Adaptive labor supply as driver of the business cycle

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Abstract

The interplay between profits, real wages and investment has long been attributed a central role in the business cycle. To model this interplay we introduce adaptive wages, which means the labor supply curve shifts up (down) when recent hours worked has been high (low). In an endogenous markup model with costly entry and Cournot competition such adaptive wages result in an endogenous business cycle (or limit cycle) under Rational Expectations. This cycle is driven by an intuitive mechanism where an increase in hours worked around the steady state increases profits, and thus boosts investment. As wages increase during the expansion profits fall, lowering investment, and turning the cycle. This endogenous cycle mimics crucial features of the business cycle, such as pro-cyclical wages, profits, consumption, and investment. Especially interesting in this respect are that profits fall before the peak in output, and similarly the labor share of income peaks during the contractions phase.

1 Introduction

There is convincing evidence that business cycles should be thought of as an unstable process, where the labor market seems to play an important role (Beaudry et al., 2017). Boldrin and Horvath (1995) (see also Zarnowitz, 1992, for a more detailed exposition) give an intuitive interpretation of how labor markets shape the dynamics of profits, real wages and investment over the business cycle. These authors observed that profits typically increase in the early stage of a recovery, led by a stronger increase in productivity than the increase in real wages. While profits tend to decline in the later stages of the expansion as higher costs lower profit margins. They argue that one possible cause of higher costs is that tight labor markets push up real wages.

Following their description we introduce an adaptive labor supply function, meaning that the labor supply function shifts up (down) when recent hours

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worked are high (low). As a result the labor supply is less elastic in the long run than in the short run, and wages (for a given hours worked) are higher (lower) around the peak (trough) of the business cycle, than around the steady state.

In our model this adaptive labor supply results in an endogenous business cycle as an increase in hours worked around the steady state results in higher profits, because output increases more than the total (real) wage bill. These higher profits will increase investment demand, which propagates the expansion by shifting labor demand up. As labor utilization stays high during the expansion phase the labor supply curve shifts up over time. This eventually turns the cycle as profits go down, resulting in lower investment and lower labor demand.

The endogenous cycle matches the most basic features of the business cycle. Among these features are a peak in the labor share of income between the peak and trough of the business cycle as we observe in Figure 1, pro-cyclical investment and consumption. Furthermore, there is a positive correlation between wages and output (see for example Haefke et al. 2013).

Our limit cycle model is mainly driven by a combination of two aspects. The first factor explains the instability of the steady state. This instability is primarily caused by an elastic labor supply in the short run, which ensures that the return on investment initially increases when investment increases (starting from the steady state). Higher investment increases the return, because more investment lowers the markup, and therewith shifts the labor demand curve up.

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1A linear trend is subtracted from each series.
2The cyclicality of wages is difficult to measure because of the composition bias, but wages are in general assumed to be pro-cyclical.
The resulting increase in hours worked increases profits per firm, because output per firm increases more than total wages per firm, as wages increase little due to the elastic labor supply.

The second aspect is that the labor supply is less elastic in the long run as the labor supply shifts up (down) when past labor utilization rates are high (low). This mechanism explains the turning points in the cycle. In Figure 2 we see that the labor supply shifts up from early in the expansion (lower right corner) till the early recession (upper left corner), and shifts down during the remainder of the cycle (from point B to point A). This is in line with empirical evidence that during business cycle expansions real wages increase for given hours worked.

Figure 2: Labor demand and supply over the business cycle

Beaudry et al. (2017) provide some evidence for the role of the labor market in endogenous business cycles. Their evidence indicates that aggregate labor market variables result in the occurrence of limit cycles with a duration of roughly 9 years, and an amplitude of about 4 to 5%. In addition, Karabarbounis (2014) shows how wedges between the wage and both (1) the marginal rate of substitution between consumption and leisure, and (2) the marginal productivity of labor shift over the business cycle. He finds that the wedge on the labor supply side is affected the most by the business cycle, which indicates a shifting labor supply curve could be an important driver of the business cycle.

In our model the shifts in the labor demand curve over the cycle result from changes in the markup. Firms compete in Cournot style within each industry as in Gali and Zilibotti (1995), where the markup falls when the number of firms increases. This does not result in multiple equilibria for the parameters
setting in our model though: our model has one unique stable steady state (and one unstable steady state) for a given labor supply curve and given beliefs. As investment and the capital stock are pro-cyclical in our model the markup is counter-cyclical in line with empirical evidence (Rotemberg and Woodford, 1991).

This paper can be classified as a limit cycle model within the Real Business Cycle (RBC) framework. It is set apart from a larger literature that relies on indeterminacy or sunspots to generate endogenous business cycle (see Benhabib and Farmer (1999) for an overview). Although research by Gali (1994) indicates that the markup may play a role in endogenous business cycles, the cycles in his model are purely driven by self-fulfilling expectations regarding the composition of future demand.

