Abstract

Conventional business cycle analysis interprets economic fluctuations as high frequency variations around an exogenous trend. In contrast to this approach, we include two sources of growth (ideas and knowledge) to determine the endogenous trend of an economy, and examine its quantitative potential in a standard medium scale New Keynesian model. We estimate this model on the US data between 1950q1-2018q4 with an occasionally binding constraint on the nominal rate. We find that the endogenous trend has been sharply declining since 1970, thus corroborating the secular stagnation theory. This dynamic is captured by a slowdown in the accumulation technology reflecting the low productivity of the R&D sector. While the contribution of human capital has been remarkably stable, the financial crisis deteriorated its contribution over the last decade.

JEL codes: E3, O3.
1 Introduction

In modern models of the business cycle, economic fluctuations are interpreted as high frequency oscillations around a trend growing at an exogenous rate. This conception is questionable given the strong body of evidence in empirical macroeconomics showing that the trend of the US economy is time-varying (Nelson and Plosser (1982)) and reducing over time.\(^1\) Despite this evidence, most of recent medium scale macroeconomic models, incarnated by the Smets and Wouters (2007) model, assumes a fixed slope of growth, or exogenous drifts to productivity (e.g., Christiano et al. (2014)). The resulting interpretation of business cycles is at odd with the evidence of structural changes in the long run growth of an economy observed over the last decades. In particular, the underlying factors that are jointly driving low frequency changes in macroeconomic time series are usually swept out by business cycle filters, or erroneously captured by exogenous disturbances. This calls for a framework that takes the determinants about long-run growth into a macroeconomic model seriously.

The main goal of this paper is thus to develop a quantitative model that features an endogenous slope of growth, referred to as an endogenous trend.\(^2\) Guided by the endogenous growth theory, the trend at which the economy is growing at a low frequency has two possible roots based on the accumulation of ideas and knowledge. For the first engine of growth based on the accumulation of ideas, the endogenous productivity mechanism we develop is based on Comin and Gertler (2006), which uses the approach to connect business cycles to growth. This model of Comin and Gertler (2006) is itself a variant of Romer (1990)’s expanding variety model of technological change, modified to include a friction on the endogenous probability of technology adoption. We include a sticky rate of adoption to capture a congestion externality in the diffusion of new technologies. The second engine of growth is based on the accumulation of knowledge (i.e., “experience” or “skill”), through a model of human capital à la Lucas Jr (1988). Each period, firms engage a fraction of their labor inputs into vocational training in order to produce human capital. We modify the Lucas framework to allow for an endogenous rate of adoption of new skills, along with endogenous human capital formation. By doing so, we are able to allow for empirically reasonable diffusion lags but still generate endogenous medium-term swings in productivity.

\(^1\)For a long run perspective on growth, see Antolin-Diaz et al. (2017) For recent papers after the Great Recession documenting the slowdown of the US economy, see Fernald and Jones (2014) and (Gordon, 2012, 2017).

\(^2\)These cyclical movement are interpreted by Comin and Gertler (2006) as medium term fluctuations. In this paper, I interpret these fluctuations as persistent changes in the growth rate of the economy that affects key macroeconomic variables.
We then estimate the model with an endogenous trend on a sample spanning from 1950q1 up to 2018q4 using Bayesian techniques. The solution method employed to estimate the model features an occasionally binding constraint on the nominal rate. We then use the model to assess the slowdown of long term growth, in particular following the onset of the Great Recession. Based on the estimated model, our key result is that we corroborate the thesis of a strong decline in the long term trend of the US economy. Among the two sources of growth examined in the paper, the slowdown mainly is induced by the technology engine reflecting a decline in the productivity of creation of new technologies since 1960. This finding tends to favor the Gordon (2012) theory stating that the US growth has strongly declined since 1970. In addition, we find that a standard macro-model with exogenous growth erroneously captures low frequency changes in economic growth by highly persistent macroeconomic shocks. In contrast, the model featuring an endogenous trend successfully captures this low frequency fluctuations.

In addition to the literature cited above, there are several other papers related to our analysis. Anzoategui et al. (2016) estimates a macroeconomic model with one source of growth for the US economy. They evaluate the role of R&D in the productivity slowdown following the financial crisis, they find that the reduction in productivity is induced by a reduction in the adoption rate of technology. Moran and Queralto (2017) complete this analysis by including the role of monetary policy, in particular when the ZLB is binding. Queralto (2013) combines R&D expenditures with financial frictions to examine the role of financial factors in the slowdown of growth for the US economy. An alternate approach of Garcia-Macia (2017) stresses misallocation between tangible and intangible capital following a financial crisis. Annicchiarico and Pelloni (2016) inspect the implications of the endogenous growth on the optimal conduct of monetary policy.

The rest of the paper is organized as follows. section 2 presents a New Keynesian Model with two sources of endogenous growth. section 3 is devoted to the estimation of the model using Bayesian econometrics. section 4 evaluates the consequences an endogenous rate of growth on the transmission of TFP shock, and the role of a time-varying trend on the cross-correlation of observable variables. section 5 studies the contribution of the accumulation of technologies and knowledge in the historical evolution of the long run growth rate of the US economy since 1950. section 6 evaluates the role of the zero lower bound on the economic contraction during the financial crisis. section 7 concludes.
2 A New Keynesian Model with Two Sources of Endogenous Growth

This section describes the theoretical framework, consisting of a standard medium-sized New Keynesian model augmented to include endogenous creation and adoption of new technologies and human capital. Sub-sections 2.2.3 and 2.4 constitute the main departures from other medium-sized DSGE models found in the literature. The rest of the subsection provides the conventional ingredients of the a New Keynesian model similar to Smets and Wouters (2007).

2.1 Households

The preferences of the $j^{th}$ family are given by:

$$E_t \left\{ \sum_{\tau=0}^{\infty} \beta^{\tau} \left[ \frac{(c_{jt+\tau} - h c_{jt-1+\tau})^{1-\sigma_C}}{1 - \sigma_C} \exp \left( \left( \frac{\sigma_C - 1}{1 + \sigma_L} \right) \chi_L (1+\tau) \right) \right] \right\}$$

(1)

where $E_t$ denotes the expectation operator and $\beta \in (0, 1)$ is the discount factor. The consumption index $c_{jt}$ is subject to external habits with degree $h \in [0; 1)$ with $c_{t-1}$ the aggregate lagged consumption, while $\sigma > 0$ is the risk aversion coefficient. Parameter $\sigma_L > 0$ shapes the consumption-leisure trade-off, while $\chi_L > 0$ is a shift parameter pinning down the steady state amount of hours worked. The disutility of working is weighted by the rate of employment $n_{jt}^s$ so that only employed workers face disutility costs.

The representative household’s budget constraint is given by:

$$c_{jt} + b_{jt} = b_{jt-1} / \pi_t + w_{jt} h_{jt} l_{jt} - t_{jt},$$

(2)

The income of the representative household is made of labor income with real wage $w_t$ and human capital $h_{jt}$, interest payments $r_{t-1} / \pi_t$ from real government bonds $b_{jt}$, with inflation rate given by $\pi_t = P_t / P_{t-1}$.

