Trend Inflation and Asset Pricing in a DSGE Model: Comment

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Abstract

We show that the ability of the Rudebusch and Swanson (2012) model to match a large and variable term premium without compromising the model’s ability to fit key macroeconomic variables relies on zero trend inflation – an assumption that contradicts recent empirical findings which emphasize low-frequency movements in inflation as a key determinant of term structure. We show that a model version that corrects for the presence of trend inflation delivers implausible business cycle and bond price dynamics. This result is driven by the behavior of price dispersion, which is i) counterfactually high and ii) highly inaccurately approximated. We document several model extensions which can to some degree restore the earlier success of the Rudebusch and Swanson model.

1 Introduction

In a highly influential paper, Rudebusch and Swanson (2012) (henceforth, RS) show that a New Keynesian model with Epstein-Zin preferences and long-run inflation risks can match risk premia in bond prices without compromising the fit of macro moments. RS thus offer a resolution to a long-standing puzzle in the macro-finance literature, the ”bond premium puzzle” (cf. Backus, Gregory, and Zin (1989), and Den-Haan (1995)), which had struggled to explain why the term structure of interest rates slopes upward. The mechanism of RS model which generates a sizable and time-varying term premium in a standard macroeconomic DSGE model relies mostly on technology shocks which give rise to large inflation risk for bond holders at business cycle frequencies.

Yet, a growing body of asset pricing literature documents that it is low-frequency movements in inflation that play the most important role in explaining U.S. bond price movements in inflation that play the most important role in explaining U.S. bond price

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† National Bank of Slovakia. We have benefited great deal from the discussions with Larry Christiano.
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dynamics. For instance, Cieslak and Povala (2015) document that the large portion of movement in Treasury bonds risk premia at business cycle frequencies can be attributed to low-frequency movements in inflation, i.e. trend inflation. Bauer and Rudebusch (2017) show that accounting for time-varying trend inflation (and not the variation in the cyclical component of inflation) stands as the key element in understanding the empirical dynamics of U.S. treasury yields. Figure ??, which reproduces figure 1 of Rudebusch and Bauer (2017), summarizes these findings visually, by plotting time series for the ten-year yield, an estimate of trend inflation and the equilibrium nominal and real short rate. The empirical literature, in both macro and finance, has long treated the inflation trend as constant. Stock and Watson (2007) provide strong evidence that the dynamics of inflation have been largely dominated by the trend component. Further, Cogley et al. (2010) and Ascari and Sbordone (2014) demonstrate that inflation innovations account for a small fraction of the unconditional variance of inflation, implying that most of the volatility stems from the trend component of inflation. Similarly, also the theoretical literature rarely accounts for trend inflation explicitly. Noteworthy exceptions are Ascari and Rossi (2012), Ascari, Castelnuovo and Rossi (2011), Ascari and Ropele (2009), who show that trend inflation represents an important factor to consider in the design of monetary policy conduct. However, most of the existing state-of-the-art macro-finance models are solved around a zero inflation steady state. In light of the recent evidence though, it appears of high importance to realign current macro-finance workhorse-model frameworks with the empirical findings and incorporate positive trend inflation as a firm model element.

We find that introducing trend inflation into the baseline RS model generates unrealistic business cycle and bond price dynamics. Moments from model simulated data become implausible, the implied output losses of price dispersion rise to unrealistic values, and price dispersion is inaccurately approximated. The poor model performance in explaining the data arises when positive trend inflation meets Calvo pricing, and decreasing returns to scale in the production function, and is not specific to the (asset price related features of the) RS model. In fact, as we show throughout the paper, similar results are obtained from a standard New Keynesian model of Clarida, Gali and Gertler (1999), albeit to a lesser degree. We can understand these outcomes as coming from two directions. First, the presence of trend inflation introduces steady state costs of inflation which amplify the effect of productivity shocks. Firms which are not able to reset their prices will produce an inefficient amount of output, even in the steady state. This effect is further magnified in case of a concave production function. With trend inflation firms that are stuck with a (too) low price will produce an inefficiently large amount of output, and, as they move along the concave production function to the right, these firms’ marginal cost will increase compared to aggregate marginal costs. Second, we demonstrate that price dispersion is on the one hand counterfactually high, and that on the other hand the Calvo price dispersion equation is poorly captured by local perturbation methods.

1However, while the problem is not specific to the macro-finance literature, the use of decreasing returns to scale production function (or, to be precise, a production function with fixed capital) in a macro-finance model is no coincidence, as it contributes strongly to being to match term premia; relaxing this assumption quickly leads to much smoother stochastic discount factors, and lower generated term premia.

2We should note, that the problem of counterfactually price dispersion and its poor approximation, is present already in the original RS specification, without trend inflation. Positive steady state inflation, however, aggravates the problem substantially, up to the point that simulated model moments stop making sense.
We propose a number of model devices that provide a remedy to the unrealistic business cycle and bond price moments, as well as the accuracy to the price dispersion equation. In particular, an otherwise equivalent setup with Rotemberg price adjustment costs instead of Calvo pricing, constant-returns-to-scale instead of decreasing-returns-to-scale production, or the introduction of inflation indexation can to a large degree restore the performance of RS model in matching a data.

The rest of the paper proceeds as follows. Section 2 documents in detail how simulated model moments are affected by the incorporation of trend inflation, and shows how the resulting complications can be rationalized through additional non-linearities that increase output losses of price dispersion. Section 3 further develops the role of price dispersion in driving these results and studies its numerical properties.

2 Comparing the Model with Trend Inflation to the Data

To explain why the yield curve slopes upward RS\textsuperscript{3} assume a setting with zero trend inflation, \( \bar{\pi} = 0 \). Large inflation risks and inflation risk premia stand behind the success of the RS model. Inflation risks with zero trend inflation necessary imply, however, that the mean of the ergodic distribution of inflation is negative. Therefore, the RS model links inflation risks closely to periods of deflation. To overcome this unsatisfactory implication, and to give the channels through which trend inflation affects bond prices a chance, that recent empirical contributions emphasized, an obvious fix to have positive average inflation is to introduce steady state inflation into the model. However, table \textsuperscript{4} illustrates that relaxing the assumption of \( \bar{\pi} = 0 \), and, instead, allowing for \( \bar{\pi} > 0 \) in the RS model, produces unreasonable, largely inflated macro\textsuperscript{4} and finance unconditional second moments. The first column of table \textsuperscript{4} displays the targeted empirical moments. The following columns are model-based unconditional moments, calculated from third order approximated and pruned model simulations. The second column replicates the RS model results using their best fit calibration from table \textsuperscript{3}. The third column reports results for the RS model with an annualized steady state inflation of 160 bps\textsuperscript{6}. Even this very moderate level of inflation blows up the model moments to unrealistic values. What drives these findings? Is this feature of steady state inflation leading to unreasonable simulated moments specific to the RS model? The forth and fifth column reports model moments when the modeling features of fixed capital (forth column) or a time-varying inflation target (fifth column) are removed from the RS model. These two modeling features additionally contribute to the amplification of the volatilities, as the moments slightly drop when fixed capital and time-varying inflation target are off but overall the volatilities stays in the unreasonable range.

