Social Learning about Monetary Policy at the Zero-Lower Bound *

Jasmina ARIFOVIC† Alex GRIMAUD‡ Isabelle SALLE§ Gauthier VERMANDEL¶

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Abstract

We develop a general equilibrium model in which heterogeneous expectations follow an evolutionary process and may lose their anchorage to the central bank target. We explore the implications for monetary policy. We jointly estimate the structural and the learning parameters of the model by matching moments from both macroeconomic and SPF data. Within the resulting framework, the sole coordination of expectations on pessimistic outlooks is able to endogenously produce persistent but stable dynamics at the zero-lower bound (ZLB). We quantify the welfare costs of belief dispersion with respect to the rational expectation benchmark. The central bank is able to reduce this cost by communicating the targeted path, which improves coordination between agents and reduces the occurrence as well as the length of ZLB episodes.

Keywords: Learning, Heterogeneous expectations, ZLB, Communication.

JEL Classification: C51, C60, D83, E31, E52, E58, E70.

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†Department of Economics, Simon Fraser University, Burnaby, BC, CA (arifovic@sfu.ca)
‡Università Cattolica del Sacro Cuore and Universiteit van Amsterdam (a.b.p.grimaud@uva.nl)
§Bank of Canada, Ottawa, ON, CA and Universiteit van Amsterdam, Amsterdam School of Economics (ISalle@bank-banque-canada.ca)
¶Université Paris-Dauphine and France Stratégie (gauthier.vermandel@dauphine.fr)
1 Introduction

We develop a general equilibrium model in which heterogeneous expectations may lose their anchorage to the central bank (CB) target, and explore the implications for monetary policy.

The Great Recession in the US and Europe has revived the interest in the analysis of liquidity trap episodes: persistent below-target inflation together with poor economic performances have forced CBs to cut the interest rates and hit the zero-lower bound (ZLB). Meanwhile, inflation expectations have been declining steadily, as depicted on Figure 1, which puts at risk the long-run anchorage of expectations.

This narrative is hard to unfold within standard workhorse macroeconomic models, namely the New Keynesian (NK) class of models. Solving those models at the ZLB under rational expectations (RE) is particularly challenging (Guerrieri & Iacoviello 2015) and zero interest rates generate implausible macroeconomic volatility (Benhabib et al. 2001a, b), which is at odds with the recent experience.

Furthermore, the standard assumption of common information set and common beliefs leaves little room for expectations to be persistently off the target and play any autonomous role in driving business cycles. Recessive episodes are entirely generated by exogenous and persistent technology or financial shocks. In particular, the possibility of long-lasting coordination of expectations on pessimistic outlooks, that turns self-confirming if agents behave accordingly, is ruled out.1 Introducing boundedly rational and learning agents into macroeconomic models has tackled some of those challenges (Sargent 1993, Woodford 2013). Yet, under learning, explosive dynamics result once interest rates are pegged to zero (Evans et al. 2008).2

Additionally, the workhorse models generally overlook important stylized facts of real-world expectations, such as positive auto-correlation in forecast errors or pervasive and time-varying heterogeneity.3 Nonetheless, heterogeneity in agents’ expectations is not anecdotal given the challenge that it may pose to the CB

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1 One exception is Angeletos et al. (2018) who investigate the role of strategic uncertainty in presence of heterogeneous information within a general equilibrium model. However, those authors use an RBC model, which leaves out monetary policy.

2 One exception that is discussed below may be Arifovic et al. (2018).

3 There is a large literature documenting deviations from RE. We refer to, inter alia, Mankiw et al. (2003), Negro & Eusepi (2010), Branch (2004). On heterogeneity particularly, see Hommes (2011) for evidence using lab forecasting experiments, Mankiw et al. (2003) in survey data from professional forecasters and Cavallo et al. (2017) from households. A burgeoning literature on Heterogeneous Agent New Keynesian (HANK) models investigate the consequences of heterogeneity in expectations on monetary policy design (Debortoli & Gali 2018, Kaplan et al. 2018, Bilbiie 2018), but heterogeneity is rarely time-varying, and its scope is generally limited to a few, usually two, types, see e.g. Hommes & Lustenhouwer (2019).
when attempting to coordinate the private sector on the desirable inflation target. Forecast dispersion has been directly related to macroeconomic uncertainty (Rossi & Sekhospyan 2015) and has been proven to induce adverse effects on the economy (Jo & Sekkel 2017).

This paper jointly addresses these two dimensions, namely how dynamic and persistent heterogeneity in expectations affects the dynamics at the ZLB. We incorporate heterogeneous expectations in an otherwise standard monetary policy model. We add that agents form beliefs about the long-run values of inflation and output. Those beliefs are revised in an evolutionary manner, which allows us to explicitly model the issue of expectation anchorage. Our choice is motivated by the simplicity and parsimony of this class of learning models, as well as their ability to match experimental findings (Arifovic & Ledyard 2012). In particular, heterogeneity is rooted into the functioning of the learning process, which allows agents to collectively adapt to an ever-changing environment, in which their own expectations contribute to shape the macroeconomic environment that they are trying to forecast. This feature is well-suited to self-referential economic systems, such as standard macroeconomic models.

We consider a model with white noise fundamental shocks only, so that the only persistence may come from the learning process, which allows us to identify the amplifier role of expectations in driving business cycles. The RE homoge-
neous agent benchmark is nested into our expectation model, so as to envision heterogeneity as a friction, and quantify the ensuing welfare cost with respect to the RE outcome.

In a novel effort within this literature,\textsuperscript{4} we take our model to the data and show that our simple model can simultaneously reproduce the properties of the Survey of Professional Forecaster (SPF) and the main US macroeconomic time series between 1968-2017, including the duration of the ZLB episode. We add two contributions to the related literature: we provide an estimation routine under heterogeneous expectations, and an empirical discipline device to learning models, by offering estimated values of the learning parameters for which we have no observed counterparts.

We then analyze the dynamic properties of the model and show how our model offers a reading of the recent economic experience as a long-lasting coordination of agents on pessimistic expectations in the aftermath of severe adverse shocks. Our model features persistent but stable dynamics during ZLB episodes, followed by inflation-less recoveries. From there, the transition back to the target can be particularly long if the transitory adverse shock has unanchored expectations which, then, per their self-fulling nature, nurture the bust. The forces underlying our narrative are reminiscent to the earlier Keynesian concept of animal spirits.

We next quantify the contribution of the dispersion of expectations to the loss function of the CB and find that heterogeneous expectations entail a consumption loss of almost 7\% with respect to the RE allocation. Then, following the intuition of the Tinbergen principle, namely that to each and every policy target must correspond at least one policy tool, we introduce CB communication as an additional monetary policy instrument next to the interest rate and investigate whether it may offset the effects on price stability of the forecast dispersion with respect to the RE homogeneous expectation benchmark. Communication is redundant in a world of RE and perfect information, but constitutes a natural policy option when agents are learning and expectations are heterogeneous (Blinder et al. 2008).

We show that announcing the targeted equilibrium path to the agents may help enforce coordination of agents’ expectations along this path. As coordination on pessimistic outlooks is the source of aggregate propagation of shocks in our model, communicating the target reduces the occurrence as well as the duration of ZLB episodes and cuts the welfare loss due to heterogeneous expectations by nearly 2.5\%, which brings the CB closer, but nonetheless below, the RE outcome.

Only if the CB’s announcements are immediately and fully integrated to

\textsuperscript{4}Negro & Eusepi (2010) attempt to replicate expectation data with RE models. Milani (2007) fit an adaptive learning NK model to macroeconomic time series only. Closer to our contribution, Slobodyan & Wouters (2012b,a) estimate a NK model on both macroeconomic and expectation times series by using Bayesian learning and exogenous auto-correlated shocks on expectations.
agents’ expectations may the CB achieve the RE outcomes. Such an implausible strong effect hinges entirely on the assumption of perfect credibility. While not directly transportable into the terms of the debate about the so-called ‘forward-guidance puzzle’, our model gives us a clue to this puzzle: the dramatic effects that are triggered by any announcement about the future in standard RE models are at odds with the much milder effects suggested by the empirical literature (Carlstrom et al. 2012, Campbell et al. 2016) because those models assume perfect credibility. By contrast, in our model, agents need to ‘see it to believe it’: if the CB’s announcements are too at odds with the realized inflation dynamics, they start discarding these announcements, and the CB communication has milder effects than under the full credibility assumption underlying most RE models.\footnote{Other solutions in the literature rely on weakening the effect of expected real interest rates on consumption by adding frictions such as liquidity constraints, limited asset-market participation or habit formation Del Negro et al. (2012). A few related contributions explain the puzzle by considering bounded rationality, under which agents switch between two simple forecasting heuristics (Goy et al. 2018), use k-level reasoning (Farhi & Werning 2017) or pay limited attention (Gabaix 2016).}

Our treatment of communication adds to the existing literature on communication under learning by modeling endogenous credibility.\footnote{The learning literature usually concludes that communication is stabilizing under learning – see e.g. Orphanides & Williams (2005) in the case of the announcement of the target under adaptive learning – but does so by assuming perfect credibility.} The closest to our concept of endogenous credibility is the paper by Hommes & Lustenhouwer (2019) that derives the stability conditions of the targeted equilibrium in a NK model with ZLB, where agents’ expectations switch to follow past inflation, may the target be missed.

We borrow from Arifovic et al. (2013) and Arifovic et al. (2018) a similar social learning (SL) mechanism to model expectations within a NK model. However, our present work differs substantially. Among others, those two papers study the long-run stability of the model as defined by the convergence towards a particular equilibrium under SL, while we focus on the short-term interplay between macroeconomic and learning dynamics. None of their model is taken to the data, only Arifovic et al. (2018) introduce the ZLB but use exogenous shocks to trigger liquidity trap episodes. They do not contemplate CB communication nor do they measure welfare implications of that departure from RE.

The rest of the paper proceeds as follows. In Section 2, we develop the model, the estimation is presented in Section 3, the dynamic properties of the model are analyzed in Section 4, Section 5 discusses the effects of CB communication and Section 6 concludes.
2 A behavioral general-equilibrium model

We first describe the building blocks of the model, then the solution under the RE benchmark and finally explain our implementation under SL.

2.1 A piecewise linear New Keynesian model

Our model builds on the workhorse three-equation NK model developed by, *inter alia*, Woodford (2011). The three equations describe aggregate demand, aggregate supply and monetary policy. All variables below are expressed in deviation from their steady state level as targeted by the CB.

Aggregate demand is described by the IS curve:

\[ \hat{y}_t = E_j^{t} \hat{y} + (\hat{i}_t - E_{t}^{t} \hat{\pi} + \hat{g}_t) \]

where \( \hat{y} \) is the output gap, \( \hat{i} \) the nominal interest rate set by the CB, \( \hat{\pi} \) the deviation of the inflation rate from the target (hence, \( \hat{i}_t - E_{t}^{t} \hat{\pi} \) represents the real interest rate), \( \sigma > 0 \) the inter-temporal elasticity of substitution of consumption (based on a CRRA utility function), and \( E_j^{t} \) the (possibly boundedly rational) expectation operator based on information available at time \( t \). The subscript \( j \) is introduced to suggest the possibility of heterogeneous expectations, where each agent type \( j = 1, ..., N \) forms her own expectation (with \( N \) the number of agent-types).

\( \hat{g} \) is an exogenous real disturbance.

