Worker Surveillance: Job Rationing and Wage Suppression

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CERGE-EI

January 2019

Preliminary: Do not circulate or cite

Abstract

I present a search-and-matching model of unemployment with moral hazard and imperfect monitoring. Job rationing arises endogenously as the incentive compatibility constraints on wages prevent the market from reaching full employment even in the absence of matching frictions. We use the framework to investigate enhanced surveillance in the workplace; a trend in the modern labor market. Whether firms chose efficiency wages, deferred payment, or a combination of both, improvements in monitoring technology lower wages and limit wage growth. Using British workplace survey data, we find evidence that wages in firms that have increased their monitoring levels have experienced smaller wage growth, and that these firms have increased their presence in the labor market after the great recession. Our findings contribute to the understanding of the post recession puzzling disconnect between wage growth and other labor market indicators.

Keywords: Wages, Labor Markets, Search and Matching, Moral Hazard

JEL Classification: J41, J64

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1 Introduction

Search-and-matching models have become the workhorse tool for the analysis of unemployment and labor dynamics due to their analytical tractability and rich comparative statics. Recent literature has pointed out that these models can be further enhanced to better replicate some empirical observations by lodging sources of unemployment other than matching frictions. Michaillat (2012) points out that during deep recessions, workers queueing outside factories to get a job is an observation at odds with standard search models, which predict that if firms could hire workers right on the spot, at minimum recruitment costs, the rate of unemployment should be low. This inconsistency suggests that other factors besides matching frictions intervene in the job creation process and must be accounted for in the modeling so two sources of unemployment can be captured: job rationing, defined as the level of unemployment that would prevail in a frictionless market, and frictional unemployment, defined as the additional unemployment due to matching frictions.

This paper presents a search-and-matching model of unemployment that incorporates worker moral hazard to microfoundate job rationing. Building on Mortensen (1989), Mortensen and Pissarides (1999a), and Rocheteau (2001), I embed the Shapiro and Stiglitz (1984) efficiency wages framework into a search-and-matching labor market and show that worker moral hazard and imperfect monitoring impose incentive constraints on wages that prevent the market from reaching full employment even in the absence of matching frictions.1 The specific form and consequences of efficiency wages vary across the literature but they generally make wages include a premium above the market average to motivate workers. However, the average wage is set by firms themselves so in equilibrium all firms pay the same premium which results in higher wages and therefore higher unemployment. Thus, the wage incentives firms give to make the inside job offer more attractive are reinforced through a deterioration of the worker’s outside options; firms open fewer vacancies so workers experience longer unemployment spells. Consequently, job rationing arises as a disciplinary device without assumptions of decreasing returns to scale or extremely low productivity.

The model predicts that job rationing is more severe during economic downturns since low productivity disables the use of high wages as motivation, so employees can be incentivized only by potentially longer unemployment spells.

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1 I decided to use the term efficiency wages to relate my work with the literature but given the way wages work in the model they are better described as incentive wages. Phelps et al. (1994) already recognizes that this title is more accurate.
Unlike previous work, the model predicts some degree of job rationing even in high-productivity states. This novel result is a direct consequence of moral hazard. In the model, the worker’s decision to shirk depends on his valuation of a job; workers won’t risk their job by shirking if their employment surplus is large enough. The surplus of a current job depends on the wage and outside opportunities, high wages and potential long unemployment spells make workers value their job and avoid shirking. In high-productivity states firms open more vacancies, raising the exit rate of unemployment and driving down its expected length. In this new environment, workers find it easier to get a job so current matches are less valuable; a wage adjustment is necessary to ensure worker’s effort. This is the role of the efficiency wage, it creates a wage floor that adjusts with the exit rate of unemployment to compensate workers for their improved job finding conditions. In a very tight market, workers would find a job almost instantaneously so the wage required to motivate them would have to be so large that they could not be hired profitably. Firms must ration jobs even in high productivity states to keep the efficiency wage affordable. Therefore, job rationing persist as a motivational device even in high productivity states.

Another novel insight of the model is the way shifts in productivity affect job rationing relative to frictional unemployment. According to previous work, the share of total unemployment due to job rationing always decreases with higher productivity. In contrast, I show that the way this share changes with productivity depends on the elasticity of the matching function with respect to unemployment. When the matching function is relatively inelastic, an increase in vacancies due to higher productivity greatly improves the exit rate of unemployment and creates disincentivizing effects that counteract the incentives of higher wages, job rationing must remain relatively unchanged as an incentive device; higher productivity disproportionally reduces frictional unemployment. If the matching function is very elastic, more vacancies do not generate large disincentivizing effects so the higher wages from higher productivity are enough to incentivize workers. Job rationing decreases relative to frictional unemployment because it is no longer needed.

When ex-ante worker-skill heterogeneity is introduced, the model offers a tractable and versatile framework to analyze asymmetric outcomes across the labor force such as diverse unemployment rates, unemployment volatilities, and wages. The model implies that low-skilled workers have higher unemployment rates that wildly fluctuate with productivity, whereas high-skilled workers experience much lower unemployment rates that remain relatively constant, both

\footnote{Michaillat (2012) and Ferraro (2018).}
well documented facts in empirical studies.\textsuperscript{3} Also consistent with the data is the prediction that the wages of high-skilled workers are relatively more volatile than those of the low-skilled. An insight of the model is that at the core of these asymmetries is the composition of the idiosyncratic unemployment rates, low-skilled unemployment has a larger share of job rationing, which is more elastic to productivity shocks than frictional unemployment. Thus, in low aggregate productivity states, the unemployment pool is characterized by a distribution of skills that is skewed to the left; low skilled workers are over represented. The skewness degree depends on aggregate productivity; when aggregate productivity is high, the distribution of skills in the unemployment pool closely replicates the skill distribution in the workforce.\textsuperscript{4} The diversity in labor market outcomes across the labor force generated by the model creates an ideal framework for the analysis of policies affecting workers differently according to their skills and productivity.

We use our theoretical framework to study the impact of enhanced worker surveillance on wage growth. According to our model, better monitoring technology would lower wages and make the trajectory of wage growth less steep. Using WERS workplace survey British data we find that evidence that the intensity of monitoring has increased within firms and that it has a negative impact on wages. Moreover, after the great recession the representation of these firms increased.

The reminder of the paper is organized as follows. The next section presents an overview of the related literature. Section 3 introduces a search-and-matching unemployment model with moral hazard. I introduce the basic features of the model assuming a representative worker and show the existence of job rationing. Section 4 introduces skill heterogeneity among workers and characterizes the behavior of job rationing for workers with different skills. Section 5 calibrates the model and presents numerical simulations to quantitatively assess the predictions of the model. Section 6 presents concluding remarks. Proofs and derivations are included in the appendix.

\textsuperscript{3}The fact that different kinds of workers experience different unemployment dynamics is well documented. The literature usually classifies workers by age and education and in general finds that younger workers with less education experience higher and more volatile unemployment levels. See Ferraro (2018) and Grossman (2013).

\textsuperscript{4}Clark and Summers (1980), Jaimovich and Siu (2009), Gomme et al. (2004) and Lindquist (2004) show that less skilled individuals have much more volatile hours of work than more skilled workers. Lower-skilled individuals also reset their wages less frequently and have less volatile. Daly et al. (2012) conclude that less skilled individuals have stickier wages than high-skilled individuals. Ferraro (2018) finds that low-skilled workers experience higher unemployment rates and that they account for most of the variation in aggregate unemployment.
2 Related Literature

There are several strands of the literature on unemployment to which the current work relates to. Firstly, the works motivating this paper which share the purpose of proposing search-and-matching models of unemployment that can explain its asymmetric behavior at different stages in the cycle, or more precisely, present mechanisms that explain why jobs are rationed in recessions. Michaillat (2012) shows that the Pissarides (2000) and Hall (2005) models predict asymptotic convergence to full employment in the absence of matching frictions, which makes them unsuitable for the analysis of recessionary unemployment which is characterized by an acute job shortage regardless of the matching environment. He proposes a model where job rationing arises during recessions as the result of combining decreasing returns to scale and exogenous wage rigidity. Ferraro (2018) evinces the lack of job rationing in models with endogenous separation rates a la Mortensen and Pissarides (1994). He presents a model that relies on worker heterogeneity and extremely low levels of productivity to generate job rationing at an aggregate level. In his model, during severe economic downturns productivity is so low that the total surplus of a match with low-skilled workers is negative, leaving them completely out of work.

An important theoretical contribution of my model is the way job rationing arises as a result of the constraints that moral hazard and imperfect monitoring impose on wages, making the assumptions of decreasing returns to scale or extremely low productivity unnecessary. In both of the models above mentioned, the presence of job rationing is contingent upon bad states of the economy, meaning that in economic expansions matching frictions are the only cause of unemployment. In contrast, in my model the existence of job rationing does not depend on the state of the economy. The inability of wages to achieve full employment persists regardless of productivity; it is rather its intensity that fades as productivity increases. The risk of moral hazard is always present so in any equilibrium there is always the need of unemployment as a disciplinary device. However, the unemployment rate necessary to motivate a worker decreases with the product of the match. This means that both rationing and frictional unemployment always coexist but their shares in total unemployment change depending on productivity. This is another contrasting feature of my model; unlike the previous works, whether the share of job rationing increases or decreases with productivity depends on the matching function, that is, on the way the probabilities of finding a job and filling a vacancy affect the incentives of workers and firms. This characteristic highlights important mechanisms behind the incentives that both sides of the market have. This is
a new insight in the literature.

To generate the wage constraints that create job rationing, I follow the canonical shirking-efficiency wages framework in Shapiro and Stiglitz (1984).\(^5\) Other works have embedded the shirking model in a job search environment. The model I present builds on Mortensen (1989) where potential causes for the indeterminacy and persistence of unemployment are explored under a search equilibrium framework with different assumptions about wage determination. A subtle difference between this model and mine is that instead of having a utility of shirking equal to their utility of leisure, I assume that production is costly for the worker so workers choosing to shirk save the disutility of labor. In his model, the frequency of inspections is an endogenous variable whereas in my model, for simplicity, it is exogenously given. Mortensen and Pissarides (1999b) present a version of the same model with a fixed inspection rate. The model I present closely follows Rocheteau (2001), where the Pissarides (2000) and the Shapiro and Stiglitz (1984) models are combined and worker heterogeneity is introduced. Other models that consider efficiency wages in a search-and-matching environment are Sattinger (1990), Larsen and Malcomson (Larsen and Malcomson).

Since efficiency wages are a very important component of my model, it is important to underscore the literature that provides evidence of the implementation of efficiency wages. Evidence of efficiency wages in practice comes in the form of a negative relationship between higher wages and alternative ways of regulating employee’s effort (supervision). Zaharieva (2010) presents a statistical summary of a large European data set collected by the researchers and shows that about 50% of firms prefer to dismiss workers rather than to reduce base wages in response to an output shock. At the same time one of the two major reasons for avoiding wage reduction is to maintain high effort and working morale. Georgiadis (2013) finds evidence for the implementation of efficiency wages in low-skilled jobs. He exploits a natural experiment provided by the 1999 introduction of the UK National Minimum Wage to test for efficiency wage considerations in a low-wage sector. He finds evidence consistent with a wage-supervision trade-off for non-managerial employees that provides

\(^5\)Although their model does not include matching frictions, they introduce the idea that unemployment is a multifactorial phenomenon and that a convincing unemployment model must try to integrate all these components. About their own works they write: “The type of unemployment studied here is not the only or even the most important source of unemployment in practice. We believe it is however, a significant factor in the observed level of unemployment, especially in lower-paid, lower-skilled, blue collar occupations. It may well be more important than frictional or search unemployment in many labor markets”. This remark is completely in the spirit of my research.
support to the shirking model. Krueger and Summers (1988) find evidence supporting the presence of efficiency wages by looking at the wage differentials of equally skilled workers across industries. More recently, the literature in management control has explored the effect of efficiency wages on employee behavior and social norms finding evidence suggesting the implementation of some sort of efficiency wage. For example, Chen and Sandino (2012) find that high levels of employee compensation can deter employee theft.

