When Old Meets Young? Germany’s Population Ageing and the Current Account

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Abstract

In a three-region New Keynesian perpetual youth model calibrated to Germany, the Euro area (without Germany) and the rest of the world, we analyze the impact of population ageing in Germany on its net foreign asset and current account developments. Assuming unsynchronized demographic trends by taking those of Germany as given and constant population everywhere else, we are able to generate German current account surpluses of more than 8% of GDP during the first half of this century. However, projected demographic trends from 2000 to 2080 in Germany, the rest of the Euro area and the remaining OECD countries (plus China in a supplementary analysis) are much more synchronized. Feeding these into our model, simulations suggest that the average annual German current account surplus from 2000 to 2018 reduces to only 1.5% (0.33%) of GDP (when taking into account China), turning negative around 2030 already.

Keywords: Population Ageing, Net Foreign Assets, Global Imbalances, DSGE Models (JEL: E43, E44, E52, E58)

1. Introduction

The issue of global imbalances has returned to public (policy) debates with momentum. A prominent example often mentioned in these debates is the high and persistent German current account surplus. During the 1990s, it fluctuated around -1% of GDP. At the turn of the millennium, it started improving, reaching a level of 7% of GDP by 2008, further increased to 9% in 2015 and still remains high, currently standing at 8% of GDP.

The surplus has long been criticized by the European Commission’s Macroeconomic Imbalance Procedure, the IMF’s External Imbalance Assessment and others, and many urge the German government to cut it down (see EC 2016; IMF 2018; The Economist 2017). Last year, the US administration even threatened to impose tariffs on imports from Germany (and other surplus countries) to deal with the issue.

To date, the German government has remained rather intractable in dealing with its current account surplus. It argues that there are no obvious policy failures and
that the surplus is an outcome of market-based adjustment to developments outside the control of the government (BMF, 2017). Population ageing is said to be one of the most important drivers of these developments (Busl et al., 2012; Felbermayr et al., 2017; Bundesbank, 2018). This statement can qualitatively be supported by findings in the academic literature dealing with global imbalances, not necessarily focusing on Germany, however (see, for example, Blanchard and Milesi-Ferretti, 2010; Ferrero, 2010; Backus et al., 2014; Kollmann et al., 2015; Dao and Jones, 2018).

In this paper, we analyze the contribution of population ageing in Germany on its net foreign asset and current account positions based on a three-region New Keynesian perpetual youth model. Two of the regions (Germany and the rest of the Eurozone) form a monetary union. The third region represents the rest of the world. We find that population ageing can indeed be a significant driver of net foreign asset and current account developments, with the potential of generating surpluses of around 8% of GDP, a value that we actually see in the data. However, this only happens when the demographic trends of the ageing region and its trading partners are rather unsynchronized. Put more vividly, “when old meets young”.

Assuming Germany’s most important trading partners to be the other Euro area member states and the remaining OECD countries (plus China in a supplementary analysis), we see in Figure 1, which we will discuss in more detail below, that population dynamics are far from being unsynchronized. All region face population ageing, and those regions that are young(er) today will age more quickly until 2080. But indeed, the pattern of ageing differs. The baby boomer generation is about 10 years older in Germany than it is in the other regions. And the decrease in total population in the other regions will be less severe than in Germany.

Feeding these developments into our model, simulations suggest that Germany will export capital (i.e. have a positive net foreign asset position) until beyond mid-century and experience a positive current account surplus until about 2030, broadly a result of the baby boomers reaching retirement age earlier. These results are in line with Marchiori (2011), who shows that Western (European) economies are capital exporters during the first half of the century and that this may change thereafter. The average yearly current account of GDP ratio from 2000 to 2015 in our model simulations is just slightly above 1.5% (or 0.33% when taking into account China). Relative to the average value of 4.3% from IMF data, our model simulations thus suggest that, at maximum, German population ageing can only explain about one third of the actual developments.

Furthermore, note that the average yearly current account of GDP ratio from 2000 to 2015 of 1.5% comes pretty close to the values reported by the IMF until 2003. Thereafter, it starts increasing rapidly according IMF data, but not in our model simulations. Taking these results serious, something else must have happened to boost the Germany current account surplus. And indeed, 2003 was the year in which Germany initiated significant labor market reforms by increasing labor market matching efficiency and by lowering the generosity of unemployment benefits. Hochmuth et al. (2019) show in a model with cross-sectional heterogeneity and a precautionary savings motive that these
reforms could be responsible for another quarter of the current account surplus.\footnote{Kollmann et al. (2015) estimate a conventional DSGE model to assess the drivers of the German current account surplus. Executing a historical shock decomposition, they find that negative wage markup shocks (which they relate to the labor market reforms) and shocks to the discount rate (which they relate to population ageing) account for about 60% of the current account developments. Adding up our findings and those by Hochmuth et al. (2019), these values come pretty close. Empirical evidence for positive current account effects of labor market liberalization is provided by Bertola and Lo Prete (2015). Other reasons could also be, for example, financial integration (Attanasio et al., 2006; Mendoza et al., 2009) and economic growth in emerging markets (Caballero et al., 2008).}

Hence, the main takeaway of our paper is that, while certainly an important driver, attributing too much of the German current account developments to ageing may be jumping too short. Further research to fully understand the effects at play is certainly in order.

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**Figure 1: Population developments**

**Notes:** Figure plots (projected) population developments for Germany (blue solid lines), the rest of the Euro area excluding Germany (red dashed lines), the remaining OECD countries (greed dotted lines) and the remaining OECD countries plus China (cyan crossed lines) from 2000 to 2080; source: OECD (2017). Population in 2000 is normalized to one in the first three subplots to make results comparable more easily. Total population was 81.488 million (Germany), 237.726 million (rest of the Euro area), 840.982 million (remaining OECD countries) and 2,124.181 million (remaining OECD countries plus China) in 2000.

The model we use for analyzing the question is a three region-version of New Keynesian OLG model presented by Kara and von Thadden (2016), who provide a monetary extension of the Gertler (1999)-OLG model. In contrast to the standard open-economy New Keynesian modelling framework with an infinitely-lived representative agent,
our OLG setting generates steady-state determinacy and stationarity of net foreign assets as well as an endogenous world interest rate (see Ghironi, 2008; Ferrero, 2010; Di Giorgio and Nistico, 2013; Di Giorgio et al., 2018; Di Giorgio and Traficante, 2018, for an in-depth discussion). This allows for a thorough analysis of the effects of population ageing on current account and net foreign asset positions. International trade and asset flows are introduced in a similar way as in Ferrero (2010). By additionally including regionally differentiated goods and home bias in consumption and investment, we are not only able able to analyze the effects on the trade balance and net foreign assets, but we can also address the impact of ageing on international competitiveness.

The literature has shown that population ageing can indeed be a significant driver of net foreign asset and current account positions (see, among others, Poterba, 2001; Börsch-Supan et al., 2003; Krueger and Ludwig, 2007; Börsch-Supan and Ludwig, 2009; Carvalho et al., 2016). First of all, the level of aggregate savings within an economy tends to increase in economies with an ageing society. This is because of two reasons. First, given a longer life expectancy, the time span over which households receive the relatively lower pension income increases. For consumption smoothing reasons, individuals prepare for this by increasing individual savings efforts. Second, as the composition between old and young changes, and because elderly people tend to have more assets, the amount of aggregate savings within an economy increases “by construction”. Moreover, less capital is needed in societies with a declining working-age population as fewer people need less (productive) capital. In a closed economy, this leads to a fall in the “natural rate of interest” as the price of capital falls. In our model, too, these mechanisms are present and are an important driver of behavioral changes and the economic effects.

In an open economy with an ageing society, higher supply and lower demand of capital within an economy lead to capital exports. This increases net foreign assets and the current account (Blanchard and Milesi-Ferretti, 2010; Ferrero, 2010; Backus et al., 2014; Börsch-Supan et al., 2006, 2014; Eugeni, 2015). Furthermore, it potentially mitigates the negative interest rate effect unless the ageing economy’s trading partners also increase their savings sufficiently to overcompensate for this effect. In a calibrated multi-region model, Brooks (2003) finds that European countries and Japan become capital exporters with respect to North American and African countries because those regions have a much younger society. However, as is discussed above and also mentioned in Dao and Jones (2018), one prerequisite for this to happen is that the demographic trends of the ageing region and its trading partners must be rather unsynchronized.

Figure 1 plots the (projected) population developments from 2000 to 2080 in Germany, the rest of the Euro area and the remaining OECD countries (plus China), which we consider to be the most relevant trading partners to Germany and Europe in our benchmark analysis. The data source is OECD (2017). Population is aged 20 to 100 (hence, we exclude younger individuals), and the old age dependency ratio (OADR) is defined as the population above 65 divided by the population between 20 and 65. We can observe the following. Societies in Germany and in the rest of the Euro area were and are indeed older than those in the remaining OECD economies. German working-
age population declined since the beginning of the 2000s and is projected to steadily decline until 2080. There is a similar process in the rest of the Euro area economies starting in 2010, while a decline in the working-age population in the remaining OECD countries has not yet started. It is projected to do so by 2030, but the decline is significantly less severe. Total population falls in Germany, slightly increases in the rest of the Euro area before falling back to its initial value in 2080 and increases sharply in the remaining OECD economies. The latter, however, is largely driven by a disproportionate increase in the population aged 65 and above. This can also be seen in the OADR. It basically increases by the same amount in the remaining OECD countries (from 20% in 2000 to 56.2% in 2080) compared to the increase in Germany (from 26.5% to 62.8%) and the rest of the Euro area (from 26.7% to 63%).

The pattern of the increase in the OADR is somewhat different, however. It is rather steady in the remaining OECD economies and contains wiggles in Germany during the periods 2000 to 2010 and 2030 to 2040. The former can be explained by a relatively sharp decrease in fertility and the latter by the retiring baby boomers. This latter baby boomer effect is also present in the rest of the Euro area with a ten year delay. If we take on board China, we see that total population in the rest of the world is projected to also start falling by around 2030, working-age population is projected to start falling quickly by around 2020 and the elderly population is projected to increase even more. This can also be seen by the much faster increase in the projected OADR (from 14.8% in 2000 to 58.1% in 2080).

While the demographic trends across these regions are certainly not entirely synchronized, there is a common pattern: population becomes older. And, at least when taking into account the entire projection period until 2080, it does so equally or even more quickly for the regions that are younger today. The ageing “problem” there is just postponed. When feeding in these population processes into our model and performing a perfect foresight simulation, we get the results described above and find that, while population ageing indeed contributed/s to the current account developments in Germany, they are most likely not the one most important driver.

The rest of the paper is structured as follows. Section 2 describes the model. Its calibration is explained in Section 3. The analysis is undertaken in Section 4 and Section 5 concludes. An appendix with some more details on the model and its computation as well as some supplementary analyses is added.

2. The model

In this section, we build a New Keynesian three-region perpetual youth model. Regions are indexed by $i = a, b, c$. Two of the regions, $a$ and $b$, form a monetary, while the third region, $c$, represents the rest of the world. In order to introduce population ageing, we follow Fujiwara and Teranishi (2008) and Kara and von Thadden (2016) to extend the non-monetary overlapping-generations model of Gertler (1999). Each region $i$ produces differentiated goods that are tradeable across countries. They are purchased by households according to their preferences in their consumption and investment baskets.
along the lines of Di Giorgio and Nistico (2013) and Di Giorgio and Traficante (2018). Regions differ in size, their demographic developments and other structural parameters. This implies that net foreign asset positions and the world interest rate are determined endogenously, also in steady state. Later on, the model and simulations will be solved in a fully non-linear fashion under perfect foresight.