In the search for the mechanism that is responsible for such poor model performance

\textsuperscript{3}To keep the exposition of this comment concise, we refrain from a section that describes the model in detail. We refer the interested reader to the original article or to the detailed model solution in the technical appendix accompanying this paper.

\textsuperscript{4}Note that there is upper bound on inflation in our model (see appendix). Our results are, however, not implied by the maximum inflation constraint.

\textsuperscript{5}Note that results are sensitive to seed of random numbers even for very long simulations (> 500 000).

\textsuperscript{6}At a reasonable value of 2% annualized inflation rate no solution can be obtained any longer.
<table>
<thead>
<tr>
<th>Unconditional</th>
<th>US data</th>
<th>RS1</th>
<th>RS2</th>
<th>RS3</th>
<th>RS4</th>
<th>RS5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moment</td>
<td>1961-2007</td>
<td>π = 0%</td>
<td>π = 1.6%</td>
<td>$A_t N_t^{1-\theta}$</td>
<td>$A_t K_t^\theta N_t^{1-\theta}$</td>
<td>π = 1.6%</td>
</tr>
<tr>
<td>SD(dC')</td>
<td>2.69</td>
<td>0.72</td>
<td>8.29</td>
<td>7.83</td>
<td>5.89</td>
<td>7.42</td>
</tr>
<tr>
<td>SD(C')</td>
<td>0.83</td>
<td>0.88</td>
<td>12.88</td>
<td>11.13</td>
<td>447.44</td>
<td>9.60</td>
</tr>
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<td>SD(N)</td>
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<td>38.16</td>
<td>30.99</td>
<td>482.68</td>
<td>27.27</td>
</tr>
<tr>
<td>Mean(\bar{i})</td>
<td>5.72</td>
<td>3.06</td>
<td>0.46</td>
<td>4.12</td>
<td>-787.40</td>
<td>1.98</td>
</tr>
<tr>
<td>SD(\bar{i})</td>
<td>2.71</td>
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<td>49.39</td>
<td>41.39</td>
<td>2212.94</td>
<td>34.86</td>
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<td>Mean(\bar{\pi})</td>
<td>3.50</td>
<td>-0.54</td>
<td>-2.12</td>
<td>1.42</td>
<td>-791.14</td>
<td>-0.66</td>
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<tr>
<td>SD(\bar{\pi})</td>
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<td>3.06</td>
<td>40.84</td>
<td>36.47</td>
<td>2211.99</td>
<td>29.83</td>
</tr>
<tr>
<td>SD(\bar{\bar{i}}^{(40)})</td>
<td>2.41</td>
<td>2.33</td>
<td>31.25</td>
<td>29.62</td>
<td>2215.71</td>
<td>23.67</td>
</tr>
<tr>
<td>Mean(NTP^{(40)})</td>
<td>1.06</td>
<td>0.91</td>
<td>2.50</td>
<td>3.41</td>
<td>0.55</td>
<td>3.23</td>
</tr>
<tr>
<td>SD(NT^{(40)})</td>
<td>0.54</td>
<td>0.42</td>
<td>7.21</td>
<td>6.63</td>
<td>5.94</td>
<td>6.27</td>
</tr>
<tr>
<td>Mean(R^{(40)} - \bar{R})</td>
<td>1.43</td>
<td>0.88</td>
<td>2.72</td>
<td>3.24</td>
<td>0.98</td>
<td>2.90</td>
</tr>
<tr>
<td>SD(R^{(40)} - \bar{R})</td>
<td>1.33</td>
<td>1.59</td>
<td>26.57</td>
<td>21.95</td>
<td>36.31</td>
<td>19.98</td>
</tr>
<tr>
<td>Mean(S^{-1})</td>
<td>1.06</td>
<td>0.99</td>
<td>1.05</td>
<td>1.01</td>
<td>1014.74</td>
<td>1.01</td>
</tr>
<tr>
<td>SD(S^{-1})</td>
<td>-</td>
<td>0.01</td>
<td>0.82</td>
<td>0.75</td>
<td>13078.28</td>
<td>0.49</td>
</tr>
</tbody>
</table>

*Notes:* Model based moments are calculated from the simulated series (500,000 simulations). RS replicates RS model using their best fit calibration in table 3 of RS. RS1 relaxes RS model by allowing trend inflation $\pi = 1.6$. RS2 is RS1 with labor only DRS production function. RS3 is the RS without time varying inflation target.

In matching the data, we identify several ways how to restore the model data fit in terms of moments, which are reported in Table 2. Table 2 presents simulated moments from versions of the trend-inflation-augmented-RS model, where (i) the assumption about how prices are set in the economy is modified to Rotemberg pricing (instead of Calvo pricing), where (ii) we use linear production function instead of decreasing return to scale (DRS), or where (iii) we remove the effects of trend inflation by inflation indexation to either steady state inflation or last period inflation. These model features that fix the problems with exploding moments documented in Table 4 have a common mechanism: they mitigate the dispersion of prices in the economy. What follows describes the mechanism in play.

Price dispersion, $S_t = \int_0^1 \left( \frac{P_t(j)}{P_t} \right)^{\frac{1}{1-\theta}} dj$, drives a wedge between the inputs needed to produce a given level of output. The aggregate production function is given by,

$$N_t = \left( \frac{Y_t}{A_t K_t^\theta} \right)^{\frac{1}{1-\theta}} \int_0^1 \left( \frac{P_t(j)}{P_t} \right)^{-\frac{1}{1-\theta}}. \tag{1}$$

In equilibrium, price dispersion depends on a ratio of two price indexes, the reset price $P_t^*$ and the aggregate price index $P_t = \left[ \int_0^1 P_t^{1-\epsilon}(j) \right]^{\frac{1}{1-\epsilon}}$, where

$$\frac{P_t^*}{P_t} = \frac{\epsilon}{\epsilon - 1} \sum_{k=0}^{\infty} \phi_{t+k} MC_{t+k}(j) \tag{2}$$

The ratio of price indexes is pined down by a weighted average of firm’s current and
figure 1 illustrates that with positive trend inflation price dispersion quickly decreases the RS model with an annualized inflation rate of zero ($\pi$ from a long model simulation) changes with parameters and order of approximation for of parameter values. The first two rows in figure 1 shows how (mean) price dispersion calibrated. We thus first confirm that the patterns from table 4 hold across a wide range by unit of labor will be dropping. This can be seen also in terms of real marginal costs, due to the concavity of production function (DRS), $Y_{\int \pi}$.