2.2 Intermediate Firms

2.2.1 Intermediate goods composite

There exists a continuum of measure $A_t$ of monopolistically competitive intermediate goods firms that each make a differentiated product. The endogenous variable $A_t$ is the stock of types of intermediate goods adopted in production, i.e., the stock of
adopted technologies. We assume that one firm produced one type of variety such that \( i \in [0, A_t] \) both refers to a good or an intermediate firm. Each firm produces output \( x_{it} \) at a selling price \( p_{it}^x \). The intermediate goods composite is the following CES aggregate of individual intermediate goods:

\[
X_t = \left[ \int_0^{A_t} x_{it}^{(\theta-1)/\theta} \, di \right]^{\theta/(\theta-1)},
\]

(3)

where parameter \( \theta > 1 \) is the degree of imperfect substitution between varieties allowing intermediate firms to make profits. The aggregate price index is given by:

\[
P_t^x = \left[ \int_0^{A_t} (p_{it}^x)^{1-\theta} \, di \right]^{1/(1-\theta)}.
\]

The optimal demand for the \( i \)-th varieties is given by:

\[
x_{it} = (p_{it}^x / P_t^x)^{-\theta} X_t.
\]

(4)

### 2.2.2 Production technology

There is a continuum of \( i \) firms that produces an homogenous good by combining labor inputs, capital inputs and technology. The \( i \)-th firm has the following Cobb-Douglas technology:

\[
x_{it} = \varepsilon_t^A \left[ (1 - e_{it}) l_{it}^d h_{it}^u \right]^{\alpha} \left[ u_{it} k_{it-1} \right]^{1-\alpha} h_t^r
\]

(5)

where (exogenous) AR(1) technology is \( \varepsilon_t^A \), hours worked demand \( l_{it}^d \), human capital \( h_{it} \), capital inputs, and technology. The utilization rate of physical capital and \( k_{it-1} \) is the physical capital. Parameter \( \alpha \in [0, 1] \) measure the labor intensity in the firms technology. Workers may spend a fraction \( e_{it} \) of their time acquiring skills. That is, they can learn to use more advanced capital goods. As in Lucas, parameter \( \tau \geq 0 \) is the intensity of human capital internalized by the worker employed by the firm, while \( \gamma \geq 0 \) is the external effect which benefits to the overall economy. According to Mincer (1974), an additional year of schooling or an additional year of experience should increase wages proportionally. To incorporate this mechanism in the model, we assume increasing returns on human capital, with elasticity \( \omega > 1 \).

Real profits are given by:

\[
d_{it}^x = \frac{P_t^x}{P_t} x_{it} - w_t h_{it} l_{it}^d - \varepsilon_{it}^l \left( 1 + S_h \left( z_{it}^H / z_{it-1}^H \right) \right) z_{it}^H - S_{ih} \left( z_{it}^H / z_{it-1}^H \right) z_{it}^H - z_{it}^H h_{it}^u
\]

(6)

where \( \varepsilon_{it}^l \) is a stochastic process which captures exogenous changes in the value of physical capital, regarding adjustment cost functions \( S_{ax} (x_t) = \chi_a (x_t - \bar{x})^2 \) with \( \chi_a \geq 0 \).
is the adjustment cost parameter.

For clarity purpose, we separate production and labor decisions in the following subsections.

2.2.3 Production and adoption of skills

As in Lucas Jr (1988), we assume that there firms can spent a fraction \( e_{it} \) of working to the accumulation of human capital while \((1 - e_{it}) l_{it}^d \) is the skill-weighted man-hours devoted to current production. The rise in more skilled worker does not necessary translate into immediate growth of output. We capture this pattern by assuming that all skills in the economy are not necessarily adopted by firms, this can interpreted as an “education inflation”. Let us assume there is a stock of unadopted human capital, denoted \( h_{it}^u \), given by:

\[
h_{it}^u = (1 - \delta_H) \left[ F_H \left( e_{i, t-1}, z_{it}^H \right) + (1 - p_{t-1}^H) h_{it-1}^u \right]
\]

(7)

where \( \delta_H \) is the obsolescence rate of a skill, \( F_H(.) \) is the production function of new human capital and \( p_{t-1}^H \) is the endogenous probability of adoption of a skill by the \( i \)-th firm. Regarding the adoption probability of a skill, our goal is to capture the notion that adoption takes time on average, but allow for adoption intensities to vary procyclically. These considerations lead us to the following formulation for the functional form:

\[
p_{H_{it}} = \varsigma_{H_{it}} \left( s_{H_{it}} \right)^{\kappa_{H_{it}}}
\]

where \( \varsigma_{H_{it}} \) is a scaling factor that pins down the steady state in the balanced growth path, \( s_{H_{it}} \) are the adoption expenditures in units of final goods.

As in Jones et al. (1993), human capital creation is a Cobb-Douglas function that combines education hours \( e_{it} \) and education expenditures \( z_{it}^H \):

\[
F_H \left( e_{i,t}, z_{it}^H \right) = \xi_{t}^H \left( e_{it} \right)^{1-\upsilon} \left( z_{it}^H \right)^{\upsilon}
\]

(8)

where \( \xi_{t}^H \) is a productivity parameter that pins down the steady state in the balanced growth path, \( \upsilon \) is a technology parameter determining the intensity of education expenditures in the production of knowledge. For \( \upsilon = 0 \), the model reads as in Lucas, while for \( \upsilon > 0 \), the model is similar to the setup of Jones et al. (1993).

The law of motion of adopted skills, or effective human capital, is given by:

\[
h_{it} = (1 - \delta_H) \left[ p_{t-1}^H h_{it-1}^u + h_{it-1} \right]
\]

(9)

Intermediate firms maximize their profits under Equation 6, the supply constraint 5, the demand constraint 4 and law of motions 7 and 9. Letting \( v_{it}^U \) and \( v_{it}^H \) denote
the Lagrangian multipliers associated with laws of motion of unadopted and adopted human capital respectively. They represent the current marginal value of unadopted and adopted skills, respectively.

The optimal fraction of hours worked spent in education $e_{it}$ is given by:

$$
\frac{p_t^e}{\mu_\vartheta \alpha} \frac{x_{it}}{1 - e_{it}} = (1 - \delta_H) E_t \left\{ m_{t,t+1} V_{t+1}^U \right\} F'_{H,t},
$$

(10)

where $F'_{H,t}$ is the derivative in education of the production function of knowledge and $V_{t}^U$ is the value of unadopted skills. The left hand side of Equation 10 is the productivity loss of increasing $e_{it}$, while the right hand side denotes the expected marginal product of unadopted skills. Parameter $\mu_\vartheta = \vartheta/(\vartheta - 1)$ is the markup over the marginal cost of producing intermediate goods.