Our model comes closer to the limit cycle under Rational Expectations as described by Beaudry et al. (2015). They show that strategic complementarities can give rise to limit cycles, and then calibrate a Dynamic Stochastic General Equilibrium model with a demand complementarity. This demand complementarity results in an increase in demand when demand by others is low, because periods of high demand are associated with low unemployment and therefore a lower perceived risk.

The next section describes the model, in Section 3 the calibration is described, Section 4 describes the results and finally Section 4 concludes.

2 Model

Our model is of the RBC type. Consumer choose their labor supply, consumption and investment in new firms. Both the investment good and consumption are a final good, which is produced using a constant returns to scale production function that takes goods of each intermediate sector \( j \) as input goods. Within each intermediate sector firms produce output using labor input, which has a constant marginal productivity. Each firm has to pay a fixed cost for operating the firm. Within each sector firms compete in Cournot style as in Gali and Zilibotti (1995) such that the markup falls when the number of firms increases. To start a new firm households have to invest a certain amount of final goods. To ensure that expectations have no influence on the labor supply curve we use a labor supply curve that is derived from the type of preferences used in Greenwood et al. (1988) (GHH-preferences).

2.1 Final Goods sector

The final good is a composite good produced using differentiated inputs \( X_j \) of sector \( j \):
\[ Y = \left( \int_0^1 X_j^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}} \]  

with the range of intermediate goods normalized to one.

The total costs of production are \( \int_0^1 p_j X_j \) where \( p_j \) is the price of good \( X_j \). Cost minimization of the final goods producers results in the standard demand function for the intermediate good, \( X_j \), produced by industry \( j \):

\[ X_j = \left( \frac{P_j}{P} \right)^{-\epsilon} Y \]  

where \( P \) is the aggregate price level \( P \equiv \left( \int_0^1 p_j^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}} \)

### 2.2 Intermediate Good sector

In the intermediate sector there is a fixed number of industries, with Cournot competition between firms belonging to the same industry as in Gali and Zilibotti (1995). Firm \( i \) in industry \( j \) thus sets its quantity to maximize profits. Every firm knows how its quantity will affect the total supply and the price within the sector. The market clearing price for an industry given the quantity is \( p_j \) as determined by (2). Firms realize that total supply in its industry is:

\[ X_j = x_{i,j} + (n_j - 1) \bar{x}_j \]  

where \( \bar{x}_j \) is the average output per firm in industry \( j \), \( n_j \) is the number of competitors in the industry.

The objective of firm \( i \) in industry \( j \) is to maximize its current profits (equal to dividends):

\[ d_{i,j} = \frac{p_j x_{i,j} - W l_{i,j}}{P} \]  

where \( x_{i,j} \) is the firm’s output, \( p_j \) is the market clearing price for industry \( j \) as determined jointly by (2) and (3), \( W \) is the aggregate nominal wage, \( l_{i,j} \) is the firm’s labor input, and \( P \) is the aggregate price level. The optimization is subject to production constraint:

\[ x_{i,j} \leq l_{i,j} - \phi \]  

which means there is constant labor productivity, and a fixed cost \( \phi \) per period of operating.

Writing the problem as a Lagrangian yields:
\[
L = \frac{p_j x_{i,j} - W_{i,j}}{P} + \zeta \left( \frac{p_j}{P} \right)^{-\varepsilon} Y - X_j + \varphi \left[ l_{i,j} - \phi - x_{i,j} \right] + \vartheta \left[ X_j - x_{i,j} - (n_j - 1) \bar{x}_j \right]
\]

The First Order Conditions (FOCs) with respect to the price \( p_j \), the quantity \( x_{i,j} \), labor input \( l_{i,j} \), and industry level of output \( X_j \) are:

\[
\frac{x_{i,j}}{P} = \zeta \frac{1}{\varepsilon} \left( \frac{p_j}{P} \right)^{-1} Y \quad (6)
\]
\[
\frac{p_j}{P} = \varphi + \vartheta \quad (7)
\]
\[
\frac{W}{P} = \varphi \quad (8)
\]
\[
\zeta = \vartheta \quad (9)
\]

The combination of these FOCs and assuming symmetry within the industry yields the labor demand:

\[
\frac{W}{P} = \frac{n_j \varepsilon - 1}{n_j \varepsilon}
\]