Although there is evidence of the implementation of some sort of efficiency wage, the theory is not free of criticism. The most important being the so-called bonding critique succinctly described in Carmichael et al. (1985). The critique states that the involuntary nature of unemployment in efficiency wages models can disappear under more complex wage schemes such as employment fees or bonds posted by workers when initially hired and forfeited if found cheating. Other possible solutions are performance bonds, entrance fees, upward sloping age-earnings profiles, pensions and other deferred compensation schemes. These complex hiring mechanisms can reduce wages to market-clearing levels or at least reduce unemployment to a level where the marginal worker is indifferent between being employed and the alternative thus making all remaining unemployment voluntary. There are theoretical objections to these hiring schemes, mainly regarding moral hazard on the side of the firm and capital constraints on workers. Akerlof and Katz (1986) show that in the most natural framework of the shirking model, in the absence of upfront bonding, upward sloping wage profiles generate wages above market clearing level. This means that in the absence of perfect capital markets, bonding and deferred payment could only reduce the severity of unemployment. An even more important objection to the critique is the fact that these complex contracts are rarely observed in practice. The purpose of the present paper is finding mechanisms whereby unemployment arises for reasons other than matching frictions, the voluntary or involuntary nature of the unemployment is no relevant for the analysis, even if some complex hiring mechanism eliminated the involuntary nature of unemployment, the qualitative results of the model would still hold. For this reason I leave issues related to the bonding critique for future research.

Finally, my model is related to several other models that explore worker heterogeneity in search-and-matching environments. A great deal of these works focus on the way heterogeneity affects the volatility and cyclicality of unemployment and vacancies. Pries (2008) addresses the Shimer (2005) puzzle by introducing worker heterogeneity, a modification that makes the model exhibit considerably greater volatility. In the same spirit is Epstein (2012) introduces worker heterogeneity in production capacity to explain the strong
procyclicality of the vacancy-unemployment ratio. In accordance with these results, when worker heterogeneity is incorporated in my model, it shows that the predicted aggregate unemployment is larger and more volatile the larger the variance of the skill distribution.

Another issue that models with worker heterogeneity investigate is the asymmetries of unemployment experiences across the labor force. Grossman (2013) focuses on how models with heterogeneous labor can affect trade patterns and how globalization affects the distribution of wages and unemployment across the complete spectrum of worker types via its impact on sorting and matching in the labor market. Shi (2002) analyzes an economy with workers with heterogeneous skills and skilled-based technologies. The model generates wage inequality among identical unskilled workers, as well as between-skill inequality. The fact that in my model heterogeneity is modelled as an ex-ante phenomenon, allows it to generate idiosyncratic unemployment rates with a unique composition of frictional and rationing unemployment for workers with different productivities. This fact helps the model account for the differences in unemployment volatility and wages observed across different workers.

3 Job Rationing and Moral Hazard

3.1 The Basic Model

In this section I present a search-and-matching unemployment model with moral hazard and imperfect monitoring based on Mortensen and Pissarides (1999b), Mortensen and Pissarides (1999b), and Rocheteau (2001). I show that, unlike conventional models, the model features job rationing as the result of incentive compatibility constraints imposed on wages. The environment is similar to the basic Pissarides (2000) model with the important difference that shirking is allowed and there is imperfect monitoring. Time is continuous and it is denoted by \( t \). Agents are risk neutral and there is a unit mass of workers and a continuum of firms which can be matched with one worker at most.

For simplicity, there are two levels of work intensity (disutility of work) \( e \) or 0, and this is known by both firms and workers. If the worker exerts effort, the per-unit of time product of a match is \( ay \), where \( y \in \mathbb{R}^+ \) is worker productivity and \( a \in \mathbb{R}^+ \) is an aggregate technology parameter. If the worker shirks, the product of a match is zero. The effort exerted by the worker is observable only after an inspection, which obeys a Poisson process with an
exogenous arrival rate $\lambda \in \mathbb{R}^+$. If the worker is caught shirking the match is terminated. There are no reputational effects so upon meeting a worker, firms do not know whether the worker has a shrinking history or not.

The number of matches made per-unit of time is given by the constant-returns matching function $h(u, v)$, differentiable and increasing in both arguments where $u$ is the fraction of workers who are unemployed and $v$ is the number of vacancies as a fraction of the labor force. Labor market tightness is defined as $\theta \equiv v/u$. The rate at which an unemployed worker finds a job is defined as $f(\theta) \equiv h(u, v)/u = h(1, \theta)$. Similarly, the rate at which a vacancy is filled is given by $q(\theta) \equiv h(u, v)/v = h(1/\theta, 1)$. Unemployed workers find a job more easily in a tighter market, that is, when there are more vacancies relative to the job seekers. Similarly, firms fill vacancies faster when there are more unemployed workers relative to vacancies. The flow cost of an open vacancy is $\gamma \in \mathbb{R}^+$. Matches can be terminated by an exogenous shock following a Poisson process with parameter $s \in \mathbb{R}^+$. There is no on-the-job search, therefore only unemployed workers can search for a job.

### 3.1.1 Worker Behavior

Once matched, it is the worker’s decision to exert effort or shirk. This decision is made based on the lifetime expected utility of taking these actions. A non-shirker is a worker who chooses not to shirk in all periods while his current job lasts. He gets a wage $w$ and suffers a disutility $e$, per unit of time, and could have his exogenously job terminated with probability $s$. The lifetime expected utility of a non-shirker, $E$, obeys the asset-pricing equation

$$rE = w - e + s(U - E),$$

where $r \in \mathbb{R}^+$ is the time rate preference and $U$ is the lifetime expected utility of an unemployed worker. If $E$ represents the asset value of employment, (1) states that the opportunity cost of holding a job without shirking is equal to the current income flow minus the disutility of effort plus the expected capital loss from a change of state.

The expected lifetime utility of someone who chooses to shirk, $S$, during a length of time $dt$, satisfies

$$S = wdt + \exp(-rdt) \left\{ \Pr [\min(\tau_s, \tau_\lambda) \leq dt] U + (1 - \Pr [\min(\tau_s, \tau_\lambda) \leq dt]) E \right\}.$$  

(2)
where \( \tau_\lambda \) is the length of time until the next inspection and \( \tau_s \) is the duration of a job.\(^6\) According to (2), during the time interval \( dt \) a shirker receives a real wage \( w dt \) and has no disutility from work, he loses his job if he is caught shirking or if the match is terminated by an idiosyncratic shock. If neither of these two events occur during the time interval \( dt \), the employed worker stops shirking in all subsequent periods. Notice that, all else remaining equal, if it is optimal for a worker to shirk during a length of time \( dt \), it will be optimal to keep shirking for the next length of time \( dt \), so if a worker decides to shirk he will do it for all subsequent periods. A worker will chose to exert effort over shirking if and only if the lifetime expected value of not shirking is greater the lifetime expected value of doing so. After some manipulation, as \( dt \) approaches zero we have that

\[
E \geq S \iff E - U \geq \frac{e}{\lambda},
\]

(3)

this is the no-shirking condition (NSC) and its derivation is shown in appendix A. It states that in order to encourage a worker to exert effort in the production process, his surplus must be at least equal to \( e/\lambda \), the expected disutility from working before the next inspection. When a worker decides to shirk he saves the disutility of effort \( e \) but has an expected capital loss of \( \lambda(E - U) \). In equilibrium, workers will never have an incentive to shirk since firms will never hire a worker if they cannot guarantee their effort, so their lifetime expected utility of unemployment satisfies

\[
r U = b + f(\theta)(E - U),
\]

(4)

where \( b \) represents the income in unemployment. According to (4), when an unemployed worker finds a job he becomes a non-shirker. It is important to remark that the permanent income of unemployed workers is increasing with market tightness since the probability of coming into contact with a firm increases with more vacancies per worker, making the average duration of unemployment smaller. Combining (1) and (4) we get an expression for the worker’s surplus of a match

\[
E - U = \frac{w - e - b}{r + s + f(\theta)}.
\]

(5)

A worker will accept a match if and only if \( E - U \) is positive and, according

\(^6\)These two processes are characterized by an exponential distribution with parameters \( \lambda \) and \( s \) respectively.
with the NSC (3), will choose not to shirk if and only if it is greater than $e/\lambda$.

### 3.1.2 Firm Behavior

The present discounted value of expected profits from a vacant job ($V$) must satisfy the Bellman equation

$$rV = -\gamma + q(\theta)(J - V),$$

where $\gamma$ is the per-unit of time cost of keeping a vacancy open, and $J$ is the value function of a filled vacancy. This last equation states that the capital cost of an open vacancy has to be exactly equal to the rate of return of the vacancy, i.e., the flow costs of recruiting plus the expected capital gain. The asset value of an occupied vacancy satisfies a similar asset-price equation:

$$rJ = ay - w + s(V - J).$$

The capital gain of a filled vacancy is equal to the income flow, $ay - w$, plus the expected capital capital loss when the match is destroyed. The free-entry condition implies that there are no exploitable profits left from opening another vacancy so $V = 0$. Thus, from (6) we get $J = \gamma/q(\theta)$. By substituting this expression into (7) we get the vacancy supply condition (VSC)

$$ay - w = (r + s) \frac{\gamma}{q(\theta)}.$$  

Equation (8) uniquely defines equilibirum market tightness and captures the essence of the model’s dynamics. Vacancies are posted by employers up to the point where the expected surplus of forming a match is exactly offset by the expected recruiting costs. Expected recruiting costs rise due to a congestion effect, more firms posting vacancies rises market tightness making the instant proobability of finding a job decrease. These are the negative search-externalities that firms looking for workers have on each other.

### 3.1.3 Wage Schedule

Wages will be specified to satisfy the NSC at all times. This creates an additional constraint on the bargaining process. Assuming wages are settled via Nash-bargaining with $\beta$ as the worker’s bargaining power, we have
\[ w = \arg \max (E - U) J^{1-\beta} \text{ s.t. } E - U \geq \frac{e}{\lambda}. \] (9)

When the NSC is not binding, the wages are determined in the same way as in conventional models. Using (5) and (7), and solving for \( w \), the expression for the Nash-bargaining wage is

\[ w_{NB} = ay \left[ \frac{\beta(r + s + f(\theta))}{r + s + \beta f(\theta)} \right] + \left[ \frac{(r + s)(1 - \beta)}{r + s + \beta f(\theta)} \right] (b + e). \] (10)

Under Nash-bargaining the worker’s surplus of a match takes the form

\[ E - U = \frac{\beta(ay - e - b)}{r + s + \beta f(\theta)} \equiv E(\theta) - U(\theta)_{NB}. \] (11)

I will refer to the worker surplus generated by Nash-bargaining wages as \( E(\theta) - U(\theta)_{NB} \). Notice that under Nash-bargaining, worker surplus asymptotically goes to zero as the market gets tighter, that is, \( \lim_{\theta \to \infty} E(\theta) - U(\theta)_{NB} = 0 \). From (10) we can observe that the worker’s threat point in wage bargaining increases with market tightness, at the limit \( w_{NB} = ay \).

![Figure 1: Worker Surplus and the No-Shirk Condition (NSC)](image)

Notes: According to the NSC, a worker will participate in the production process only if the surplus he gets from a match is at least equal to \( e/\lambda \). Firms know this so they will only hire a worker if they can guarantee at least that level of surplus. Under Nash bargaining worker surplus, \( E(\theta) - U_{NB} \), drops to zero as the economy converges to full employment, making it necessary to implement efficiency wages, which guarantees a worker surplus of \( e/\lambda \), after some level of employment \( n^S \).
This means that under Nash-bargaining wages, when the market is too tight, the NSC is inevitably violated as worker surplus gets closer to zero. Since firms will hire workers only if the participation of the worker can be guaranteed, when NSC cannot be satisfied with Nash-bargaining wages, firms will pay higher wages to incentivize workers. The minimum wage that a firm must pay to induce effort from the worker is the wage that makes the NSC binding. Substituting (5) into the NSC and solving for \( w \) with an equality, we get that the efficiency wage is

\[
w_E = b + e + \frac{e}{\lambda} (r + s + f(\theta)).
\]

This is the minimum wage required to encourage workers to exert effort. Without the threat of moral hazard, i.e., if perfect monitoring was possible, the unconstrained Nash-Bargaining wage would be enough to guarantee the worker’s participation in the production process, however if his actions are not perfectly observable, guaranteeing his participation requires a moral hazard premium defined as the difference between the efficiency wage and the unconstrained Nash-Bargaining. It can be showed that this premium is inversely related to productivity and it increases with the relative value of the worker’s outside options. If his current job is likely to end or if the expected length of unemployment is short, the outside options are relatively more valuable so the moral hazard premium must be larger. Consistent with the efficiency-wage literature, the no-shirking wage is higher when the effort to be exerted is larger or the detection probability is lower. Notice that the efficiency wage is an increasing function of market tightness just like the Nash-bargaining wage but unlike it, it is not bounded above. This is the result of the fact that the moral hazard premium goes to infinity along with market tightness. If the worker’s valuation for his job must be kept above a threshold to prevent shirking, as the market tightens, the wage premium must get increasingly large to compensate the worker for his outside options.

