Entry, markup and unemployment in an estimated DSGE model

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Abstract

How important is the role of endogenous market structure to the fluctuations in unemployment? Do search and matching frictions in the labor market generate counter-cyclical markups? We estimate a monetary DSGE model with endogenous firm entry as in Bilbiie et al. (2012) and search and matching friction in the spirit of Mortensen and Pissarides (1994) for the labor market using Bayesian methods. The model is estimated using US data over the period 1992Q3 to 2016Q4. We find that the presence of the net business formation has a key role in jointly determining labor market variables (vacancies, finding rate and the Beveridge Curve). Furthermore, based on a historical and variance decomposition we find that entry shocks contribute to explaining unemployment fluctuations. Finally, we also find that the presence of labor market frictions enhances the counter-cyclical nature of markups.

1 Introduction

Firm entry accounts for a large fraction of job creation (destruction) in the U.S. economy due to the birth (death) of firms. As pointed by Jaimovich and Floetotto (2008), the average fraction of quarterly job-gain (losses) that can be explained by the opening (closing) of establishments is about 20%. Despite this evidence, the current state-of-art macroeconomic models with firm entry only are estimated on the intensive margin of labor, thus leaving the empirical nexus between labor frictions and firm entry yet undocumented. The main

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The topic of this paper is thus to close this gap by formulating and estimating a quantitative macroeconomic model that incorporates both labor and goods market frictions. The model is estimated on US time series from 1992Q3 to 2016Q4. The objective of the paper is to explore the interaction of goods and labor market frictions, and assess quantitatively the importance of firm entry in shaping the aggregate fluctuations of (un)employment for the US economy.

In the literature, two important papers related to the entry-unemployment nexus are close to our approach. First, Colciago and Rossi (2011) provides an important contribution based on a friction on firm entry limiting the entry to startups that have reached the labor size of incumbents. Using the entry mechanism and search and labor market frictions, they find a strong amplification channel of technology shocks on labor market variables which partially solve the Shimer (2005) puzzle. We complement their analysis by having a time-varying markup determined endogenously by the number of producers operating in the market. The fraction of the labor force employed by incumbents is challenged by the number of firms willing to enter the market. According to the terminology of Etro et al. (2007) and Lewis and Stevens (2015), we refer the effect of entry on markup and by extension on economic activity to the ‘competition effect’ in the rest of the paper. The second paper is Cacciatore et al. (2016) where as for the first cited paper, find the same amplification channel. However, their paper has a clear focus on the impact of market reforms on labor and product and their welfare implications.

In these two main papers, the empirical dimension is not examined by full information methods, limiting the analysis to a calibration exercise, with an external validation assessed by the comparison of second moments statistics with the data. In this paper, we formulate an original model with entry and labor market frictions, and take the model to the data through Bayesian techniques. By doing so, we formally evaluate the quantitative implication of entry to the overall model performance, an answer two questions: (i) how important is the role of endogenous market structure to the fluctuations in unemployment? (ii) do search and matching frictions in the labor market generate countercyclical markup?

Our first question relates to the dynamics of unemployment in the traditional medium-scale DSGE models to study business cycle fluctuations. In this branch of the literature, unemployment is introduced using the setup of Mortensen and Pissarides (1994) (e.g., Gertler et al. (2008)). In these models, a positive demand shock rise the labor demand, firms post
vacancies and through the matching process, hire new employees. In the case of a TFP shock, firms are more productive which reduces the marginal cost of production and rises the marginal gain of hiring. Through the matching process, firms engage in hiring new workers. However, the presence of endogenous firm creation significantly shakes this conventional propagation mechanism. Under firm entry, unemployment is proportional to the number of firms operating in the market, which reshapes the overall dynamics following demand and supply shocks. Since operating firms are the sum of incumbents and startup, then their combination affects employment dynamics. Second, as in Colciago and Rossi (2011) and Cacciatore and Fiori (2016), entry costs depends on the incumbent workforce size and the probability for firms to find a worker. The modification of these two components following a shock can drop the entry cost acts as an amplification mechanism on entry and employment volatility. In the case of an expansionary shock boosting entry, the number of producers rises but potentially the size of incumbent workforce can drop due to the competition effect. In the case of an expansionary shock that crowds out entry, the number of producers drops but the competition effect can lead to an expansion of the incumbent labor force. Therefore, changes in the number of producers may have conflicting effects on unemployment.

The second question is related to the study of markup. From an empirical point of view, markups of price over marginal costs are unobserved and therefore hard to measure, or simply unobservable. Important uncertainties remain on the cyclicality of markups. For example, Bils (1987) and Rotemberg and Woodford (1999) find evidence of countercyclical markups, while Nekarda and Ramey (2013) obtain the opposite result on the correlation link. Since there is no well-established consensus on the cyclicality of markups in the data or even conditional on shock, the estimated model can thus be employed to contribute to the measure of markup, and investigate their procyclical link. Thus, the choice of search and matching friction in the vein of Mortensen and Pissarides (1994) introduce a marginal cost depending on hiring costs and the intertemporal value of employment. Then, this marginal cost is susceptible to be possibly affected, which would change markup cyclicality, with respect to a intensive margin labor as in Lewis and Stevens (2015) or Bilbiie et al. (2012).

The main findings of the paper read as follows. First, we find that entry shock have important implications in shaping unemployment dynamics. This result is based on a historical and variance decomposition. Second, we obtained a countercyclical markup with a less important competition effect than previous studies. Third, we find that the model with
firm entry outperforms the model without business dynamics in terms of empirical fit. Finally, after conducting some robustness analysis with alternative hypothesis, we find that our model is preferred to other specifications.

2 The model

In this section we present the model. Our model combines the entry mechanism proposed by Bilbiie et al. (2012) and search and matching frictions in the labor market in the spirit of Mortensen and Pissarides (1994). We include a set of real and nominal frictions as in Christiano et al. (2005) and Smets and Wouters (2007) such as habit formation, investment adjustment costs and variable capital utilization. These features are necessary in order to replicate the dynamics of investment and consumption as observed in the data. We consider an economy populated by households, firms which can be a producer or a new entrant and national authorities with a central bank and a government. Thus, we first describe translog preferences as in Feenstra (2003) of the representative household and its optimal decisions regarding investment and consumption. After that, we display the production sector by separating incumbents and new entrants. The first type of firms minimize their cost (labor and capital) and after that they fix their price subject to nominal rigidities à la Rotemberg (1982). The second type needs to pay an entry cost which is dependant of the firm-level of incumbent. Third, we describe the wage setting resulting from a Nash bargaining between incumbent and his workforce. Finally, in two different sections, we describe the role of national authorities and the general equilibrium of the economy.

2.1 Household

Each household is conceived as a large extended family containing a continuum of members along a unit interval. The number of family members currently employed is $L_t$. Following Andolfatto (1996) and Merz (1995), the family provides perfect consumption insurance for its members such that there is no ex-post heterogeneity across individuals in the household.

2.1.1 Translog preferences and product turnover

Preferences The final consumption basket $Y^C_t$ is defined over a continuum $\tilde{N}$ where only a subset of goods are available at time $t$: $N_t \in \tilde{N}$. We follow Feenstra (2003) assuming that
the final consumption basket $Y_t^C$ which includes the consumption bundle $C_t$ takes a translog form\(^1\). Translog preferences are characterised by defining the unit expenditure function i.e. the welfare-based price index $P_t^C$ associated with the basket good $Y_t^C$. Let $P_{f,t}$ be the nominal price for the good $f \in [0, N_t]$, the unit expenditure function on the basket good $Y_t^C$ is:

$$\ln P_t^C = \frac{N - N_t}{2\sigma\varepsilon'_t NN_t} + \frac{1}{N_t} \int_{f \in N_t} \ln P_{f,t} df + \frac{\sigma\varepsilon'_t}{2N_t} \int_{f \in N_t} \int_{f' \in N_t} \ln P_{f,t}(\ln P_{f,t} - \ln P_{f,t}) df df', \quad (1)$$

where $\sigma > 0$ denotes the price-elasticity of the spending share on an individual good and $\varepsilon'_t$ an exogenous price markup shock. The assumption of translog preferences allows us to model endogenous markup through product\(^2\). Translog preferences give an elasticity of demand $\varepsilon_t$ which varies with the number of goods: $\varepsilon_t = 1 + \sigma\varepsilon'_t N_t$ and the following optimal demand address for one good,

$$N_t P_{f,t} y_{f,t} = P_t^C Y_t^C, \quad (2)$$

where $y_{f,t}$ is the output of producer $f$. Note that we use the terms ‘goods’, ‘firms’ and ‘producers’ interchangeably assuming that each firm produces exactly one differentiated variety. According to Bilbiie et al. (2012), each unit in the model is interpreted as a production line that could be part of a multi-product firm whose boundary is left undetermined. Thus, household preferences make the demand more elastic when the number of producer rises.

**Product Turnover** Each period all firms are affected by an exogenous exit shock $\delta^N \in [0, 1]$ which affects both entering and existing firms. Then, the law of motion of firms in the economy reads:

$$N_t = (1 - \delta^N) \left( N_{t-1} + (1 - AC_{t-1}^{E}) N_{t-1}^E \right), \quad (3)$$

\(^1\)Since the aggregate demand $Y_t^C$ includes sources others than household consumption $C_t$, this assumption ensures that the consumption price index is also the price index for the aggregate demand.

\(^2\)They are various ways to introduce endogenous markup with demand-side complementarities as in this chapter or supply-driven competition effects working through changes in the market structure. As in Bilbiie et al. (2012) and Lewis and Stevens (2015), we choose the first tool for its capacity to reproduce key features of the business cycle. Moreover, a supply-driven tool needs to replace the monopolistic competition by another type such as Cournot or Bertrand’s competition (Lewis and Poilly (2012)) and complicate the comparison with other estimated DSGE models.
where \( AC_{t-1}^E \) denotes the failure probability of startups. Following Lewis and Stevens (2015), we assume that not all startups are successful and a fraction \( AC_{t-1}^E \) of new firms does not succeed in starting their business. Formally, \( AC_{t-1}^E = \frac{\varphi^E}{2} \left( \frac{\varepsilon_t^E N_{t-1}^E}{N_{t-2}^E} - 1 \right)^2 \) with \( \varphi^E \geq 0 \) the degree of rigidity and \( \varepsilon_t^E \) an exogenous shock. Thus, the failure probability of startups rises with the number of competitors acting as a congestion cost. As for investment costs, this cost helps to catch the dynamics of entry over the business cycle.

### 2.1.2 Budget and intertemporal decisions

The representative households maximise the expected intertemporal utility function brings by the discount factor parameter \( \beta \in [0,1] \):

\[
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \varepsilon^B_t \left( C_t - h^C C_{t-1} \right)^{1-\sigma^C} \right\}.
\]  

(4)

where \( \varepsilon^B_t \) is an exogenous preference shock, \( h^C \in [0,1] \) represents external habits and \( \sigma^C > 0 \) is the risk-aversion parameter.

The households’ budget constraint in real terms reads:

\[
\frac{W_t}{L_t} L_t + (1 - L_t) b + R_{t-1} \frac{B_{t-1}}{P_t} + X_t^K + (1 - \delta^N) (d_t + e_t) (x_{t-1} + (1 - AC_{t-1}^E)N_{t-1}^E)
= C_t + T_t + \frac{B_t}{P_t} + N_t^E \phi_t^E + e_t x_t.
\]  

(5)

The income of the representative household is made of labor income with \( \frac{W_t}{L_t} L_t \) for employed members and \( (1 - L_t) b \) for unemployed members, return on riskless bonds \( R_{t-1} \frac{B_{t-1}}{P_t} \) with \( R_{t-1} \) the nominal interest rate, the net return of owning the capital stock \( X_t^K \) and return on share holdings \( x_t \) and on successful startup \((1 - AC_{t-1}^E)N_{t-1}^E\).

The expenditure side includes consumption \( C_t \), taxes \( T_t \), bonds purchases \( B_t \), investment on startup \( N_t^E \phi_t^E \) and shares purchases \( e_t x_t \) (where \( e_t \) stands for the market price of a share \( x_t \)). The goal of the representative household is to maximize its utility (Eq.4) subject to its budget constraint (Eq.5) to choose the optimal amount of consumption and saving in the form of bonds, the optimal amount of share holding and investment in startup and optimal decisions concerning the capital stock such as investment and optimal utilization rate. Thus, we describe the behavior of the representative household in this order.
Consumption and saving on bond decisions  The optimal consumption choice reads:

$$
\lambda_t^C = \epsilon_t^B \left(C_t - h^C C_{t-1}\right)^{-\sigma^C} - \mathbb{E}_t \beta h^C \epsilon_{t+1}^B \left(C_{t+1} - h^C C_t\right)^{-\sigma^C},
$$

(6)

where $\lambda_t^C$ is the marginal utility of consumption. The First Order Condition (FOC hereafter) with respect to $B_t$ gives the Euler equation for bonds:

$$
\frac{\lambda_t^C}{\beta \mathbb{E}_t \lambda_{t+1}^C} = \frac{R_t}{\mathbb{E}_t \pi_{t+1}^C},
$$

(7)

where $\pi_{t+1}^C = \frac{p_{t+1}^C}{p_t}$ is the welfare-based inflation rate. This condition determines the optimal consumption path.

Share holdings and investment in start-ups  Successful startup $(1-AC_{t-1}^E)N_{t-1}^E$ and shares purchased in the previous period $x_{t-1}$ pays dividends $d_t$ and are worth $e_t$ conditionally to the exogenous survive probability $1 - \delta^N$. To get this income, the representative households need to spend resources for the investment in startups at cost $\mathbb{E}_t e_t$ which describes latter and purchase shares at the price market $e_t$. Then, the optimal share purchasing leads to the Euler condition on shares:

$$
e_t = \mathbb{E}_t \beta_t \left(1 - \delta^N\right) \left(d_{t+1} + e_{t+1}\right),
$$

(8)

with $\beta_t = \beta \mathbb{E}_t \lambda_{t+1}^C$ the stochastic discount factor. Put in a recursive form (Eq.8 becomes $e_t = \mathbb{E}_0 \sum_{t=1}^{\infty} \beta_t \left(1 - \delta^N\right)^t d_t$), the current value of shares is equal to the discounted sum of expected dividends as in the standard corporate finance theory. The FOC with respect to new firms $N_t^E$ gives the free entry condition:

$$
\phi_t^E = e_t \left(1 - \frac{\partial AC_t^E N_t^E}{\partial N_t^E}\right) - \mathbb{E}_t e_{t+1} \frac{\partial AC_{t+1}^E N_{t+1}^E}{\partial N_t^E}.
$$

(9)

Assuming that the probability of success for new business is equal to one such that $\varphi^E = 0$, the free entry condition is the same as in Bilbiie et al. (2007) ($\phi_t^E = e_t$). Thus, in this case entry occurs until the firm value is equalized with the entry cost $\phi_t^E$. Note that the adjustment cost (including exogenous shock $\epsilon_t^E$) drives a wedge between the equity value $e_t$ and the cost to settle new firms ($\phi_t^E$).

