Timmermann (2018).
improves, especially near recessions [Rapach et al (2010)]. In addition, in univariate forecasting regressions, imposing restrictions from economic theory on the regressions improves predictability [Petenuzzo et al (2014)]. Given the uncertainty about how interest rates affect the equity risk premium, we are uncertain about which direction the risk-premium channel will go in our empirical work. For illustrative purposes in our theoretical analysis we rely on an estimate of how Treasury-bill rates affect the equity premium in a univariate regression with economic restrictions based on Petenuzzo et al (2014). They report the following relationship towards the end of their estimation period\textsuperscript{15}:

\begin{equation}
\lambda = .004 - .007 \times r_f , \quad (5)
\end{equation}

Where $r_f$ is the yield on 3-month U.S. Treasury bills, which we will interpret as the risk-free rate. Inserting this expression for $\lambda$ inside equation (4) shows that the short-rate effects risk-taking through the funding ratio and the equity premium.

As a first cut for providing intuition on how the funding ratio and interest rates affect risk-taking, we assume the RC has a utility function for time $t$ that has a power utility form:

$$U_t(C_t) = -(C_t)^{-k} \text{ for } k \geq 1.$$  

We focus on the case of $k=10$, and for now focus on the case of the State Debt to Income Ratio = 3%, and we assume that state debt is riskfree and will not be defaulted upon. We then numerically solve for the optimal portfolio choices and risk-taking as a function of the funding ratio and the risk-free rate. The main results from the numerical analysis are presented in Figure 2. The figure shows that risk, here measured by the proportion of the portfolio invested in risky assets, increases in pension underfunding, i.e. risk increases as the funding ratio decreases. This finding is consistent with reach for yield behavior since a lower risk free rate would lower the funding ratio and hence increase risk. In addition, holding underfunding fixed, lower risk-free rates are associated with more risk-taking through the risk-premium channel from [equation (5)]. Figure 2 also shows there is an interaction effect: The marginal effect of underfunding on risk-taking is larger when interest rates are lower, and the marginal effect that interest rates have on

\textsuperscript{15} The estimates reported in the published version of Petenuzzo et al (2014) end in the mid-1980s. We thank the authors for providing us with estimates from December 2010, the end of their sample, and the middle of our sample. We use their constrained estimates above. Their estimates for the unconstrained equation are $\lambda = .008 - .09 \times r_f$ where $r_f$ is the yield on 3-month Treasury bills.
risk-taking is more pronounced when underfunding is higher. This shows the reach for yield and risk premium channels interact, and one should account for the interaction that these different channels have on risk taking. These findings provide justification for our empirical specification that studies how funding ratios, interest rates, and their interactions affect risk taking.

Equation (4) also shows state finances measured by debt to be paid as fraction of state income, \( SDI_t \), also affects risk-taking. To investigate the role of state finances on risk-taking, we consider two circumstances, the first is when the state will not default on its debt. We model this with our assumption that its debt will be covered by taxes on income at date \( t \). Under our assumptions, Figure 3 shows a lower funding ratio increases risk, for a range of levels of state debt. This is consistent with the reaching for yield evidence in Figure 2. The figure also shows that greater state debt relative to income leads to reduced risk taking by the state’s pension fund. The intuition for this result is when the state is more indebted, then the taxes it faces if the pension fund performs poorly have a much greater effect on the utility of the representative citizen than it does if the state is less indebted. To avoid the more severe consequences the pension fund takes less risk if the taxpayers have to make up the shortfall for large losses by the pension fund.

A more general model of state finances would account for the possibility that some states might actually default on and/or renegotiate their debt and that all else equal larger pension liabilities increase the risk of default or renegotiation [see Boyer (2018)]. To model this in a simple way, we will assume that when a state defaults on its debt, it defaults on all of its debt, and when it does so it incurs a penalty measured in utility terms that is proportional to the amount of its debt. Specifically the representative citizen receives the following utility when it does not and does default at date \( t \):

\[
U_t(.) = \begin{cases} 
U_t(Y_t - D_t - T_t) & \text{No Default} \\
U_t(Y_t - T_t) - \gamma \times D_t & \text{Default}
\end{cases}
\]

where \( \gamma \times D_t \ (\gamma > 0) \) is the penalty for defaulting, and \( T_t \) represents the taxes if any that need to be paid to pension beneficiaries at date \( t \). The state will choose to default on its debt if doing so raises its utility. Algebra shows that to a first order approximation default is optimal, when...
\[
\gamma \leq U_t'(Y_t - T_t) - 0.5 * U_t''(Y_t - T_t) * D_t.
\]

Because utility is increasing and concave, this means default occurs when the representative citizen’s income \(Y_t\) is low, or when the taxes it needs to pay for its pension fund are too high. If the state can default on its nonpension debt, this shift some of the downside risk of its pension fund assets to debt holders which reduces the potential downside consequences of taking more risk.

A related mechanism for risk-shifting comes from recalling that pension claims are low risk to beneficiaries because they are very senior in the state’s debt structure. As noted by Ivanov and Zimmerman (2018) when states are more indebted, they tend to issue more senior debt (bank debt) to increase their debt capacity, and the borrowers that do this are high risk. Because pension liabilities are very senior, they represent another way to subordinate state bond holders and shift risk on to them. To examine how the possibility of shifting risk affects the pension funds investment decisions we solved for the pension funds optimal investments when the state can default on its debt. The results are presented in Figure 4. They show that for low funding ratios as the amount of debt to income increases from 0, risk-taking goes down, just as it did in Figure 3. But, unlike Figure 3, when debt to income increases enough, risk-taking suddenly becomes much higher. This non-monotonicity is due to the risk-shifting effect. Moreover, the Figure shows an interaction between state debt to income and pension underfunding: risk-shifting occurs for lower levels of state debt when a pension plan is more underfunded. An additional interaction effect is between state debt to income, and the risk free rate. More specifically, the effect that changes in interest rates have on risk-taking can depend on the state debt to income ratio. This interaction is present when the state cannot default on its debt (not shown) and it is also present in some circumstances when the state can default on its debt. In particular, Figure 5 shows that the response of risk-taking to interest rate changes through the risk-premium channel can very strongly depend on the level of state debt to income.

In our empirical examination of the role of state finances in pension fund risk-taking, we examine whether higher state debt to income is associated with higher pension fund risk-taking as would be consistent with state pension funds shifting risk to state debt. In addition, we examine if there is an interaction effect between the ratio of debt to income and interest rates.
In summary, four main results emerge from our theoretical analysis:

1. The effect that the funding ratio has on risk-taking captures the effect of reach for yield. In our regression analysis we interpret the coefficient on the funding ratio as capturing reach for yield effects.
2. After controlling for funding ratios, interest rates may also affect risk-taking because they affect risk-premia. We interpret the coefficient on interest rates in our regression analysis as the risk-premium effect.
3. The reach-for-yield and risk-premium effects interact in theory. We allow for their interaction in our empirical specification.
4. How state-finances affect risk-taking depends on whether states shift risk to their debt holders. If they do shift risk, then when state debt is large enough, it leads to higher risk taking, especially for underfunded pension plans. If states don’t shift risk, then greater state debt is predicted to lead to lower risk.
5. The effect that state finances have on risk-taking interacts with the risk premium effects of interest rate changes in determining risk taking.

The appendix provides further comparative statics results for the case when state debt is risk free.

The next section of the paper describes our data, and the methodology we use to measure pensions funds risk and their funding ratio.

3. Data and Risk Measurement

3.1 Data

Publicly available data on PPFs’ investment performance, risk-taking, and the value of liabilities is limited and incomplete. In this section we describe our data on PPFs and the methods we use to measure the PPFs’ risk and underfunding despite the data limitations. Our main data set on state and local public pension plans is the Public Plans Database (PPD) from the Center for
Retirement Research at Boston College.\textsuperscript{16} The PPD currently contains plan-level annual data from 2001 through 2016 for 170 public pension plans: 114 administered by states and 56 administered locally. This sample covers 95 percent of public pension plan membership and fund assets nationwide.\textsuperscript{17} The data set includes annual (by fiscal year) observations on the returns on each fund’s assets, the percentage of the fund’s portfolio invested in six main asset categories (equities, fixed income, real estate, cash, alternatives, and other), the market (fair) value of funds’ assets, and the actuarial value of funds’ liabilities.\textsuperscript{18}

Descriptive statistics on our PPD data are contained in Table 1. On average across time and funds, the largest asset holdings were equities and fixed income (54 and 27 percent of total assets, respectively), followed by alternatives and real estate (10 and 5 percent, respectively). The value of assets represented only 80 percent of the actuarial value of liabilities, pointing to substantial underfunding. Figure 1 also shows the evolution of these variables over time: As seen in panel (b), the ratio of actuarial assets to liabilities (henceforth funding ratio, or FR) declined from almost 100 percent in 2011 to little more than 70 percent in 2016. In panel (c), the shares of equities and fixed income assets in funds’ portfolios declined, while the share of alternative assets rose steeply from 5 percent in 2001 to almost 20 percent in 2016.

The data on funds’ risk exposures is coarse. Therefore, to infer funds’ risk, in the next subsection we bring in additional information by assuming that funds’ returns in each asset category can be decomposed into the return on an asset-category index and fund-specific risk. Furthermore, we assume that the category index returns are linear combinations of tradable market indices and estimate their linear combination. Although we could choose a wider set of tradeable indices for our analysis, for now we rely on the 17 tradable indices, which are detailed in Table 2.