2.1. Demographic structure

In the spirit of Gertler (1999), population in each region $i$ consists of two distinct groups: workers, $N_{i}^{w,i}$, and retirees, $N_{i}^{r,i}$, where the superscripts $w$ and $r$ denote variables/parameters relevant for the corresponding group. Each individual is born as a new worker. The working-age population grows at rate $n_{i}^{w,i}$. Conditional on being a worker in the current period, an individual faces a probability $\omega_{i}^{i}$ of remaining a worker in the next period. Retirees face a survival probability $\gamma_{i}^{i}$ and die with probability $(1 - \gamma_{i}^{i})$. As these two states are successively reached by individuals, our model gives rise to a life-cycle pattern.

In order to facilitate aggregation within each group, we assume that the probabilities of retirement and death are independent of age (Blanchard, 1985; Weil, 1989). However, the probabilities of retirement and death as well as the working-age population growth rate can be time-varying. Hence, the laws of motion for workers and retirees in region $i$ are

$$N_{i+1}^{w,i} = \left(1 - \omega_{i}^{i} + n_{i}^{w,i}\right) N_{i}^{w,i} + \omega_{i}^{i} N_{i}^{w,i} = \left(1 + n_{i}^{w,i}\right) N_{i}^{w,i},$$

$$N_{i+1}^{r,i} = \left(1 - \omega_{i}^{i}\right) N_{i}^{r,i} + \gamma_{i}^{i} N_{i}^{r,i}.$$

Defining the old-age dependency ratio as $\Psi_{i}^{i} = N_{i}^{r,i} / N_{i}^{w,i}$, its law of motion can be calculated as

$$\Psi_{i+1}^{i} = \frac{1 - \omega_{i}^{i}}{1 + n_{i}^{w,i}} + \frac{\gamma_{i}^{i}}{1 + n_{i}^{w,i}} \Psi_{i}^{i}. \quad (1)$$

Because growth of the labor force and ageing across regions may be different over time, the relative size of the labor force between region $i$ and $j$, defined as $r_{i}^{j} = N_{i}^{w,i} / N_{i}^{w,j}$, evolves according to $r_{i+1}^{j} = (1 + n_{i}^{w,i}) / (1 + n_{i}^{w,j}) r_{i}^{j}$. Relative total population size is given by $r_{i+1}^{j,\text{tot}} = (1 + \Psi_{i}^{i}) / (1 + \Psi_{j}^{i}) r_{i}^{j}$. In steady state, it must thus hold that $n_{i}^{w,a} = n_{i}^{w,b} = n_{i}^{w,c}$ as, otherwise, one region would eventually disappear. But $\omega^{i}$ and $\gamma^{i}$ can be structurally different across regions. The growth rate of the retiree population satisfies $N_{i+1}^{r,i} / N_{i}^{r,i} = (1 + n_{i}^{r,i}) = (1 - \omega_{i}^{i}) / \Psi_{i}^{i} + \gamma_{i}^{i}$ which, along a balanced growth path, implies $n_{i}^{w,i} = n^{r,i}$. 
2.2. Decision problem of retirees and workers

Let \( V_{z,i}^{t} \) denote the value function associated with the life-cycle states \( z = \{w, r\} \) in region \( i \). Households maximize their expected recursive life-time utility function from consumption, \( c_{t}^{z,i} \), consumer price index-deflated real money balances, \( m_{t}^{z,i} \) and leisure, \((1 - l_{t}^{z,i})\):

\[
V_{t}^{z,i} = \left\{ \left[ \left( c_{t}^{z,i} \right)^{\psi} \left( m_{t}^{z,i} \right)^{\psi} \left( 1 - l_{t}^{z,i} \right)^{\psi} \right] + \beta^{\psi} E_{t} \left[ V_{t+1}^{i} | z \right] \right\}^{\frac{1}{\rho}},
\]

\[
\beta^{w} = \beta, \quad \beta^{r} = \beta \gamma_{i},
\]

\[
E_{t} \left[ V_{t+1}^{i} | w \right] = \omega_{i} V_{t+1}^{w,i} + (1 - \omega_{i}) V_{t+1}^{r,i},
\]

\[
E_{t} \left[ V_{t+1}^{i} | r \right] = V_{t+1}^{r,i},
\]

where \( \rho \) determines the intertemporal elasticity of substitution and \( \psi c, \psi m \) as well as \( \psi l \) define the marginal rate of transformation between consumption, real money balances and leisure. It holds that \( \psi c + \psi m + \psi l = 1 \). The conditional expectations operator \( E_{t} \) depends on the states \( z = \{w, r\} \), and workers and retirees have different discount factors to account for the probability of death.

As discussed by, among others, [Gertler (1999), Ferrero (2010) and Carvalho et al. (2016)], the model is analytically tractable because the transition probabilities from working age to retirement and, then, to death are independent of age. However, to avoid a strong precautionary saving motive for young agents, which is at odds with data, this requires assuming a utility function similar to Epstein and Zin (1989). Recursive non-expected utility can be used to separate risk aversion from intertemporal substitution (i.e. risk-neutral preferences with respect to income fluctuations prevent counterfactual excess of young agents’ savings; see Farmer, 1990; Heiberger and Ruf, 2019). Separating the coefficient of intertemporal substitution, \( \sigma = 1/(1 - \rho) \), from risk aversion, as done in the utility function, helps to reproduce reasonable responses of consumption and savings to interest rate variations.

**Retirees:** In period \( t \), the representative retiree, indexed by \( j \), maximizes

\[
V_{t}^{r,i,j} = \left\{ \left[ \left( c_{t}^{r,i,j} \right)^{\psi} \left( m_{t}^{r,i,j} \right)^{\psi} \left( 1 - l_{t}^{r,i,j} \right)^{\psi} \right] + \beta \gamma_{i} \left( V_{t+1}^{r,i,j} \right) \right\}^{\frac{1}{\rho}},
\]

with respect to real consumption, \( c_{t}^{r,i,j} \), real money holdings, \( m_{t}^{r,i,j} \), labor supply \( l_{t}^{r,i,j} \), and
real assets excluding money holdings, \( \bar{a}^{r,ij}_t \), subject to the nominal flow budget constraint

\[
P^i_t \ c^{r,ij}_t + P^i_t \ m^{r,ij}_t + P^i_t \ d^{r,ij}_t = \frac{1 + \iota_{t-1}}{\gamma_{t-1}^i} \cdot P^i_{t-1} \bar{a}^{r,ij}_{t-1} + \xi^i \cdot P^i_t w^i_t \cdot l^{r,ij}_t + P^i_t e^{ij}_t
\]

\[
+ \frac{P^i_{t-1} \ m^{r,ij}_{t-1}}{\gamma_{t-1}^i},
\]

where \( P^i_t \) is the consumer price index (CPI) of region \( i \). It will be derived in detail below. The retiree receives real old-age benefits \( e^{ij}_t \) and faces an effective real wage rate \( \xi^i w^i_t \). Here, we have assumed that a retiree may still be working. The parameter \( \xi^i \in (0, 1) \) captures the productivity difference between the old and the young. As is standard in the literature, we will choose \( \xi^i \) such that \( l^{r,ij}_t \) is close to zero. \( i^*_t \) denotes the nominal interest rate in region \( i \).

The real return on asset investments for a retiree who has survived from period \( t-1 \) to \( t \) is \((1 + \iota_{t-1}^i) \left( \frac{P^i_t}{P^i_{t-1}} \right) / \gamma^i_{t-1} \), and the return on his nominal money holdings is \((P^i_{t-1} / P^i_t) / \gamma^i_{t-1} \cdot (P^i_t / P^i_{t+1}) \). We implicitly assume that, for retirees, a perfectly competitive mutual fund industry invests the proceeds and pays back a premium over the market return to compensate for the probability of death (see Yaari, 1965; Blanchard, 1985).\(^3\) Defining \( r_t \) as the real world interest rate that clears international capital markets (therefore, the lack of the superscript), dividing by \( P^i_t \) and using the Fisher relation \((1 + r_t) = (1 + \iota^*_t) \left( \frac{P^i_t}{P^i_{t+1}} \right) \), which we will verify to hold in section \( 2.5 \) yields

\[
c^{r,ij}_t + m^{r,ij}_t + d^{r,ij}_t = \frac{1 + r_{t-1}}{\gamma_{t-1}^i} \cdot \left[ a^{r,ij}_{t-1} + \frac{m_{t-1}^{r,ij}}{1 + \iota^*_t} \right] + \xi^i \cdot w^i_t \cdot l^{r,ij}_t + e^{ij}_t.
\]

Let us now define \( \bar{a}^{r,ij}_t = a^{r,ij}_t + 1/(1 + \iota^*_t)m^{r,ij}_t \) as the retiree’s real savings including money holdings. This allows us to restate the budget constraint in real terms as:

\[
c^{r,ij}_t + \frac{\iota_t^i}{1 + \iota^*_t} \cdot m^{r,ij}_t + a^{r,ij}_t = \frac{1 + r_{t-1}}{\gamma_{t-1}^i} \cdot \bar{a}^{r,ij}_{t-1} + \xi^i \cdot w^i_t \cdot l^{r,ij}_t + e^{ij}_t.
\]

As discussed in detail in Kara and von Thadden (2016), setting up the problem like this has two advantages. First, the term \( c^{r,ij}_t + \iota_t^i/(1 + \iota^*_t)m^{r,ij}_t \) can be interpreted as “total consumption expenditures” of a retiree in period \( t \) when \( \iota_t^i/(1 + \iota^*_t)m^{r,ij}_t \) is interpreted as the “consumption of real money balances”. This allows us to simplify the decision problem with respect to \( m^{r,ij}_t \). Second, it allows us to derive the consumption and value

\(^3\)In our model, national funds of region \( i \) only operate in their home region. This prevents equalization of returns in the insurance market, which would otherwise dampen the effects of life expectancy differences across regions significantly (see Ferrero, 2010).
functions in a similar fashion as is done in [Gertler (1999)]