Note also that larger mark-up allows the firm to accommodate bigger deviations from the optimal price. When the optimal price, $P^*_t$, drifts away due to trend inflation the firm will hire more labor than the amount that would be optimal if it were allowed to reset its price optimally, $N^*_t$, and it produces more output. However, due to the concavity of production function (DRS), $Y_t(j) = A_tK_t^\theta N_t(j)^{1-\theta}$ the product by unit of labor will be dropping. This can be seen also in terms of real marginal costs,

$$MC_t^* = \left( \frac{P_t^*}{P_t} \right)^{-\frac{\theta}{1-\theta}} MC_t = \frac{\left( \frac{P_t^*}{P_t} \right)^{-\frac{\theta}{1-\theta}}}{\int_{0}^{1} \left( \frac{P_t(j)}{P_t} \right)^{-\frac{\theta}{1-\theta}} dj}$$

Trend inflation adds a drift into the evolution of prices and thus drives the distribution of prices further apart from the aggregate price index $P_t$. This implies that $\left( \frac{P_t^*}{P_t} \right)^{-\frac{\theta}{1-\theta}} < \left[ \int_{0}^{1} \left( \frac{P_t(j)}{P_t} \right)^{-\frac{\theta}{1-\theta}} dj \right]$. The same logic holds for the firm $j$ (which is not setting its price).

The firm hires $N_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\frac{\theta}{1-\theta}} N^*_t$. When the optimal price, $P^*_t$, drifts away due to trend inflation the firm will hire more labor than the amount that would be optimal if it were allowed to reset its price optimally, $N^*_t$, and it produces more output. However, due to the concavity of production function (DRS), $Y_t(j) = A_tK_t^\theta N_t(j)^{1-\theta}$ the product by unit of labor will be dropping. This can be seen also in terms of real marginal costs,

$$MC_t^* = \left( \frac{P_t^*}{P_t} \right)^{-\frac{\theta}{1-\theta}} MC_t = \frac{\left( \frac{P_t^*}{P_t} \right)^{-\frac{\theta}{1-\theta}}}{\int_{0}^{1} \left( \frac{P_t(j)}{P_t} \right)^{-\frac{\theta}{1-\theta}} dj}$$

This shows that aggregate marginal costs will be higher than optimal real marginal costs with trend inflation. A linear production function, $\theta = 0$, can therefore mitigate the effects of trend inflation by decreasing the wedge between optimal and aggregate marginal costs. A second option how to decrease the wedge between marginal costs is by full inflation indexation. A third option is to increase the monopolistic mark-up (decrease $\epsilon$); having larger mark-up allows the firm to accommodate bigger deviations from the optimal price. Note also that $\theta$ and $\epsilon$ increases the non-linearity of model equilibrium conditions.

In what follows we focus our analysis on the dispersion of prices in the economy. As the RS calibration method targets the moments for $\pi = 0$, the model might be not well calibrated. We thus first confirm that the patterns from table 4 hold across a wide range of parameter values. The first two rows in figure 1 shows how (mean) price dispersion (from a long model simulation) changes with parameters and order of approximation for the RS model with an annualized inflation rate of zero ($\pi = 0\%$). The first picture in figure 1 illustrates that with positive trend inflation price dispersion quickly decreases.

\[\text{See the formal proof in the appendix}\]
Table 2: Empirical and Model-Based Unconditional Moments

<table>
<thead>
<tr>
<th>Unconditional Moment</th>
<th>RS</th>
<th>RS*</th>
<th>RS**</th>
<th>RS***</th>
<th>RS****</th>
</tr>
</thead>
<tbody>
<tr>
<td>π = 1.6%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SD(dC)</td>
<td>8.29</td>
<td>0.43</td>
<td>0.45</td>
<td>0.71</td>
<td>0.49</td>
</tr>
<tr>
<td>SD(C)</td>
<td>12.88</td>
<td>0.49</td>
<td>0.53</td>
<td>0.89</td>
<td>0.68</td>
</tr>
<tr>
<td>SD(N)</td>
<td>38.16</td>
<td>1.45</td>
<td>1.39</td>
<td>2.50</td>
<td>1.85</td>
</tr>
<tr>
<td>Mean(i)</td>
<td>0.46</td>
<td>3.16</td>
<td>4.80</td>
<td>5.73</td>
<td>4.83</td>
</tr>
<tr>
<td>SD(i)</td>
<td>49.39</td>
<td>2.09</td>
<td>2.46</td>
<td>3.43</td>
<td>3.07</td>
</tr>
<tr>
<td>Mean(π)</td>
<td>-2.12</td>
<td>-0.48</td>
<td>1.05</td>
<td>2.22</td>
<td>1.42</td>
</tr>
<tr>
<td>SD(π)</td>
<td>40.84</td>
<td>2.14</td>
<td>2.33</td>
<td>2.98</td>
<td>2.58</td>
</tr>
<tr>
<td>SD(i(40))</td>
<td>31.25</td>
<td>1.54</td>
<td>1.54</td>
<td>2.37</td>
<td>1.84</td>
</tr>
<tr>
<td>Mean(NTP(40))</td>
<td>2.50</td>
<td>0.83</td>
<td>0.64</td>
<td>1.08</td>
<td>1.23</td>
</tr>
<tr>
<td>SD(NTP(40))</td>
<td>7.21</td>
<td>0.36</td>
<td>0.10</td>
<td>0.55</td>
<td>0.03</td>
</tr>
<tr>
<td>Mean(R(40) − R)</td>
<td>2.72</td>
<td>0.84</td>
<td>0.61</td>
<td>1.11</td>
<td>1.27</td>
</tr>
<tr>
<td>SD(R(40) − R)</td>
<td>26.57</td>
<td>1.03</td>
<td>1.13</td>
<td>1.61</td>
<td>1.59</td>
</tr>
<tr>
<td>Mean(S−1)</td>
<td>11</td>
<td>0.00</td>
<td>1.00</td>
<td>0.99</td>
<td>1.00</td>
</tr>
<tr>
<td>SD(S−1)</td>
<td>11</td>
<td>0.00</td>
<td>0.00</td>
<td>0.01</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Notes: RS replicates RS model using best fit calibration in table 3 of RS with steady state inflation. RS* is RS but assuming Rotemberg pricing. RS** is RS model but with linear production function. RS*** shows RS with indexation to average inflation (ι = 0) and past inflation (ι = 1) and for higher order approximation quickly becomes economically unfeasible. The other subplots show how price dispersion changes with other key model parameters. The first two rows in figure 1 show how price dispersion varies across the parameter space and order of approximation for RS model with an annualized inflation rate of one percent (π = 1%).