The optimal education spending $z_{H, it}$ reads as:

$$
1 + \frac{\partial S_{H,t} H_{it-1}}{\partial z_{it}} + E_t \left\{ m_{t,t+1} \frac{\partial S_{H,t+1} z_{H,t}}{\partial z_{it}} \right\} = (1 - \delta_H) E_t \left\{ m_{t,t+1} V_{t+1}^U \right\} F'_{H,t},
$$

(11)

Similarly to the optimal education, the left hand side denotes the marginal cost of education spending and the right hand side is the expected marginal product of unadopted skills.

In addition, the optimal amount of adopted human capital $h_{it}$ is given by:

$$
V_{t}^H = \frac{p_t^H}{\mu_\vartheta} \omega \frac{y_{it}}{h_{it}} - w_t l_{it} + (1 - \delta_H) E_t \left\{ m_{t,t+1} V_{t+1}^H \right\}
$$

(12)

The current value of human capital $V_{t}^H$ is determined by its marginal productivity, net of wage payments, and the expected value of the adopted skill if the human capital does not depreciate.

The value of unadopted skills:

$$
V_{t}^U = (1 - \delta_H) E_t \left\{ m_{t,t+1} \left[ p_t^H V_{t+1}^H + (1 - p_t^H) V_{t+1}^U \right] \right\}
$$

(13)

Finally the optimal demand for $s_{it}^H$ is given by:

$$
(1 - \delta_H) E_t \left\{ m_{t,t+1} \left[ V_{t+1}^H - V_{t+1}^U \right] \right\} p_t^H = 1
$$

(14)

where $p_t^H$ is the derivative of the probability of adoption with respect to the quantity of goods $s_{it}^H$ spent in adoption of skills. The term on the right is the marginal gain from adoption expenditures: the increase in the adoption probability $p_t^H$ times the
discounted difference between the value of an adopted versus an unadopted skill. The
right side is the marginal cost. The term $V_{t+1}^H - V_{t+1}^U$ is pro-cyclical, given the greater
influence of near term profits on the value of adopted skills relative to unadopted ones.

2.2.4 Capital decisions

Intermediate firms maximize their profits under Equation 6 under the supply con-
straint 5 and the demand constraint 4 and the following law of motion of capital:

$$k_{it} = i_{it} + (1 - \delta (u_{it})) k_{i-1},$$

where $\delta (u_{it}) = \delta_c + \frac{b}{1+\psi} u_{it}^{1+\psi}$. In this function $\delta_c > 0$ is the fixed part of the deprecia-
tion, while the time-varying part is a function of the utilization rate of capital. $\psi \geq 0$
is the elasticity of the depreciation with respect to utilization. Parameter $b \geq 0$ is a
shift parameter which allows to pin down the steady state utilization rate.

The first order condition determining the shadow value of investment goods is
given by:

$$q_t = \varepsilon_t (1 + \frac{\partial i_{it} S (i_{it}/i_{i-1})}{\partial i_{it}}) + m_{t,t+1} \varepsilon_t \frac{\partial S (i_{it+1}/i_{it}) i_{it+1}}{\partial i_{it}},$$

where $q_t$ is the Lagrangian multiplier associated with the law of motion of physical
capital.

The optimal demand for physical capital is given by:

$$q_t = E_t \left\{ m_{t,t+1} \left[ \frac{P_{t+1}}{\mu_\theta} (1 - \alpha) \frac{y_{it+1}}{k_{it}} - (1 - \delta (u_{it+1})) q_{t+1} \right] \right\}.$$

The optimal utilization rate is given by:

$$(1 - \alpha) \frac{y_{it} P_t}{u_{it} \mu_\theta} = \delta' (u_{it}) q_t$$

2.3 Final firms

Each period firm $i$ is not allowed to re-optimize its price with probability $\theta_P$ but price
increases by $\xi_P \in [0; 1)$ with respect to the previous period’s rate of price inflation,
$P_t = \pi_{t-1}^{\xi_P} P_{t-1}$. The $i^{th}$ firm allowed to modify its selling price with a probability $1 - \theta_P$
chooses \( P_t^* \) to maximize its discounted sum of profits:

\[
\max_t \left\{ \mathbb{E}_t \sum_{s=0}^{\infty} \theta_t^s m_{t,t+s} \left[ \frac{P_{t+s}^* \Xi \theta_{t+s}^s - \varepsilon_{t+s}^P P_{t+s}^x}{P_{t+s}} \right] Y_{t+s} \right\},
\]

s.t. \( y_{it+s} = (p_{it}^* \Xi \theta_{t+s}^s / P_{t+s})^{-\epsilon_P} Y_{t+s} \)

where \( \varepsilon_{t+s}^P \) is an ad hoc cost-push shock to the inflation equation following an AR(1) process which captures exogenous inflation pressures. \( \Xi_{t+s}^P = \prod_{j=1}^{s} \eta_{t+j} \bar{\eta}_{t-1+j} \bar{\eta}_{1-\xi_w} \) for \( s > 0 \), while \( \Xi_{t+s}^P = 1 \) for \( s = 0 \).

Once goods are produced and prices are set, final firms act as goods packers: they combine differentiated goods to produce the homogenous final good sold mainly to households.\(^3\)

### 2.4 INNOVATORS

We model technology following Comin and Gertler (2006), which is in turn based on the expanding-variety framework due to Romer (1990). Innovations in the model take the form of new patents \( Z_t \) which are discovered endogenously as a result of private R&D spending. As Comin and Gertler (2006), patents are subject to a “time-to-adop” friction: a new technology does not necessarily give birth immediately to a new variety of intermediate goods. Converting a patent into a new variety is costly for innovators and create a lag between the creation of a new technology and its translation into an stronger rate of growth for the economy.

Let us assume that there are \( z \in [0; \eta] \) different innovators populating the economy.\(^4\) Each innovator owns a stock of existing patents, denoted \( Z_{zt} \), representing the technological frontier in the economy. These technologies are subject to exogenous obsolescence, which occurs with probability \( \delta_A \). Let \( x_{it}^A \) denote the R&D expenditures (in units of final output) devoted to the creation of a new patent, denoted \( \nu(x_{it}^A) \), the law of motion of patents (or the “technological frontier”) are given by:

\[
Z_{zt} = (1 - \delta_A) \left( Z_{zt-1} + \Phi_A \left( x_{zt-1}^A \right) \right). \tag{19}
\]

Here, both existing and new patents are subject to the obsolescence shock, this implies

3Goods packers are perfectly competitive and maximize profits, \( P_t Y_t - \int_0^1 P_{it} y_{it} \, di, \) under their packing technology constraint, \( Y_t = \int_0^1 (Y_{it} \varphi_{(\epsilon_P - 1)} / \epsilon_P) \, di / (\epsilon_P - 1). \) Here, \( P_t \) is the production price, \( Y_t \) is the aggregate demand (or the resource constraint) and \( \epsilon_P \) is a substitution parameter. The first order condition which determines the optimal demand for the \( i^{th} \) good is, \( y_{it}^* = (p_{it} / P_t)^{-\epsilon_P} Y_t, \) \( \forall i. \) Thus the aggregate price index of all varieties in the economy emerges from the zero-profit condition: \( P_t = \left[ \int_0^1 P_{it}^{1-\epsilon_P} \, di \right]^{1/(1-\epsilon_P)} \).