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\[3\text{Formally, this corresponds to } (1 - \delta^N) (d_t + e_t) \left(x_{t-1} + (1 - AC_{t-1}^E)N_{t-1}^E\right) \text{ in the income side of the budget constraint (Eq.5).}\]
Optimal decisions for capital  The representative household owns the total stock of capital of the economy $K_t$ and choose the capital utilization rate $\nu_t$, which transforms physical capital into effective capital $K'_t$ according to:

$$K'_t = \nu_t K_{t-1}. \quad (10)$$

Modifying the utilization rate is costly for the representative household and they have to pay $(\nu_t)$ per unit of physical capital if they want to change this rate. Thus, as in Christiano et al. (2005) we impose that in the steady-state $\nu = 1$, $\psi (\nu) = 0$ and $\psi' (\nu) = r^K$. More formally, we have $\psi(\nu_t) = \frac{1-\Psi}{\Psi} r^K \left( e^{1-\Psi (\nu_t - 1)} - 1 \right)$ with $\Psi \in [0,1]$ the utilization elasticity$^4$. Additionally to this cost, the investment in physical capital is costly i.e. the representative household faces an adjustment cost $AC^I_t$ on investment, such that $AC^I_t = \varphi^t \left( \varepsilon^t - \varepsilon_{t-1} - 1 \right)^2$ where $\varphi^t \geq 0$ is the degree of rigidity and $\varepsilon_t$ an exogenous shock on investment costs. The capital stock in the economy thus evolves according to:

$$K_t = (1 - \delta^K) K_{t-1} + (1 - AC^I_t) I_t. \quad (11)$$

According to these costs, effective capital is rented to the firms at price $r^K_t$ in a competitive capital market. Then, the net return of owning the capital stock $X^K_t$ in the budget constraint (Eq.5) reads:

$$X^K_t = r^K_t \nu_t K_{t-1} - \psi(\nu_t) K_{t-1} - I_t, \quad (12)$$

where $r^K_t \nu_t K_{t-1}$ represents the household’s earnings from supplying capital services.

Integrating the net return for capital in the budget constraint and derive with respect to capital $K_t$ by considering the capital law of motion (Eq.11), gets:

$$q^K_t = \beta_t (r^K_{t+1} \nu_{t+1} - \psi(\nu_{t+1}) + q^K_{t+1} (1 - \delta^K)), \quad (13)$$

where $q^K_t$ denotes the shadow value of capital (in units of consumption)$^5$. This shadow value is defined by the FOC for investment $I_t$:

$^4$For $\psi \to 1$, the cost for changing the utilization rate is very high and the utilization rate does not vary. For $\psi \to 0$, the marginal cost of changing the capital utilization rate is constant and as a result the rental rate of capital does not vary.

$^5$q^K_t corresponds to the Lagrange multiplier associated with the capital law of motion (Eq.11) normalized by the marginal utility of consumption $\lambda^C_t$. 

8
Then for $\varphi^I \neq 0$ the real shadow value of capital is non-constant and the exogenous shock $\varepsilon^I_t$ with fluctuations in investment drives the value of capital $q^K_t$.

Finally, the optimal utilization rate for capital is defined as,

$$\psi'(\nu_t) = r^K_t,$$

where it is optimal for the household to choose a capital utilization rate equals to the marginal product of using capital i.e. the renting price.

### 2.2 Production

There is a continuum of monopolistically competitive firms, each producing a differentiated variety $f \in [0, N_i]$ and a continuum of new entrants $e \in [0, N^E_t]$. Prior to entry, new firms need to pay a sunk entry cost $\phi^E_{e,t}$ to be specified later on. As in Cacciatore (2014), all firms that enter the market start producing at the next period.

#### 2.2.1 Incumbent

Incumbent face a two-stage problem. First, they minimize their cost subject to labor and capital market specificities. Second, they fix price subject to Rotemberg (1982) adjustment costs according to the monopolistic competition in this sector.

**Minimization Cost** For incumbents, production requires labor and capital. For labor a representative firm is subject to matching frictions for hiring workers. To create a new job, a producer needs to post a vacancy, incurring a real cost $f^V$. The probability to find a worker depends on a constant return to scale matching technology, which converts aggregate unemployed workers $U_t$ and aggregate vacancies $V_t$ into aggregate news jobs $M_t = m (V_t)^\zeta (U_t)^{1-\zeta}$ with $m > 0$ the degree of efficiency and $\zeta \in [0,1]$ the elasticity of matches with respect to vacancies. Thus, the probability of filling a vacancy is given by: $q_t = \frac{M_t}{V_t}$ and the probability for an unemployed worker to find a job is: $f_t = \frac{M_t}{U_t}$. From the perspective of an individual firm, the level-employment $l_{ft}$ is the sum of the inflow of new workers represented by $q_t V_{ft}$ and the outflow of workers due to exogenous separation $\delta^L_t \in [0, 1]$. Formally, we have:
According to our timing assumption unemployed workers looking for a job is given by the difference between unity (the total population of workers) and the number of employed workers at the end of period \(t - 1\): 
\[ U_t = 1 - L_{t-1}. \]

Thus, the total output of firm \(f\) is given by:
\[
y_{f,t} = \varepsilon_t^Z (L_{f,t})^\alpha (k_{f,t}^V)^{1-\alpha},
\]

where \(\alpha \in [0, 1]\) is the part of labor used in the production and \(\varepsilon_t^Z\) is an exogenous productivity shock. Thus, the goal of the firm is to choose the optimal amount of labor and vacancies subject to Eq.16 for the labor market and the optimal amount of capital given the renting rate \(r^K_t\). Formally, they solve
\[
\max_{l_{f,t}, v_{f,t}, k_{f,t}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (1 - \delta^N) \left\{ \frac{P_{f,t}}{P_t} y_{f,t} - (1 + AC_{f,t}^W) \frac{W_t}{P_t} l_{f,t} - f^V v_{f,t} - r^K_t k_{f,t} \right\},
\]
with \(AC_{f,t}^W\) a quadratic adjustment cost for nominal wage à la Rotemberg (1982) describes latter\(^7\).

Starting with the optimal amount of labor force given by:
\[
\mu_{f,t}^L = \alpha \frac{mc_{f,t} y_{f,t}}{l_{f,t}} - (1 + AC_{f,t}^W) \frac{W_t}{P_t} + \mathbb{E}_t \beta_t (1 - \delta^N) (1 - \delta^L) \mu_{f,t+1}^L,
\]

with \(\beta_t = \beta \mathbb{E}_t \frac{\lambda_{f,t}^N}{\lambda_{f,t}^L}\) denotes the stochastic discount factor of households, who are assumed to own domestic firms and \(mc_{f,t}\) the real marginal cost associated with production. This condition consists of the revenues from output, net of wages and their adjustment costs, and the expected continuation value of the job next period, accounting for the probability of both separations (from the labor market \(\delta^L\) and from the product market \(\delta^N\)). Optimizing with respect to vacancies leads to define the value of hiring a new worker \(\mu_{f,t}^L\) \(^8\):

\(^6\)Gertler et al. (2008) and Gertler et al. (2008) among others use a similar timing.

\(^7\)Since the wage is determined through a Nash bargaining and not in the present optimization program, we describe this property in the section 2.3.

\(^8\)Mathematically speaking, the value of hiring a new worker \(\mu_{f,t}^L\) correspond to the Lagrange multiplier associated with the law of motion of employment at the firm-level (Eq.3).
\[ \mu^{L}_{f,t} = \frac{fV}{q_t}. \] (19)

This value depends negatively on the probability that the vacancy is filled. Intuitively, the firm valued the hiring when it is difficult for him to find employees.

Finally, the FOC with respect to effective capital reads:

\[ r^K_t = (1 - \alpha) \frac{mc_{f,t} y_{f,t}}{k^\nu_{f,t}}. \] (20)

Having seen the optimal condition for both markets (capital and labor), we turn to the second step namely the price setting of firms.

**Price setting** In the second step, the representative firms operate monopolistically and set price according to Rotemberg (1982) technology. The quadratic adjustment cost is given by: \[ AC^P_{f,t} = \frac{\kappa^P}{2} \left( \frac{P_{f,t}}{P_{f,t-1}} - 1 - \lambda^P (\pi_{t-1} - 1) \right)^2 \frac{P_{f,t}}{P^P_t} \] with \( \kappa^P \) the degree of rigidity and \( \lambda^P \) the indexation on past inflation. Given this price adjustment cost specification, the problem of the firm is to choose his optimal price \( P_{f,t} \) to maximize the expected steam of profits given by:

\[ \max_{P_{f,t}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_t (1 - \delta^N) \left\{ \left( \frac{P_{f,t}}{P^C_t} - mc_{f,t} - AC^P_{f,t} \right) y_{f,t} \right\}. \] (21)

Since the amount of firm-specific output \( y_{f,t} \) is demand-determined in response to its relative price \( \rho_{f,t} = \frac{P_{f,t}}{P_t} \) (Eq.2), the optimal pricing policy is:

\[ \rho_{f,t} = \frac{mc_{f,t}}{mc_{f,t}} \left( 1 - \frac{\mu_{D,t}}{1 - AC^P_{f,t}} + \kappa^P \psi^P_{f,t} \right), \] (22)

where \( \mu^D_t \) is the desired markup defined in the household preferences \( \mu^D_t = 1 + \frac{1}{\sigma_{f,t} N_t} \) which depends on the number of varieties due to translog preferences and \( \psi^P_{f,t} \) is an auxiliary variable that depends on the Rotemberg cost of adjustment\(^9\). The optimal pricing induces that the price is set as a markup over the real marginal cost of production,

\[ \mu_{f,t} = \frac{\rho_{f,t}}{mc_{f,t}}. \] (23)

Note that in the absence of rigidity for nominal prices: \( \kappa^P = 0 \), the markup in Eq.22 is

\[ \psi^P_{f,t} = \pi_{f,t} (\pi_{f,t} - 1 + \lambda^P (1 - \pi_{t-1})) - \mathbb{E}_t \left( 1 - \delta^N \right) \beta_t \frac{y_{f,t+1} \psi^C_{f,t}}{y_{f,t+1} \psi^C_{f,t+1}} (\pi_{f,t+1} - 1 + \lambda^P (1 - \pi_t)) (\pi_{f,t+1})^2 . \]
equal to the desired markup $\mu_{f,t} = \mu_{D,t}$. As a consequence, more diversity for consumers conduces to a lower desired markup for firms.

2.2.2 New entrants

As mentioned before, an entrant needs to pay a sunk entry cost to enter the market $\phi^E_{e,t}$. As in Cacciatore and Fiori (2016), we adopt the following form:

$$\phi^E_{e,t} = \frac{1}{\epsilon_t} \left( f^E + f^V_{V^E_{e,t}} \right).$$

(24)

As standard practice in the literature, we assume that the technological shock for incumbent also affect new entrants since new technology leads to lower costs in both sectors.

The first term ($f^E$) represents technological requirements such as the research and development and the cost in terms of goods and services imposed by administrative barriers to market entry. The second term of the entry cost corresponds to the recruitment of workers with $v^E_{e,t}$ vacancies posted by the representative new firm. The optimal hiring policy of new firms, which have no initial workforce, consists in posting at time $t$ as many vacancies as required to hire workers and to have the same workforce as incumbent $l_{f,t}$. Therefore, a new entrant posts:

$$v^E_{e,t} = \frac{(1 - \overline{A_{C_{t}^E}}) \left(1 - \delta^N\right) \left(1 - \delta^L\right) l_{f,t}}{\overline{E}_{q_{t+1}}}. \quad (25)$$

Then, $(1 - \overline{A_{C_{t}^E}}) \left(1 - \delta^N\right)$ accounts for the probability of success for an entrant to produce in $t+1$ and $(1 - \delta^L)$ represents the exogenous probability that the employment relationship succeed. Note that the probability to find a worker is in $t+1$ since vacancies posted by new firms need a time lag to be hired$^{10}$.

Thus, the cost to enter the market depends positively on the labor incumbent force and negatively of the expected probability to find a worker. The formulation of the entry cost is, furthermore, discussed in the sensitivity analysis made in section 5.2.

$^{10}$The assumption of time lag for hiring people for new entrants and not for incumbent is necessary otherwise new entrants need to pay wages.
2.3 Wage setting

We assume that nominal wages are determined through a Nash bargaining process between workers and firms who maximise the joint surplus of an employment relationship. The bargaining solution is determined by the following program,

$$\arg \max_{\mu_{f,t}} \left( \mu_{f,t}^W \right)^{\eta \epsilon_t^L} \left( \mu_{f,t}^L \right)^{1-\eta \epsilon_t^L},$$

(26)

where $\eta \in [0, 1]$ is the exogenous bargaining power of the worker and $\epsilon_t^L$ is the attached exogenous shock. $\mu_{f,t}^L$ is the firm’s surplus from Eq.18 and the surplus of a worker from employment at firm $f$ is given by:

$$\mu_{f,t}^W = \frac{W_{f,t}}{P_t} - b + E_t \beta_t \mu_{f,t+1}^W \left( (1-\delta^N) (1-\delta^L) - f_{t+1} \right)$$

(27)

which corresponds to the net value of being employed (wage minus transfert to unemployed workers) and the expected continuation value of the job in the next period accounting for destruction for both market (labor and product) and less the expected probability to find a job for an unemployed worker. Intuitively, more the probability to find a job ($f_{t+1}$) in the future is high, more the worker can easily find another job and thus reduce the incentive to keep his job at firm $f$.

The FOC of the Nash bargaining is given by:

$$E_t \omega_t \mu_{f,t}^L = E_t (1-\omega_t) \mu_{f,t}^W.$$  

(28)

Due to wages adjustment costs in the expected value of a job for firms (Eq.18) denoted by $AC_{f,t}^W = \frac{\psi_{f,t}}{2} \left( \frac{W_{f,t}}{W_{t+1}} - 1 - \lambda^W (\pi_{t+1}^C - 1) \right)^2$ with $\kappa^W \geq 0$ the degree of rigidity and $\lambda^W \in [0, 1]$ the indexation on welfare-based price inflation, the effective bargaining power of the worker denoted by $\omega_t$ is time-varying and reflect the evolution of current and expected wage adjustment costs. This expression is,

$$\omega_t = \frac{\epsilon_t^L \eta}{\epsilon_t^L \eta + (1-\epsilon_t^L \eta) (1+AC_{f,t}^W + E_t (\kappa^W \psi_{f,t}^W))},$$  

(29)

where $\psi_{f,t}^W$ is an auxiliary variable that depends on the Rotemberg cost of adjustment\footnote{Due to the value of the firm which is linear in its employment level, all workers are the same at the margin, and the wage negotiation is between the firm and the marginal worker.}

\footnote{More precisely $\psi_{f,t}^W = \pi_{f,t}^W \left( \pi_{f,t}^W - 1 + \lambda^W (1-\pi_{t+1}^C) \right) - E_t \frac{\alpha(1-\delta^N)(1-\delta^L)}{\pi_{t+1}^C} \left( \pi_{f,t+1}^W - 1 + \lambda^W \left( 1 - \pi_{t+1}^C \right) \right) \left( \pi_{f,t+1}^W \right)^2$}
similar in his form to the one obtained for optimal price (Eq. 22). With adjustment costs the bargaining power becomes state-dependent. During periods of rising wages, $\frac{\partial \omega}{\partial W_{f,t}} < 0$, the effective bargaining power of workers decline (respectively during declining wages, the bargaining power increase). Intuitively, when wages decline workers capture a larger fraction of the joint surplus of the employment relationship and smaller when they rise.

