The Optimal Joint Design of Unemployment and Pension Insurance

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Preliminary and Incomplete. This Version: February 15, 2019

Abstract

We characterize the optimal joint design of public unemployment and pension insurance systems. We start with a simple three period model to illustrate analytically interactions between both systems. We then develop a quantitative multi-period life-cycle model with incomplete markets, endogenous labor supply and retirement decisions in general equilibrium that features a restricted set of policy instruments to represent public unemployment and pension insurance schemes. Our main objective is to quantitatively characterize the optimal mix of both systems by calibrating the model to the German economy, which features generous public insurance systems.

This paper is incomplete. We are currently at the stage of diagnosing the quantitative model, which provides important insights into desirable refinements of the model’s specification. Very preliminary results based on an earlier model variant suggested that unemployment pay should be more generous and pension insurance more redistributive than for the German status quo.

JEL Classification: E61, H55, J65.

Keywords: Ramsey Problem, Unemployment Insurance, Pension Insurance

*We thank Dirk Krueger and seminar participants at Goethe University Frankfurt for helpful comments. We also gratefully acknowledge financial support by the Research Center SAFE, funded by the State of Hessen initiative for research LOEWE.

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1 Introduction

Unemployment insurance provides insurance against the cost of consumption fluctuations during working life, but distorts labor supply decisions. It also sets incentives for unemployed individuals to return to work. Pension insurance mainly insures longevity risk, but distorts labor supply and savings decisions that are muted by earnings related linkages of pension payments in retirement. It may also feature redistributive components to insure earnings risk.

In this paper we characterize the optimal joint design of both insurance systems. We focus on their interactive nature, which arises because they both may provide insurance against earnings risk and set incentives. Explicitly modeling these interactions we then ask how much insurance to optimally provide through each system.

We address this question in two steps. We start with a simple three period model with two working periods and one retirement period to illustrate how both systems interact. We then develop a realistically calibrated incomplete markets life-cycle model to quantitatively characterize the optimal policy mix. The model is populated by agents that are ex-ante heterogeneous with respect to education specific earnings abilities and face idiosyncratic income shocks and unemployment risk. There is a one-period risk-free asset that serves as a self-insurance device and cannot be shortened. A restricted set of policy instruments reflects the main features of the public unemployment and pension insurance schemes.

A core element of our model is the endogeneity of extensive margin labor supply decisions. Unemployed individuals choose job search effort which affects the probability to receive a job-offer. Employed individuals exogenously receive job-offers. Conditional on having received a job-offer, agents decide whether to accept the job or not. Additionally, for a certain retirement window, agents choose when to enter into retirement.

The model is disciplined by matching model moments to key moments of German data. Focussing on Germany provides an ideal laboratory for our question as it features generous unemployment and pension insurance systems. The standard old-age pension, however, has only mild redistributive components. Thus, one emphasis of our analysis in the calibrated model lies on the question whether an explicit redistributive component should be introduced into the German public pension system.

To provide an answer to this normative question and to characterize the opti-
mal policy mix of both insurance schemes, we assume an ex-ante Utilitarian social welfare function and optimize over a restricted set of instruments. First—and very preliminary—results based on an earlier model variant suggest that the optimal fiscal constitution should feature more generous unemployment insurance and more progressivity of the pension system.

**Related literature.** In structural macroeconomics there is a large literature focusing on the optimal design of tax and insurance systems given a restricted set of policy instruments, which is typically referred to as the Ramsey approach to optimal taxation. Core element of this line of research is to investigate the trade-off between providing (labor supply) incentives and to provide insurance or a more equal distribution. Our paper bridges two literatures. The first explicitly studies (partial) reforms of social security systems in environments with idiosyncratic risk (İmrohoroğlu et al. 1995; Huggett and Ventura 1999; Nishiyama and Smetters 2007; Nishiyama and Smetters 2008) or characterizes optimal pension policy (Fehr et al. 2013; Golosov et al. 2013) given a restricted set of instruments. The second scrutinizes the optimality of unemployment insurance systems (Hansen and İmrohoroğlu 1992; Wang and Williamson 2002; Abdulkadiroğlu et al. 2002; Lentz 2009; Rendahl 2012; Koehne and Kuhn 2015). More recent literature extends the literature on optimal progressive taxation to account for interactions with different social insurance schemes. For instance, Krüger and Ludwig (2016) study the optimal design of income progressivity and education subsidies. Huggett and Parra (2010) find potentially large welfare gains from simultaneously optimizing the income tax system and the social security system. Relative to this literature, our focus is on the interaction between unemployment insurance and pension insurance, and we highlight the importance of extensive margin labor supply decisions. Our stylized model shares some flavor with papers that study the optimal design of social insurance systems by means of sufficient statistics (Baily 1978; Chetty 2008; Chetty 2009; Michelacci and Ruffo 2015; Badel and Huggett 2017). Finally, our work complements the Mirrleesian literature on optimal taxation. In that literature, constraints to the planning problem are not imposed by a restricted set of

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1There is also an earlier literature studying the optimality of unemployment insurance systems focusing on moral hazard problems induced by unobserved job search effort. They rely on mechanism design techniques and are more theoretical in its nature. (Shavell and Weiss 1979; Hopenhayn and Nicolini 1997; Shimer and Werning 2007).

instruments but rather arise from asymmetric information problems (Golosov et al. 2016; Golosov and Tsyvinski 2015).\(^3\)

The remainder of the paper is structured as follows. Section 2 is devoted to introducing the stylized model highlighting the interactive nature of unemployment and pension system. Section 3 outlines the quantitative model used for our counterfactual policy experiments. The model is mapped into German data in Section 4. We analyze various policy reforms by counterfactual experiments in Section 5. Finally, Section 6 concludes.

2 A Three-Period Life-Cycle Model

In this section we present a simple three period life-cycle model which provides intuition for the interactive nature of public unemployment and pension insurance. We analyze under which assumptions unemployment and pension insurance are perfect substitutes and how perfect substitutability is affected once we relax these assumptions.

2.1 The Problem

An individual (an agent) lives for \( J = 3 \) years and enters the economy at age \( j = 1 \) as unemployed. She is endowed with initial assets \( a_1 > 0 \) and exercises search effort which affects her probability of being in the labor market as being employed \((m_j = w)\) or unemployed \((m_j = u)\) in the second period where \( j = 2 \). In the third period the agent is retired \((m_3 = r)\) and dies with certainty at the end of the period.