To measure underfunding, we would like to compare the market value of funds’ assets with the market value of funds’ liabilities. However, as noted in the introduction, the actuarial value of

\begin{itemize}
  \item \textsuperscript{16} The PPF data is available at: \url{http://publicplansdata.org/public-plans-database/download-full-data-set/}.
  \item \textsuperscript{17} The sample of plans is a carry-over from the Public Fund Survey (PFS), which was constructed with an eye toward the largest state-administered plans in each state, but also includes some large local plans such as New York City ERS and Chicago Teachers.
  \item \textsuperscript{18} The data on holdings in some asset categories are further subdivided into foreign and domestic subcategories. Because the subcategories are not well populated, we combine like subcategories into the six broader categories noted above.
\end{itemize}
liabilities is measured using GASB standards that discount liability cash flows based on the properties of the funds’ assets, not their liabilities. This approach to liability valuation is inconsistent with finance theory and has been widely criticized (see for example Brown and Wilcox, 2009). We adjust plan liabilities using a discount rate that is more reflective of the true riskiness of PPF obligations. Following Rauh’s (2017) methodology, we infer the discount rates that funds should have used from the U.S. Treasury yield curve.\(^{19}\) We then use information on funds’ interest rate sensitivities to revalue the liabilities. The information on the sensitivity of funds’ liabilities to interest rate changes comes from GASB Statement 67 or from funds’ Comprehensive Annual Financial Report (CAFR). This sensitivity information is only available starting in 2014, when GASB 67 required funds to report interest rate sensitivities.\(^{20}\) GASB 67 data was available for 108 of the 170 funds in the Boston College dataset.\(^{21}\) As a robustness check, following Lucas (2017), we also revalue the liabilities with discount factors based on a high-quality corporate bond yield curve.\(^{22}\)

We provide further information on our data in the discussion of results. The following two subsections discuss how we measure funds’ risk and how we rediscount (i.e. revalue) their liabilities.

### 3.2 Risk Measurement

There are many possible measures of funds’ portfolio risk that could conceivably be used in our analysis. We have chosen to focus on funds’ 5% annual value-at-risk (VaR), which measures the most a fund could lose over a one-year horizon except with 5% probability. For example, if the probability a fund could lose 12% or more over the next year is 5%, then the funds’ 5% value-at-risk is 12%. An advantage of using VaR to measure portfolio risk is that it is comprehensive: it

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\(^{19}\) The data on the Treasury Yield Curve, which is updated daily, is provided by the Federal Reserve Board at: [https://www.federalreserve.gov/pubs/feds/2006/200628/200628abs.html](https://www.federalreserve.gov/pubs/feds/2006/200628/200628abs.html).

\(^{20}\) GASB Statement 67 disclosures require plans to disclose their Net Pension Liability (NPL) under alternative assumptions of the discount rate being 1 percentage higher and 1 percentage lower.

\(^{21}\) We gathered CAFRs from public pension fund websites. In many instances, GASB 67 information was not included in the materials posted on the website.

\(^{22}\) We use the return on the Citigroup Treasury Model Curve, which is created through a multistep process, starting with the Citigroup Corporate Index and taking corporate bonds rate AA-, AA, and AA+ by S&P. The data includes yields ranging from half a year to 30 years, reported monthly.
depends on the joint distribution of the returns on all of the assets in the fund’s portfolio. In
addition, VaR changes through time as the joint distribution of asset returns changes.

By contrast, some of the other risk measures that have been used in the literature on pension
funds’ risk-taking are less comprehensive or do not capture time variation in risk. For example,
some papers in the literature have measured risk as the share of equities in a fund’s portfolio.
This measure is not comprehensive because it does not consider all portfolio assets. In addition,
it does not capture time variation in the risk of portfolio assets. For example, two portfolios that
have the same equity shares have different value at risk in 2006 and 2009 because conditions in
2009 were much more volatile than in 2006. A risk measure that only focused on the share of
equity holdings would be insensitive to this difference. Moreover, the risk profile in an “equity
bucket” could vary greatly for two PPFs with identical shares of equity (one could be invested in
domestic large capitalization stocks while the other in emerging markets equities, which tend to
be riskier than domestic equities, all else equal).

In fairness, an important part of the reason VaR has rarely been used in the academic literature
on pension fund risk is because computing funds’ VaR requires data on both funds’ risk
exposures and on the returns of the assets the funds are exposed to. The data available are much
more limited; we know funds’ annual return and portfolio weights in six asset categories:
equities, fixed income, real estate, cash, alternatives, and other. Another complicating factor is
the data is not time-synchronous because returns and weights are both measured at the end of
each fund’s fiscal year.

An important contribution of our paper is that we develop a methodology to measure funds’ VaR
despite the data limitations. To do so, we make the following assumptions:

\textit{Assumption 1:} Each fund i’s return for asset category c at time t, \( r_{c,i,t} \), can be expressed as the
projection of the return onto a risk-category index \( r_{c,t} \) that is common across funds plus a
residual return \( \epsilon_{c,i,t} \) that is not correlated with any of the category return indices.

\[
r_{c,i,t} = \alpha_{c,i} + r_{c,t} + \epsilon_{c,i,t} \quad (7)
\]
Assumption 2: The return of each risk-category index \( r_{c,t} \) is spanned by the return of publicly available asset return indices indexed by \( j \):

\[
r_{c,t} = \alpha_c + \sum_j r_{j,t} \theta_{c,j} \quad (8)
\]

Assumption 1 is strong. It assumes within each risk category, funds returns for that category can be decomposed into a category index that is common across funds, plus a fund specific component that is uncorrelated with all of the category return indices. Assumption 2 is weaker. It assumes the returns for each category index can be spanned by the returns of publicly available indices. Assumption 2 should be satisfied if the category return indices are well diversified, so they only depend on pervasive risk-factors (risk factors that affect a significant part of the economy), and if the publicly available indices are well diversified and if the same factors that drive the returns on the category indices also drive the returns of the publicly available indices.

Using the arbitrage pricing theory from finance, the intercepts in equations (7) and (8) should be nonzero only if they represent compensation for nondiversifiable risks that are priced by the market but not included as regressors in the equation; i.e. it is compensation for the risks in the residuals of the equations. Because the residuals in equation (7) are by assumption fund specific and not captured by the category return indices, they should receive a zero price, hence \( \alpha_{c,i} = 0 \) for all \( c \) and \( i \). Relatedly, in equation (8) because there are no residuals by assumption, \( \alpha_c = 0 \) for all \( c \). The implications of this reasoning are summarized in the following assumption:

Assumption 3: \( \alpha_c = 0 \) for all \( c \), and \( \alpha_{c,i} = 0 \) for all \( c \) and \( i \).

To illustrate how these assumptions make it possible to compute funds risk, note each fund \( i \)'s asset return can be written as the sum of its portfolio weights times its asset return in each category.

---

23 Our analysis admits a slightly more general framework in which funds return within each category have the form:

\[
r_{c,i,t} = \alpha_{c,i} + \beta_c r_{c,t} + \epsilon_{c,i,t},
\]

with \( \beta_c \) common across funds. Without loss of generality, we have normalized \( \beta_c \) to 1 in our analysis.
\[ r_{i,t} = \sum_c w_{i,c,t} r_{c,i,t}. \]

Substituting in for each funds category return from assumptions 1, and the decomposition of the category return from Assumption 2, this equation can be rewritten as a regression equation in which the right hand side variables are an intercept term and funds’ portfolio weights interacted with the returns on the publicly available indices:

\[
\begin{align*}
    r_{i,t} &= \sum_c w_{i,c,t} (\alpha_{c,i} + r_{c,t} + \epsilon_{c,i,t}) \\
    &= \sum_c w_{i,c,t} \left[ \alpha_{c,i} + \left( \alpha_c + \sum_j r_{j,t} \theta_{c,j} \right) + \epsilon_{c,i,t} \right] \\
    &= \sum_c w_{i,c,t} \left[ \alpha_{c,i} + \alpha_c \right] + \sum_c \sum_j w_{i,c,t} r_{j,t} \theta_{c,j} + \sum_c w_{i,c,t} \epsilon_{c,i,t} \\
    &= \alpha_{i,t} + \sum_c \sum_j w_{i,c,t} r_{j,t} \theta_{c,j} + \epsilon_{i,t} \\
    &= \alpha + \sum_{c=1}^{C} \sum_{j=1}^{J} w_{i,c,t} r_{j,t} \theta_{c,j} + u_{i,t}, \quad (9)
\end{align*}
\]

where,

\[
u_{i,t} = \epsilon_{i,t} + (\alpha_{i,t} - \bar{\alpha}_{i,t}), \quad (10)\]

and \( \alpha = \bar{\alpha}_{i,t} \) is the average value of \( \alpha_{i,t} \) when averaged over all pension funds and time periods.

Equation (9) is a regression equation with a fairly large number of regressors. In particular, in what follows we estimate the intercept in equation (9) and 102 \( \theta_{c,j} \) coefficients corresponding to 17 publicly traded indices (\( J=17 \)) interacted with the portfolio weights for 6 categories of assets (\( C=6 \)).

The estimated \( \theta_{c,j} \) coefficients identify the stochastic part of the category return indices denoted by \( \tilde{r}_c \), whose time \( t \) realization is given by \( \tilde{r}_{c,t} = \sum_j r_{j,t} \theta_{c,j} \). The regression residuals \( u_{i,t} \) that
are recovered from estimation of the regression consist of two pieces. The first piece is \( \epsilon_{i,t} \), which is the stochastic part of each funds returns that is not explained by the category return indices. Additionally, \( \epsilon_{i,t} \) is a component of the risk of fund’s investments. The second piece is \( (\alpha_{i,t} - \bar{\alpha}_{i,t}) \), which is a function of fund’s portfolio weights and of the regression intercepts in equations (7) and (8). Importantly, the second piece of the residual is not a source of investment risk for the funds, hence if the residual has this component, then the funds risk will be slightly overstated because of it. It turns out the second piece of the residual will be uniformly equal to zero if the regression intercepts in equations (7) and (8) are zero as assumed using finance theory in Assumption 3. This implies that under Assumptions 1 – 3, the regression residuals in equation (9) will only contain the stochastic part of funds investment risks that are not explained by the category return indices. In summary, our methodology can estimate the stochastic part of the funds category returns and residual risks. Given these estimates, we can compute funds value at risk through time. In what follows we describe how we estimate funds value at risk and how we estimate the regression parameters in equation (9).

To estimate funds value at risk, we make the following assumptions on the return of the category return indices and the idiosyncratic risks.

