Bitcoin and its mining on the equilibrium path

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Abstract

Bitcoin as a major cryptocurrency has come up as a shooting star of the 2017 and 2018 headlines. After exploding its price twenty times just in the twelve months of 2017, the tone has changed dramatically in 2018 after major price corrections and increasing concerns about its mining power consumption and overall sustainability. The dynamics and interaction between Bitcoin price and its mining costs have become of major interest. Here we show that these two quantities are tightly interconnected and they tend to a common long-term equilibrium. Mining costs adjust to the cryptocurrency price with the adjustment time of several months up to a year. Current developments suggest that we have arrived at a new era of Bitcoin mining where marginal (electricity) costs and mining efficiency play the prime role. Presented results open new avenues towards interpreting past and predicting future developments of the Bitcoin mining framework.

Keywords: Bitcoin, digital mining, cryptocurrency, electricity

1 Introduction

Bitcoin (Nakamoto, 2008) emerged in the aftermath of the global financial crisis in 2008 as a decentralized alternative to standard (fiat) currency systems, which are under scrutiny of central banks. The decentralization stems from the blockchain as a public ledger where all verified transactions are being recorded. The verification itself is not conducted by any central authority but by a large network of nodes that undertake and solve complex mathematical problems (hashes). This cryptographical element has given the name to the whole family of cryptocurrencies. As a reward for the verification, a pre-specified number of bitcoins is emitted forming an algorithmically given monetary supply (or growth). In parallel to fiat money being historically backed up by gold, which needs to be physically mined, the process of Bitcoin creation is also referred to as mining as work of the network is needed to verify the transactions. As all the transactions are being recorded, the system boasts transparency, even though the actual addresses of sending and receiving parties are represented by alphanumerical chains that cannot be linked to a specific geographic location or a person. In addition, creating a new address (wallet) is trivial and free so that hypothetically, a new one can be created for every single transaction.

Such anonymity had formed the topics of the early Bitcoin (and cryptocurrency in general) studies that dealt primarily with safety and legal issues (Barber et al., 2012;...
Reid and Harrigan, 2013; Velde, 2013). And even though the legality of its early adoption and utility can be and has been questioned repeatedly (Jacobs, 2011; Barratt, 2012), it started to be traded first over the counter and then at specialized Bitcoin exchanges. As the economic saying goes, when supply meets demand on a market, there is a price and an asset. Combining the decentralization, anonymity, little or no regulation, and thus also extreme risk, the Bitcoin price dynamics has never been close to stable. Research on drivers of the price fluctuations logically followed (Kristoufek, 2013; Kondor et al., 2014; Garcia et al., 2014; Kristoufek, 2015; Ciaian et al., 2016) and showed that even though Bitcoin price is strongly driven by speculations, there are fundamental components in the price formation process. These results have been further validated by newer studies (Bouri et al., 2017; Baur et al., 2018; Phillips and Gorse, 2018; Mai et al., 2018; Ciaian and Rajcaniova, 2018).

And even though public awareness of and interest in Bitcoin and crypto-world in general have been tightly connected to its bubble and bust cyclical dynamics, the year of 2017 and mainly its second half experienced a literal crypto-craze, when not only Bitcoin increased its price twenty-fold in a year but also other cryptocurrencies (altcoins) and tokens (derived from smart contract wielding cryptocurrencies such as majorly Ethereum) went through massive gains (Ethereum price grew more than 100 times, Ripple almost 400 times, and Litecoin almost 70 times to name a few). Late 2017 saw an influx of the speculative capital and the FOMO (“fear of missing out”) effect was tangible across groups of eventual investors that perhaps had never heard about cryptocurrencies just few months back.