We denote the employment probability in the second period by \( \pi(e) \), where \( e \) is the search effort, and assume that \( \pi_e(e) > 0, \pi_{ee}(e) < 0, \pi(e) \) is strictly monotonically increasing with \( \lim_{e \to 0} \pi_e(e) = \infty \) so that optimal search is interior. Search is costly in terms of utility units. We denote additively separable search costs by \( \Psi(e) \) with \( \Psi_e(e) > 0 \) and \( \Psi_{ee}(e) > 0 \). Through choice of function forms of \( \pi(e) \) and \( \Psi(e) \) we further assume that optimal search effort \( e^* \) satisfies \( e^* < \bar{e} \) and that for any \( e^* \), \( \pi(e^*) < 1.\(^4\) This implies a technological constraint to full re-employment chances in

\(^3\)Findeisen and Sachs (2017) and Golosov et al. (2013) combine elements of both approaches.

\(^4\)For example \( \pi(e) = e^\phi \) for \( \phi \in (0,1) \) and \( \Psi(e) = \frac{1}{1-e} \) for which \( \bar{e} = 1 \) and \( \Psi(\bar{e}) = \infty \) so that \( e^* = \bar{e} \) cannot be optimal.
the second period. We rule out insurance against this risk which constitutes the only form of market incompleteness we consider in this simple setup.

In case of working \((m_2 = w)\) in the second period, the agent earns an exogenous wage income \(\omega > 0\). In the competitive laissez faire equilibrium, there is no income in other states. However, a Ramsey government may pay unemployment insurance \(b^u\) if \(m_2 = u\). In the third period, the Ramsey government pays pension insurance which may be contingent on the employment history. Accordingly, pension income in period three in case of unemployment in the second period is \(b^p(|m_2 = u) = b^{p,u}\) and in case of employment in the second period it is \(b^p(|m_2 = w) = b^{p,w}\). Observe that we do not pose a sign restriction on all these instruments. Summarizing the income earned in states \(m_j, j \geq 2\) we get

\[
y_{m_j}^{m_2} = \begin{cases} 
  b^u & j = 2 \land m_2 = u \\
  \omega & j = 2 \land m_2 = w \\
  b^{p,u} & j = 3 \land m_2 = u \\
  b^{p,w} & j = 3 \land m_2 = w 
\end{cases} \quad (1)
\]

Let \(a_3^{m_2}\) denote the savings decision of the agent at age \(j = 2\) and in respective employment status \(m_2 \in \{w, u\}\). \(u(\cdot)\) denotes the agent’s instantaneous utility with \(u_c(\cdot) > 0\) and \(u_{cc}(\cdot) < 0\). Normalizing the real interest rate factor as well as the subjective discount factor to \(R = \beta = 1\) the agent’s problem writes as:

\[
V = \max_{a_2, a_3^{m_2}, a_3} \{u(a_1 - a_2) + \pi(e)(u(\omega + a_2 - a_3^{w}) + u(b^{p,w} + a_3^{w})) + (1 - \pi(e))(u(b^u + a_2 - a_3^{u}) + u(b^{p,u} + a_3^{u})) - \Psi(e)\} \quad (2)
\]
2.2 Analysis

Competitive Equilibrium. We denote optimal household decisions in the competitive equilibrium by *. For the optimal search effort we get:

\[ e^* = \arg \max_e \pi(e) \left( (u(\omega + a_2^* - a_3^w) + u(b_{p,w}^* + a_3^w)) - (u(b_u^* + a_2^u - a_3^u) + u(b_{p,u}^* + a_3^u)) \right) - \Psi(e) \]

from which we get \( \pi^* \equiv \pi(e^*) \). One can show that \( e^* \) is increasing in \( V \), i.e., if the difference between the present value of utility out of employment and unemployment is increasing agents exercise more search effort. Given that the agent’s effort choice is not observable by the government equation (3) describes the moral hazard problem faced by the government.

The Ramsey Problem. The government maximizes the agent’s life-time utility subject to its budget constraint and the agent’s economic choices with the inherent moral hazard problem induced by unobserved search effort. For sake of simplicity we assume the government’s revenue is exogenously given by \( T \). \(^5\) We consider a local analysis, i.e., we evaluate the government’s objective at the optimal decisions by the households (which are themselves a function of policy). The government’s expenses are \( (1 - \pi^*)(b_u^* + b_{p,u}^*) + \pi^*(b_{p,w}^*) \). Let \( \Xi \) be the set of policy instrument over which the government can optimize. The government’s objective function is given by the indirect utility function as:

\[ \max_{\Xi} V(c_1^*, c_2^w, c_2^u, c_3^w, c_3^u, b_u^*, b_{p,w}^*, b_{p,u}^*). \]  

Maximization is subject to the resource constraint

\[ T = (1 - \pi^*)(b_u^* + b_{p,u}^*) + \pi^*b_{p,w}^* \]

\(^5\)Note that endogenizing \( T \) would not change our qualitative results but would distract from our main point. Thus, we keep this simplifying assumption.
We proceed by characterizing the conditions of optimality for each policy instrument. In our interpretation of these conditions, we focus on highlighting the interactions of both insurance schemes. All formal details of the derivations are contained in Appendix A.

**Exogenous Flat Pension Benefits.** Let us start by assuming that $\Xi = \{b^u\}$ and by restricting $b^{p,u} = b^{p,\omega}$. Denote by $\lambda$ the Lagrange multiplier of the problem. Using the envelope theorem, and assuming $b^{p,u} = b^{p,\omega}$ we get as the optimality condition

$$\lambda = \frac{\frac{\partial u(\cdot)}{\partial b^u}}{1 + \epsilon_{1-P,b^u}}$$  \hspace{1cm} (6)

where $\epsilon_{1-P,b^u}$ is the elasticity of the unemployment probability with respect to unemployment benefits. Since in equilibrium the unemployment rate is $1 - P$ this is equal to the percentage increase of the unemployment rate in response to a percent increase in unemployment benefits.

The intuition for equation (6) is as follows. The numerator contains the *insurance effect* as the marginal increase in utility caused by an increase of unemployment benefits $b^u$. The denominator captures the *moral hazard effect*, i.e., adverse incentive effects in response of an increase in $b^u$. Thus, according to equation (6) under constant $\lambda$, a tighter moral hazard friction—i.e., an increase of $\epsilon_{1-P,b^u}$—calls for a reduction of unemployment benefits.