Depending on whether the NSC is binding, equilibrium wages are determined either using Nash-bargaining or with the expression for efficiency wages so the solution to (9) can be specified as

\[
w(\theta) = \begin{cases} 
    w_{NB}(\theta), & E(\theta) - U(\theta)_{NB} \geq \frac{e}{\lambda}, \\
    w_E(\theta), & E(\theta) - U(\theta)_{NB} < \frac{e}{\lambda}.
\end{cases}
\]

It can be verified that this is a continuous function of market tightness and
it is strictly increasing with $\theta$. This wage schedule guarantees that whenever Nash-bargaining wages cannot induce effort from the worker, the worker is paid the minimum to do so. Figure 1 shows the logic behind the wage schedule. Using (11) we can verify that $\lim_{\theta \to \infty} E(\theta) - U(\theta)_{NB} = 0$. For $e > 0$ and $\lambda < \infty$, this implies that $\lim_{\theta \to \infty} w(\theta) = \lim_{\theta \to \infty} b + e + (r + s + f(\theta)) e^\lambda / \lambda = \infty$, so under the wage scheme described in (13), equilibrium wages increase to infinity as the economy converges to full employment.

This is the gist of the departure from conventional models; in the presence of moral hazard and imperfect monitoring, full employment is unattainable. In the standard Pissarides Model, where equilibrium wages are determined using Nash-bargaining, as the economy converges to full employment the equilibrium wage converges to the product of the match, extracting all the surplus from the firm as a result of increased bargaining power for the workers. In the presence of moral hazard, wages must always be established considering the incentive compatibility constraint, participation of the worker in the production process must be ensured at all times. As market tightness increases, the benefits from shirking grow higher so to encourage a worker to exert effort, a higher wage must be offered. As the labor market is closer to full employment, shirking becomes so attractive that no wage can be large enough to encourage workers to exert effort. The firm’s surplus will turn negative before full employment can be achieved.

There are essentially two endogenous variables in the model: $\theta$ and $u$. Equilibrium market tightness is determined by (8). To determine equilibrium unemployment we use the fact that at a steady state the inflow and outflow from unemployment must be equal. The size of the unemployed population is given by $u = 1 - n$ at all times so the equality of inflow and outflow of unemployed workers gives

$$u = \frac{s}{s + f(\theta)},$$

this expression tell us that for a given separation rate and market tightness, there is a unique equilibrium unemployment rate.

**Definition 1.** A steady-state equilibrium is a collection $(u, \theta, w) \in \mathbb{R}^{3+}$ satisfying (14), (8),and (13).

I am implicitly assuming that all matches, new and old, set wages according to (13) at all times, as if would be the case if wages were being constantly renegotiated. In equilibrium neither workers nor firms will have incentives to break the matches and workers will never have incentive to shirk.
3.2 Job Rationing

Now I show that the presence of moral hazard and imperfect monitoring creates job rationing. In neoclassical models, job rationing is the unemployment arising from the failure of equilibrium wages to clear the labor market. This definition must be adapted to a search-and-matching environment where bilateral meetings take place and wages are usually assumed to be the product of a Nash-bargain between employer and employee. In this scenario, following Michaillat (2012), I define job rationing as the unemployment that persists when recruitment costs are zero. In standard search-and-matching models unemployment arises due to matching frictions which increase the expected cost of opening a vacancy as market tightens. If opening a vacancy was a costless activity, there would be convergence to full employment since Nash-bargaining guarantees that the wage remains in the bargaining set, defined as the interval between the minimum wage acceptable to the worker and the maximum wage acceptable to the firm, at all times. Without recruitment costs, the existence of job rationing requires that the settled wage is above the bargaining set so although workers would be willing to fill a vacancy, firms cannot hire them profitably. In Michaillat (2012) job rationing arises from wages only partially adjusting to productivity shocks and the assumption of decreasing returns to scale. In Ferraro (2018) the bargaining set disappears in economic downturns due to extremely low productivity. In his model, if productivity is low enough, the product of a match is less than the worker’s outside option so the total surplus of a match is negative for entire portions of the labor force. An important theoretical contribution of the present paper is the way job rationing arises as a result of the constraints imposed on wages due to moral hazard.

To see if job rationing exists it is necessary to determine what the unemployment rate would be if job creation was entirely determined by productivity and wages just as it would happen in a frictionless environment.

**Definition 2.** Job rationing is defined as the unemployment that persists as recruitment costs disappear. That is to say, job rationing $u^R$ is given by

$$u^R = \frac{s}{s + f(\theta^R)}$$

where $\theta^R$ is such that

$$ay - w(\theta^R) = \lim_{\gamma \to 0} (r + s) \frac{\gamma}{q(\theta^R)}.$$
conceptually different from a frictionless economy, their equilibrium effects on the labor market are equivalent. When opening vacancies and keeping them open is free, firms will open infinitely many vacancies to completely overcome the frictions in the matching process.

Proposition 1: There exists job rationing in the model.
Proof: Appendix 1.

Proposition 1 tells us that even if opening a vacancy was a costless activity, at some level of employment \( n^R = 1 - u^R < 1 \), firms could not profitably keep hiring workers due to the incentive constraint on wages; not enough vacancies would be opened to make the economy converge to full employment. This is the role of equilibrium unemployment as a disciplinary device as described in Shapiro and Stiglitz (1984). There are two forces affecting the worker’s decision to shirk, the wage they receive and their job opportunities in the market. Since encouraging workers only through wages becomes impossible (no finite wage by itself would be enough), unemployment arises to reduce the worker’s job opportunities and make them cherish their current job enough to exert effort. Figure 2 provides a graphical representation of the situation.

![Figure 2: Nash Bargaining and Efficiency Wages](image)

According to the wage function (13), as the economy converges to full employment, at some point \( n^S \), Nash-bargaining wages \( w_{NB} \) can no longer guarantee workers’ participation, making the use of efficiency wages \( w_E \) necessary. Efficiency wages are designed to encourage workers by compensating them for their increasing outside opportunities as the market gets tighter. At the limit, the average spell of unemployment is close to zero, wages must be infinite to compensate the worker. This means that even without recruiting costs, the profits of the firm are equal to zero at employment level \( n^R \).

An expression for unemployment rationing can be derived combining equa-
tions (8) and (14). As $\gamma \to 0$ equilibrium unemployment converges to

$$u^R = 1 - n^R = \frac{(e/\lambda)s}{ay - b - e - (e/\lambda)r}.$$  \hspace{1cm} (15)

Notice the importance of the separation rate in the magnitude of job rationing. According to the NSC, a higher separation rate increases the incentive to shirk since it makes a job less valuable, it will be terminated rather soon. When the separation rate is close to zero, job rationing is close to zero, firms do not ration many jobs because workers do not have strong incentives to shirk, the almost unlimited duration of jobs makes them very valuable to risk by shirking. Comparative statics allow us to understand better the forces behind job rationing.

$$\frac{\partial u^R}{\partial ay} < 0, \frac{\partial u^R}{\partial e} > 0, \frac{\partial u^R}{\partial \lambda} < 0, \frac{\partial u^R}{\partial b} > 0, \frac{\partial u^R}{\partial s} > 0, \frac{\partial u^R}{\partial r} > 0.$$

An increase in productivity reduces unemployment rationing since it enables firms to pay higher wages instead of cutting down jobs to motivate workers. All the changes in the parameters that make unemployment more attractive than exerting effort in a job, such as a higher permanent income in unemployment, less effort necessary for production, or lower average duration of employment, have a positive impact on unemployment rationing. For a given productivity of a match, the less undesirable unemployment is, the more firms will have to restrict vacancies to encourage workers.

Now that I have laid out the essentials of the model I turn my attention to the source composition of unemployment. Determining the job rationing rate allows us to decompose equilibrium unemployment rate according to its causes. Equilibrium unemployment $u$ is implicitly determined by the VSC (8) and the Beveridge curve (14). Together, these equations give a positive relationship between unemployment and the cost of opening a vacancy. For a $\gamma > 0$, we have that $u > u^R$. The extra amount of unemployment caused by the costs of opening a vacancy will be called frictional unemployment ($u^f$), so it can be defined as the difference between equilibrium unemployment and job rationing unemployment. That is, $u = u^f + u^r$.

To quantify the job rationing component of unemployment I focus on its share in equilibrium unemployment. The share of unemployment rationing is given by

$$R \equiv \frac{u^R}{u^R + u^F}.$$  \hspace{1cm} 16
Figure 3: Composition of Equilibrium Unemployment

Equilibrium unemployment level, $u^*$, is given by the Beveridge curve BC, and the vacancy supply condition VS. In the absence of matching frictions, the VS is vertical since for the equality to hold in (8), the RHS must be zero so wages must be equal to $ay$. The wage schedule specified in (13) pins down the equilibrium wage and therefore equilibrium market tightness. The unemployment level in this situation is $u^{R}$. In the presence of matching frictions, the additional unemployment due to frictions appears as VS rotates to the left. The additional unemployment is frictional unemployment.

Previous models analyzing how the composition of unemployment changes across the business cycle conclude that the share of job rationing is countercyclical, job rationing plays a more prominent role during recessions. Michaillat (2012) finds that when the aggregate level of technology is high, matching frictions account for all unemployment, but when technology is low, both job rationing and matching frictions contribute to unemployment. In his model, once technology has reached a certain level, firms always find it profitable to hire workers even in the absence of frictions. In Ferraro (2018), job rationing is a result of the skill heterogeneity in workers. In an economic downturn the productivity of the least skilled workers could be so low that is not enough to cover their reservation wages, firms could not profitably hire those workers.\(^7\) Both models predict that if the product of a match is high enough, all unemployment is due to frictions in the market, and that during a recession

\(^7\)Notice that in the strict sense this is not job rationing, unemployment does not arise from some sort of wage rigidity but it is entirely due to low productivity.
both frictional and rationing unemployment are at work. In my model this is not the case, there is always a job rationing component in unemployment regardless of the state of technology. Since efficiency wages must be infinitely large as the economy converges to full employment, no matter how high the product of a match is, there will always exist job rationing, only disappearing asymptotically as the product of a match gets infinitely large. The state of the economy is here represented by the aggregate productivity parameter $a$.

**Proposition 2:** The share of unemployment rationing decreases with productivity if and only if the elasticity of rationing unemployment is more negative than the elasticity of frictional unemployment, that is,

$$\frac{\partial R}{\partial a} < 0 \iff \varepsilon_{a}^{R} < \varepsilon_{a}^{F}.$$ 

Proof: Appendix 2.

As previously noted, job rationing decreases as the product of a match increases. Proposition 2 states that to have job rationing decreasing faster than total unemployment, job rationing must be more sensitive to technology shocks than frictional unemployment. Whether the increase in the product of a match has a larger effect on frictional unemployment or job rationing depends on whether it alleviates the moral hazard problem more than it helps to overcome the frictions in the labor market. The magnitude of these two effects is linked to the matching function since it determines the elasticity of the instant probability of finding job and filling a vacancy. These two elasticities are inversely related, if $\eta \in [0, 1]$ is the elasticity of $f(\theta)$, then $\eta - 1$ is the elasticity of $q(\theta)$, when $f(\theta)$ is very elastic, $q(\theta)$ is very inelastic.

With a positive productivity shock, matches become more profitable so firms decide to create more vacancies. A tighter market dampens the initial impulse to create vacancies in two different ways. On the one hand, wages in-

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8 Notice that the results of proposition 2 tells us that the elasticity of rationing unemployment must be less than the elasticity of frictional unemployment, we know that the elasticity of frictional unemployment is negative in the model, however, the derivative of frictional unemployment with respect to technology is indeterminate. It can be shown that the sign of this derivative goes from positive to negative as technology increases. This rather curious phenomenon is an algebraic consequence of the definition of frictional unemployment so it is also a feature of previous models, namely, Michaillat (2012) and Ferraro (2018). Although this result is curious it is not relevant as long as total unemployment is decreasing with productivity, as it is the case.
crease whether they are determined using Nash-bargaining or efficiency wages. In the first case, higher tightness increases the effective bargaining power of the worker, and in the second case, it increases his outside options making a larger remuneration necessary. This is the wage effect and it is related with job rationing. On the other hand, the expected cost of keeping a vacancy open increases due to the augmented search externalities within firms, this is the cost effect, and it is entirely related with frictional unemployment. Given the relationship between elasticities, when the wage effect is large the cost effect must be small.