The first-order conditions with respect to labor and real money holdings are given by

\[
(1 - I_t^{r,i,j}) = \frac{\psi_i^{l}}{\psi_i^{c}} \cdot \frac{c_t^{r,i,j}}{c_t^{l}} \cdot \sigma_i^{w_t} \cdot \eta^{r,i,j},
\]

\[
m_t^{r,i,j} = \frac{\psi_m}{\psi_i^{l}} \cdot \frac{1 + \gamma_t^{r,i,j}}{\gamma_t^{l}}
\]

while the consumption-Euler equation of the retiree’s maximization problem turns out to be

\[
c_{t+1}^{r,i,j} = \left[ \left( \frac{1 + \gamma_{t+1}^{r,i,j}}{1 + \gamma_t^{r,i,j}} \right) \cdot \left( \frac{w_t^{i}}{w_{t+1}^{i}} \right) \cdot \beta (1 + r_t) \right]^{\sigma} \cdot \frac{c_t^{r,i,j}}{c_t^{l}},
\]

where \( \sigma = \frac{1}{1 - \rho} \). If we define \( \pi_t^{i} \) as the marginal propensity of retirees to consume out of wealth, we can derive the consumption function and the law of motion of the retiree’s marginal propensity to consume:

\[
c_t^{r,i,j} = \frac{\eta_t^{r,i,j}}{1 + \gamma_t^{r,i,j}} \cdot \frac{\psi_i^{l}}{\psi_i^{c}} \cdot \left( \frac{1 + \gamma_t^{r,i,j}}{1 + \gamma_t^{l}} \right) \cdot \left( \frac{w_t^{i}}{w_{t+1}^{i}} \right) \cdot \beta (1 + r_t)
\]

and

\[
\epsilon_t^{r,i,j} = 1 - \left[ \beta \cdot \alpha x_t^{r,i,j} \right]^{\sigma} \cdot \left( (1 + r_t) \right)^{\sigma - 1} \cdot \gamma_t^{l} \cdot \frac{\epsilon_t^{r,i,j}}{\epsilon_t^{l} \cdot \pi_t^{l}}
\]

where

\[
h_t^{r,i,j} = \sigma_i^{w_t^{i}} \cdot \gamma_t^{r,i,j} + \epsilon_t^{r,i,j} + \frac{\gamma_t^{r,i,j}}{1 + r_t} \cdot h_{t+1}^{r,i,j}
\]

is the recursive law of motion of human capital (i.e., life-time income from wages and pension benefits at time \( t \)). These expressions allow us to derive an analytical expres-

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4We follow the literature with this definition, where \( \pi_t^{r,i,j} \) is the marginal propensity to consume for workers (see below) and \( \epsilon_t^{r,i,j} \) is the markup on that value for retirees. Also note that we think about “gross consumption” including money balances, \( c_t^{r,i,j} + i_t^{i} / (1 + i_t) \cdot m_t^{r,i,j} \) (as in [Kara and von Thadden (2016)].
sion for the value function $V_{r,i,j}^t$, which will be a key input for the decision problem of the representative worker. Getting to these expressions from the above model description is a tedious but – down to the present day – a relatively standard calculation, well described in (the technical appendices of) Gertler (1999), Fujiwara and Teranishi (2008), Ferrero (2010), Carvalho et al. (2016) and Kara and von Thadden (2016), among others. As we do exactly the same here, we refer the reader to these works for the formal derivations.

Workers: In period $t$, the representative worker, again indexed by $j$, maximizes

$$V_{w,i,j}^t = \left\{ \left[ \left( c_{t,i,j}^{w,i,j} \right)^{v_c} \left( m_{t,i,j}^{w,i,j} \right)^{v_m} \right] \right\}^{\frac{1}{\rho}} + \beta \left[ \left( \omega_i^t V_{r,i,j}^t + (1 - \omega_i^t) V_{r,i,j}^{t+1} \right) \right]^{\frac{1}{\rho}};$$

with respect to real consumption, $c_{t,i,j}^{w,i,j}$, real money holdings, $m_{t,i,j}^{w,i,j}$, labor supply $l_{t,i,j}^{w,i,j}$, and real assets, $a_{t,i,j}^{w,i,j}$, defined analogously to those of retirees, subject to the flow budget constraint

$$c_{t,i,j}^{w,i,j} + \frac{i_t}{1 + r_t} \cdot m_{t,i,j}^{w,i,j} + a_{t,i,j}^{w,i,j} = (1 + r_{t-1}) \cdot a_{t-1,i,j}^{w,i,j} + w_i^t \cdot l_{t,i,j}^{w,i,j} + f_{t,i,j}^j - \tau_{t,i,j}.$$

In contrast to the retiree, the worker has a different discount factor (because he cannot die) and takes into account the fact that he may stay a worker or become a retiree next period. Furthermore, the return on assets is different from retirees as there is no mutual fund operated for workers (i.e. gross interest on asset investments are no longer divided by the probability of death), workers do not receive pension benefits but obtain firm profits, $f_{t,i,j}^j$, instead, and they have to pay lump-sum taxes, $\tau_{t,i,j}$. They also receive the full wage $w_i^t$. The decision problem of the worker solves:

$$\omega_i^t c_{t+1,i,j}^{w,i,j} + (1 - \omega_i^t) \left( c_{t+1,i,j}^i \right)^{1/(1-\sigma)} \left( \frac{1}{\sigma} \right)^{v_i} c_{t+1,i,j}^{w,i,j} = \left[ \beta \left( 1 + r_t \cdot \Omega_{t+1,i,j}^{i} \cdot \text{aux}_{t+1,i,j}^i \right) \right]^{\frac{1}{\sigma}} c_{t,i,j}^{w,i,j},$$

as the consumption-Euler equation, with

$$\Omega_{t+1,i,j}^i = \omega_i^t + (1 - \omega_i^t) \left( c_{t+1,i,j}^i \right)^{1/(1-\sigma)} \left( \frac{1}{\sigma} \right)^{v_i},$$

and the first-order conditions with respect to labor and real money holdings:

$$f_{\ell,t,i,j} = \frac{v_i^t}{v_c^t} \cdot \frac{c_{t,i,j}^{w,i,j}}{w_i^t},$$

$$m_{t,i,j}^{w,i,j} = \frac{v_m^t}{v_c^t} \cdot \frac{1 + i_t}{i_t^j} \cdot c_{t,i,j}^{w,i,j}. $$
Again, the reader is referred to (the technical appendices of) Gertler (1999), Fujiwara and Teranishi (2008), Ferrero (2010), Carvalho et al. (2016) and Kara and von Thadden (2016) for the formal derivations.

Defining \( \pi_i^t \) as the marginal propensity of workers to consume out of wealth, again including real money holdings, the worker’s consumption function and the law of motion of the worker’s marginal propensity to consume are

\[
c_w^{w,i,j} + \frac{v_i^t}{1 + \pi_i^t} m_w^{w,i,j} = \pi_i^t \cdot \left( 1 + r_{t-1} \right) \cdot a_w^{w,i,j} + h_w^{w,i,j},
\]

and

\[
\pi_i^t = 1 - \left[ \beta \cdot \text{aux}_{x_{i,t+1}} \right]^{\sigma} \cdot \left[ (1 + r_t) \cdot \frac{1 + \tau_i^t}{1 + \Omega_i^{t+1}} \right]^{\sigma - 1} \pi_i^{t+1},
\]

where

\[
h_w^{w,i,j} = w_i^t \cdot l_w^{w,i,j} + f_i^j + \frac{\omega_i^j}{1 + \tau_i^{t+1}} h_w^{w,i,j} - \left( 1 - \frac{\omega_i^j}{\Omega_i^{t+1}} \right) h_r^{r,i,j} \frac{1}{1 + r_t}.
\]

One can show that retirees have a higher marginal propensity to consume than workers, \( \epsilon_i^t > 1 \forall t \), implying \( \Omega_i^t > 1 \forall t \) which, in turn, indicates that workers discount future income income streams at an effective rate \( (1 + r_t)\Omega_i^{t+1} > (1 + r_t) \). This reflects the expected finiteness of their life, making the future less valuable relative to a conventional New Keynesian setting with infinite lives. This also suspends Ricardian equivalence, even for lump-sum taxes (see, among others, Gertler, 1999, and Kara and von Thadden, 2016, for a discussion).

2.3. Aggregation of households’ decisions

To characterize aggregate variables, we drop the index \( j \) and carry on using the previous notation. Given the numbers of retirees and workers in each period \( t \), \( N_r^{i,t} \) and \( N_w^{i,t} \), the aggregate labor supply schedule can be derived from the individual ones as

\[
l_w^{w,i} = N_w^{w,i} \cdot l_w^{w,i,j} = N_w^{w,i} - \frac{v_i^t}{w_i^t} \cdot c_w^{w,i},
\]

\[
l_r^{r,i} = N_r^{r,i} \cdot l_r^{r,i,j} = N_r^{r,i} - \frac{v_i^t}{c_i^t} \cdot c_r^{r,i},
\]

\[
l_i^t = l_w^{w,i} + l_r^{r,i},
\]

where \( c_z^{z,i} = N_z^{z,i} \cdot c_z^{z,i,j} \) with \( z = \{ w, r \} \) denotes aggregate consumption levels of workers and retirees, respectively. Using the respective equations for retirees and workers, these
are given by
\[ c^{w,i}_t \left(1 + \frac{v^i_m}{v^i_c} \right) = \pi^i_t \left[ (1 + r_{t-1}) (1 - \lambda^i_{t-1}) a^{i}_{t-1} + h^{w,i}_t \right], \tag{8} \]
\[ c^{r,i}_t \left(1 + \frac{v^i_m}{v^i_c} \right) = \epsilon^i_t \pi^i_t \left[ (1 + r_{t-1}) \lambda^i_{t-1} a^{i}_{t-1} + h^{r,i}_t \right]. \tag{9} \]
We define aggregate consumption as \( c^i_t = c^{w,i}_t + c^{r,i}_t \) and get \( m^i_t = m^{w,i}_t + m^{r,i}_t = (1 + i^i_t) / \omega^i_t \cdot v^i_m / v^i_c \cdot c^i_t \). We have also used \( a^i_t = a^{w,i}_t + a^{r,i}_t \) and the definition \( \lambda^i_t = a^{r,i}_t / a^i_t \), which is the share of (financial) wealth held by retirees over total wealth.\footnote{Note that this implies that \( c^i_t \left(1 + \frac{v^i_m}{v^i_c} \right) = \pi^i_t \left\{ [1 + (c^r_t - 1) \lambda^i_{t-1}] (1 + r_{t-1}) a^{i}_{t-1} + \epsilon^i_t h^{r,i}_t + h^{w,i}_t \right\} \).}

To determine the aggregate stocks of human capital, \( h^{r,j}_t = N^{r,j}_t h^{r,i,j}_t \) and \( h^{w,j}_t = N^{w,j}_t h^{w,i,j}_t \), we have to take into account population dynamics described in section 2.1. This yields
\[ h^{r,j}_t = \xi^j_t \cdot \omega^j_t \cdot l^{r,j}_t + e^j_t + \frac{\gamma^j_t}{(1 + n^{r,j}_t)(1 + r_t)} h^{r,j}_{t+1}, \tag{10} \]
\[ h^{w,j}_t = \omega^j_t \cdot l^{w,j}_t + f^j_t - \tau^j_t + \frac{\omega^j_t \cdot h^{w,i,j}_{t+1}}{(1 + n^{w,j}_t)(1 + r_t) \Omega^i_{t+1}} - \left( 1 - \frac{\omega^j_t}{\Omega^i_{t+1}} \right) \frac{h^{r,j}_{t+1}}{(1 + n^{r,j}_t)(1 + r_t) \Psi^i_t}, \tag{11} \]
where \( e^j_t = N^{r,j}_t e^{i,j}_t, f^j_t = N^{w,j}_t f^{i,j}_t \) and \( \tau^j_t = N^{w,j}_t \tau^{i,j}_t \). These equations take into account the respective population growth rates \( n^{r,j}_t \) and \( n^{w,j}_t \). The absence of \( \gamma^j_{t-1} \) in equation (9) relative to individual human wealth for retirees reflects the competitive insurance/annuity market that takes into account death probabilities. While it is relevant for the individual household \( j \), it does not affect the aggregate consumption of retirees (see also Blanchard, 1985, and Yaari, 1965).