**Assumption 4**: Let \( R_{c,t} \) denote the \( C \times 1 \) vector of category return indices in year \( t \) and let \( \epsilon_{i,t} \) denote the residual return on fund \( i \)’s investment portfolio in year \( t \). Then,

\[
R_{c,t} \sim N(\mu_t, \Sigma_t)
\]

\[
\epsilon_{i,t} \sim N(0, \sigma^2_{\epsilon})
\]

\[
Cov(R_{c,t}, \epsilon_{i,t}) = 0.
\]

Note: The category return indices and the residual returns are modeled as Gaussian for simplicity. Additionally, for simplicity the residual returns are for now modeled as independently and identically distributed (i.i.d.) across funds and time, and by Assumption 1 their covariance with the category indices is 0. There is scope to relax the conditions in Assumption 4 if needed.
In this paper we have chosen to measure funds value-at-risk as the 5th percentile of the unexpected component of a funds return distribution, and then express this quantity as a loss. This is best illustrated using an example. If fund $i$'s return has distribution $r_i \sim N(\mu_i, \sigma_i^2)$, then the unexpected component of the funds return is the return less its expected value $r_i - \mu_i$. Furthermore the 5th percentile of the funds unexpected return distribution is $\Phi^{-1}(0.05)\sigma_i = -1.65\sigma_i$. Expressed as a loss, $VaR(.05) = 1.65 \sigma_i$.

Using analogous reasoning, a fund with portfolio weights $w_{i,t}$ at the beginning of time $t$ has annual 5 percent value at risk given by

$$VaR_{i,t}(5\%) = 1.65 \sqrt{w_{i,t}' \Sigma_t w_{i,t} + \sigma^2_\varepsilon} \quad (11)$$

In order to compute VaR, we need measures of $\Sigma_t$ and $\sigma^2_\varepsilon$. Because our data on pension funds returns is annual and has less than 20 annual time series observations, it would not be possible to estimate a time-varying $\Sigma_t$ matrix using the short span of annual data on publicly available indices that is used to estimate equation (9). To overcome this problem, we estimate $\Sigma_t$ using daily data on the public indices returns. In particular, using the estimated coefficients for $\theta_{c,j}$, and daily data on public indices returns, we construct daily series of the returns on the category indices. Let $R_{c,d,t-1}, ..., R_{c,d,N,t-1}$ denote the vector of estimated daily returns on the category indices for each trading day of the year (or fiscal year) that ends on date $t - 1$. Our estimate of $\Sigma_t$, the annual volatility in year $t$ conditional on daily returns in year $t - 1$ is:

$$\Sigma_t = 250 \times \frac{1}{N} \sum_{k=1}^{N} R_{c,d_k,t-1} * R_{c,d_k,t-1}' \quad (12)$$

$\Sigma_t$ is equal to the conditional variance-covariance of daily category index returns scaled up by 250, the number of business days per year, to make it a variance covariance matrix of annualized category index returns.\(^{24}\)

\(^{24}\) In estimating $\Sigma_t$ we made an assumption that expected daily returns are equal to 0. This assumption approximates the reality that expected returns at a daily frequency are close to 0. Because high frequency estimation of expected returns is very noisy, when estimating the variance-covariance matrix of high frequency returns it is better to set expected returns to 0 rather than trying to estimate them.
Although we can estimate $\Sigma$ using daily returns for the estimated category indices, we cannot estimate $\sigma^2$ using daily data because we only observe pension funds returns annually and hence can only observe annual residuals from equation (9). Therefore, to estimate $\sigma^2$ we simply rely on the estimated residuals from equation (9) and estimate $\sigma^2$ as the sample variance of the residuals.

As a robustness check for our analysis, we also estimate an unconditional version of $\Sigma$ that is equal to the variance covariance matrix of monthly returns scaled up to represent the variance covariance matrix of annual returns. This matrix is unconditional because it is based on the average variance covariance of matrix of the monthly return on the category return indices, and does not change over time.

Our approach for estimating each funds value at risk has many advantages. The first and most important is we estimate the category return indices that best explain funds annual returns and portfolio weights. This improves on other approaches which don’t measure time variation in risk or which assume that the returns in different asset categories are the returns of a particular traded index.

The second advantage is by using daily data we are able to overcome some of the limitations in estimating funds’ risk on the basis of annual data. In particular, our approach produces estimates of funds risk that vary through time because of changes in the variance-covariance matrix, and because of changes in funds’ asset composition.

The third advantage of our approach is we can produce estimates of fund risk that vary by fiscal year. For example if a fund’s fiscal year ends in March, using our approach we can estimate the risk of each fund’s portfolio over each of its fiscal years. This is important when studying pension funds because their fiscal years end on different dates.

To estimate our risk measures, it is necessary to estimate the $\theta_{c,f}$ coefficients from equation (9). This requires the estimation of a large number of parameters with a relatively small sample of data. This requires finding a way to avoid statistical problems associated with overfitting. Based on the fact that to some extent, some of these 17 indices are correlated with each other, and that there might be some common factors driving these indices, we assume that the dependent
variable in equation (9) can be closely approximated by using a small subset of these indices for each asset category, which is the approximate sparsity assumption in our paper.\(^{25}\) This assumption allows us to use a penalized estimation method to estimate the model parameters. We use a two-step procedure to estimate the model in equation (9). First we use a penalized regression method to select the most relevant subset of indices for each asset category; this is then followed by an estimation based on the selected indices. In the first step, we use LASSO (Least Absolute Shrinkage and Selection Operator) (see references: Frank and Friedman (1993), Tibshirani (1996) and James et al (2013))\(^{26}\) regression in equation (9) to select the most relevant indices to be used in estimation and shrinks the coefficients on the other variables to 0, essentially eliminating them from the regression.\(^{27}\) After using LASSO, the number of relevant variables to use shrinks considerably. Then in the second step, ordinary least squares estimation (OLS) is applied.

This two-step procedure estimator, termed as OLS post-Lasso estimator, is well known in the literature of high-dimensional sparse models. It has been shown that the OLS post-LASSO estimator performs at least as well as LASSO in terms of the rate of convergence and has the advantage of a smaller bias (see Belloni and Chernozhukov (2013)).\(^{28}\) This nice performance still holds even if the LASSO-based model selection is not perfect. More specifically, if LASSO omits some components, as long as these components have relatively small coefficients, this nice performance still holds. Regarding the penalized selection method for the first stage, LASSO is a popular and powerful approach but not the only one. Researchers

\(^{25}\) As discussed by Hastie et al. (2015), there are two settings of the sparsity condition. One is the so-called hard sparsity, in which only a small number of the true coefficient parameters are nonzero. This assumption is overly restrictive, so they also consider the other one, which is the so-called weak sparsity where the true coefficient parameters can be closely approximated by vectors with few nonzero entries, in order words, coefficients can be estimated based on a subset of the explanatory variables and letting the coefficients of the rest explanatory variables being zero. This weakly sparsity is more general and has been widely used in the literature of Lasso-type penalized methods. The approximate sparsity condition used in both this paper and Belloni and Chernozhukov (2013) is the weakly sparse condition.

\(^{26}\) In James et al (2013), the main description of LASSO can be found on pages 219 through 227. Useful introductory notes can also be found at: [https://onlinecourses.science.psu.edu/stat857/node/158/](https://onlinecourses.science.psu.edu/stat857/node/158/)

\(^{27}\) Lasso has been used and its properties have been researched in many papers, for instance, Bickel et al. (2009), Meinshausen and Yu (2009), Van de Geer (2008), Zhang and Huang (2008), and so on.

\(^{28}\) Belloni and Chernozhukov (2013) investigate the properties of OLS post-LASSO in the mean regression problem; Belloni and Chernozhukov (2011) also studies the post-penalized procedures, but different problem of median regression.
can choose different penalized methods from a variety of choices, for example, threshold LASSO, the Dantzig selector and etc.29

Finally, once we have the estimated coefficients using the OLS post-LASSO estimators (as reported in Table 3), then from assumption 2 and data on the returns of the publicly traded indices we know the returns of the category indices, and how they vary through time. Moreover, we can identify funds’ residual risk, and then use the estimated joint dynamics of the category indices and the residual risks to compute funds value at risk as described above.

Figure 6 shows plots of funds’ VaR through time when the variance-covariance matrix is estimated on a conditional or unconditional basis. The figures show that both conditional and unconditional VaRs change over time, but conditional VaR changes more. The reason is that conditional VaR changes through time due to changes in both of economic conditions and portfolio weights, while unconditional VaR only changes due to portfolio weights.

3.3 Measurement of Underfunding

In this subsection, we use the approach of Rauh (2017) to rediscount the PPFs’ liabilities and then use them to recompute the PPFs’ extent of underfunding. Because PPF cash flows to liability holders are nearly risk-free, Rauh (2017) proposes to rediscount PPFs’ liabilities using the yield of a U.S. Treasury bond whose duration matches the duration of the funds’ liabilities. The adjustment in the liability’s value can then be found by taking into account the sensitivity of PPFs’ liabilities to changes in interest rates, as discussed below.