**Exogenous History Dependent Pension Benefits.** Next, maintain the assumption that $\Xi = \{b^u\}$ and consider the case where $b^{p,u} \neq b^{p,\omega}$. Now the condition of optimality with respect to $b^u$ changes to

$$\lambda = \frac{\frac{\partial u(\cdot)}{\partial b^u}}{1 + \epsilon_{1-P,b^u} \left(1 + \frac{b^{p,u} - b^{p,\omega}}{b^u}\right)}$$  \hspace{1cm} (7)

and the effect of an change in unemployment benefits $b$ is no longer independent of the pension insurance system. For instance, under the assumption that pension payments increase in life-time earnings, i.e., if $b^{p,\omega} > b^{p,u}$, then $\frac{b^{p,u} - b^{p,\omega}}{b^u} < 0$ and the moral hazard friction is muted. An increase of pension payments conditional on employment $b^{p,\omega}$ thus leads to an increase of optimal unemployment pay $b^u$. Hence, the pension system exerts an additional positive labor supply incentive effect to the
agent.

**Full Set of Policy Instruments.** Now, consider the full set of policy instruments available to the Ramsey government of \( \Xi = \{ b^u, b^{p,u}, b^{p,\omega} \} \). In addition to (7) we get the corresponding first-order conditions for \( b^{p,u} \) and \( b^{p,\omega} \), respectively, so that the full set of optimality conditions write as

\[
\lambda = \frac{\partial u(\cdot)}{\partial b^u} \left( 1 + \epsilon_{1-P,b^u} \left( 1 + \frac{b^{p,u} - b^{p,\omega}}{b^u} \right) \right) \quad (8a)
\]

\[
\lambda = \frac{\partial u(\cdot)}{\partial b^{p,u}} \left( 1 + \epsilon_{1-P,b^{p,u}} \left( 1 + \frac{b^u - b^{p,\omega}}{b^{p,u}} \right) \right) \quad (8b)
\]

\[
\lambda = \frac{\partial u(\cdot)}{\partial b^{p,\omega}} \left( 1 + \epsilon_{P,b^{p,\omega}} \left( 1 - \frac{b^u + b^{p,u}}{b^{p,\omega}} \right) \right) \quad (8c)
\]

The government’s optimum is characterized by equating the right-hand-sides of the equations in (8). Equation (8a) is the same as (7). Equation (8b) states that under constant \( \lambda \) optimal pension payments to previously unemployed agents \( b^{p,u} \) decrease if the elasticity of the unemployment probability with respect to pension payments conditional on unemployment \( \epsilon_{1-P,b^{p,u}} \) increases. This moral hazard friction is muted if unemployment benefits \( b^u \) decrease.

Finally, equation (8c) states that an increase of the elasticity of the job finding probability—which in equilibrium is equal to the employment rate \( P \)—to pension payments conditional on employment \( b^{p,\omega} \) denoted by \( \epsilon_{P,b^{p,\omega}} \) leads to a reduction of optimal pension payments conditional on employment \( b^{p,\omega} \). If unemployment insurance pay \( b^u \) or pension pay conditional on unemployment \( b^{p,u} \) decrease, then the effects of the elasticity \( \epsilon_{P,b^{p,\omega}} \) are amplified such that optimal pension payments conditional on employment \( b^{p,\omega} \) decrease as well.

**Discounting.** Finally, we introduce a notion of discounting. We normalize the overall length of life stages 2 and 3 to \( T = 1 \). Let \( 1 - t \) denote the relative length of life stage 2 and \( t \) denote the relative length of life stage 3.\(^6\) By introducing this notion

\(^6\)Instead of using \( \beta \) which is usually used as a subjective discount rate, we decided to use this framework as we want to highlight the broader interpretation of discounting in this setting. For instance, \( t \) can describe the length of the remaining life-time in stage 2 as compared to stage 3. Thus, \( t \) can be thought of a measure of age. Thus, \( t \) is comprised of horizon such effects due to age, subjective discounting as well as survival risk.
of discounting, or, more generally, of horizon effects, equation (7) changes to

$$\lambda = \theta_b^u \equiv \frac{\partial u(\cdot)}{\partial b} \left( \frac{b^{p,u} - b^{p,\omega}}{b^u} \right) \text{ (9)}$$

We see that the importance of the effect of retirement for the moral hazard effect is increasing in the relative length of stage 3. One interpretation is that the pension insurance is relatively more important when the agent get older. Equations (8b) and (8c) change analogously.

### 3 Quantitative Model

We develop a realistically calibrated life-cycle model that features the main incentives inherent in public unemployment and pension insurance systems. The model serves the purpose of answering the normative question of how to optimally design unemployment and pension insurance systems jointly.

#### 3.1 Demographics

We consider a stationary economy in general equilibrium which is populated by overlapping generations. Agents in this economy start their economic lives at age $j = 1$ and live for at most $J$ years. They face exogenous age-dependent survival risk denoted by $\psi_j$. Individuals that die may leave accidental bequests. These bequests are distributed across all agents younger than the minimum retirement age as a lump-sum payment. We abstract from population growth (which is a realistic approximation to the German economy) and conveniently normalize the total population size to one so that aggregate and per capita variables are identical.

Agents start being unemployed or employed (c.f. Section 4 for more details on initial unemployment distribution) with zero assets and zero earning points, the latter reflecting pension entitlements, see below. Further, they differ in the education level, $s$. Education is constant over the life-cycle and affects labor productivity. Core element of the model is the explicit modeling of an individual’s labor market status.

There are three labor market states, working, unemployed and retired, hence $m \in \{w, u, r\}$, whereby state $r$ can only be reached after some retirement age $j = j_r$ or older. Each age period employed agents receive a job-offer with an exogenous
probability that depends on age $j$ and education $s$ (schooling), $\pi_{j,s}^o$ (superscript $o$ stands in for offer). When unemployed, agents exercise some search effort $e$ that affects their probability to generate a job offer in the next period $\pi(e)$, as in the simple model of Section 2. Conditional on having received a job-offer, agents decide on whether to accept the job or not. By not accepting a job offer the individual voluntarily transits to (remains in) unemployment. For a certain retirement window, ages $j = \{j_r, \ldots, j_r-1\}$, individuals may choose to retire and we model retirement as an absorbing state. At age $j = j_r$ we force all agents into retirement. All agents make consumption and savings decision. Employed agents also decide how much to work.

3.2 Preferences

Agents aged $j$ derive utility from consumption $c_j$ and disutility from labor $\ell_j$ (employment) or effort choice $e_j$ (unemployment). Additionally, we assume that there is a fixed time cost to labor market participation which is age $j$ and education $s$ dependent and given by $\mathbb{I}_{\ell>0}\kappa(j,s)$, where $\mathbb{I}_{\ell>0}$ denotes the indicator function for working and $\kappa(j,s)$ are the participation costs. Thus, current period utility reads as

$$u(c_j, \ell_j, e_j, \mathbb{I}_{\ell>0}\kappa(j,s)).$$

Agents maximize expected life-time utility, discount the future with raw time discount factor $\beta$ and with the probability to survive to the next period $\psi_j$.