### 2.3 Household & Law of Motion for firms

The objective for the household is the maximization of expected utility:

\[
E_0 \left[ \sum_{t=0}^{\infty} \beta^t \left( \frac{\varrho C_t - \psi_{t-1} L_t^\theta}{1 - \nu} \right)^{1 - \nu} - 1 \right]
\]

where \( \beta \) is the discount factor, \( C \) is consumption, \( \varrho \) is a scaling factor, \( \nu \) is the risk aversion parameter, \( L \) is labor supply, and \( \psi_{t-1} \) is the adaptive variable in the labor supply, which is crucial for the model. Central to the adaptive labor supply are normal working hours \( \bar{L} \), which is the steady state labor supply in a Rational Expectations equilibrium with preferences \( U (C_t, L_t) = \frac{(\varrho C_t - \psi_t L_t^\theta)^{1 - \nu} - 1}{1 - \nu} \) where \( \psi_t \) is the base value of \( \psi \) (see below). The adaptive variable \( \psi_t \) measures how much past working hours deviated from normal working hours. A positive (negative) deviation from the norm results in a higher (lower) disutility of working. In effect, past labor market conditions influence current wages, with high past labor utilization resulting in higher current wages. The idea is that during a business cycle expansion (recession) wages tend to increase (decrease) for a given hours worked.

The current deviation from the norm is \( \eta_t \) and is defined as:

\[
\eta_t = \bar{\psi} + \zeta_1 \left( \frac{L_t - L}{L} \right) + \zeta_3 \left( \frac{L_t - \bar{L}}{L} \right)^3
\]

where the parameters \( \zeta_1 \) and \( \zeta_3 \) determine how strong the deviation from the norm affects the disutility of working, and \( \bar{\psi} \) determines the steady state labor
supply $\bar{L}$. The current adaptive variable is a weighted average of the current deviation from the norm $\eta_t$, and the adaptive variable in the previous period:

$$ \psi_t = \gamma \eta_t + (1 - \gamma) \psi_{t-1} \quad (11) $$

How these preferences affect the labor supply curve is described in more detail in Subsection 2.4.

The household can invest by starting new firms $N_E$. The real budget constraint for the household is:

$$ F_E N_{E,t} + C_t = d_t N_t + w_t L_t $$

where $F_E$ is the costs of starting a new firm, $N_{E,t}$ is the number of newly set up firms, $N_t$ is the number of firms at the start of the period, $d_t$ are dividends of each firm, $w_t$ is the real wage $W_t^P$, and $L_t$ is hours worked. The Law of Motion for the number of firms is:

$$ N_{t+1} = (1 - \delta) (N_t + N_{E,t}) $$

so a fraction $\delta$ of newly setup firms fails before starting operation as is standard (see for example Bilbiie et al., 2012).

The problem can be written as a Lagrangian:

$$ L = E_t \sum_{t=0}^\infty \beta^t \left[ \left( \beta C_t - \psi_{t-1} L_t^{\theta} \right)^{\beta-1} \right] + \lambda_t \left\{ d_t N_t + w_t L_t - (F_E N_{E,t} + C_t) \right\} $$

The FOCs for consumption $C_t$, labor supply $L_t$, new firms $N_{E,t}$, and the number of firms in the next period $N_{t+1}$ are:

$$ \psi_t - \psi_{t-1} L_t^{\theta} = \lambda_t $$

$$ w_t = \psi_{t-1} \theta L_t^{\theta-1} $$

$$ \lambda_t F_E = \kappa_t (1 - \delta) $$

$$ \kappa_t = \beta E_t \{ \lambda_{t+1} d_{t+1} + \kappa_{t+1} (1 - \delta) \} $$

where $\lambda$ has been substituted out of the second equation. Combining the FOCs for $N_E$ and $N_{t+1}$ yields the Euler equation:

$$ \lambda_t F_E = \beta (1 - \delta) E_t \{ \lambda_{t+1} (d_{t+1} + F_E) \} $$

2.4 Labor supply

The adaptive labor supply curve shifts up and down over the business cycle, which results in a different labor supply in the Short Run and Long Run. The labor supply in the Short Run and Long Run are shown in Figure 3. As a
reference we also show a labor demand curve and the labor supply curve with a standard Frisch elasticity of 2.5. The figure shows that the labor supply in the Short Run ($\psi_{t-1}$ is held constant) is more elastic than the reference labor supply or the labor supply in the Long Run. The Short Run labor supply function will shift up and down as $\psi_{t-1}$ adjusts based on past hours worked. The Long Run labor supply is obtained by setting $\psi_{LR} = \bar{\psi} + \zeta_3 \left( \frac{L_{LR}}{\bar{L}} - 1 \right)^3$. The figure shows that also the Long Run labor supply is more elastic around the steady state, but less elastic further away from the steady state (relative to a standard Frisch elasticity of 2.5).

2.5 Market clearing

The aggregate resource constraint is given by:

$$Y_t = L_t - N_t \phi = C_t + F_E N_{E,t}$$

with $L_t = \sum_j \sum_i l_{i,j,t}$ being the aggregate labor utilization. Furthermore, the number of firms in each industry is equal to the aggregate number of firms: $n_{j,t} = N_t$.