Specifically, we compute funding ratios (FR) to measure the PPF’s underfunding status as follows:

\[
Funding \ Ratio \ (FR) = \frac{Actuarial \ Assets \ (AA)}{Total \ Pension \ Liabilities \ (TPL)}
\]

(13)

with a lower ratio reflecting greater underfunding. We use two measures of total pension liabilities (TPL) to compute funding ratios. One is the amount of \(TPL_r\) reported by the PPFs

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29 Belloni and Chernozhukov (2013) also consider using the threshold LASSO for the first stage. For more details about the Dantzig selector, readers can refer to Bickel et al. (2009), Candes and Tao (2007), and among many others.
themselves, which are discounted using their own expected rates of returns $r$. The other measure is the one we obtain following the approach in Rauh (2017) $^{30}$, denoted as $TPL_r$, in equation (14), which takes into account both the duration and convexity of liabilities with respect to changes in the discount rate, as well as the difference $\Delta r$ between the original and our duration-matched discount rate:

$$TPL_{r'} = TPL_r - TPL_r \times \text{Duration} \times \Delta r + 0.5 \times TPL_r \times \text{Convexity} \times (\Delta r)^2 \quad (14)$$

where $\Delta r = r' - r$, with $r'$ denoting the duration-matched Treasury yield $^{31}$ and $r$ the funds’ original discount rate. $^{32}$ Finally, the duration and convexity are obtained as:

$$\text{Duration} = -\frac{TPL_{(r + 0.01)} - TPL_{(r - 0.01)}}{0.02 \times TPL_r} \quad (15)$$

$$\text{Convexity} = \frac{TPL_{(r + 0.01)} + TPL_{(r - 0.01)} - 2 \times TPL_r}{(0.01)^2 \times TPL_r} \quad (16)$$

where $TPL_{(r+1\%)}$ is the reported new TPL if the discount rate is 1 percentage higher which is $(r + 1\%)$; and $TPL_{(r-1\%)}$ is the new TPL if the discount rate is 1 percentage lower which is $(r - 1\%)$.

With the PPFs having reported the sensitivity of liabilities to interest rate changes under GASB accounting standards only recently, we use data on funding status for the year 2015 to illustrate the importance of rediscounting the PPFs’ liabilities using appropriate discount rates. In Figure 7, panel (a), if rediscounting made no difference, all observations would line up along the 45 degrees line. However, most observations are situated above the 45 degree line, showing that rediscounting at the more realistic, lower rates boosts the present value of Total Pension Liabilities to almost double the reported amounts. Similarly, in panel (b), the observations fall below the 45 degrees line, showing that rediscounting boosts liabilities and, as a result, reduces the funding ratios by almost half. Also, the observations in panel (b) do not suggest a linear

---

$^{30}$ The main idea is that the new computed liability is approximated by the 2nd order Taylor expansion with respect to changes in the discount rate $r$. As noted, GASB 67 requires plans to disclose the net pension liability under alternative assumptions of the discount rate being 1 percentage point higher ($TPL_{(r + 1\%)}$) and 1 percentage point lower ($TPL_{(r - 1\%)}$).

$^{31}$ $r'$ is the rate on the Treasury yield curve that matches the duration of each PPFs’ liabilities.

$^{32}$ The discount rate is available in the GASB statement 67 reports as “Current Discount Rate.”
relationship between the original and rediscounted funding ratios, showing that our rediscounting method also offsets differences in discount rates \( r \) across PPFs.

The following section uses our measures of risk and underfunding to analyze how they are related to each other and to reach for yield.

4. Determinants of Risk-Taking Behavior

Using the measures of risk and underfunding developed before, in this section we relate the PPFs’ risk-taking behavior to fund- and state-specific characteristics in the cross section, as well as to the evolution of interest rates on safe assets over time. We do so in three separate frameworks. First, in a cross-sectional framework, we study the relation between PPFs’ riskiness and lagged funding ratios across funds at one point in time (i.e., year 2016) for which the best data is available from the GASB Standard 67 reports. These include the reported sensitivity of PPFs’ liabilities to changes in interest rates, which are necessary to compute appropriate measures of underfunding. Second, we estimate the cross-sectional relation between riskiness and funding ratios for each year in the sample. Third, using a difference-in-difference approach in a panel data context, we study the importance of funding ratios across funds, the level of risk-free interest rates over time, and the fiscal condition of the funds’ state sponsors, as well as interactions among them, as determinants of riskiness. Thus, our empirical analysis tests the model implications discussed in Section 2.

4.1 Riskiness vs. Underfunding: Cross-section Results for 2016

For our cross-sectional results, Figure 8 shows the relation between the PPFs’ riskiness measured for 2016 (on the vertical axis) and the one-year lagged funding ratios (on the horizontal axis), using either the original \( TPL_r \) reported by the funds (panel a) or the rediscounted \( TPL_{r^*} \) (panel b) to recompute funding ratios.

Several conclusions emerge from the comparison of the two panels. First, rediscounting liabilities shifts the entire distribution of funding ratios to the left, as discussed earlier. Second, the link between PPFs’ riskiness and the one year-lagged funding ratios is negative and statistically significant in both cases, i.e., PPFs with ex-ante lower funding ratios displayed
higher riskiness. Third, the approach with rediscounted liabilities results in a steeper slope coefficient and higher statistical significance for the link between riskiness and funding ratios (panel b). In particular, the increased statistical significance is consistent with the view that rediscounting the liabilities of all funds at the same discount rate provides for a more consistent comparison of the true extent of underfunding across funds.

4.2 Riskiness vs. Underfunding: Cross-section Results over Time

When examining the relation between PPF riskiness and funding ratios over the entire sample period, data availability constrains our ability to compute rediscounted funding ratios going back in time. Since the PPFs have reported the sensitivity of their liabilities to interest rate changes only since 2014 under the new GASB accounting standards, the measures of duration and convexity are available only for the most recent years. Hence, to overcome the data limitations and compute rediscounted funding ratios over time, we adopt two strategies. First, we use the rediscounted funding ratio for 2015 computed like in equation (5) as a time-invariant proxy for the PPFs’ underfunding status over the entire sample period, based on evidence that the extent of underfunding has been persistent over time. Second, we assume that the measures of duration and convexity from 2014-16 are informative about the past, like in Andonov et al. (2017), and use the their values—adjusted for the evolution of PPF characteristics and state-level demographics over time—along with the duration-matched Treasury rates to rediscount Actuarial Liabilities (instead of Total Pension Liabilities), and hence to recompute funding ratios going back to 2001 (Figure 9). While neither approach is without criticism, each provides insight about the funds’ reach-for-yield behavior through the interaction between risk-free rates and funding ratios over time.

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33 Using the PPFs’ own reported funding ratios computed as Actuarial Assets divided by Actuarial Liabilities, we find that the average funding ratios for 2014-16 are highly informative about the average funding ratios for 2001-06 in the cross-section of funds. Regressing the latter on the former, we obtain a slope coefficient close to unit (0.83), statistically significant at the 1 percent level, and an R-squared value of 0.43.

34 To adjust duration and convexity for trends in fund characteristics and state-level demographics, we first take the mean of duration and convexity over the period 2014-2016. Then we estimate cross-sectional regressions for duration and convexity on the funds’ beneficiaries-to-members ratio and the states’ life expectancy at birth in 2015. For both duration and convexity, we find negative and statistically significant coefficients for the beneficiaries-to-members ratio, and positive and significant coefficients for life expectancy (i.e., less beneficiaries but higher life expectancy are associated with higher duration). Finally, we use the cross-sectional estimates from 2015 and historical fund- and state-level data to obtain time series for duration and convexity over 2001-2015.
Using the original and rediscounted funding ratios obtained under the two approaches, Table 4 shows the cross-sectional link between PPFs’ riskiness and funding ratios for each year during 2002-2016. In panels (a) and (b), using the funding ratios fixed at their 2015 values, the results show a negative link between PPFs’ riskiness and funding ratios. Similarly, using the time-varying funding ratios in panels (c) and (d), the results show a negative link between riskiness and lagged funding ratios. However, the link is statistically significant only for the interval 2012-2016, which largely coincides with the post-crisis period of low risk-free rates.

4.3 Riskiness vs. Underfunding, Rates, and State Finances: Panel Data

For our panel analysis, we adopt a difference-in-difference approach to examine the link between PPFs’ riskiness (as a dependent variable) and the following explanatory variables: the PPFs’ funding ratios; proxies for the level of risk-free rates (i.e., Treasury yields and an indicator variable for the post-crisis period of low risk-free rates); and measures of the sponsor states’ public finances (i.e., debt-to-income ratios and state bond ratings). These variables mimic the drivers of risk-taking behavior considered in the model in Section 2.

Like in Section 4.2, we use two approaches to overcome data limitations when rediscounting funding ratios for the entire sample period. For the approach with fixed funding ratios, we use the following specification:

\[ VaR_{pt} = \alpha + \beta * FR_p + \gamma * TrYield_t + \delta * FR_p * TrYield_t + \epsilon_{pt} \]  

(17)

where \( p \) denotes a fund, \( t \) denotes the year, and \( VaR_{pt} \) is the conditional measure of risk defined in Section 3.2. Among the explanatory variables, \( FR_p \) is the time-invariant funding ratio computed as actuarial assets divided by actuarial liabilities (both original and rediscounted). Since the funding ratio is time-invariant, \( FR_p \) is equivalent to a fund fixed effect. Also, \( TrYield_t \) is the 1-year or 10-year Treasury yield, or alternatively is replaced by a post-crisis dummy variable for the 2009-2016 period of low interest rates. To avoid the type of reverse causation potentially driven by recessions, when stock price declines coincide with monetary policy easing, the crisis years are excluded from the sample (2002, 2003, and 2009). We cluster the standard errors at both the fund and year level.
For the approach with time-varying funding ratios, the specification is:

\[
VaR_{pt} = \alpha + \beta \cdot FR_{pt-1} + \gamma \cdot TrYield_t + \delta \cdot FR_{pt-1} \cdot TrYield_t + \mu_p + \epsilon_{pt} \quad (18)
\]

which is similar to equation (17) in most respects, except that \(FR_{pt-1}\) is the time-variant funding ratio computed as actuarial assets divided by actuarial liabilities (both original and rediscounted). The time-variant funding ratio is demeaned relative to the cross-sectional mean from each year. Like before, we cluster the standard errors by both fund and year level. This time, we also include a fund fixed effect \(\mu_p\).