3.3 Endowments

Labor income. Gross labor income of an agent is given by

$$y^w = w \cdot e(f(j,s), \eta) \cdot \ell$$

where $w$ is the aggregate wage rate and $e(f(j,s), \eta)$ captures individual wage heterogeneity. $f(j,s)$ depicts variation in wage as a function of age $j$ and education $s$. $\eta$ is a residual stochastic income component. We assume that the residual income

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7 Note that this second layer of endogeneity was absent in the simple model.

8 We use the term individual, household, and agent interchangeably.
component η follows a Markov process with states η ∈ 𝒯 and age-specific transition probabilities πη(η′ | η).

Agents pay income taxes on gross labor income according to the tax function 𝑇(𝑦^w). They also pay contributions to unemployment and pension insurance, with contribution rates denoted by 𝜏^u and 𝜏^p, respectively. In accordance with German legislation, we assume that there is a contribution ceiling of 𝜉^w · ̄𝑦^w where ̄𝑦^w is average gross labor income and 𝜉^w is a fixed calibration parameter.

Net labor income is therefore given by

\[ y^w = y^w - T(y^w) - \min(y^w, \gamma^w \bar{y}^w) \times (\tau^p + \tau^u) \]  \hspace{1cm} (12)

**Pension benefits.** In retirement agents’ regular pension income is the product of the accumulated stock of earning points, 𝑝ₐ, an adjustment factor 𝜈(𝑗ᵣ), and the current earning point equivalent, 𝐶𝐸𝐸. An individual receives the maximum out of regular pension income and some minimum pension income 𝑏^p.

\[ y^r = \max\left( b^p, p_j \times \nu(j^r) \times CEE \right) \]  \hspace{1cm} (13)

Below follows a description of each of the three components that define regular pension income: Non-retired individuals accumulate earning points.\(^9\) One earning point is related to average gross labor income ̄𝑦^w so that an individual earns exactly one earning point if she earns average income. Earning points are also capped at some 𝜉^p,w > 1, which is a fixed calibration parameter. Thus, an earning point reflects an individual’s income position in a given year only up to 𝜉^p,w times the average earnings. Accordingly, the pension stock when working accumulates according to

\[ p_{j+1} = p_j + \min\left\{ \frac{y^w}{\bar{y}^w}, \gamma^{p,w} \right\}. \]

During unemployment, individuals also accumulate earning points in the pension stock. The factual legislation in Germany is very complex and there were several changes over the past years.\(^10\) We approximate this by a symmetric treatment of gross incomes earned during the working period and the period of unemployment,\(^9\)

\(^9\)This is very similar to the AIME, the average indexed monthly earnings, known from the US system.

\(^10\)See “Portal Sozialpolitik – Arbeitslosigkeit und Rente”.

11
accordingly we have
\[ p_{j+1} = p_j + \min\left\{ \frac{y_u}{\gamma w}, \gamma^{p,u} \right\}. \] (14)

The adjustment factor reflects a penalty, \( v^-, \) (extra-benefit, \( v^+ \)), for entering retirement at age \( j^r \) prior (after) the normal retirement age \( j^{NRA} \). Hence, the adjustment factor reads as
\[ v(j^r) = \begin{cases} 
(1 - (j^{NRA} - j^r) \times v^-), & \text{if } j^r < j^{NRA}, \\
(1 - (j^r - j^{NRA}) \times v^+), & \text{else.} 
\end{cases} \] (15)

Additionally, the overall pension income formula comprises the current earning point equivalent, CEE, reflecting the actual monetary amount that is received per accumulated earning point.\(^{11}\)

**Unemployment benefits.** Unemployed agents receive unemployment benefits that depend on the duration of unemployment \( d \). According to legislation, unemployment benefits also depend on net labor income in the period before unemployment \( (y_u^u) \). Define \( \tilde{b} = \min \{ b^u, b^u \times y_u^u \} \). No taxes are paid on unemployment benefits, and thus, gross benefits \( y_u^u \) equal net benefits \( y_u^u \). The exact level of unemployment benefits hinge on a combination of unemployment duration, age, and level of last net labor income. Unemployment benefits are calculated as follows:
\[ y_u^u = y_u^u = \begin{cases} 
\max \{ b^u, \tilde{b} \}, & \text{if } d^u = 1 \\
0.75 \times \tilde{b}^u + 0.25 \times \max \{ b^u, \tilde{b} \}, & \text{if } d^u = 2 \land 50 \leq j < 55 \\
0.50 \times \tilde{b}^u + 0.50 \times \max \{ b^u, \tilde{b} \}, & \text{if } d^u = 2 \land 55 \leq j < 58 \\
\max \{ b^u, \tilde{b} \}, & \text{if } d^u = 2 \land 58 \leq j \\
\tilde{b}^u, & \text{else.} 
\end{cases} \] (16)

In our model we can calculate the deterministic part of the last net labor income with \( j, d \) and \( s \). Thus, we additionally have to save all possible combinations of \( \ell \) and \( \eta \).

\(^{11}\)Note that in a dynamic setting the CEE would at least have to reflect that changes in the old-age ratio affect the generosity of pension benefits and that CEE are a function of its past values. Given in this paper we conduct a steady state analysis we can ignore these institutional details.
3.4 Market Incompleteness

There do not exist private markets for insurance against idiosyncratic productivity shocks $\eta$, the exogenous job offer probability $\pi_{j,s}$, and the job finding probability $\pi(e)$. Also, agents face strict zero borrowing limits so that asset holdings $a_j$ are weakly positive in every period. Each period non-retired individuals receive accidental lump-sum bequest $tr_j$. Finally, there are no insurance markets against the survival risk $\psi_j$ so that the dynamic budget constraint in labor markets state $m \in \{w,u,r\}$ writes as

$$a_{j+1} = a_j(1 + r(1 - \tau^k)) + y_j^m + tr_j - c_j(1 + \tau^c) \geq 0,$$

where $r$ is the market interest rate. The capital income tax rate $\tau^k$ and the consumption tax rate $\tau^c$ are exogenous. They are included in the model for calibration reasons. Note that for $m = r$ $tr$ is zero.