This was the most evident manifestation of the herding behavior but in the background, also the mining community was expanding. Gamers might never forget Winter Holidays of 2017 when the graphical cards were either out of stock completely or available with large markups due to Bitcoin and Ethereum miners expanding their mining rigs or just getting into the mining business. As a result, the competition in the crypto-mining industry became fiercer, more computational power was needed to be rewarded the same amount of bitcoins as before, and the overall power demand of the mining network was increasing rapidly. The increase has been so immense that the power consumption of the cryptocurrency mining network now comes into the spotlight. Recent results of Krause and Tolaymat (2018) indicate that crypto-mining consumes more energy than mining of copper, gold, platinum and rate earth oxides to produce an equivalent market dollar value and Mora et al. (2018) even suggest that Bitcoin mining could contribute to global warming and by itself increase temperatures by two degrees centigrade within less than three decades. We contribute to the discussion on the relationship between Bitcoin mining and its price formation by finding an equilibrium relationship between the two and showing that mining costs are driven (in addition to the technical factors such as electricity price, mining efficiency, and mining network power consumption) by Bitcoin price but not vice versa forming a hysteresis-like dynamics when a (possibly bubble/speculation induced) price increases are being caught up by increasing mining costs which then form a new support level for potential future price increases.

In the following section, details on the dataset construction are given as it forms an essential part of the whole problem solution. The next section describes the methodological framework, specifically the cointegration and vector error-correction model, and the model selection procedure. The results section is split into three parts. First, the esti-
mated mining costs and profitability are presented. Second, the equilibrium relationship between mining costs and Bitcoin price is established. And third, interactions within the error-correction model are discussed together with the short-term and long-term causality. We conclude with a general discussion of Bitcoin mining market and its possible future developments.

2 Data sources and dataset construction

There are two variables of interest relationship of which is being quantified in this study – Bitcoin price and costs of mining/creating a single bitcoin. The former one is rather simple to obtain as there are various Bitcoin (and cryptocurrency in general) exchanges that provide needed data to analyze. As the price might differ (but not considerably) across exchanges, we utilize the Bitcoin Price Index, which is based on an average price across the most liquid exchanges. The details and downloadable series can be found on https://www.coindesk.com/price. As for the latter, the situation is a bit more complicated. The necessary steps are described in the following sections.

2.1 Marginal/operational costs specification

Bitcoins are mined as a reward for transactions verification to miners who contribute their computational power to the network. The awarded bitcoins are distributed among the miners practically proportionally\(^2\) with respect to the computation power they have delivered to the system. The production cost of a single bitcoin can be thus quantified as costs towards its mining. There are two major costs – purchasing costs of a miner and electricity costs spent on the mining itself. From the standard economics perspective, the former can be seen as a fixed cost and the latter can be seen as a marginal cost. The troubling part of the miners purchasing costs is data availability. Even though prices can be found for specific miners (e.g. https://shop.bitmain.com), there are at least three issues. First, the presented prices are not final as they do not include tariffs, taxes and similar additional costs. Second, the presented prices do not include the costs of power cords, SD cards, and other necessary hardware costs. And third, the prices change in time. The prices of the same miner at the top of the late 2017 bull run were as high as ten times the prices now and it is not possible to keep track of these changes, more so connected with the previous two points. Rumor has it (crypto-community speculates) that producers of the miners set the prices so that the initial costs are amortized in 10 months\(^3\). As we do not want to include such speculations into our analysis, as it would be an important parameter in calculating the costs and it could influence the outcomes strongly, we take a standard economics/finance approach and consider only marginal/operational costs into the price formation dynamics. The purchasing costs are

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\(^1\)The complete dataset is appended to this article.

\(^2\)Higher hashrate implies higher chance of getting rewarded. That is why miners join in mining pools that have higher chances of being rewarded. Mining pool reward is then proportionally distributed among the pool members (minus a cut for the pool operators).

\(^3\)Still this number cannot be reasonably calculated as amortization depends on profits, which in turn depend on electricity costs that vary considerably across miners. In addition, future profits of course depend on the Bitcoin future price that is rather hard to predict.

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then assumed to be a fixed percentage markup to the marginal costs included in the intercept of the final log-log cointegration model.

2.2 Network power consumption

The operational mining costs are not directly available. However, they can be inferred from available data and knowledge of the mining procedures. Mining rewards are given for verification of each block of transactions and the number of bitcoins rewarded is known. 12.5 BTC at the moment and the reward halves every 210,000 blocks. Block time is approximately 10 minutes. This makes a halving period of around 4 years. Power consumption of the network is represented as a number of hashes per second, i.e. the hashrate of the network. Each mining hardware, apart from its own hashrate (power), has a given power consumption, which put together form a miner efficiency measured in joules per hash. Combining these, we are able to get the amount of joules (kilowatt-hours) spent on creation of a single bitcoin. Putting this together with the price of a kilowatt-hour, we arrive at the final operational mining costs. Out of these, all variables are available for the Bitcoin network on https://blockchain.info but two – electricity price and miners’ efficiency.