4The number of innovator pins down the steady state of R&D spending to GDP ratio.
that some new technologies are abandoned and never translate into new intermediate goods. Regarding the production of a new technology using R&D spending, we assume \( \Phi_A(x_A^{zt}) = \varepsilon_A^{zt} \xi_A^{zt} (x_A^{zt})^{\varepsilon_A} \) where \( \varepsilon_A \) is a technology parameter, and \( \xi_A^{zt} \) pins down the growth rate of technology in the balanced growth path. As suggested by Griliches (1990), the production of new patent has decreasing return to scale (i.e. \( \varepsilon_A < 1 \)) that captures a congestion effect raises the cost of developing new products as the aggregate level of R&D intensity increases. This effects is usually referred to as the “stepping on toes”: i.e. the obvious new ideas are discovered first and it gets increasingly difficult to find the next new one (e.g., see ??).

Recall that \( A_{zt} \) is the number of varieties of intermediate goods, thus any point on the real line between 0 and \( A_{zt} \) represents a distinct variety of intermediate goods. With a time-to-adopt assumption, there is a gap between numbers of available and adopted technologies, both denoted \( Z_{zt} \) and \( A_{zt} \) respectively. This gap, denoted \( A_u^{zt} \), is referred to as the stock of unadopted technologies and has the following law of motion:

\[
A_u^{zt} = (1 - \delta_A) \left( \Phi_A \left( x_A^{zt-1} \right) + (1 - p_A \left( s_A^{zt-1} \right)) A_u^{zt-1} \right).
\] (20)

In this expression, \( p_A(\bullet) \) denotes the speed of adoption of an unadopted technology, that is an increasing function of R&D spending \( s_A^{zt-1} \) in units of final goods.\(^5\) If the adopter is not successful, he may try again in the next period. Thus, under our formulation there is slow diffusion of technologies on average that varies positively with the intensity of adoption expenditures. This endogenous mechanism of adoption reproduces the cyclicality of technology diffusion that is observed in the micro data, as shown by Anzoategui et al. (2016).

The remaining set of technologies \( A_{zt} \) that are effectively converted into an intermediate goods are given by the following law of motion:

\[
A_{zt} = (1 - \delta_A) \left( p_A \left( s_A^{zt-1} \right) (1 - S_A \left( p_A^{zt-1} / p_A^{zt-1} \right)) A_u^{zt-1} + A_{zt-1} \right)
\] (21)

where \( S_A \left( p_A^{zt-1} / p_A^{zt-1} \right) \) denotes an adjustment cost on rising the probability of adoption with \( S_A(x_t) = 0.5 \chi_A (x_t - \bar{x})^2 \) similar to Christiano et al. (2005). This cost function is new with respect to the literature and has two goals. First, it captures another congestion externality à la Romer (1990) on the adoption of a new technology: firms trying to get a new product to market face a lower probability success. Second, this cost aims at capturing the low frequency nature of \( A_{zt} \): an higher value for \( \chi_A \) implies a lower frequency for the growth of technology \( \chi_A \). We are thus free to estimate this

\(^5\)The functional form for \( p_A \left( s_A^{zt-1} \right) = \varsigma_A^{zt} (s_A^{zt-1})^{\kappa_A} \) is taken from Comin and Gertler (2006), parameter \( \kappa_A \) is the elasticity of adoption with R&D spending \( s_A^{zt-1} \) while \( \varsigma_A^{zt} \) is a scaling factor pining down the steady state in the balance growth path of the model.
cost parameter to match the evidence by setting a diffuse prior distribution on this parameter. The fit exercise of Moran and Queralto (2017) shows that adjustment cost on R&D expenditures are much larger than for investment goods.

The real profit of the innovator is given by:

\[
\Pi^A_{zt} = A^t_{zt} / P_t - \frac{P^I_t}{P_t} A^u_{zt} \]  

(22)

where \( \Pi^e_{zt} \) is the monopoly rent that the innovator obtain from selling an amount \( A^t_{zt} \) of varieties of intermediate goods. At every stage of the innovation process, the innovator successfully adopting a new technology exploits the competitive advantage and monopolize the market as in Aghion and Howitt (1996). The innovator must pay cost of adoption \( A^u_{zt} \) and R&D expenditures \( x^A_{zt} \) in units of investment goods at market price \( P^I_t / P_t \).

Each period maximizes the discounted sum of profits Equation 22 using control variables \( x^A_{zt} \), \( s^A_{zt} \), \( A^t_{zt} \), \( A^u_{zt} \) and \( p^A_{zt} \) under technology law of motions Equation 20 and Equation 21. Anticipating symmetry, and letting \( v^U_t \) and \( v^A_t \) denotes the real shadow values of unadopted and adopted technologies respectively, the value of adopted technologies is the present discounted value of profits from producing the good:

\[
v^A_t = \Pi^e_t / P_t + (1 - \delta_A) E_t \{ m_{t,t+1} v^A_{t+1} \} . \]

(23)

While the value of unadopted technologies are determined by:

\[
v^U_t = - \frac{P^I_t}{P_t} s^A_t + (1 - \delta_A) E_t \{ m_{t,t+1} \left[ v^U_{t+1} (1 - p^A_t) + v^A_{t+1} \left( 1 - S \left( p^A_{t+1} / p^I_t \right) \right) \right] \} \]

(24)

Firm invest \( x^A_t \) units of final goods in R&D until the expected marginal product of discovering a new patent reaches the marginal cost of production:

\[
\frac{P^I_t}{P_t} = (1 - \delta_A) \Phi^t_A \left( x^A_t \right) E_t \{ m_{t,t+1} v^U_{t+1} \} . \]

(25)

The marginal cost of rising the adoption rate, denoted \( q^A_t \), reads as follows:

\[
\frac{P^I_t}{P_t} A^u_t = q^A_t \Phi^t_A \left( x^A_t \right) . \]

(26)

Finally, optimal adoption rate is given by:

\[
\frac{q^A_t}{A^u_t} + \Psi_t = (1 - \delta_A) E_t \{ m_{t,t+1} \left[ v^A_{t+1} - v^U_{t+1} \right] \} \]

(27)
The left hand side of this equation reflects the current marginal cost of adopting a technology, while the right hand side is the discounted benefits in the next period. Innovators increase their adoption expenditures until the marginal cost of adopting is equal to the expected marginal gain. As Comin and Gertler (2006), this marginal gain is $v_{i+1}^A - v_{i+1}^U$ is pro-cyclical, given the greater influence of near term profits on the value of adopted technologies relative to unadopted ones.