3.5 Agent’s Problem: Recursive Formulation

We here summarize the recursive formulation of the individual’s problem depending on the individual’s current labor market state $m \in \{w,u,r\}$. The state vector of a working individual is given by $x_w = (j, s, a, p, \eta)$, for an unemployed it is $x_u = (j, s, d, y^w, a, p, \eta)$ and for a retired person it is $x_r = (j, j^r, a, p)$. To write down the agents’ problem in recursive form we introduce the notational convention that the state vector of an individual with labor market state $m \in \{w,u,r\}$ is $x_m$ and the value function writes as $V^m(x_m)$. Additionally, we denote by $\mathbb{E}$ expectation which is taken with respect to the realization of the productivity shock $\eta' | \eta$. The timing is such that individuals learn about their productivity levels $\eta$ and job offer at the beginning of the period. All other decisions are made thereafter.

**Employed individuals.** Employed individuals choose consumption, the number of hours to be worked, and – conditional on receiving a job offer – they decide whether to remain employed, become unemployed or enter retirement. It is understood that the retirement option is not available for $j + 1 < j_r$ and that for ages $j + 1 \geq j_r$, retirement is the only option available in the next period. To nest this in our following description of the dynamic problem we let $V^r(x_r) = -\infty$ for $j + 1 < j_r$ and $V^w(x_w) = V^u(x_u) = -\infty$ for $j + 1 \geq j_r$. Accordingly, the problem of a working individual writes
as

\[ V^w(x_w) = \max_{c, \ell, e, p} U(c, \ell, e, \mathbb{I}_{\ell>0} \kappa(j, s)) + \beta \psi_j \mathbb{E} \left[ \pi_{j,a}^o \max \left( V^w(x'_w), V^u(x'_u), V^r(x'_r) \right) \right] \]

subject to\(^{12}\)

\[ \begin{align*}
  a' &= a(1 + r(1 - \tau^k)) + y^w + tr - c(1 + \tau^c) \geq 0 \\
p' &= p + \min \left\{ \frac{y^u}{y^w}, \gamma_{p,u} \right\}.
\end{align*} \tag{19a, 19b} \]

**Unemployed individuals.** Unemployed individuals choose consumption, search effort and whether to opt for retirement, hence their problem writes as

\[ V^u(x_u) = \max_{c, e, a'} U(c, e, \mathbb{I}_{e>0} \kappa(j, s)) + \beta \psi_j \mathbb{E} \left[ \pi(e) \max \left( V^w(x'_w), V^u(x'_u), V^r(x'_r) \right) + (1 - \pi(e)) \max \left( V^u(x'_u), V^r(x'_r) \right) \right] \]

subject to

\[ \begin{align*}
  a' &= a(1 + r(1 - \tau^k)) + y^u + tr - c(1 + \tau^c) \geq 0 \\
p' &= p + \min \left\{ \frac{y^u}{y^w}, \gamma_{p,u} \right\}.
\end{align*} \tag{20a, 20b} \]

**Retired individuals.** Retirees face the simpler dynamic problem:

\[ V^r(x_r) = \max_{c, a'} U(c) + \beta \psi_j V^r(x'_r) \]

subject to

\[ \begin{align*}
  a' &= a(1 + r(1 - \tau^k)) + y^r - c(1 + \tau^c) \geq 0 \\
p' &= p.
\end{align*} \tag{22a, 22b} \]

\(^{12}\)In order to facilitate notation we will drop subindices indicating age and use e.g. \(a'\) and \(a_{j+1}\) interchangeably.
3.6 Production

There is a standard neoclassical representative firm which produces output $Y$ by employing aggregate capital $K$, aggregate labor $L$, with a constant returns to scale technology $F(\cdot)$:

$$Y = AF(K, L) \quad (24)$$

where $F \in C^2$ features positive and diminishing marginal products.

3.7 Government

We distinguish the general tax, the unemployment and the pension system.

**General Tax and Transfer System.** The government collects taxes on labor income, capital income and consumption from individuals. It finances fixed government expenditures $G$. Surpluses of the unemployment and pension systems are denoted $S^u$ and $S^p$ respectively. These surpluses (or deficits if negative) enter the general budget of the government. Additionally, let $\mathbb{I}_{b^u}$ and $\mathbb{I}_{b^p}$ describe indicator functions for whether individuals are eligible for minimum unemployment or minimum pension benefits, respectively. In line with German legislation these payments are financed via the tax system.

$$\int T(x)d\Phi(x) + \tau^c \int c(x)d\Phi(x) + \tau^k r \int a(x)d\Phi(x) + S^p + S^u$$

$$= \frac{b^u}{\mu} \int \mathbb{I}_{b^u}d\Phi(x) + \frac{b^p}{\mu} \int \mathbb{I}_{b^p}d\Phi(x) + G \quad (25)$$

Given surpluses (or deficits) pension and unemployment benefit systems are balanced.

**Pension system.** The pension system runs a balanced budget each period: We accordingly have

$$S^p + \int (1 - \mathbb{I}_{b^p})y^p(x)d\Phi(x) = \tau^p \left( \int \min(y^u, \gamma^u y^u)d\Phi(x) \right) \quad (26)$$
Unemployment system. Analogously, the balanced budget of the unemployment insurance system writes as:

\[ S^u + \int (1 - \mathbb{I}_0^x)y^u(x)d\Phi(x) = \tau^u\left(\int \min\left(y^u, \gamma^u y^u\right)d\Phi(x)\right) \]  

(27)

3.8 Competitive Equilibrium

Given exogenous parameters and government policy variables, \{S^u, S^p, G, T(\cdot), \tau^k, \tau^c, \tau^u, \tau^p, \gamma^\omega, \gamma^{p\omega}, \gamma^{p,u}\}, a competitive stationary equilibrium consists of individuals’ decision rules \{c, \ell, e, a', p'\} for each state vector \(x_m\), factor prices \{r, w\}, and the measures of individuals over the state space \(X(x)\) such that:

1. Household Maximization: Given prices \((r, w)\), government policies, (accidental) bequests, the value function \(V(x)\) satisfies the bellman equation and \((c, \ell, e, a', p')\) are the policy functions associated with the problem.

2. Firm maximization: The firm’s optimal choices satisfy

\[
\begin{align*}
    r &= AF_K(K, L) - \delta_k \\
    w &= AF_L(K, L)
\end{align*}
\]  

(28)

(29)


4. Market Clearing:

(a) The labor market clears

\[ L = \int \ell(x)e(f(j, s), \eta)d\Phi(x) \]  

(30)

(b) The capital market clears

\[ K = \int a'(x)d\Phi(x) \]  

(31)

(c) The goods market clears

\[ Y = \int c(x)d\Phi(x) + \delta_k K + G \]  

(32)
5. *Accidental bequests* and inheritances are balanced. Total bequests $B$ are re-distributed lump-sum among surviving agents younger than the minimum pension age. $B$ writes as:

$$B = \int (r(1 - \tau^k))(1 - \psi)a'(x)d\Phi(x)$$

(33)

6. *Consistency of Probability Measure:* The distribution of individuals across the state space is stationary, that is $X_{t+1}(x) = X_t(x)$.