2.3 Electricity price

We take an average electricity price over countries that are considered to be the main players in the mining industry and they have available and variable electricity prices4 – Canada, Estonia, Georgia, Sweden, and the USA. Electricity price series are available at respective agencies and providers – Independent Electricity System Operator (IESO, http://www.ieso.ca) for Canada, Nordpool (https://www.nordpoolgroup.com) for Estonia and Sweden, Electricity Market Operator (ESCO, https://www.nordpoolgroup.com) for Georgia, and U.S. Energy Information Administration (EIA, https://www.eia.gov) for the USA. The necessary exchange rates have been obtained from the National Bank of Georgia (https://www.nbgo.gov.ge) and the Federal Reserve Bank of St. Louis (https://www.stlouisfed.org). These prices are available with monthly frequency which sets the frequency for the overall analysis. Other variables that are available on daily basis are then taken as monthly averages.

2.4 Mining efficiency

Information about efficiency of specific chips and miners, especially their introduction date and actual use, is not easily available as some mining chips are being kept secret (at least for a period of time) before being made available for public and bulk purchases. However, power consumption with respect to performed hashes is a crucial piece of the puzzle in calculating the marginal mining costs of cryptocurrencies. We use the data available at https://en.bitcoin.it/wiki/List_of_Bitcoin_mining_ASICS. These have been

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4These restrictions disqualify China and Russia from the set. However, this should not influence the final analysis much as the final mining costs are considered in logarithms in the final equation, i.e. the proportional differences are reflected only in the intercept of the final regression, which is not of interest for final interpretations.
also checked with respect to the available information at the respective manufacturers. Mining chips for the analyzed period are listed in Tab. 1. The mining efficiency evolution is then illustrated in Fig. 1 where, in addition to the specific miners’ efficiency, we also add information about the best possible miner at the time as well as an informative hyperbolic fit to the efficiency development. The best available mining chip for a given month is then used in the marginal mining costs calculation.

Table 1: Efficiency of mining chips. Efficiency is quantified in joules per gigahash, i.e. the lower the value the more efficient the mining chip is considered. Mining chips are ordered chronologically with respect to the introduction date.

<table>
<thead>
<tr>
<th>Chip</th>
<th>Manufacturer</th>
<th>Introduced</th>
<th>J/Gh</th>
</tr>
</thead>
<tbody>
<tr>
<td>BE200</td>
<td>ASICMiner</td>
<td>Mar-14</td>
<td>0.66</td>
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<tr>
<td>BF864C55</td>
<td>BitFury Group</td>
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<tr>
<td>FB1600</td>
<td>DigBig</td>
<td>Mar-14</td>
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<td>Spondoolies-Tech LTD</td>
<td>Mar-14</td>
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<tr>
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<td>Avalon Project</td>
<td>Apr-14</td>
<td>0.89</td>
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<td>Apr-14</td>
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<tr>
<td>Neptune</td>
<td>KnCMiner</td>
<td>Jun-14</td>
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<td>Aug-14</td>
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<td>Nov-16</td>
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<td>Ebang</td>
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<td>Bitmain Technologies Ltd.</td>
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<td>DW1228</td>
<td>Ebang</td>
<td>Dec-17</td>
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</tr>
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</table>

3 Cointegration and vector error-correction model

We investigate the relationship between two variables – Bitcoin price and costs of mining of a single bitcoin. As these series are very likely non-stationary, the cointegration analysis is the most intuitive starting point. As it turns out, the series are cointegrated. In this section, we provide some basic notions of the cointegration analysis need for the detailed dynamics examination.