The results for equation (17) are presented in Table 5. In all columns, the coefficients on funding ratios are negative and statistically significant, showing that underfunding is associated with higher riskiness. In addition, the coefficients on the 1-year Treasury yields are negative and statistically significant (columns 1-2), while the coefficients on the post-crisis dummy variable are positive and significant (columns 5-6), showing that PPFs became more risky in the low-interest rate environment. The coefficients on the interaction between the Treasury yields and funding ratios are positive and statistically significant (columns 2-4), while those on interactions between the post-crisis dummy and funding ratios are negative and statistically significant (columns 5-6). These results suggest that, during episodes of low interest rates, it was especially the PPFs with low funding ratios that took more risk. Also, the steeper slopes for rediscounted funding ratios (columns 2, 4, and 6) suggest that rediscounting provides a better measure of the true extent of underfunding, which in turn determines the extent of risk-taking, and a more consistent comparison across funds. Overall, these results provide strong empirical support for the model implications in Section 2 regarding the roles of underfunding and risk-free rates as drivers of risk, as illustrated in Figure 2.

Table 6 shows similar results for equation (18). The coefficients on the lagged funding ratios are mostly negative, and statistically significant in the case with 10-year Treasury yields and rediscounted liabilities (column 4). In addition, the coefficients on Treasury yields are negative and statistically significant (columns 1-2), while those on the post-crisis dummy are positive (columns 5-6), indicating that PPFs took more risk when the risk-free rates were lower. The interactions between Treasury yields and funding ratios are positive and statistically significant (columns 2-4), while those between the post-crisis dummy and funding ratios are negative.
(column 6), showing reach-for-yield behavior during periods with low interest rates especially by the PPFs that were more underfunded. Notably, the coefficients on interacted terms become larger and gain statistical significance in the case with rediscounted funding ratios, highlighting the importance of measuring underfunding appropriately. Once again, these results support the model implications discussed in Section 2 and illustrated in Figure 2.

To examine the role of state public finances as a driver of risk, we include a relevant measure of state finances $X_{pt}$ in equations (17) and (18), which consists of the sponsor states’ debt-to-income ratios and state bond ratings. Variable $X_{pt}$ enters the regressions both in levels and interacted with the funding ratios and the risk-free rates, in line with the model implications discussed in Section 2.

The results for the role of state public finances are shown in Table 7. On the first two rows, the coefficients for state debt-to-income ratios and state bond ratings have the expected sign and are statistically significant in most specifications. They show that PPFs sponsored by states with higher debt-to-income ratios or worse bond ratings displayed more risk. In addition, the interactions between Treasury yields and state-specific measures of public finances suggest that PPFs from states with more debt and worse bond ratings took more risk especially during episodes with low interest rates. Overall, these results provide support for the model implications with risky state debt discussed in Section 2 and illustrated in Figure 5. Namely, higher state debt and lower interest rates are associated with higher risk-taking by the PPFs. In addition, the negative coefficient on the interacted term is consistent with the model implication that for a level of state debt that is large enough, lower risk-free rates provide an incentive for PPFs to take more risk while their sponsor states shift the risk to debtholders.

Also in Table 7, the results in columns 2-3 and 5-6 show that while controlling for public finances, lower funding ratios coincide with higher risk-taking by the PPFs. Once again, this empirical result is consistent with the model implications illustrated in Figures 3 and 4. The interaction coefficients between state finances and funding ratios are positive and statistically significant, supporting the model implications with risk-free debt (Figure 3).
4.4 Robustness Checks

We perform a set of robustness tests for the results presented in Sections 4.1 and 4.3. Our tests involve using alternative measures of risk as the dependent variable, such as the conditional VaR computed with asset weights that abstract from valuation changes, or the unconditional VaR measure. They also involve alternative explanatory variables, such as funding ratios obtained by rediscourting liabilities with duration-matched corporate bond yields instead of Treasury yields.

For the cross-sectional relation between the PPFs’ riskiness and funding ratios, the earlier results are preserved when using the conditional VaR with weights that abstract from valuation changes (Figure 10, panel a), the unconditional VaR (panel b), or funding ratios rediscourted with corporate bond yields (panel c).

The difference-in-difference results are supported by our robustness tests. In Table 8, the results hold when the conditional VaR is computed with asset weights that abstract from valuation changes (columns 1, 4, 7, and 10), and also when funding ratios are rediscourted with corporate bond yields (columns 3, 6, 9, and 12). In various specifications, the coefficients on the fixed and time-variant funding ratios remain negative and statistically significant. The same is the case for the 1-year Treasury yield. Notably, the interacted term between funding ratios and Treasury yields remains positive and statistically significant, for either fixed or variable funding ratios, and for either 1-year or 10-year Treasury yields. For the most part, the results do not hold for the unconditional VaR (columns 2, 5, 8 and 11).

We also consider the use of measures of risk based only on the relative composition of PPF portfolios. The difference-in-difference results become considerably weaker when using more traditional measures of risk as the dependent variable, such as the share of alternatives in Table 9. With the share of alternatives as the new dependent variable, only the negative coefficients on

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35 We decompose the change in portfolio weights $\Delta w_{pat}$ (i.e., fund $p$’ portfolio share of asset type $a$ held at time $t$) into two parts: (1) a more passive change component driven by valuation changes, and (2) a residual change component that more closely resembles active portfolio reallocations that involve trading. The valuation-driven weight change is defined as: $Valuation\,Change_{pat} = w_{p(a,t-1)} \times \left( \frac{1 + R_{at}}{\sum_{j=1}^{n} w_{p(j,t-1)}(1 + R_{jt})} \right) - w_{p(a,t-1)}$, where $R_{at}$ is the net return of asset class $a$. The reallocation change is defined as: $Reallocation_{pat} = w_{pat} - Valuation\,Change_{pat}$. Subsequently, the cumulate the reallocation changes to obtain the weights that abstract from valuation changes. See Ahmed et al. (2016) for similar decompositions applied to international portfolio flows.
Treasury yields and the positive coefficients on the post-crisis dummy preserve statistical significance, which is consistent with the growing share of alternatives in PPF portfolios over time. However, the coefficients on funding ratios or state debt-to-income ratios are not statistically significant, and neither are the interaction coefficients between Treasury yields and funding ratios.

5. Economic Significance and Implications for Financial Stability

This section quantifies the contributions of underfunding and low risk-free rates to risk-taking by PPFs, and also infers their potential implications for state and municipal finances.

5.1 The Magnitude of Risk-Taking Behavior

To quantify the contributions of drivers to PPFs’ risk, we use our cross-section and panel regression results from Section 4.3 to decompose the conditional VaR measure into two components: one that is related to underfunding and low risk-free rates, and a residual that abstracts from the two drivers. We follow two approaches to perform this decomposition.

First, the cross-sectional results in Figure 8 (panel b) imply the following relation between risk and the funding ratio: $\text{VaR}_p,2016 = \alpha + \beta \times FR_p,2015$, with $\beta = -0.0319$. Since a lower funding ratio is associated with higher risk, we assume that the risk component associated with underfunding is measured as follows:

$$\text{VaR}^{RFY}_{p,2016} = 0.0319 \times (1 - FR_p,2015),$$

where the brackets capture the gap between a fully funded counterfactual status (i.e. FR=1) and each fund’s actual funding ratio. The residual risk is the component that abstracts from the underfunding-related component of risk:

$$\text{VaR}'_{p,2016} = \text{VaR}_{p,2016} - \text{VaR}^{RFY}_{p,2016}$$

To convert our measure of risk into the dollar loss that each fund would have suffered if a severe stress event were to materialize in 2016 (i.e., with returns in the bottom 5 percentile of
realizations), we multiply the funds’ actuarial assets from end-2015 by their corresponding VaR. We aggregate the dollar losses across funds in each state, and express them a fraction of the state income for scaling purposes.\(^{36}\) Using this metric, we illustrate the risk component related to underfunding (the red portion of the bars in Figure 11, panel a), and the residual component (the yellow bars). On average across states, the risk component corresponding to underfunding represented about 12 percent of the total risk as of 2016.

Second, we repeat the exercise using the panel results from Table 5, column 2. We decompose the total risk into a component driven by both underfunding and the low level of Treasury yields in 2016 (compared to the pre-crisis average of Treasury yields) vs. the residual component.\(^{37}\) Once again, we translate risk into dollar losses normalized by state income. In Figure 11 (panel b), the risk component associated with underfunding and low risk-free rates (the red portions of the bars) represented on average about 33 percent of the total risk as of 2016.

Overall, we find that the PPFs’ risk-taking behavior related to underfunding was responsible for about 12 percent of their total risk, and the risk-taking behavior related to both underfunding and low levels of risk-free rates accounted for about 1/3 of their total risk at the end of our sample.

### 5.2. Implications of Risk-Taking for State Finances

The results of our study have implications for the state and local-level public finances. The shift of PPFs into riskier investments raises concerns regarding the potential impact of sharp declines in asset values or low returns on state and local finances.\(^{38}\) Sponsors of these plans typically have little ability to alter benefit levels or the terms of retirement plans.\(^{39}\) As a result, most of the downside risks associated with a decline in asset values or lower investment returns is likely to be borne by the taxpayers of the jurisdiction sponsoring the plan. In light of the results in Section 4.1, the burden of possible PPF losses on state finances could be sizeable: As shown in

\[^{36}\text{We rescale VaR losses take into account the fact that, in some states, the data on duration and convexity necessary to rescale liabilities is only available for a sub-set of funds. In such cases, we scale up the predicted 5-percent VaR losses by the inverse of the fraction of fund liabilities from each state for which duration and convexity data are available.}\]

\[^{37}\text{Given the results in Table 5, column 2, the risk component associated with both underfunding and lower rates post-crisis is: } VaR^{\text{RPY}}_{p,2016} = -0.018 \times (1yrTrYield_{2016} - 1yrTrYield_{2001-07}) + 0.0057 \times (1yrTrYield_{2016} \times FR_{p} - 1yrTrYield_{2001-07}).\]

\[^{38}\text{Boyd and Yin (2017).}\]

\[^{39}\text{Munnell and Quimby (2012).}\]
Figure 7, the potential loss implied by the 5-percentile VaR risk, which corresponds to a severe stress event occurring once in 20 years, would have represented about 3 percent of state income on average across the U.S. states in 2015. Compared to the average state debt-to-income ratio of 7 percent, the potential loss associated with the 5-percent VaR would have represented almost half of the state-level public debt in 2015, with up to 1/3 driven by reach for yield.