It remains to characterize the law of motion for \( \lambda^i_t \), i.e. the fraction of wealth over total wealth held by retirees. In doing so, we realize that the fraction of total wealth held by the working-age population evolves according to \( (1 - \lambda^i_t) a^i_t = \omega^i_t \left[ (1 - \lambda^i_{t-1}) (1 + r_{t-1}) a^i_{t-1} + \omega^i_t \cdot l^{w,i}_t + f^i_t - \tau^i_t - c^w_t \left(1 + v^i_m / v^i_c \right) \right] \). It increases by the savings of those workers who remain workers in the next period. Analogously, the fraction of total wealth held by retirees increases by the savings of those retirees who do not die (bearing in mind that savings of those who die are redistributed through the competitive annuity market) plus the savings of those workers who become retirees: \( \lambda^i_t a^i_t = \lambda^i_{t-1} (1 + r_{t-1}) a^i_{t-1} + \xi^i_t \cdot \omega^i_t \cdot l^{w,i}_t + e^i_t - c^r_t \left(1 + v^i_m / v^i_c \right) + (1 - \omega^i_t) \left[ (1 - \lambda^i_{t-1}) (1 + r_{t-1}) a^i_{t-1} + \omega^i_t \cdot l^{w,i}_t + f^i_t - \tau^i_t - c^w_t \left(1 + v^i_m / v^i_c \right) \right] \). Combining these...
expressions and using equations (8) and (9), we get

\[
\lambda_i^{t} a_i^{t} = \omega_i^{t} \left\{ \left(1 - \epsilon_i^{t} \right) \left[ (1 + r_{t-1}) \lambda_i^{t-1} a_i^{t-1} + h_i^{r} - \alpha_i^{t} \right] - \left( h_i^{r} - \epsilon_i^{t} \right) \right\} + (1 - \omega_i^{t}) a_i^{t}.
\]

(12)

2.4. Production

The production side follows the conventional structure of New Keynesian models in the literature (see, for example, [Gertler et al. 1999, Christiano et al. 2005, or Smets and Wouters 2003, 2007]). This implies that the production sector is partitioned in a final and an intermediate goods sector. As it is standard, we will keep its description brief.

**Final goods:** We assume that, in each country \( i \), there is a measure-one continuum of firms in the final goods sector. Firms are owned by the working-age population as in Fujiwara and Teranishi (2008) and Kara and von Thadden (2016). Each final goods producer purchases a variety of differentiated intermediate goods, bundles these and sells them to the final consumer under perfect competition. The producer price index (henceforth, PPI) of goods produced in country \( i \) and sold in \( j \) is defined as \( P_{i}^{f} \). We assume that the law of one price holds across regions, so firms in country \( i \) set their price \( P_{i}^{f} \) for all markets (Di Giorgio and Nistico, 2013; Di Giorgio et al., 2018; Di Giorgio and Traficante, 2018). Multiplying with the nominal exchange rate \( S_{i}^{f} \) then yields the price of country-\( i \) goods charged in \( j \): \( P_{i}^{f} = S_{i}^{f} P_{i}^{f} \), where \( S_{i}^{f} \) is defined as country-\( j \) currency per unit of country-\( i \) currency. Within the monetary union, it holds by definition that \( S_{i}^{f,a} = S_{i}^{f,b} = 1 \forall t \). It must then hold that \( S_{i}^{f,c} = S_{i}^{f,b} \equiv S_{f} \), where \( S_{f} \) is the nominal exchange rate between the monetary union and the rest of the world (expressed in country-\( c \) currency per unit of the monetary union currency); see, for example, Gadatsch et al. (2016) for a discussion.

Assuming a Dixit and Stiglitz (1977)-aggregator on the interval \( \tilde{j} \in [0, 1] \), the final good in region \( i \) is, as usual, given by \( y_{i}^{t} = \left[ \int_{0}^{1} y_{i}^{t}(\tilde{j})^{(\theta_{p}^{t}-1)/\theta_{p}^{t}} d\tilde{j} \right]^{\theta_{p}^{t}/(\theta_{p}^{t}-1)} \). \( \theta_{p}^{t} > 1 \) is the elasticity of substitution between differentiated intermediate goods. Demand for a an intermediate good \( \tilde{j} \) is given by \( y_{i}^{t}(\tilde{j}) = \left[ P_{i}^{f}(\tilde{j}) / P_{i}^{f} \right]^{-\theta_{p}^{t}} y_{i}^{t} \). The PPI of region \( i \) is given by \( P_{i}^{f} = \left[ \int_{0}^{1} P_{i}^{f}(\tilde{j})^{1-\theta_{p}^{t}} d\tilde{j} \right]^{1/(1-\theta_{p}^{t})} \).

**Intermediate goods:** The representative intermediate good producer \( \tilde{j} \) operates with production technology \( y_{i}^{t}(\tilde{j}) = tf p^{i} \cdot [X_{i}^{t} \cdot l_{i}^{t}(\tilde{j})]^{\alpha^{i}} \cdot [k_{i-1}^{t}(\tilde{j})]^{1-\alpha^{i}} \). Here, \( tf p^{i} \) is an exogenously given parameter scaling production across regions and \( X_{i}^{t} \) denotes an exogenously given level of labor-augmenting technological progress. Following the literature (Gertler 1999, Kara and von Thadden 2016), we assume that \( X_{i}^{t} \) grows at a constant rate \( x^{i} \geq 0: X_{i}^{t} = (1 + x^{i}) X_{i-1}^{t} \). \( \alpha^{i} \) is the Cobb-Douglas share of labor in production and \( l_{i}^{t}(\tilde{j}) \)
and \( k_{i-1}^j \) are the inputs of labor and capital in production by producer \( j \). Taking prices for labor (CPI-deflated real wages \( w^i_t \)) and capital (CPI-deflated real capital interest \( r^k_{i,t} \)) as given, firm \( j \)'s cost minimization problem yields the following capital-to-labor ratio

\[
\frac{l^i_t}{k_{i-1}^j} = \frac{\alpha^i}{1 - \alpha^i} \cdot \frac{r^k_{i,t}}{w^i_t}.
\]  

(13)

which, as can easily be seen, must be equal across all intermediate goods producing firms for given wages and capital interest rates (as symmetry applies, we dropped the index \( j \) for convenience). Hence, CPI-deflated real marginal costs are given by

\[
m^i_c = \left( \frac{w^i_t}{\alpha^i X^i_t} \right)^{\alpha^i} \cdot \left( \frac{r^k_{i,t}}{1 - \alpha^i} \right)^{1 - \alpha^i}.
\]  

(14)

Following the convention in the New Keynesian literature, we assume that, each period, a randomly chosen fraction of firms \( \kappa^i_p \in [0, 1] \) cannot re-optimize their price (Calvo, 1983). In a symmetric equilibrium, the price of those firms \( j \) who can set their price in period \( t \) is equal across firms, i.e. \( P^{i,i,*}_t(j) = P^{i,i,*}_t \). This profit maximizing price is given by

\[
\frac{P^{i,i,*}_t}{P^i_t} = \frac{\kappa^i_p}{\kappa^i_p - 1} \cdot \frac{\sum_{z=0}^{\infty} \left( \kappa^i_p \beta \right)^z \cdot DF^i_{t,z} \cdot y^i_{t,z} \cdot m^i_c \cdot \left( \frac{P^{i,i,*}_t}{P^i_t} \right)^{\kappa^i_p}}{\sum_{z=0}^{\infty} \left( \kappa^i_p \beta \right)^z \cdot DF^i_{t,z} \cdot y^i_{t,z} \cdot \left( \frac{P^{i,i,*}_t}{P^i_t} \right)^{\kappa^i_p - 1}}.
\]  

(15)

As shown by Fujiwara and Teranishi (2008), the discount factor of firms is given by

\[
DF^i_{t,z+1} = \partial V^{w^i} / \partial c^{w^i}_{t+1} = (\pi^i_{t+1})^{-1/\rho} \left( v^i_{m} / v^i_{c} \cdot \left( 1 + i^i_{t+1} \right)^{\rho} / i^i_{t+1} \right)^{v^i_{m}} \left( v^i_{i} / v^i_{c} / w^i_{t+1} \right)^{v^i_{i}}.
\]

This is a result of the fact that we assume workers to be the owners of firms. Producer prices in region \( i \) hence evolve according to

\[
P^{i,i}_{t} = \left[ \kappa^i_p \cdot \left( P^{i,i}_{t-1} \right)^{1 - \rho^{i,i}_p} + \left( 1 - \kappa^i_p \right) \cdot \left( P^{i,i,*}_{t} \right)^{1 - \rho^{i,i}_p} \right]^{1/(1 - \rho^{i,i}_p)},
\]  

(16)

while \( D^{i}_{t} = \kappa^{i}_{p} \cdot \left( P^{i,i}_{t} / P^{i,i}_{t-1} \right)^{\rho^{i,i}_p} \cdot D^{i}_{t-1} + \left( 1 - \kappa^{i}_{p} \right) \cdot \left( P^{i,i}_{t} / P^{i,i,*}_{t} \right)^{\rho^{i,i}_p} \) is the resulting measure of price dispersion, expressed recursively. Aggregate firm profit are \( f^i_t = \left( P^{i,i}_{t} / P^{i,i}_{t} - m^i_c \right) y^i_{t} \).

2.5. Investment funds and financial market clearing

Following Fujiwara and Teranishi (2008), an investment fund in each region \( i \) collects deposits from households, \( a^i_t \), and invests these into physical capital, domestic government bonds and international assets. The decision about real money balances is
undertaken by the household as described above. Government bonds and international assets are assumed to pay a nominal interest \( \tilde{i}_t \) and \( \tilde{d}_t \) next period, respectively. The financial investor pays the households a real interest \( r_t \) on the deposited assets. The investment fund hence aims to maximize

\[
   f_{t+1}^{\text{fund},i} = k_t \cdot k_{t+1} + \left( 1 - \tilde{i}_t \right) \frac{p_t}{p_{t+1}} \cdot b_t + \left( 1 - \tilde{d}_t \right) \frac{p_t}{p_{t+1}} \cdot d_t - i_t
   + a_t - \left( 1 + r_t \right) a_t,
\]

where \( b_t \) and \( d_t \) are CPI-deflated real government bonds and net foreign assets, respectively, and \( r^{k,t+1} \) is the ex-ante uncertain rate of return on capital. Capital follows the conventional law of motion:

\[
   k_{t+1} = (1 - \delta^i)k_t + \left[ 1 - \frac{\kappa^i_{\text{inv}}}{2} \left( \frac{i_{\text{inv},t}}{i_{\text{inv},t-1}} - 1 \right) \right] \cdot \text{inv}_t, \tag{17}
\]

where \( \delta^i \) denotes capital depreciation and \( S^i(\cdot) \) denote capital adjustment costs in line with Christiano et al. (2005). This implies that the conventional no-arbitrage conditions must hold:

\[
   (1 + r_t) = \left( 1 + i_{\text{G},t} \right) \frac{p_t}{p_{t+1}} = \left( 1 + \tilde{i}_t \right) \frac{p_t}{p_{t+1}} = \frac{r^{k,t+1} + Q^{l+1}_{t+1}(1 - \delta^i)}{Q_t} = (1 + \tilde{i}_t) \frac{p_t}{p_{t+1}}, \tag{18}
\]

where the last equality on the right-hand-side of that equation results from the fact that, in order for households to be indifferent between asset and money holdings, the interest rate \( \tilde{i}_t \) must adjust accordingly (see also section 2.2). \( Q^i_t \) is the shadow price of capital, also known as Tobin’s Q. It evolves according to

\[
   1 = Q^i_t \cdot \left[ 1 - S^i(\cdot) - S'^i(\cdot) \cdot \frac{i_{\text{inv},t}}{i_{\text{inv},t-1}} \right] + \frac{Q^{l+1}_{t+1}}{1 + r_{t+1}} \cdot S'^i(\cdot) \left( \frac{i_{\text{inv},t+1}}{i_{\text{inv},t}} \right)^2. \tag{19}
\]