The supply side is summarized by the forward looking NK Phillips Curve:

\[ \hat{\pi}_t = \beta E_j^{t} \hat{\pi} + \kappa \hat{y}_t + \hat{u}_t \]

where \( 0 < \beta < 1 \) represents the discount factor, \( \kappa > 0 \) a composite parameter capturing the sensitivity of inflation to the output gap, and \( \hat{u}_t \) an exogenous cost-push shock.

In the RE literature, the shocks \( \hat{g} \) and \( \hat{u} \) are usually assumed to be AR(1) processes:

\[ \hat{g}_t = \rho^{g} \hat{g}_{t-1} + \varepsilon^{g}_t \]

\[ \hat{u}_t = \rho^{u} \hat{u}_{t-1} + \varepsilon^{u}_t \]

We follow here most of the learning literature and introduce heterogeneity in the reduced-form models rather than in the micro-foundations (see, *inter alia*, Bullard & Mitra (2002), Arifovic et al. (2013), Hommes & Lustenhouwer (2019)). We are well aware of the conceptual limitation of this approach. Nonetheless, while the complications of the alternative are clear (see e.g. Woodford (2013)), the benefits in terms of qualitative results are uncertain. For instance, in an asset-pricing model, Adam & Marcet (2011) show that, under a sophisticated form of adaptive learning, the infinite-horizon pricing equation reduces to a myopic mean-variance equation. Bearing those caveats in mind, we proceed within the reduced-form model.

As it will be useful in the estimation stage in Section 3, the slope \( \kappa \) in fact reads as \( \kappa = (\delta + \sigma)(1 - \theta) \frac{1 - \theta^\theta}{\theta^\theta} \) with \( \theta \) the Calvo updating parameter and \( \delta > 0 \) the Frish labor elasticity.
\[ \hat{u}_t = \rho^u \hat{u}_{t-1} + \varepsilon^u \]  
where \( 0 \leq \rho^u, \rho^g < 1 \) measure the persistence of the shocks, and \( \varepsilon^g, \varepsilon^u \) are i.d.d. with respective standard deviations \( \sigma^g \) and \( \sigma^u \).

Monetary policy implements a flexible inflation targeting regime, subject to the ZLB constraint, which results in the following non-linear forward-looking Taylor rule:

\[ \hat{\pi}_t = \max \{ -\bar{\tau}; \phi^\pi \mathbb{E}_t \hat{\pi}_{t+1} + \phi^y \mathbb{E}_t \hat{y}_{t+1} \} \]  
where \( \phi^\pi \) and \( \phi^y \) are, respectively, the reaction coefficients to the inflation and the output gaps, and \( \bar{\tau} \equiv (1 + \pi^T) \beta^{-1} - 1 \) the steady state level of interest rate associated to the inflation target \( \pi^T \).

We now solve the model under the benchmark of RE and then detail how we introduce social learning in the expectation formation process of the agents.

### 2.2 The model under rational expectations

In this section, we consider RE and impose \( \mathbb{E}_{t,t}(\cdot) = E(\cdot | I_t) \) to be the rational expectation operator given the information set \( I_t \) common to all agents in period \( t \). We solve for the minimal state variable (MSV) solution using the method of undetermined coefficients.

It is well-known that the ZLB introduces a non-linearity in the Taylor rule and generates an additional deflationary steady state, next to the target (Benhabib et al. 2001a, b). This ZLB steady state corresponds to a liquidity trap, in which agents are indifferent between risk-free bonds and money and the deflation rate matches the discount factor. Hence, expressing the model in reduced form is challenged by this non-linearity and we need to disentangle two pieces, one around the target and one when the ZLB is binding.

A short digression through the one-dimensional Fisherian model easily illustrates this configuration. Figure 2 displays inflation and interest rate dynamics, abstracting from the production side: the social optimum or inflation target corresponds to \( (\hat{y} = 0, \hat{\pi} = 0) \) and the deflationary steady state to \( (\hat{y}_{zlb}, \hat{\pi}_{zlb}) \). Provide that \( \hat{\pi}_{zlb} \leq 0 \leq \pi^T \), the two equilibria co-exist.

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9This method gives a second-best estimate of the dynamics around the deflation state, because log-linearizing the model around this second steady state would result in a MSV solution involving an additional state variable, namely the price dispersion (Ascari & Rossi 2012) and, hence, additional coefficients to learn under SL, see Section 2.3. The benefits in terms of qualitative results are unlikely to outweigh the costs of such a complication of the learning process of the agents. We instead follow the related literature and impose the ZLB constraint in the log-linearized model around the targeted steady state to describe the dynamics around the low inflation state, see, *inter alia*, Nakov (2008), Guerrieri & Iacoviello (2015).
Figure 2: Co-existence of two steady states under the ZLB constraint.

Notes: We can write the log approximated Fisher equation as follows: \( \hat{i} = \beta^{-1} \hat{\pi} \). At the targeted steady state (on green), no deviation occurs: \( \hat{i} = \beta^{-1} \hat{\pi} = 0 \). At the ZLB (on red), we can derive an equilibrium such that: \( -\tau = \beta^{-1} \hat{\pi}_{ZLB} \Rightarrow \hat{\pi}_{ZLB} = -\tau \beta \). Provide that \( \hat{\pi}_{ZLB} \leq 0 \leq \pi^T \), the two equilibria co-exist. The grey area is indeterminate under RE and unstable under adaptive learning.

Coming back to the two-dimensional model, Appendix A shows that the REE at the target, that we denote by a star superscript, reads as:

\[
z_t = a^T + c^T \hat{g}_t + d^T \hat{u}_t
\]

with \( z_t = \begin{bmatrix} \hat{y}_t \\ \hat{\pi}_t \end{bmatrix} \), \( a^T = (I-B^T)^{-1} \alpha^T \), \( c^T = (I-B^T \rho^g)^{-1} \chi^g \) and \( d^T = (I-B^T \rho^u)^{-1} \chi^u \), where the detailed expressions of matrices \( B^T, \alpha^T \) can be found in Appendix A.

In the sequel, we follow Arifovic et al. (2018) and consider white noise real and cost-push shocks, i.e. \( \rho^g = \rho^u = 0 \). In this case, the REE solution reduces to \( z_t = a \), with \( a^T = (I - B^T)^{-1} \alpha^T \) at the target, and \( a^{ZLB} = (I - B^{ZLB})^{-1} \alpha^{ZLB} \) at the ZLB. With this simplification at hand, we now introduce the expectation formation mechanism under SL.

### 2.3 Expectations under social learning

Under SL, we relax the assumption of homogeneous agents endowed with RE, and consider instead a population of \( N \) heterogeneous and interacting agents, indexed by \( j = 1, \ldots, N \). We now define \( E_{j,t}(\cdot) = E_{j,t}^{SL}(\cdot | I_{j,t}) \) to be the expectation operator under social learning (SL) given the information set \( I_{j,t} \) available in period \( t \) for agent \( j \). The information set is agent-specific as it contains the
current and past individual strategies that need not be shared with the whole population.

Following Arifovic et al. (2013, 2018), we assume that agents are endowed with the same form of the forecasting rule, consistent with the MSV solution as described by Equation (20), but with agent-specific coefficients \( a_{j,t} \) that they revise over time. In any period \( t \), each agent \( j \) is therefore entirely described by a two-coefficient strategy \( (a_{1,j,t}, a_{2,j,t})' \) and her expectations read as:

\[
\begin{align*}
E_{j,t}^S \hat{y}_{t+1} &= a_{1,j} \\
E_{j,t}^S \hat{\pi}_{t+1} &= a_{2,j} 
\end{align*}
\]

These pairs of coefficients find an appealing interpretation. In the absence of shocks, \( a_{1,t} \) corresponds to her long-run output gap forecast and \( a_{2,t} \) to her long-run inflation forecast. In the presence of i.d.d. shocks, those coefficients correspond to the average output gap and inflation gap forecasts of agent \( j \). Under either of these interpretations, the strategies of the agents represent their beliefs about the steady state values of the inflation and output gaps, which allows us to intuitively model expectations anchorage or misanchorage by simply evaluate the distance between those coefficient values and their targeted counterparts \( a^T \).

Individual expectations (7) are aggregated using the mean:

\[
E_t^S \hat{z}_{t+1} = \frac{1}{N} \sum_{j=1}^{N} E_{j,t}^S \hat{z}_{t+1}
\]

and inserted into the reduced-form model (18). This process fully describes the dynamics of endogenous variables under SL. Let us now detail how individual expectations are formed.

Agents collectively explore the space of possible parameter values for coefficients \( (a_1, a_2) \), and we are interested in whether, and under which conditions, they may coordinate on the targeted REE values \( a^T \). Specifically, this class of learning models utilizes two operators. The first one is an innovation stochastic process, or mutation, that allows for a constant exploration of the solution space outside the existing population of strategies. In each period, with an exogenously fixed probability \( \mu_x \), each agent’s coefficient is modified as:

\[
x_{j,t+1} = x_{j,t} + \iota \xi^x
\]

where \( x_{j,t} \equiv a_{1,j,t}, a_{2,j,t} \), \( \iota \) is a random draw from a standard normal distribution and \( \xi^x \) the standard deviation associated to coefficient \( x \). Mutation can be interpreted as an innovation, a trial-and-error process or a control error in the computation of the corresponding expectations.
The second operator, the tournament, is the selection force of the learning process and allows better-performing strategies to spread among agents at the expense of least-performing ones. Performance of any forecasting rule \((a_{1,j,t}, a_{2,j,t})\) is measured with its resulting forecast errors over the whole past history of the economy, not solely over the last period, because the environment is stochastic (see Branch & Evans (2007)).

For each agent \(j\), the first component \(a_{1,j,t}\) is assessed regarding output gap forecast errors and its fitness is computed as:

\[
F_{y,t}^y = -\sum_{\tau=1}^{t} (\rho_y)^\tau (\hat{y}_{t-\tau} - a_{1,j,t})^2
\]  

and the second component \(a_{2,j,t}\) is assessed in the same way using past inflation data as:

\[
F_{\pi,t}^\pi = -\sum_{\tau=1}^{t} (\rho_{\pi})^\tau (\hat{\pi}_{t-\tau} - a_{2,j,t})^2
\]  

The terms \(\hat{y}_{t-\tau} - a_{1,j,t}\) and \(\hat{\pi}_{t-\tau} - a_{2,j,t}\) correspond, respectively, to the output and inflation gap forecast errors that agent \(j\) would have made in period \(t - \tau - 1\), had she used her current coefficients \(a_{1,j,t}\) and \(a_{2,j,t}\) to predict the output and inflation gaps in period \(t - \tau\). The smaller the forecast errors, the higher the fitness of a forecasting component \(a_{1,j,t}\) or \(a_{2,j,t}\).

Parameter \(0 < \rho^x \leq 1, \ x = a_1, a_2\), represent memory. In the nested case where \(\rho^x = 0\), the fitness of each strategy is completely determined by the forecasting error of the most recent observable data, i.e. \(t - 1\). For any \(0 < \rho^x \leq 1\), all past forecast errors impact the fitness of the rule but with exponentially declining weights, while for \(\rho^x = 1\), all past errors have an equal weight in the computation of the fitness.\(^{10}\) This memory parameter allows the agents to discriminate between a one-time lucky draw and persistently good forecasting performances.

In the tournament, agents are randomly paired (the number of agents is conveniently chosen even), their fitness on inflation and output gaps are each compared and the one with the lowest fitness copies the strategy component of the other. There is one tournament for inflation and one tournament for output gap forecasting strategies.