When the elasticity of the instant probability of finding a job is small, the moral hazard premium remains relatively unaltered by changes in market tightness. This means that in the event of a positive productivity shock, firms can open many vacancies without having large repercussions on wages. The other side of this effect is that if the elasticity of \( f(\theta) \) is small, the elasticity of \( q(\theta) \) must be large. An increase in market tightness largely increases the expected cost of vacancies, so the cost effect is strong, frictional unemployment will barely be reduced. The overall outcome of these effects is a large change in the composition of unemployment; the share of rationing unemployment will be largely reduced. A small elasticity of \( f(\theta) \) makes unemployment rationing very elastic and frictional unemployment relatively inelastic. The opposite is true when the elasticity of \( f(\theta) \) is large. This result is formalized in the next proposition where for the sake of exposition a Cobb-Douglas matching function is assumed but, as shown in the appendix, it can be generalized to any matching function.

**Proposition 3**: Let \( h(u,v) = \mu^\alpha u v^{1-\alpha} \). The elasticity of the probability of filling a vacancy, \( \alpha \), determines how the share of job rationing changes with productivity.

a) If \( \alpha \geq \frac{1}{2} \), then \( \frac{\partial R}{\partial \alpha_y} < 0 \).

b) If \( \alpha \in (0, \frac{1}{2}) \), there exist a threshold \( \alpha^* \in (0, \frac{1}{2}) \) determined by all the rest of the parameters such that, if \( \alpha > \alpha^* \) the share of job rationing decreases with productivity, if \( \alpha = \alpha^* \) it is not affected, and if \( \alpha < \alpha^* \), it increases with productivity.

c) If \( \alpha = 0 \), The share of job rationing increases with productivity, that is, \( \frac{\partial R}{\partial \alpha_y} > 0 \).

Proof: Appendix 3.

Proposition 3 gives us conditions that can be empirically checked to see whether the share of unemployment rationing is countercyclical. Petrongolo and Pissarides (2001) perform a survey of the empirical literature related to the estimation of the matching function and find that when a Cobb-Douglas form
is assumed, the different estimations of $\alpha$ place it in the range $\alpha \in (0.5, 0.7)$, which would imply that the share of job rationing decreases with productivity. As simulations in section 4 indicate, the share of job rationing is procyclical only at a neighborhood around $\alpha = 0$.

In this section I have laid out a basic search-and-matching model that incorporates worker moral hazard. I showed that the presence of moral hazard creates a wage constraint which, even in the absence of matching frictions, makes the equilibrium wage fail to adjust to full employment level, a situation referred to as job rationing. When matching frictions exists, equilibrium unemployment is composed of rationing unemployment and frictional unemployment. My model departs in two important ways from previous results in the literature: i) The model predicts coexistence of both rationing and frictional unemployment regardless of the aggregate productivity level, and ii) the share of job rationing is heavily determined by the matching function, more specifically, by the elasticity of the instant probability of finding a job. This is a new insight of the model that helps to understand the nature of unemployment rationing and frictional unemployment. When this probability is very elastic, the original incentive to open vacancies as a result of higher product of a match is mostly offset by higher wages to compensate the worker for his amelioration of outside options as opposed to being mostly offset by an increase in costs of opening a vacancy. The share of job rationing remains relatively unchanged or could even increase. When the instant probability of finding a job is relatively inelastic, the share of job rationing is strongly reduced by a positive productivity shock.

According to the literature estimating the elasticity values of the matching function, the share of job rationing is countercyclical. During a recession, a larger proportion of unemployment is due to job rationing compared to what happens during an expansion. This results suggest that the effectiveness of policies seeking to reduce unemployment are dependent on the state of the economy, during a recession policies reducing the outside options of a worker are more effective, whereas policies aimed to reduce the search frictions may be more effective during an economic expansion.

4 Worker Heterogeneity

Another important issue this paper addresses is the employment experience of workers with ex-ante different productivities. The extension of the model incorporating worker-skill heterogeneity closely follows Rocheteau (2001). The
environment is the same as in section 3 with the exception that there is an ex-ante distribution of workers with different skills. Firms are identical so the production flow of a match depends entirely on the worker. Search is undirected so a firm can meet workers with any skill level which is perfectly observable upon meeting.\footnote{There are other search-and-matching models where search is undirected such as Acemoglu (1999), Shimer (2001), Dolado et al. (2009) and Pries (2008). Acemoglu (1999) makes a case for undirected search by pointing out that skill is imperfectly correlated with observable characteristics, such as years of education and age, making it difficult for employers to recruit workers with a particular skill level.}

There are $n$ different kinds of workers with productivities $(a_{y_1}, ..., a_{y_n})$ satisfying $0 < a_{y_1} < ..., < a_{y_n}$. A worker with productivity $a_{y_i}$ will be referred to as a worker of type $i$ and is hired by a firm upon meeting with an average probability of $\Pi(a_{y_i})$, which depends on how profitable the match is for the firm and will be discussed later. The present-discounted value of the expected income stream of an unemployed worker with idiosyncratic productivity $a_y$ satisfies

$$rU_{a_y} = b + f(\theta)\Pi(a_{y})\{E_{a_y} - U_{a_y}\},$$

where $U_{a_y}$ is the value to be unemployed and $f(\theta)\Pi(a_{y})$ is the exit rate of unemployment. The value to be employed ($E_{a_y}$) satisfies

$$rE_{a_y} = w_{a_y} - e + s[U_{a_y} - E_{a_y}],$$

where $w_{a_y}$ is the wage. By assumption, all workers face the same production effort, unemployment benefit, discount rate and separation rate. The worker’s surplus of a match is

$$E_{a_y} - U_{a_y} = \frac{w_{a_y} - e - b}{r + s + \Pi(a_{y})f(\theta)}. \quad (16)$$

It has two idiosyncratic components, wages and the exit rate of unemployment. By default, wages are established under Nash-bargaining unless the NSC binds in which case the worker gets a wage that guarantees him the minimal surplus to keep him from shirking, i.e., the efficiency wage. According to (3) a worker’s surplus of a match in equilibrium must always satisfy

$$E_{a_y} - U_{a_y} = \max\{\frac{e}{\lambda}, E_{a_y} - U_{a_yNB}\},$$

where $E_{a_y} - U_{a_yNB}$ is the worker’s surplus when wage is determined via Nash-
4.1 The firm’s behavior under heterogeneity.

The value of a vacant job for a firm satisfies the following bellman equation:

$$rV = -\gamma + q(\theta)\left[\sum_i \Pi(ay_i)\mu_i(J_{ayi} - V)\right],$$

(17)

where $\mu_i$ is the fraction of unemployed workers with productivity $ay_i$. The firm’s asset value of an occupied job by a worker with productivity $ay_i$, $(J_{ayi})$, must satisfy

$$rJ_{ayi} = ay_i - w_{ayi} + s[V - J_{ayi}].$$

(18)

Given the assumption that in equilibrium all profit opportunities from new jobs are exploited driving rents from vacant jobs to zero, $V = 0$, and combining equations (17) and (18) the VSC is derived:

$$\sum_i \Pi(ay_i)\mu_i(ay_i - w_{ayi}) = (r + s)\frac{\gamma}{q(\theta)}.\quad (19)$$

With worker-skill heterogeneity, the profitability of a match depends on the worker so the firm’s best hiring response when coming into contact with a specific worker will depend on his productivity. This worker-specific hiring response gives rise to wage dispersion and different exit rates of unemployment. The firm’s hiring strategy is derived after Rocheteau (2001) and a detailed derivation is included in the appendix. Before presenting the results of the optimal hiring schedule, it is necessary to introduce some definitions.

**Definition 3**: For a given $\theta$, define the segments of the real line: $C_0 \equiv (-\infty, y_L]$, $C_1 \equiv (y_L, y_M(\theta)]$, $C_2 \equiv (y_M, y_H(\theta)]$, and $C_3 \equiv (y_H, \infty)$. Where $y_L = b + e + (r + s)^\frac{\xi}{\lambda}$, $y_M = b + e + (r + s + f(\theta))^\frac{\xi}{\lambda}$, and $y_H = b + e + (r + s + f(\theta)\beta)^\frac{\xi}{\lambda\beta}$.

This definition delimits four intervals in the real line which will classify workers according to their productivity $ay_i$. Notice that the limits of the interval do not depend on the aggregate productivity parameter $a$, this characteristic allows workers to transition between intervals in response to productivity shocks. With this classification, the equilibrium hiring schedule can be specified. In the presence of heterogeneity there is the possibility of having hiring probabilities upon meeting different from zero or one.
**Proposition 4**: For a given market tightness $\theta$, consider a worker with productivity $ay$:

1. If $ay \in C_0$, then the worker is never hired, $\Pi(ay) = 0$.
2. If $ay \in C_1$, then the worker is hired with a probability $\Pi(ay) = (ay - b - e - (r + s)\frac{\lambda}{5}) / f(\theta)\frac{\lambda}{5}$, and is paid $w = ay$.
3. If $ay \in C_2$, then the worker is always hired, $\Pi(ay) = 1$, and is paid efficiency wages, $w = w_E$.
4. If $ay \in C_3$, then the worker is always hired, $\Pi(ay) = 1$, and is paid Nash-bargaining wages, $w = w_{NB}$.

Proof: Appendix 4.

The rationale behind proposition 4 is summarized in Figure 4.

Figure 4: The firm’s hiring strategy.

For a given $\theta$, upon contact with a worker the firm’s hiring response ($w_{ay}$, $\Pi(ay)$) will depend on the product of the match. If $ay \in C_0$, the productivity of a worker is so low that the total surplus of a match ($TS$) is not enough to guarantee his effort ($TS < \frac{\lambda}{5}$), no match will be made. If $ay \in C_1$, the worker is barely employable and will be discriminated with a hiring probability $\Pi(ay) \in (0, 1)$ that will reduce his outside options to the point his wage, $w_{ay} = ay$, is just enough to guarantee his effort ($TS = \frac{\lambda}{5}$). If $ay \in C_2$, the worker’s productivity is high enough to generate a positive surplus for the firm ($J_{ay} = TS - \frac{\lambda}{5} > 0$) so he will always be hired ($\Pi(ay) = 1$). However, it is not large enough to guarantee his effort under Nash-bargaining so he will get the efficiency wage, $w = w_E$. If $ay \in C_3$ the match will generate positive surplus for a firm ($\Pi(ay) = 1$) and the productivity of a worker is high enough to guarantee his participation with Nash-bargaining wages, $w = w_{NB}$.

Proposition 4 describes the maximizing behavior of the firm. It specifies the
firm’s best response upon coming into contact with a worker with productivity $ay$. For a given market tightness, it describes the average hiring probability of a worker and his wage. If the product of a match is high enough ($ay \in C_3$) wage is set using Nash-bargaining since it is high enough to generate surplus for both, worker and firm, without violating the NSC. Worker surplus is above the no-shirking threshold so he will be incentivized to work, and the surplus that a firm gets is positive so it will hire the worker with probability one. When productivity is not so high ($ay \in C_2$) then the Nash-bargaining wage is not high enough to prevent a worker from shirking, efficiency wages are necessary to ensure that the match is productive. The firm still gets a positive surplus so the worker is hired with probability one. Workers in $C_2$ and $C_3$ will be referred to as “perfectly employable” since firms can always hire them and get a strictly positive match surplus.

When $ay \in C_1$, the efficiency wage is greater than the product of a match ($w_E = b + e + (r + s + f(\theta))^{\frac{\xi}{\lambda}} > ay$), the worker can still be encouraged to work with a wage equal to his productivity, $w = ay$, if his outside options are eroded by a lower probability of transition out of unemployment. If $\Pi(ay)$ were equal to one, the no-shirking wage would have to be superior to the worker’s productivity so firms would never hire them. Conversely, if $\Pi(ay)$ were equal to zero, the worker would generate positive profits for his employer so he would always be hired. The equilibrium answer to this conundrum is that employers adopt a mixed strategy, they hire the worker with a probability proportional to his productivity, that is $\Pi(ay) = (ay - b - e - (r + s)^{\frac{\xi}{\lambda}}) / f(\theta)^{\frac{\xi}{\lambda}}$. The decrease of the exit rate of unemployment can be interpreted as a disciplinary device for less productive workers. Notice that firms hiring these workers do not get any surplus from being matched so they are indifferent between hiring them or not, this is the reason I will refer to these workers as “barely employable”. If the productivity of a worker is extremely low ($ay \in C_0$), then even if the worker is fully discriminated, his no-shirking wage would have to be larger than the product of his match so he will never be hired. This differentiated treatment to workers creates wages dispersion and different unemployment rates, shares in the pool of unemployed and exit rates of unemployment.