Cointegration relationship (Engle and Granger, 1987) in its bivariate form states a relationship between two series that share a common stochastic trend and they tend to a common equilibrium. From the statistical perspective, two series are cointegrated if
they are integrated of the same order $d$ and there exists their linear combination that is integrated of order $d - b, b > 0$. Specifically, we have series $x$ and $y$ such that

$$y_t = \beta_0 + \beta_1 x_t + \varepsilon_t \tag{1}$$

with time index $t, t = 1, \ldots, T$ where $x \equiv \{x_t\}_{t=1}^T$ and $y \equiv \{y_t\}_{t=1}^T$ are integrated of order $I(d)$ and $\varepsilon \equiv \{\varepsilon_t\}_{t=1}^T$ is an error term integrated of order $I(d - b)$. In the very simplest form, we have $d = b = 1$ but the statistical properties translate into the more general case as well. If the series are cointegrated, the parameters $\beta_0$ and $\beta_1$ can be consistently estimated even for non-stationary series $x$ and $y$. Parameter $\beta_1$ then quantifies the long-term relationship between the two series. As Eq. 1 represents an equilibrium relationship, the error term $\varepsilon$ is a deviation from this equilibrium. In the traditional setting, which is also the case for the dataset we analyze, $d - b < 0$, i.e. it is stationary and mean-reverting so that the overall relationship never systematically deviates from the long-term equilibrium.

This idea is integrated into the (vector) error-correction model – (V)ECM – which uses the ideal cointegration setting when $d = 1$ so that $x$ and $y$ are unit roots and their differences are by definition $I(0)$, i.e. weakly stationary. The error-correction model is then written as

$$\Delta y_t = \omega_0 + \omega_1 \Delta x_t + \eta(y_t-1 - \hat{y}_{t-1}) + \epsilon_t \tag{2}$$

where $\hat{y}_{t-1}$ is a lagged fitted value from Eq. 1 and the whole term $y_{t-1} - \hat{y}_{t-1}$ shows the lagged deviation from the long-term equilibrium. Parameter $\eta$ represents how fast and if at all the system tends towards the equilibrium. When $\eta < 0$, the equilibrium deviation is corrected for (hence the error-correction model label) and the system is stable. When $\eta > 0$, the system diverges.

It is more practical to rewrite the error-correction models into the vector autoregression (VAR) representation, which covers both short-term (VAR) and long-term (cointegration) dynamics of the whole system. Eq. 2 is then rewritten to

$$\Delta x_t = \omega_{10} + \omega_{11} \Delta x_{t-1} + \omega_{12} \Delta y_{t-1} + \eta_1(y_{t-1} - \hat{y}_{t-1}) + \epsilon_{1t}, \tag{3}$$

$$\Delta y_t = \omega_{20} + \omega_{21} \Delta x_{t-1} + \omega_{22} \Delta y_{t-1} + \eta_2(y_{t-1} - \hat{y}_{t-1}) + \epsilon_{2t}. \tag{4}$$

With a system of multiple endogenous variables, the representation can get naturally more complex. As the bivariate version suffices for our analysis here, we redirect an interested reader to Banerjee and Hendry (1992), Ericsson et al. (1998), Hendry and Juselius (2001), Juselius (2006), and Hoover et al. (2008) for more details on multivariate cases where different types and specifications of the vector error-correction models (un/restricted constant/trend specifications) are discussed in detail.

As the procedure of selecting a proper model specification does not need to be completely straightforward, we provide the following step-by-step procedure for transparency and possible replications:

1. Test for the unit roots in the examined series using the augmented Dickey and Fuller (1979) test. Appropriate number of lags is selected based on the Bayesian information criterion (BIC) (Schwarz, 1978) with the maximum lag of 12 as we have monthly series. If the unit roots are not rejected, we proceed.
2. Using the VAR representation of the VECM model, identify the optimal number of lags in the model using the BIC with the maximum lag of 12 again. Check whether the time trend is significant in this model specification. If it is, it points to either the restricted or the unrestricted version of VECM.

3. Using the Johansen tests (both trace and $L_{max}$) (Johansen, 1991, 1995), verify whether the series are cointegrated. Use specifications hinted during the previous steps.

4. Estimate the final VECM specification.

5. Test whether the error-correction term from the final model contains a unit root. If the unit root hypothesis is rejected, we have arrived at the final model for further interpretation.