Financial markets must clear, which implies that

\[
   a_t = Q^i_t k_t + b_t + d_t + \frac{1}{1 + \tilde{i}_t} \cdot m_t. \tag{20}
\]
2.6. Fiscal policy

The government’s budget constraint in region \( i \) in CPI-deflated real terms is given by

\[
b_i^t + m_i^t + \tau_i^t = (1 + r_{i-1}) \left( b_{i-1}^t + \frac{m_{i-1}^t}{1 + i_{i-1}^t} \right) + \frac{p_{i,i}^t}{p_i^t} \cdot g_i^t + e_i^t, \tag{21}
\]

where use has been made of equation (18). Hence, the government must finance real government expenditures, \( g_i^t \), which may follow an AR(1)-process, aggregate real pension benefits, \( e_i^t \), and interest payments on outstanding debt, \((1 + r_{i-1})b_{i-1}^t\), by lump-sum taxes, \( \tau_i^t \), issuance of new debt, \( b_i^t \), and seigniorage, \( m_i^t - P_i^{t-1}/P_i^t \). Following Stähler and Thomas (2012) and Gadatsch et al. (2016), we assume full home bias in government consumption, which requires the PPI/CPI correction. The assumption is based on the observation that the import share in government consumption is, in general, significantly lower than in private consumption or investment (Brülhart and Trionfetti, 2001, 2004, and Trionfetti, 2000).

The path of aggregate real pension benefits is determined by the replacement rate \( \mu_i^t \) between individual benefits and real wages, that is

\[
\mu_i^t = \frac{e_i^{i,j}}{w_i^t} \Rightarrow e_i^t = e_i^{i,j} \cdot N_i^{r,i} = \mu_i^t \cdot w_i^t \cdot N_i^{r,i}. \tag{22}
\]

To close the system, we assume that fiscal policy follows a so-called fiscal reaction function that stabilizes a certain target level of debt, say, a certain fraction \( \phi b_i^t \) of GDP, \( y_i^t \), by variations in the remaining free fiscal instrument \( \tau_i^t \) such that

\[
\log \left( \frac{\tau_i^t}{\tau_i^t} \right) = \rho^{\tau,i} \log \left( \frac{\tau_{i-1}^t}{\tau_i^t} \right) + \xi^{b,i} \log \left( \frac{p_{i,j}^t \cdot b_{i-1}^t}{p_{i-1,j}^t \cdot \phi b_i^t \cdot y_{i-1}^t} \right), \tag{23}
\]

where the omission of the time subscript \( t \) indicates steady-state values, \( \rho^{\tau,i} \) is an autocorrelation parameter and \( \xi^{b,i} > 0 \) is a direct feedback parameter to counteract deviations of debt from its target. Similar fiscal rules have been discussed by, among others, Schmitt-Grohe and Uribe (2007) and Kirsanova and Wren-Lewis (2012). Because real government debt is deflated by CPI, while GDP by PPI, as outlined in section 2.4, we need to correct for \( P_i^t / P_{i-1}^j \). Closing the model in terms of alternative fiscal instruments (such as \( g_i^t \) or \( \mu_i^t \), for example) and target variables (for example, including reactions to output deviations or alike) can easily be done by replacing equation (23) accordingly (see, for example, Mitchell et al., 2000, for a discussion).

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6In principle, it is straightforward to explicitly model a different home bias in public consumption, too. Nevertheless, this would come at the cost of additional relative prices. Because this assumption is irrelevant for the analysis at hand, we simplify the model here.
2.7. International linkages, monetary policy and market clearing

International trade in goods and assets implies that the three regions \( i = a, b, c \) are linked together, which not only affects the net foreign asset position but also the market clearing conditions. Furthermore, we have to bear in mind that there is a common monetary policy for regions \( a \) and \( b \), while the one for region \( c \) is solely undertaken for that region. We will describe these linkages in more detail below.

**International trade, prices and net foreign assets:** We assume that households in region \( i \) consume goods produced in any of the three regions. The corresponding consumption bundle is given by

\[
c^i_t = \left( \frac{\theta^{i}_a}{\theta^{i}_a} \right)^{1-\eta^i} \left( c^i_{a,t} \right)^{\eta^i} + \left( \frac{\theta^{i}_b}{\theta^{i}_b} \right)^{1-\eta^i} \left( c^i_{b,t} \right)^{\eta^i} + \left( \frac{\theta^{i}_c}{\theta^{i}_c} \right)^{1-\eta^i} \left( c^i_{c,t} \right)^{\eta^i} \right].
\]

Here, \( c^i_{j,t} \) denotes goods produced in \( j \) and consumed in \( i \) and \( \eta^i \in (-\infty, 1) \) governs the elasticity of substitution between these goods, which equals \( 1/(1-\eta^i) \). As \( \eta^i = 0 \), the function boils down to a Cobb Douglas aggregator. \( \theta^{i}_j \) denotes the consumption bias of region \( i \)-households towards goods produced in \( j \). Hence, \( \theta^{i}_j \) can be interpreted as the home bias of region \( i \). We assume that \( \theta^{i}_a + \theta^{i}_b + \theta^{i}_c = 1 \). Cost minimization of nominal consumption expenditures, \( P^i_t c^i = P^i_t c^i_{a,t} + P^i_t c^i_{b,t} + P^i_t c^i_{c,t} \), implies

\[
c^i_{j,t} = \theta^{i}_j \left( \frac{P^i_j}{P^i_t} \right) ^{-\frac{1}{1-\eta^i}} c^i_{t}.
\]

The consumer price index (CPI) results to be

\[
P^i_t = \left[ \frac{\theta^{i}_a}{\theta^{i}_a} \cdot \left( P^i_t \right)^{-\eta^i/(1-\eta^i)} + \frac{\theta^{i}_b}{\theta^{i}_b} \cdot \left( P^i_b \right)^{-\eta^i/(1-\eta^i)} + \frac{\theta^{i}_c}{\theta^{i}_c} \cdot \left( P^i_c \right)^{-\eta^i/(1-\eta^i)} \right]^{-\frac{1}{\eta^i}}.
\]

We assume that an analogous aggregator holds for investment goods such that we can derive analogous equations for \( inv^i_t \) and \( inv^i_{j,t} \). CPI-deflated net exports in region \( i \), \( nx^i_t \), are hence given by

\[
nx^i_t = \frac{P^i_t}{P^i_i} \cdot (c^i_{i,t} + inv^i_{i,t}) + \frac{\tilde{P}^i_j}{P^i_t} \cdot (c^i_{j,t} + inv^i_{j,t}) - \frac{P^i_i}{P^i_t} \cdot (c^i_{j,t} + inv^i_{j,t}) - \frac{P^i_j}{P^i_t} \cdot (c^i_{j,t} + inv^i_{j,t}),
\]

where \( i, j, \tilde{j} = a, b, c \), and \( i \neq j \neq \tilde{j} \). Alternatively, net exports can also be written as domestic production minus domestic demand: \( nx^i_t = \frac{p^i_t}{P^i_t} (y^i_t - g^i_t) - c^i_{i,t} - inv^i_{i,t} \).
Setting it up as in the former expression, however, allows us to disentangle the bilateral trade balance between regions. Remembering the discussion in section 2.4, we note that $P_{j}^{i,j} = P_{j}^{j,j}$ whenever the regions belong to the monetary union (i.e., $i, j = a, b$). Whenever a monetary union-country imports from the rest of the world, $P_{j}^{i,c} = P_{j}^{c,j} / S_t$, and when the rest of the world imports from the monetary union, $P_{j}^{c,i} = S_t \cdot P_{j}^{i,j}$, with $i = a, b$.

Given net exports and using the no-arbitrage conditions (18), we get that net foreign assets in region $i$ evolve according to

$$d_t^i = (1 + r_{t-1}) d_{t-1}^i + nx_t^i.$$  \hspace{1cm} (27)

Because international assets traded between regions are in zero net supply for the entire world, it must hold that $P_{a}^{a} d_{a}^a + P_{b}^{b} d_{b}^b + P_{c}^{c} d_{c}^c = 0$.

At this juncture, it may be noteworthy that the standard (multi-country) representative agent model, in general, entails steady-state indeterminacy and non-stationary dynamics of net foreign assets. To overcome this problem, modelers assume additional frictions in the international financial markets (for example, a risk premium on international asset holdings or some asset adjustment costs) whenever holdings of net foreign assets exceed some exogenously fixed reference level. That introduces a link between consumption and the net foreign asset position and pins down the steady-state level of international financial assets uniquely. However, it does so independent of policy or structural economic changes. An in-depth discussion of this issue can be found in Schmitt-Grohe and Uribe (2003), Hunt and Rebucci (2005), Lubik (2007) and Benigno (2009). As discussed by, for example, Ghironi (2008), Ghironi et al. (2008) and Di Giorgio and Nistico (2013), such an “extra” assumption is not needed in our framework due to the endogenously arising elastic asset demand curve resulting from the old-age savings motive discussed in section 2.2.

**Monetary policy:** Following Ghironi (2008), Di Giorgio and Nistico (2013) and Kara and von Thadden (2016), monetary policy is modelled through a Taylor-type feedback rule (Taylor, 1993), assuming a target of gross output price inflation of one. According to the Taylor rule, the nominal interest rate set by the central bank, $i_t^i$, is a function of output price inflation deviations from target, $\log{\text{inf}_t^i}$, where $\text{inf}_t^i = P_{t}^{i,i} / P_{t-1}^{i,i}$, the output gap, $\log{\text{gap}_t^i}$, where $\text{gap}_t^i = y_t^i / y_t^i$ (the omission of the time-subscript again denotes the steady-state value), and the previous value of the nominal interest rate. Given that regions $a$ and $b$ form a monetary union with a common monetary policy, we follow, among others, Stähler and Thomas (2012) and Gadatsch et al. (2016), and assume that the monetary policy rate in the union, denoted by $i_t^u = i_t^a = i_t^b$, reacts to a population-weighted average of inflation deviations and output gap. Denoting union-wide aggregates by the superscript $u$, these are given by $\text{inf}_t^u = rs_t^{a,b} \left( P_{t}^{a,a} / P_{t-1}^{a,a} \right) + (1 - rs_t^{a,b}) \left( P_{t}^{b,b} / P_{t-1}^{b,b} \right)$ and $\text{gap}_t^u = rs_t^{a,b} \cdot y_t^a / y_t^a + (1 - rs_t^{a,b}) \cdot y_t^b / y_t^b$. Hence, monetary pol-
icy in $i = u, c$ is described by

$$\log \left( \frac{i^i}{\hat{i}^i} \right) = \rho^{mp,i} \log \left( \frac{i^i_{t-1}}{\hat{i}^i_{t-1}} \right) + \zeta^{\pi,i} \log \left( \inf_i^i \right) \zeta^y,i \log \left( \text{gap}_i^i \right),$$

where $\rho^{mp,i}$ is an autocorrelation parameter, and $\zeta^{\pi,i} > 0$ is a direct feedback parameter to counteract deviations of inflation from target, and $\zeta^y,i > 0$ the direct feedback parameter of monetary policy to the output gap.