Note that the RE benchmark is nested in our heterogeneous-agent model: as soon as the inflation and output gap expectations of all agents are initialized at

\(^{10}\)Considering the increasingly demanding computational power of testing every agent’s strategy over every past periods, we define a minimum weight \(\epsilon = .01\) beyond which past periods become negligible in the computation of the fitness and are discarded. Precisely, the sums in Equations (10)-(11) are implemented from \(\tau = 1\) to \(\tau = t_{max}\), where \(t_{max} \equiv \ln(\epsilon)/\ln(\rho)\). After extensive sensitivity tests, as long as \(\epsilon < .01\), there is no effect on the dynamics under SL.
the REE values and mutation is switched off (i.e. \( \xi^u, \xi^\pi = 0 \)), the dynamics boil down to the RE benchmark.

Our learning model involves a few parameters, namely the probabilities of mutation, the sizes of those mutations and the memory of the fitness function. We now detail how we estimate those parameter values.

3 Estimation of the model under social learning

We jointly estimate the learning and the structural parameters of the model. We do so by using macroeconomic US time series as well as forecast data from the Survey of Professional Forecasters (SPF). We first detail our choice and construction of the datasets, then discuss our estimation method and comment on the results.

3.1 Dataset

Data for output, price index and nominal rates are taken from the FRED data base, while forecast data are taken from the Survey of Professional Forecasters (SPF) from the Federal Reserve of Philadelphia.\(^\text{11}\) The latter spans the period 1968 to 2018 on a quarterly basis. To make the dataset stationary, we divide output by both the working age population and the price index, and compute the percentage deviation from its linear trend to obtain a measurement of the output gap. The inflation rate is measured by the growth rate of the GDP deflator.

Following the related literature, we obtain time series of forecast errors by computing the difference between the one-quarter-ahead forecast by all the professional forecasters of the SPF and the next quarter now-cast. The use of the now-cast, rather than the actual realization of the variable, also enables us to remain consistent with the timing of observability of the variables implicit in the NK given that actual realizations are given that actual realizations are generally available with a up-to-three-year delay. A positive value of the forecast error implies that the professional forecasters overestimated the realized data, while an increase in the cross-sectional dispersion of forecasts means that disagreement between forecasters has increased.

\(^{11}\)We use SPF data, as opposed to alternatives such as the Michigan survey, because the related literature argues that those data provide a good approximation of the private sector expectations that are implicitly involved in the New Keynesian micro-foundations (Negro & Eusepi 2010, Slobodyan & Wouters 2012a).
Values | Sources
---|---
$\sigma$ | risk aversion | 1 | Galí (2015)
$\varphi$ | Frish labor elasticity | 1 | Galí (2015)
$\phi^\pi$ | policy stance on inflation | 1.50 | Galí (2015)
$\phi^y$ | policy stance on output | 0.125 | Galí (2015)
$\pi$ | inflation target | 1.005 | Official Fed Target
$N$ | number of agents | 300 | Arifovic et al. (2018)

Table 1: Calibrated parameters (quarterly basis)

### 3.2 Estimation method

With those data at hand, we proceed by matching the statistics from empirical moments with their simulated counterparts under SL. Appendix C provides the technical details of our estimation method. To do so, we employ the Simulated Moments Method (SMM) as initially developed by McFadden (1989), which provides a rigorous basis to evaluate whether the model is able to replicate salient business cycle properties.

To avoid identification issues, the number of estimated parameters has to coincide with the exact number of matched moments, so that each estimated parameter can be directly mapped onto one empirical moment. Hence, we first calibrate some of the parameters, namely the monetary policy and the preference parameters, as standard in the related literature, see Table 1.

We are left with four structural parameters from the NK model, namely the size of the shocks $\sigma^g$ and $\sigma^u$, the discount factor $\beta$ and the price rigidity parameter à la Calvo $\theta$, which pins down the slope of the NK Phillips curve given that we have fixed the structural parameters $\sigma$ and $\phi$ (see Footnote 8). As for the SL parameters, we need not estimate common values for the inflation and the output gap expectation processes, as the two tournaments are separated, and the two time series are likely to behave differently and exhibit different properties, both in reality and in the model. For instance, estimating inflation and output gap-specific memory parameters $\rho^\pi$ and $\rho^y$ may translate the fact that agents learn that one variable may be more persistent than the other. Hence, we estimate six learning parameters, namely the mutation sizes and frequencies $\xi^m$ and $\mu_m$ as well as the memory of the fitness $\rho^m$ for $m = \{\pi, y\}$.

We now discuss how we map those parameters to the moments that we have chosen to match. Firstly, the standard deviations of the shocks $\sigma^g$ and $\sigma^u$ naturally capture the empirical volatility of output and inflation. Secondly, the discount factor $\beta$ aims to match the ZLB probability: increasing the discount factor mechanically lowers the nominal interest rate through the Euler equation, and
increases the probability of hitting the ZLB. Finally, $\theta$, which tunes the slope of Phillips curve, captures the correlation between the output and inflation gaps.

As for the SL parameters, the memories of the fitness function $\rho^y$ and $\rho^\pi$ tune the sluggishness of the expectations, as they determine the weights of recent versus past shocks in the computation of forecasting performances. The higher $\rho^y$ and $\rho^\pi$, the longer the memory of the agents, the less reactive the learning process and the more sluggish the expectations. Hence, those two parameters are matched with the autocorrelation of, respectively, the output and the inflation gaps.

We use four moments from the SPF data, namely the auto-correlation of inflation and output gap forecast errors $\rho(\hat{y}_t - \mathbb{E}_{t-1}\hat{y}_t, \hat{y}_{t-1} - \mathbb{E}_t\hat{y}_{t-1})$ and $\rho(\hat{\pi}_t - \mathbb{E}_{t-1}\hat{\pi}_t, \hat{\pi}_{t-1} - \mathbb{E}_{t-1}\hat{\pi}_{t-1})$ as well as their standard deviations $\sigma(\hat{y}_t - \mathbb{E}_{t-1}\hat{y}_t)$ and $\sigma(\hat{\pi}_t - \mathbb{E}_{t-1}\hat{\pi}_t)$ to match the four mutation parameters. Sensitivity analysis of the objective function with respect to the estimated parameter values shows that $\mu^y$ and $\mu^\pi$ match most of the autocorrelation of forecast errors, while $\xi^y$ and $\xi^\pi$ capture most of the standard deviations of the same forecast errors.

Finally, in the same vein as Ruge Murcia (2007), we impose prior restrictions on the estimated parameters and treat them as an additional moments in the objective function. The priors for the structural NK parameters are taken from the literature on Bayesian estimation of DSGE models (Smets & Wouters 2007). As for the SL parameters, we impose a Beta distribution for the mutation probabilities $\mu^m$ and the fitness persistence $\rho^m$, $m = \{\pi, y\}$ with a prior mean and standard deviation that deliver posterior results close to the initial parametrization of Arifovic et al. (2013). We further impose to the sizes of mutation $\xi^m$ a positive support with a diffuse prior through an inverse gamma distribution with mean 0.1 and standard deviation of 5.

### 3.3 Estimation results

The matched moments, together with their observable counterparts, are reported in percentage points in Table 2, and Table 3 gives the corresponding estimated values of the parameters.

We first point out that, overall, our model is able to account for a substantial share of the selected moments: almost all ratios between theoretical and observed moments are relatively close to one. Remarkably, we succeed in capturing a substantial share of the persistence in macroeconomic variables with a model that employs only white-noise shocks. Specifically, we reproduce almost all the output gap persistence, and close to two-third of the inflation persistence. Hence, learning acts as an endogenous propagation mechanism that may amplify the effects of

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12 To see why, note that the ZLB binds when $i_t = 0 \iff \hat{r}_t = -\tau = \frac{1+\hat{\pi}}{\beta} - 1$, which is decreasing in $\beta$. 
Moments Ratios

Matched moments Empirical $M_O$ Simulated $M_S$ $M_S/M_O$

<table>
<thead>
<tr>
<th>Moments</th>
<th>$\sigma(y_t)$ - output sd.</th>
<th>4.51</th>
<th>4.510</th>
<th>1.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho(y_t, y_{t-1})$ - output autocor.</td>
<td>0.98</td>
<td>0.8170</td>
<td>0.83</td>
<td></td>
</tr>
<tr>
<td>$\sigma(\pi_t)$ - inflation sd.</td>
<td>0.60</td>
<td>0.9313</td>
<td>1.55</td>
<td></td>
</tr>
<tr>
<td>$\rho(\pi_t, \pi_{t-1})$ - inflation autocor.</td>
<td>0.89</td>
<td>0.5230</td>
<td>0.59</td>
<td></td>
</tr>
<tr>
<td>$\rho(y_t, \pi_t)$ - infl-output correlation</td>
<td>0.09</td>
<td>0.0565</td>
<td>0.63</td>
<td></td>
</tr>
<tr>
<td>$\sigma(\text{Err}_{yt})$ - forecast err. output sd.</td>
<td>0.94</td>
<td>0.7699</td>
<td>0.81</td>
<td></td>
</tr>
<tr>
<td>$\rho(\text{Err}<em>{yt}, \text{Err}</em>{yt-1})$ - forecast err. output autocorr.</td>
<td>0.47</td>
<td>0.4203</td>
<td>0.89</td>
<td></td>
</tr>
<tr>
<td>$\rho(\text{Err}<em>{yt}, \text{Err}</em>{yt-1})$ - forecast err. infl. autocorr.</td>
<td>0.69</td>
<td>0.0729</td>
<td>0.11</td>
<td></td>
</tr>
<tr>
<td>$P(i_t &gt; 0)$ - probability not at ZLB</td>
<td>0.86</td>
<td>0.6726</td>
<td>0.78</td>
<td></td>
</tr>
</tbody>
</table>

| Objective function | $\times$ | 0.74 | $\times$ |

Table 2: Comparison of the (matched) theoretical moments with their observable counterparts

<table>
<thead>
<tr>
<th>Estimated Parameters</th>
<th>Prior Distributions</th>
<th>Posterior Results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Shape</td>
<td>Mean</td>
</tr>
<tr>
<td>$\sigma^g$ - demand shock std</td>
<td>Invgamma</td>
<td>.1</td>
</tr>
<tr>
<td>$\sigma^u$ - supply shock std</td>
<td>Invgamma</td>
<td>.1</td>
</tr>
<tr>
<td>$\beta$ - discount factor</td>
<td>Beta</td>
<td>.99</td>
</tr>
<tr>
<td>$\theta$ - Calvo probability</td>
<td>Beta</td>
<td>.5</td>
</tr>
<tr>
<td>$\mu^y$ - mutation rate for $\text{E}_y$</td>
<td>Beta</td>
<td>.2</td>
</tr>
<tr>
<td>$\mu^\pi$ - mutation rate for $\text{E}_\pi$</td>
<td>Beta</td>
<td>.2</td>
</tr>
<tr>
<td>$\xi^y$ - mutation var. for $\text{E}_y$</td>
<td>Invgamma</td>
<td>.1</td>
</tr>
<tr>
<td>$\xi^\pi$ - mutation var. for $\text{E}_\pi$</td>
<td>Invgamma</td>
<td>.1</td>
</tr>
<tr>
<td>$\rho^y$ - fitness decay rate for $\text{E}_y$</td>
<td>Beta</td>
<td>.5</td>
</tr>
<tr>
<td>$\rho^\pi$ - fitness decay rate for $\text{E}_\pi$</td>
<td>Beta</td>
<td>.5</td>
</tr>
</tbody>
</table>

Table 3: Estimated parameters using the simulated moment method matching the SPF data (1968-2017)

i.d.d. shocks and lead to persistent dynamics. We detail those dynamics in Section 4.2. Furthermore, our model succeeds in producing positive autocorrelation in forecast errors, and matches particularly well the strong value observed in the output gap forecast data. This is an important step forward in the modeling literature, as RE models do not address this important empirical dimension.