In addition, the impact of PPFs’ reach for yield on state finances is likely to be skewed, with states with weaker finances likely to be hit more. In Section 3.2, we find that PPFs from states that are more financially constrained are more likely to assume additional levels of risk. That is, reach for yield behavior is most pronounced among fund sponsors with the least ability to bear additional risk.

6. Conclusion

This paper examined the determinants of risk-taking behavior by U.S. public pension funds. To study PPFs risk, we developed a new methodology for inferring PPF’s asset exposures and risk (measured by Value-at-Risk) on the basis of limited public data. In addition, to create meaningful measures of underfunding, we adjusted the discount rate used to value their liabilities so that the discount rates better match the risk of the liabilities. We find evidence consistent with reach-for-yield behavior by PPFs in which PPFs take more risk in response to a low rate environment. The effects of low rates on risk-taking were especially pronounced for PPFs that were more underfunded, received lower contributions from state sponsors, or are affiliated with states with weaker public finances. We also quantified the risk to public finances that are attributable to risk-taking by PPFs.

Our measure of risk also allow us to compute the losses that would be suffered by PPFs under a severely adverse economic scenario. Based on our results, we infer that the potential loss transferable to the states if a 1-in-20 years adverse return were to occur in 2016 would have been on average about 3% of states’ income, or about 36% of states’ debt. We attribute between 12% and 33% of the losses to reach for yield behavior by the funds.
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Appendix

A. Comparative Static Analysis for Theoretical Model.

In this appendix we provide comparative statics analysis for the benchmark theoretical model in section 2. To better understand the generality of our results with power utility, we use comparative static analysis to derive how risk taking is related to funding ratios, state finances, disposable income, and the equity premium when state debt is riskfree. For the comparative static analysis we assume that the optimal portfolio choice is an interior solution, and we assume the utility function is twice differentiable in wealth. To simplify the comparative statics, we assume that the value of pension fund asset portfolios at date \( t \) will be less than pension fund assets with probability 1, and that therefore any difference must be covered by taxes.\(^40\) This assumption implies the terms involving Max(.,0) in the equations above are always positive, which vastly simplifies the analysis. The optimization for choosing the portfolio, after applying the simplification takes the form:

\[
Max_{\omega} \ E_0 \ U_t[Y_t(1 - SDI_t - PDI_t \left(1 - FR_0 \left( 1 + \omega(\bar{R} - 1) \right) \right))],
\]

Where, \( \bar{R} = e^{(\lambda - 5\sigma^2)t + \sigma \sqrt{t} \epsilon} \).

Note: we suppress the dependence of the funding ratio \( FR_0 \) and the risk premium \( \lambda \) on the risk-free rate for simplicity since we assume each of these arguments is monotone in the risk-free rate.

The first order condition for choice of \( \omega \) reduces to the expression

\[
E_0 \ U'_t[\cdot](\bar{R} - 1) = 0.
\]

Given this first order condition, our comparative statics results use very mild regularity conditions, which require the funding ratio does not exceed 1, and the sum of state debt to income and pension debt to income is less than 1, which appears to be satisfied for all U.S. states.\(^41\) To sign the comparative statics, additional assumptions are required about the representative citizen’s preferences. We assumed the representative citizen is strictly risk-averse. This implies \( U'_t[\cdot] < 0 \). As shown in the proof, the comparative static results also roughly depend on how the absolute risk aversion of the representative citizen at date \( t \) (measured by \( -U''_t[\cdot]/U'_t[\cdot] \)) changes with his consumption. Theory does not pin down how

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\(^40\) This is equivalent to assuming that the funding ratio is chosen to ensure that assets at date \( t \) do not exceed liabilities at date \( t \). This is reasonable since the fund would like to avoid providing beneficiaries with more payments at date \( t \) then the beneficiaries are owed. The mathematical form of the assumption is \( 1 - FR_0(t_f) \left( 1 - \omega + \omega e^{(\lambda - 5\sigma^2)t + \sigma \sqrt{t} \epsilon} \right) \geq 0 \).

\(^41\) An exception is The Commonwealth of Puerto Rico, where primary “state” debt to income exceeded 1 in 2014 (see Norcross and Gonzalez (2016)).
risk preferences change with consumption, therefore we consider both possibilities when reporting the comparative statics results.

Our main finding from the comparative statics is they cannot always be signed, but they can be signed more often when the absolute risk aversion of the representative citizen positively covaries with consumption, which we interpret as the representative citizen wants the state to take less risk as the representative citizen becomes wealthier. In this circumstance, the comparative statics show that risk increases as funding ratios decline, consistent with reach for yield. In addition, risk decreases with the representative citizen’s income, and risk decreases with state debt to income. Our findings for the relationship between risk and \( \lambda \) is ambiguous. Intuitively, this is because when the reward for risk-taking increases, it can result in taking less risk to obtain investment objectives or it can result in more taking because it is better rewarded. Because of the ambiguity, our results show it is possible that lower rates can increase risk premia but lead to lower risk taking through the risk premium effect, while at the same time lead to more risk-taking through the reach for yield effect. Which directions these effects go is an empirical question.

The details on our comparative statics results are as follows:

Proposition A1. If \( 0 < FR_0 < 1, 0 < PDI_t < 1, Y_t > 0, SDI_t + PDI_t < 1, \) and the maximizing value of \( \omega \in (0,1) \), then

A. If the absolute risk aversion of the representative citizen is monotone increasing in consumption, then \( \frac{d\omega}{dFR_t} < 0, \frac{d\omega}{dSDI_t} < 0, \frac{d\omega}{dY_t} < 0, \) and \( \frac{d\omega}{d\lambda} \) has ambiguous sign.

B. If the absolute risk aversion of the representative citizen is monotone decreasing in consumption, then \( \frac{d\omega}{dSDI_t} > 0, \frac{d\omega}{dFR_t} \) has ambiguous sign, \( \frac{d\omega}{dY_t} \) has ambiguous sign, and \( \frac{d\omega}{d\lambda} \) has ambiguous sign.

Proof:

Taking the total differential of the first order condition with respect to \( Y_t, SDI, FR_0 \), and \( \lambda \) produces the equation:

\[
0 = E_0 U''[\cdot](\bar{R} - 1)^2 Y_t PDI_t FR_0 \ d\omega \\
+ E_0 U''[\cdot](\bar{R} - 1) \left[ (1 - SDI_t - PDI_t \left( 1 - FR_0 \left( 1 + \omega(\bar{R} - 1) \right) \right) \right] \ dY_t \\
+ E_0 U''[\cdot] Y_t (\bar{R} - 1) d SDI_t \\
+ E_0 U''[\cdot] (\bar{R} - 1) \left[ Y_t PDI_t (1 + \omega(\bar{R} - 1)) \right] dFR_0 \\
+ (E_0 U'[\cdot] \bar{R} t + E_0 U'[\cdot] (\bar{R} - 1) Y_t PDI_t FR_0 \omega \bar{R} t) d\lambda
\]
Rearrangement leads to the following comparative static results:

\[
\frac{d\omega}{dY_t} = \frac{-E_0 U''_.t](\bar{R} - 1) \left[ (1 - SDI_t - PDI_t \left(1 - FR_0 \left(1 + \omega(\bar{R} - 1)\right)\right) \right]}{E_0 U''_.t](\bar{R} - 1)^2 Y_t PDI_t FR_0} \\
\frac{d\omega}{dSDI_t} = \frac{-E_0 U''_.t] Y_t (\bar{R} - 1)}{E_0 U''_.t](\bar{R} - 1)^2 Y_t PDI_t FR_0} \\
\frac{d\omega}{dFR_0} = \frac{-E_0 U''_.t](\bar{R} - 1)[Y_t PDI_t(1 + \omega(\bar{R} - 1))]}{E_0 U''_.t](\bar{R} - 1)^2 Y_t PDI_t FR_0} \\
\frac{d\omega}{d\lambda} = \frac{-(E_0 U''_.t][\bar{R} t + E_0 U''_.t](\bar{R} - 1)Y_t PDI_t FR_0 \omega \bar{R} t)}{E_0 U''_.t](\bar{R} - 1)^2 Y_t PDI_t FR_0}
\]

To sign the comparative statics note that the denominator in the expressions for all of the comparative statics is negative. Furthermore, the numerator of the expression for \(\frac{d\omega}{dY_t}\) can be written as the sum of the terms \(-E_0 U''_.t](\bar{R} - 1)[(1 - SDI_t - PDI_t(1 - FR_0)])\) and \(-E_0 U''_.t](\bar{R} - 1)^2 PDI_t FR_0 \omega\). The second of the terms is unambiguously positive. If the first term is also unambiguously positive, it confirms the sign of \(\frac{d\omega}{dY_t}\) in case A. In the first term, \([(1 - SDI_t - PDI_t(1 - FR_0)]) > 1 - SDI_t - PDI_t > 0\). Therefore, the first term is greater than zero if \(-E_0 U''_.t][\bar{R} - 1]\) > 0. This condition can be rewritten as \(E_0 \frac{-u''_.t[}{u'_.t[} \times U'_t[\bar{R} - 1]] > 0\). This can further be written as:

\[
E_0 \left\{ \frac{-u''_.t[}{u'_.t[} \times E_0 \left\{ U'_t[\bar{R} - 1]]\right\} + Cov \left\{ \frac{-u''_.t[}{u'_.t[}, U'_t[\bar{R} - 1]]\right\} > 0 \right. (A.1)
\]

From the first order condition for choice of \(\omega\), \(E_0 \left\{ U'_t[\bar{R} - 1]]\right\} = 0\). Therefore, the first term in equation (A.1) above is zero. In the second term above \(-\frac{u''_.t[}{u'_.t[}\) is absolute risk aversion, and the second term is roughly speaking increasing in consumption since consumption is higher when \(\bar{R}\) increases, and algebra shows the second term is increasing for most plausible values of \(\bar{R}\). Therefore, when absolute risk aversion and consumption are positively correlated, then

\[^{42}\text{For the purposes of calibration of the model, the time horizon } t \text{ is assumed to be one year. For longer horizons it would be necessary to recalibrate risk aversion to the appropriate horizon. } U'_t[\bar{R} - 1] \text{ is clearly increasing for } \bar{R} \leq 1 \text{ since the representative citizen is assumed to be strictly risk averse. For } \bar{R} \geq 1, \text{ differentiation and rearrangement shows the expression is increasing in } \bar{R} \text{ if } \frac{u''_.t[}{u'_.t[} \leq \frac{1}{k-\sigma}. \text{ If we choose a coefficient of relative risk aversion } (= \frac{u''_.t[}{u'_.t[}) \text{ of 10 which is plausible for the representative citizen, and a pension debt to income ratio of 0.7 (the maximum value in Norcross and Gonzalez (2016) and a value of } \omega = 1, \text{ and an } FR_0 = 1, \text{ both of which are unrealistically high, then the term is increasing if } \bar{R} \leq 114 \text{ percent.}}\]

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risk taking is decreasing in income. The proof for \( \frac{d\omega}{dFR_t} \) in case A and for \( \frac{d\omega}{dSDI_t} \) are similar in case A.