**Solving the Model.** Computational complexity is induced by the existence of two endogenous state variables ($a$ and $p$) and the non-convexity of the value function due to the existence of discrete labor market choices. In order to cope with both problems our algorithm relies on a variant of the *hybrid method* as in Ludwig and Schön (2018) and an *upper envelope refinement step* similar to Druedahl and Jorgensen (2017), c.f. Appendix C for more details.

**4 Mapping the Model to Data**

If not otherwise noted, in the initial equilibrium we match the model to population moments based on 2015 data of the German Socio Economic Panel (G-SOEP).

**4.1 First Stage Calibration**

In this section we describe the parameters that are set outside the model.

**4.1.1 Demographics**

In our model, one model period covers one year. For the classification of education we use the International Standard Classification of Education (ISCED) of the UNESCO. By this we get three education levels, i.e. $\mathcal{E} = \{1, 2, 3\}^{13}$. We use the same sample as for estimating labor productivity profiles, c.f. below. We assume agents in the lowest two education groups start their economically active life at the age of 20 ($j = 1$).

---

13For low education we aggregate levels 0-2 (primary and lower secondary education), levels 3 and 4 (higher-secondary and post-secondary education) are grouped into middle education and levels 5 and 6 (tertiary education) comprise highly educated agents.
Agents in the highest education group start their economically active lives at the age of 25 ($j = 6$). All agents live to a maximum of 90 years ($J = 71$). In our sample used for the labor income profiles (c.f. below) at the age of 25 28%, 61%, and 11% are high, middle, and low educated individuals, respectively. We take male survival probabilities from the Human Mortality Database (HMD, 2015). Initially, everyone starts with zero assets and earnings points. The probability to be employed in the first period of the economic life is taken as exogenous and matched with the observed employment numbers in our SOEP sample. The initial employment rates are 87.4%, 89.5%, and 76.8% respectively.

4.1.2 Preferences

Our period utility function is assumed to be given by

$$U(c, \ell, e, \mathbb{I}_{\ell_j > 0}\kappa(j, s)) = \frac{c^{1-\sigma_c}}{1 - \sigma_c} - \frac{\lambda(\ell + e)^{1+\sigma}}{1 + \sigma} - I_{\ell_j > 0}\kappa(j, s)$$

(34)

The risk aversion parameter is exogenously set to its standard value in the literature of $\sigma_c = 2$. In order to obtain an empirically plausible Frisch elasticity of 0.6, we set $\sigma = 1.67$. We also set $\lambda = 0.25$. In this version of the paper $\kappa(j, s)$ is set to 0 for all ages and education groups, see below for a discussion of this choice.

4.1.3 Labor Productivity & Choice

In order to estimate labor productivity profiles we pool G-SOEP waves 1992-2015. We focus on working male household heads excluding civil servants, self-employed persons and those in military\ community services, or education. We take gross labor income as in the data, divide it by hours worked and transform it to its full-time work yearly equivalent. Annual hours of full time work are defined to be $40 \times 52 = 2080$. As described in the previous section processes are composed of a deterministic and a stochastic part.\footnote{We estimate an deterministic and stochastic process for each education group as described below. In order to enhance readability we refrain from using the usual subindex $s$.} We continue by first describing the deterministic part $f(\cdot, j, s)$, i.e. the predictable component, and then the residual part that evolves stochastically. We estimate the predictable component via an Ordinary Least Squares (OLS) regression equation comprising an intercept, wave-dummies and a third-order polynomial in age for the lowest two education groups and a second-order polynomial for the highest
education group\textsuperscript{15}. Estimation of the stochastic part $\eta$ follows a strategy very similar to Nardi et al. (2018), Kaas et al. (2017), and Grevenbrock (2018) which relies on estimating Markov processes from the data. We get median residual income values and average yearly transition matrices as described below: First, we define three residual income bins $\eta_{pc} \in \{\eta_1, \eta_2, \eta_3\}$. Second, to each bin we assign its respective median residual income value. Third, we calculate yearly Markov transition matrices as the proportions of individuals that move from bin $pc$ at age $j$ to bin $pc'$ at age $j+1$. To capture some age-dependency in the transition probabilities, we allow the transition matrices to be age dependent.\textsuperscript{16} In order to make the transition matrices uniformly stationary we translate the transition matrices from the data into doubly-stochastic matrices following a Sinkhorn-Knopp algorithm (Sinkhorn 1964) as in Kaas et al. (2017). We discretize the labor choice to two values, working full-time or part-time i.e. $\ell \in \{1.0, 0.5\}$. Income values are normalized such that $y^m = 1.0 \equiv €1000$ (2015).

4.1.4 Production

We assume a standard Cobb-Douglas production function $F(K, L) = AK^\alpha L^{1-\alpha}$. We set $\alpha = 0.36$ and calibrate $A$ such that $w = 1$ in the initial steady state.

4.1.5 Government Policies

If not otherwise noted we use 2015 as the baseline year for calibrating our initial steady state. Regarding the labor income tax schedule $T(\cdot)$ we are guided by the German legislation as outlined in the income tax law (EStG). Capital income tax we set to $\tau^k = 0.25$ and consumption tax to $\tau^c = 0.19$. Contribution rates for pension insurance and unemployment insurance are set to $\tau^r = 0.187$ and $\tau^u = 0.03$, respectively. Caps for contributions to social insurances is set to $\gamma^c = 2.0$ analogously to Fehr et al. (2012). In this version of the paper we set the contribution rate in case of unemployment to $\gamma^{p,u}$ reflecting no accumulation of pension points during unemployment.

The unemployment insurance net replacement rate we set to $b^u = 0.60$ mimicking ALG I. Minimum unemployment benefits are chosen in accordance with ALG II.

\textsuperscript{15}The fit of the second-order polynomial turned out to be better for the highest education group.

\textsuperscript{16}We allow for four age groups: $[20, 29]$, $[30, 39]$, $[40, 49]$, $[50, 90]$
"Hartz IV") in 2015, i.e. $4.884 = 12 \times \frac{407}{1000}$. We set $d$ to have a maximum value of three. The maximum unemployment benefit level is given by $12 \times b^u \times \frac{72600}{1000}$ where 72600 maximum annual net labor income in 2015 counting towards unemployment benefits. We set $v^+$ to 0.06 and $v^-$ to 0.036. We restrict the minimum pension level to $b^p = b^u$. We set $j_r = 44$, $j^{NRA} = 46$ and $j^{f} = 48$ corresponding to ages 63, 65, and 67.