Finally, by replacing the marginal value of posted a vacancy (Eq. 18) and its marginal value (Eq. 19), the expression of wage is:

$$\frac{W_t}{P_t^C} \left(1 + \omega_t AC_{f,t}^W\right) = \omega_t \left(\frac{mc_{f,t} l_{f,t}}{l_{f,t}}\right) + (1 - \omega_t) b$$

$$+ \mathbb{E}_t (r_{1-t}^f Y_{1-t}^V) \left(\omega_t - \frac{(1-\omega_t) \omega_{t+1}}{(1-\omega_{t+1})} \left(1 - \frac{f_{t+1}}{(1-\delta^R)(1-\delta^E)}\right)\right)$$

(30)

Note that in the absence of distortion due to wage adjustment costs or exogenous shock ($\kappa^W = 0$ and $\varepsilon_t^f = 0$), the real wage is simply an average of the highest wage that the firm can pay and the minimum that household accept to have:

$$\frac{W_{f,t}}{P_t^C} = \eta \left(\frac{mc_{f,t} y_{f,t}}{l_{f,t}} + \mathbb{E}_t \beta_t f_t^V f_{t+1}^{1-q_{t+1}}\right) + (1 - \eta) b.$$  

(31)

### 2.4 Authorities

The central bank sets the interest rate following a standard Taylor rule,

$$\frac{r_t}{\bar{r}} = \left(\frac{r_{t-1}}{\bar{r}}\right)^{\rho^R} \left(\frac{\pi_{t}^{\phi^C}}{\pi_{t}^{\phi}}\right)^{\phi^Y} \left(\frac{Y_{t}^{C}}{Y_{t-1}^{C}}\right)^{(1-\rho^R)}$$

(32)

where $\rho^R$ is the weight according to the past interest rate, $\phi^Y$ the emphasis for the GDP growth where we use $Y_t^C$ the aggregate demand defined below as a proxy for GDP, $\phi^\pi$ the parameter for inflation dynamics and $\varepsilon_t^R$ correspond is the exogenous monetary policy shock.

Concerning the government, he finances public spending and compensation for unemployed households by collecting lump-sum taxes $T_t$ from households as well as issues one-period bonds $B_t$. The total amount of public spending $G_t$ is assumed to evolve according to an exogenous process such that $G_t = \varepsilon_t^G g^Y$ where $g^Y$ is the steady-state ratio of public spending to GDP ($g^Y = g/Y^C$). Thus, the balance sheet of government corresponds to,

$$\varepsilon_t^G g^Y + (1 - L_t) b + R_{t-1} B_{t-1} = T_t + B_t.$$  

(33)

with $\pi_{f,t}^W = \frac{W_{f,t}}{W_{f,t-1}} \pi_{t}^{C}$ is the inflation of nominal wages for workers employed by a producer $f$. 


2.5 Shocks, aggregation and equilibrium condition

In this model, there are eight exogenous shocks process defined by \( \varepsilon_t^i = \rho^i \varepsilon_{t-1}^i + \eta_t^i \) for \( i = \{Z, E, C, G, I, P, L, R\} \) and where \( \rho^i \) are autoregressive roots (AR(1)) of the exogenous variables, \( \eta_t^i \) are standard errors that are mutually independent, serially uncorrelated and normally distributed such that \( \eta_t^i \sim \mathcal{N}(0, \sigma^i) \) with \( \sigma^i \) the variance. Price and bargaining shocks are augmented with a moving average (MA(1)) term denoted \( \mu^l \) for \( l = \{P, L\} \) as in Smets and Wouters (2007) such that \( \varepsilon_t^m = \rho_m \varepsilon_{t-1}^m + \eta_t^m - w^m \eta_{t-1}^m \). Finally, we also follow Smets and Wouters (2007) assuming that the spending shock is affected by the productivity innovation by \( \rho^GZ \) i.e \( \varepsilon_t^G = \rho^G \varepsilon_{t-1}^G + \rho^G \eta_t^Z + \eta_t^G \).

Aggregate all agents and varieties in the economy and imposing market clearing for all markets lead to the definition of aggregate labor \( L_t = \int_0^{N_t} l_{f,t} df \) and capital \( K_t = \int_0^{N_t} k_{f,t} df \), while aggregate vacancies also depends on the number of new entrants such that \( V_t = N_t v_t + N_{E,t} E_t + w_l L_t A C_t^W + \rho L_t A C_t^P \).

After (i) aggregating all agents and varieties in the economy, (ii) imposing market clearing on all markets and (iii) substituting the relevant demand functions, the resource constraint for the economy, also defined as the GDP through the demand approach reads as follows:

\[
Y_t^C = C_t + I_t + \Psi(v_t)K_{t-1} + g^Y \varepsilon_t^G + N_t f^V v_t + N_{E,t} E_t^E + w_l L_t A C_t^W + \rho L_t A C_t^P, \quad (34)
\]

which equals the sum of consumption, investment, the cost of using capital, vacancy costs and product creation expenses\(^{13}\). Using the optimal demand (Eq.2) and equalize supply with demand we obtain,

\[
\rho_t Y_t = Y_t^C. \quad (35)
\]

Concerning prices rearranging the translog expenditure function (Eq.1) and imposing symmetry among producers, the relative price \( \rho_t = \frac{P_t}{P^f} \) emerges,

\(^{13}\)They also include adjustment costs on wages and prices. However, at a first order condition these two rigidities didn’t affect the GDP.
\[
\rho_t = \exp \left( -\frac{\tilde{N} - N_t}{2 \sigma P \tilde{N} N_t} \right). 
\]

Using this price index we deduct the welfare-based inflation \( \pi_t^C \),

\[
\frac{\rho_t}{\rho_{t-1}} = \frac{\pi_t}{\pi_t^C}.
\]

3 Estimation

We estimate the model using Bayesian techniques as in Smets and Wouters (2007, 2003). In this section, we present the data sources and transformations, before turning to our choice of prior and to the posterior distributions of the model parameter.

3.1 Data

The model is estimated with Bayesian methods on US quarterly data from 1992Q3 to 2016Q4. Our sample is rather short since samples on business establishments are discontinued and short as pointed by Lewis and Stevens (2015). The dataset includes output, consumption, investment, nominal interest rate, inflation, real wage, unemployment and net business formation. The first six data are the same as Smets and Wouters (2007). Output is measured by GDP, consumption by personal consumption expenditures and investment with fixed private investment which abstracts from changes in inventories. The nominal interest rate is defined by the federal fund rates expressed on a quarterly basis i.e. divided by 4. The inflation is defined as the first log-difference of the GDP deflator. Since our model allows for the extensive margin of employment, we use the unemployment rate rather than hours. Finally, net business formation is defined as the ratio between the number of establishment births to establishment deaths from the Bureau of Labor Statistics (BLS hereafter).

The goal of each data is to match with our theoretical counterpart. In the model, nominal variables are deflated using the welfare price index \( P_t^C \) which is unobserved\(^{14}\). Thus, we strip the variety effect i.e. the welfare price index by multiplying each real variable by \( P_t^C \) and dividing by the product price \( P_t \) which corresponds to the GDP price deflator. As in Lewis and Stevens (2015), for any real variable \( a_t \) in the model the data-consistent counterpart of

\(^{14}\)The consumer price index computed by the Bureau of Labor Statistics (BLS) doesn’t reflect the welfare product turnover represented by the translog expenditure function.
any real theoretical variables is given by \( a^R_t = \frac{a_t}{\rho_t} \) where \( \rho_t \) correspond to the price index defined in Eq.36.

With \( \hat{a}_t = 100 \log \left( \frac{a_t}{\bar{a}} \right) \) which denotes the log-deviation of \( a_t \) from its steady-state, the measurement equation reads as follows,

\[
Y_t = \left\{ \hat{Y}^R_{agg,t}, \hat{C}^R_t, \hat{I}^R_t, \hat{\bar{w}}^R_t, \bar{U}_t - \bar{\bar{U}}_t, R_t - \bar{R}_t, \bar{\pi}_t, NBF_t \right\}.
\] (38)

where the net business formation is defined as the difference between entry \( N^E_t \) and exit \( \delta^N (N_t + N^E_t) \). Then, in log deviation we have: \( NBF_t = (1 - \delta^N) \left( \hat{N}^E_t - \bar{N}_t \right) \).

Figure 1: Observable variables used in the estimation

We have chosen to focus on short run macroeconomic fluctuations and to neglect long run effects involved by trends\(^{15}\). Thus, series are transformed in order to map non-stationary data to a stationary-model. All nominal variables are deflated with the GDP deflator and aggregate real variables are expressed in per capita terms by dividing by the Civilian Non-institutional Population over 16. The series for GDP, consumption, investment, wages and net business formation are taking in log and detrended using an HP filter \(^{16}\). The rest of series namely unemployment, interest rate and inflation are demeaned by subtracting their respective sample averages. The transformed series used in the estimation are then plotted in Fig.1.

\(^{15}\)Lewis and Stevens (2015) and Poutineau and Vermandel (2015) use the same approach for a similar fit exercise on firms’ entry.

\(^{16}\)As in Hodrick and Prescott (1997) we choose a smoothing value for the filter to 1600 due to quarterly data.
3.2 Calibration and prior distribution of parameters

It is standard practice to estimate certain parameters while keeping other fixed at their calibrated values (Smets and Wouters (2003)). In particular, there are a number of parameters which are well identified on the basis of long-run averages and great ratios but which little information is contained in the HP-filtered data we use in the estimation. Then our calibration is summarized in Tab. 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total labor separation</td>
<td>$\rho = .1$</td>
<td>Spending to GDP ratio</td>
<td>$g^Y = 0.21$</td>
</tr>
<tr>
<td>Labor share</td>
<td>$\alpha = 0.67$</td>
<td>Exit rate Producers</td>
<td>$\delta^N = 0.025$</td>
</tr>
<tr>
<td>Discount factor</td>
<td>$\beta = .9925$</td>
<td>Capital depreciation rate</td>
<td>$\delta^K = 0.025$</td>
</tr>
<tr>
<td>Probability to find a job</td>
<td>$f = .95$</td>
<td>Probability to find an employee</td>
<td>$q = .75$</td>
</tr>
<tr>
<td>Matching elasticity vancancies</td>
<td>$\zeta = .6$</td>
<td>Share of potential producers</td>
<td>$\frac{N}{\tilde{N}} = 0.95$</td>
</tr>
<tr>
<td>Fixed barrier entry</td>
<td>$f^E = 0.4$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

We use standard values for all the parameters that are conventional in the business cycle literature. These include the share of labor in the Cobb-Douglas production function $\alpha = .67$, the discount factor $\beta = 0.9925$, the steady-state of government expenditures in output $g^Y = 0.21$. As in Bilbiie et al. (2012) the exogenous exit rate $\delta^N$ has the same value than the capital depreciation rate with $\delta^N = \delta^K = .025$.

For the labor market parameters, we set the elasticity of matches to unemployment, $\zeta$ to 0.6 in the range of estimation provides by Petrongolo and Pissarides (2001). The total job separation rate $1 - \rho$ (with $1 - \rho = 1 - (1 - \delta^N) (1 - \delta^L)$) due to product turnover $\delta^N$ and exogenous separation by firms $\delta^L$ is calibrated to 0.1 matching the average job duration of two and a half years in the US\textsuperscript{17}. The steady-state job finding probability $f$ is set to 0.95 as in Shimer (2005). Then, we have a steady-state value of unemployment equal to 9.52%. This is higher than the means of unemployment rate over the period which is 6% (1992Q3 to 2016Q4). As emphasized by Krause and Lubik (2007), the assumption of higher steady-state value for unemployment than reality is made to account for potential participants in the matching market such as discouraged workers and workers loosely attached to the labor force. Finally, the steady-state job filling rate $q$ is set to 0.75 as in Den Haan et al. (2000).

\textsuperscript{17}Since the exogenous exit rate for firms is $\delta^N = .025$, we have a steady-state value for exogenous separation within firms of 7.69% using $\delta^L = \frac{\delta^N}{1 + \delta^N}$. 


barrier upon entry \( f^E \). For the first, we follow Lewis and Stevens (2015) with a share of potential producers equals to 95%. For the last, as shown in the appendix, for any positive value of \( f^E > 0 \), this barrier only influence the steady-state number of goods available in the economy and the price-elasticity of the spending share on an individual good \( \sigma \) without affecting GDP ratios. Then, we choose the same calibration for the last parameter than Lewis and Stevens (2015) with \( \sigma = 0.61 \) implying a value for the fixed barrier of \( f^E = 0.4 \).

Concerning prior, for the majority of new Keynesian models’ parameters i.e. parameters for households’ utility (\( h^C, \sigma^C \)), price indexation for prices and wages (\( \lambda^P, \lambda^W \)), the Taylor rule (\( \rho^R, \phi^\pi, \phi^\Delta Y \)), rigidity on investment and capital adjustment (\( \varphi^I, \Psi \)) we use the prior distributions chosen by Smets and Wouters (2007). Regarding the endogenous market structure i.e for adjustment costs on extensive investment (\( \varphi^E \)) and the demand elasticity (\( \theta \)), we use the prior of Lewis and Stevens (2015). For Rotemberg adjustment cost on wages and prices (\( \kappa^P \) and \( \kappa^W \)), we choose a reasonable loose prior using a gamma distribution with mean 50 and standard deviation of 7.5.

For the labor market, two important parameters can be estimated namely the negotiation power for workers \( \eta \) and the steady-state value of unemployment \( \tilde{b} \). As in Gertler et al. (2008), we define the steady-state value of unemployment as a fraction of the contribution of the worker to the job:

\[
\tilde{b} = \frac{b}{\alpha \frac{mcY}{L}},
\]

where \( \alpha \frac{mcY}{L} \) is the marginal product of labor\(^{18}\). Thus, we use the same prior of Gertler et al. (2008) for these two parameters.

\[\text{3.3 Posterior estimates}\]

In this subsection, we discuss our posterior estimates and we contrast them, when possible, with the existing empirical evidence from the existing literature. The Tab.2 reports our baseline estimation which summarizes means and the 5th and 95th percentiles of the posterior

\(^{18}\)However, we don’t have the same cost structure as Gertler et al. (2008). In their paper, they use hiring costs more than vacancy cost.s As a consequence, in our model firms value of the job (Eq.18) does not depend on the discounted savings on adjustment costs. Thus, in Gertler et al. (2008) we have \( \tilde{b} = \frac{b}{\alpha \frac{mcY}{L} + \beta \frac{x^V}{x^V(x)^2}} \) with \( \beta x^V (x)^2 \) the steady-state value of saving on adjustment costs and \( x \) the steady-state value of hiring.
price contract duration as this requires a constant population of price setters. stickiness with NKPC with Walk Metropolis Hastings algorithm using dynare, see. This higher value suggests that wages are more sensitive to movements in the shadow value (the estimation of Gertler et al. 2008). In a Calvo analog, we have a higher value for the negotiation power (η = 0.78) than the range provided by the literature which is typically between 0.5 and 0.7. This higher value suggests that wages are more sensitive to movements in the shadow value.