### 2.5 Authorities

Concerning federal monetary policy, the general expression of the central bank’s rate follows a standard Taylor rule:

$$r_t = \rho r_{t-1} + \left( \frac{\pi_t - \bar{\pi}}{\bar{\pi}} \phi_\pi \right) \left( \frac{Y_t}{\Gamma_t} \phi_Y \right) \left( \frac{Y_t/Y_{t-1}}{g} \right)^{(1-\rho)} \phi_G \varepsilon_t^R,$$

where $\varepsilon_t^R$ is a monetary policy shock common to the monetary union members, $\phi_\pi \geq 1$ is the inflation stance, $\phi_Y$ is the policy stance on deviations from the trend $\Gamma_t$ while $\phi_G$ is another stance on deviations of production growth from its steady state $g$. Recall that here, changes in the medium term component, denoted $g_t$, is affecting the nominal rate as long as $\phi_G \neq 0$.

However, a ZLB constraint on the nominal rate generates a wedge between the desirable interest rate for the economy and the effective one. The effective rate, denoted $\underline{r}_t$, determining the rate of return of government bonds reads as:

$$\underline{r}_t = \max \left( r_t, 1 \right)$$

Regarding the government, it consumes $G_t$ units of final goods. The government supports these expenditures by issuing one-period debt securities, $b_t$, and charging a lump-sum tax to household, $T_t$. The government budget balance reads as: $G_t + b_{t-1}r_{t-1}/\pi_t = b_t + T_t$. I assume that along the balance growth path, the share of government purchases in output, denoted $s_g$, is constant over time. To this end, I impose $G_t = \Gamma_t s_g \bar{Y} \varepsilon_t^G$, where $\Gamma_t$ is the trend component of output, $s_g \bar{Y}$ is the steady state of public spending and $\varepsilon_t^G$ is an exogenous AR(1) capturing exogenous changes in aggregate demand. The presence of $\Gamma_t$ ensures that government spending and output are cointegrated.

---

6The term $\Psi_t$ denotes the adjustment cost that must be paid by the innovator that makes the adoption rate sluggish: $\Psi_t = v_t^A (1 - \delta_A) p_t^A \frac{\partial s(p_t^2/p_{t-1}^2)}{\partial p_t^A} \frac{\partial s(p_{t+1}^2/p_{t+1}^2)}{\partial p_t^A} + (1 - \delta_A) v_{t+1}^A \frac{\partial m_{t+1}^A}{\partial p_t^A}$. 

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2.6 Market clearing conditions

The aggregate constraint on final goods market is given by:

\[ \frac{Y_t}{\Delta_P^t} = C_t + I_t \left(1 + S_I(\cdot)\right) + \frac{P_I^t}{P_t^t} \left(I_t^H + I_t^A\right) + G_t \]  \hspace{1cm} (30a)\n
where \( \Delta_P^t \) is the price dispersion term induced by the Calvo pricing scheme.

Aggregate expenditures in R&D and educations are given by:

\[ I_t^H = Z_t^H (1 + S_H(\cdot)) + H_t^u X_t^H \quad \text{and} \quad I_t^A = \eta \left(Z_t^A + A_t^u X_t^A\right) \]  \hspace{1cm} (31)

Recall that aggregating the price index: \( 1 = A_t \left(P_{x_t}^x/P_{x_t}^x\right)^{(1-\theta)} \), the equilibrium on the intermediate market is given by the demand function:

\[ Y_t = A_t^{\theta-1} \epsilon_t^A \left(H_t^\mu L_t\right)^\alpha (u_t K_{t-1})^{1-\alpha} H_t^\gamma \]

where \( K_{t-1} = \int_0^{A_t} K_{it-1} di \) and \( L_t = \int_0^{A_t} L_{it} di \). Here, \( Y_t \) is interpreted as an average firm given by \( Y_t = X_t/A_t \).

2.7 Balanced growth path

Empirically, the growth of education and R&D expenditures have both been secularly increasing twice faster than output. To capture this upward trend in the expenditure side of the GDP without structurally modifying key supply side ratios in output, we introduce a common investment-specific trend, denoted, \( \Upsilon_t \), which grows at a fixed gross rate \( \bar{\gamma}_x = \Upsilon_t/\Upsilon_{t-1} \). These investment goods \( I_t^H \) and \( I_t^A \) are produced from final goods by means of a linear technology whereby \( 1/\Upsilon_t \) units of final goods yield one unit of investment goods. The slope of this investment-specific trend crucially appears in the measurement equation of the model and is estimated in the fit exercise.

This economy features three sources of permanent growth: two are endogenous (\( A_t \) and \( H_t \)) and one is exogenous \( \Upsilon_t \). As a result, a number of variables, such as output, will not be stationary. We therefore perform a change of variable in order to obtain a set of equilibrium conditions that involves only stationary variables. Along the balanced growth path, per capita output \( \{Y_t\} \), per capita expenditure categories \( \{Z_t^H/\Upsilon_t, Z_t^A/\Upsilon_t, A_t^H X_t^A/\Upsilon_t, I_t, C_t\} \) and per capita capital stocks \( \{K_{t-1}\} \), per capita income categories \( \{W_t\} \) and government expenditures and lump sum transfers \( \{} \) grow at the same rate. This growth rate is equal to:

\[ \gamma_t = \Gamma_t/\Gamma_{t-1} \quad \text{with} \quad \Gamma_t = \left[A_t^{\beta-1} H_t^{\mu_\alpha+\tau}\right]^{1/\alpha}. \]
The growth rate given in Equation 31 depends on technology parameters $\mu$, $\alpha$ and $\gamma$; competition parameter in intermediate markets $\vartheta$; and stocks of adopted technologies $A_t$ and skills $H_t$. These stocks grow both at rates $g_{H,t} = H_t / H_{t-1}$ and $g_{A,t} = A_t / A_{t-1}$.

### 3 Estimation

#### 3.1 Solution method

To take into account the zero lower bound constraint on the nominal rate, I employ the solution method developed by Guerrieri and Iacoviello (2015). It applies a first order perturbation approach in a piecewise fashion in order to handle occasionally binding constraints. In this model, the presence of the ZLB is treated as a second regime that occasionally binds when the state variable in Equation 28 is below zero, otherwise the constraint is slack. The piecewise linear solution method maps these two different regimes in the same model by using first order approximation of each regime around the same steady state. The solution of the model is non-linear as decision rules parameters depend on the value of the nominal rate. Unlike global methods, this piecewise solution is fast enough to allow the estimation with full information methods of models with many state variables.

Because the solution is state-dependent, the Kalman filter cannot be employed to compute the smoothed sequence of shocks. We follow the estimation method of Guerrieri and Iacoviello (2017) by replacing the Kalman filter by an inversion filter in order to construct the log-likelihood function. As shown by Kollmann (2017), this filter extracts the sequence of innovations recursively by inverting the observation equation. The drawback of this approach lies in the number of shocks that has to be exactly the same as the number of innovations to allow the recursive inversion of the observation equation. Given this limitation, the model is estimated on 8 observable macroeconomic time series and are jointly replicated by the model through the joint realization of 8 corresponding innovations.