2.6 Solution method

To solve the model under Rational Expectations we employ the perturbation method described by Galizia (2018), which allows us to solve models that are not
saddle-path stable, but feature a limit cycle. For limit cycle models the extra requirement is that we need to check if the solution satisfies the Transversality Condition\(^3\).

3 Calibration

The parameter values are chosen such that the model displays a limit cycle. The depreciation rate and fixed costs of a firm are set at relatively high values such that a limit cycle occurs for a larger parameter range. The parameters and some basic statistics are listed in Table 1, where one period in the model represents 6 months. One critical parameter is \(\zeta_1\) as low values of this parameter can result in multiple equilibria\(^4\). We set \(\zeta_1\) high enough such that such multiple equilibria do not occur.

4 Simulation

The resulting cycles are relatively long, with a duration of just under 30 periods, meaning about 15 years compared to the 9 years that Beaudry et al. (2017) find in the data. Part of a simulation is shown in Figures 4 and 5. Each cycle

\[\text{Table 1: Parameters & statistics}\]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor ((\beta))</td>
<td>0.975</td>
</tr>
<tr>
<td>Risk aversion ((\nu))</td>
<td>1.00</td>
</tr>
<tr>
<td>Depreciation ((\delta))</td>
<td>0.15</td>
</tr>
<tr>
<td>Fixed costs ((\phi))</td>
<td>0.0040</td>
</tr>
<tr>
<td>Cost of starting new firm ((F_E))</td>
<td>0.0066</td>
</tr>
<tr>
<td>Price elasticity ((\epsilon))</td>
<td>3.50</td>
</tr>
<tr>
<td>Scaling consumption ((\varphi))</td>
<td>100.00</td>
</tr>
<tr>
<td>Frisch el. in short run ((\theta))</td>
<td>1.10</td>
</tr>
<tr>
<td>Scaling lab. supply ((\psi))</td>
<td>1.00</td>
</tr>
<tr>
<td>Adaptive par. (1st ord.) ((\zeta_1))</td>
<td>0.050</td>
</tr>
<tr>
<td>Adaptive par. (3rd ord.) ((\zeta_3))</td>
<td>25.00</td>
</tr>
<tr>
<td>Weight of (\eta_t) ((\gamma))</td>
<td>0.15</td>
</tr>
<tr>
<td>Number of firms in s.s. ((N_{ss}))</td>
<td>1.27</td>
</tr>
<tr>
<td>Investment rate</td>
<td>0.059</td>
</tr>
<tr>
<td>Labor share</td>
<td>0.931</td>
</tr>
<tr>
<td>Fixed cost share</td>
<td>0.167</td>
</tr>
<tr>
<td>Standard deviation (Y) ((%))</td>
<td>0.042</td>
</tr>
</tbody>
</table>

\(^3\)This is not necessary for the perturbation solution of a saddle-path stable model.

\(^4\)A bifurcation occurs where the Long Run steady state looses its stability, where the Long Run refer to the situation where all time-subscripts and expectation operators are dropped. At this bifurcation two steady states are generated that are stable in the Long Run.
is almost identical (and also corresponds to the limit cycle depicted earlier in Figure 2) as the simulations are completely deterministic, meaning there are no shocks.

In the first panel of Figures 4 we see that wages are pro-cyclical in line with empirical results (see Haefke et al., 2013). This feature is difficult to generate in a model without technology shocks. The pro-cyclicality is mainly the result of changes in the markup, which depends inversely on the number of firms: during the expansion phase the number of firms increases, which increases competition and lowers the markup. This counter-cyclical markup is observed in the data (Rotemberg and Woodford, 1991).

In the middle panel of Figure 4 we see how profits fluctuate over the cycle, with profits leading the cycle, which is intuitively realistic. In the lower panel of Figure 4 we see how the labor share of income has its peak between the peak and trough in output as is observed in the data. The labor share stays high for a relatively long period during the recession though, which we do not observe in the data (see Figure 1)\(^5\).

In Figure 5 we can see that the correlation between Consumption and Output is close to one. A related issue is that investment has a lower standard deviation than output. A intuitive feature is that investment falls before the peak in output, which is caused by the fall in profits that we saw in Figure 4.

\(^5\)To get a labor share that falls faster after the peak in output we could introduce an asymmetry in the process that shifts the labor supply curve.
5 Conclusion

We have shown how an adaptive labor supply can drive an endogenous business cycle. Although the model is relatively simple and therefore not suitable for a full calibration it shows how a shifting labor supply curve can be an important factor for the occurrence of business cycles. Furthermore, an endogenous business cycle as generated by our model has important implications for macroeconomic policy as favorable conditions during the expansion phase sow the seeds for the subsequent contraction.

References


