The Great Recession and the Zero Lower Bound
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Abstract
We investigate the drivers of the Great Recession and analyse the effects of interest rate policies in the aftermath of the crisis. For that purpose we estimate and filter a medium-scale DSGE model, accounting for an endogenously binding zero lower bound (ZLB). We find that the Great Recession was caused by an increase in the risk premium, which persisted after the recession and weighed heavily on real activity. Our analysis suggests that the long duration of the zero lower bound was a reaction to the weak economic development as opposed to a commitment by the central bank to actively keep interest rates lower than a Taylor-rule implied rate. Nonetheless, the sharp cut in the Fed Fund rate prior to the ZLB period attenuated the fall in output by roughly 1 percent. At the ZLB, expansionary forward guidance shocks had an strengthening effect on output, whereas the exit from the ZLB was too early and had recessionary effects. Inflation dynamics are largely untouched by interest rate policy due to a flat Phillips Curve. Furthermore, we find that the cost of the ZLB was substantial and lowered output by two percent.

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1 Introduction
In recent years, the US economy has been gradually recovering from the Great Recession and the accompanying liquidity trap. With the recovery in sight, a controversially debated question in the last year was, when and how to return to "normal" monetary policy, i.e. to raise interest rates from the so-called effective zero-lower bound on nominal interest rates (ZLB) and to return to a state of affairs in which adjustments of the policy interest rate reflect changes in economic fundamentals.

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In this paper, we analyze the economic development in the US during the Great Recession and in its aftermath and focus on its causes and consequences for our understanding of economic dynamics and economic policy. A brief look at the development of the US economy raises three broad questions: What caused the long duration of the zero lower bound and how much impact did it have?, What were the effects of forward guidance and the interest rate policy?, and in how far has the economic environment changed in the last decade compared to the pre-crisis era?

To answer these questions, we employ a medium scale DSGE model of the US economy, which is a variation of the model by Smets and Wouters (2007)\(^1\) and estimate the series of structural shocks at the zero lower bound (ZLB) while taking account of the nonlinearities imposed by the ZLB using the novel IPAS filter described in Boehl (2019). This approach enables us to decompose the macroeconomic dynamics into the contributions of the different exogenous drivers. Given the series of exogenous shocks we create counterfactuals by switching off certain shocks or shock-particles at particular moments. Secondly, we analyse the role of monetary policy in this period. In particular, we identify forward guidance shocks at the zero lower bound and discuss their effects. Third, we investigate on the cost of a binding ZLB for the US economy.

Our results suggest that the Great Recession was initially caused by an exogenous increase of the risk premium, which persistently stayed high during the rest of our sample. The prolonged period of positive risk premiums stymied economic demand and forced the nominal interest rate endogenously to the ZLB. The long duration of the ZLB was therefore not driven by dovish behaviour of the FED, but consistent with its interest rate rule. Despite the nominal interest rate being constant at the ZLB, the FED influenced the real economy by managing agents’ expectations of the duration of the ZLB. Absent this active forward guidance policy, which kept expectations of future interest rates low and thereby stimulated aggregate demand, the economy would have suffered from a longer and more severe recession.

We further argue that the occasionally binding constraint imposed by ZLB imposes nonlinearities that are not only non-negligible for direct policy advice but also for the estimation. Ignoring it during the estimation procedure bears the risk of severe misspecification. We find that the Phillips Curve got quite flat in the recent years, implying that inflation was detached from economic fundamentals. This finding can also be interpreted as such that our economic models miss the crucial connection between inflation and fundamentals. Lastly, we are able to put a number on the costs of a binding ZLB and find that this is up to 3% of GDP.

Hence, we give an account of the dynamics of real activity, inflation and the interest rate from the onset of the Great Recession until the rates returned to normal levels at the end of 2017 with a focus on the role of forward guidance. We are not the first to study the events of the Great Recession and the role of the zero lower bound. Gust et al. (2017) globally solve a smaller version of the Smets and Wouters model and estimate it using a particle filter. Our approach differs from theirs methodologically to the extent that we use a linearized model, solve it at the ZLB and employ the IPAS-filter. This procedure allowing us to analyse the dynamics in a full medium-scale model with a high-

\(^{1}\)Our main departure from the model developed by Smets and Wouters (2007) is the use of a utility function, which is separable in consumption and leisure. We find that this helps us in avoiding regions, in which the duration of the ZLB is indeterminate.
dimensional state space. While our results are similar to theirs in several respects, our estimated cost of the ZLB are larger than what Gust et al. (2017) find. Linde et al. (2017) estimate a linearized variation of the model by Smets and Wouters (2007) at the ZLB. While our model differs in some aspects (different preferences, different number of shocks), and they employ regime-switching methods, our estimated parameters are in line with theirs. In contrast to their paper, we lay a strong focus on the decomposition of the dynamics and a discussion of the role of monetary policy. Another approach is taken by Fratto and Uhlig (2014), who commit to using only methods that have been developed prior to the crisis, and conduct an analysis of the zero lower bound episode in a linear setting. Our results suggest that this approach delivers distorted estimates for some parameters and fails to capture relevant economic effects.