At the steady state, the idiosyncratic unemployment rates ($u_1, ..., u_n$) are constant so the equality between flows out and into unemployment give the unemployment rate of workers of type $i$ as

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10 We can verify that this is indeed a probability, that is $\Pi(ay) \in [0, 1]$ by observing that it is the solution to the equation $ay_i = (1 - x)y_L + xy_M$. And by assumption $ay_i \in C_1 \equiv (y_L, y_M)$. Full derivation is included in the appendix.
\[ u_i = \frac{s}{s + \Pi(ay_i) f(\theta)}, \]
moresover,
\[ \frac{\partial u_i}{\partial ay_i} < 0, \quad \frac{\partial u_i}{\partial b} > 0, \quad \frac{\partial u_i}{\partial e} > 0, \quad \frac{\partial u_i}{\partial s} > 0, \quad \frac{\partial u_i}{\partial r} > 0, \quad \frac{\partial u_i}{\partial \lambda} < 0 \quad \forall i. \]

Total unemployment is given by
\[ u = \sum_i p_i u_i, \]
where \( p_i \) is the proportion of workers type \( i \) in the labor force.\(^{11}\) The fraction of unemployed workers with productivity \( ay_i \) is given by
\[ \mu_i = \frac{p_i u_i}{u}. \quad (20) \]

Substituting the wages and unemployment rates into equations (20) and (19), after some algebraic manipulation the VSC can be expressed as
\[ \sum ay_i \in C_2 \left[ ay_i - b - e - \frac{e}{\lambda} (r + s + f(\theta)) \right] + \sum ay_i \in C_3 \left[ \frac{(1 - \beta)(ayj - b - e)(r + s)}{r + s + \beta f(\theta)} \right] \]
\[ = u \left[ \frac{\gamma (r + s) (s + f(\theta))}{q(\theta) s} \right], \quad (21) \]
where
\[ u = \sum ay_i \in C_0 \left[ \frac{s (e/\lambda)}{ary - b - e - r (e/\lambda)} \right] + \sum ay_i \in C_1 \left[ \frac{s}{s + f(\theta)} \right] + \sum ay_i \in C_2 \left[ \frac{s}{s + f(\theta)} \right] + \sum ay_i \in C_3 \left[ \frac{s}{s + f(\theta)} \right]. \quad (22) \]

Equation (21) uniquely determines equilibrium market tightness \( \theta^* \). It can be verified that

\(^{11}\)The entire worker population as been normalized to 1 so \( \sum p_i = 1 \)
\[ \frac{\partial \theta^*}{\partial e} < 0, \quad \frac{\partial \theta^*}{\partial \lambda} \geq 0, \quad \frac{\partial \theta^*}{\partial b} < 0, \quad \frac{\partial \theta^*}{\partial s} < 0, \quad \frac{\partial \theta^*}{\partial r} < 0, \]

and

\[ \frac{\partial \theta^*}{\partial ay_i} \geq 0 \quad i = 1, \ldots, n. \]

Equilibrium market tightness is strictly increasing in the productivity of all the hirable workers and it is unaffected by small changes in the productivity of workers that are not hirable. In an undirected market, the fact that increasing an idiosyncratic productivity level has a positive effect on equilibrium market tightness means that wages and unemployment rates of all workers change due to a general equilibrium effect. Given the hiring schedule, the magnitude of the derivative varies across workers depending on their productivity. Since the effect of a change in the idiosyncratic productivities on market tightness depends on the proportion of workers with that productivity, it is difficult to compare them without making assumptions about the worker-skill distribution. To overcome this problem we can compute the per-worker productivity effect on market tightness defined as

\[ W_i \equiv \frac{\partial \theta^*}{\partial ay_i} p_i. \]

**Proposition 5:** The per-worker productivity effect on market tightness is decreasing with worker-skill, that is

\[ W_i > W_j > W_k \quad \forall ay_i \in C_1, \; ay_j \in C_2, \; ay_k \in C_3. \]

Moreover,

\[ \frac{\partial W_i}{\partial ay_i} < 0 \quad \forall ay_i \in C_1. \]

Proof: Appendix 5.

The proposition states that the impact on equilibrium market tightness of an increase in the productivity of a single hirable worker is greater the lower his productivity. This implies that increasing the productivity of the least-skilled side of the labor force has a greater per-worker impact on market tightness than increasing the productivity of those at the top end. Increments in the productivity of a specific group of workers has positive effects on the whole labor
force, higher market tightness means higher wages and lower unemployment rates, however this effect is not symmetric.

Increasing the productivity of any hirable worker in the labor force has a positive effect on market tightness, however not all workers in the labor force benefit in the same way from this general equilibrium effect. For barely employable workers, an increase in market tightness does not translate into higher wages or lower unemployment rates. These workers are being paid exactly their productivity so their wages are not affected by a tighter market. This leaves the hiring probability upon meeting as the only mechanism of adjustment to satisfy the NSC. To satisfy the NSC, since wages are fixed, their outside opportunities must be fixed too. Higher market tightness increases their instant probability of being matched with a firm but this higher arrival rate is offset by a lower probability of being hired upon meeting, their unemployment rates do not change with market tightness. This means that increments in the productivity of perfectly employable workers will not affect the wage or the unemployment rate of barely employable workers. However, when there is an increase in the productivity of barely employable workers, it improves the unemployment rate and wages of all employable workers. Moreover, the size of this effect is strictly decreasing with productivity, meaning that the highest per-worker productivity effect on market tightness is achieved by increasing the productivity of those with the lowest productivities. As it will be highlighted in the next section, behind these asymmetries is the fact that the unemployment experienced by low-skilled workers is essentially different from the unemployment experienced by high-skilled workers. The unemployment of the barely employable is entirely due to job rationing.

The model can also replicate the empirical observation that low-skilled workers are overrepresented in the poll of the unemployed with respect to their share in the worker population. Using (20) we can corroborate that $\mu_i < p_i$ if and only if $u_i < u$. The aggregate unemployment rate in the economy is given by (22), so it can be interpreted as the average unemployment rate. Any group of workers with an unemployment rate below the average will be over represented in the pool of unemployed. The intensity of this over and underrepresentation in the pool of the unemployed depends on the spread of the skill distribution, a large disparity in productivities will generate strong positive skewness in the unemployment rates.
4.2 Job Rationing among different workers.

This subsection shows that the model incorporating heterogeneity also features job rationing for all the workers and the labor force as a whole.

**Proposition 6:** The equilibrium job rationing unemployment rates for a worker with productivity $a y_i$ is:

$$u_i^R = \begin{cases} 
\frac{s(e/\lambda)}{a y_i - b - e - r(e/\lambda)}, & a y_i \notin C_0, \\
1, & a y_i \in C_0.
\end{cases} \quad (23)$$

Thus, $u_1^R \geq u_2^R, ..., > u_n^R > 0$. Moreover, if the worker with the highest productivity is hirable, $a y_n \notin C_0$, firms will open vacancies to the point where the efficiency wage is equal to the product of a match with a worker with the highest productivity. So equilibrium market tightness $\theta^*$ is such that $a y_n = b + e + (r + s + f(\theta^*)) \frac{e}{\lambda}$.

Proof: Appendix 6

Proposition 6 states that when there are no recruitment costs, firms will open vacancies to the point of driving the income flow of every match to zero, that is, they will open vacancies until every worker in the economy is barely employable. Every type of worker is paid his productivity, $w_i = a y_i$, and faces an unemployment rate given by (23). The job rationing unemployment rate for each type of worker is inversely related to his productivity. Notice that every kind of worker will experience some degree of job rationing regardless of the aggregate level of technology, this result contrasts with Ferraro (2018) where only the least skilled workers experience job rationing. We can compute the share of job rationing for each type of worker as:

$$R_i \equiv \frac{u_i^R}{u_i}.$$

**Corollary to proposition 6:** High-skilled workers experience lower shares of job rationing, i.e.,

$$R_1 > R_2 > ... > R_n.$$

Moreover, the idiosyncratic shares of job rationing are decreasing with productivity, i.e.,
The corollary tells us that the composition of the equilibrium unemployment of workers varies according to their productivity. The more skillful the worker is, the larger the role of frictions in the unemployment he experiences. Total job rationing is given by $u^R = \sum p_i u^R_i$. So the aggregate share of job rationing is given by

$$R \equiv \frac{u^R}{u}.$$  

When heterogeneity is added, characterizing the behavior of the share of job rationing, either at an idiosyncratic or an aggregate level, is not as straightforward as in the case of a representative worker. The analysis depends heavily on the underlying worker-skill distribution. Characterization of the results for an arbitrary distribution are left for further research. For the present work I rely on the numerical simulations performed in section 5. According to these results, obtained assuming a uniform distribution, both the idiosyncratic share of job rationing and the aggregate share of job rationing decrease with aggregate unemployment. As the aggregate technology level increases, frictions play a larger role in the generation of unemployment.

In this section I have extended the basic model described in section 3 to include heterogeneity among workers. In an undirected market firms can come into contact with any type of worker and their best response upon meeting a worker will depend on the worker’s productivity. Workers with a high-enough productivity will always be hired and depending on the NSC they could be paid an efficiency wage or Nash-bargaining wage. In equilibrium, low-skilled workers will be discriminated by firms to reduce their outside options so the low wage corresponding to their productivity is enough to encourage them to exert effort. The model with heterogeneity also exhibits job rationing at an idiosyncratic and aggregate levels, how they are affected by productivity shocks depends on the skill distribution.

5 SIMULATION

This section presents the simulations of the model with a representative worker and the model with worker-skill heterogeneity. First, the parametriza-
tion of the model is discussed and then the results of the simulations are analyzed. This section concludes with some results that underscore the importance of considering heterogeneity in the model and the selection of a specific worker-skill distribution.

5.1 Calibration

Table 1 summarizes the baseline parameter values used in the following simulations. The objective is to illustrate the effect of changes in the parameters on the behavior of the steady state equilibrium. When assessing these effects, only the parameter of interest changes while the rest are equal to their baseline specification. For the choice of matching function I follow the literature and assume a Cobb-Douglas 

\[ h(u, v) = \mu u^\alpha v^{1-\alpha}, \]

therefore

\[ f(\theta) = \mu \theta^{1-\alpha} \quad \text{and} \quad q(\theta) = \mu \theta^{-\alpha}. \]

I set the elasticity of the probability of filling a vacancy to \( \alpha = 0.6 \) which according to Petrongolo and Pissarides (2001) is at the middle of the rage of the parameter values, [0.5, 0.7], estimated across the literature. Following Shimer (2005) I consider quarters as the time unit. The parameter for the efficacy of matching is set to \( \mu = 1.355 \) from the fact that a worker finds a job with a 0.45 probability per month so the flow arrival rate is approximately 1.35 on a quarterly basis. I set the discount rate to \( r = 0.012 \), equivalent to an annual discount factor of 0.953. The quarterly separation rate is set to \( s = 0.10 \) so the mean duration of a job is 2.5 years. The value of leisure is set to \( b = 0.4 \), and the bargaining power of the workers is set to satisfy the Hosios (1990) rule, that is, \( \beta = 0.6 \). The cost of opening vacancy is set to \( \gamma = 0.213 \) after Shimer (2005).

There are two new parameters in the model, the worker effort requirement and the arrival rate of inspections. The lack of empirical evidence to identify either of these parameters has been a serious limitation of the efficiency wages literature. Following Pissarides (1998), I set \( e = 0.05 \). He sets \( \lambda \) so that on average a worker is detected shirking after 17 months, this implies a value of \( \lambda = 1.75 \).

---

\(^{12}\)The calibration of the base line model closely follows Shimer (2005) given the importance this paper has gained in the literature.

\(^{13}\)See Shimer (2005) for further details.