#### 3.2 Data

The model is estimated with Bayesian methods on US quarterly data over the sample time period 1950Q1 to 2018Q4 and are all taken from FRED. Our sample is rather large because to capture growth patterns.

Concerning the transformation of series, the point is to map non-stationary data to a stationary model (namely, the GDP, consumption, investment, R&D and educations expenditures). Following Smets and Wouters (2007), data which exhibit a trend or
unit root are made stationary in two steps. First, I divide the sample by the working age population. Second, data are taken in logs and we use a first difference filtering to obtain growth rates. Real variables are deflated by GDP deflator price index. Following ?, who underline the limited coverage of the nonfarm business sector compared to GDP, we multiply the index of average hours for the nonfarm business sector (all persons) by civilian employment. The inflation rate is computed from the log variations of the GDP deflator, while the nominal rate is measured by the effective fund rate. The latter is divided by 4 to be in a quarterly basis. Interest rate data prior 1955 are taken from Olson and Enders (2012). The effective FF rate is not the central bank target, but an average interest rate charged by depository institutions on money market. The use of this series with no prior transformation would make the ZLB not to bind in the model as the FF rate was slightly above zero during the ZLB period. To portray exactly the ZLB, we set the nominal rate data to zero when the lower limit of the federal funds target established by the Federal Open Market Committee reached zero.

To measure the empirical contribution of endogenous growth, we use a cost-based approach by including two new time series with respect to the benchmark model of Smets and Wouters (2007). First, R&D expenditures are observable which allows to characterize the unobserved growth if technology. Second, I measure investment in education through personal consumption expenditures in education services. However this series is in an annual basis, so I apply the temporal disaggregation method of Fernandez (1981). This method makes the use of the information obtained from related indicators observed at the desired higher frequency. We use health expenditure as the latter is the most correlated time series with education expenditures among all sub-elements constituting personal consumption expenditures. Finally, these two new time series are transformed using the same scheme as output.

Measurement equations are given by:

\[
\begin{bmatrix}
100 \times \log \bar{\gamma} \\
0 \\
100 \times \log \bar{\gamma} \\
100 \times \log \bar{\gamma} \\
100 \times \log \bar{r} \\
100 \times \log (\bar{\gamma} \cdot \bar{\gamma}_X) \\
100 \times \log (\bar{\gamma} \cdot \bar{\gamma}_X)
\end{bmatrix}
\begin{bmatrix}
\hat{\gamma}_t \\
0 \\
\hat{\gamma}_t \\
\hat{\gamma}_t \\
0 \\
\hat{\gamma}_t \\
\hat{\gamma}_t
\end{bmatrix}
+ \begin{bmatrix}
\Delta \hat{y}_t \\
\Delta \hat{\ell}_t \\
\Delta \hat{c}_t \\
\Delta \hat{i}_t \\
\Delta \hat{\ell}^A_t \\
\Delta \hat{i}_t^H
\end{bmatrix},
\]

(32)

where the hat over the variables’ names denotes the percentage deviations of these variables from their steady state. Measurement equations are mapped to the vector of
transformed series $Y^\text{obs}_t$ directly through $Y_t = Y^\text{obs}_t$. A striking feature of this model with respect to other estimated macroeconomic models is the existence of a common endogenous trend. We note that $\Delta \hat{y}_t, \Delta \hat{c}_t, \Delta \hat{t}_t, \Delta \hat{A}_t$ and $\Delta \hat{H}_t$ are cointegrated with $\hat{\gamma}_t$, thus the endogenous determination of $\hat{\gamma}_t$ is key as it affects jointly most of observed variables.

### 3.3 Calibration and Prior Distributions

Calibrated parameters are reported in Table 1. As Christiano et al. (2014), the discount factor is set as to 0.9987, the depreciation rate of physical capital is 2.5% and the government spending to GDP ratio is 20%. As in most real business cycles models, steady state working hours are given a value of 1/3. Given the high value of the discount factor, we impose $\alpha = 0.2$ for the capital intensity parameter in the production function to obtain an investment to GDP ratio close to 20%. Substitution on final goods market is set to 10 as in Smets and Wouters (2007) thus implying a 11% percent steady state markup. For intermediate goods, the elasticity of substitution is set to 3.85 as Anzoategui et al. (2016) to be in line with the estimate of Broda and Weinstein (2006). Steady state adoption rate for technology is set to 0.2/4 as Anzoategui et al. (2016) to get an average time lag to adopt of five years. The calibration of human capital adoption rate is more problematic as human capital is an unobservable variable. We impose an adoption rate of 0.33/4 in order to mimic the graduation of a bachelor degree in 3 years. Regarding the elasticity of patents creation to R&D expenditures, we follow the calibration strategy of Comin and Gertler (2006) by borrowing the lower bound interval value estimated by Griliches (1990). Finally, R&D expenditures in GDP are set to 3% to match postwar US data.

Table 2 and 3 report prior distributions of shock and structural parameters, respectively. Common parameters with Smets and Wouters (2007) are given prior distributions similar or close to this benchmark paper. Regarding the adoption elasticity to final goods inputs, papers featuring an endogenous technology such as Comin and Gertler (2006) typically calibrate this parameter to 0.95. To get an estimated parameter in the same range, we impose a beta distribution with prior mean of 0.8 and standard deviation of 0.05. We impose the same prior distribution for human capital. Lucas Jr (1988) found that the external effect of human capital $\tau$ is 0.415, we thus impose a prior distribution with mean of 0.4 and standard deviation of 0.2 in order to get an estimated parameter in the same range.\footnote{Lucas discussed that the estimated value for this parameter may not be plausible and cannot be verified. As a result, we adopt a conservative approach by setting a diffuse prior distribution to allow this parameter to be close to zero.} For congestion costs on adopting new technologies, Moran and Queralto (2017) suggest that adjustment cost of R&D
are higher than those of investment. I do not make any strong prior assumption on this cost by setting the same prior information as investment adjustment costs. This prior is not informative and will let the data be informative about their posterior values. For the percentage growth rate of human capital $\bar{\gamma}_H$, Lucas Jr (1988) calibrates this parameter to 0.014 using the estimation of Denison Edward (1962). This would correspond to a 0.35% quarterly growth rate that would abnormally drive all the contribution to the growth in the model.\footnote{This result is not surprising as Lucas' model only include one source of growth.} I thus impose on $\bar{\gamma}_H$ a gamma distribution with mean of 0.2 and standard deviation of 0.15: this prior is diffuse enough to allow the data to decide whether one engine drives all the observed growth of output. The skill premium $\omega$ determining the productivity gain from human capital inputs is assigned a gamma distribution of mean 1.5 and standard deviation of .3. This prior allows the skill premium to lie at 95% in the interval [1;2] found by Alon et al. (2018) for the US economy. The remaining set of parameters are which are not estimated nor calibrated are determined endogenously.