**Product market clearing:** Product market clearing implies that whatever is produced in region $i$ must be consumed/used somewhere around the world. Formally, we get

$$D_i^i \cdot y_i^i = \left( c_i^i + inv_i^i \right) + \left( c_j^j + inv_j^j \right) + \left( \hat{c}_i^j + \hat{inv}_i^j \right) + g_i^i.$$  \hspace{1cm} (29)

2.8. General equilibrium and detrending

The previous sections complete the model description. At equilibrium, government actions and optimizing decisions of workers, retirees, investment funds and firms must be mutually consistent at the aggregate level. As we allow for an exogenous labor-augmenting technological progress and exogenously given, time-varying population dynamics, the economy may be subject to ongoing exogenous growth. Following Kara and von Thadden (2016), we therefore consider a detrended version the model. Any unbounded model variable $\psi_i^i$ detrended through

$$\bar{\psi}_i^i = \frac{\psi_i^i}{\bar{\psi}_i^i}, \quad \bar{\psi}_i^{w,i} = \frac{\psi_i^{w,i}}{\bar{\psi}_i^{w,i}}, \quad \bar{\psi}_i^{r,i} \cdot \Psi_i = \frac{\psi_i^{r,i}}{\bar{\psi}_i^{r,i} \cdot \Psi_i},$$

where the bar denotes the detrended variable. For the labor-market related variables, we get

$$\bar{\psi}_i^i = \frac{\psi_i^i}{X_i^i}, \quad \bar{\psi}_i^{w,i} = \frac{\psi_i^{w,i}}{\bar{\psi}_i^{w,i}}, \quad \bar{\psi}_i^{r,i} \cdot \Psi_i = \frac{\psi_i^{r,i}}{\bar{\psi}_i^{r,i} \cdot \Psi_i}.$$ 

Using these definitions allows us to express our model, described through equations (2) to (29), in terms of efficiency units per worker. An equation summary of the fully detrended system, as well as a description about how to solve for the initial steady state, can be found in the appendix.

3. Calibration

Table XX reports the values of the parameters and the steady state variables that we use as exogenous targets. When calibrating our model, we follow the literature and assume that the time period is one year. Individuals are born at the age of 20, stay on
average $1/(1 - \omega^i)$ years in the labor force and live on average $1/(1 - \gamma^i)$ years after retirement. We calibrate our model to Germany (region $a$), the rest of the Euro area (region $b$) and the rest of the world (region $c$). The latter reflects the remaining OECD countries (excluding Germany and Euro area member states). We choose $\omega^i = 0.97778$, with $\beta = a, b, c$, to reflect an average retirement age of 65. The survival probabilities $\gamma^i$ are derived to match all region-$i$ old-age dependency ratios of the year 2000, which we take as the base year for our steady-state derivation. All pension-system related data and especially the population dynamics, including the old-age dependency ratios ($\Psi^a = 0.2647$, $\Psi^b = 2.2670$ and $\Psi^c = 0.2001$), come from OECD (2017) and the related data appendices. We assume that, in steady state, there is neither population nor technological growth. The initial relative sizes of the working-age population are $rs^{a,b} = 0.343$, $rs^{a,c} = 0.092$ and $rs^{b,c} = 0.268$. As population growth rates will vary differently across regions in our simulations, these will change along the transition.

For the general model calibration, we follow Ferrero (2010) and set standard values from the business cycle literature (see also Cooley and Prescott, 1995). This implies that we set $\beta = 0.98$, choose a labor share in production of $2/3$, assume that capital depreciates at an annual rate of 10% and set the elasticity of intertemporal substitution to $\sigma = 0.5$. As discussed in Ferrero (2010), the latter somewhat low value has become standard in this class of models since Auerbach and Kotlikoff (1987). The investment adjustment cost parameter is set to 4.5, which is a standard value in the DSGE literature.

In order to make our simulations comparable to the literature, we follow Kara and von Thadden (2016) and set $v^i_m = 0$. As in Kara and von Thadden (2016), assuming a small positive value here does not alter our results (which we show in the appendix by assuming $v^i_m = 0.05$). The choice of the relative productivity parameter $\xi^i$ as well as $v^i_c$ then ensures that the participation rate of workers is 0.70 (0.67) in the remaining Euro area (OECD) economies, and that the one of retirees is 0.01 (remember that $v^i_l = 1 - v^i_c - v^i_m$). The replacement rate for pension benefits $\mu^i$ is set to 0.48. The government spending-to-GDP ratio is set to 0.18 in all regions, which is a standard value. The debt-to-GDP ratio is 59.8%, 68.2% and 50.53% for Germany, the rest of the Eurozone and the rest of the world in line with Eurostat and OECD data for the year 2000.

As we work with a nominal extension of the Gertler (1999)-model, we also have to specify price markups and monetary policy. We assume a steady-state price markup
of 30%, which gives $\theta_p^i = 4.3$. We also assume a standard Calvo parameter of 0.75. In the Taylor rule, we opt for an autocorrelation parameter of 0.85 and an inflation coefficient of 1.5. These values hold for the monetary union as well as for the rest of the world. Autocorrelation in the fiscal policy rule is assumed to be 0.75 and the reaction to deviations of the debt-to-GDP ratio from target (the initial steady-state value in our baseline simulations) is 0.3. These values are standard values from the literature; see Schmitt-Grohe and Uribe (2007), Kirsanova and Wren-Lewis (2012) and Kara and von Thadden (2016).

As regards international trade, we assume that, in the initial steady state, relative prices between all regions equal one. Given this, plus the calibration choices made so far, we can then endogenously solve for each region’s net foreign asset-to-GDP ratio and the world interest rate in the initial steady state. Assuming that European economies have a home bias parameter of 0.6 and a bias towards the goods produced in the other other European region of 0.1 allows us to derive the biases towards the different regional consumption/investment goods in the rest of the world that meet the net foreign asset positions which we just calculated. Table YY shows the values of selected endogenous variables in the initial steady state.

4. Analysis

As is common in the literature (for example, Ferrero, 2010), we assume that demographic variables change unexpectedly in 2000 (the initial steady state). After the initial period, agents perfectly anticipate the evolution of the exogenous variables, which become constant again in 2080. To match the demographic changes shown in Figure 1, we assume that working-age population growth, $n_{w,i}$, and survival probabilities, $\gamma_i$, adjust accordingly (see also section 2.1). In the main text, we will show the simulation results for a version of the model calibrated to Germany, the rest of the Euro area and the remaining OECD countries. We briefly discuss the implications of calibrating the model differently below. The demographic processes that we feed into our model are depicted in Fig 2.

When households become aware of the rise in life expectancy in 2000, they start increasing their savings effort and reduce consumption (see Figure 3). This effect is stronger for retired agents than for the working-age population. As a result of the increased live expectancy (see Figure 2), the former group expects to live longer on average. For consumption smoothing reasons, they cut consumption today to have more to consume tomorrow. Working-age agents also save more to prepare for their longer lives, but to a lesser extent. As we see in Figure 3, the increase in per-capita savings is strongest in Germany on impact, they start increasing significantly around 2010 in the rest of the Euro area (eventually exceeding German per-capita savings), and per-capita

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9This simulation design implies that there will be a disproportionate jump in some endogenous variables on impact because agents are fully surprised by the new demographic trends, which is probably somewhat unrealistic. Therefore, the jump on impact should be interpreted with care.
Figure 2: Exogenous demographic processes

Notes: Figure plots exogenous processes for working-age population growth, $n_{ij}^{w}$, and survival probabilities, $\gamma_{ij}$, for Germany (blue solid lines), the rest of the Euro area excluding Germany (red dashed lines) and the remaining OECD countries (greed dotted lines) to reproduce (projected) population dynamics of Figure 1.
savings in the remaining OECD countries actually fall. As Figure 2 reveals, the increase in projected life-expectancy in Germany is strongest on impact. Furthermore, growth of the working-age population is negative, while it is positive in the rest of the Eurozone until about 2010 and in the remaining OECD countries until about 2030. As new workers enter the economy with zero assets, positive population growth may compensate for the higher savings efforts of the elderly population in terms of per-capita savings. This is the case in the rest of the Euro area (until about 2010) and in the remaining OECD countries.

Per-capita consumption falls in all regions.

Variables related to the production side of the economy are depicted in Figure 4. As ageing advances, per-capita labor decreases because a larger fraction of the population is in its retirement phase. The scarcity of labor increases wages, which implies that individual labor supply also rises (see equations (5) and (6)). Higher wages and lower

Notes: Figure plots per-capita consumption and savings in percentage deviations from initial steady state for Germany (blue solid lines), the rest of the Euro area excluding Germany (red dashed lines) and the remaining OECD countries (green dotted lines).

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10 The individual effort to build up savings by reducing consumption significantly slows down around 2030 (2040) in Germany (the rest of the Euro area), especially for retirees. However, their number is still rising. Because that latter increase is higher than the rise in individual savings, savings per capita start falling, which explains the corresponding turning points in Figure 3. Note, however, that world asset supply still increases, which is also a result of relatively higher individual savings and population growth in the remaining OECD countries. This can be verified by the world interest rate staying below its initial steady-state value, see Figure 5.

11 We do not explicitly show the evolution of wages here. But real wages increase in Germany and the
capital interest (see Figure 5), the latter a result of higher aggregate savings, means that production becomes more capital-intensive. Capital per worker rises. The aggregate stock of capital per capita, however, falls with decreasing GDP per capita because less productive capital is needed in production.

![Graphs of GDP per capita, Labor per capita, Capital per worker, Capital per capita](image)

**Figure 4: Production sector reactions**

**Notes:** Figure plots production related variables in percentage deviations (percentage point deviation for per-capita labor) from initial steady state for Germany (blue solid lines), the rest of the Euro area excluding Germany (red dashed lines) and the remaining OECD countries (green dotted lines).

In Figure 5 we show the evolution of international variables. As explained above, the increase in savings augments the world supply of assets and reduces the real asset return. This leads to a fall in the world interest rate, also in the long run (see also Carvalho et al., 2016, for a discussion). Because savings increase more on impact – when people start realizing that they live in an ageing world –, the drop is higher on impact. But the interest rate still stays 0.2 percentage points below its initial steady-state value in the new steady state. Because of the disproportionately high increase in individual savings to insure against longevity, Germany and the rest of the Euro area become capital exporters, reflected by an increase in their net foreign asset and current account-to-GDP ratios. The current account is positive until about 2030 (2040) in Germany (the rest of the rest of the Euro area until about 2035. Thereafter, wages fall and remain below their initial steady-state values because demand for labor has fallen substantially, a result of lower GDP. In the remaining OECD countries, wages remain high until about 2080. Individual labor supply increases to partly compensate for the loss in aggregate labor. It more or less follows the evolution of capital per worker in Figure 4 at lower levels.
Euro area), which corresponds to the turning point in their net foreign asset over GDP positions and the time when per-capita savings start falling again (see Figure 3 and the corresponding explanation). International competitiveness between Germany/the Euro area and the OECD economies falls because of relatively more younger people (workers) and, therefore, labor supply in the OECD countries. As ageing in Germany proceeds more quickly in the beginning, its competitiveness falls vis-à-vis the Euro area but recovers during the transition as the rest of the Euro area ages.