4 Results

4.1 Mining costs and profitability

We examine the relationship between the mining costs and price of Bitcoin between January 2014 and August 2018. As the latter is rather trivial to obtain, we focus in more detail on the former one. Mining costs of Bitcoin are driven by several factors while the core ones are three – electricity price, power/computational demands of the mining network, and efficiency of chips/miners. Mining efficiency is represented by power demand efficiency of mining chips which is summarized in Fig. 1. In the examined period, the efficiency improved from approximately 0.65 joules per a gigahash (J/GH) at the beginning of 2014 to 0.1 J/GH in the half of 2018. Even though this is a six times improvement, the efficiency has not improved much since the second half of 2014 when it already reached the levels of 0.2 J/GH. As illustrated in Fig. 1, a simple hyperbolic fit suggests a decay of $-0.7$, i.e. slower than a linear increase in efficiency. The other two main factors – electricity price and network hashrate – are illustrated in Fig. 2. Electricity price remains relatively stable and even though there is an evident decreasing trend between 2014 and 2016 followed by a milder increasing trend till the end of the analyzed sample, the price ranges between $0.04$ and $0.07$ per kWh for the whole period. Network computational demands have experienced much more rapid development – starting at around 20 PH/s and reaching around 80 EH/s, i.e. increasing around 4,000 times over the examined less than four years.

We thus see three intertwined forces forming the final marginal costs of mining a new bitcoin – mining efficiency, which has been improving only slightly over the analyzed period\textsuperscript{5}, electricity costs, which have remained rather stable, and power demands of the mining network, which have been exploding over the studied four years\textsuperscript{6}. Putting these dynamics together, we arrive at the estimated marginal costs of mining a single bitcoin

\textsuperscript{5}As of the final writing of this text, Bitmain has announced new S15 and T15 miners, which should reach efficiency of 0.05 J/GH. Their shipping should start by the end of 2018.

\textsuperscript{6}As of the final writing of this text, the power needs of the mining network have stabilized for last few weeks and maybe even two months or so, which suggests that the explosive behavior of the mining difficulty has its rational boundaries. We discuss this in more detail later.
shown in Fig. 3. These costs started at around $3 at the beginning of 2014 shooting up to around $3000. In the similar manner, the Bitcoin price has increased from around $1,000 at the beginning of 2014 going through a bear market in 2014 and 2015 to reach the all-time highs of around $20,000 by the end of 2017 and correcting to the levels of around $6,000 in the middle and second half of 2018. To better illustrate the mining profitability with respect to expenses associated with electricity consumption, we also present the mining margin simply put as a return on investment. The margin starts at rather absurd levels of around 10,000% at the beginning of 2014, then going down to touch quite reasonable 200% in the second half of 2016 and the first half of 2017. This level was distorted by the explosive bull run of the second half of 2017 when the mining profitability peaked at around 2000% at the break of 2017 and 2018. Since the beginning of 2018, the mining profitability has been on a stable downward trend, currently bottoming touch above 100%, i.e. with Bitcoin price at $6,000 and mining costs between $2,000 and $3,000.

![Energy efficiency of miners](image)

**Figure 1:** Energy consumption is measured in joules per gigahash here. The higher the energy consumption, the lower the efficiency. The chart is based on the miners data in Table 1. The circles represent specific miners, the black curve represents the most efficient miner available at the time and the red dashed curve shows the power-law fit to the miners available at the time. The estimated power-law exponent of $-0.69$ suggests that the increase in mining efficiency is slower than linear in time.