\(^{14}\)Pissarides (1998) considers an environment where efficiency wages are paid, however this model does not consider frictions but instead workers have to be called to a job in the
Table 1: Baseline Parameter Values in Simulations of the Model

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount rate</td>
<td>0.012</td>
<td>Shimer (2005)</td>
</tr>
<tr>
<td>Separation rate</td>
<td>0.1</td>
<td>Shimer (2005)</td>
</tr>
<tr>
<td>Unemployment-elasticity of matching</td>
<td>0.6</td>
<td>Petrongolo and Pissarides (2001)</td>
</tr>
<tr>
<td>Match productivity</td>
<td>1</td>
<td>Shimer (2005)</td>
</tr>
<tr>
<td>Unemployment benefits</td>
<td>0.4</td>
<td>Shimer (2005)</td>
</tr>
<tr>
<td>Worker bargaining power</td>
<td>0.6</td>
<td>Petrongolo and Pissarides (2001)</td>
</tr>
<tr>
<td>Technology level</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Effort intensity</td>
<td>0.05</td>
<td>Pissarides (1998)</td>
</tr>
<tr>
<td>Inspection rate</td>
<td>0.175</td>
<td>Pissarides (1998)</td>
</tr>
<tr>
<td>Recruiting cost</td>
<td>0.213</td>
<td>Shimer (2005)</td>
</tr>
</tbody>
</table>

For the worker-skill distribution, I follow Ferraro (2018) by assuming that the distribution is uniform and I set the number of different skills in the population to $n = 200$. This is a discrete uniform distribution over the interval $[0.612, 1.388]$ meaning that $y_1 = 0.612$ and $y_{200} = 1.388$. The choice of these limits was made to comply with the observation made by Syverson (2011) that within four-digit SIC industries in the U.S. manufacturing sector the average difference in logged total productivity between plants in the 90th and 10th percentile is 0.651. In our setting this means that $\ln(y_{90th}/y_{10th}) = 0.651$. This would imply that the workers at the 90th percentile of the distribution are almost twice as productive as the worker at the 10th percentile. The distribution was also chosen to satisfy a mean productivity of one since the rest of the values of the parameters were taken from Shimer (2005) where the product of a match in normalized to one.

---

market. He mentions that the role of unemployment in this setting is to discipline workers into not shirking on the job where as in other settings, with job frictions, unemployment serves to curve wage demands. In the presence of Nash-bargaining, if there was no unemployment the worker would get all the production. In his model, wages also depend negatively on equilibrium unemployment. He recognizes the limitations of this model; the absence of empirical evidence on the effort level and the inspection rate.

---

15 The choice of the number of different categories of worker-skills does not affect the results of the simulations as long as it is a large number. For $M \geq 100$ the results are qualitatively the same.
5.2 Representative worker simulation.

In this subsection I present the results of the model when a representative worker is considered. I am interested in how the steady-state outcomes of the model behave across the economic cycle. To simulate the economic cycle I compute and plot the steady-state outcomes of the model at different values of the aggregate technology level $a$, leaving the rest of the parameters constant. Michaillat (2012) constructs a deseasonalized and detrended series for $a$ using U.S. data and estimates it as a residual. He creates quarterly data for the period 1964 to 2009, during this period the parameter fluctuates roughly between 0.94 and 1.06. To clearly appreciate the contrasts, I consider a very low technology level ($a = 0.08$) and a state of high technology ($a = 1.2$). The simulations are made over a grid of 100 equally spaced points.

Equilibrium Unemployment Across the Cycle.

Figure 5 shows the behavior of equilibrium unemployment, the composition of unemployment by its source, and wages for different values of the technology parameter. It can be observed that equilibrium unemployment decreases as the economy moves to a better state. The abrupt change of slope in the curve corresponds to the change in wage determination described in the wage function (13). When the product of a match is not high enough to guarantee the participation of the worker under Nash-Bargaining, efficiency wages are
implemented. As the analytical results suggested, market tightness is more elastic under efficiency wages reflected here by a steeper unemployment line. As predicted, larger realizations of technology decrease unemployment and change its composition making it more frictional, the expected cost of opening a vacancy plays a more prominent role in the decision of opening vacancies. The results are in accordance with those from previous studies with the difference that there is always some amount of rationing unemployment regardless of productivity levels. Also in accordance with the analytical results, the gap between wages and productivity widens as matches become more profitable.

To illustrate the analytical predictions about the importance of the elasticity of the matching function, I perform simulations of the model for different values of $\alpha$ and present it in Figure 6.

Figure 6: The importance of the elasticity of the matching function.

When the probability of finding a job is relatively inelastic (high $\alpha$) the share of job rationing is decreasing with productivity and aggregate unemployment remains relatively flat. These results are interrelated, low elasticity implies that more vacancies do not translate into a higher probability of finding a job, so although an increase in the productivity of a match makes firms open more vacancies, this does not translate into more employment. The outside options of an employed worker barely improve so the efficiency wage remains relatively constant. However, the inter-firm externalities are strong, so the expected costs of opening a vacancy increase so much that leave unemployment relatively unchanged. These forces make unemployment more frictional. When the probability of finding a job is very elastic (low $\alpha$) the creation of vacancies responding to a positive productivity shock translates into more workers finding a job, hence unemployment is more responsive to increments in productivity. The expected costs of opening a vacancy remain constant so opening a vacancy becomes relatively cheaper, more vacancies are open and the unemployment attributed to frictions is reduced as much as the unemployment attributable to moral hazard. The share of unemployment rationing
stays relatively constant. In the extreme case of $\alpha = 0$, the share of rationing unemployment is procyclical, an increase in productivity reduces the frictional unemployment more than it reduces job rationing.

5.3 Simulation of the model with worker heterogeneity.

Now I present the simulations of the model when heterogeneity is introduced. Once the skill distribution has been specified, I compute the equilibrium market tightness, idiosyncratic unemployment rates, shares of job rationing, and wages. 200 different types of workers are assumed so the results are presented in terms of averages across deciles, the first decile includes the least skilled workers and the tenth decile the most skilled.

Figure 7: Equilibrium Outcomes with Heterogeneity.

Figure 7 shows the behavior of the average unemployment rates, shares in the pool of the unemployed, job rationing shares and wages by deciles for different aggregate technology levels. Table 2, included in appendix B shows the details. During a recession, the average unemployment rate of those in the bottom decile is almost 3 times as high as those in the top decile. As the economy moves to a better state, unemployment rates improve for all in the labor force and the unemployment rate gap between the bottom and the top
deciles reduces to being only 1.2 times higher. Table 3 presents the standard deviations of these outcomes by deciles. It shows that the standard deviation of the unemployment rates of those in the bottom deciles is almost 6 times the standard deviation of the top decile. Unemployment for the low-skilled is much more volatile than unemployment for the high-skilled, decreasing much more in expansions and increasing much more in recessions. This phenomenon has repercussions in the composition of the pool of the unemployed. In a low state of the economy those in the bottom decile represent 22% of the unemployed whereas the top decile represents only 7%. The skill distribution in the unemployment pool approximates the population skill distribution as the economy moves to a better state. In a very good state, the bottom decile represent 13% of the unemployed and the top decile 9%. One of the insights of the model is that the asymmetries in the behavior of unemployment across deciles is a consequence of the differences in the nature of the idiosyncratic unemployment rates. These predictions are corroborated by the simulation, we can observe that job rationing is a larger component of the unemployment of those less skilled, in fact, unemployment rationing is the sole cause of unemployment for those in the bottom two deciles. In general, all deciles see a diminishing share of unemployment rationing as technology improves and, unlike other models, both kinds of unemployment persist throughout the cycle for all types of workers, having job rationing disappearing only asymptotically as the product of the match approximates infinity.

<table>
<thead>
<tr>
<th>Table 2: Standard Deviations Across the Cycle by Deciles</th>
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<tbody>
<tr>
<td><strong>1st</strong></td>
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<tr>
<td>--------</td>
</tr>
<tr>
<td><strong>Average Unemployment Rate</strong></td>
</tr>
<tr>
<td><strong>Share of pool of unemployment</strong></td>
</tr>
<tr>
<td><strong>Job Rationing Share</strong></td>
</tr>
<tr>
<td><strong>Average Wage</strong></td>
</tr>
</tbody>
</table>

We can observe that although the average wage of all deciles is procyclical, those of the most skilled vary more. The standard deviation of the wages in the top decile is two times the the standard deviation of those in the bottom decile. This is a direct result of the incentive constraints on the wages of least skilled workers. During a recession the wages of high-skilled workers can adjust accordingly to Nash-bargaining but the wages of low-skilled workers are constrained by the NSC.

These results can be empirically corroborated. Ferraro (2018) uses CPS micro data to analyze the behavior of groups of workers by age and education as proxies for skills, and finds that i) young and least educated workers experience average unemployment rates that are up to nine times that of primed-aged
educated workers, and ii) they account for approximately 70 percent of the
time series variation in the U.S. unemployment rate. Gervais et al. (2016) find
a similar situation. Sparber and Fan (2012) finds that over the last decade,
the average U.S. unemployment rate among individuals with a high-school
degree or less education was 7.8 % whereas unemployment equaled just 4.0 %
for those with some college degree or more education. He also finds that the
standard deviation of monthly unemployment rates was 2.6 percentage-points
for less-educated individuals, nearly twice the figure for college educated labor.

A new insight of the model is the causes behind these differences. The
model suggests that for the lowest skilled workers job rationing is the sole
factor creating unemployment whereas for workers with higher productivity
matching frictions also contribute to their unemployment.

5.4 The importance of the skill distribution.

In this subsection I introduce a topic for future research, the importance
of worker-skill distribution in the behavior of unemployment. Recent litera-
ture emphasizes the role of heterogeneity in the labor force to understand the
fluctuations in the economy. For example, Pries (2008) and Bils et al. (2011)
analyze how introducing worker heterogeneity into search-and-matching mod-
els helps to emulate the observed volatility of aggregate unemployment. They
find that heterogeneity does increase predicted volatility but not enough to
match the volatility observed in the data.

There is also an increasing literature relating demographics and skills with
the observed volatility in macroeconomic variables. Jaimovich and Siu (2009)
investigate the consequences of demographic change for business cycle analysis.
They find that changes in age composition, which could be a proxy for skills,
account for a significant fraction of the variation in business cycle volatility
observed in major economies. They show that the cyclical volatility of the
labor market is U-shaped as a function of age. These results suggest that
the age composition of the labor force is potentially a key determinant of the
responsiveness of an economy to business cycle shocks, when the labor force is
mostly composed of individuals with high observed volatility in hours of work,
such as the young and those close to retirement, there is more propagation in
the business cycle.

Other works that emphasize how critical it is to understand cyclical move-
ments of low-skilled workers to explain the large fluctuations in the labor
market and the economy are Champagne (2015), Champagne and Kurmann
(2013), and Gorry (2013). This research underscores how important it is to
incorporate the right distribution of skills into the analysis of the model with heterogeneity.

I perform the simple exercise of computing the equilibrium total unemployment across the cycle considering different worker-skill distributions. I consider uniform distributions with mean one and different variances. The results are presented in figure 8.

This figure shows the behavior of equilibrium unemployment for different productivity levels for skill distributions with different variances. All the distributions are uniform with mean one. When the variance is equal to zero the model reduces to the representative worker case with a productivity of one. As the variance of the distribution increases, total unemployment becomes more elastic. Aggregate unemployment increases for every realization of the technology parameter. These predictions expose the peril of the assumption of a representative agent and the importance of the skill distribution for the calibration of the model.

When the variance is equal to zero, the model reduces to the representative worker case presented in subsection 4.2. As workers become more diverse aggregate unemployment increases for any technology level and also becomes more sensitive to its changes as we can observe in the steeper unemployment curve. These results suggest that i) if heterogeneity must be incorporated into the model, the calibration of the model must be changed accordingly to match the empirical moments, and ii) incorporating heterogeneity does have a great impact on the volatility predicted by the model. Future extensions of this paper attempting to replicate the historic behavior of unemployment must consider how the skill distribution changes over time.