Notes: Figure plots deviations of international variables from initial steady state for Germany (blue solid lines), the rest of the Euro area excluding Germany (red dashed lines) and the remaining OECD countries (green dotted lines). Interest rates and international competitiveness, defined as $P_j^t / P_i^t$ between regions $i$ and $j$, are in percentage point deviations, net foreign asset and current account-to-GDP ratios in levels. The current account-to-GDP ratio is defined as $CA_i^t = (d_i^t - d_i^{t-1}) / y_i^t$.

Inspecting Figure further, we note the following. Germany and the rest of the Euro area are capital exporters with positive current account balances during the first half of the century which changes around 2030 (2040) in Germany (the rest of the Euro area). These results are in line with Marchiori (2011). The average German annual current account surplus from 2000 to 2018 is 1.52% of GDP. According to the IMF, a surplus of around 1.5% of GDP can also be observed in the data at the beginning of the millennium. Around 2003, however, the surplus in the data started increasing rapidly and steadily, which it does not in our model simulations. From 2000 to 2018, the average annual surplus of GDP according to the data is roughly 4.3%. According to our model simulation, ageing therefore only accounts for about one third of the actual surplus and, from 2003 onwards, something else must have been responsible for the sharp increase. One
potential candidate for these developments may be the German labor market reforms implemented in 2003 that significantly increased German international competitiveness (see Hochmuth et al., 2019, for a discussion). But further research to understand the effects at play is certainly in order.

In order to check how important population dynamics of Germany’s trading partners are for the evolution of net foreign assets and current account balances, we perform two additional simulations. First, as a rather extreme case, we assume that constant population dynamics around the world except for Germany (i.e. we assume zero population growth and the survival probabilities to remain at their initial steady-state levels in the rest of the world). Second, we feed in the population dynamics of the remaining OECD countries plus China. German current account and net foreign asset positions resulting from these simulations are presented in Figure 6. The other economic mechanisms are analogous to those described above.

![Figure 6: German current account effects for differently synchronized population dynamics](image)

Notes: Figure plots German net foreign asset and current account-to-GDP ratios for different population dynamics and compares these to the baseline simulations that we presented above. The baseline scenario is represented by blue solid lines, constant population dynamics for Germany’s trading partners by red dashed lines and population dynamics of OECD countries including China by green dotted lines.

As we can see, a constant world population significantly increases capital exports in Germany, driving up the current account surplus to 8% of GDP until beyond 2030. The reason is that the incentive to save more because of ageing is only present in Germany, and not around the world. Hence, as German asset supply increases disproportionately and domestic asset demand does not rise sufficiently, this leads to higher capital ex-
When taking into account the quicker ageing process in China, relative to our baseline simulations, in our second additional simulation, the opposite happens. This is a result of the fact that asset supply in the rest of the world now also increases sharply, driving down the world interest rate further, and leading to more negative demand effects (on the world level). The pattern of the current account-to-GDP ratio is similar to the one of our baseline simulation, but about one to 1.5 percentage points lower. As these simulations show, the (dis-)synchrony of the ageing process between regions is crucial for their net foreign asset and current account developments. Taking the demographic trends of Germany’s most relevant trading partners into account, ageing seems to only explain a minor part, at maximum one third, of the developments since the start of the millennium.

5. Conclusions

In this paper, we develop a three-region New Keynesian perpetual youth model calibrated to Germany, the Euro area and the rest of the world to analyze the impact of population ageing in Germany on its net foreign asset and current account developments.

Assuming unsynchronized demographic trends by taking those of Germany as given and constant population everywhere else, we are able to generate German current account surpluses of more than 8% of GDP during the first half of this century. However, projected demographic trends from 2000 to 2080 in Germany, the Euro area and the remaining OECD countries (plus China in a supplementary analysis) are much more synchronized. Feeding these into our model, simulations suggest that the average annual German current account surplus from 2000 to 2018 reduces to only 1.5% of GDP, turning negative around 2030 already. Hence, while indeed playing a significant role, our simulations suggest that German population ageing should not be seen as the one most important driver of the currently observed high and persistent German account surpluses. Further research to fully understand the reasons and effects at play is certainly in order.

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Note that, even though the rest of the world is not subject to demographic change, its economy will also be affected. This is a result of a fall in the world interest rate and reduced demand from Germany. However, as ageing only boosts asset supply in Germany, the effects on the world interest rate will be muted. And, given its relative size, German demand effects affecting the rest of the world are also limited.
Appendix

In this appendix, we provide a summary of the detrended model equations as described in Section 2.8. The bar indicates the respective detrended variable. For each region $i = a, b, c$, the marginal propensities to consume are given by

$$
e_i^t \pi_t^i = 1 - \left[ \beta \cdot a \tilde{u} x_{i,t+1}^i \right]^{\sigma} \cdot [(1 + r_t)]^{\sigma - 1} \cdot \gamma_i^t \cdot \frac{e_i^t \pi_t^i}{e_{i+1}^t \pi_{i+1}^t}, \quad (A.1)$$

and

$$\pi_t^i = 1 - \left[ \beta \cdot a \tilde{u} x_{i,t+1}^i \right]^{\sigma} \cdot \left[(1 + r_t) \cdot \Omega_{t+1}^i\right]^{\sigma - 1} \cdot \frac{\pi_t^i}{\pi_{i+1}^i}, \quad (A.2)$$

with

$$\Omega_{t+1}^i = \omega_i^t + (1 - \omega_i^t) \left(e_{t+1}^i \right)^{1/(1 - \sigma)} \left(1 / \xi^i \right) \nu_i^j, \quad (A.3)$$

where

$$a \tilde{u} x_{i,t+1}^i = \left[ \left(1 + i_{i+1}^t \right) \cdot \frac{v_m^i}{\nu_c^i} \cdot \nu_m^i \cdot \left(1 + x_i^t \right) \frac{w_i^j}{\tilde{w}_{i+1}^j} \right] \cdot \left(1 + x_i^t \right) \frac{w_i^j}{\tilde{w}_{i+1}^j}. \quad (A.4)$$

Consumption is given by

$$\bar{c}_{t}^{w,i} \left(1 + \nu_m^i / \nu_c^i \right) = \pi_t^i \left[ \frac{(1 + r_{t-1})}{(1 + n_i^{w,i}(1 + x_i^t))} \left(1 - \lambda_{i-1}^i \right) \tilde{a}_{i-1}^i + \tilde{h}_{i}^{w,i} \right], \quad (A.5)$$

$$\bar{c}_{t}^{r,i} \left(1 + \nu_m^i / \nu_c^i \right) = \pi_t^i \left[ \frac{(1 + r_{t-1})}{(1 + n_i^{w,i}(1 + x_i^t))} \lambda_{i-1}^i \tilde{a}_{i-1}^i + \tilde{h}_{i}^{r,i} \right], \quad (A.6)$$

$$\bar{c}_t^i = \bar{c}_{t}^{w,i} + \Psi_t^i \cdot \bar{c}_t^i, \quad (A.7)$$

implying

$$\bar{m}_t^i = (1 + \tilde{i}_t^i) / \nu_m^i \cdot \nu_c^i \cdot \bar{c}_t^i. \quad (A.8)$$
Human wealth is

\[
R^i_t = \bar{c}_i \cdot \bar{w}_i \cdot \bar{P}^i_t + \bar{c}_i + \frac{(1 + x^i) \cdot \gamma^i_t}{(1 + n^i_t)(1 + r^i)} \cdot R^i_{t+1},
\]

(A.9)

and financial wealth

\[
\lambda^i_t \bar{a}^i_t = \omega^i_t \left\{ (1 - c^i_t \kappa^i_t) \left[ \frac{(1 + r_{t-1})}{(1 + n_{t-1}^{w,i})(1 + x^i)} \lambda^i_{t-1} a^i_{t-1} + \bar{H}^i_t \right] - \left( \bar{H}^i_t - \bar{c}_i \bar{w}_i \bar{P}^i_t - \bar{c}_i \right) \right\}
\]

+ (1 - \omega^i_t) \bar{a}^i_t,

(A.10)

with

\[
\bar{a}^i_t = Q^i_t \bar{k}^i_t + \bar{b}^i_t + \bar{d}^i_t + \frac{1}{1 + i^i_t} \cdot \bar{m}^i_t.
\]

(A.12)

Labor supply by households is

\[
\bar{l}^w,i_t = 1 - \frac{v^i_t}{\bar{v}_c^i} \cdot \bar{c}^w,i_t, \quad (A.13)
\]

\[
\bar{l}^r,i_t = 1 - \frac{v^i_t}{\bar{v}_c^i} \cdot \bar{c}^r,i_t, \quad (A.14)
\]

\[
\bar{l}^i_t = \bar{l}^w,i_t + \bar{l}^r,i_t + \bar{c}_i \cdot \Psi^i_t. \quad (A.15)
\]

Firms produce

\[
\bar{y}^i_t = tf p^i \left( \bar{y}^i_t \right)^{\alpha^i} \left( \frac{\bar{k}^i_{t-1}}{(1 + n_{t-1}^{w,i})(1 + x^i)} \right)^{1-\alpha^i}, \quad (A.16)
\]
implying

\[
(1 + n_{i-1}^j)(1 + x^i) \cdot \frac{\bar{p}_i^j}{\bar{k}_{i-1}} = \frac{\alpha^i}{\bar{\alpha}^i} \cdot \frac{r_{i}^{k,i}}{\bar{\omega}_i^i},
\]

(A.17)

\[
mc_i^j = \left(\frac{\bar{\omega}_i^j}{\alpha^i}\right)^{\alpha^i} \cdot \left(\frac{r_{i}^{k,i}}{1 - \alpha^i}\right)^{1-\alpha^i}
\]

(A.18)

and

\[
\bar{f}_i^j = (\frac{p_{i}^{l,i}}{p_i^j} - mc_i^j) \bar{y}_i^j
\]

(A.19)

as the capital-labor ratio, the marginal cost function and aggregate profits, respectively.