Our model also features positive autocorrelation in inflation forecasts, albeit of a milder amplitude than in the SPF data. This underestimate stems from the functional form of the NK Phillips curve itself: given the close-to-unity value of $\beta$, the low value of the slope $\kappa$, and the i.d.d. structure of the shocks, inflation is almost unit-root (under backward-looking expectations), and hence inflation expectations are almost self-fulfilling. Auto-correlation in the errors may not be large. The same feature explains why we need relatively large cost-push shocks
to match inflation volatility.

Turning to the estimated values of the parameters in Table 3, we first remark that the estimated Calvo probability $\theta$ is in line with recent measures in the literature, see e.g. Gourio et al. (2018), that argue for a structural flattening of the Philips Curve. Furthermore, $\beta$ is fairly close to one and implies a realistic value of 2.5% for the natural interest rate, given the 2% inflation target. We also note that the estimated values of the SL parameters are in line with the values usually employed in numerical simulations in the related literature (Arifovic et al. 2013).

Lastly, the estimated values of $\rho_y$ and $\rho_\pi$ imply that agents’ memory amounts to roughly five quarters for forecasting the output gap and seven quarters for forecasting inflation.\footnote{Considering that observations weighting less than 1% are discarded, we have $0.531^8 < 0.01$ and $0.4223^6 < 0.01$.} It is interesting to note that those estimates are broadly in line with empirical estimates from forecasting lab experiment (Anufriev & Hommes 2012) and micro data (Malmendier & Nagel 2016). In empirical macroeconomics, only few lags, often four, are typically considered in estimations of VAR models that are the benchmark for short-term forecasting. All those observations comfort the estimates of a short memory of the SL process.

Since our model is able to capture salient business cycle statistics of both macroeconomic and forecast data, we now analyze the propagation mechanism induced by SL.

4 Dynamics under social learning

This section first describes the reaction of the endogenous variables and expectations to a negative real shock using impulse response functions (IRFs), and then discusses the global dynamics of the model under SL.

4.1 Impulse response functions to a negative real shock

As we are interested in ZLB dynamics, we study the reaction of the model to a large negative real shock that contracts output gap by $-30%$.\footnote{Admittedly, this size of the shock is particularly large, but larger shocks are necessary to produce ZLB dynamics in the case of i.d.d. compared to the case of AR(1) shocks with high auto-correlation, as usually considered in the related literature. An alternative is to trigger a series of small shocks. Results are unchanged.}

The IRFs are obtained in the following manner (see also Arifovic et al. (2013, 2018)). A history of 100 periods is generated at the targeted steady state (Equation (19)). We then draw the initial population of strategies from a normal distribution centered around the targeted steady state values (the standard deviation
being given by the estimated sizes of mutation), apply the shock on \( g \) and compute the corresponding inflation and output gap. Each strategy is then evaluated over the entire past history, including the 100-period initialization, and the simulation proceeds for 600 periods. We perform 1000 Monte Carlo simulations. The IRFs report the median realization (blue line), the 95% percentile (top red dotted line) and the 5% percentile (bottom red dotted line), as well as the average (black line).
Figure 3 reports the IRFs. It is striking that despite the i.d.d. nature of the shock, the downturn is particularly persistent under SL. It should be recalled that the RE counterpart would simply feature a one-period deviation of the endogenous variables from the steady state, with no movement in the expectations, causing the system to return immediately to the target from the next period on. Hence, it is not displayed on Figure 3. In contrast, expectations under SL, and the resulting realization of inflation and output gap take several hundreds of periods to return to steady state.

The sluggish nature of expectations enables the model to endogenously sustain negative inflation gaps along binding ZLB. On average, the ZLB binds for more than 150 periods after the shock. Hence, our model offers a stylized representation of the observed loss of anchorage of long-run inflation expectations depicted in Figure 1 in the introduction.

In our model, prolonged low inflation and ZLB environments are not a result of highly persistent exogenous shocks but stem from the failure of heterogeneous agents to coordinate back onto the socially desirable equilibrium after a one-time—albeit large—shock. The underlying mechanism may be unraveled as follows. First recall that expectations under SL are heterogeneous in any point in time, due to the noise introduced by the mutation process. This means that, even at the steady state, some agents have more pessimistic views, i.e. have lower coefficients $a_{1,j,t}$ and $a_{2,j,t}$, than others, while the average expectations across agents gravitate around the MSV solution $a_{1}^{T}$ and $a_{2}^{T}$. Once a large negative shock hits, the most pessimistic agents, i.e. the agents with the lowest coefficients, realize the best forecasting performances because their forecasts are closer to the negative output and inflation gaps generated by the shock in that period than those of agents whose forecasts are closer to steady state. Hence, the selection pressure of the SL algorithm, i.e. the tournament, will propagate those pessimistic expectations among the population of agents, so as to validate those pessimistic views through the self-fulfilling nature of expectations in the NK model. This process endogenously introduces persistence in the model along with long-lasting low-inflation episodes.

This type of dynamics is reminiscent of the idea of learning as an amplification mechanism, which has been described extensively in the related literature. The SL model provides an endogenous propagation mechanism of aggregate fluctuations through shifts in individual expectations. We stress that recessive episodes are the sole product of coordination of agents’ expectations on pessimistic outlooks.

Looking into the dynamics of expectations in more detail, the long period of accommodating monetary policy in the aftermath of the shock (see the IRF of $\hat{i}$) eventually results in a positive output gap (see the IRF of $\hat{y}$) driven by above steady-state output gap expectations (see the IRF of the average $a_{1} =$
The response of the average output gap expectations overshoots, which gives rise to a prolonged period of expansion with below-target inflation and zero interest rates. This low-inflation boom in the aftermath of the ZLB episode is particularly interesting as it corresponds to the recent experience of the ‘inflation-less recoveries’ in the US and in the Eurozone after the Great Recession, while it is difficult to capture within the RE counterpart of our model (Galí & Gertler 1999). This prolonged period of positive output gap may also suggest that the economy may really settle back to equilibrium only after full tapering by the CB.

Finally, we can quantify the heterogeneity in forecasts in any period or, in other words, the level of disagreement between agents, and look at how it adjusts to the shock. On Figure 3, we denote by $\Delta^\pi$ and $\Delta^y$ the dispersion of individual expectations given by the cross-sectional standard deviation of, respectively, individual inflation and output gap forecasts. The measurements $\Delta^\pi_t$ and $\Delta^y_t$ quantify macroeconomic uncertainty (Rossi & Sekhposyan 2015).

Interestingly, our model reproduces another stylized fact discussed in Mankiw et al. (2003). As clear from the IRF of $\Delta^\pi_e$ and $\Delta^y_e$, a recession is associated with an increase in the dispersion of forecasts among agents. The rise in dispersion does not last because the selection pressure of the learning algorithm pushes agents’ forecasts back towards homogenization in the aftermath of the shock. The level of heterogeneity between agents then returns to its long-run value, dictated by the size of the mutations.

We now turn to a global analysis of the model under SL.

### 4.2 Dynamics under social learning

We examine here the long-run behavior of the model over the entire state space of the endogenous variables $(\hat{\pi}, \hat{y})$. We proceed through Monte-Carlo simulations.

Figure 4a represents the phase diagram of the model, where the average inflation expectation (the average $a_2$ coefficients across agents) is given on the x-axis and the average output gap expectation (the average of the $a_2$ coefficients) on the y-axis. The simulations are initialized in a similar manner as for the IRFs: a history of 100 periods is generated at the targeted steady state, the initial population of strategies is then drawn from a normal distribution centered around a given point $(a_1, a_2)$ of the state space (which is a way to represent an expectational shock), and the corresponding inflation and output gaps are computed. Each strategy is then evaluated over the entire past history, and the simulation proceeds for 1,000 periods. We perform 100 simulations per point of the state space.

---

On our dataset, such a correlation between output gap and output gap forecast dispersion reaches $-.34$ and $-.33$ between output gap and inflation forecasts dispersion.
The phase diagram 4a shows that the model converges to either the target - in grey-shaded areas - or diverges along a deflationary spiral - in white areas. The first interesting message is that the basin of attraction of the target under social learning is larger than under adaptive learning, and is larger than the determinacy region of the targeted steady state.

To see that, it should be recalled that the targeted steady state is locally determinate under RE and locally stable under adaptive learning, while the ZLB state is locally indeterminate, and takes the form of a saddle under learning (see Appendix B for detail and references). Graphically, above the stable manifold associated with the low inflation saddle point under recursive learning (in red on Figure 4a), the targeted steady state is determinate and below, the target is indeterminate. The stable manifold is also the frontier of the basin of attraction of the target under adaptive learning: any point on the right-hand side of it converges to the target, any point on the left-hand side diverges along deflationary spirals (Evans et al. 2008).

Figure 4a shows that there exists a locus of points on the left-hand side of the stable manifold for which the model does converge back to the target under SL, while it is indeterminate under RE and diverges along a deflationary spiral under adaptive learning. This discrepancy is due to a key difference between social and adaptive learning (see also the discussions in Arifovic et al. (2013, 2018)).

An adaptive learning algorithm merely iterates the most recent forecast error, without concerns for their magnitude or alternative forecasting solutions. By contrast, under SL, only the strategies that give raise to the lowest forecast errors over past periods (and not just the most recent one) survive and feed back into the dynamics of the endogenous variables. Hence, a single inflation and output gap data point in the unstable region caused by a one-period pessimistic expectation shift is not enough to steer the whole population of strategies towards a deflationary path. After the pessimistic shift, some strategies are below the target, but still lie above the stability frontier. Because they perform better than those already lying in the unstable area when it comes to forecasting on average over the last few periods – which includes pre-shock dynamics, those mild pessimistic expectations may spread out, eliminate the most pessimistic forecasts, and steer the economy back to the target.

As long as the pessimistic shock is not too large and the population of strategies is not entirely thrown beyond the stable manifold, such a recovery is possible. By contrast, under adaptive learning, a single data point in the unstable area is enough to lead the agents to iterate further their forecast errors, revise downward their expectations and drive the economy along a deflationary spiral. Yet, our model may also lead to self-sustaining deflationary spirals when shifts in expectations are large enough. In such an extreme case, all strategies are pushed beyond
Notes: The targeted steady state is denoted by the green dot, the deflationary steady state by the red one. The ZLB frontier (yellow dashed line) is the locus of points for which \(-\tau = \phi^\sigma \hat{\pi} + \phi^y \hat{y}\); on the left-hand side, the ZLB binds. The stable manifold associated to the saddle low inflation steady state (red line) is computed under recursive learning and corresponds to the stable eigenvector of \(B^{zlb}\); on the left-hand side, the model is indeterminate under RE and E-unstable. For each point of the grid, we run 100 Monte-Carlo simulations over 1,000 periods. The empty area represents pairs of expectations for which the model diverges along a deflationary spiral. Left-hand side: The darker, the higher the probability to converge back to the steady state. Right-hand side: The darker, the faster the convergence back to the steady state.
the region of stability and the deflationary trend kicks in. However for this to happen, as shown by the white area on Figure 4a, the shift on expectations has to be really large.