3 Approximation Accuracy of Price Dispersion

This section examines the role of price dispersion in generating the explosive dynamics from the point of view of the numerical accuracy of the approximation. We calculate a more accurate measure of price dispersion, using the fact that the price dispersion can be written recursively as

\[ S_{t-1}^{1/\gamma} = (1 - \theta) \left[ \frac{1 - \zeta (\pi_t)^{\gamma - 1}}{1 - \zeta} \right]^{(\gamma - 1)/(\gamma - \Theta)} + \Theta (\pi_t)^{\gamma - \Theta} S_{t-1}^{1/\gamma}. \]  

We then proceed as follows. First, we use \( S_{t-1} \) from the approximated model as starting point. Second, we iterate the equation forward to get an exact solution conditional on the model-approximated time path of \( \pi_t \). Third, we compare this more exact measure of price dispersion with its counterpart from the third order approximation.

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8 Price dispersion is bounded by one. See the technical appendix or Schmitt-Grohe and Uribe (2007).
9 We very much thank Larry Christiano for suggesting to look at the problem in this way.
Figure 1: Parameters sensitivity RS (2012) model.
Notes: RS replicates RS model using best fit calibration in table 3. RS $\pi = 1$ is RS with 1% annualized steady state inflation. RS $Y_t = A_t N_t$ is RS with linear production function. RS 1st order approx is the RS approximated only up to first order. RS $\pi = 1$ indexation is the RS with indexation to past inflation ($\iota = 1$). RS $\pi = 1$ and $Y_t = A_t N_t$ is RS with steady state inflation and linear production function.
Figure 2 demonstrates that local methods deliver a poor approximation of the price dispersion equation. Further, the figure shows that the more accurate measure of price dispersion is unreasonably large for the RS model and implies that the correct model does not match the macro and finance stylized facts. The first picture stresses this finding by showing that the approximation of price dispersion is poor even for the original RS model as the deviations between third order and 'exact' solution are large. The perturbation methods do an even poorer job in case of positive trend inflation. In addition, in case of positive inflation the third order approximation generates states of the worlds which are economically infeasible as $1/S_t$ exceeds one, which would mean that more resources are spent than produced, $Y_t < C_t + I_t + G_t$. The first picture in the second row shows that a first order approximation delivers smaller approximation errors.

The approximation errors for the cases of indexation to past inflation and constant return to scale in labor are negligible. The RS model with positive steady state inflation and indexation delivers both small price dispersion and negligible approximation errors (as can be observed by the almost complete overlay). However, note that in case of linear production function the more exact measure of price distortion is still large. There are states of the world when the price dispersion implies an almost 10% quarterly output loss which is at odds with empirical evidence (see for example Nakamura, Steinsson, Sun, and Villar (2016)).

4 Conclusion

This note emphasizes that an attempt to realign the current macro-finance workhorse modeling framework of RS with recent empirical evidence should consist in incorporating positive trend inflation into such framework. We document that pricing assets in models which are based on the Calvo price mechanism can lead to very counterfactual model dynamics, once trend inflation is present; we then propose a number of directions how to overcome such complications. This way, we contribute to providing guidance along the path of finding a new, empirically well-motivated and consistent modeling framework.

5 Appendix A

The appendix A shows the detailed exposition of $\Theta$ and $\Phi$ model. We mirror the calculations of the main text here for the $\Phi$ model (presented in the subsection ??).

5.1 Basic New Keynesian Model

This section of the appendix outlines the basic New Keynesian model and presents results analogous to the one in the main paper. The model closely follows the sticky price model

\footnote{Price dispersion is expressed here in terms of the output loss due to dispersion of prices in the economy, $1/S_t$.}

\footnote{Note that the original Rudebusch and Swanson (2012) results are sensitive to seed of random number generator even for very long simulations, putting doubts on the numerical properties of the model.}
of Clarida, Gertler, Galí (CGG), with two exceptions: one, we use a production function that is assumed to be of the DRS-labor-only type as our baseline, such that we can document the problems arising from it when steady state inflation is positive, paralleling our findings based on the RS model. Two, we assume that productivity shocks are difference-stationary, as in this case the inaccuracies in price dispersion is much more pronounced and easier to document than in the case of trend-stationary shocks (where they are mild).

Otherwise, the model features are standard, firms are monopolistically competitive, face nominal rigitities a la Calvo, and the monetary authority follows a standard Taylor rule. Below we derive the model’s first order and equilibrium conditions.

5.1.1 Households

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left( \frac{C_t}{A_t} \right)^{1-\tau} - \frac{N_{t+1}}{1 + \varphi} \right\},
\]

(6)

subject to

\[
P_tC_t + B_t \leq B_{t-1}R_{t-1} + W_t N_t + T_t.
\]

(7)

The first order conditions are given by:

w.r.t. \(C_t\):

\[
C_t^{1-\tau} A_t^{(1-\tau)} = \lambda_t P_t,
\]

(8)

w.r.t. \(N_t\):

\[
\xi_t N_t^\varphi = \lambda_t W_t,
\]

(9)

w.r.t. \(B_t\):

\[
\lambda_t = \beta E_t \lambda_{t+1} R_t,
\]

(10)

w.r.t. \(\lambda_t\):

\[
P_tC_t + B_t = B_{t-1}R_{t-1} + W_t N_t + T_t.
\]

(11)

Combining gives:

\[
\xi_t N_t^\varphi C_t \left( A_t^{(1-\tau)} \right) = \frac{W_t}{P_t},
\]

(12)

\[
C_t^{(1-\tau)} A_t = \beta E_t C_{t+1}^{(1-\tau)} A_t \left( \frac{P_t}{P_{t+1}} \right) R_t,
\]

(13)

\[
P_tC_t + B_t = B_{t-1}R_{t-1} + W_t N_t + T_t.
\]