In case B, there are two terms in the numerator for \( \frac{d\omega}{dY_t} \), and they have opposite signs. Hence the sign of \( \frac{d\omega}{dY_t} \) is ambiguous. The same is true for \( \frac{d\omega}{dFR_0} \) hence it too has an ambiguous sign.

Finally, to prove the results for \( \frac{d\omega}{d\lambda} \), note that by adding and subtracting \( E_0U''_{t}[.] (\bar{R} - 1)Y_t PDI_t FR_0 \omega t \) to the numerator, and rearranging the numerator can be rewritten to have three terms. The first term is \( -(E_0U'_t[.]\bar{R} t) \), which from the first-order condition is equal to \( -E_0U'_t[.]t \), which is less than zero. The second term is \( -E_0U''_{t}[.]Y_t PDI_t FR_0 \omega (\bar{R} - 1)^2 t \), which is positive. The third term is \( -E_0U''_{t}[.](\bar{R} - 1)Y_t PDI_t FR_0 \omega t \). This term is also positive in case A as shown above, which makes the sign in case A ambiguous. In case B, the sign of third term is negative, and the sign of the overall expression remains ambiguous. ■
Table 1: Summary Statistics of Public Pension Fund Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Median</th>
<th>St Dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>Returns (Annual)</td>
<td>.062</td>
<td>.085</td>
<td>.109</td>
</tr>
<tr>
<td>% Equities</td>
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<td>.551</td>
<td>.111</td>
</tr>
<tr>
<td>% Fixed Income</td>
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<td>.262</td>
<td>.090</td>
</tr>
<tr>
<td>% Real Estate</td>
<td>.054</td>
<td>.053</td>
<td>.045</td>
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<tr>
<td>% Cash</td>
<td>.024</td>
<td>.013</td>
<td>.037</td>
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<tr>
<td>% Alternatives</td>
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<td>.073</td>
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<tr>
<td>% Other</td>
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<td>6.682</td>
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<td>Actuarial liabilities ($ mil)</td>
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<td>Funding ratio, rediscounted</td>
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<td>.492</td>
<td>.1537</td>
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<tr>
<td>Percent ARC paid</td>
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<td>State debt-to-income ratio</td>
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<td>.079</td>
<td>.039</td>
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<tr>
<td>State bond rating</td>
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<td>3</td>
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Table 1 is based on the Public Plans Data dataset, from the Center for Retirement Research (CRR) at Boston College, and the Center for State and Local Government Excellence (SLGE). The data is publicly available at www.publicplansdata.org. The data summarized in the table above is as follows.

Returns (Annual) is the InvestmentReturns_1yr variable in the dataset, which reports each fund’s returns in a given fiscal year. Asset allocation shares denoted by %AssetClass, shows the percentage allocation to a particular asset class. There are total six categories of asset classes—equities, fixed income, real estate, cash, alternatives, and other. Their allocation shares are found as equities_tot, FixedIncome_tot, RealEstate, CashAndShortTerm, alternatives, and other (note capitalization) respectively in the original data.

Actuarial assets and liabilities are reported as ActAssets_GASB and ActLiabilities_GASB in thousands of dollars. The original funding ratio comes from the dataset’s ActFundedRatio_GASB variable, which is ActAssets_GASB divided by ActLiabilities_GASB. Our rediscounted funding ratio is computed as ActAssets_GASB divided by a rediscounted ActLiabilities_GASB, where the rediscounting uses duration and convexity variables computed as follows: we take the average of duration and convexity in the years 2014-2016 for each fund. GASB data is only consistently available for these three years. We extrapolate the average 2014-2016 duration and convexity to the entire sample period of 2001-2016, and then rediscount funding ratio.

Percent ARC paid is PercentARCPaid in the dataset, and reports the percent of the actuarial required contribution paid by each fund in each year.

State debt-to-income ratio and state bond rating are not in Public Plans Data, but are easily accessible data. We collect state income from the Bureau of Economic Analysis, and state debt from the United States Census Bureau, and calculate the ratio for each state and year. State bond rating is the Standard and Poor’s grade by state year, coded numerically as AAA = 1 through BBB = 8.
Table 2: List of Indices

<table>
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<th>Index name</th>
<th>Symbol</th>
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<td>BCOM</td>
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<td>Bloomberg Commodities total returns</td>
<td>BCOMTR</td>
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<tr>
<td>Thomson Reuters/CoreCommodity Index total returns</td>
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<td>HEDGNAV</td>
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<tr>
<td>Barclays Hedge Fund</td>
<td>BGHSHEDG</td>
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<tr>
<td>ICE USD LIBOR 3 Mon</td>
<td>ICELIBOR3Mon</td>
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<tr>
<td>SIFMA Minu Swap Index</td>
<td>MUNIPSA</td>
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<tr>
<td>S&amp;P 500 Index</td>
<td>SPX</td>
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<td>Russel 3000</td>
<td>Russel3000</td>
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<tr>
<td>FTSE All World Excluding US</td>
<td>FTAW02</td>
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<tr>
<td>Dow Jones Global Index</td>
<td>W1DOW</td>
</tr>
<tr>
<td>ICE BoAML US Broad Market Ind</td>
<td>US00</td>
</tr>
<tr>
<td>ICE BofAML Global Broad Market</td>
<td>GBXD</td>
</tr>
<tr>
<td>Citi World Government Bond Ind</td>
<td>SBWGU</td>
</tr>
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<td>FTSE NAREIT All Equity REITS I</td>
<td>FNER</td>
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*Notes:* The table provides information on the set of publicly traded indices that are used to estimate funds category-index risk exposures. See section 3 of the text for further details.
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<td>(0.033)</td>
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<td>0.30***</td>
<td>(0.10)</td>
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<td>(2.29)</td>
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<tr>
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<td>(2.56)</td>
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Observations 2,473  
R-squared 0.897  

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1
Table 4. Risk and underfunding, cross-sectional relation over time

### (a) Fixed funding ratio, original

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### (b) Fixed funding ratio, rediscounted

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<td>-0.013*</td>
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<td>0.11***</td>
<td>0.17***</td>
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### (c) Time-varying funding ratio, original

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### (d) Time-varying funding ratio, rediscounted

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Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1
Table 5. Results with fixed funding ratio

Specification: \( \text{VaR}_{pt} = \alpha + \beta \cdot FR_p + \gamma \cdot TrYield_t + \delta \cdot FR_p \cdot TrYield_t + \epsilon_{pt} \)

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<td>FR</td>
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<td>1 yr Tr Yield</td>
<td>-0.018** (0.0069)</td>
<td>-0.018** (0.0069)</td>
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<tr>
<td>1 yr Tr Yield * FR</td>
<td>0.00077 (0.00052)</td>
<td>0.0057*** (0.0013)</td>
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<tr>
<td>10 yr Tr Yield</td>
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<td>10 yr Tr Yield * FR</td>
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<td>0.054* (0.030)</td>
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<td>0.19*** (0.027)</td>
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<td>0.237</td>
<td>0.238</td>
<td>0.069</td>
<td>0.070</td>
<td>0.182</td>
<td>0.183</td>
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Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1
Table 6. Results with the time-varying funding ratio

Specification: \( VaR_{pt} = \alpha + \beta \times FR_{pt-1} + \gamma \times TrYield_{t} + \delta \times FR_{pt-1} \times TrYield_{t} + \mu + \epsilon_{pt} \)

<table>
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<th></th>
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<th></th>
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</thead>
<tbody>
<tr>
<td>FR (t-1)</td>
<td>-0.0079 (0.030)</td>
<td>-0.060 (0.045)</td>
<td>-0.045 (0.028)</td>
<td>-0.13** (0.056)</td>
<td>0.0042 (0.020)</td>
<td>-0.037 (0.030)</td>
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<tr>
<td>1 yr Tr Yield</td>
<td>-0.017** (0.0067)</td>
<td>-0.017** (0.0067)</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 yr Tr Yield * FR (t-1)</td>
<td>0.0028 (0.0024)</td>
<td>0.0099** (0.0038)</td>
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<tr>
<td>10 yr Tr Yield</td>
<td>-0.015 (0.011)</td>
<td>-0.015 (0.011)</td>
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</tr>
<tr>
<td>10 yr Tr Yield * FR (t-1)</td>
<td>0.0099*** (0.00041)</td>
<td>0.022*** (0.0046)</td>
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</tr>
<tr>
<td>Post-crisis</td>
<td></td>
<td></td>
<td>0.053* (0.029)</td>
<td>0.053* (0.029)</td>
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<tr>
<td>Post-crisis * FR (t-1)</td>
<td>-0.015 (0.013)</td>
<td>-0.045** (0.018)</td>
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</table>

FR rediscouted        | No  | Yes | No  | Yes | No  | Yes |
| Observations         | 1,289 | 1,289 | 1,289 | 1,289 | 1,289 | 1,289 |
| Number of funds      | 111  | 111  | 111  | 111  | 111  | 111  |
| Fund fixed effects   | Yes  | Yes  | Yes  | Yes  | Yes  | Yes  |
| R-squared            | 0.254 | 0.255 | 0.088 | 0.089 | 0.199 | 0.201 |