Note: B, beta; G, gamma; N, normal; P1, mean and P2 standard deviation for all distributions.

The estimated Taylor rule parameters are consistent with existing evidence with a substantial interest rate smoothing (ρ = 0.788), a response coefficient on inflation that satisfies the Taylor Principle (ρφ = 2.24) and the influence response of the growth rate (φΔY = 0.15). Concerning wages and prices rigidities, they are in line with the estimation of Smets and Wouters (2007) with higher rigidity for wages than price. Stickiness parameters ϕP and ϕW are respectively estimated to 55.59 and 36.24. In a Calvo analog, the rotemberg price rigidity is associated with a probability of 0.59 to reset prices which correspond to an average contract duration of approximately two and a half quarters for prices.

The negotiation power of households η and the unemployment value are in line with the estimation of Gertler et al. (2008). We have a higher value for the negotiation power (η = 0.78) than the range provided by the literature which is typically between 0.5 and 0.7. This higher value suggests that wages are more sensitive to movements in the shadow value.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Prior (P1, P2)</th>
<th>Mean</th>
<th>5%, 95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>σφC</td>
<td>Consumption utility</td>
<td>N(1.50, 0.375)</td>
<td>1.98</td>
<td>[1.42, 2.55]</td>
</tr>
<tr>
<td>hφC</td>
<td>External habit</td>
<td>B(0.70, 0.10)</td>
<td>0.77</td>
<td>[0.67, 0.87]</td>
</tr>
<tr>
<td>ϕI</td>
<td>Investment adjustment cost</td>
<td>N(4.00, 1.50)</td>
<td>6.61</td>
<td>[4.70, 8.47]</td>
</tr>
<tr>
<td>ψ</td>
<td>Capacity utilisation cost</td>
<td>B(0.50, 0.15)</td>
<td>0.86</td>
<td>[0.78, 0.95]</td>
</tr>
<tr>
<td>ϕE</td>
<td>Entry adjustment cost</td>
<td>N(4.00, 1.50)</td>
<td>4.07</td>
<td>[2.31, 5.78]</td>
</tr>
<tr>
<td>˜θ</td>
<td>Demand elasticity</td>
<td>N(4.00, 1.50)</td>
<td>11.34</td>
<td>[9.93, 12.79]</td>
</tr>
<tr>
<td>κP</td>
<td>Price rigidity</td>
<td>G(50.0, 7.50)</td>
<td>36.24</td>
<td>[28.8, 43.49]</td>
</tr>
<tr>
<td>λP</td>
<td>Indexation price</td>
<td>B(0.50, 0.15)</td>
<td>0.17</td>
<td>[0.06, 0.27]</td>
</tr>
<tr>
<td>κW</td>
<td>Wage rigidity</td>
<td>G(50.0, 7.50)</td>
<td>55.59</td>
<td>[42.63, 68.49]</td>
</tr>
<tr>
<td>λW</td>
<td>Indexation wage</td>
<td>B(0.50, 0.15)</td>
<td>0.51</td>
<td>[0.27, 0.74]</td>
</tr>
<tr>
<td>η</td>
<td>Negotiation power household</td>
<td>B(0.50, 0.1)</td>
<td>0.78</td>
<td>[0.67, 0.89]</td>
</tr>
<tr>
<td>b</td>
<td>Unemployment value</td>
<td>B(0.50, 0.1)</td>
<td>0.83</td>
<td>[0.73, 0.94]</td>
</tr>
<tr>
<td>ρR</td>
<td>Interest rate smoothing</td>
<td>B(0.70, 0.10)</td>
<td>0.85</td>
<td>[0.82, 0.88]</td>
</tr>
<tr>
<td>ϕP</td>
<td>Policy inflation</td>
<td>N(2.025)</td>
<td>2.10</td>
<td>[1.74, 2.46]</td>
</tr>
<tr>
<td>ϕΔY</td>
<td>Policy lagged output</td>
<td>G(0.50, 0.25)</td>
<td>0.38</td>
<td>[0.22, 0.54]</td>
</tr>
</tbody>
</table>

The posterior moments are computed using 600,000 draws from the distribution simulated by the Random Walk Metropolis Hastings algorithm using dynare, see Adjemian et al. (2011) for further information.

In the Calvo analog of the model, we have the coefficient \(\frac{1-\bar{\xi}^P}{1-\bar{\xi}^P}\) on the markup gap in the NKPC with \(\xi^P\) the Calvo probability of resetting price. With Rotemberg adjustment costs, this coefficient corresponds to \(\frac{\bar{\theta}^{-1}}{\bar{\xi}^P}\). Using this relation for calculate the Calvo analog, we have the duration of price stickiness with \(\frac{1}{1-\bar{\xi}^P}\). However, as emphasized by Lewis and Stevens (2015) we cannot compute an average price contract duration as this requires a constant population of price setters.
of labor, and thus less sensitive to employment. Together with wage rigidity they confirm that wages are sensitive to movement in productivity for low and medium frequencies and dependant on the past value for high frequency.

Turning to rigidity parameters on intensive investment, our results are in line with business cycle models without entry (Christiano et al. (2005), Smets and Wouters (2007)) with $\psi^I = 6.61$ and $\psi = 0.86$. For the extensive part, we find a reasonable value for the rigidity on entry ($\varphi^E = 4.07$).

We find a higher value for the demand elasticity with $\theta = 11.34$. That delivers a steady-state markup of approximatively 9.6% and a ratio of entry to GDP of $6.79\% \left( \frac{N_{E}^E \varphi^E}{\psi} \right)$. For the first value, this in line with the literature without entry (Smets and Wouters (2007) among others use a value of $\theta = 10$). However, this is almost twice than Lewis and Stevens (2015) due to our cost structure. Since, our entry cost is only dependent on a fixed component and vacancy posting, a higher value for $\theta$ is essential to catch the volatility of entry. For the second value, the proportion of entry cost to GDP is in the bottom range of empirical estimates. Barseghyan and DiCecio (2011) estimate two different ratios using the ratio of entry to operating costs and the evolution of firms’ productivity over time. The first gives a ratio between entry costs and output per worker of $20.8\%$. The second is lower with $12.15\%$. The World Bank reports that legal fees to register a business amount to $1.4\%$ of per capita income in the US for the year 2011.

Finally, steady-state value of employment and job creation accounted by new entrants are in line with empirical evidence. The share of overall employment due to startups is equal to $2.3\% \left( \frac{N_{E}^E \varphi^E}{\psi} \right)$ as in Haltiwanger et al. (2010)$^{21}$. The average job creation accounted to new firms $\left( \frac{N_{E}^E \varphi^E}{\psi} \right)$ is equal to $17.31\%$ in line with Jaimovich and Floetotto (2008)$^{22}$.

$^{21}$They find that business startups account for roughly 3 percent of U.S. total employment in any given year between 1992 and 2005.

$^{22}$Using employment data at the establishment level, they estimate that the average fraction of quarterly job-gain that can be explained by the opening of establishments is about 20 percent.
Table 3: Prior and posterior distributions of shock processes

<table>
<thead>
<tr>
<th>Shocks AR(T), MA(T)</th>
<th>Symbol</th>
<th>Description</th>
<th>Prior (P1,P2)</th>
<th>Mean [5%, 95%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho^Z$</td>
<td>AR - Productivity</td>
<td>$B(0.50, 0.20)$</td>
<td>0.85 [0.80, 0.90]</td>
<td></td>
</tr>
<tr>
<td>$\rho^C$</td>
<td>AR - Risk premium</td>
<td>$B(0.50, 0.20)$</td>
<td>0.43 [0.25, 0.61]</td>
<td></td>
</tr>
<tr>
<td>$\rho^G$</td>
<td>AR - Spending</td>
<td>$B(0.50, 0.20)$</td>
<td>0.86 [0.80, 0.93]</td>
<td></td>
</tr>
<tr>
<td>$\rho^I$</td>
<td>AR - Investment</td>
<td>$B(0.50, 0.20)$</td>
<td>0.65 [0.56, 0.74]</td>
<td></td>
</tr>
<tr>
<td>$\rho^E$</td>
<td>AR - Entry</td>
<td>$B(0.50, 0.20)$</td>
<td>0.13 [0.04, 0.21]</td>
<td></td>
</tr>
<tr>
<td>$\rho^R$</td>
<td>AR - Monetary policy</td>
<td>$B(0.50, 0.20)$</td>
<td>0.56 [0.46, 0.65]</td>
<td></td>
</tr>
<tr>
<td>$\rho^P$</td>
<td>AR - Price markup</td>
<td>$B(0.50, 0.20)$</td>
<td>0.69 [0.58, 0.81]</td>
<td></td>
</tr>
<tr>
<td>$\rho^L$</td>
<td>AR - Firm’s bargaining</td>
<td>$B(0.50, 0.20)$</td>
<td>0.54 [0.32, 0.76]</td>
<td></td>
</tr>
<tr>
<td>$\rho^{GZ}$</td>
<td>AR - Productivity-spending</td>
<td>$B(0.50, 0.20)$</td>
<td>0.61 [0.41, 0.85]</td>
<td></td>
</tr>
<tr>
<td>$u^P$</td>
<td>MA - Price</td>
<td>$B(0.50, 0.20)$</td>
<td>0.41 [0.22, 0.61]</td>
<td></td>
</tr>
<tr>
<td>$u^L$</td>
<td>MA - Firm’s bargaining</td>
<td>$B(0.50, 0.20)$</td>
<td>0.48 [0.29, 0.67]</td>
<td></td>
</tr>
</tbody>
</table>

Innovations

$\sigma^Z$ Productivity          $\mathcal{IG}(0.1, 2)$  0.50 [0.44, 0.56] 
$\sigma^C$ Risk premium          $\mathcal{IG}(0.1, 2)$  4.29 [2.56, 5.95] 
$\sigma^G$ Spending              $\mathcal{IG}(0.1, 2)$  2.28 [1.97, 2.59] 
$\sigma^I$ Investment            $\mathcal{IG}(0.1, 2)$  2.07 [1.69, 2.43] 
$\sigma^R$ Monetary policy       $\mathcal{IG}(0.1, 2)$  0.09 [0.08, 0.10] 
$\sigma^E$ Entry                 $\mathcal{IG}(0.1, 2)$  0.07 [0.06, 0.08] 
$\sigma^P$ Price markup          $\mathcal{IG}(0.1, 2)$  0.51 [0.37, 0.65] 
$\sigma^L$ Bargaining            $\mathcal{IG}(0.1, 2)$  0.10 [0.04, 0.15] 

Note: $B$, beta; $\mathcal{IG}$, inverse gamma; $P1$, mean and $P2$ standard deviation for all distributions.

4 Labor and goods market dynamics

In this section, we analyse the dynamics of employment in the presence of endogenous entry as predicted by the model. First, we study the employment’s behavior in this extensive and intensive view using bayesian impulse response function. Second, we examine the source of employment dynamics based on a variance and historical decomposition. Finally, we look at the dynamics of markup in our setup.

4.1 Impulse response analysis

All shocks displayed in this subsection generate a rise in output to facilitate comparison between them.

Supply shocks  Fig.2 focuses on the three supply shocks namely TFP, bargaining power and entry costs shocks.
Figure 2: Bayesian IRF : supply shocks

Notes: The x-axis corresponds to quarters and y-axis to the percentage standard deviation from steady-state. Median IRF and 5th and 95th percentiles are based on 300 random draws from the posterior distribution.

A positive technology shock leads to a short-run decline in employment in line with Galí et al. (2012). This shock lower firms’ real marginal cost and due to price stickiness, rise markups. Since the markup and demand rises, dividend increases (Eq. 8). The fall in the entry cost (measured as the value of adding a worker time the incumbent labor force i.e. \( v_t^E \)) combines with the increase in dividends boost entry on goods market (Eq. 9). Since entry rises, employment picks up in its extensive part and compensate the short decline due to the reduction in the intensive margin.

For the negative bargaining power shock i.e. a decrease on the worker’s bargaining power, employment rises in its extensive and intensive margin. The immediate drop in wage creates an incentive for firms to postulate more vacancies (Eq. 18) by boosting the marginal value of adding a worker. Lower wage reduces marginal costs and increase profits. Entry occurs in good markets since the market value rises more than the entry cost.

Entry costs shock directly raises entry via Eq. 9. As a consequence, during first periods there is a reallocation between intensive and extensive activity in favor of the last. The rise...
in extensive margin activity leads to a rise in demand via the definition of GDP (Eq.34) and simulates marginal cost. Then, the fall in the desired markup through the competition effect combined with inflation leads the markup to fall.

**Demand shocks**  Fig.3 depicts the Bayesian impulse responses of the three demand shocks that refer to the government spending, time-preference and investment shocks. All of them, induce variation in the same direction for all variables which simplify their descriptions.

![Bayesian IRF: demand shocks](image)

*Notes:* The x-axis corresponds to quarters and y-axis to the percentage standard deviation from steady-state. Median IRF and 5th and 95th percentiles are based on 300 random draws from the posterior distribution.

The increase in demand raises the marginal production cost conducing to inflation and a reduction of markup. This type of shock leads to a strong crowding out effect at the extensive margin. The monetary tightening i.e. the rise in interest rate in reaction to the demand shock leads to lower firm value through Eq.8. Combined with the increase in the marginal value of adding a worker to satisfy the demand, entry falls as describes by Eq.9.
First, despite the drop in the number of goods, the employment is stimulated via the job creation condition for incumbent. Second, new entrants need to post more vacancies to have the same workforce than incumbent, which support employment.

**Monetary and Price markup shock**  Fig. 4 shows the Bayesian impulse response for monetary policy and price markup shock.

![Bayesian IRF: Monetary and price markup shocks](image)

Notes: The x-axis corresponds to quarters and y-axis to the percentage standard deviation from steady-state. Median IRF and 5th and 95th percentiles are based on 300 random draws from the posterior distribution.

As referred in the literature, an expansionary monetary policy generates two opposing effects on profit. First, the decline of interest rate increases marginal cost and with price stickiness the markup decreases. Together, they depress profits. Second, this shock has expansionary effects on aggregate demand and raises profits. As in Bilbiie et al. (2007) and Lewis and Stevens (2015) the first effect dominates. Despite the positive effect on profits, entry falls at the difference of Bergin and Corsetti (2008) during first periods. Even if the decline in interest rate entails a decrease in the expected return on shares to eliminate arbitrage across assets, the difference between the firm value and entry cost is negative. This negative gap is due to the entry cost (Eq. 9) which is not directly dependant from marginal cost and increases with lower interest rate. Even with the initial drop in the extensive margin, total employment rises.
An expansionary price markup shock leads to reduce prices via the NKPC (Eq. 22). The reduction in price leads to more demand from households and boosts the GDP. Since the markup of firms decreases, their dividends follow the same path. As a consequence, firms become less attractive and combine with greater entry cost the number of goods produced decreases. As for demand shocks, employment drops in its extensive part but the overall effect remains positive due to the intensive part.