### 4.2 Equilibrium relationship

The mining profitability is thus in a global decreasing trend which was disturbed only by the bull run of late 2017. This suggests that the mining industry, even though quite slowly,
Figure 2: Bitcoin network daily hashrate averaged over the given month in petahashes per second (left) and average electricity price in USD cents per kilowatt-hour. Hashrate is given in the semi-log scale, electricity price is shown non-transformed. The electricity price had started the examined period slightly below 7c/kWh and decreased down to touch 4c/kWh in the first quarter of 2016. Since then, the price is in a slight increasing trend but mostly varies between 4c and 6c per kWh. The network hashrate during the examined period can be split into three phases. First, ending around the middle of 2014, signifies the end of the ASIC miners booming introduction, followed by the second one signifying a temporary satiation of the mining network. And the third – increasing – phase is connected to the inversion of the bear market into the bull run that culminated at the end of 2017.

behaves like a standard business where the long-term profitability is slowly drained out by an influx of miners and new competition. It also seems quite clear that the increasing price motivates more miners to participate in the business which leads to higher competition and higher network hashrate which in turn increases the mining costs. To see whether this is in fact true in the statistical sense, we look at the relationship between Bitcoin price and its mining costs through the lenses of cointegration analysis which allows to study both long-term relationship between the series but also short-term dynamics utilizing the vector error-correction model (VECM).

The potential cointegration relationship here is the relationship between mining costs and price of Bitcoin\textsuperscript{7}. Based on the dynamics observed in Fig. 3, we construct the model

\textsuperscript{7}All estimations were performed in gretl 2016d using the base functions of the software and therefore, they should be easy to replicate. All necessary steps are described in the text.
Figure 3: Bitcoin price in USD and marginal costs of mining a single bitcoin in USD (both left) and their margin in percent (right) defined as margin = \( \frac{\text{price}}{\text{cost}} - 1 \). All three time series are represented in the semi-log scale. Bitcoin starts the examined period at around $1,000 and stays in a bear market for around two years before starting a rally towards its all-time highs around $20,000 by the end of 2017. The year of 2018 sees a correction towards levels around $6,000. Marginal mining costs have been catching up the price surge the whole examined period, narrowing the gap with a rather constant rate with the exception of the second half of 2017 as represented by the margin series.

so that Bitcoin price is the impulse variable and mining costs represent the response variable\(^8\). Both variables are of the same level of integration as they both contain a unit root \((p = 0.98 \text{ and } p = 0.96 \text{ in the Augmented Dickey-Fuller test with the null hypothesis of a unit root, respectively for Bitcoin price and mining costs})\). Following by estimation of the long-term connection between the series, we find a highly significant \((p \ll 0.01)\) elasticity of 1.30 based on the Stock and Watson (1993) dynamic least squares estimator. This shows that in the long run, mining costs strongly react to the increasing price of Bitcoin and eventually catch up, the costs even overreact to the increasing prices as the estimated elasticity is above unity. Part of this effect might be due to very cheap mining costs compared to prices at the beginning of the examined period. Further examination is needed to uncover the speed of this adjustment.

Following the procedure described in the previous sections, we estimate the vector autoregression representation of the possible cointegration relationship and identify the optimal number of lags based on the Bayesian information criterion. For both types of

\(^8\)Results do not differ qualitatively when defined the other way around, the interpretation of results remains exactly the same.
models (with and without a time trend), the optimal number of lags is a single one. Detailed inspection into the model uncovers that the time trend is statistically significant for the mining costs equation ($t = 2.7481$ and $p = 0.0084$). We then proceed by testing for cointegration relationship using the set of Johansen tests with this model specification. Out of two possibilities – cointegration with a restricted trend, and an unrestricted trend – the testing procedure selects the specification with the restricted trend, which is further confirmed by the Engle and Granger (1987) testing procedure ($p = 0.0934$). We can thus proceed to the final VECM estimation.

The resulting vector error-correction model is summarized in Tab. 2. The estimated effects suggest that the short-term dynamics is not dominant in the relationship. Apart from a relatively weak auto-correlation element in the Bitcoin price dynamics, there are no significant effects. Importantly, the negative and statistically significant effect of the error-correction term in the marginal costs equation and the insignificant error-correction term in the price equation validate the equilibrium relationship between Bitcoin price and its mining costs. Even though the significance is quite lacking in this VECM system, we see that the coefficient of determination for the marginal costs equation is decent for practically financial data and price evidently plays an important role in setting marginal costs in future. In the next section, we further discuss the interactions and causality between the variables and show that the model still delivers reasonable implications.