### 3.4 Posterior Distribution

In addition to priors distributions, Table 2 and 3 also report the posterior mean drawn from 10,000 iterations of Metropolis-Hasting algorithm, with an acceptance ratio close to 30%. To contrast the result with the fixed trend assumption of Smets and Wouters (2007), an alternative version of the model was estimated with the same prior distribution but with a fixed trend. Two data and shocks related to the two engines of growth are thus discarded from the estimation, while the ZLB is preserved. This difference in the number of observable time series between the two models does not allow us to compare likelihood ratios.

Regarding the model with an endogenous trend, standard parameters from the workhorse New Keynesian model are rather consistent with previous findings such as Christiano et al. (2005) and Smets and Wouters (2007).

Regarding parameters specific to the two endogenous growth engine, I find that shocks which are the most persistent are those related to the accumulation of technologies and knowledge, these shocks are probably the main source of persistence in the model with endogenous trends, and generate desired low frequency variations for the endogenous trend.

For parameters related to technology, the obsolescence rate of technology is 0.75% in a quarterly basis, which is consistent with the 3% annual obsolescence rate of Comin and Gertler (2006). In the same vein, the adoption rate elasticity is strikingly the close to the one of Comin and Gertler (2006). Regarding the sluggishness of
the adoption rate, the cost parameter is much higher than for investment goods as suggested by Moran and Queralto (2017).

Next we turn to the parameter related to the accumulation of knowledge. First, we find a quarterly obsolescence rate of knowledge of 0.4% that lies in the ballpark of the 1.5% annual rate of Jones et al. (1993), while the technology of skill creation is more intensive in goods compared to the same benchmark paper. In addition, the external effect of knowledge is twice lower than the one computed by Lucas Jr (1988).

Finally by comparing the models with endogenous versus exogenous growths, we find that low frequency fluctuations are not correctly accounted by the exogenous growth model, and are thus captured by more persistence in the shocks processes.

4 Macroeconomic implications of endogenous trends

4.1 Inspecting the propagation mechanism

To understand how the two engines of growth shape the propagation of the economy, Figure 1 reports the linear response of the model following a standard productivity shock in the production function.

The rise in the productivity makes both labor and physical capital more productive, leading to a decline of the inflation rate combined with a rise in the rate of growth of output. As in the workhorse New Keynesian model, monetary reacts to the decline in inflation by lowering the nominal rate. The decline in the real rate lowers the incentive to save for household, and thus rise in turn the consumption. In the meantime, the cost
of physical is lower and allows intermediate firm to investment more. The investment cost function thus generate an hump-shape response of investment, consistent with the VAR model as shown by Christiano et al. (2005).

The trend requires up to 10 quarters to rise, the origin of this persistence is the sluggishness of adoption rates of unadopted technologies and knowledge. The rise in productivity increases the marginal product of human capital, and in turn enhances the value of unadopted knowledge. Firms thus engage their employees into vocational training which rises the share of the labor force into education. Accumulating knowledge takes on average 3 years, and adopting a new skill is sticky, thus the rise in education efforts takes time to translate into effective units of human capital.

For the growth of technology engine, the propagation of a TFP shock is rather similar to the other engine of growth. When intermediate firms are more productive, they can produce more with less inputs, and thus implies a rise in intermediate profits. Through a standard Schumpeterian effect, higher profits represent more monopoly rent for innovators, thus drives upward the value of adopted technologies. Innovators have more incentive to innovate and adopt to monopolize the rent, in turn they rise their R&D spending which enhances the demand of final goods. The combination of the two growth engine thus increase the common trend at which aggregate variables are growing.

4.2 Business cycle analysis

In this subsection, we evaluate how the persistence and the correlation links between observable variables are affected by the endogenous trend. Figure 7 reports dynamic correlations between observable variables using the estimated models with endogenous and exogenous trends.

On average, the overall persistence is not strikingly affected by the presence of an endogenous trend. Correlations between variables are reported in the diagonal of Figure 7. Both models fails at replicating the persistence of investment, while the persistence of hours worked is better accounted by the endogenous trend model, while the opposite situation is observed for inflation where both models fails at replicating this moment. The same situation is observed for the nominal rate.

For the cross-correlations, there is also no important differences between these two models. The exogenous trend model does a slightly better job in capturing the correlation link for nominal rate-investment and inflation-nominal rate. In contrast, the endogenous trend model clearly outperforms cross-correlations for hours-inflation and hours-nominal rate with respect to the exogenous trend model.
Figure 2: Historical path of the endogenous trend between 1950q1 to 2018q4.

### 5 Why economic growth has declined since WWII?

Figure 2 reports the time-path of this medium term component measured by the estimated model. Recall that this component jointly rise growth rates of key macroeconomic aggregates such as consumption, investment and GDP. Over the postwar, the US economy has experienced sizable medium frequency oscillations. From 1950 up to 1970, the endogenous trend has been continually increasing upward despite small recessionary episodes. This period is characterized by a persistent increase in R&D expenditures, thus leading the trend to peak up to 3% at the end of the 60s. In decades following the 70s, the trend growth rate have been declining synchronously with the different recessions hitting the US economy. Recessions induced by oil price shocks in the 70s and the Great Recession clearly damaged the engine of growth. If at first sight the trend to be volatile, Figure 2.b shows that these fluctuations are less volatile compared to the annual fluctuations of real output. Thus the endogenous trend clearly replicates a fraction of the low frequency volatility in macroeconomic time series. Antolin-Diaz et al. (2017) employs a dynamic factor model to track changes in the long run growth rate of GDP, by separating them from their cyclical counterpart. Their sample span a period as long as the one used in the fit exercise and thus allows us to examine any similarity between their estimates of the long run growth with the endogenous trend. Both models seems to generate close estimates of the long run growth, which confirms that the endogenous growth model is able to successfully capture low frequency variations.

Why has the trend reduced over time? Unlike Anzoategui et al. (2016) who consider only one source of growth, the present framework allows to disentangle con-
tributions induced by the growth of knowledge (human capital) from those induced by the growth of ideas (technology). Figure 3 reports on the left the common endogenous trend of the economy $\gamma_t$, that is a non-linear function of $\gamma_A^t$ and $\gamma_H^t$ (see Equation 31). Over the sample period, it’s striking to notice that the growth of skills has been remarkably stable over time while the main source of variations of the endogenous trend since 1950 has been the growth of technology. From 1950, the growth rate of technology peaked up to 4.5% but started to decline prior to the two coming recessions induced by rising oil prices.

We next explore the relative importance of the two sources of growth on the common trend. Equation 31.b reports the percentage contribution of each source of growth using a linear approximation of Equation 31. This figure confirms that the R&D engine accounts for much of the cyclical variation in the endogenous trend, as it has contributed on average up to 60% of the variation of the trend. The downward pressure on the trend has clearly been driven by variations in technology since 1970. While before the financial crisis, the growth of knowledge was driving up the trend, the financial crisis worryingly reversed the contribution of human capital.