(14)
Detrending and deflating, that is, rewriting the above conditions in terms of stationary allocations and real (relative) prices, denoted by lowercase variables, i.e.: \( c_t = \frac{C_t}{A_t}, y_t = \frac{Y_t}{A_t}, \)
\( \pi_t = \frac{P_t}{P_{t-1}}, w_t = \frac{W_t}{P_tA_t}, b_t = \frac{B_t}{P_tA_t}, t_t = \frac{T_t}{P_tA_t}, \)
gives:

\[
\xi_t N_t^\varepsilon c_t^\varepsilon = w_t, \quad (15)
\]
\[
c_t^\varepsilon = \beta E_t c_{t+1}^{\varepsilon+1} \frac{R_t}{\pi_{t+1} dA_{t+1}}, \quad (16)
\]
\[
c_t + b_t = \frac{b_{t-1}}{dA_t \pi_t} R_{t-1} + w_t N_t + t_t, \quad (17)
\]

### 5.1.2 Final good firms

Final good firms have production technology

\[
Y_t = \left[ \int_0^1 Y_t(i) \frac{i^\varepsilon}{\varepsilon} \, di \right]^{\frac{1}{1-\alpha}}, \quad (18)
\]
and buy \( Y_t(i) \) at prices \( P_t(i) \) and sell \( Y_t \) at \( P_t \). Their profit maximization problem results in the following optimality conditions:

\[
Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t, \quad (19)
\]
with

\[
P_t = \left[ \int_0^1 P_t(i)^{1-\varepsilon} \, di \right]^{1-\varepsilon}. \quad (20)
\]

### 5.1.3 Intermediate goods firms

An intermediate good firm’s problem can be split into a (static) cost minimization and a (dynamic) profit maximization problem. The cost minimization problem reads

\[
\min_{N_t(j)} \left\{ W_t N_t(j) + MC_t(j) \left[ Y_t(j) - A_t N_t(j)^{1-\alpha} \right] \right\},
\]
with first order condition with respect to \( N_t(j) \) and \( MC_t(j) \):

\[
MC_t(j) = \frac{1}{1-\alpha} W_t^{\frac{1}{1-\alpha}} A_t^{\frac{1}{1-\alpha}} Y_t(j)^{\frac{\alpha}{1-\alpha}} \quad (21)
\]

\[
Y_t(j) = A_t N_t(j)^{1-\alpha} \quad (22)
\]
The firm’s profit maximization problem, having substituted in for the demand function the firm faces for its product, equation [19], is

\[
\max_{P_t(j)} \sum_{k=0}^{\infty} \theta^k \Omega_{t,t+k} \left\{ [P_t(j) - MC_t(j)] \left( \frac{P_t(j)}{P_t} \right)^{-\varepsilon} Y_t \right\},
\]

with first order condition:

\[
\frac{P_t^*(j)}{P_t} = \frac{\varepsilon}{(\varepsilon - 1)} \frac{E_t \sum_{k=0}^{\infty} (\beta \theta)^k \frac{uc_{t+k,j}}{uc_{t+j}} \frac{MC_{t+k}(j)}{P_{t+k}} \left[ \left( \frac{P_t}{P_{t+k}} \right)^{1-\varepsilon} Y_{t+k} \right]}{E_t \sum_{k=0}^{\infty} (\beta \theta)^k \frac{uc_{t+k,j}}{uc_{t+j}} \left[ \left( \frac{P_t}{P_{t+k}} \right)^{1-\varepsilon} Y_{t+k} \right]}.
\]

Wanting to get rid of individual firm \(j\)’s marginal cost and writing everything in terms of average variables, we rewrite marginal cost of firm \(j\) as:

\[
MC_{t+k}(j) = \frac{1}{1 - \alpha} W_{t+k} \left( \frac{1}{A_{t+k}} \right)^{\frac{1}{\alpha}} Y_{t+k}(j) \left( \frac{1}{Y_{t+k}} \right)^{\frac{\alpha}{1-\alpha}}
\]

\[
= \frac{1}{1 - \alpha} W_{t+k} \left( \frac{1}{A_{t+k}} \right)^{\frac{1}{\alpha}} Y_{t+k}(j) \left( \frac{P_t^*}{P_{t+k}} \right)^{-\varepsilon} \left( \frac{\alpha}{\varepsilon - \alpha} \right),
\]

\[
= MC_{t+k} \left( \frac{P_t^*}{P_{t+k}} \right)^{-\varepsilon} \left( \frac{\alpha}{\varepsilon - \alpha} \right),
\]

where \(MC_{t+k}\) is average marginal cost, given by

\[
MC_{t+k} = \frac{1}{1 - \alpha} W_{t+k} \left( \frac{1}{A_{t+k}} \right)^{\frac{1}{\alpha}} Y_{t+k}^{\frac{\alpha}{1-\alpha}}.
\]

Using the above relation to substitute out for \(MC_{t+k}(j)\) and substituting in for \(u_{C_{t+k}} = C_{t+k} A_{t+k}^{-1(1-\gamma)}\), detrending and deflating, i.e., defining \(p_t^* = \frac{p_t(j)}{P_t}, mc_t(j) = \frac{MC_t(j)}{P_t}, y_t = \frac{Y_t}{A_t}\), the optimal pricing equation can be rewritten as:

\[
p_t^* \left( 1 + \frac{\alpha}{\varepsilon - \alpha} \right) = \frac{\varepsilon}{(\varepsilon - 1)} \frac{E_t \sum_{k=0}^{\infty} (\beta \theta)^k \frac{uc_{t+k,j}^{\gamma_{t+k}}}{c_t^{\gamma_{t+k}}} mc_{t+k} \left( \frac{P_t}{P_{t+k}} \right)^{\frac{\gamma_{t+k}}{1-\gamma}} y_{t+k}}{E_t \sum_{k=0}^{\infty} (\beta \theta)^k \frac{uc_{t+k,j}^{\gamma_{t+k}}}{c_t^{\gamma_{t+k}}} \left( \frac{P_t}{P_{t+k}} \right)^{1-\varepsilon} y_{t+k}}.
\]

Define auxiliary variables \(aux_{1t}\) and \(aux_{2t}\). 