As we work with only 56 observations, it is crucial to check the essential assumptions of such time series analysis, specifically remaining serial correlation, heteroskedasticity, and normality of error-terms (residuals) of the final model. There is no remaining serial correlation (according to the Rao-type serial correlation test (Lutkepohl, 2005) with $F(48,52) = 1.428$ and $p = 0.1044$ for up to 12 lags), no heteroskedasticity (according to the Lagrange Multiplier test (Engle, 1982) with $\chi^2(108) = 117.86$, $p = 0.2430$ up to 12 lags), and normality is not rejected either (according to the Doornik and Hansen (2008) test with $\chi^2(4) = 6.9673$ and $p = 0.1376$). As the model passes all these tests, the results can be considered valid and reliable.

4.3 Interactions and causality

To further examine the interaction between Bitcoin price and its mining costs, we test for Granger causality between the series in two dimensions – short-term and long-term. The short-term Granger causality shows no causality from either side ($t = 0.3943$ and $p = 0.6951$ for “price not causing mining costs” null hypothesis, and $t = 0.4759$ and $p = 0.6363$ for “mining costs not causing price” null hypothesis). These testing statistics are the t-stats in Tab. 2. In the long-term, the Toda and Yamamoto (1995) approach uncovers strong causality from price towards mining costs ($F(2,49) = 4.6883$ and $p = 0.0137$) and mild causality from the other side ($F(2,49) = 2.9354$ and $p = 0.0625$). These results are further supported and illustrated by impulse-response functions and variance decompositions in Fig. 4.

The impulse-response functions represent a reaction of one variable to a single standard deviation shock to another variable. Here we see that in both directions, there is only a small reaction in the short term. The overall effect of shocks to mining costs onto Bitcoin price are small even for longer period and the effect is insignificant almost everywhere. From the other side, there is a small reaction by mining costs to shocks in
Table 2: Vector error-correction model. Results are shown for separate marginal costs and price equations. Estimates, standard errors, \(t\)-statistics, and \(p\)-values are reported and accompanied by (adjusted) coefficients of determination and serial correlation statistics (\(\hat{\rho}\) and Durbin-Watson statistics).

**Marginal costs equation**

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<th>Estimate</th>
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<th>(p)-value</th>
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<td>(\Delta Costs_{t-1})</td>
<td>-0.0627</td>
<td>0.1196</td>
<td>-0.5239</td>
<td>0.6027</td>
</tr>
<tr>
<td>(\Delta Price_{t-1})</td>
<td>0.0606</td>
<td>0.1538</td>
<td>0.3943</td>
<td>0.6951</td>
</tr>
<tr>
<td>(EC_{t-1})</td>
<td>-0.2477</td>
<td>0.0857</td>
<td>-2.8898</td>
<td>0.0057</td>
</tr>
</tbody>
</table>

\[ R^2 \quad 0.2568 \quad \bar{R}^2 \quad 0.1962 \]

\[ \hat{\rho} \quad 0.0456 \quad \text{D-W stat} \quad 1.8725 \]

**Bitcoin price equation**

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>SE</th>
<th>(t)-stat</th>
<th>(p)-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>0.0038</td>
<td>0.1009</td>
<td>0.0373</td>
<td>0.9704</td>
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<tr>
<td>(\Delta Costs_{t-1})</td>
<td>0.0644</td>
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<td>0.4759</td>
<td>0.6363</td>
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<td>(\Delta Price_{t-1})</td>
<td>0.3154</td>
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<td>0.0758</td>
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<td>-0.0186</td>
<td>0.0969</td>
<td>-0.1919</td>
<td>0.8486</td>
</tr>
</tbody>
</table>

\[ R^2 \quad 0.1081 \quad \bar{R}^2 \quad 0.0353 \]

\[ \hat{\rho} \quad 0.0049 \quad \text{D-W stat} \quad 1.9799 \]
Figure 4: On the top, the generalized impulse-response functions representing the effect of a single standard deviation shock in mining costs (Bitcoin price) on Bitcoin price (mining costs) shown on the left (right). Black curve represents the estimated shock magnitude, the grey lines represent one standard deviation confidence intervals. On the bottom, the forecast error-variance decomposition (FEVD) is shown for Bitcoin price (left) and mining costs (right). The effect of shocks in mining onto Bitcoin price are tiny and perish quickly. However, price dynamics plays an important role in mining costs.
prices in the short-term but after approximately three months, the mining costs start correcting for the shock and the correction stabilizes only after approximately one year. Forecast variance decomposition tells a very similar story as the mining costs play only a marginal role in the future dynamics of price and it forms only around 10% of the total price variance. The results for the mining costs reflect the dynamics of its impulse-response function – for three months, price forms only around 10% of the mining costs variance but it increases up to around 70% after 12 months. Bitcoin price thus plays a directing role in the dynamics of its mining costs and there is a strong tendency towards the common equilibrium.