To sum up, the markup is procyclical in the case of a technology shock and for the bargaining shock as in Lewis and Stevens (2015). Considering employment dynamics, even in the case where entry is countercyclical (demand shocks), total employment rises. Thus, when entry is countercyclical the incumbent labor force increases sufficiently to rises the total employment. In the case of procyclical entry (supply shocks and price markup shock), the total employment rises through these two components i.e. the number of active firms times the labor force of incumbent (recall that $L_t = N_t l_t$).

### 4.2 Unemployment dynamics

Since our goal is to study unemployment dynamics with endogenous entry, it is important to test if the estimated model account for these dynamics.

**Empirical fit** We start by conducting an external validation exercise to assess the reliability of the model in fitting time series that were not used as inputs in the estimation. Such an exercise is of particular interest since it addresses the critique that DSGE models can do a good job at fitting the data in the sample, but have poor performance otherwise. Two important series related to unemployment dynamics are not taking into account in the estimation: vacancies ($V_t$) and the finding probability ($f_t$).

**Figure 5**: External validation: model simulated (smoothed estimates) versus actual data

**Notes**: The solid lines plot model simulated series and the dashed line actual data.
We contrast in Fig. 5 the model’s simulated time series for these two measures against their data counterparts. These two data series are constructed respectively using the methodology of Barnichon (2010) and of Shimer (2007). Then, in order to establish a comparison we took this series in log and detrended using an HP filter. Both simulated series are closer from their data counterparts with pointing to a large and persistent decline around the financial crisis. The fit of vacancy is due to the capacity of the model to reproduce the correlation between vacancies and unemployment i.e. the Beveridge curve. In the data we have a negative correlation of $-0.92$ against $-0.89$ in the model.

Finally, concerning the empirical fit of the model, the Table 4 reports the standard deviations of key variables (normalized by the standard deviation of the output).

<table>
<thead>
<tr>
<th>Variable</th>
<th>$Y_t$</th>
<th>$w_t$</th>
<th>$\pi_t$</th>
<th>$U_t$</th>
<th>$V_t$</th>
<th>$\theta_t$</th>
<th>$N^p_t$</th>
<th>$f_t$</th>
<th>$I_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>1</td>
<td>0.93</td>
<td>0.20</td>
<td>10.41</td>
<td>10.94</td>
<td>20.95</td>
<td>5.91</td>
<td>10.30</td>
<td>4.55</td>
</tr>
<tr>
<td>Model</td>
<td>1</td>
<td>1.18</td>
<td>0.16</td>
<td>9.63</td>
<td>10.27</td>
<td>18.86</td>
<td>6.61</td>
<td>10.11</td>
<td>3.25</td>
</tr>
</tbody>
</table>

Notes: All standard deviation are normalized by the standard deviation of GDP ($Y_t$). Note that in order to establish comparisons, all of these series are expressed in log and detrended using an HP filter with smoothing parameter of 1600 (except for the inflation rate $\pi_t$).

The real wage is more volatile than the data (1.18 versus 0.93). However, one important improvement is that the unemployment and vacancies are closely from their data counterparts even with a more responsive wage. When wages are more responsive, unemployment and vacancy tend to be less volatile. This relation is highlighted by Krause and Lubik (2007) and Shimer (2005) where the rigidity for wages is central to fit the volatility of unemployment and vacancies. For the rest of the series, the model do a decent job to replicate the data with closer standard deviation for the inflation, net entry and investment.

Since the model has good performances to replicate the data, we now turn to the study of unemployment dynamics. Then, we perform a variance decomposition and a historical decomposition of the unemployment rate.

---

23Since the Job Openings and Labor Turnover Survey (JOLTS) measure of job openings starts from December 2000 and our period from 1984, I use a composite index based on “print” and “online” help wanted index as in Barnichon (2010).

24The approximation of Shimer (2007) gives the following definition of this probability: $f_t = 1 - \frac{u_{t+1} - u^S_{t+1}}{u_t}$, where $u^S_{t+1}$ correspond to unemployed workers less than 5 weeks and $u_{t+1}$ the number of unemployed people. See appendix for more details.
Table 5: Variance decomposition for unemployment at different horizons

<table>
<thead>
<tr>
<th></th>
<th>Technology</th>
<th>Entry Demand</th>
<th>Bargaining</th>
<th>Monetary</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>($\varepsilon_Z^t$)</td>
<td>($\varepsilon_N^t$)</td>
<td>($\varepsilon_L^t, \varepsilon_C^t$)</td>
<td>($\varepsilon_P^t$)</td>
</tr>
<tr>
<td>$Q1$</td>
<td>25.88</td>
<td>24.45</td>
<td>46.09</td>
<td>0.94</td>
</tr>
<tr>
<td>$Q2$</td>
<td>20.45</td>
<td>24.40</td>
<td>48.71</td>
<td>1.12</td>
</tr>
<tr>
<td>$Q4$</td>
<td>15.03</td>
<td>21.95</td>
<td>49.81</td>
<td>2.52</td>
</tr>
<tr>
<td>$Q10$</td>
<td>13.77</td>
<td>18.52</td>
<td>44.88</td>
<td>5.03</td>
</tr>
<tr>
<td>$\infty$</td>
<td>15.53</td>
<td>17.44</td>
<td>43.8</td>
<td>5.11</td>
</tr>
</tbody>
</table>

Note: Variance decomposition at different horizons where $Q$ corresponds to quarters.

Variance decomposition The Tab.5 reports the forecast error variance decomposition for unemployment at different horizons. As observed, demand shocks explain almost half of variations for unemployment for the short run ($Q1$ to $Q4$) as for the medium run ($Q10$) and the long run ($\infty$ corresponds to the unconditional variance decomposition). On impact ($Q1$) entry and technology shock contributes to the half of variation for unemployment. As a consequence, markup price, bargaining and monetary shock affects marginally the variance of the unemployment. Quarter after quarter, the sources of unemployment fluctuations become more mixed with a diminishing role for entry and technology and a more prominent role for markup, bargaining and monetary. Thus, at the unconditional horizon ($\infty$) the two supply shocks (entry and technology) explains 32.97% of unemployment variations while the three other 23.23% (against 3.58% at $Q1$).
Notes: Supply corresponds to the TFP shock ($\varepsilon^Z_t$), Entry ($\varepsilon^E_t$), Demand ($\varepsilon^B_t, \varepsilon^I_t, \varepsilon^G_t$ i.e. respectively preferences, investment and public spending), Monetary ($\varepsilon^R_t$), MUprice ($\varepsilon^{P_t}$) and MUwage ($\varepsilon^{L_t}$). The solid line depicts the unemployment rate demeaned by substrating his respective sample average as in Fig.1.

**Historical decomposition** We perform a historical decomposition of unemployment in the Fig.6. The solid line depicts the time path of unemployment from its sample mean (as used in the estimation), while bars depict the size of shocks in the corresponding point deviation. For both recessions (beginning of the new millennium and the financial crisis of 2007), the rise of the unemployment rate is mostly due to demand shock. However, during the financial crisis (2007 to 2009), the entry shock plays a major role to explain the rise of unemployment as opposed to the precedent recession. This is in line with the data where net business formation collapses as for unemployment during this period. Then, the deep negative effect on unemployment is driven by entry and demand shocks and marginally by monetary and supply shocks. The relative recovery of employment after 2009 is driven by monetary policy shocks (the zero lower bound) and supply shocks. Furthermore, the wage markup shock seems to alter the recovery after the financial turmoil while during other periods they act as an amplifier of unemployment’s decrease. As we have seen in the IRF
analysis (Fig.2) this shock impact the extensive margin of activity and labor force for incumbent in the same direction. Then, they have a deep impact on unemployment through these two components.

4.3 Cyclicality of the markup with search and matching frictions in the labor market

An interesting question is to study the impact of search and matching frictions for the markup’s behavior of firms. Two mechanisms differ from the traditional approach of DSGE models to studied markups. First, the marginal cost (Eq. 18) depends on the expectation of future hires and then affects the transmission of shocks to inflation. Second, we find a higher elasticity of substitutions for goods than Lewis and Stevens (2015), which implies a desired markup less sensitive to variation of the number of goods. Then, as Lewis and Stevens (2015) and Bilbiie et al. (2012) we study the unconditional cyclicality of the markup implied by the model.

At a first order approximation, the markup given by Eq.22 is:

\[
\hat{\mu}_t = \frac{-\hat{\mu}_t}{\hat{\mu}_t} \left( \frac{\hat{\mu}_t + \hat{\mu}_t}{\hat{\mu}_t} \right) + \hat{\mu}_t \left( 1 - \delta^N \right) \left( \pi_t + \lambda^P \pi_t - \pi_t + \lambda^P \pi_t - \pi_t \right).
\]  

(40)

The first line corresponds to the desired markup which depends on the number of goods due to translog preferences and to exogenous markup shock \( \hat{\mu}_t \). The second line corresponds to the sticky-price component of markup with \( \kappa^P \) the Rotemberg cost. In order to see how component affects the cyclicality of the markup we decompose them into three components. The first correspond to the baseline model \( \mu_t \). We note by \( \mu_{tWE} \) the markup component without endogenous entry by assuming a large entry cost \( \varphi^E \to \infty \). Finally, we note by \( \mu_{tSP} \) the sticky-price component which abstracts from entry and exogenous markup shock i.e the second line of Eq.40.

Thus, the Fig.7 displays the means of cross-correlation between the different type of markup and GDP at different leads and lags (i.e. \( corr \left( \mu_t, Y_{t+\tau} \right) \), \( corr \left( \mu_{tWE}, Y_{t+\tau} \right) \) and \( corr \left( \mu_{tSP}, Y_{t+\tau} \right) \) for \( \tau = -5, ..., 0, ..., 5 \).
The markup obtained in the estimation $\mu_t$ is countercyclical at all leads and lags. If we drop firm entry dynamics, the sign of cyclicality is maintained and slight slower than the baseline. This slight difference is due to the high degree of substitutions between goods which implied a markup less sensitive to entry. The sticky price component remains countercyclical since they imply a difference between the desired prices for firms and the actual price due to price adjustment costs.

Then, the entry in our model amplifies the markup cyclicality but with an order of magnitude less important than Lewis and Stevens (2015). Furthermore, as pointed by Krause and Lubik (2007), the marginal cost induces by search and matching frictions in the labor market is more pro-cyclical than Walrasian labor market. This difference in the form of the marginal cost explains our countercyclical markup even if we drop the entry and price-markup shock.

5 Sensitivity analysis

In this section, we studied alternative hypothesis to test the robustness of our results.

5.1 Wage rigidity

First, we examine the role of wage rigidity in our model by comparing the baseline model with an alternative model in which the Rotemberg wage parameter ($\kappa^W$) is set to 0. Then, the bargaining power in Eq.29 is made endogenous only due to the presence of the exogenous bargaining shock ($\varepsilon^N_t$). The Tab.6 presents the results where estimated parameters which differs from the baseline model are in bold. Concerning parameters the bargaining power decreases (0.78 to 0.65) and the unemployment value increases (0.83 to 0.953). This is consistent with the results of Gertler et al. (2008) i.e. when wage rigidity is absent the
unemployment value tends is close to unity such as the labor supply is very elastic and the bargaining power is less important. A smaller value of $\eta$ make wages less responsive to shocks and thus the more sensitive is employment. We also note that the preference shock tends to be more persistent ($\rho^C = 0.43$ in the baseline against 0.54) and less volatile. In the same way, the bargaining power shock is half as much as the baseline. The volatility drop for those shocks is in line with the effect of the volatility for wage to employment.

The model comparison in Tab. 6 reports the log-marginal density and the posterior model probability i.e. which model is preferred by comparing log-marginal density. In this case, the baseline model is clearly preferred to the model without wage rigidity.

<table>
<thead>
<tr>
<th>Table 6: Comparison: Flexible Wages vs Baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Symbol Description</strong></td>
</tr>
<tr>
<td><strong>STRUCTURAL PARAMETERS</strong></td>
</tr>
<tr>
<td>$\sigma^C$</td>
</tr>
<tr>
<td>$h^C$</td>
</tr>
<tr>
<td>$\phi^I$</td>
</tr>
<tr>
<td>$\psi$</td>
</tr>
<tr>
<td>$\phi^E$</td>
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<tr>
<td>$\theta$</td>
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<tr>
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<tr>
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<tr>
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<tr>
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<tr>
<td>$\phi^\pi$</td>
</tr>
<tr>
<td>$\phi^{\Delta Y}$</td>
</tr>
<tr>
<td>$\phi^{AR(1), MA(1)}$</td>
</tr>
<tr>
<td>$\phi^{AR - Risk premium}$</td>
</tr>
<tr>
<td>$\phi^{AR - Spending}$</td>
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<tr>
<td>$\phi^{AR - Investment}$</td>
</tr>
<tr>
<td>$\phi^{AR - Entry}$</td>
</tr>
<tr>
<td>$\phi^{AR - Monetary policy}$</td>
</tr>
<tr>
<td>$\phi^{AR - Price markup}$</td>
</tr>
<tr>
<td>$\phi^{AR - Firm’s bargaining}$</td>
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<td>$\phi^{AR - Productivity-spending}$</td>
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<td>$u^L$</td>
</tr>
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<td>$u^{L_{MA - Firm’s bargaining}}$</td>
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<tr>
<td><strong>INNOVATIONS</strong></td>
</tr>
<tr>
<td>$\sigma^Z$</td>
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<td><strong>MODEL COMPARISON</strong></td>
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<td>Log marginal Data Density</td>
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<tr>
<td>Prior probability</td>
</tr>
<tr>
<td>Posterior model probability</td>
</tr>
</tbody>
</table>
5.2 Entry and translog preferences

Second, in the model, the fixed cost for entry is measured as a combination of an exogenous shock weighted by $f^E$ and a cost for posting vacancies (Eq. 24). Since the number of vacancies depends on the labor used by incumbent firms $l_t$ and the marginal value of adding a worker, this cost introduces a direct link between the labor and goods market. We evaluate this link by considering $f^E_t = f^E$ without vacancy posting. The result is presented in the third column of the Tab. 7 as Fixed entry. Any of the parameter estimates change significantly with this other specification.

Another point is to evaluate the role of the competition effect in the model. This effect plays a key role in the analysis of markup since they induce a counter-cyclical desired markup in the case of increasing entry. However, this effect can also drop the incentive for incumbent firms to increase their labor force (Eq. 18). This latter impact can explain the relatively higher demand elasticity. To measure the importance of the competition effect, we consider a Constant Elasticity of Substitution (CES hereafter) rather than a translog function. The fourth column of Tab. 7 as CES shows the result of the estimation. Dropping the competition effect results in a higher degree of price Rotemberg adjustment cost. Since the desired markup in Eq. 22 becomes independent of entry, higher rigidity for price is needed to account for inflation dynamics. In the same way, the degree of price indexation increase. Rigidity for the intensive $\varphi^I$ and extensive $\varphi^E$ margin becomes higher in line with the estimation of Poutineau and Vermandel (2015) which use CES preferences.