This section has presented some numerical results of the models with a representative worker and worker skill heterogeneity. The numerical results backup the analytical predictions of previous sections and unveil new insights about the importance of the matching function in the moral hazard problem and the relevance of the skill distribution for the calibration.
6 FINAL REMARKS

I have presented a framework that integrates moral hazard and worker heterogeneity into a standard search-and-matching model of unemployment. The broad contribution of this paper is to show that these features greatly enrich the predictions of an otherwise standard search-and-matching model. Moral hazard and imperfect monitoring introduce job rationing into the model, making it more versatile and suitable to analyze the labor market situations where matching frictions have been reduced but unemployment is still persistent. When combined with worker heterogeneity, moral hazard diversifies the outcomes of the labor market across the labor force. It creates a broad classification of workers with important repercussions. There are employable workers, who generate benefits for the firm and it is the possibility of being randomly matched with one of these workers that makes firms enter the market, and there is also barely employable workers, very low-skilled workers who do not generate profits for the firm but do create search externalities that make the market less efficient. Without the presence of moral hazard and imperfect monitoring, although wage dispersion could still be generated, the diversity in the unemployment rates and exit rates of unemployment would not exist. It is due to the diversity of incentive combinations necessary to align worker-firm interests at a minimum cost that all these results arise. The policy implications of the model are that the effectiveness of labor market and macroeconomic policies depends on aggregate productivity and the skill distribution in the workforce since they determine the overall and idiosyncratic share of job rationing and therefore the dynamics of unemployment. The most relevant implication has to do with policies targeting the unemployment of a particular sector in the labor force. It can also offer recommendations for programs targeting the productivity of workers such as training programs. According to the results, the general equilibrium effect of increasing the productivity of workers on the low-side of the skill distribution is greater than effect of an increase in the productivity of the most skilled workers. This suggests that during an economic downturn, programs aimed to enhance the productivity of those with lowest skills are optimal to stimulate the economy. The model also implies that programs aimed to reduce search frictions do not affect the employment situation of low-skilled workers and their overall efficacy depends on productivity.

Also, the framework of the model makes it ideal to study the effects of employment policies that are believed to have asymmetric effects across the labor force such as minimum wages laws. This is the specific policy discussed in the paper but future extensions could consider other aspects such as the
effects of unemployment insurance and participation decisions in the labor market, both are aspects that most likely are determined by the productivity of the worker. The results on minimum wages emphasize that determining what the discouragement effects of a higher minimum wage are is essential to quantify the welfare consequences of a wage floor. According to the model, the only participation effect that a minimum wage can have is discouragement. If search intensity was incorporated for the analysis of minimum wages, as it is done in many other studies of the welfare repercussions of minimum wages, the model could deliver even more powerful implications for the role of a minimum wage. Further extensions of the model should investigate the matter empirically and try to provide evidence of the effects of minimum wages on participating decisions in low-wage labor markets and try to determine how that decision varies across workers with different skills.

Appendix A: Derivations and Proofs

Appendix 1: Proof of Proposition 1

First, it is necessary to prove that when recruiting costs go to zero, efficiency wages are paid. From the VSC (8) we get an inverse relationship between equilibrium market tightness $\theta^*$ and recruitment costs $\gamma$. As recruitment costs go to zero, market tightness goes to infinity. Accordingly to (13), for very large values of $\theta$, efficiency wages must be paid.

With efficiency wages, the job creation condition (8) becomes

$$ay - b - e - \frac{e}{\lambda}(r + s + f(\theta)) = (r + s)\frac{\gamma}{q(\theta)}.$$ 

From this expression it can be verified that as $\gamma \to 0$, $\theta \to \theta^R$, where $\theta^R = f^{-1}\left(\frac{ay - b - e - \frac{e}{\lambda}(r + s)}{e/\lambda}\right)$.

In the absence of recruiting costs, $\theta^R$ is the equilibrium market tightness. Using the Beveridge curve (14), we have that the equilibrium employment level in the absence of recruitment costs is $n^R = \frac{f(\theta^R)}{s + f(\theta^R)} < 1$, the economy does not converge to full employment. □
Appendix 2: Proof of Proposition 2

\[
\frac{\partial S^R}{\partial ay} = \frac{\partial u^R}{\partial ay} u^F - \frac{\partial u^F}{\partial ay} u^R < 0, \quad \iff \quad \frac{\partial u^R}{\partial ay} u^F < \frac{\partial u^F}{\partial ay} u^R \iff \frac{\partial u^R}{\partial ay} < \frac{\partial u^F}{\partial ay}
\]

\[< \quad 0, \quad \iff \quad \frac{\partial u^R}{\partial ay} < \frac{\partial u^F}{\partial ay} \]

Appendix 3: Proof of Proposition 3

Proposition 4 follows from the next theorem:

Theorem 1:

Let \( \eta(\theta) \in [0, 1] \) be the elasticity of \( f(\theta) \). A sufficient condition to have a countercyclical share of job rationing, i.e. \( \frac{\partial S^R}{\partial ay} < 0 \), is

\[
\eta(\theta) < \frac{s + f(\theta)}{s + 2f(\theta)}. \tag{24}
\]

Proof of Theorem 1:

Taking all the parameters as fixed, the equilibrium market tightness is given by the piecewise implicitly defined function:

\[
\theta^*(ay) = \begin{cases} 
ay - b - c - \frac{r + f(\theta)}{q(\theta)} & = 0, \quad E - U = \frac{\xi}{\chi}, \\
(1-\beta)(ay-b-c)(r+s) & = 0, \quad E - U < \frac{\xi}{\chi}, \\
\frac{(1-\beta)(r+s)}{q(\theta)} - \frac{\gamma(s+r)}{q(\theta)} & = 0.
\end{cases}
\]

The reader can verify that this is a continuous function. This is a piecewise differentiable function and the derivative is given by
\[
\frac{\partial \theta^*}{\partial ay} = \begin{cases} 
\frac{1}{2}f'\left(\theta^*\right) - \frac{\gamma(\theta^*)^2 q'\left(\theta^*\right)}{q\left(\theta^*\right)}, & E - U = \frac{e}{\lambda}, \\
\frac{(1-\beta)}{(1-\beta) (ay - b - e) q'\left(\theta^*\right) - \gamma(\theta^*) q'\left(\theta^*\right)}, & E - U > \frac{e}{\lambda}.
\end{cases}
\]

Rationing share of unemployment can be expressed as:
\[
S^R = \frac{1 - n^R}{1 - n(\theta^*)}.
\]  
(25)

Let \( n^* \equiv n(\theta^*) \). Taking the derivative of (25):
\[
\frac{\partial S^R}{\partial ay} = \frac{(1 - n^*)(-\frac{\partial n^R}{\partial ay}) - (1 - n^R)(-\frac{\partial n^*}{\partial ay})}{(1 - n^*)^2}.
\]  
(26)

Using \( n(\theta^*) = f(\theta^*)/(s + f(\theta^*)) \), and \( n^R \) we get the expressions:
\[
\frac{\partial n(\theta^*)}{\partial ay} = \frac{f'(\theta^*)s}{s + f(\theta^*)} \frac{\partial \theta^*}{\partial ay} = (1 - n^*(\theta^*)) \frac{f'(\theta)}{s + f(\theta)} \frac{\partial \theta}{\partial ay},
\]
\[
\frac{\partial n^R}{\partial ay} = \frac{(e/\lambda)s}{(ay - b - e - (e/\lambda)r)^2} = \frac{1 - n^R}{(ay - b - e - (e/\lambda)r)^2}.
\]

Substituting these derivatives into (26), we get
\[
\frac{\partial S^R}{\partial ay} = \frac{(1 - n^R)}{(1 - n^*)} \left[ \frac{f'(\theta)}{s + f(\theta)} \frac{\partial \theta}{\partial ay} - \frac{1}{(ay - b - e - (e/\lambda)r)^2} \right].
\]

This expression is negative if and only if
\[
f'(\theta) \frac{\partial \theta^*}{\partial ay} \left[ ay - b - e - \frac{e}{\lambda} \right] - (s + f(\theta)) < 0. \]  
(27)

Depending on whether the NSC is binding or not, \( \theta^* \) is given by different equations. First consider the case where the NSC is binding, \( E - U = \frac{e}{\lambda} \). In this case we have
\[
f'(\theta) \frac{\partial \theta^*}{\partial ay} = \frac{e}{\lambda} - \frac{\gamma(r + s)q'(\theta)}{q(\theta)^2 f'(\theta)}. \]  
(28)
At equilibrium \( \frac{\gamma(r+s)q'(\theta)}{q(\theta)^2} = ay - b - e - \frac{e}{\lambda}(r+s + f(\theta^*)) \), we also have \( \frac{q'(\theta)}{q(\theta)f'(\theta)} = \frac{1-\eta(\theta)}{\eta(\theta)f(\theta)} \), substituting into (28) we have that (27) is satisfied if and only if

\[
\frac{(ay - b - e - \frac{e}{\lambda}(r + s + f(\theta^*))\eta(\theta)f(\theta) - (1 - \eta(\theta))(s + f(\theta))}{(ay - b - e - \frac{e}{\lambda}(r + s + f(\theta^*)))(1 - \eta(\theta))} - (s + f(\theta)) < 0.
\]

After some manipulation this expression becomes

\[
\frac{(ay - b - e - \frac{e}{\lambda}(r + s + f(\theta^*)))(\eta(\theta)f(\theta) - (1 - \eta(\theta))(s + f(\theta))}{\frac{e}{\lambda}\eta(\theta)f(\theta) + (ay - b - e - \frac{e}{\lambda}(r + s + f(\theta^*)))(1 - \eta(\theta))} < 0,
\]

further manipulation gives

\[
\eta(\theta) < \frac{s + f(\theta)}{s + 2f(\theta)}.
\]

Now consider the case when the NSC is not binding, \( E - U > \frac{e}{\lambda} \). Making all the substitutions and using the fact that at an equilibrium \( \frac{(1-\beta)(ay - b - e)(r + s - \beta f(\theta))}{r + s + \beta f(\theta)} = \frac{\gamma(s+r)}{q(\theta)} \) we can express (27) as

\[
\frac{(r + s)(ay - b - e - \frac{e}{\lambda}r)(r + s + \beta f(\theta))\eta f(\theta)}{(ay - b - e)[\beta \eta f(\theta) + (r + s)(1 - \eta)(r + s + \beta f(\theta))]} - (s + f(\theta)) < 0,
\]

after some manipulation the expression becomes

\[
(ay - b - e) [(r + s)(r + s + \beta f(\theta))(\eta f(\theta) - (s + f(\theta))(1 - \eta)) - (s + f(\theta))\beta\eta] - \frac{e}{\lambda} r \eta f(\theta)(r + s + \beta f(\theta)) < 0.
\]

The only term with undefined sign is \( (r + s)(r + s + \beta f(\theta))(\eta f(\theta) - (s + f(\theta))(1 - \eta)) \), the rest are negative, so a sufficient condition to have the whole expression being negative is

\[
\eta f(\theta) - (s + f(\theta))(1 - \eta) < 0,
\]
or differently expressed:

\[ \eta(\theta) < \frac{s + f(\theta)}{s + 2f(\theta)}. \]

This is a sufficient condition to have the derivative being negative regardless of whether the NSC is binding or not. □

Theorem 1 gives sufficient conditions for the share of job rationing to be countercyclical without any assumption about the specific form of the matching function or any specific parameter values. The proof of Proposition 4 follows from theorem 1. By assumption the matching function takes the form

\[ h(v, u) = \mu v^{1-\alpha} u^\alpha. \]

With this specification the elasticity of the instant probability of finding a job is constant an equal to \( \eta(\theta) = 1 - \alpha \). Condition (24) Takes the form:

\[ 1 - \alpha < \frac{s + \mu \theta^{1-\alpha}}{s + 2\mu \theta^{1-\alpha}}. \]

(30)

a) If \( \alpha > \frac{1}{2} \), condition (30) always holds.
b) If \( \alpha \in (0, \frac{1}{2}) \) condition (30) depends on the equilibrium market tightness which depends on the rest of the parameters.
c) If \( \alpha = 0 \), condition (30) never holds. If the NSC is binding, this is a sufficient condition to have \( \frac{\partial S_R}{\partial ay} > 0 \). If the NSC does not bind then eq.(29) is always positive so \( \frac{\partial S_R}{\partial ay} > 0 \). □

**Appendix 4: Proof of Proposition 4**

Consider the firms’ hiring decision in an undirected market with workers with heterogeneous skills. I only consider symmetric Nash equilibria (NE). Each employer takes the strategy of others employers as given and chooses a probability \( \Pi(y) \) of recruiting the worker with idiosyncratic productivity \( y \) in order to maximize his expected profits. Because in equilibrium the value of a vacancy is zero \( (V = 0) \), the best response function of an employer satisfies the following rule:

- \( J_y > 0 \implies \Pi(y) = 1 \)
- \( J_y < 0 \implies \Pi(y) = 0 \)
- \( J_y = 0 \implies \Pi(y) \in [0, 1] \).

The employer accepts to recruit a worker with probability one if this worker generates positive profits for the firm, if the profits are negative the firm never
hires the worker and if it makes no profits the the firm is indifferent between hiring and not.