How does the model account for the decline in the growth rate of technology? ?? plots the detrended evolution of main state variables determining the aggregate evolution of technology. During the first twenty years of the sample, the growth of R&D investment has been high enough (figure b) to fuel an high rate of entry of new patents (figure e), the latter were mostly effectively adopted and thus converted into new intermediate goods (figure a). In the meantime, the monopoly was declining but did not translated into lower expected technology value (figure f) as expectations about
future monopoly rents were high. However the 70s recessions irrevocably damaged the main engine of growth and announce the beginning of a slowdown.

For the post 70s period, (Gordon, 2012, 2017) argues that technological advancement has been slowing and translates into slower growth over time. The model tends provides a theoretical formulation of Gordon’s narrative that explains this reduction in the rate of growth of the US economy. The model captures this decline in the growth rate by a reduction in the entry rate of new patents, that measures the productivity of innovators during 1950. This result is corroborated by the estimated model of Anzoategui et al. (2016) that finds a similar path for the R&D productivity. Recessions in the 70s reduced the monopoly rents (figure c), and thus reduced the value of adopted technologies (figure f). Thus, the incentive for the innovator to adopt a technology became low (figure a). This reduction in the adoption rate of technology rose the stock of unadopted technologies until 1985. After this date, the stock of unadopted technologies has been critically falling, mainly because the creation rate of new patent and the R&D expenditures were declining. Despite an improvement of the situation in the 2000s, the financial crisis broken this recovery in the engine of growth through a large contraction of the monopoly rent.

6 QUANTIFYING THE EFFECT OF THE ZERO LOWER BOUND

We now explore how important was the presence of the zero lower bound on the economic contraction of the US economy during the Great Recession. ?? compares the observed data against the outcome from the same model without the constraint on the nominal rate.
Figure 5: Macroeconomic implications of the zero lower bound during the Great Recession.

When the ZLB started to bind in 2009, monetary policy could not accommodate further the nominal rate to dampen the recession. As a consequence, real interest rates were abnormally high which, through the Euler equation, artificially increased both the marginal utility of consumption and the cost of capital renting, and in turn it weakened aggregate demand. Our results show that without the ZLB the annual growth rate of output would have been 1.5% higher in 2009. In addition, the ZLB has amplified the deflation mechanism, this translates into year-on-year inflation differential in 2009 of 1%, and 0.3% in 2014 and 2016.

Using the estimated model, we can also gauge the effect of the zero lower bound on the two engines of growth in the economy. *Equation 31* provides the annualized growth rate of the medium term component. This component is itself a combination of adopted technologies and adopted skills depicted in subfigures e and d. According to the model, the role of the ZLB on growth is trivial as the trend decline was quick and negligible: the trend reduced of 0.1 pp in 2009 before recovering quickly with no persistent effect. The main contributor to this modest drop is the accumulation process of human capital that is temporary damaged by the high rates.
A natural question at hand is to the possible causal relation between the slowdown in economic growth and the zero lower bound. As Orphanides (2003) emphasized, real-time misperceptions about the long-run growth of the economy can play a large role in monetary policy mistakes. In the standard workhorse New Keynesian, monetary policy stabilizes short run fluctuations of output and inflation without having any concerns about possible long-term changes in the growth patterns of the economy. Here, we are going to do a counterfactual exercise and see whether the monetary policy reaction to long term growth strongly has lead the nominal rate to reach the zero lower bound. Figure 6 reports a counterfactual interest rate that does not respond to the endogenous trend. When monetary policy does not respond to long term change in the growth rate, the nominal rate is higher because it does not respond the slowdown of the endogenous trend. An higher interest rate induces a reduction of inflation and actually increases the zero lower bound probability. Thus the macroeconomic situation is worse when monetary policy does not respond to long term growth as a ZLB binds for more quarters.

7 Conclusion

We have estimated a non-linear DSGE model that originally features a time varying trend driven by two sources of endogenous growth. We then used the model to assess the slowdown of long term growth, in particular following the onset of the Great Recession. Based on the estimated model, our key result is that we corroborate the thesis of a strong decline in the long term trend of the US economy. Among the two sources of growth examined in the paper, the slowdown mainly is induced by the technology engine reflecting a decline in the productivity of creation of new technologies since 1960. This finding tends to favor the Gordon (2012) theory stating that
the US growth has strongly declined since 1970. In addition, we find that a standard macro-model with exogenous growth erroneously captures low frequency changes in economic growth by highly persistent macroeconomic shocks. In contrast, the model featuring an endogenous trend successfully captures this low frequency fluctuations.
References


<table>
<thead>
<tr>
<th>Calibrated parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$ Discount factor</td>
<td>0.9989</td>
</tr>
<tr>
<td>$l$ Labor supply</td>
<td>1/3</td>
</tr>
<tr>
<td>$s_A$ Public spending share in output</td>
<td>0.20</td>
</tr>
<tr>
<td>$\bar{u}$ Capital utilization rate</td>
<td>1</td>
</tr>
<tr>
<td>$\alpha$ Labor intensity</td>
<td>0.80</td>
</tr>
<tr>
<td>$\varepsilon_A$ Patent production function</td>
<td>0.60</td>
</tr>
<tr>
<td>$\phi$ Substitution intermediate goods</td>
<td>3.85</td>
</tr>
<tr>
<td>$\delta$ Capital depreciation rate</td>
<td>0.025</td>
</tr>
<tr>
<td>$p_A (\bar{x}^A)$ Technology adoption rate</td>
<td>0.20/4</td>
</tr>
<tr>
<td>$p_H (\bar{x}^H)$ Skill adoption rate</td>
<td>0.33/4</td>
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<tr>
<td>$\bar{I}^A / \bar{Y}$ R&amp;D expenditures to GDP</td>
<td>0.03</td>
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<td>$\epsilon$ Substitution final goods</td>
<td>10</td>
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Table 1: Calibrated parameter values (quarterly basis)
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<tr>
<th></th>
<th>Prior distributions</th>
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<th>Posterior distributions mean</th>
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<tr>
<td>Std. markup</td>
<td>$100 \times \sigma_P$</td>
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<tr>
<td>Std. investment</td>
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<tr>
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<td>AR(1) productivity</td>
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<td>AR(1) human capital</td>
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Marginal log-likelihood 398.2815 186.2354

Table 2: Prior and Posterior distributions of shocks

<table>
<thead>
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<td>Shape</td>
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<td>$\xi$</td>
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<tr>
<td>Capital utilization elasticity</td>
<td>$\psi$</td>
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<td>Adoption congestion cost</td>
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<td>Adoption congestion cost</td>
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Marginal log-likelihood 398.2815 186.2354

Table 3: Prior and Posterior distributions of structural parameters.
The shaded areas and red dashed lines represent 90% intervals for the dynamic correlation computed from estimated models with and without endogenous trends, respectively.

Notes: The shaded areas and red dashed lines represent 90% intervals for the dynamic correlation computed from estimated models with and without endogenous trends, respectively.

Figure 7: Dynamic correlations between macroeconomic time series