Table 1: First Stage Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Short description</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J$</td>
<td>Number of periods</td>
<td>61</td>
<td>see text</td>
</tr>
<tr>
<td>$\Psi_j$</td>
<td>Survival rates</td>
<td>see text</td>
<td>HMD (2015)</td>
</tr>
<tr>
<td>Preferences</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma^c$</td>
<td>Risk aversion</td>
<td>2.0</td>
<td>Standard value</td>
</tr>
<tr>
<td>$\sigma^\ell$</td>
<td>Frisch elasticity of 0.6</td>
<td>1.67</td>
<td>Kindermann and Krueger ('17)</td>
</tr>
<tr>
<td>$\kappa_{i,s}$</td>
<td>Fix working costs</td>
<td>0.0</td>
<td>see text</td>
</tr>
<tr>
<td>Endowments</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f(j,s)$</td>
<td>Deterministic income part</td>
<td>see text</td>
<td></td>
</tr>
<tr>
<td>$\eta_{pc}$</td>
<td>Residual income percentile values</td>
<td>see text</td>
<td></td>
</tr>
<tr>
<td>$\pi(\eta_{pc}</td>
<td>\eta_{pc}')$</td>
<td>Residual income transition matrices</td>
<td>see text</td>
</tr>
<tr>
<td>$\ell$</td>
<td>Labor choice</td>
<td>$\ell \in {0.5, 1.0}$</td>
<td>see text</td>
</tr>
<tr>
<td>Policies</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tau^p$</td>
<td>Contribution rate pension insurance</td>
<td>0.187</td>
<td>German legislation ('15)</td>
</tr>
<tr>
<td>$\tau^u$</td>
<td>Contribution rate unemployment insurance</td>
<td>0.03</td>
<td>German legislation ('15)</td>
</tr>
<tr>
<td>$T(\cdot)$</td>
<td>Income tax system</td>
<td></td>
<td>German legislation ('15)</td>
</tr>
<tr>
<td>$\gamma^w$</td>
<td>Contribution cap as multiple of avg. income</td>
<td>2.0</td>
<td>Fehr et al. ('12)</td>
</tr>
<tr>
<td>$\gamma^{p,w}$</td>
<td>Cap on earning point accum. (working)</td>
<td>2.0</td>
<td>Fehr et al. ('12)</td>
</tr>
<tr>
<td>$\gamma^{p,u}$</td>
<td>Cap on earning point accum. (unemployed)</td>
<td>0.0</td>
<td>see text</td>
</tr>
<tr>
<td>$j_r$</td>
<td>Earliest retirement age</td>
<td>63</td>
<td>German legislation ('15)</td>
</tr>
<tr>
<td>$j^{NRA}$</td>
<td>Normal retirement age</td>
<td>65</td>
<td>German legislation ('15)</td>
</tr>
<tr>
<td>$j^{f}$</td>
<td>Latest retirement age</td>
<td>67</td>
<td>German legislation ('15)</td>
</tr>
<tr>
<td>$v^+$</td>
<td>Late retirement reward</td>
<td>6%</td>
<td>German legislation ('15)</td>
</tr>
<tr>
<td>$v^-$</td>
<td>Early retirement penalty</td>
<td>3.6%</td>
<td>German legislation ('15)</td>
</tr>
<tr>
<td>$b^u$</td>
<td>Minimum unemployment benefits</td>
<td>4.884</td>
<td>German legislation ('15)</td>
</tr>
<tr>
<td>$b^p$</td>
<td>Minimum pension benefits</td>
<td>4.884</td>
<td>see text</td>
</tr>
<tr>
<td>$b^{u*}$</td>
<td>Unemp. benefits, % of last net labor income</td>
<td>0.6</td>
<td>German legislation ('15)</td>
</tr>
<tr>
<td>$\tau^k$</td>
<td>Capital income tax</td>
<td>0.25</td>
<td>German legislation ('15)</td>
</tr>
<tr>
<td>$\tau^c$</td>
<td>Consumption tax</td>
<td>0.19</td>
<td>German legislation ('15)</td>
</tr>
</tbody>
</table>

4.2 Second Stage Calibration

In a next step, we use the model to pin down parameters that govern transition of unemployment into employment and vice versa as well as the overall generosity of the pension system relative to the overall wage level.
4.2.1 Job-Finding Probabilities

We match transition rates between employment states as observed in the data. Transition rates vary over the life-cycle. We match our model to transition probabilities for individuals younger than 54. Transition from employment into unemployment in our model is a combination of an exogenous job offer probability and—conditional on having received a job-offer—an endogenous decision to be unemployed or working. In this calibration of the paper all $\pi_{j,s}$ are constrained to be the same for all education groups and ages. We match this exogenous job offer probability to transition rates from employment into unemployment in the data. The targeted transition rate is $0.1908$.

In the model transition from unemployment to employment is affected by two endogenous decisions. In contrast to transition from employment into unemployment, now the component of receiving a job offer induces a second layer of endogeneity, i.e. search effort that affects the probability to receive a job-offer. We impose the functional form assumption that

$$\pi(e) = 1 - \exp(-\zeta e) \quad (35)$$

$\zeta$ can be interpreted as the search efficiency. Analogously to matching $\pi_{j,s}$ we now match $\zeta$ to yearly transition rates from unemployment into employment. The targeted transition rate is $0.0201$.$^{18,19}$

4.2.2 Current Earning Point Equivalent, CEE

Note that $CEE$ governs the generosity of the pension system, i.e. the translation of an earning point into its monetary value. We calibrate $CEE$ such that we match the

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$^{17}$We thank Moritz Kuhn for providing us with data on transition probabilities. He estimates quarterly transition probabilities from employment into unemployment and vice versa using data from the Institut für Arbeit und Beschäftigung (IAB). We take these rates and transform them into yearly counterparts.

$^{18}$Note that provided transition rates imply a steady state unemployment rate of $9.05\%$, a number in line with the long-run average unemployment rate as provided by the Federal Statistical Office (www.destatis.de).

$^{19}$Lentz (2009) have for search effort the optimality conditions as we have, i.e. $e = \zeta \beta \pi_e(e)[V_e - V_u]$. Lentz and Tranaes (2005) show that separability between consumption and search in the utility function will result in a decreasing search intensity choice in wealth. The sufficient conditions then might ensure concavity of the value function. See p.40 in Lentz (2009) for more details.
standard pension level (‘Standardrentenniveau’) of 47.7\%^{20}. We take data from the German Rentenversicherung\cite{21}. 