Calvo pricing gives

\[
D_i^j = \frac{p_{i}^{l,i}}{p_i^j} = \kappa_p^i \left(\frac{p_{i}^{l,i}}{p_{i-1}^{l,i}}\right)^{\theta_p^i} D_{i-1}^j + (1 - \kappa_p^i) \left(\frac{p_i^j}{p_{i-1}^{l,i,*}}\right)^{\theta_p^i}
\]

(A.20)

with

\[
\frac{p_{i-1}^{j,i,*}}{p_i^j} = \frac{\kappa_p^i}{\kappa_p^i - 1} \cdot \frac{q_{1_i}^i}{q_{2_i}^i}
\]

(A.21)

where

\[
q_{1_i}^i = DF_i^j mc_i^j \bar{y}_i^j + \beta \kappa_p^i \left(\frac{p_{i}^{l,i}}{p_{i-1}^{l,i}}\right)^{\theta_p^i} \cdot q_{1_i}^j_{i+1},
\]

(A.22)

\[
q_{2_i}^j = DF_i^j \bar{y}_i^j \cdot \frac{p_i^j}{p_{i-1}^{l,i}} + \beta \kappa_p^i \left(\frac{p_{i}^{l,i}}{p_{i-1}^{l,i}}\right)^{\theta_p^i-1} \cdot q_{2_i}^j_{i+1},
\]

(A.23)

and

\[
DF_{i,t+1}^j = \left(\pi_{t+1}^i\right)^{-1/p} \left(v_m^i/v_c^i \cdot (1 + i_{t+1}^i)/i_{t+1}^i\right) u_m^i \left(v_f^i/v_c^i / ((1 + x^i)\bar{\omega}_{i+1}^i)\right)^{v_f^i}.
\]

(A.24)
Capital evolves according to

\[
\bar{k}_t^i = \frac{(1 - \delta^i)}{(1 + n^w \cdot \delta)} \cdot \bar{k}_{t-1}^i \cdot \left[1 - \frac{k_{i,\text{inv}}^i}{2} \left(\frac{i_{\text{inv}}^i}{t-1} - 1\right)^2\right] \cdot \bar{\nu}_i,
\]

(A.25)

and the no-arbitrage conditions imply

\[
(1 + r_t) = \frac{r_{t+1}^k + Q_{t+1}^i (1 - \delta^i)}{Q_t^i} = (1 + i_t^i) \frac{p_t^i}{p_{t+1}^i},
\]

(A.26)

with

\[
1 = Q_t^i \cdot \left[1 - S_i^i(\cdot) - S_i^i(\cdot) \cdot \frac{i_{\text{inv}}^i}{\text{inv}_{t-1}}\right] + \frac{Q_{t+1}^i}{1 + r_{t+1}} \cdot S_i^i(\cdot) \left(\frac{i_{\text{inv}}^i}{\text{inv}_{t-1}}\right)^2.
\]

(A.27)

The fiscal authorities behave according to

\[
\bar{b}_t^i + \bar{m}_t^i + \bar{\tau}_t^i = \frac{(1 + r_{t-1})}{(1 + n^w \cdot \delta)} \cdot \left(\bar{b}_{t-1}^i + \frac{m_{i-1}^i}{1 + i_{t-1}^i}\right) + \frac{p_t^i}{p_t^i} \cdot \bar{g}_t^i + \bar{e}_t^i,
\]

(A.28)

with

\[
\bar{g}_t^i = \bar{g}_t^i,
\]

(A.29)

\[
\mu_t^i = \mu_t^i,
\]

(A.30)

and

\[
\log \left(\frac{\tau_t^i}{\tau_t^i}\right) = \rho^{\tau,i} \log \left(\frac{\tau_{t-1}^i}{\tau_t^i}\right) + \zeta^{\tau,i} \log \left(\frac{p_t^i \cdot \bar{b}_{t-1}^i}{p_{t-1}^i \cdot \omega^{\tau,b,i} \cdot \bar{y}_{t-1}^j}\right),
\]

(A.31)

where \(\bar{e}_t^i = \mu_t^i \cdot \bar{w}_t^i \cdot \Psi_1^t\) as described in the main text.

For the international part, we get

\[
\bar{x}_t^i = \left[\left(\theta_a^i\right)^\eta^i \left(\bar{x}_{a,t}^i\right)^{\eta^i} + \left(\theta_b^i\right)^\eta^i \left(\bar{x}_{b,t}^i\right)^{\eta^i} + \left(\theta_c^i\right)^\eta^i \left(\bar{x}_{c,t}^i\right)^{\eta^i}\right]^\frac{1}{\eta^i},
\]

(A.32)

\[
\frac{\bar{x}_{a,t}^i}{\bar{x}_{b,t}^i} = \theta_a^i \left(\frac{p_{t}^{b,i}}{p_{t}^{a,i}}\right)^\frac{1}{1-\eta^i},
\]

(A.33)
and

\[
\frac{\bar{x}_{i,t}}{\bar{x}_{c,t}} = \frac{\partial_i}{\partial_c} \left( \frac{p_{i,t}^{c,i}}{p_{i,t}^{c,i}} \right)^{\frac{1}{1-\eta_i}}, \quad (A.34)
\]

where \(\bar{x} \in \{c, inv\}\) and \(p_{i,t}^{c,i} = S_{i,t}^{c,i} p_{i,t}^{i,i}\) with \(S_{i,t}^{c,i} = S_{i,t}^{a,b} = S_{i,t}^{b,a} = 1\) as described in the main text. For relative prices, this implies

\[
\frac{p_i^i}{p_i^{b,i}} = \left[ \frac{\vartheta_a^i + \vartheta_b^i \cdot \left( \frac{p_{i,t}^{b,i}}{p_{i,t}^{c,i}} \right)^{\eta_i/(1-\eta_i)} + \vartheta_c^i \cdot \left( \frac{p_{i,t}^{c,i}}{p_{i,t}^{c,i}} \right)^{\eta_i/(1-\eta_i)} }{\eta_i} \right]^{\frac{1-\eta_i}{\eta_i}}, \quad (A.35)
\]

\[
\frac{p_i^i}{p_i^{b,i}} = \left[ \vartheta_a^i \cdot \left( \frac{p_{i,t}^{a,i}}{p_{i,t}^{b,i}} \right)^{\eta_i/(1-\eta_i)} + \vartheta_b^i + \vartheta_c^i \cdot \left( \frac{p_{i,t}^{c,i}}{p_{i,t}^{c,i}} \right)^{\eta_i/(1-\eta_i)} \right]^{\frac{1-\eta_i}{\eta_i}}, \quad (A.36)
\]

and

\[
\frac{p_i^i}{p_i^{b,i}} = \left[ \vartheta_a^i \cdot \left( \frac{p_{i,t}^{a,i}}{p_{i,t}^{c,i}} \right)^{\eta_i/(1-\eta_i)} + \vartheta_b^i \cdot \left( \frac{p_{i,t}^{b,i}}{p_{i,t}^{c,i}} \right)^{\eta_i/(1-\eta_i)} + \vartheta_c^i \right]^{\frac{1-\eta_i}{\eta_i}}. \quad (A.37)
\]

For net foreign assets in regions \(\tilde{i} \in \{a, b\}\), we get

\[
f_{\tilde{i}} = \frac{(1 + r_{\tilde{i}-1})}{(1 + n_{\tilde{i}-1})(1 + \chi^i)} \cdot \tilde{f}_{\tilde{i}}^{\tilde{i}-1} + \frac{p_{\tilde{i}}^{i,i}}{p_{\tilde{i}}^{i,i}} (\tilde{g}_{\tilde{i}} - \tilde{g}_{\tilde{i}}^i) - \tilde{c}_{\tilde{i}} - i\nu_{\tilde{i}}, \quad (A.38)
\]

and

\[
rs_{\tilde{i}}^{a,c} \cdot f_{\tilde{i}}^{a} = -rs_{\tilde{i}}^{b,c} \cdot \frac{p_{a}^{b}}{p_{a}^{b}} \cdot f_{\tilde{i}}^{b} - \frac{p_{b}}{p_{b}} \cdot f_{\tilde{i}}^{b}, \quad (A.39)
\]

where the latter equation is a result of the fact that net foreign assets must be in zero net supply on the world level. Product market for regions \(i \in \{a, b, c\}\) clearing implies

\[
D_{j} \tilde{y}_{j} = \tilde{c}_{j}^{i} + i\nu_{j}^{i} + \tilde{g}_{j}^{i} + rs_{j}^{i,i} \cdot \left( c_{j}^{i} + inv_{j}^{i} \right) + rs_{j}^{i,i} \cdot \left( c_{j}^{i} + inv_{j}^{i} \right), \quad (A.40)
\]

where \(j, \tilde{j} \in \{a, b, c\}\) and \(i \neq j \neq \tilde{j}\).

Monetary policy reacts as described in the main text (see Section 2.7 as well as equation (28)), where inflation rates and the output gap are defined accordingly. Population
dynamics are given by

\[
\Psi_i^t = \frac{1 - \omega_i^{t-1}}{1 + n_i^{w,i}} + \frac{\gamma_{i-1}^t}{1 + n_i^{w,i-1}} \Psi_i^{t-1},
\]

(A.41)

where population growth rates as well as retirement and survival probabilities are given by the exogenous processes

\[
(1 + n_i^{w,i}) = (1 + n_i^{w,j}) + \epsilon_i^{n,w,i},
\]

(A.42)

\[
\omega_i^t = \omega^t + \epsilon_i^{\omega,i}
\]

(A.43)

and

\[
\gamma_i^t = \gamma^t + \epsilon_i^{\gamma,i}.
\]

(A.44)

Here, the \(\epsilon\)'s represent exogenous iid shock processes. Relative (working-age) population between regions \(i\) and \(j\) evolves according to

\[
rs_{i,j}^t = \frac{(1 + n_i^{w,i})}{(1 + n_i^{w,j})} \cdot rs_{i,j}^{t-1}.
\]

(A.45)

For the relative total population size, we have to take into account the evolution of the OADR as described in Section 2.1 of the main text.

Given the targets and the parameter values described in Section 3, it is then straightforward to numerically \textbf{derive the steady state} based on the above equation system evaluated at steady state. After some rearranging and making use of the recursiveness of the above equation system, that boils down to solving a system of 12 equations in 12 unknowns. For \(i \in \{a, b, c\}\), the 12 equations we need to solve for are

\[
\bar{c}_i = \bar{c}_w,i + \Psi_i \cdot \bar{c}_r,i,
\]

(B.1)

\[
\bar{c}_r = \frac{1 - [\beta \cdot a\bar{x}i]^{\sigma} \cdot [(1 + r)]^{\sigma-1} \cdot \gamma_i}{1 - [\beta \cdot a\bar{x}i]^{\sigma} \cdot [(1 + r) \cdot \Omega_i]^{\sigma-1}}
\]

(B.2)

\[
\bar{f}_w,i = 1 - \frac{v_j^i}{v_c^i} \cdot \frac{\bar{c}_w,i}{\bar{w}^i},
\]

(B.3)

\[
\bar{f}_r,i = 1 - \frac{v_j^i}{v_c^i} \cdot \frac{\bar{c}_r,i}{\bar{w}^i}.
\]

(B.4)

The unknowns are the world interest rate \(r\), the net foreign asset positions \(\bar{f}_a\) and \(\bar{f}_b\) – where, by equation (A.39) evaluated steady state, we can also solve for \(\bar{f}_c\) directly –, and
the markup on the propensity to consume of retirees $c^i$, with $i \in \{a, b, c\}$. The remaining (“unknown”) parameters to solve for are $\xi^i$ and $\upsilon^i$, making use of $\upsilon^i = 1 - \upsilon_c^i - \upsilon_l^i$ for the latter one. Given the recursiveness of the system of equations (A.1) to (A.45), it is straightforward and computationally not demanding to solve for all steady-state values.

As stated in the main text, we also show the simulation results when assuming a positive utility of real money balances by setting $\upsilon^i_m = 0.05$. Figures C.1 to C.3 are analogous to Figures 3 to 5 in the main text. As we can see, the general economic effects are analogous, too. However, as the interest rate falls by a bit more, the level of savings increases a bit. This also holds for the net foreign asset positions and the current account.

Figure C.1: Consumption and savings reactions when $\upsilon^i_m = 0.05$
Figure C.2: Production sector reactions when $\nu_{im} = 0.05$

Figure C.3: International transmission when $\nu_{im} = 0.05$
References