12
\[ \text{aux}_{t+1} = m_c y_t + E_t \beta \pi_{t+1} \frac{\varepsilon c_{t+1}^{\gamma}}{c_t^{\gamma}} \text{aux}_{t+1}, \quad (29) \]

\[ \text{aux}_{2t} = y_t + E_t \beta \pi_{t+1} \frac{c_{t+1}^{\gamma}}{c_t^{\gamma}} \text{aux}_{2t+1}, \quad (30) \]

we can rewrite the optimal price setting equation recursively as

\[ p_t^{(1+\frac{\varepsilon}{1-\alpha})} = \frac{\varepsilon \text{aux}_{1t}}{(\varepsilon-1) \text{aux}_{2t}}. \]

If we were to have modeled that firms get a production subsidy that offsets the distortion from monopolistic competition, the above option price setting equation reads (where \( \nu = \frac{1}{\varepsilon} \)):

\[ p_t^{(1+\frac{\alpha}{1-\alpha})} = \left(1 - \nu \right) \frac{\varepsilon \text{aux}_{1t}}{(\varepsilon-1) \text{aux}_{2t}}. \quad (31) \]

### 5.1.4 Aggregation and market clearing

Using the production function to express \( N_t(j) \),

\[ N_t(j) = \left[ \frac{Y_t(j)}{A_t} \right]^{\frac{1}{1-\alpha}}, \]

and pugging it into demand for variety \( j \),

\[ Y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\varepsilon} Y_t, \]

and integrating over all varieties gives

\[ \int_0^1 N_t(j) \, dj = \left[ \frac{Y_t}{A_t} \right]^{\frac{1}{1-\alpha}} \int_0^1 \left( \frac{P_t(j)}{P_t} \right)^{\frac{-\varepsilon}{1-\alpha}} \, dj, \]

\[ \Delta_t Y_t = A_t N_t^{1-\alpha}, \]

which in terms of detrended variables becomes

\[ \Delta_t y_t = N_t^{1-\alpha}. \]

Price dispersion is defined recursively:

\[ \Delta_t^{\frac{1}{1-\alpha}} \equiv \int_0^1 \left( \frac{P_t(j)}{P_t} \right)^{\frac{-\varepsilon}{1-\alpha}} \, dj \]

\[ = (1 - \theta) \left( \frac{P_t^*(j)}{P_t} \right)^{\frac{-\varepsilon}{1-\alpha}} + \theta (1 - \theta) \left( \frac{P_{t-1}^*(j)}{P_t} \right)^{\frac{-\varepsilon}{1-\alpha}} + \theta^2 (1 - \theta) \left( \frac{P_{t-2}^*(j)}{P_t} \right)^{\frac{-\varepsilon}{1-\alpha}} + \ldots \]

\[ \Delta_t^{\frac{1}{1-\alpha}} \equiv (1 - \theta) (p_t^*)^{\frac{-\varepsilon}{1-\alpha}} + \theta (\pi_t)^{\frac{\varepsilon}{1-\alpha}} \Delta_t^{\frac{1}{1-\alpha}}. \quad (32) \]
Finally, from the aggregate price index, equation (20), we can derive the law of motion for prices:

\[ p_t^* = \left[ \frac{1 - \theta \pi_t^{-1}}{1 - \theta} \right]^{\frac{1}{1-\varepsilon}}. \]

5.1.5 Flexible price output

The flexible price level of output can be obtained in closed form as a function of parameters and exogenous shocks only:

labor supply:

\[ \xi_t N_t^\varepsilon c_t^\tau = w_t, \]

labor demand:

\[ w_t = (1 - \nu) \frac{\varepsilon}{(\varepsilon - 1)}, \]

resource constraint:

\[ c_t = y_t, \]

production function:

\[ y_t = N_t^{1-\alpha}, \]

Combining:

\[ \xi_t N_t^\varepsilon c_t^\tau = (1 - \nu) \frac{\varepsilon}{(\varepsilon - 1)}, \]

\[ \xi_t y_t^{1-\alpha} y_t^\tau = (1 - \nu) \frac{\varepsilon}{(\varepsilon - 1)} \]

\[ y_t^{\text{flex}} = \left[ (1 - \nu) \frac{\varepsilon}{(\varepsilon - 1)} \frac{1}{\xi_t} \right]^{\frac{1-\alpha}{\varepsilon+\varepsilon(1-\alpha)}} \]

(33)

5.1.6 List of equilibrium conditions

\[ \xi_t N_t^\varepsilon c_t^\tau = w_t \]

(34)

\[ c_t^{\tau} = \beta E_t c_{t+1}^{\tau} \frac{1}{dA_{t+1}} \frac{R_t}{\pi_{t+1}} \]

(35)

\[ p_t^{(1+\varepsilon\alpha)/(1-\alpha)} = (1 - \nu) \frac{\varepsilon}{(\varepsilon - 1)} \frac{aux_{1,t}}{aux_{2,t}} \]

(36)
\[ \text{aux}_{1t} = mc_t y_t + E_t \beta \pi_{t+1} \frac{c_{t}^{\tau}}{c_{t}^{\tau}} aux_{1t+1} \]  
(37)

\[ \text{aux}_{2t} = y_t + E_t \beta \pi_{t+1} \frac{c_{t}^{\tau}}{c_{t}^{\tau}} aux_{2t+1} \]  
(38)

\[ \Delta_t y_t = N_t^{1-\alpha} \]  
(39)

\[ \Delta_t^{\frac{1}{1-\alpha}} \equiv (1 - \theta) (p^*_t)^{\frac{\gamma}{1-\alpha}} + \theta (\pi_t)^{\frac{\epsilon}{1-\alpha}} \Delta_t^{\frac{1}{1-\alpha}} \]  
(40)

\[ p^*_t = \left[ \frac{1 - \theta \pi_t^{\frac{\epsilon}{1-\alpha}} - 1}{1 - \theta} \right] \]  
(41)

\[ mc_t = \frac{1}{1 - \alpha} w_t y_t^{\frac{n}{1-\alpha}} \]  
(42)

\[ c_t = y_t \]  
(43)

\[ \frac{R_t}{R} = \left( \frac{R_t}{R} \right)^{\rho_R} \left[ \left( \frac{\pi_t}{\pi} \right)^{\rho_p} \left( \frac{y_t}{y_t^{\text{flex}}} \right)^{\rho_y} \right]^{1-\rho_R} e^{\varepsilon_R,t} \]  
(44)

\[ y_t^{\text{flex}} = \left[ (1 - \nu) \frac{\epsilon}{(\epsilon - 1)} \xi_t \right]^{\frac{1}{\phi + \tau(1-\alpha)}} \]  
(45)

\[ \log (dA_t) = \rho_A \log (dA_{t-1}) + (1 - \rho_A) dA + \varepsilon_{dA,t} \]  
(46)

\[ \log (\xi_t) = \rho_\xi \log (\xi_{t-1}) + (1 - \rho_\xi) \xi + \varepsilon_{\xi,t} \]  
(47)