Comparing the three models, our baseline specification is preferred to the rest of specification with a posterior model probability of 0.9836 for the baseline model.
6 Conclusion

In this paper, we have developed and estimated a DSGE model with unemployment and endogenous entry. Using Bayesian econometrics, we have found evidence of the key role of the net business formation as an amplifying mechanism for employment dynamics. In particular, our model reveals that even when entry is counter-cyclical, the employment level for the economy is procyclical. Using search and matching frictions more than Walrasian labor market conduces to counter-cyclical markup even in the absence of entry due to the
strong procyclical aspect of marginal cost in our setup.

In the paper, firms enter the market after paying sunk entry cost and cost for posting vacancy. For future research, it would be interesting to incorporate capital and wages as a determinant of entry. This new setting have two potential effect. First, they can reconcile our paper with regulation in the goods and labor market as in Cacciatore and Fiori (2016) since in our setup $f^E$ doesn’t alter steady-state ratio but only the number of active firms. Furthermore, this would be affected the elasticity of substitutions between goods which in our estimation is high compared to Lewis and Stevens (2015).
A Data sources

- Nominal GDP: Gross Domestic Product, Billions of Dollars, Seasonally Adjusted Annual Rate from the FRED database https://fred.stlouisfed.org/series/GDP.

- Inflation: defined as the log-difference of the Implicit Price Deflator, with Index 2012=100, Seasonally Adjusted from the FRED database https://fred.stlouisfed.org/series/GDPDEF, ($\pi_t$ in all chapters).

- Unemployment: Civilian Unemployment Rate, in Percent, Seasonally Adjusted from the Fred database https://fred.stlouisfed.org/series/UNRATE/, ($U_t$ in all chapters).

- Vacancies: data from Barnichon (2010) which combine job openings from the JOLTS data set, the Help-Wanted Online Advertisement Index published by the Conference Board, and the Help-Wanted Print Advertising Index that was discontinued in October 2008 and it was also constructed by the Conference Board. (https://sites.google.com/site/regisbarnichon/research, Composite Help-Wanted Index), ($V_t$ in all chapters).

- Job Finding Probability: we apply the methodology of Shimer (2007) by defining the probability to find a job for an unemployed worker as:

$$f_t = 1 - \frac{u_{t+1}^S - u_{t+1}^S}{u_t}$$

where $u_{t+1}^S$ correspond to unemployed workers less than 5 weeks (from the Bureau of Labor Statistic, BLS hereafter) and $u_{t+1}$ the number of unemployed people. We convert the obtained serie in a quarterly basis, ($f_t$ in all chapters).

- Labor market tightness: Ratio of vacancies to unemployment defined below.

- Interest rate: Effective Federal Funds rate from the FRED database https://fred.stlouisfed.org/series/FEDFUNDS, converted in a quarterly basis ($r_t$ in all chapters).

- Consumption: Personal Consumption Expenditures, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate from the FRED database https://fred.stlouisfed.org/series/PCE.
• Wage: the Nonfarm Business Sector: Compensation Per Hour from the FRED database, seasonally adjusted (https://fred.stlouisfed.org/series/COMPNFB) as a proxy for wage.


• New firms: Number of establishment births, Total private, Seasonally Adjusted from the BLS (Employment Business Dynamics)

• Establishment deaths: Number of establishment deaths, Total private, Seasonally Adjusted from the BLS (Employment Business Dynamics)

• Net Business Formation: define as the ratio between private sector establishment Births and private sector establishment deaths, seasonally adjusted (In thousands) from the BLS.

Data in nominal terms are converted using the GDP deflator. Furthermore, in order to create per capita series we divide them by Civilian Noninstitutional Population over 16 from the FRED database https://fred.stlouisfed.org/series/CNP16OV.

In this appendix, we describe the main features of the chapter I with the complete set of First Order Condition (FOC hereafter) and the analytic steady-state.

B Non-linear model

Here we present the complete non-linear model of chapter I.

B.1 Households

B.1.1 Translog preferences

Recall that we use the term “good”, “producer” and “firms” are used interchangeably. Each individual firms produce one differentiated intermediate good indexed by \( f \in [0, N_t] \) where \( N_t \) is the mass of producers. Firms’ output \( y_{f,t} \) are bundled into a final good \( Y_t^C \) which is sold at price \( P_t^C \) to households. We follow Feenstra (2003) assuming that the final consumption basket \( Y_t^C \) a translog form. Translog preferences are characterised by defining the unit expenditure function i.e. the price index \( P_t^C \) associated with the preference aggregator.
Denote by $P_{ft}$ the nominal price for the good $f \in [0, N_t]$, the unit expenditure function on the basket good $Y_t^C$ is:

$$\ln P_t^C = \frac{\bar{N} - N_t}{2\sigma \varepsilon^P_t NN_t} + \frac{1}{N_t} \int_{f \in N_t} \ln P_{ft} df + \frac{\sigma^P_t}{2N_t} \int_{f \in N_t} \int_{f' \in N_t} \ln P_{ft} (\ln P_{ft} - \ln P_{ft}) df df', \quad (41)$$

where $\sigma > 0$ scales the demand elasticity and $\varepsilon^P_t$ the exogenous price-markup shock. This expenditure function gives several properties (see Lewis and Stevens (2015) for a complete demonstration):

The optimal demand addressed to a firm $f$ is given by:

$$P_t^C Y_t^C = P_{ft} N_t y_{ft}, \quad (42)$$

The price elasticity of demand is:

$$\theta_{f,t} = 1 + N_t \sigma^P \varepsilon^P_t$$

and finally by imposing symmetry among producers (such as $P_{ft} = P_{ft}$) and applied exponentiel to the translog expenditure function, we obtained the price index,

$$\rho_{f,t} = \exp \left( -\frac{\bar{N} - N_t}{2\sigma \varepsilon^P_t NN_t} \right),$$

where $\rho_{f,t} = \frac{P_{ft}}{P_t}$. 

**B.1.2 Household budget constraint and optimal decisions**

As customary in the literature, family members perfectly insure each other against variation in labor income due to employment status, so that there is no ex-post heterogeneity across individuals (Andolfatto (1996); Merz (1995)). The problem faced by the representative household can be summarized as:
\[
\begin{align*}
\max_{C_t, B_t, x_t, N_t^E, K_t, I_t, \nu_t} & \quad E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \varepsilon_t^B \left( C_t - h^C C_{t-1} \right)^{1-\sigma^C} \right\} \\
\text{s.t.} & \quad w_t L_t + (1 - L_t) b + R_{t-1} \frac{B_{t-1}}{L_{t-1}} + r_t^K \nu_t K_{t-1} + (1 - \delta^N) \left( d_{t+1} + \varepsilon_t \right) \left( x_{t+1} + \left( 1 - AC_{t-1}^E \right) N_{t-1}^E \right) \\
& \quad = C_t + \varepsilon_t x_t + \frac{B_{t+1}}{L_{t+1}} + T_t + N_t^E \phi_t^E + \psi(\nu_t) K_{t-1} + I_t \\
\text{s.t.} & \quad N_t = (1 - \delta^N) \left( N_{t-1} + \left( 1 - AC_{t-1}^E \right) N_{t-1}^E \right) \\
\text{s.t.} & \quad K_t = (1 - \delta^K) K_{t-1} + \left( 1 - AC_{t-1}^I \right) I_t
\end{align*}
\]

(43)

where the first line correspond to the utility function of the household (4), the second line his budget constraint in real terms (5), the third line the law of motion of firms and finally the law of motion of capital (Eq.11). The First Order Condition (FOC hereafter) with respect to consumption leads to the marginal utility \( \lambda_t^C \):

\[
\lambda_t^C = \varepsilon_t^B \left( C_t - h^C C_{t-1} \right)^{-\sigma^C} - \mathbb{E}_t \beta h^C \left( C_{t+1} - h^C C_t \right)^{-\sigma^C} \quad (44)
\]

Combining it with the FOC with respect to \( B_t \) leads to the Euler condition on bonds,

\[
\frac{\lambda_t^C}{\beta \mathbb{E}_t \lambda_{t+1}^C} = \frac{R_t}{\mathbb{E}_t \pi_{t+1}^C},
\]

(45)

where \( \pi_{t+1}^C = \frac{p_{C_{t+1}}}{p_{t+1}} \) is the welfare-based inflation.

In the same way, we obtained the Euler condition on share using the FOC with respect to \( x_t \),

\[
e_t = \mathbb{E}_t (1 - \delta^N) \beta_t \left( d_{t+1} + \varepsilon_{t+1} \right),
\]

(46)

where \( \beta_t = \beta \mathbb{E}_t \frac{\lambda_t^C}{\lambda_{t+1}^C} \) is the stochastic discount factor of the representative household.

The free-entry condition is obtained using the FOC with respect to new entrants \( N_t^E \),

\[
\phi_t^E = \mathbb{E}_t \beta_t (1 - \delta^N) \left( d_{t+1} + \varepsilon_{t+1} \right) \left( 1 - \frac{\partial AC_{t+1}^E N_{t+1}^E}{\partial N_{t+1}^E} \right) \\
- \mathbb{E}_t \frac{2}{N_{t+1}^E} \beta_t^2 (1 - \delta^N) \left( d_{t+2} + \varepsilon_{t+2} \right) N_{t+1}^E \left( \frac{\partial AC_{t+1}^E}{\partial N_{t+1}^E} \right)
\]

(47)

and combine it with the firm value equation (Eq.47):

\[
\phi_t^E = \varepsilon_t \left( 1 - \frac{\partial AC_{t+1}^E N_{t+1}^E}{\partial N_{t+1}^E} \right) - \mathbb{E}_t \beta_t \varepsilon_{t+1} \frac{\partial AC_{t+1}^E}{\partial N_{t+1}^E} N_{t+1}^E
\]

(48)
as in the text (Eq.9). Using the shape of the failure probability \( AC_t^E = \frac{e^E}{2} \left( \frac{N_t^E}{N_t^{E-1}} - 1 \right)^2 \), we have \( \frac{\partial AC_t^E}{\partial N_t^E} = AC_t^E + \frac{N_t^E}{N_t^{E-1}} \varphi^E \left( \frac{N_t^E}{N_t^{E-1}} - 1 \right) \) and \( \frac{\partial AC_t^{E+1}}{\partial N_t^E} = -\frac{N_t^{E+1}}{N_t^E} \varphi^E \left( \frac{N_t^{E+1}}{N_t^E} - 1 \right) \). Then, the entry condition in its complete form reads:

\[
\phi_t^E = e_t \left( 1 - AC_t^E + \frac{N_t^E}{N_t^{E-1}} \varphi^E \left( \frac{N_t^E}{N_t^{E-1}} - 1 \right) \right) - \mathbb{E}_t \beta_t e_{t+1} \left( \frac{N_t^{E+1}}{N_t^E} \right)^2 \varphi^E \left( \frac{N_t^{E+1}}{N_t^E} - 1 \right).
\]

Turning to capital supply, the representative household choose the optimal amount of capital \( K_t \) such that:

\[
q_t^K = \mathbb{E}_t \beta_t \left( r_{t+1}^{Kt+1} - \psi(\nu_{t+1}) + q_{t+1}^{Kt} (1 - \delta^K) \right),
\]

where \( q_t^K \) denotes the shadow value of capital i.e. the Lagrange multiplier associated with the capital law of motion and normalized by the marginal utility of consumption\(^{25} \). This shadow value is defined by the FOC for investment \( I_t \):

\[
1 = q_t^K \frac{\partial (1 - AC_t^I)I_t}{\partial I_t} - \mathbb{E}_t q_{t+1}^K \beta_t \frac{\partial AC_t^{I+1}I_{t+1}}{\partial I_t}.
\]

Then in its complete form the shadow value is:

\[
1 = q_t^K \left( 1 - AC_t^I - \varepsilon_t^I \frac{I_t}{I_{t-1}} \varphi^I \left( \varepsilon_t^I \frac{I_t}{I_{t-1}} - 1 \right) \right) + \mathbb{E}_t q_{t+1}^K \beta_t \varepsilon_{t+1}^I \left( \frac{I_{t+1}}{I_t} \right)^2 \varphi^I \left( \frac{I_{t+1}}{I_t} - 1 \right).
\]

Finally, the optimal utilization rate for capital is defined as,

\[
\psi' (\nu_t) = r_t^K.
\]

Using the definition of the cost for changing the utilization rate: \( \psi(\nu_t) = \frac{1 - \psi}{\varphi} r_t^K \left( e^{\varphi (\nu_t - 1)} - 1 \right) \),

we can rewrite the optimal utilization rate as:

\[
r_t^K \left( e^{\varphi (\nu_t - 1)} \right) = r_t^K
\]

\(^{25}q_t^K \) corresponds to the Lagrange multiplier associated with the capital law of motion (Eq.11) normalized by the marginal utility of consumption \( \lambda_t^E \).
As demonstrated below, the labor market law of motion from the household perspective writes: \( L_t = (1 - \delta^N) (1 - \delta^L) L_{t-1} + f_t(1 - L_{t-1}) \). Include this equation into the problem faced by the representative household (Eq. 43) and optimize with respect to \( L_t \), we have:

\[
\mu_t^W = \frac{W_t}{P^C_t} - b + \mathbb{E}_t \beta_t \mu_{t+1} \left( (1 - \delta^N) (1 - \delta^L) - f_{t+1} \right) .
\]

(52)

which correspond to the surplus from employment in the production sector for the representative household.

Note that in the case where new entrants doesn’t post vacancies i.e when \( \phi_t^E = f^E \), the surplus according by households for employment in any firms is given by :

\[
\mu_t^W = \frac{W_t}{P^C_t} - b + \mathbb{E}_t \beta_t \mu_{t+1} \left( \frac{N_{t+1}}{N_t} (1 - \delta^L) - f_{t+1} \right) .
\]

B.2 Producers

B.2.1 Cost minimization

Here we consider the maximisation problem solved by producer \( f \in [0, N_t] \). In a first step, the firm chooses the optimal level of employment \( l_{f,t} \), the number of vacancies to be posted \( v_{f,t} \) and the optimal amount of capital \( (k_{f,t}^U) \) subject to the production function \( () \) and the employment level \( () \). Formally, they solve:

\[
\begin{align*}
\max_{k_{f,t}^U, l_{f,t}, v_{f,t}} & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_t (1 - \delta^N)^t \left\{ \frac{P_{f,t}}{P^C_t} y_{f,t} - (1 + AC_{f,t}^W) \frac{W_{f,t}}{P^C_t} l_{f,t} - r_l^F k_{f,t}^U - f^V v_{f,t} \right\} \\
\text{s.t} & \quad l_{f,t} = (1 - \delta^L)l_{f,t-1} + q_t v_{f,t} \\
\text{s.t} & \quad y_{f,t} = \varepsilon_t^Z (l_{f,t})^\alpha (k_{f,t}^U)^{1-\alpha}
\end{align*}
\]

(53)

The FOC with respect to the workforce is given by:

\[
\mu_{f,t}^L = \alpha \frac{mc_{f,t} y_{f,t}}{l_{f,t}} - (1 + AC_{f,t}^W) \frac{W_{f,t}}{P^C_t} + \mathbb{E}_t \beta_t (1 - \delta^L) (1 - \delta^N) \mu_{f,t+1}^L ,
\]

(54)

where \( mc_{f,t} \) is the real marginal cost for the representative firms and \( \mu_{f,t}^L \) the marginal utility to get a new worker (mathematically speaking they correspond respectively to the
Lagrange multiplier associated with the production function \( y_{f,t} \) and the labor market law of motion \( l_{f,t} \). \( \beta_t \) represents the stochastic discount factor of households: \( \beta_t = \beta \mathbb{E}_t \frac{\lambda^{t+1}}{\lambda_t} \).