Another interesting, related, observation is given in Figure 4b. Using the same state space as Figure 4a, the figure reports the speed of convergence to the target for each pair of initial average expectations. The darker the area, the faster the convergence. It is striking to see that the closer to the targeted steady state, the faster the convergence. In general, there is a locus of points spiraling around the target where convergence is fast, which is consistent with the complex eigenvalues associated with that steady state. In contrast, the further from the target, the slower the convergence. For extremely pessimistic expectations, the model diverges along a deflationary spiral (white area), in line with Figure 4a.

Most interestingly, on Figure 4b, the area in the southwest side from the target, beyond the stable manifold, is depicted in light gray. This means that for those severely pessimistic inflation and output gap expectations, the model under SL does converge back to the target, contrary to the model under RE or adaptive learning, but does so at a particularly slow speed. This area is beyond the ZLB frontier (yellow dashed line), which indicates that the ZLB is binding, but yet the model does not diverge along a depressive downward spiral.

Those observations show how our model can produce persistent but non-diverging episodes at the ZLB. Those episodes are triggered by non-correlated shocks that are amplified by the resulting pessimistic shifts in expectations. During those episodes, the population of strategies and the realized output gap and, particularly, inflation gap, are below the target but do not diverge along a deflationary spiral.

Those dynamics partly resolve the issue of explosive and diverging dynamics that arise at the ZLB in standard related models (see the discussion in the introduction). Our model can feature losses in expectation anchorage from the target but not easily sustain deflationary spirals because, in a way, SL agents are smarter than adaptive learners and need first to persistently experience explosive dynamics to coordinate on them. This characteristic of SL agents, namely that they need to ‘see it to believe it’, also influences how communication of the CB may help stabilize the economy. We now analyze this question.
5 Central Bank Communication Experiments

So far, we have shown that SL agents may coordinate on pessimistic expectations which occasionally generate large and persistent contractionary episodes despite the i.d.d. nature of the shocks. From an allocation perspective, these waves of pessimism leave the economy into second-best equilibria with respect to the benchmark model under RE.\(^{16}\) In this section, we first show that the welfare cost entailed by those miscoordination frictions is substantial with respect to the RE allocation. We then introduce an additional monetary policy instrument, namely CB communication, and discuss how it may help enforce the price stability objective.

5.1 Welfare cost of coordination failures

How costly is the presence of miscoordination in the standard workhorse NK macroeconomic model? To evaluate this cost, we use the welfare function which has become the main microfounded criterion to compare alternative policy regimes. Following Michael (2002), we consider a second-order approximation to this criterion and use the unconditional mean to express this criterion in terms of inflation and output volatility. The detailed derivations are deferred to Appendix B. The corresponding micro-founded welfare function reads as:

\[
E[\bar{W}_t] \simeq \bar{W} - \lambda^y \left( E[\hat{y}^2_t] + \lambda^\pi E[\hat{\pi}^2_t] \right),
\]

(12)

where \(\bar{W}\) is the steady state level of welfare, \(\lambda^y\lambda^\pi\) and \(\lambda^y\) are, respectively, the elasticities of the loss function with respect to the variance of inflation \(E[\hat{\pi}^2_t]\) and output \(E[\hat{y}^2_t]\) (see Appendix D.3 for the explicit forms). It is straightforward to notice that macroeconomic volatility reduces the welfare of households.

While in standard macroeconomic models based on the representative agent assumption the loss function is unique, it may be expressed in an agent-specific manner in an heterogeneous-agent framework. Since aggregation of agents is done after linearization of the model, we proceed in the same way with the welfare function, that we linearize up to the second order.\(^{17}\) The welfare criterion provides a metric to compare macroeconomic performances under SL and under RE. Comparing these two allocations results in a measurement of the welfare costs of miscoordination which can be expressed in permanent consumption equivalents (Lucas 2003). Using a standard no-arbitrage condition between the SL and the

\(^{16}\) We refer to the RE counterpart of the NK model as the first-best equilibrium, as we do not study the welfare implications of the price rigidities vs. the first-best allocation under flexible prices, see Woodford (2011).

\(^{17}\) Explicitly, the welfare index is computed as \(E[\bar{W}_t] = \frac{1}{N} \sum_{j=1}^{N} E[\bar{W}_{j,t}]\)
Moments | \(\hat{\pi}_t\) inflation var. | \(\hat{y}_t\) output var. | \(\Delta \pi_t\) forecast dispersion | \(\Delta y_t\) forecast dispersion | \(E[\mathcal{W}_t]\) welfare | \(\lambda\) welfare cost | \(P[r_t=1]\) ZLB probability
---|---|---|---|---|---|---|---
RE | 0.14 [0.0017] | 0.07 [0.0008] | 0 | 0 | -290 [0.1095] | \(\times\) | 0 [0]
SL | 0.86 [0.0548] | 20.35 [2.970] | 0.18 [0.001] | 0.7 [0.0126] | -336 [5.7966] | 6.97% | 0.32 [0.031]

Notes: Average statistics over 5,000 Monte-Carlo runs of 200 periods (50 series of shocks repeated 100 times) under \(\text{SL}\), and average over the same 50 series of shocks under \(\text{RE}\). Standard errors between brackets. Confidence intervals for an average \(X_1\) with standard error \([\sigma_1]\) at 5% can be approximated by \(X_1 \pm 2\sigma_1\).

Table 4: Welfare and business cycles statistics under \(\text{RE}\) and \(\text{SL}\) using estimated parameters.

RE allocations, the fraction of consumption \(\lambda\) that \(\text{SL}\) households are willing to pay to live in a \(\text{RE}\) world solves the following conditions on utility streams:

\[
\sum_{\tau=0}^{\infty} \beta^\tau \frac{1}{N} \sum_{j=1}^{N} \mathcal{U} \left( (1 + \lambda) C^{SL}_{jt+\tau}, H^{SL}_{jt+\tau} \right) = \sum_{\tau=0}^{\infty} \beta^\tau \mathcal{U} \left( C^{RE}_{t+\tau}, H^{RE}_{t+\tau} \right)
\]

where \(x_t^{SL}\) and \(x_t^{RE}\) denote the endogenous variables resulting from the same sequence of shocks under two different expectations schemes (see Appendix D.4).

Table 4 reports the corresponding business statistics of both the rational and social learning expectation schemes using the same estimated parameters as in Table 3. This exercise allows us to disentangle between the contribution of exogenous fluctuations in the \(\text{RE-NK}\) model and those induced by \(\text{SL}\).

The main message from Table 4 may be summarized as follows. Under \(\text{RE}\), there is no uncertainty about the expected levels of inflation and output, which shuts down the propagation channel that is at work under \(\text{SL}\). Hence, under \(\text{RE}\), self-fulfilling ZLB episodes cannot occur, and macroeconomic volatility is negligible. By contrast, under \(\text{SL}\), heterogeneous expectations exacerbate macroeconomic volatility and may trigger endogenous ZLB episodes, as explained in Section 4.2. These self-fulfilling recessions substantially deteriorate the welfare of households.

In absence of an authority coordinating the private sector on the fundamentals, the resulting cost of \(\text{SL}\) expectations with respect to \(\text{RE}\) reaches up to 7% percent of permanent consumption. This welfare cost is high with respect to the real business cycle literature, in particular under CRRA preferences.\(^{18}\) Hence, the

\(^{18}\text{Lucas (1991) finds that the overall welfare cost of business cycles is as low as 0.05\% with}\
contribution of SL on the welfare loss appears non-trivial. The presence of miscoordination of expectations on out-of-target paths questions the effectiveness of monetary policy based on the sole setting of the nominal interest rate and leaves room for an additional monetary policy instrument.

5.2 Communication as an additional policy instrument

We now introduce an additional policy instrument to reduce the welfare-based consumption gap between the SL and the RE regimes. In doing so, we are motivated by the Tinbergen (1952) principle stating that each policy objective requires (at least) one policy instrument. Precisely, we consider CB communication as policy tool that aims to anchor inflation expectations close to the CB target. We now describe the implementation of communication in our model, and then evaluate how it is effective in steering the economy closer to the RE allocation.

5.2.1 Implementing communication

How can the central bank affect the expectations of agents? In real life situations, central banks organize press conferences on a regular basis to announce their policy decisions as well as mere forecasts about future states of the economy. These announcement are found to have significant effects in shaping expectations according to survey data, see e.g. Altavilla & Giannone (2017). In this section, we provide a parsimonious formalization of these announcements and explore their quantitative potential in bringing the economy closer to the RE equilibrium.

Formally, communication is modeled as the CB signaling the MSV solution associated to the socially optimal steady state. It does so even during ZLB episodes, which implies that the CB’s signal does not necessarily correspond to the actual realizations of the endogenous variables. In that sense, and especially when the ZLB is binding, the communication of the CB is rather a promise or a bet. It could only be fully credible in the absence of ZLB, where the dynamics of the model would always imply stability at the targeted steady state, independently from the shocks.

Announcing the MSV solution translates into communicating to the agents the inflation target.\(^{19}\) To integrate this piece of information to the expectation formation process of agents, we modify the social learning algorithm as in Arifovic et al. (2016), albeit in a simpler game.

CRRA preferences. By contrast, with alternative preference schemes, this cost lies between 1 to 25% (Campbell & Cochrane 1999, Tallarini 2000).

\(^{19}\)Central banks usually have no explicit target for the output gap, as the latter has no directly observable counterpart, contrary to the inflation target. Hence, we do not model the communication of an output gap value.
In any period $t$, besides the two components $a_{1,j,t}$ and $a_{2,j,t}$ related, respectively, to the steady state expectations of the output and inflation gaps, the strategy set of each agent $j$ is augmented by a third component, denoted by $\psi_{j,t}$. The component $0 < \psi_{j,t} < 1$ stands for the probability for agent $j$ of following the signal from the CB. If she does so, she anchors her inflation forecasts at the MSV solution. With a probability $1 - \psi_{j,t}$, she does not follow the signal, and set her inflation forecasts as previously. The determination of output gap forecasts is unchanged. Under the communication scenario, the expectation formation process given by (7) is modified as:

$$E_{j,t}^{SL}\{\hat{\pi}_{t+1}\} = \begin{cases} a_{2}^{T} = 0 \text{ with probability } \psi_{j,t} \\ a_{2,j,t} \text{ with probability } 1 - \psi_{j,t} \end{cases}$$ (14)

$$E_{j,t}^{SL}\{\hat{y}_{t+1}\} = a_{1,j,t}$$ (15)

The augmented inflation forecast strategy $\{\psi_{j,t}, a_{2,j,t}\}$ undergoes the same mutation and tournament processes as the the output gap forecast strategy $a_{1,j,t}$. The only difference from the algorithm used so far lies in the computation of the fitness of each inflation forecast strategy. Equation (11) is now modified as follows to account for the two options, namely following the CB’s signal or using their own strategy $a_{2,j,t}$:

$$F_{j,t}^{\pi} = -\psi_{j,t} \sum_{\tau=1}^{t} (\rho^{\pi})^{\tau} (\hat{\pi}_{t-\tau} - a_{2}^{T})^{2} - (1 - \psi_{j,t}) \sum_{\tau=1}^{t} (\rho^{\pi})^{\tau} (\hat{\pi}_{t-\tau} - a_{2,j,t})^{2}$$