### 5.1.7 Steady state

Flexible price output:

\[ y_t^{\text{flex}} = \left[ (1 - \nu) \frac{\epsilon}{(\epsilon - 1)} \xi_t \right]^{\frac{1}{\phi + \tau(1-\alpha)}} \]  
(48)

Taylor rule at stst:

\[ \pi = \pi^{\text{target}} \]  
(49)
Euler eq. at stst:

\[ R = \frac{d\Lambda}{\beta} \]  

\( p^* \) at stst

\[ p^* = \left[ \frac{1 - \theta \pi^{-1}}{1 - \theta} \right]^{\frac{1}{\varepsilon}} \]  

Price dispersion at stst

\[ \Delta \equiv \left[ \frac{1}{1 - \theta (\pi)^{\frac{1}{1-\alpha}}} (1 - \theta) (p^*)^{-\frac{\varepsilon}{1-\alpha}} \right]^{1-\alpha} \]  

Pricing equation and auxiliary variables, to obtain stst mc:

\[ p^* (1 + \frac{\varepsilon}{\alpha}) = (1 - \nu) \frac{\varepsilon}{(\varepsilon - 1)} aux_1 aux_2 \]

\[ aux_1 = \frac{mc y + \beta \theta \pi^{\frac{1}{1-\alpha}} aux_1}{1 - \beta \theta \pi^{\frac{1}{1-\alpha}}} \]

\[ aux_2 = \frac{y + \beta \theta \pi^{\frac{1}{1-\alpha}} aux_2}{1 - \beta \theta \pi^{\frac{1}{1-\alpha}}} \]

\[ p^* (1 + \frac{\varepsilon}{\alpha}) = (1 - \nu) \frac{\varepsilon}{(\varepsilon - 1)} \frac{1}{1 - \beta \theta \pi^{\frac{1}{1-\alpha}}} \frac{mc y}{y} \]

\[ mc = p^* (1 + \frac{\varepsilon}{\alpha}) \frac{\varepsilon - 1}{\varepsilon (1 - \nu)} \frac{1}{1 - \beta \theta \pi^{\frac{1}{1-\alpha}}} \]  

Definition of marginal cost, labor supply, resource constraint, and production function:

\[ mc = \frac{w}{1 - \alpha} y^{\frac{1}{1-\alpha}} \]

\[ \xi N^\varepsilon c^\varepsilon = w \]
\[ c = y \]

\[ \Delta y = N^{1-\alpha} \]

\[ mc = \frac{\xi N^\varphi e^\tau}{1 - \alpha} y^{\frac{\alpha}{1-\alpha}} \]

\[ mc = \frac{\xi [\Delta y]^{\frac{\varphi}{1-\alpha}} y^\tau}{1 - \alpha} y^{\frac{\alpha}{1-\alpha}} \]

\[ mc = \frac{\xi \Delta^{\frac{\varphi}{1-\alpha}}}{1 - \alpha} y^{\frac{\alpha}{1-\alpha}} y^{\frac{\varphi}{1-\alpha}} + \frac{\alpha}{1-\alpha} \]

\[ y = \left[ \frac{(1 - \alpha) mc}{\xi \Delta^{\frac{\varphi}{1-\alpha}}} \right]^{\frac{1-\alpha}{\alpha + \tau(1-\alpha)+\varphi}} \]

\[ y = \left[ \frac{(1 - \alpha) mc}{\xi \Delta^{\frac{\varphi}{1-\alpha}}} \right]^{\frac{1-\alpha}{\alpha + \tau(1-\alpha)+\varphi}} \]  \hspace{1cm} (54)

\[ c = y \]  \hspace{1cm} (55)

\[ N = [\Delta y]^{\frac{1}{1-\alpha}} \]  \hspace{1cm} (56)

\[ w = \frac{(1 - \alpha) mc}{y^{\frac{\alpha}{1-\alpha}}} \]  \hspace{1cm} (57)

\[ aux_1 = \left[ \frac{1}{1 - \beta \theta \pi^{\frac{\alpha}{1-\alpha}}} \right] mc y \]  \hspace{1cm} (58)

\[ aux_2 = \left[ \frac{1}{1 - \beta \theta \pi^{\varphi-1}} \right] y \]  \hspace{1cm} (59)

5.1.8 Results

5.2 Rudebusch and Swanson Model

5.2.1 Households

The description of the households and firms’ problems below closely follows RS.
Table 3: Empirical and Model-Based Unconditional Moments

<table>
<thead>
<tr>
<th>Unconditional Moment</th>
<th>US data 1961-2007</th>
<th>NK</th>
<th>NK*</th>
<th>NK**</th>
<th>NK***</th>
<th>NK****</th>
</tr>
</thead>
<tbody>
<tr>
<td>SD(C)</td>
<td>0.83</td>
<td>5.37</td>
<td>33.25</td>
<td>4.50</td>
<td>5.37</td>
<td>7.00</td>
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<tr>
<td>SD(N)</td>
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<td>7.78</td>
<td>17.29</td>
<td>10.75</td>
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<td>-3.34</td>
<td>-2.37</td>
<td>-2.72</td>
<td>-2.38</td>
</tr>
<tr>
<td>SD((\pi))</td>
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<td>8.96</td>
<td>19.52</td>
<td>10.52</td>
<td>8.96</td>
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<tr>
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<td>-5.08</td>
<td>-3.59</td>
<td>-4.11</td>
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<tr>
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<td>10.36</td>
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<td>21.21</td>
<td>11.01</td>
<td>7.26</td>
<td>1.85</td>
</tr>
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Table 4: Empirical and Model-Based Unconditional Moments

<table>
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<tr>
<th>Unconditional Moment</th>
<th>US data 1961-2007</th>
<th>NK</th>
<th>NK*</th>
<th>NK**</th>
<th>NK***</th>
<th>NK****</th>
</tr>
</thead>
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<tr>
<td>SD(C)</td>
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</tr>
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<td>1598.03</td>
<td>56.23</td>
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</tr>
</tbody>
</table>

Table 5: Moments from the models
The household maximizes the continuation value of its utility \( V \), which is of Epstein-Zin form and follows the specification of RS:

\[
V_t = \begin{cases} 
U(C_t, L_t) + \beta \left[ E_t V_{t+1}^{1-\alpha} \right]^{1/\alpha} & \text{if } U(C_t, L_t) \geq 0 \\
U(C_t, L_t) - \beta \left[ E_t (-V_{t+1})^{1-\alpha} \right]^{1/\alpha} & \text{if } U(C_t, L_t) < 0.
\end{cases}
\] (60)

The households’ problem is subject to its flow budget constraint:

\[ B_t + P_t C_t = W_t L_t + D_t + R_{t-1} B_{t-1} \] (61)

In equation (60), \( \beta \) is the discount factor. Utility \( U \) at period \( t \) is derived from consumption \( C_t \) and leisure \((1 - L_t)\). \( E_t \) denotes expectations conditional on information available at time \( t \). As the time frame is normalized to one, leisure time \((1 - L_t)\) is what remains after spending some time working \( L_t \). \( W_t L_t \) is labor income, \( R_t \) is the return on the one-period nominal bond, \( B_t \), \( D_t \) is dividend income.