The FOC with respect to vacancies reads,

\[
\mu^L_{f,t} = \frac{fV}{q_t},
\]

(55)

The FOC with respect to capital is:

\[
(1 - \alpha) \frac{mc_{f,t}y_{f,t}}{k^V_{f,t}} = r^K_t.
\]

(56)

Given Cobb-Douglas technology and perfect capital mobility, all firms choose the same capital/output ratio and, in turn, the same capital/labor and labor/output ratios. Then the marginal cost is symmetric across firms.

**B.2.2 Price setting**

In the second step, the firms chooses the price of its product subject to Rotemberg (1982) adjustment cost. More precisely, price adjustment costs are given by:

\[
AC^P_{f,t} = \frac{\kappa^P}{2} \left( \frac{P_{f,t}}{P_{f,t-1}} - 1 - \lambda^P (\pi_{t-1} - 1) \right)^2
\]

where \( \kappa^P \) is the degree of rigidity and \( \lambda^P \) stands for indexation on past inflation.

Given the real marginal cost (\( mc_t \)), real profits can be rewritten as:

\[
d_{f,t} = \left( \frac{P_{f,t}}{P_t} - mc_t - \frac{P_{f,t}}{P_t} AC^P_{f,t} \right) y_{f,t}
\]

Thus, firms set price \( P_{f,t} \) to maximize:

\[
\max P_{f,t} \sum_{t=0}^{\infty} \beta_t (1 - \delta^t) \left( \frac{P_{f,t}}{P_t} - mc_t - \frac{P_{f,t}}{P_t} \frac{\kappa^P}{2} \left( \frac{P_{f,t}}{P_{f,t-1}} - 1 - \lambda^P (\pi_{t-1} - 1) \right)^2 \right) y_{f,t}
\]

The FOC of this problem is:

\[
\left( \frac{1}{P_t} - \frac{1}{P_{f,t}} AC^P_{f,t} - \frac{P_{f,t}}{P_t} \frac{\partial AC^P_{f,t}}{\partial P_{f,t}} \right) y_{f,t} + \left( \frac{P_{f,t}}{P_t} - mc_t - \frac{P_{f,t}}{P_t} AC^P_{f,t} \right) \frac{\partial y_{f,t}}{\partial P_{f,t}} + \mathbb{E}_t \beta_t (1 - \delta^t) \frac{\partial}{\partial P_{f,t}} \left( \frac{P_{f,t+1}}{P_{f,t+1}^{t+1}} \frac{p_{f,t+1}}{p_{f,t+1}^{t+1}} AC^P_{f,t+1} y_{f,t+1} \right) = 0.
\]

(57)

Using the definition of price elasticity for a good \( f \): \( \theta_{f,t} = -\frac{\partial y_{f,t}}{\partial P_{f,t}} \frac{P_{f,t}}{y_{f,t}} \), we can rewrite this expression as:
\[ (1 - \theta_{f,t}) (1 - AC_{f,t}^P) + \theta_{f,t} \frac{mc_t}{P_{f,t}} + P_{f,t} \frac{\partial AC_{f,t}^P}{\partial P_{f,t}} - \mathbb{E}_t \frac{\partial AC_{f,t+1}^P}{\partial P_{f,t}} \beta_t (1 - \delta^N) \frac{P_{f,t+1}P_{C,t}}{P_{t+1}} \frac{y_{f,t+1}}{y_{f,t}} = 0. \] (58)

With the markup defined as the real price \( \rho_{f,t} = \frac{P_{f,t}}{P_{C,t}} \) over the marginal cost: \( \mu_{f,t} = \frac{\rho_{f,t}}{mc_t} \) and rearrange the previous equation we have:

\[ \mu_{f,t} = \frac{\theta_{f,t}}{(\theta_{f,t-1} - 1) (1 - AC_{f,t}^P) + \frac{\partial AC_{f,t}^P}{\partial P_{f,t}} + \mathbb{E}_t \frac{\partial AC_{f,t+1}^P}{\partial P_{f,t}} \beta_t (1 - \delta^N) \frac{P_{f,t+1}P_{C,t}}{P_{t+1}} \frac{y_{f,t+1}}{y_{f,t}}} \] (59)

The FOC of the price adjustment cost in \( t \) is given by:

\[ \frac{\partial AC_{f,t}^P}{\partial P_{f,t}} = \frac{\kappa^P}{P_{f,t-1}} \left( \frac{P_{f,t}}{P_{f,t-1}} - 1 - \lambda^P (\pi_{t-1} - 1) \right), \]

and in \( t + 1 \):

\[ \frac{\partial AC_{f,t+1}^P}{\partial P_{f,t}} = - \frac{P_{f,t+1}}{(P_{f,t})^2} \kappa^P \left( \frac{P_{f,t+1}}{P_{f,t}} - 1 - \lambda^P (\pi_t - 1) \right). \]

Then, by using derivate of the adjustment cost at \( t \) and \( t + 1 \) in Eq.59, we have:

\[ \mu_{f,t} = \frac{\theta_{f,t}}{(\theta_{f,t-1} - 1) (1 - AC_{f,t}^P) + \kappa^P \psi_{f,t}}, \] (60)

where \( \psi_{f,t} \) is given by:

\[ \psi_{f,t} = \pi_{f,t} \left( \pi_{f,t} - 1 - \lambda^P (\pi_{t-1} - 1) \right) - \mathbb{E}_t \frac{y_{f,t+1}P_{C,t}}{y_{f,t}P_{t+1}} \beta_t (1 - \delta^N) (\pi_{f,t+1})^2 \left( \pi_{f,t+1} - 1 - \lambda^P (\pi_t - 1) \right), \] (61)

with the firm-level inflation \( \pi_{f,t} = \frac{P_{f,t}}{P_{f,t-1}} \) and \( \pi_t = \frac{P_t}{P_{t-1}} \) the aggregate product inflation.

**B.3 Wages determination**

Nominal wages are determined through a Nash bargaining scheme between workers and employers who maximize the joint surplus of employment by choosing the nominal wages. Formally, they solve:

\[ \arg \max_{W_{f,t}} \left( \mu_{f,t}^{W} \right)^{e_{t+1}} (\mu_{f,t}^{L})^{1-e_{t+1}} \]
with $\eta \in [0, 1]$ the negotiation power according to workers associated with an exogenous shock $\varepsilon_t^L$. $\mu^W_{f,t}$ is the employment surplus from the household perspective defined in Eq. 52. Note that we used the subscript $f$ for $\mu_{f,t}$. Due to constant returns, all workers are the same at the margin and the wage negotiation is between the firm and the marginal worker. Finally, $\mu^L_{f,t}$ is the employment surplus from the producer perspective defined in Eq. 54.

The FOC with respect to nominal wage $W_{f,t}$ implies the following sharing rule,

$$ (1 - \omega_t) \mu^W_{f,t} = \omega_t \mu^L_{f,t} $$

(62)

where $\omega_t$ is the time-varying negotiation power defined as:

$$ \omega_t = \varepsilon_t^N \frac{1}{\partial W_{f,t} \partial \mu^L_{f,t}} (1 - \varepsilon_t^N) \frac{\partial \mu^L_{f,t}}{\partial W_{f,t}} $$. 

Since the firms is subject to adjustment cost à la Rotemberg (1982) for adjusting the nominal wage $W_t$. They have to pays:

$$ AC^W_{f,t} = \frac{\kappa^W}{2} \left( \frac{W_{f,t}}{W_{f,t-1}} - 1 - \lambda^W \left( \pi^C_{t-1} - 1 \right) \right)^2 $$

where $\kappa^W \geq 0$ is the degree of rigidity and $\lambda^W \in [0, 1]$ is the indexation on past welfare-based inflation $\pi^C_{t-1}$. Derive the employment surplus for the marginal worker:

$$ \frac{\partial \mu^W_{f,t}}{\partial W_{f,t}} = \frac{1}{P^C_t} $$

and for the employment surplus for the producers,

$$ \frac{\partial \mu^W_{f,t}}{\partial W_{f,t}} = -\partial (1 + AC^W_{f,t}) W_{f,t} \frac{1}{P^C_t} + \beta_t \left( 1 - \delta^L \right) \left( 1 - \delta^N \right) \frac{\partial \mu^L_{f,t+1}}{\partial W_{f,t}} $$

$$ \Leftrightarrow - (1 + AC^W_{f,t} + W_{f,t} \partial AC^W_{f,t} \partial W_{f,t}) \frac{1}{P^C_t} - \beta_t \left( 1 - \delta^L \right) \left( 1 - \delta^N \right) \frac{\partial AC^W_{f,t+1}}{\partial W_{f,t}} \frac{1}{P^C_{t+1}} $$

where $\partial AC^W_{f,t} \partial W_{f,t} = \frac{1}{W_{f,t-1}} \kappa^W \left( \frac{W_{f,t}}{W_{f,t-1}} - 1 - \lambda^W \left( \pi^C_{t-1} - 1 \right) \right)$ and $\partial AC^W_{f,t+1} \partial W_{f,t} = -\frac{W_{f,t+1}}{(W_{f,t})^2} \kappa^W \left( \frac{W_{f,t+1}}{W_{f,t}} - 1 - \lambda^W \right)$

Then we can rewrite the FOC of the employment surplus as:
\[
\frac{\partial \mu_{f,t}^W}{\partial W_{f,t}} = - \left(1 + AC_{f,t}^W + \frac{W_{f,t}}{W_{f,t+1}} \kappa_L^W \left(\frac{W_{f,t}}{W_{f,t+1}} - 1 - \lambda^W (\pi_{t-1}^C - 1)\right)\right) + \mathbb{E}_t \beta_t (1 - \delta^L) (1 - \delta^N) \left(\frac{W_{f,t+1}}{W_{f,t}} - 1 - \lambda^W (\pi_t^C - 1)\right) \frac{E_{t+1}^C}{P_{t+1}}
\]

Then the time-varying negotiation power reads:

\[
\omega_t = \frac{\varepsilon_t^N \eta}{(\varepsilon_t^N \eta + (1 - \varepsilon_t^N \eta) (1 + AC_{f,t}^W + \mathbb{E}_t \kappa^W \psi_{f,t}^W))},
\]

where \(\psi_{f,t}^W\) is the auxiliary variable that depends on the Rotemberg adjustment cost used:

\[
\psi_{f,t}^W = \pi_{f,t}^W \left(\pi_{f,t}^W - 1 - \lambda^W (\pi_{t-1}^C - 1)\right) - \mathbb{E}_t \beta_t \frac{(1 - \delta^L) (1 - \delta^N)}{\pi_{t+1}^C} \left(\pi_{f,t+1}^W - 1 - \lambda^W (\pi_t^C - 1)\right),
\]

where \(\pi_{f,t}^W = \frac{w_t}{w_{t-1}^f} \pi_t^C\) is the nominal wage inflation rate at firm \(f\).

After presenting the time-varying negotiation power, we turn to the wage setting using the sharing rule (Eq. 62) and replace with the expression of surplus from both perspectives (firms in Eq. 54 and the marginal worker in Eq. 52) we have:

\[
\frac{W_t}{P_t^C} (1 + \omega_t AC_{f,t}^W) = \omega_t \left(\alpha_{mc} \frac{m_{f,t}^W}{f_t} \right) + \mathbb{E}_t \beta_t (1 - \delta^L) (1 - \delta^N) \mu_{f,t+1}^L + (1 - \omega_t) b - (1 - \omega_t) \mathbb{E}_t \beta_t \mu_{f,t+1}^L \left(1 - \delta^N\right) (1 - \delta^L - f_{t+1}) + (1 - \omega_t) b - (1 - \omega_t) \mathbb{E}_t \beta_t \mu_{f,t+1}^L \left(1 - \delta^N\right) (1 - \delta^L - f_{t+1})
\]

Since the sharing rules hold in \(t + 1\) \((\mathbb{E}_t \mu_{f,t+1}^W = \mathbb{E}_t \frac{w_{t+1}}{w_t} \mu_{f,t+1}^L)\), we can rewrite the previous equation:

\[
\frac{W_t}{P_t^C} (1 + \omega_t AC_{f,t}^W) = \omega_t \left(\alpha_{mc} \frac{m_{f,t}^W}{f_t} \right) + (1 - \omega_t) b + \mathbb{E}_t \beta_t \mu_{f,t+1}^L \left(1 - \delta^N\right) (1 - \delta^L) \left\{\omega_t - \left(\frac{1 - \omega_t}{1 - \omega_t} \frac{w_{t+1}}{w_t} \right) \left(1 - \frac{f_{t+1}}{(1 - \delta^L)(1 - \delta^N)}\right)\right\}
\]

and using the the free-entry condition for vacancies (Eq. 55) to replace \(\mu_{f,t+1}^L\):

\[
\frac{W_t}{P_t^C} (1 + \omega_t AC_{f,t}^W) = \omega_t \left(\alpha_{mc} \frac{m_{f,t}^W}{f_t} \right) + (1 - \omega_t) b + \mathbb{E}_t \beta_t \mu_{f,t+1}^L \left(1 - \delta^N\right) (1 - \delta^L) \left\{\omega_t - \left(\frac{1 - \omega_t}{1 - \omega_t} \frac{w_{t+1}}{w_t} \right) \left(1 - \frac{f_{t+1}}{(1 - \delta^L)(1 - \delta^N)}\right)\right\}
\]

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Then in the case of a flexible wage (absence of exogenous shock and rigidity)

\[ \frac{W_t}{P_t} = \eta \left( \alpha \frac{mc_{f,t}(y_{f,t})}{f_{f,t}} \right) + (1 - \eta) b 
+ (1 - \eta) b + \eta \frac{f_{f,t}}{y_{c,t+1}} E_{t} \beta_{t+1} f_{t+1} \]

B.4 Symetric equilibrium, new entrants and aggregation

B.4.1 Symmetric equilibrium

Given Cobb-Douglas technology and perfect capital mobility, all producers choose the same capital/output ratio and, in turn, the same capital/labor and labor/output ratios. As a consequence, the marginal cost is symmetric across firms \((mc_{f,t} = mc_t)\). Thus, equilibrium prices and quantities are identical across producers \(f\).