The probability $\psi_{j,t}$ can be easily interpreted as the credibility level in the target for agent $j$ in period $t$. If agents following the signal of the CB (i.e. with a high value of $\psi_{j}$) have lower forecast errors than agents not following the signal (i.e. with a low value of $\psi_{j}$), the strategy of following the signal will spread out in the population, and the average value of $\psi$ across the agents shall increase. The opposite holds if followers perform more poorly than non-followers. Thus, SL agents endogenously build trust or distrust in the signal of the CB according to their forecasting performance when following it or not. Another interpretation of this communication policy is that the CB sends a public signal to a fraction only of agents (a fraction $\psi$), or a fraction only of agents are able to understand its signal, which may or may not spread out into the population of agents.\footnote{Such an interpretation echoes the work by Carroll (2003) that describes inflation expectations as epidemiological because expert forecasts only gradually diffuse into the population. Baeriswyl & Cornand (2010) and Walsh (2007) discuss partial but time-invariant transparency.}

We now contrast the outcomes under this communication policy with the SL and the RE benchmarks.
5.2.2 Simulation results

Figure 5 reports the IRFs of the variables under SL with no communication (see the red dashed-dotted line, which corresponds to the median scenario on Figure 3), and under SL with communication for various levels of initial credibility, i.e. various initial distribution of the $\psi_{j,0}$ coefficients across agents (blue lines). The closer to one (resp. zero), the higher (resp. lower) the initial level of credibility in the target. The solid line corresponds to a high credibility scenario: the $\psi_j$ probabilities are initialized around 0.9, which means that on average, 90% of the agents follow the announcement of the CB in the period of the shock. The dashed line represents a mild credibility scenario, where the $\psi_j$ probabilities are initialized around 0.5, while the dotted line represents a low credibility scenario where only 10% of the agents follow the announcement when the shock hits.21 We include those different scenarios as credibility evolves endogenously in our model and the initial level of credibility may have a non-trivial effect on the ability of the CB communication to influence expectations in the aftermath of the shock.

It is clear from Figure 5 that the CB communication attenuates the persistence of the shock, all the more so as the initial level of credibility when the shock hits is high. Only in the low credibility scenario does the ZLB still bind. Yet, the most striking difference is between the no-communication treatment versus the communication treatments, which suggests that communication, even with a relatively low level of credibility, systematically limit the fall in expectations and endogenous variables in the aftermath of the shock.

The reason why this is the case may be explained as follows. When the shock hits, a fraction of agents follow the signal of the CB and keep their inflation expectations anchored to the target. In turn, inflation does not plunge as much as in the absence of those followers, which self-confirms milder pessimistic expectations than in the absence of communication. This coordination of expectations on the target despite the shock is particularly strong in the case of a high level of credibility (see how the heterogeneity of inflation expectations decreases in the wake of the shock in that scenario vs. in all others).

Convergence back to the target is also faster in the presence of communication. This is a direct implication from our analysis in Figure 4b above: as the drop in inflation and inflation expectations is milder when a fraction of the agents follow the CB’s signal, the economy stays closer to the steady state which implies a faster convergence back on target.

\footnote{In the simulations, we use a mutation rate and size of $\mu^\psi = \xi^\psi = \mu^\pi = 0.28$ for the ‘following’ strategy. A milder mutation process only delays the return of the $\psi_j$ probabilities to one after the shock but does not affect the qualitative results.}
Figure 5: Response of the estimated model to a 30% negative real shock under different communication treatments.

Notes: The median realizations over 1,000 Monte Carlo simulations are reported. The shock occurs in period 100. The red dashed-dotted line corresponds to the SL model without communication. The blue solid line reports the high credibility scenario where 90% on average of the agents follow the signal of the CB at the time of the shock, the dashed blue line a mild credibility scenario where 50% of the agents do so, and the dotted blue line a low credibility scenario where only 10% do so. All plots report the zero line. The lower horizontal line on the IRF of $\hat{r}_t$ is the ZLB.
Yet, communication is not a panacea: the drop in inflation and output gaps, albeit of smaller magnitude than in the absence of communication, eventually eliminates the strategy that consists in following the signal of the CB (see the plot of the average $\psi_j$ that invariably drops to zero around period 150). This is because the forecasting performances of the followers are worse than the ones of the non-followers, who hold more pessimistic views. Only once the system has returned to steady state may the CB build back its trust, and this process takes several hundreds periods. In other words, in our model, agents need to ‘see it to believe it’: if the CB’s announcements are too at odds with the actual inflation dynamics, they do not integrate the announcements in their expectations any longer, and the CB communication may not be trivially redundant as it is the case in RE models with perfect information (Blinder et al. 2008).

Before concluding, we provide a systematic analysis of the implications of communication in Table 5. We report the business cycle statistics of the model under the RE benchmark (column i), under SL with no communication (column ii), and under SL with communication and the various levels of initial credibility (columns iii-vi).

With respect to the RE benchmark, the inclusion of communication improves the overall macroeconomic situation, regardless of the initial level of credibility. Announcements about the next period’s inflation rate reduces the cross-sectional dispersion of forecasts in the economy, as well as inflation and output gap volatility. The credibility of the CB is strongly correlated with expectations coordination and macroeconomic stability (see the correlation lines of the average coefficient $\psi$ with various indicators of coordination and stability).

To identify the effects of communication on expectation anchorage at the target, we report an indicator, that we denote by $\Omega$, that quantifies the expectations (un)anchorage as the average squared distance of individual expectations to the targeted values of inflation and output gaps:

\[
\Omega^\pi_t = \frac{1}{N} \sum_{j=1}^{N} E^{SL}_{j,t} \left\{ \hat{\pi}_{t+1} - a_2^T \right\}^2
\]

\[
\Omega^y_t = \frac{1}{N} \sum_{j=1}^{N} E^{SL}_{j,t} \left\{ \hat{y}_{t+1} - a_1^T \right\}^2
\]

The lower those values, the stronger the anchorage of expectations.

As clear from Table 5, communication significantly and strongly anchors expectations. Communication is able to steer inflation expectations closer to the target with respect to the case without any announcement, which reduces the probability of ZLB recessions, as shown previously using the IRFs.
These effects translate in terms of welfare by a reduction of the welfare cost from 7% to 1.4% in the best case of communication policy (with the highest initial level of credibility). This improvement in the welfare measurement translates the idea stressed in the introduction that stronger disagreement among forecasters is empirically associated with higher overall uncertainty surrounding the economy because dispersion of forecasts adds to the fundamental uncertainty induced by shocks.

Finally, it is striking to see that the scenario involving an initial full level of credibility (fourth column) essentially appears indistinguishable from the RE benchmark. Of course, this particularly strong effect of communication on the model properties appears implausible, given that most CBs used to announce their inflation target before the Great Recession. This implausible effect echoes the so-
called ‘forward-guidance puzzle’. While not directly transportable to the frameworks used to study forward-guidance, especially the announcements about future paths of interest rates, the dynamics of credibility observed in our model gives us a clue to the puzzle. The dramatic effects triggered by any policy announcement in standard RE models hinge on the perfect credibility of those announcements. Once imperfect credibility is accounted for, the effects of any announcements may be attenuated. Hence, imperfect credibility is a credible direction to close the bridge between the model outcomes and the empirical observations that has led to the puzzle.

6 Conclusion

We introduce heterogeneous expectations through a social learning mechanism into an otherwise baseline workhorse macroeconomic model with a monetary policy rule subject to the zero-lower bound (ZLB) and white noise shocks. Fluctuations arise from shifts in expectations which may lose their anchorage to the target as soon as agents coordinate on pessimistic outlooks following an adverse, although transitory, shock. Anchoring expectations back on the target may take a long time. This phenomenon is the only source of persistence in our model, which conveniently allows us to isolate the effect of heterogeneous expectations on monetary performances. In particular, our framework nests the RE benchmark, so as to enable a direct quantification of this effect.

We provide three main contributions. First, we take our model to the data and show how it can jointly capture moments from macroeconomic and forecast data, including the duration of the ZLB episode. In that respect, we provide a routine to estimate macroeconomic models under heterogeneous expectations. Second, our model endogenously produces long episodes of ZLB, followed by inflation-less recoveries, that are only caused by the dynamics of misanchored expectations. This narrative echoes the recent US and European experience while being hard to unfold within traditional RE models. Third, we show how communication of the CB’s target may help mitigate this expectation desanchorage following adverse shocks, provide that the level of credibility of the CB is high enough.

Our model offers a simple framework that yet opens up the possibility for the analysis of a rich set of monetary policy alternatives. As for our estimation routine, it may be applied to a wide range of standard workhorse models that could then be explored under heterogeneous expectations. Those research avenues are left for further work.
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A Solution with rational expectations

In what follows, we solve the model with rational expectations using the method of undetermined coefficients, and give the determinacy properties of the two equilibriums with and without the zero lower bound.

First, inserting Equation 5 into Equation 1 removes an endogenous variable, and thus provide a reduced form expression of the linearized model:

\[ z_t = \alpha + BE_t z_{t+1} + \chi^g \hat{g}_t + \chi^u \hat{u}_t, \quad (18) \]

with a couple of endogenous variables \( z_t = \left[ \hat{y}_t; \hat{\pi}_t \right]' \); matrices \( \chi^g, \chi^u \) are related to shocks while \( \alpha \) and \( B \) are matrices related respectively to the model’s steady state and forward looking variables. Here, \( \alpha \) and \( B \) are key matrices as their values switch when the zero lower bound is binding or not.

Given Equation 18, the general form of the MSV solution reads as:

\[ z_t = a + c \hat{g}_t + d \hat{u}_t, \quad (19) \]

where coefficients in matrices \( a, c \) and \( d \) are also switching when the ZLB is binding or not.

Taking expectations based on the specification of the processes Equation 3 - 4 yields:

\[ E(z_{t+1}) = a + c \rho^g \hat{g}_t + d \rho^u \hat{u}_t \quad (20) \]

Inserting Equation 20 back into Equation 18 uniquely identifies the Minimum State Variables (MSV) solution as:

\[ z_t = \alpha + B a + g_t (B c \rho^g + \chi^g) + u_t (B d \rho^u + \chi^u), \quad (21) \]

with \( a = (I - B)^{-1} \alpha, c = (I - B \rho^g)^{-1} \chi^g \) and \( d = (I - B \rho^u)^{-1} \chi^u \), in which the coefficient values of matrices \( B \) and \( \alpha \) depend on the steady state considered.

First, we consider the RE at the targeted steady state, that we denoted by a star superscript. We insert the specification of the Taylor rule (5) when the ZLB is not binding, i.e. \( \hat{\pi}_t = \phi^\pi \hat{\pi}_{t+1} + \phi^y \hat{y}_t + \phi^u \hat{u}_t \) into (1) and obtain the expression for matrices \( \alpha^T = \left[ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right] \) and \( B^T = \left[ \begin{bmatrix} 1 - \sigma^{-1} \phi^y & \sigma^{-1} (1 - \phi^\pi) \\ \kappa (1 - \sigma^{-1} \phi^y) & \beta + \sigma^{-1} (1 - \phi^\pi) \kappa \end{bmatrix} \right] \).