To be consistent with balanced growth, RS imposes the following functional form on \( U \):

\[
U(C_t, L_t) = \frac{C_t^{1-\varphi}}{1 - \varphi} + \chi_0 Z_t^{1-\varphi} \frac{(1 - L_t)^{1-\chi}}{1 - \chi}, \quad \varphi, \chi > 0,
\] (62)

where \( Z_t \) is an aggregate productivity trend, and \( \varphi, \chi, \chi_0 > 0 \). The intertemporal elasticity of substitution (IES) is \( 1/\varphi \), and the Frisch labor supply elasticity is given by \((1 - \bar{L})/\chi \bar{L} \), where \( \bar{L} \) is the steady state of hours worked.

### 5.2.2 Firms

Final good firms operate under perfect competition with the objective to minimize expenditures subject to the aggregate price level \( P_t = \left( \int_0^1 P_t(i) i^{1-\lambda} \, di \right)^{-\lambda} \), where \( P_t(i) \) is the price of intermediate good produced by firm \( i \), using the technology \( Y_t = \left( \int_0^1 Y_t(i) i^{1+\lambda} \, di \right)^{1+\lambda} \).

The final good firms aggregate the continuum of intermediate goods \( i \) on the interval \( i \in [0, 1] \) into a single final good. Parameter \( \lambda \) pins down the markup coming from monopolistic competition and it determines the elasticity of substitution between goods variety, \( \epsilon = \frac{1+\lambda}{\lambda} \).

The cost-minimisation problem of final good firms deliver demand schedules for intermediary goods of the form:

\[
Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\frac{1+\lambda}{\lambda}} Y_t
\] (63)

A continuum of intermediate firms operates in the economy. Intermediate firm \( i \) uses the Cobb-Douglas technology The capital share of output is controlled by \( \theta \). The aggregation across firms, yields:

\[
S_t Y_t = A_t \bar{K}^\theta (Z_t N_t)^{1-\theta}
\] (64)
$K$ refers to the fact that firms have fixed capital\textsuperscript{12} and $S_t$ is the cross-sectional price dispersion.

In equation (64) technology follows the autoregressive process:

$$\log A_t = \rho_A \log A_{t-1} + \sigma_A \epsilon^A_t$$

where $\epsilon^A_t$ is an independently and identically distributed (iid) shock with zero mean and constant variance.

Intermediate firms maximize the present value of future profits facing Calvo contracts by choosing price, $P_t(i)$,

$$E_t \left\{ \sum_{k=0}^{\infty} \zeta^k Q_{t,t+k} \left[ P_t(i) Y_{t+k}(i) - W_{t+k} N_{t+k}(i) \right] \right\}$$

where $Q_{t,t+j}$ is the stochastic discount factor from period $t$ to $t + k$ which is given by equation (??). The term $W_{t+j} N_{t+j}(i)$ represents the cost of labor. The optimal price is a weighted average of current and future expected nominal marginal costs,

$$P_t(i) = (1 + \lambda) \sum_{k=0}^{\infty} \mu_{t+k} MC_{t+k}(i)$$

Where $\mu_{t+k} = \frac{E_t \zeta^k Q_{t,t+k} Y_{t+k}(i)}{E_t \sum_{k=0}^{\infty} \zeta^k Q_{t,t+k} Y_{t+k}(i)}$ is the time varying mark-up implied by price rigidity and $1 + \lambda_{t+k}$ is the mark-up implied by monopolistic competition.

The average real marginal cost is defined as

$$MC_t = \frac{1}{1 - \theta} \left( \frac{W_t}{A_t} \right) \left( \frac{Y_t}{KA_t} \right)^{\theta \bar{\theta}}$$

Fiscal Policy and Monetary Policy. Government spending follows the process:

$$\log \left( \frac{g_t}{\bar{g}} \right) = \rho_G \log \left( \frac{g_{t-1}}{\bar{g}} \right) + \epsilon^G_t, \quad 0 < \rho_G < 1,$$

where $\bar{g}$ is the steady-state level of $g_t \equiv G_t / Z_t$, and $\epsilon^G_t$ is an iid shock with mean zero and variance $\sigma^2_G$.

The model is closed by a monetary policy rule:

$$4i_t = 4\rho_i i_{t-1} + \left[ 4 \log \left( \frac{\tilde{\pi}}{\pi^*_t} + \pi^*_t \right) + \phi_\pi \left( 4\pi^*_t - \pi^*_t \right) + \phi_Y \left( \frac{\mu_t Y_t}{\bar{\mu}Y^*} - 1 \right) \right]$$

where $i_t$ is the policy rate, $\tilde{\Pi}_t$ is a four-quarter moving average of inflation (defined below), and $Y^*_t$ is the trend level of output $\bar{Y} Z_t$ (where $\bar{Y}$ denotes the steady-state level

\textsuperscript{12}Firm-specific capital can be interpreted as a model with endogenous investment that features high adjustment costs in investment.
of $Y_t/Z_t$. Here, $\bar{R}$ is the steady-state gross interest rate, which equals $\log(\Pi^*/\bar{\beta})$, $\Pi^*$ is the target rate of inflation, and $\varepsilon^*_i$ is an iid shock with mean zero and variance $\sigma^2_i$. $\rho_i$ captures the motive for interest rate smoothing.

The four-quarter moving average of inflation ($\Pi_t$) can be approximated by a geometric moving average of inflation:

where $\theta_{\pi} = 0.7$ ensures that the geometric average of inflation has an effective duration of approximately four quarters.

The inflation target $\pi^*_t$ is time varying and driven by following process,

$$
\pi^*_t = (1 - \rho_{\pi^*}) 4 \log(\bar{\pi}) + \rho_{\pi^*} \pi^*_{t-1} + \zeta_{\pi^*} 4 (\pi^*_{avg} - \pi^*_t) + \sigma_{\pi^*} \varepsilon_{\pi^*}^t
$$

(71)

5.2.3 Big summary table

6 References