B.4.2 New entrants

To enter the goods market, the representative new entrant \(e (e \in [0, N_t^E])\) need to cover the following cost:

\[ \phi_{e,t}^E = \frac{f^E}{\varepsilon_t^E} + f^V v_{e,t}^E, \]

where the first part stands for technological requirement subject to an exogenous shock \(\varepsilon_t^E\) and the second the vacancy posting cost. Since, new firms need to post as many vacancy to reach the size of the labor force of an incumbent \(f\) and considering that they only produce in the next period, \(v_{e,t}^E\) reads:

\[ v_{e,t}^E = \frac{(1 - AC_t^E) (1 - \delta^N) (1 - \delta^L) l_{f,t}}{E_{t} q_{t+1}}. \]

Since all producers choose the same amount of labor \(l_{f,t} = l_t\), the cost for entering the market is identical accross new entrants. Thus, the firm’s value are identical across producers.

B.4.3 Aggregation

**Labor market**  Recall that for an individual firm we have the following employment-level:

\[ l_{f,t} = (1 - \delta^L) l_{f,t-1} + q_t v_{f,t}. \]
At the aggregate level, we have \( L_t = \int_0^{N_t} l_{f,t} df \) and \( V_t = \int_0^{N_t} v_{f,t} + \int_0^{N_{E,t-1}} v_{E,t-1} de \). Using the fact that all producers choose the same amount of labor and vacancies \((L_t = N_t l_t \text{ and } N_t v^C_t)\) and new entrants the same amount of vacancies \((N_{E,t-1} v^E_{t-1})\), we can rewrite the previous equation as:

\[
L_t = \frac{N_t}{N_{t-1}} \left(1 - \delta^L\right) L_{t-1} + q_t \left(V_t - N_{E,t-1} v^E_{t-1}\right)
\]

Using the law of motion for variety with \( \frac{N_t}{N_{t-1}} = (1 - \delta^N) \left(1 + (1 - AC^E_{t-1}) \frac{N_{E,t-1}}{N_{t-1}}\right) \), we have:

\[
L_t = (1 - \delta^N) \left(1 - \delta^L\right) L_{t-1} + q_t V_t + (1 - AC^E_{t-1}) \frac{N_{E,t-1}}{N_{t-1}} \left(1 - \delta^N\right) \left(1 - \delta^L\right) L_{t-1} - q_t N_{E,t-1} v^E_{t-1}
\]

Using the total amount of vacancies posted by a new firm \( v^E_{t} = (1 - AC^E_{t}) \frac{(1 - \delta^L)(1 - \delta^N)L_t}{E_{t,q} N_t} \) the last two terms cancel each other out, leading to the aggregate labor law of motion:

\[
L_t = (1 - \delta^N) \left(1 - \delta^L\right) L_{t-1} + q_t V_t.
\] (63)

Then it is important that new firms that enter the market only filling vacancy in the next period on the contrary of incumbent. Without this assumption, the aggregate level of employment cannot be read as \( L_t \neq N_t \) and then we cannot obtain the previous form for aggregate labor law of motion.

**Aggregate production** The aggregate production of goods \( Y_t \) reads as follows:

\[
Y_t = \varepsilon_t^Z (L_t)^\alpha (K_t^\nu)^{1-\alpha},
\]

where \( Y_t = \int_0^{N_t} y_{f,t} df \), \( L_t = \int_0^{N_t} l_{f,t} df \) and \( K_t^\nu = \int_0^{N_t} k_{f,t}^\nu df \). In a symmetric equilibrium this simplifies to:

\[
\Leftrightarrow N_t y_t = \varepsilon_t^Z (L_t)^\alpha (K_t^\nu)^{1-\alpha}
\]

with \( L_t = N_t l_t \) and \( K_t^\nu = N_t k_t^\nu \).
National Income Accounts  To define the GDP, we need to aggregate the household budget constraint:

\[ w_t L_t + (1 - L_t) b + R_{t-1} \frac{B_{t-1}}{P_{t-1}} + r^K_t \nu_t K_{t-1} + (1 - \delta^N) (d_t + e_t) (x_{t-1} + (1 - AC_{t-1}^E) N_{t-1}^E) \]

\[ = C_t + e_t x_t + \frac{B_{t-1}}{P_{t-1}} + T_t + N_t^E \phi_t^E + \psi(\nu_t) K_{t-1} + I_t \]

with \( x_t = N_t \) and using the law of motions for firms \((N_t = (1 - \delta^N) (N_{t-1} + (1 - AC_{t-1}^E) N_{t-1}^E))\), we have:

\[ \iff \quad w_t L_t + (1 - L_t) b + R_{t-1} \frac{B_{t-1}}{P_{t-1}} + r^K_t \nu_t K_{t-1} + N_t d_t \]

\[ = C_t + \frac{B_{t-1}}{P_{t-1}} + T_t + N_t^E \phi_t^E + \psi(\nu_t) K_{t-1} + I_t \]

Using the definition of effective capital \( K_t^e = \nu_t K_{t-1} \), the definition of individual profits \( d_t = \rho_t y_t - w_t l_t (1 + AC_{t}^W) - r^K_t k_t^e - \rho_t y_t AC_{t}^P - f^V_t v_t \), aggregate labor \( L_t = n_t l_t \), aggregate capital services \( K_t^e = n_t k_t^e \) and the budget government constraint (Eq.33), we have:

\[ \rho_t y_t N_t = C_t + g^C t + \psi(\nu_t) K_{t-1} + I_t + f^V_t v_t N_t + N_t^E \phi_t^E + w_t L_t AC_{t}^W + \rho_t y_t N_t AC_{t}^P \quad (64) \]

where the right side of the equation corresponds to the final consumption basket \((Y_t^C)\) using the optimal demand in Eq.42.

Prices Identities  Concerning prices, rearranging the translog expenditure function (Eq.1) and imposing symmetry among producers, the relative price \( \rho_t = \frac{P_t}{P_{t-1}} \) emerges,

\[ \rho_t = \exp \left( -\frac{\tilde{N} - N_t}{2 \sigma^P \varepsilon_{t}^P \tilde{N} N_t} \right) \quad (65) \]

Thus, the relative price \( \rho_t = \frac{P_t}{P_{t-1}} \), the product price inflation \( \pi_t = \frac{P_t}{P_{t-1}} \) and welfare-base inflation \( \pi_t^C = \frac{P_t^C}{P_{t-1}^C} \) can be linked through:

\[ \frac{\rho_t}{\rho_{t-1}} = \frac{\pi_t}{\pi_t^C} \quad (66) \]

Since, we have imposing symmetry among producers, the wage nominal inflation rate \( \pi_t^W \) is given by:
\[ \pi^W_t = \frac{w_t}{w_{t-1}} \pi^C_t, \]

with \( w_t \) the real wage.

\section{Steady state}

\subsection{The steady state}

From the Euler condition, we have the steady state value for interest rate, \( r = \frac{1}{\beta} - 1 \) and by definition we have \( \pi = 1, q^K = 1, \pi^W = 1 \) and \( \pi^C = 1 \).

Start with the law of motion for aggregate employment described in the general equilibrium conditions but from the household perspective:

\[ L = (1 - \delta^N) (1 - \delta^L) L + fU. \]

with \( U = 1 - L \) the aggregate unemployment rate. Using the calibration in Tab.1 for \( \delta^N, \delta^L \) and \( f \), we have the steady-state value for labor at the aggregate level:

\[ L = \frac{f}{(1 - (1 - \delta^N)(1 - \delta^L) + f)}, \]

and by extension the aggregate unemployment rate \( U \). Using the definition of the probability for a firm to find a worker \( q = \frac{M}{V} \) and an unemployed worker to find a job \( f = \frac{M}{U} \), we obtained the aggregate vacancy such as: \( V = \frac{LU}{q} \). Thus, we have the flow of new hires through \( fU = qV = M \) stands for the matching function. We deduce the matching efficiency on the matching function \( m = \frac{M}{\sqrt{U+\pi}} \). Since new entrants need to post vacancy to reach the same workforce size than incumbent, we have: \( v^E = \frac{(1-\delta^N)(1-\delta^L)q}{\pi} \).

Concerning the product market, we start with the dispersion price. After some arrangement to make appear the ratio of incumbent to potential producers \( \frac{N}{N} \) which is calibrate in the expression of the relative price (Eq.36), we have:

\[ \rho = e^{(\frac{1}{\pi^{\frac{1}{\beta-1}}}(1-\frac{N}{N}))}. \]

Using the definition of markup: \( \mu = \frac{\theta}{\delta - 1} \), we get the expression of marginal cost using Eq.23 \( mc = \frac{\rho}{\mu} \).
Using the optimal capital (Eq. 13), we can obtain the amount of capital which is equivalent to the capital used at the steady state \((\nu = 1, \psi(\nu) = 0 \text{ and } K = K^\nu)\),

\[
K = \left( \frac{1 - (1 - \delta^K)\beta}{L^{1-\alpha} \beta (1 - \alpha) mc} \right)^{-\frac{1}{\alpha}}
\]

and by extension the production function \((Y = (L)^\alpha (K)^{1-\alpha})\).

Thus, we define the value of unemployment as in Gertler et al. (2008) by \(\tilde{\delta} = \frac{bL}{mcY}\).

In order to obtain the steady-state value of wage \(w\) and the vacancy cost \(f^V\), we use the job creation condition (Eq.18 and Eq.55) and the equation of Nash bargaining (Eq.31) as a system:

\[
\begin{align*}
\frac{f^V}{q} &= \alpha mcY_L - w + \frac{f^V}{q} \beta (1 - \delta^V) (1 - \delta^L) \\
w &= \eta \left( \alpha \frac{mcY}{L} + \beta \frac{f^V L}{q} \right) + (1 - \eta) b
\end{align*}
\]

\[
W = \eta \left( \alpha \frac{mcY}{L} + (1 - \eta) b + \eta \frac{f^V}{q} \beta \{ f - (1 - \delta^L) \delta^N \} \right)
\]

rearrange the system and make appear the steady-state value of unemployment:

\[
\begin{align*}
f^V &= \phi_1 \left( \alpha mcY_L - w \right) \\
w &= \eta \left( \alpha \frac{mcY}{L} + \beta \frac{f^V L}{q} \right) + (1 - \eta) \tilde{b} \alpha mcY_L
\end{align*}
\]

with \(\phi_1 = \frac{q}{(1 - \beta (1 - \delta^V) (1 - \delta^L))}\). Then, replacing the second line in the first gets the steady-state value of posting vacancy:

\[
f^V =\alpha mcY \frac{\phi_1 (1 - \eta) \left( 1 - \tilde{b} \right)}{\left( 1 + \eta \beta \frac{L}{q} \right)}.
\]

and then the transfert to unemployed workers is given by : \(b = \tilde{b} \frac{mcY}{L}\).

For the number of producers, we use the Euler condition on equity (Eq.46), the equity value (Eq.48) and the form of dividends to get the number of incumbent:

\[
N = \frac{1 - \frac{1}{\mu} Y_p - f^V \frac{(1-\delta^V)(1-\delta^N)L}{q}}{f^E},
\]

After that, we have the number of new entrants \(N^E = \frac{\delta^N}{(1-\delta^N)} N^E\), \(\tilde{N} = N/0.95\) and
Using the aggregate vacancy defines in the general equilibrium. \( Y^C = \rho Y \), \( I = \delta^K K \) and \( C = (1 - g^Y)Y^C - I - N f^V v - N^E \left( f^E + f^V v^E \right) \).

Now, we show that the proportion of barrier to entry \( f^E \) with respect to the equity value is high in the model whatever the amount specified. We start with the equity value given by Eq.24 by making appear the ratio of barrier with respect to equity \( \tau^E = \frac{\mu^E}{\epsilon} \):

\[
e = f^E + \frac{(1 - \delta^L) \left(1 - \delta^N\right) f^V L}{N q}
\]

Use the expression of the steady-state number of firms \( N \):

\[
\tau^E = 1 - \frac{f^E f^V L}{\epsilon} \left(1 - \frac{1}{\mu} Y \rho q - (1 - \delta^L) \left(1 - \delta^N\right) f^V L\right)
\]

make appears \( \tau^E = \frac{f^E}{\epsilon} \):

\[
\Leftrightarrow \tau^E = 1 - \frac{(1 - \beta \left(1 - \delta^N\right)) f^V L}{\beta (1 - \delta^N) \left(1 - \frac{1}{\mu} Y \rho q\right)}
\]

which is independant of \( f^E \).

Now, we show that in the same way, the calibration of barrier cost doesn’t alter the proportion of the extensive margin in the definition of the aggregate demand (Eq.34)

\[
1 - g^Y - \frac{C}{Y^C} - \frac{I}{Y^C} - N f^V v - \frac{N f^V v}{Y^C} = \frac{N^E \epsilon}{Y^C}.
\]

Make appears the proportion of barrier cost with respect to the equity value \( \tau^E = \frac{\mu^E}{\epsilon} \) and replace \( N^E = \frac{\delta^N}{(1 - \delta^N)} N \) using the steady-state value of \( N \) (Eq.67):

\[
1 - g^Y - \frac{C}{Y^C} - \frac{I}{Y^C} - N f^V v - \frac{N f^V v}{Y^C} = \left(\frac{\beta (1 - \delta^N)}{(1 - \beta (1 - \delta^N))} \left(1 - \frac{1}{\mu} Y \rho - f^V L \frac{q}{\epsilon}\right)\right) \frac{\delta^N}{Y^C \tau^E (1 - \delta^N)}
\]

with \( Y^C = \rho Y \) and \( f^V = \phi_1 (\alpha m c Y \frac{Y}{T} - w) \)

\[
1 - g^Y - \frac{C}{Y^C} - \frac{I}{Y^C} - N f^V v - \frac{N f^V v}{Y^C} = \left(\frac{\beta (1 - \delta^N)}{(1 - \beta (1 - \delta^N))} \left(1 - \frac{1}{\mu}\right) - \frac{\alpha m c Y - Lw}{\rho Y (1 - \beta (1 - \delta^N) (1 - \delta^L))}\right) \frac{\delta^N}{\tau^E}
\]

Note by \( \tau^{NE} \) the part of the extensive margin of activity in aggregate demand: \( \tau^{NE} = \frac{N^E \epsilon}{Y^C} \).
we have:

$$\tau^{NE} = \left( \frac{\beta(1 - \delta^N)}{(1 - \beta(1 - \delta^N))} \left( 1 - \frac{1}{\mu} \right) - \frac{\alpha mcY - Lw}{\rho Y (1 - \beta (1 - \delta^N) (1 - \delta^L))} \right) \delta^N \tau^E,$$

which is independant from the barrier cost to entry $f^E$ as for $\tau^E$ and relatively unsensitive to various calibration.

References


Colciago, A. and L. Rossi (2011). Endogenous market structures and labour market dynamics. 1


Gertler, M., L. Sala, and A. Trigari (2008). An estimated monetary DSGE model with unemployment and staggered nominal wage bargaining. *Journal of Money, Credit and Banking 40*(8), 1713–1764. 1, 6, 3.2, 3.2, 18, 3.3, 5.1, C.1


Rotemberg, J. J. (1982). Monopolistic price adjustment and aggregate output. *The Review of Economic Studies* 49(4), 517–531. 2, 2.2.1, 2.2.1, 2.2.1, B.2.2, B.3

Rotemberg, J. J. and M. Woodford (1999). The cyclical behavior of prices and costs. *Handbook of macroeconomics* 1, 1051–1135. 1


3, 3.2