**Lemma 4.1** A match will never form if the NSC is not satisfied.

**Proof:** Assume the NSC is not satisfied. Then, by assumption $ay = 0$ which implies that the value of a match is $J = -\frac{w}{r+s}$. A firm will only accept the match if $w = 0$. If $w < b$ the worker will not accept the match. By assumption $b > 0$ so at $w = 0$ workers will not accept the match. Since both worker and firm must accept the match, if the NSC cannot be satisfied the match will never form.□

**Lemma 4.2** If $ay < y_L \equiv b + e + (r + s)\frac{\epsilon}{\lambda}$ then the NSC can never be satisfied.

**Proof:** Assume that $ay < y_L$ and that the NSC is satisfied, that is:

$$E_y - U_y = \frac{w_y - e - b}{r + s + \Pi(y)f(\theta)} \geq \frac{e}{\lambda}$$

For parameters $e$, $b$, $\lambda$, $r$, $s$, and taking $\theta$ as given. Considering the restrictions $w_y \leq ay$ and $\Pi(y) \in [0, 1]$. The largest the RHS of the inequality can be is

$$\frac{ay - e - b}{r + s} \geq \frac{e}{\lambda} \iff ay \geq e + b + \frac{e}{\lambda}(r + s).$$

By assumption $ay < e + b + \frac{e}{\lambda}(r + s)$ which contradicts the statement above. □

**Proof of Proposition 4.**

1. If $ay \in C_0$, then the worker is never hired, $\Pi(y) = 0$.

   **Proof:** If $ay \in C_0$ then $ay < y_L \equiv b + e + (r + s)\frac{\epsilon}{\lambda}$ so by lemma 5.2 the NSC cannot be satisfied. By lemma 5.1 a match will never form, $\Pi(ay) = 0$ □

2. If $ay \in C_1$, then the worker is hired with a probability $\Pi(y) = (ay - b - e - (r + s)\frac{\epsilon}{\lambda}) / f(\theta)\frac{\epsilon}{\lambda}$, and is paid $w = ay$.

   **Proof:** We must prove that to have a match forming it must be the case that $w = ay$ and $\Pi(y) = (ay - b - e - (r + s)\frac{\epsilon}{\lambda}) / f(\theta)\frac{\epsilon}{\lambda}$.

   Assume that $w < ay$, this would imply $J_y > 0$, so $\Pi(y) = 1$. Then

$$E_y - U_y = \frac{w - e - b}{r + s + f(\theta)} < \frac{ay - e - b}{r + s + f(\theta)} \leq \frac{y_M - e - b}{r + s + f(\theta)} = \frac{b + e + (r + s + f(\theta))\frac{\epsilon}{\lambda} - e - b}{r + s + f(\theta)} = \frac{e}{\lambda}$$

The NSC is violated.
If \( w > ay \) then \( J_y < 0 \), matches will never form. This proves that if a match must form with \( ay \in C_1 \) then it must be that \( w = ay \).

The probability of hiring a worker with this productivity is the result of a symmetric Nash Equilibrium. Firms know what the average probability of hiring this worker is and each firm takes it as given. Notice that with \( w = ay \) and \( \Pi(y) = \frac{(ay - b - e - (r + s)\frac{e}{\lambda})}{f(\theta)\frac{e}{\lambda}} \) we have \( E_y - U_y = \frac{\gamma}{\lambda} \). If \( \Pi(y) > \frac{(ay - b - e - (r + s)\frac{e}{\lambda})}{f(\theta)\frac{e}{\lambda}} \) then a firm coming into contact with the worker would not hire him because the NSC is violated and cannot encourage him by paying a higher wage. If \( \Pi(y) < \frac{(ay - b - e - (r + s)\frac{e}{\lambda})}{f(\theta)\frac{e}{\lambda}} \) then the firm could pay a worker \( w = ay - \epsilon \) to make a profit and still not violate the NSC, but since every firm would do the same then \( \Pi(y) = 1 \), so the NSC could not be satisfied.

We can verify that this is indeed a probability, that is \( \Pi(y) = \frac{(ay - b - e - (r + s)\frac{e}{\lambda})}{f(\theta)\frac{e}{\lambda}} \in [0, 1] \) by observing that it is the solution to the equation \( ay_i = (1 - x)y_L + xy_M. \) And by assumption \( ay_i \in C_1 \equiv (y_L, y_M). \)

Appendix 5: Proof of Proposition 5

Making use of (19) and implicit differentiation we can compute:

\[
W_i \equiv \frac{\partial \theta / \partial ay_i}{p_i} = \frac{\gamma(r+s)(s+f(\theta))}{q(\theta)} \frac{s}{s} \left[ \frac{s(e/\lambda)}{(ay_i - b - e - r(e/\lambda))^2} \right] \quad \forall ay_i \in C_1
\]

\[
W_j \equiv \frac{\partial \theta / \partial ay_j}{p_j} = \frac{1}{d\theta} \quad \forall ay_j \in C_2
\]
\[ W_k \equiv \frac{\partial \theta / \partial ay_k}{p_k} = \frac{(1-\beta)(r+s)}{r+s+\beta f(\theta)} \text{ } d\theta \quad \forall ay_k \in C_3 \]

where,
\[ d\theta = \sum_{C_2} p_k \frac{e}{\lambda} f' + \sum_{C_3} p_j \left(1-\beta\right)(ay_j - b - e)(r + s)\beta f' \]

\[ + \sum_{C_0} \left( \sum_{C_1} \frac{p_i s(e/\lambda)}{ay_j - b - e - r(e/\lambda)} - \sum_{C_1} p_i u_i (s + f(\theta)) \right) \left( \frac{q'(\theta)}{q(\theta)^2} \right) \]

We have that \( W_j > W_k \) \( \forall ay_j \in C_2, ay_k \in C_3 \) if and only if
\[ 1 > \frac{(1-\beta)(r+s)}{r+s+\beta f(\theta)} \iff f(\theta) > -(r + s) \]

which is always the case. Also \( W_i > W_j \) \( \forall ay_1 \in C_1, ay_j \in C_2 \) if and only if
\[ \frac{\gamma(r+s)}{q(\theta)} > \frac{e}{\lambda}(s + f(\theta)) \quad (31) \]

From equation (19) we get that at an equilibrium
\[ \frac{\gamma(r+s)}{q(\theta)} > \frac{e}{\lambda}(s + f(\theta)) \]

since \( su < 1 \), this condition implies (30). □

Appendix 6: Proof of Proposition 6

Assume that \( ay_m \notin C_0 \). When \( \gamma \to 0 \), by (21) and (22), the equilibrium market tightness increases to the point of the RHS of the equation is close to zero. This happens at the point where \( ay_m = w_c = b + e + (s + r + f(\theta))\frac{e}{\lambda} \). By definition 3, this implies that any hirable worker with productivity \( ay_i \) must be in Class 1. □
Appendix 7: Proof of Proposition 7

To prove this proposition first it is necessary to see the VSC can be expressed as

\[ \sum_{i} G_i(\theta, m) \]
\[ \frac{U(\theta, m)}{q(\theta)} = (r + s) \gamma \]

(32)

With the introduction of a minimum wage, all the components of (19) can be seen as a function of \( \theta \) and \( m \).

The Probability of being hired upon meeting is given by

\[ \Pi_i(\theta, m) = \begin{cases} 
0 & ay_i < \max \{m, y_L\} \\
\frac{ay_i - b - e - (r + s)(e/\lambda)}{f(\theta)(e/\lambda)} & \max \{m, y_L\} \leq ay_i < y_M(\theta) \\
1 & \max \{m, y_M(\theta)\} \leq ay_i 
\end{cases} \]

The unemployment rate is given by

\[ u_i(\theta, m) = \begin{cases} 
1 & ay_i < \max \{m, y_L\} \\
\frac{s(e/\lambda)}{ay_i - b - e - r(e/\lambda)} & \max \{m, y_L\} \leq ay_i < y_M(\theta) \\
\frac{s}{s + f(\theta)} & \max \{m, y_M(\theta)\} \leq ay_i 
\end{cases} \]

Define \( G_i(\theta, m) \equiv \Pi_i(\theta, m)p_iu_i(\theta, m)(ay_i - w_i(\theta, m)) \). It is clear that \( G_i(\theta, m) \) is increasing in \( \theta \) and decreasing in \( m \). Total unemployment is given by

\[ U(\theta, m) = \sum_i p_iu_i(\theta, m) \]

and it is clear that it is decreasing in \( \theta \) and increasing in \( m \). So for a given \( \theta, U(\theta, m) \leq U(\theta, m') \) and \( G_i(\theta, m) \geq G_i(\theta, m') \) \( \forall i \).

The LHS of (32) is decreasing with \( m \) and \( \theta \), and the RHS is increasing in \( \theta \). Let \((\theta', m')\) and \((\theta'', m'')\) be solutions to (32). By assumption \( m' < m'' \) so it must be that \( \theta' \leq \theta'' \). From the definition of \( u_i(\theta, m) \), it must be \( w_i(\theta', m') \geq w_i(\theta'', m'') \), and from the definition of \( u_i(\theta, m) \), it must be that \( u_i(\theta', m') \geq u_i(\theta'', m'') \). \( \square \)
Appendix 8: Proof of Proposition 8

With discouragement, define
\[ G_i(\theta, m) \equiv \Pi_i(\theta, m)p_i(m)u_i(\theta, m)(ay_i - w_i(\theta, m)) \] and
\[ U(\theta, m) = \sum_i p_i(m)u_i(\theta, m). \]
It is clear that \( U(\theta, m) \) is decreasing in \( \theta \) and, contrary to the case when there is no discouragement, decreasing in \( m \). For a given \( m^* \), let \( \theta^* \) be the equilibrium market tightness that solves the VSC (32)
\[ \sum_i G_i(\theta^*, m^*) = (r + s)\gamma q(\theta^*). \]

By assumption \( m^* < m' \leq w_{E}(\theta', m') \). It follows that \( G_i(\theta^*, m^*) = G_i(\theta^*, m') \forall i, \forall \theta \), and \( U(\theta^*, m^*) \geq U(\theta^*, m') \). We have
\[ \frac{\sum_i G_i(\theta^*, m^*)}{U(\theta^*, m^*)} \leq \frac{\sum_i G_i(\theta^*, m')}{U(\theta^*, m')} \]

Let \( \theta' \) and \( m' \) be solutions to (32), that is
\[ \frac{\sum_i G_i(\theta', m')}{U(\theta', m')} = (r + s)\frac{\gamma}{q(\theta')} \]

Since the LHS of (32) is decreasing in \( \theta \), and the RHS is increasing in \( \theta \). It must be that \( \theta' \geq \theta^* \). From the definition of \( w_i(\theta, m) \) and \( u_i(\theta, m) \)

\[ w_i(\theta^*, m^*) \leq w_i(\theta', m') \quad \text{and} \quad u_i(\theta^*, m^*) \leq u_i(\theta', m') \forall i \in H(m') \]

Where \( H(m') \) is the set of hirable workers under \( m' \).

Appendix A: The no-shirking condition.

The expected lifetime utility of someone who chooses to shirk \( (S) \) during a length of time \( dt \), satisfies
\[ S = wdt + \exp(-rdt) \{ \Pr [\min(\tau_s, \tau_\lambda) \leq dt] U + (1 - \Pr [\min(\tau_s, \tau_\lambda) \leq dt]) E \}. \]
Where \( \min(\tau_s, \tau_\lambda) \) is a Poisson process with parameter \( \lambda + s \) which yields

\[
S = w dt + \exp(-rdt) \left\{ (1 - \exp(-(s + \lambda)dt)) U + (\exp(-(s + \lambda)dt) E \right\}.
\]

Using power series

\[
S = w dt + (1 - r dt + o(dt)) \left\{ [(s + \lambda)dt + o(dt)] U + (1 - (s + \lambda)dt + o(dt)) E \right\}
\]

with \( \lim_{dt \to 0} o(dt)/dt = 0 \). Rearranging

\[
S = w dt + (1 - r dt) \left\{ (s + \lambda)dt U + [1 - (s + \lambda)dt] E \right\} + o(dt)
\]

Substituting \( w dt = s E dt + edt - s dt(U - E) \).

\[
S = E + edt - \lambda dt(E - U) - r dt^2(s + \lambda)(E - U) + o(dt),
\]

. When \( dt \) approaches zero, the worker’s optimal strategy is not to shirk if and only if

\[
S - E \simeq [e - \lambda(E - U)] dt \leq 0
\]

\[
\iff
\]

\[
E - U \geq \frac{e}{\lambda}.
\]

**Appendix B:**
Table 3: Outcomes by deciles for different technology levels

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Table 4: Outcomes by deciles

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Table 5: Outcomes by deciles

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