The MSV-RE solution at the target is then given by:

\[ a^T = (I - B^T)^{-1} \alpha^T, c^T = (I - B^T \rho^g)^{-1} \chi^g \text{ and } d^T = (I - B^T \rho^u)^{-1} \chi^u. \quad (22) \]

Similarly, when the ZLB is binding, the monetary policy rule reads as \( \hat{\pi}_t = -\pi \). Inserting this expression back into Equation 1, the RE at the ZLB, that we denote with a \( zlb \) superscript, is described by:

\[ \alpha^{zlb} = \left[ \sigma^{-1} \pi, \kappa \sigma^{-1} \pi \right] \text{ and } B^{zlb} = \left[ \begin{bmatrix} 1 \\ \kappa \beta + \sigma^{-1} \kappa \end{bmatrix} \right]. \quad (23) \]
If shocks are iid., \( \rho^\delta = \rho^\nu = 0 \), and the RE solution reduces to \( z_t = a \), with 
\[
a^T = (I - B^T)^{-1} \alpha^T
\]
at the target and 
\[
a^{zlb} = (I - B^{zlb})^{-1} \alpha^{zlb}
\]
at the ZLB.

**B  Determinacy and E-stability**

The REE (22) is determinate under RE if the two eigenvalues of matrix \( B^T \) lie within the unit circle. This is the case if all three conditions 
\[
\phi^y < \sigma(1 + \beta^{-1}),
\]
\[
0 < \kappa(\sigma^x - 1) + (1 + \beta)\sigma^y < 2\sigma(1 + \beta) \text{ and } \kappa(\phi^x - 1) + (1 - \beta)\phi^y > 0
\]
hold (Bullard & Mitra 2002, p. 1121).

With our parameter values, the REE values at the target are 
\[
a^T = \begin{bmatrix} 0 \\ 0 \end{bmatrix},
\]
and the REE is determinate (the two eigenvalues are \( \lambda^T_- = .92 - .1i \) or \( \lambda^T_+ = .92 + .1i \)). Note that the same conditions ensure that this solution is E-stable, i.e. stable if agents use adaptive learning instead of forming RE (Bullard & Mitra 2002).

By contrast, the REE at the ZLB (23) is indeterminate under RE and unstable under learning. To see that, notice that the characteristic polynomial of \( B^{zlb} \) is 
\[
\beta + \lambda^2 + \lambda(-1 - \beta - \kappa\sigma^{-1}) = 0 \iff a_0 + a_1\lambda + \lambda^2.
\]
For both eigenvalues to be within the unit circle and the REE to be determinate, we need \( |a_0| < 1 \) and \( 1 + a_0 < |a_1| \). The first condition always holds as \( \beta < 1 \) but the second is always violated as \( \sigma^{-1}\kappa > 0 \). Therefore, the deflationary state is indeterminate under RE.\(^{22}\)

Note that the determinant of \( B^{zlb} - I \) (\( I \) being the identity matrix) is \( -\sigma^{-1}\kappa < 0 \), which implies that one eigenvalue of \( B^{zlb} - I \) has negative real part and one has positive real part (equivalently, one eigenvalue of \( B^{zlb} \) is within the unit circle, the other is not). Therefore, under learning, the deflationary steady state is unstable and is a saddle (see also Mccallum (2002) and Honkapohja et al. (2017)).

Under our calibration, the REE values at the ZLB are 
\[
z_t = a^{zlb} = \begin{bmatrix} -0.026 \\ -0.029 \end{bmatrix},
\]
and the two eigenvalues of \( B^{zlb} \) are \( \lambda^z_- = 0.83 \) or \( \lambda^z_+ = 1.16 \).

**C  Estimation strategy**

The standard workhorse New Keynesian model - described in Equation 1 to 5 - can be expressed in the following compact form:
\[
E_t^* \{ f_\Theta(z_{t+1}, z_t, z_{t-1}, \varepsilon_t) \} = 0 \quad \text{for } * = \{ \text{RE, SL} \}
\]
(24)
where \( z_t \) is the set of endogenous variables, \( \varepsilon_t \) the set of i.i.d. innovations and \( f_\Theta(\cdot) \) the model’s equations using calibration \( \Theta \).

\(^{22}\)Note that the ZLB corresponds to the situation of an interest rate peg as described in Woodford (2011, Chap. 2) that gives rise to indeterminacy.
In the following fit exercise, we originally consider the ZLB as an explicit objective to match along other standard business cycle moments. We use an optimized version of the algorithm of Arifovic et al. (2013) or Arifovic et al. (2018) to speed up the estimation.

We partition the parameters $\Theta$ into two sets: the first set contains mainly monetary policy and preferences parameters which we calibrate following the literature as given in Table 1. The second set $\theta \in \Theta$ contains parameters that we estimate by minimizing the distance between simulated and empirical moments.

In order to do so, let us define $m_T(x_t)$ a $p \times 1$ vector of moments calculated using stationary and ergodic$^{24}$ real data $x_t$ of sample size $T$, and $m_{s,\tau}(\hat{x}_t^\theta)$ the model-generated counterpart based on artificial series $\hat{x}_t^\theta$ of size $\tau$ generated using the set of parameters $\theta$. To get an unconditional measure of simulated moments, we exploit asymptotic properties of Monte-Carlo methods by sampling $s$ different sequences of shocks of size $\tau$ and compute the unconditional moment as an average from the $s$ moments from each sequence. To ensure that each iteration of the optimization algorithm is performed in every iteration, we randomly draw $s$ different sequences of shocks of size $\tau$ at the initialization of the fit exercise, and keep them unchanged during the optimization. Specifically, we generate artificial series of size $\tau = 300$ and drop the first 100 draws, so as to have 200 quarters to match as in our real time time series. These artificial series are drawn $s = 20$ different times to approximate the unconditional moments used in the objective function. Artificial series $\hat{x}_t^\theta$ are obtained from models in Equation 24 that are solved under social learning.

Following Ruge Murcia (2007), we include priors, denoted $P(\theta)$, into the objective function. As in Bayesian econometrics based on the optimization of a log-likelihood function, we linearize the contribution of the priors by applying a log, so that the objective function is the sum of the square distance of the moments plus the sum of the log-priors. Unlike full information methods where the number of observations is large, here the SMM methods is a weak information method that relies on a very small number of observations, making the contribution of

---

$^{23}$The mutation involved by social learning may generate unstable dynamics when one (or a combination) of mutation(s) is too large. The solution usually adopted by Arifovic et al. (2013) is to draw a large number of parallel economies and select the median of them. However, the selection process of an average stable path is computationally intensive and slows down the optimization exercise. We solve this issue by selecting the median, prior to the optimization, which allows to speed up the algorithm and makes the estimation as quick as for the RE models. This selection process of the stable median prior the estimation is comparable to selecting the stable roots in the policy function of a RE model.

$^{24}$The use of matching moment methods requires the simulated time series from the model to be ergodic. Prior estimating the model, we have checked that moments remain very similar across different chains with different starting values. This result holds as long as the sample is large enough and by dropping first draws.
priors information too important. As following Ruge Murcia (2007), we introduce
a parameter $\phi$ that allows to weight the information provided by priors.

The SMM estimator is defined as:

$$\hat{\theta}_{SMM} = \arg \min_{\theta} \left[ m_T (x_t) - m_{s,\tau} (\hat{x}_t^\theta) \right]^T W \left[ m_T (x_t) - m_{s,\tau} (\hat{x}_t^\theta) \right] + \phi P (\theta)$$

where $m_T (x_t) - m_{s,\tau} (\hat{x}_t^\theta)$ is the distance vector between the observed and the
simulated moments that we seek to minimize, and $W$ is the weighting matrix. The
matrix product in Equation 25 provides the sum of the squares of the residuals
between observed and matched moments.

We solve Equation 25 using the CMAES optimization algorithm of Hansen
et al. (2003). The CMAES algorithm is a global estimation strategy that has
the advantage of dealing with large-scale optimization problems and providing
an accurate measure of the Hessian matrix, even in the presence of bound re-
strictions and priors for control variables in Equation 25. Specifically, learning
the covariance matrix in the CMAES is analogous to learning the inverse Hessian
matrix in a quasi-Newton method which allows the algorithm to apply an efficient
evolutionary process.

\section{Welfare criterion}

In this section, we develop the approximation to the welfare criterion.

\subsection{The welfare in terms of output and prices}

Before approximating the welfare function, we first rewrite the welfare function
by expressing the utility function in terms of output and price equivalents. Recall
that:

$$\mathcal{W}_t = \sum_{\tau=0}^{\infty} \beta^T \mathcal{U} (C_{t+\tau}, H_{t+\tau})$$

with $\mathcal{U} (C_t, H_t) = \log(C_t) - \frac{\chi}{1 + \phi} H_t^{1+\phi}$

In absence of physical capital, the resource constraint reduces to:

$$\int_0^1 y_{it} \text{d}i = Y_t = C_t,$$

where aggregate production is the sum of the production of each variety $i$ of goods
in the economy. This equation allows us to substitute output for consumption in
the utility function. We know need to do the same for the hours. Let us start with the optimal demand for good $i$ which we aggregate:

$$
\int_0^1 \left( \frac{p_{it}}{P_t} \right)^{-\epsilon} Y_t \, di
$$

where $\epsilon > 1$ is the elasticity of substitution between differentiated types of goods $i$, $p_{it}$ is the price of variety $i$, and $P_t$ the aggregate price level of all varieties in the economy.

Market clearing imposes (29) to be equal to (28). Using a constant-return-to-scale production function, $y_{it} = h_{it}$ and Eq. (28) reduces to $\int_0^1 y_{it} \, di = H_t$.

In Eq. (29), the term $\int_0^1 \left( \frac{p_{it}}{P_t} \right)^{-\epsilon} \, di$ is the price dispersion across varieties $i$ induced by the price stickiness, that we rewrite as $\Delta_t$. To summarize, market clearing implies:

$$
H_t = \Delta_t Y_t
$$

Substituting consumption $C_t$ and labor $H_t$ into the utility function using expressions (28) and (30):

$$
U(Y_t, \Delta_t Y_t) = \log(Y_t) - \frac{\chi}{1 + \phi} (\Delta_t Y_t)^{1+\phi}
$$

where $\chi = (\epsilon - 1)/\epsilon$ is the inverse of the markup in the economy.

The price dispersion $\Delta_t$ is hard to interpret and has no observable counterpart in macroeconomic time series. Following Woodford (2011), in the next section, we express price dispersion in term of inflation.

### D.2 Price dispersion

The price dispersion is induced by the Calvo probability $\theta$ that constrains firms in updating their price. Following Schmitt-Grohé & Uribe (2004), we can rewrite the price dispersion as:

$$
\Delta_t = \int_0^1 \left( \frac{P_{it}}{P_t} \right)^{-\epsilon} \, di
$$

$$
= (1 - \theta) \left( \frac{P^*_{t}}{P_t} \right)^{-\epsilon} + \theta \Delta_{t-1} \pi_t \bar{\pi}^{-\epsilon}
$$

where $\bar{\pi}$ is the rate of inflation in steady state.