5 Discussion and conclusions

Putting the results crudely, Bitcoin mining industry has been following its path towards a standard business. Even though the profits were absurd in the retrospect, the last few months have shown that the standard economic forces are at work even at the crypto-markets. The 100%-200% margin that seems to be a lower floor for the business might look very high. However, we need to keep in mind that the mining costs we consider here are the marginal/operational ones, i.e. taking into consideration only the running costs, not the costs of purchasing the miners, power cords, other hardware and equipment at the mining sites and similar further costs that would be hard to quantify. As long as the mining components manufacturers keep their margins stable, the same can be expected for the minimal margin between the operational costs and Bitcoin price. The analysis we present here can be summarized in the following points.

Bitcoin price drives the mining costs and not (or only weakly) the other way around. This is rather non-standard dynamics of the price formation as normally, the prices are formed with respect to their production costs. However, here we observe the reversed relationship which is tightly connected to an ongoing discussion about the fundamental price/value of Bitcoin or any other cryptocurrency or token. The fact that the mining profit margin was decreasing even during the bear market of 2014 and 2015 only shows that the mining costs catch up on the prices rather than the mining costs keeping the prices up. With the market stabilization in the second half of 2018, at least from the mining perspective, we might be able to see the relationship in a different phase of the market, i.e. a satiated one.

Also, the mining business is far from being unprofitable. Even though the times when the Bitcoin mining was profitable even for home/fan miners are probably irreversibly gone, the electricity costs we consider ensure that the mining is profitable even for electricity prices above the minimal ones. Interestingly, we are getting to the point when the standard economic competition gets into play and the prime factor in the business becomes the electricity price. This has some maybe unexpected implications. Mining will be pushed to places where the electricity prices are low. And when we get to the situation when one needs to get below $0.04/kWh, which forms a floor of our representative electricity costs, the renewable sources of energy become essential. The combination of these sources and cryptocurrency mining can help balancing the unstable systems of the renewables as mining is a non-stop business. The networks can thus become more stable as there could always be consumers for normally excessive capacities and overproduction
of electricity.

Interestingly, Bitcoin might become a victim of its own success. We see that the increasing price drives the mining costs up. These costs are mainly formed by the power consumption as the electricity prices as well as the mining efficiency remain rather stable. As it is hard to imagine much cheaper electricity prices, there would need to be a sustainable revolution in mining efficiency. Looking at the history, this is quite unlikely. Even if we consider the newly (November 2018) announced S15 and T15 miners that promise the consumption fo 0.05 J/GH, it is only less than a four-times improvement over what was available at the end of 2014. The linear or slower increase in mining efficiency is not sufficient to keep the mining power consumption in check. A new rapid (speculation induced) bull run could get the mining costs and thus also associated power consumption to hardly sustainable proportions. And even though we have shown that the market adjusts towards equilibrium, the adjustment is rather slow and the shooting up prices might threaten existence of the whole system. In a sense, the fact that the miners producers are rather slow at this moment (the waiting times are in a frame of several months) are friendly towards sustainability of the whole Bitcoin system. Should the miners be readily available in large amounts, a rocketing price would be quickly followed by a rocketing power consumption and a correction downwards would be problematic.

The Bitcoin market in general and its mining segment specifically still remain a fascinating field to examine. The second half of 2018 has seen a market stabilization and even though the speculators and fan-investors would rather see another price explosion through the whole crypto-market, it seems we are eventually getting to a stable market situation when the mining margins are well set, mining difficulty has stabilized and Bitcoin can look ahead towards its more standard times.

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