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1
Table 7. The role of public finances

Specifications:

\[ VaR_{pt} = \alpha + \beta \cdot X_{pt} + \gamma \cdot TrYield_t + \delta \cdot FR_p + \eta \cdot X_{pt} \cdot TrYield_t + \epsilon_{pt} \quad \text{(columns 1, 4)} \]

\[ VaR_{pt} = \alpha + \beta \cdot X_{pt} + \gamma \cdot TrYield_t + \delta \cdot FR_p + \eta \cdot X_{pt} \cdot FR_{pt-1} + \mu_p + \epsilon_{pt} \quad \text{(columns 2, 5)} \]

\[ VaR_{pt} = \alpha + \beta \cdot X_{pt} + \gamma \cdot TrYield_t + \delta \cdot FR_{pt-1} + \eta \cdot X_{pt} \cdot FR_{pt-1} + \mu_p + \epsilon_{pt} \quad \text{(col. 3, 6)} \]

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<td>0.0026***</td>
<td>0.0018*</td>
<td>0.012**</td>
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<td>(0.13)</td>
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<td>(0.00023)</td>
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<td>-0.018**</td>
<td>-0.017**</td>
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<td>(0.0077)</td>
<td>(0.0061)</td>
<td>(0.0070)</td>
<td>(0.0072)</td>
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<td>1 yr Tr yield * State debt/income</td>
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<td></td>
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<tr>
<td>(0.033)</td>
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<tr>
<td>1 yr Tr Yield * State bond rating</td>
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<td>-0.020***</td>
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<tr>
<td>(0.00081)</td>
<td>(0.00076)</td>
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</tr>
<tr>
<td>FR fixed</td>
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<td>-0.034***</td>
<td>-0.019***</td>
<td>-0.020***</td>
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<td>FR fixed * State debt/income</td>
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<td>(0.0070)</td>
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<td>FR (t-1)</td>
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<td>FR (t-1) * State bond rating</td>
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Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1
Table 8. Robustness with VaR-based measures of PPF riskiness

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<td>Unconditional VaR</td>
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<td>Unconditional VaR</td>
<td>Conditional VaR</td>
<td>Unconditional VaR</td>
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<td>-0.043*** (0.00047)</td>
<td>-0.034*** (0.0082)</td>
<td>-0.0078*** (0.00027)</td>
<td>-0.069*** (0.00041)</td>
<td>-0.050*** (0.011)</td>
<td>-0.013*** (0.000038)</td>
<td>-0.14** (0.060)</td>
<td>-0.0045 (0.020)</td>
<td>-0.17* (0.083)</td>
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<td>FR (t-1)</td>
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<td>0.012 (0.013)</td>
<td>-0.087 (0.054)</td>
<td>-0.14** (0.060)</td>
<td>-0.0045 (0.020)</td>
<td>-0.17* (0.083)</td>
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<tr>
<td>1 yr Tr Yield</td>
<td>-0.019** (0.0075)</td>
<td>0.00039 (0.00032)</td>
<td>-0.018** (0.0069)</td>
<td>-0.019** (0.0071)</td>
<td>0.00018 (0.00033)</td>
<td>-0.017** (0.0067)</td>
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<tr>
<td>1 yr Tr Yield * FR fixed</td>
<td>0.0080*** (0.0012)</td>
<td>0.0038** (0.0014)</td>
<td>-0.00035 (0.00057)</td>
<td>0.0099** (0.0041)</td>
<td>0.0025 (0.0017)</td>
<td>0.012*** (0.0036)</td>
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<td>10 yr Tr Yield</td>
<td>-0.018 (0.013)</td>
<td>-0.0065 (0.0069)</td>
<td>-0.015 (0.011)</td>
<td>-0.017 (0.013)</td>
<td>-0.0040 (0.0069)</td>
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<td>10 yr Tr Yield * FR fixed</td>
<td>0.012*** (0.00024)</td>
<td>0.0065** (0.0028)</td>
<td>0.0013*** (0.000066)</td>
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<td>FR rediscounted</td>
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<td>Yes</td>
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<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
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<td>Fund fixed effects</td>
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<td>0.237</td>
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<td>0.050</td>
<td>0.069</td>
<td>0.103</td>
<td>0.610</td>
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Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1
Table 9. Robustness with traditional measures of PPF riskiness

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<th>(7)</th>
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<td>Share of alternatives</td>
<td>Share of alternatives</td>
<td>Share of alternatives</td>
<td>Share of alternatives</td>
</tr>
<tr>
<td>FR fixed</td>
<td>-0.13 (0.11)</td>
<td>-0.26 (0.19)</td>
<td>0.012 (0.044)</td>
<td>-0.069 (0.080)</td>
<td>-0.090 (0.091)</td>
<td>-0.029 (0.070)</td>
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<td>-0.013 (0.078)</td>
<td>-0.026*** (0.0045)</td>
<td>-0.026*** (0.0046)</td>
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<td>1 yr Tr Yield</td>
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<td>-0.026*** (0.0044)</td>
<td>-0.026*** (0.0045)</td>
<td>-0.026*** (0.0046)</td>
<td>-0.025*** (0.0043)</td>
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<td>1 yr Tr Yield * FR fixed</td>
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<td>1 yr Tr Yield * FR (t-1)</td>
<td>-0.0078 (0.017)</td>
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<tr>
<td>10 yr Tr Yield</td>
<td>-0.048*** (0.0055)</td>
<td>-0.048*** (0.0067)</td>
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<td>Post-crisis</td>
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<td>0.10*** (0.016)</td>
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<tr>
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<tr>
<td>State debt/income</td>
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<tr>
<td>1 yr Tr Yield * State debt/income</td>
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Robust standard errors in parentheses:
*** p<0.01, ** p<0.05, * p<0.1
Figure 1. PPFs’ portfolio allocation and expected return targets

Panel (a)

Expected Return (ER) Targets

Panel (b)

Ratio of PPFs’ Assets to Projected Liabilities

Panel (c)

PPF Assets and Portfolio Allocation

Notes: The figure presents summary information on public pension funds expected return targets (panel a), the ratio of plans assets to actuarial liabilities (panel b), and how plans aggregated asset allocations have changed through time (panel c).
Notes: For the theoretical model in Section 2, when states are unable to default on their debt, the Figure presents the relationship between pension fund asset risk, the plans funding ratio, and the risk-free interest rate. Risk in the figure is measured as the proportion of risky asset in the fund’s asset portfolio. The funding ratio is the present value of fund liabilities discounted by the risk-free rate. For further details see Section 2 of the text.
Figure 3: Risk as a Function of Debt to State Income and Pension Funding Ratio When State Debt is Risk-Free.

Notes: For the theoretical model in Section 2, when states are unable to default on their debt, the Figure presents the relationship between pension fund asset risk, the plans funding ratio, and the ratio of state debt to state income. Risk in the figure is measured as the proportion of risky asset in the fund’s asset portfolio. The funding ratio is the present value of fund liabilities discounted by the risk-free rate. Debt to state income is measured as debt at date $t$ divided by state income at date $t$. For further details see Section 2 of the text.
Figure 4: Pension Fund Risk vs Pension Funding Ratio and Debt to State Income when State Debt is Risky

Notes: For the theoretical model in Section 2, when states can choose to default on their debt, the Figure presents the relationship between pension fund asset risk, the plans funding ratio, and the ratio of state debt to state income. Risk in the figure is measured as the proportion of risky asset in the fund’s asset portfolio. The funding ratio is the present value of fund liabilities discounted by the risk-free rate. Debt to state income is measured as debt at date $t$ divided by state income at date $t$. For further details see Section 2 of the text.
Figure 5: Pension Fund Risk vs Risk-Free Rate and Debt to State Income when State Debt is Risky

Notes: For the theoretical model in Section 2, when states can choose to default on their debt, the Figure presents the relationship between pension fund asset risk, the risk free rate, and the ratio of state debt to state income. Risk in the figure is measured as the proportion of risky asset in the fund’s asset portfolio. Debt to state income is measured as debt at date $t$ divided by state income at date $t$. For further details see Section 2 of the text.
Figure 6. VaR-based measures of risk

Panel (a)

Conditional VaR (5%, annual)

Panel (b)

Unconditional VaR (5%, annual)

Notes: For the Public Pension Funds in our data sample, Panel (a) present time-series of 5% Value-at-Risk (VaR) for each fund (gray lines), average of funds 5% VaR (black lines) and averages of VaR pre- and post- 2007-09 financial crisis (red lines). Panel (b) presents time-series of 5% unconditional VaR for each fund (gray lines), and the average of funds 5% VaR (black lines).
Figure 7. Rediscounthing liabilities

Figure 8. Conditional VaR vs. lagged funding ratio, 2016
Figure 9. Rediscounting actuarial liabilities over time

![Average funding ratios over 2001-2016](image)

Funding ratio = Act Assets / Act Liab

Funding ratio, original
Funding ratio, rediscounted actuarial liabilities

Figure 10. Robustness tests, cross section

Panel (a) Active-shares conditional VaR vs. lagged funding ratio, 2016

![Panel (a) Active-shares conditional VaR vs. lagged funding ratio, 2016](image)

Funding ratio = Actuarial Assets / TPL

Beta = -0.0181, T-stat = -3.6200, R-sq = 0.1085, N = 108

Beta = -0.0442, T-stat = -4.9111, R-sq = 0.1839, N = 108
Panel (b) Unconditional VaR vs. lagged funding ratio, 2016

Panel (c) Conditional VaR vs. lagged funding ratio rediscounted with corporate bond yields
Figure 11. PPFs’ riskiness associated with underfunding and low risk-free rates

Panel (a)

Panel A: RFY related to underfunding

Loss = 5% VaR * Actuarial Assets / State Income

Panel B: RFY related to underfunding and low rates

Loss = 5% VaR * Actuarial Assets / State Income