Now that we have an expression for the price dispersion, we need to rule out the optimal price $P^*_t/P_t$ from the previous expression and replace it by the inflation rate. To do so, we use the aggregation condition on prices of constrained
firms and firms that can update their price is approximated using the law of large numbers:

\[ P_t^{1-\epsilon} = \int_0^1 P_{it}^{1-\epsilon} \, di \]  

(34)

\[ P_t^{1-\epsilon} = \theta \bar{\pi} P_{t-1}^{1-\epsilon} + \frac{1-\theta}{P_t^*} (1-\epsilon) \]  

(35)

Dividing by \( P_t^{1-\epsilon} \), the price index becomes:

\[ 1 = \theta \left( \frac{\pi_{t-1}}{\bar{\pi}} \right)^{1-\epsilon} + \frac{1-\theta}{P_t^*} (1-\epsilon) \]  

(36)

From the latter expression, the relative optimal price \( \frac{P_t^*}{P_t} \) is a non-linear function of past inflation:

\[ \frac{P_t^*}{P_t} = \left( \frac{1-\theta}{\bar{\pi}^{1-\epsilon}} \right)^{1/(1-\epsilon)} \]  

(37)

Combining Equation 36 and Equation 37, the price dispersion term \( \Delta_t \) may be expressed in term of inflation:

\[ \Delta_t = (1-\theta) \bar{\pi}^{1-\epsilon} \left( \frac{1-\theta}{\bar{\pi}} \right)^{1/(1-\epsilon)} \Delta_{t-1} \left( \frac{\pi_t}{\bar{\pi}} \right)^\epsilon. \]  

(38)

In this latter expression, \( \Delta_t \) is a function of inflation rates in \( t \) and \( t-1 \) as well as previous dispersion \( \Delta_{t-1} \). It's not possible to get an expression of the price dispersion as an unique function of inflation. However up to second order, the variance is unconditional and allows to express the variance of the price dispersion as a function of inflation as Woodford (2011).

D.3 Approximation up to second order

Recall that in our setup, we have normalized to one hours worked, \( \bar{H} = 1 \), which through the production function, normalize to one the production, \( \bar{Y} = \bar{H} = 1 \), which also normalizes the consumption through the resources constraint, \( \bar{C} = \bar{Y} = 1 \). In addition, since the following exercise does not aim at determining the optimal rate of inflation, we simply normalize to one the price dispersion term \( \bar{\Delta} = 1 \) and make it independent of the steady state inflation rate.

Let us first consider the approximation of the welfare utility function in Equation 31, the left hand side up to second order reads as:

\[ \log(Y_t) \simeq \log(\bar{Y}) + \frac{1}{\bar{Y}} (Y_t - \bar{Y}) - \frac{1}{2} \frac{1}{\bar{Y}^2} \left( \bar{g}_t - \bar{Y} \right)^2 \]  

(39)

\[ \simeq \log(\bar{Y}) + \left( \bar{g}_t + \frac{1}{2} \bar{g}_t^2 \right) - \frac{1}{2} \bar{g}_t^2 \]  

(40)

\[ \simeq \log(\bar{Y}) + \bar{g}_t \]  

(41)
where $\hat{y}_t = \left( Y_t - \bar{Y} \right) / \bar{Y}$. Since there is no second order term, the unconditional mean of the utility function on consumption is simply zero:

$$E \left[ \log(Y_t) \right] \approx 0,$$

as the unconditional mean of a normally distributed random variable $x_t$ allows to get rid off the first order terms which are zero asymptotically $E \left[ x_t \right] \approx 0$.

Concerning the left hand side, up to second order:

$$\frac{X}{1 + \phi} \left( \Delta_t Y_t \right)^{1+\phi} \approx \frac{X}{1 + \phi} \left( \bar{Y} \Delta \right)^{1+\phi} + \frac{X}{h} \left( \bar{Y} \Delta \right)^{1+\phi} \left( Y_t - \bar{Y} \right) + \frac{\phi X}{2} \frac{\bar{Y} \Delta}{Y^2} \left( \bar{Y} \Delta \right)^{1+\phi} \left( \bar{Y} - \bar{Y} \right)^2$$

$$+ \frac{X}{\Delta} \left( \bar{Y} \Delta \right)^{1+\phi} \left( \Delta_t - \bar{\Delta} \right) + \frac{\phi X}{2} \frac{\bar{Y} \Delta}{\Delta^2} \left( \bar{Y} \Delta \right)^{1+\phi} \left( \Delta_t - \bar{\Delta} \right)^2$$

Recall that we have normalized to one steady state, and the parameter $\chi$ is the inverse of the markup:

$$\frac{X}{1 + \phi} \left( \Delta_t Y_t \right)^{1+\phi} \approx \frac{\epsilon - 1}{\epsilon} \left[ \frac{1}{1 + \phi} + \hat{y}_t + \frac{1 + \phi}{2} \hat{\pi}_t + \frac{1 + \phi}{2} \hat{\pi}_t^2 \right]$$

The unconditional mean of the right term of the utility function reads as:

$$E \left[ \frac{X}{1 + \phi} \left( \Delta_t y_t \right)^{1+\phi} \right] \approx \frac{\epsilon - 1}{\epsilon} \left[ \frac{1}{1 + \phi} + \frac{1 + \phi}{2} \left( E \left[ \hat{\pi}_t^2 \right] + E \left[ \hat{\pi}_t^2 \right] \right) \right]$$

Gathering Equation 42 and Equation 43, the unconditional mean of the utility function $U \left( \cdot \right)$ as defined in Equation 31 up to second order reads as:

$$E \left[ U \left( \cdot \right) \right] \approx - \frac{\epsilon - 1}{\epsilon} \frac{1}{1 + \phi} - \frac{\epsilon - 1}{\epsilon} \frac{1}{2} E \left[ \hat{\pi}_t^2 \right] + \frac{1 + \phi}{2} E \left[ \hat{\pi}_t^2 \right]$$

It’s straightforward to notice that the expression of the utility function includes the price dispersion term variance $E \left[ \hat{\pi}_t^2 \right]$. To get a closed form expression of this variance, we perform a second order approximation to the expression in Equation 38:

$$1 + \Delta_t + \frac{1}{2} \Delta_t^2 \approx (1 - \theta) + \frac{\theta \left( \hat{\pi}_t + \frac{1}{2} \hat{\pi}_t^2 \right) + \frac{1}{2} \theta \left( 2\theta + (\epsilon - 2) \right) \hat{\pi}_t^2}{(1 - \theta)}$$

$$+ \theta \left( 1 + \hat{\pi}_{t-1} + \frac{1}{2} \hat{\pi}_{t-1}^2 + \epsilon \left( \hat{\pi}_t + \frac{1}{2} \hat{\pi}_t^2 \right) + \frac{1 + \phi}{2} \hat{\pi}_t^2 \right)$$

The unconditional mean of the price dispersion allows us to obtain the expression of the variance of the price dispersion term:

$$E \left[ \Delta_t^2 \right] \approx \frac{\theta [\epsilon + 2 (1 - \theta) (\epsilon - 1)]}{(1 - \theta)^2} E \left[ \hat{\pi}_t^2 \right],$$

41
where \( E[\Delta_2^2] = E[\Delta_{t-1}^2] \) and \( E[\pi_2^2] = E[\pi_{t-1}^2] \) as the variance of a random variable is unconditional. This expression is very close to the expression of Michael (2002), except that the expression is not conditional on the value of \( \hat{\Delta}_{t-1} \) being set to zero. Woodford had made this assumption to get a more digest expression for the price dispersion by using the slope of the NK Phillips curve to obtain the expression (45). Here, we do not use this shortcut on the initial value of \( \hat{\Delta}_{t-1} \) to obtain Equation 45.

It can be shown that the first and second-order derivatives of the price dispersion terms are given by:

\[
\frac{\partial}{\partial \pi_{t-1}} (1 - \theta) \left( 1 - \theta \bar{\pi}^{1-\epsilon} \pi_{t-1}^{\epsilon-1} \right)^{1/(1-\epsilon)} \bigg|_{\pi_{t-1} = \bar{\pi}} = \frac{\theta \bar{\pi}}{\pi \epsilon} \\
\frac{\partial^2}{\partial^2 \pi_{t-1}} (1 - \theta) \left( 1 - \theta \bar{\pi}^{1-\epsilon} \pi_{t-1}^{\epsilon-1} \right)^{1/(1-\epsilon)} \bigg|_{\pi_{t-1} = \bar{\pi}} = \frac{\theta (2\theta + \epsilon - 2)}{\bar{\pi}^2 (1 - \theta)}
\]

Replacing the price dispersion term Equation 45, we get the final expression of the utility function up to second order:

\[
E[\mathcal{U}(\cdot)] \simeq -\frac{\epsilon - 1}{\epsilon} \frac{1}{1 + \phi} - \frac{\epsilon - 11 + \phi}{\epsilon} \left[ E[\hat{\gamma}_t^2] + \theta \left[ \epsilon + 2 (1 - \theta) (\epsilon - 1) \right] \frac{E[\hat{\pi}_t^2]}{(1 - \theta)^2} \right]
\]

The last step it to obtain the welfare index from the utility function. Recall that the welfare is defined by the discounted sum of upcoming utility streams. Put recursively, the welfare index reads as:

\[
W_t = \mathcal{U}(C_t, H_t) + \beta W_{t+1}.
\]

The unconditional mean of the welfare reads as:

\[
E[W_t] = \frac{1}{1 - \beta} E[\mathcal{U}(C_t, H_t)]
\]

\[\text{The Mathematica routine that we used for this purpose is available upon request.}\]
as $E[\mathcal{W}_t] = E[\mathcal{W}_{t+1}]$. Finally, replacing the utility function into the previous expression:

$$E[\mathcal{W}_t] \simeq -\frac{\epsilon - 1}{\epsilon (1 - \beta)} \frac{1}{1 + \phi} - \frac{\epsilon - 1}{\epsilon (1 - \beta)} \frac{1 + \phi}{2} \left[ E[\hat{g}_t^2] + \frac{\theta [\epsilon + 2 (1 - \theta) (\epsilon - 1)]}{(1 - \theta)^2} E[\hat{\pi}_t^2] \right]$$

$$\equiv \tilde{W} - \lambda^\pi E[\hat{g}_t^2] + \lambda^\pi \lambda^y E[\hat{\pi}_t^2]$$

(49)

with $\tilde{W} \equiv -\frac{\epsilon - 1}{\epsilon (1 - \beta)} \frac{1}{1 + \phi}$ the steady state level of welfare, $\lambda^\pi \equiv \lambda^y \frac{\theta [\epsilon + 2 (1 - \theta) (\epsilon - 1)]}{(1 - \theta)^2}$ and $\lambda^y \equiv \frac{\epsilon - 1}{\epsilon (1 - \beta)} \frac{1 + \phi}{2}$, respectively, the elasticities of the loss function with respect to the variance of inflation $E[\hat{\pi}_t^2]$ and output $E[\hat{g}_t^2]$.

**D.4 Welfare cost**

Suppose now that we want to measure the welfare cost of one regime with respect to another one. As in Lucas (2003), we look for the fraction $\lambda$ of utility that the household would be willing to pay to live regime A rather than regime B through this no arbitrage condition on welfare indexes in both regimes:

$$\sum_{t=0}^{\infty} \beta^t U \left( (1 + \lambda) C_t^A, H_t^A \right) = \sum_{t=0}^{\infty} \beta^t U \left( C_t^B, H_t^B \right)$$

(50)

Approximating up to second order this expression, using the proof in Equation 49, then the expression of the welfare cost between policy regimes are given by:

$$\lambda = \exp((1 - \beta) \left( \lambda^\pi \left[ \text{var}(\hat{\pi}_t^A) - \text{var}(\hat{\pi}_t^B) \right] + \lambda^y \left[ \text{var}(\hat{g}_t^A) - \text{var}(\hat{g}_t^B) \right] \right)) - 1$$

(51)