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 - \pi^o$</td>
<td>0.0200</td>
<td>Transition: EU</td>
<td>0.0201</td>
<td>0.0201</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>0.4471</td>
<td>Transition: UE</td>
<td>0.1908</td>
<td>0.1908</td>
</tr>
<tr>
<td>CEE</td>
<td>0.1657</td>
<td>&quot;Standardrentenniveau&quot;</td>
<td>0.4770</td>
<td>0.4770</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.9823</td>
<td>Capital-Output ratio (K/Y)</td>
<td>3.5000</td>
<td>3.4998</td>
</tr>
<tr>
<td>$\lambda$ now, exog</td>
<td>0.25</td>
<td>Average working hours</td>
<td>0.6255</td>
<td>0.0208</td>
</tr>
<tr>
<td>$\delta_k$ now, exog</td>
<td>0.048</td>
<td>Investment-Output ratio (I/Y)</td>
<td>0.3887</td>
<td>0.1880</td>
</tr>
</tbody>
</table>

Table 3: Further values in initial steady state

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r$</td>
<td>0.0549</td>
<td>real interest rate</td>
</tr>
<tr>
<td>$w$</td>
<td>1.0</td>
<td>wage rate (by adjusting $A = 0.8476$)</td>
</tr>
<tr>
<td>$\frac{G}{Y}$</td>
<td>0.2595</td>
<td>Government expenditure</td>
</tr>
<tr>
<td>$\frac{C}{Y}$</td>
<td>0.5724</td>
<td>Consumption</td>
</tr>
<tr>
<td>$\frac{I}{Y}$</td>
<td>0.1680</td>
<td>Investment</td>
</tr>
<tr>
<td>$\frac{S_u}{Y}$</td>
<td>0.0034</td>
<td>Surplus unemployment budget</td>
</tr>
<tr>
<td>$\frac{S_p}{Y}$</td>
<td>0.0378</td>
<td>Surplus pension budget</td>
</tr>
</tbody>
</table>

4.3 Model Fit

Figure 1 evaluates the model’s performance in matching unemployment rates and retirement transitions over the life-cycle. The figure shows that the current version of the model does not match this data close to the statutory retirement age of the model. There are two issues, one relates to the data, the other to the specification of the model. As to the data observations, the fraction of the retired population also reflects early retirement due to sickness, a component which is absent from the model. In the next version of the paper we will clean the data accordingly. Second, from the

\footnote{The Standardrente refers to 47.7\% of average gross labor income before taxes but after social security contributions.}  

\footnote{c.f. \url{www.deutsche-rentenversicherung.de}}
model perspective, in future versions we will improve the fit by calibrating the fixed costs of labor market participation, $\kappa(j,s)$.

Figure 1: Model Fit: Labor Market Rates over the Life-Cycle

(a) Unemployment Rate

(b) Retirement Rate

Notes: This Figure compares average life-cycle profiles of unemployment and retirement rate. Source: Own calculations.

Figure 2 displays employment rates in Panel (a) and labor supply conditional on employment in Panel (b). The employment rate in the model is basically the mirror image of the retirement rate. Thus the fit will improve by calibrating the fixed costs of labor market participation. The lack of fit to the intensive margin of labor supply conditional on employment shown in Panel (b) is likely a result of missing experience related earnings in the model. The next version will be extended accordingly.

Finally, life-cycle profiles of average consumption and assets in the model exhibit patterns well known from the literature, c.f. Figure 3.

5 Experiments

In this paper we are interested in the optimal policy mix of unemployment and pension insurance. We proceed as follows: As in the stylized model let $\Xi$ denote the set of all policy instruments over which the government can optimize. We start off by analyzing the optimality of unemployment and pension systems separately. Then we go on analyzing the joint optimal design of both systems. This procedure allows for understanding additional welfare effects induced by the interactive nature of both
Figure 2: Model Fit: Labor Market Rates over the Life-Cycle

(a) Employment Rate

(b) Intensive Labor Choice (1=full-time)

Notes: This Figure compares average life-cycle profiles of unemployment and retirement rate. 
Source: Own calculations.

Figure 3: Life-Cycle Profiles: Average Consumption and Assets

(a) Consumption

(b) Assets

Notes: This Figure compares average model life-cycle profiles of assets and consumption. 
Source: Own calculations.
systems. Throughout all experiments, we hold constant the level of transfers of the general tax system to unemployment and pension system, respectively. I all experiments unemployment and pension budgets are balanced by adjusting the respective contribution rates. The overall tax system clears by adjusting the consumption tax rate. We pin down optimal policy parameters via a grid search over the level of scrutinized policy instruments. For each policy experiment we calculate the new steady state and investigate its welfare consequences as compared to the baseline steady state. As utility is an ordinal measure a plain comparison of welfare is not suitable, and thus, we calculate consumption equivalence variations (CEV), i.e. we ask by how much an agent’s per period consumption in the baseline case has to be changed such that the agent is on average indifferent between the baseline steady state and the new steady state after the policy reform. Conceptually this is an ex-ante welfare measure drawing on a welfare comparison before the household knows his education type (i.e. evaluating her future life behind a veil of ignorance in a Rawlsian manner.) A non-zero CEV means that agents would be indifferent between the baseline steady state and the new steady state if they would be compensated with an \((1 + \text{CEV})\) change in consumption. As a result agents prefer the new steady state over the baseline steady state if \(\text{CEV} > 1\) and vice versa.\(^{22}\)

The next version of this paper will contain results from the calibrated model. Computations on the basis of an earlier model variant suggested that the optimal policy mix is characterized by higher unemployment pay as well as stronger redistribution through the pension system than the current status quo.

[TBC]

6 Concluding Remarks

In this paper we characterize the optimal joint design of unemployment and pension insurance systems. We start off highlighting the interactive nature of both insurance systems in a stylized model. We move on to developing a multi-period life-cycle model that we calibrate to match key features of German data.

This paper is incomplete. We are currently at the stage of diagnosing the quantitative model, which provides important insights into desirable refinements of the model’s specification. In future versions of the paper we will employ the quantitative

\(^{22}\)C.f. Appendix B for a derivation of the CEV measure given our utility function specification.
model to characterize the optimal policy mix of unemployment and pension insurance. Very preliminary results based on an earlier model variant suggested that unemployment pay should be more generous and pension insurance more redistributive than for the German status quo.
References