Next to the empirical investigations into the effects of the ZLB, there is an abundant body of literature outlining theoretical mechanisms that aim at capturing the interactions between the financial sector and the macroeconomy, which led to the Great Recession (see, e.g., Meh and Moran, 2010; Andrea et al., 2010; Cúrdia and Woodford, 2011; Gertler and Karadi, 2011a; Brunnermeier and Sannikov, 2014; Christiano et al., 2014) and several papers which investigate on the dynamics of the US economy in that period as for example Christiano et al. (2015), Del Negro et al. (2015b), and Del Negro et al. (2017). Our findings, that attribute the Great Recession to a hike in the risk premium confirms the gist of the bulk of this literature.

The beneficial effects of forward guidance in the presence of the ZLB were first pointed out by Eggertson and Woodford (2003) in the context of a theoretical model. Del Negro et al. (2015a) find empirically meaningful effects of forward guidance on macroeconomic dynamics, but raise the issue that the effects of forward guidance are grossly overstated in theoretical models (the so-called "Forward Guidance Puzzle"). The flat Phillips Curve that we find, implies that prices are relatively inflexible. As inflation hardly moves in response to forward guidance announcements, the effect on the real rate is small as well. As a consequence, the forward guidance puzzle is mitigated in our model.\(^2\)

The rest of this paper is structured as follows. In Section 2 we set up the medium-scale DSGE model that we use for our analysis. In Section 3 we briefly explain the data, the methodology used for solving, filtering and estimating and then present the results of our estimation. In Section 4 we lay out our quantitative analysis and decomposition of the crisis whereas Section 5 discusses some more elements of the estimation. Section 6 concludes.

2 Model

This section gives a brief description of the model. For our analysis, we employ a standard medium scale model as in Smets and Wouters (2007). Labor is differentiated by a union with monopoly power that faces nominal rigidities. In the firm sector, intermediate good producers employ labor and capital services and sell their goods to final goods firms. Final good firms are monopolistically competitive and face nominal rigidities as in Calvo (1983). Furthermore, the model features a stylized government sector and a monetary authority that sets the short-term nominal interest rate according to a Taylor Rule.

\(^2\)This is in line with Linde et al. (2017), who also find a flat Phillips curve.
Households

The model features continuum of households, which derive utility from consumption and leisure. Their utility function reads

\[
U_t = E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{(C_t(k) - C_{t-1})^{-\sigma_c} - 1}{1 - \sigma} - \nu \log(1 - L_t(k)) \right)
\]

where parameters \( \beta, h, \) and \( \sigma_c \) are, respectively, the discount factor the degree of external habit formation in consumption, and the coefficient of relative risk aversion. \( C_t(k) \) is consumption and \( L_t(k) \) are the hours worked. Habit formation is with respect to past aggregate consumption, \( C_{t-1} \). This formulation of the utility function – separability in consumption and leisure – stands in contrast to the non-separable formulation in Smets and Wouters (2007). In several trial runs, we found that the non-separable formulation has unfavorable effects on the size of the determinacy region at the ZLB. While the same model with separable preferences can, given an appropriately strong recessionary shock, easily stay at the ZLB for 25+ periods (which are of course fully anticipated on impact), in a model with non-separable preferences larger shocks c.p. are more likely to cause non-existence of an rational expectation equilibrium.

The optimizing household \( k \) earns nominal wages \( W_t \) from its labor supply \( L_t(k) \) and receives income from one-period zero coupon bonds \( B_t(k) \) that yield an interest rate \( R_t \). The return on bonds is subject to a disturbance term, \( d_{t,t} \) which follows an AR(1) process. This term drives a wedge between the interest rate controlled by the central bank and the required return on assets. Following Smets and Wouters (2007), we interpret these shocks as risk-premium shocks. Furthermore, households receive a return, \( R_k(t) \), on physical capital, \( K_t(k) \), that they accumulate and rent out to intermediate good producers. \( U_t(k) \) denotes the share of available capital that households rent out to firms and can be interpreted as a utilization rate. Furthermore, there is a cost to changing the degree of capital utilization, \( P_t a(U_t(k))K_{t-1}(k) \). Households spend their funds on consumption \( C_t(k) \), investment in physical capital, \( I_t(k) \), and the purchase of new bonds.

The budget constraint of consumers reads

\[
P_t C_t(k) + P_t I_t(k) + \frac{B_t(k)}{v_{d,t} R_t} = B_{t-1}(k) + W_t L_t(k) + R_t^K U_t(k) K_{t-1}(k) - P_t a(U_t(k)) K_{t-1}(k) - T_t + \Phi_t.
\]

Here, \( P_t \) denotes the price level at time and \( T_t \) are lump sum taxes raised by the government to finance government spending, and \( \Phi_t \) are profits of monopolistic firms.

The capital accumulation equation reads

\[
K_t = (1 - \delta) K_{t-1} + v_{i,t} \left( 1 - S \left( \frac{I_t}{I_{t-1}} \right) \right) I_t,
\]

where \( \delta \) is the depreciation rate and the function \( S(\cdot) \), indicates a cost of adjusting the level of investment. In steady state it holds that \( S = 0, S' = 0, \) and \( S'' > 0. \) and \( v_{i,t} \) follows an AR(1) process. While Smets and Wouters (2007) interpret \( e_{i,t} \) as an investment-specific technology disturbance, Justiniano et al. (2011) stress that this shock can as well be viewed as a reduced-form way of capturing financial frictions, as it drives a
wedge between aggregate savings and aggregate investment.

**Firms**

Intermediate good producers are in monopolistic competition, employ labor and capital services from households, and set their prices as markups over the marginal cost. Firm $i$ produces according to the Cobb-Douglas Function

$$Y_t(i) = e^{z_t}(K_t(i))^\alpha (\gamma_t L_t(i))^{(1-\alpha)} - \gamma^t \Phi.$$ (3)

Here, $Y_t(i)$ are intermediate goods, $z_t$ is a TFP-shock, $K_t(i)$ is the effective capital used in production ($K_t(i) = U_t(i)K_{t-1}(i)$), and parameter $\alpha$ is the output elasticity with respect to effective capital. $\gamma^t$ represents the labor-augmenting growth rate in the economy, and $\Phi$ is the fixed cost of production. The period profit function of firm $i$ is

$$P_t(i) = Y_t(i) - W_t L_t(i) - R_t K_t(i).$$

$P_t(i)$ denotes the price of intermediate good by firm $i$. Cost minimization of the firm implies that marginal cost are

$$MC_t = \alpha e^{z_t}(1-\alpha)W_t (1-\alpha)\phi e^{-z_t}.$$ (4)

Each period, firms face a constant probability of being able to optimally adjust their prices, $\zeta_p$. Those firms, which cannot optimally adjust their prices in a given period, index their prices to a weighted average of last periods inflation and steady state inflation. These assumptions give rise to the following maximization problem for firm $i$

$$\max_{P_t(i)} \sum_{s=0}^{\infty} \left( \beta \zeta_p \right)^s \frac{\Lambda_{t+s}}{\Pi_{t+s}} \left[ P_t(i) \Pi_{t+s} (\Pi_{t+s-1}^{\epsilon_t}) - MC_{t+s} \right] Y_{t+s}(i)$$ (4)

s.t. $Y_{t+s}(i) = \frac{G'}{G} \left( \frac{P_t(i) \Pi_{t+s} (\Pi_{t+s-1}^{\epsilon_t})}{P_{t+s}} \right) \tau_{t+s}.$ (5)

$P_t(i)$ is the price set by firm $i$, $\Pi_{t+s}$ is the accumulated change in the aggregate price level between periods $t$ and $t+s$, $\Lambda_{t+s}$ is the stochastic discount factor of the firm, $Y_{t+s}$ is the demand by final good firms for intermediate goods. Parameter $\epsilon_t$ is the degree of price indexation. Function $G$ governs how the relative price of firm $i$ affects the amounts of goods it can sell. We make the same assumptions on $G$ as Smets and Wouters (2007).³

Furthermore, $\tau_{t+s} \equiv \int_0^1 G' \left( \frac{Y_{t+s}(i)}{Y_{t+s}} \right) \frac{Y_{t+s}(i)}{Y_{t+s}} di$. The aggregate price index is in this case

³That is that $G' > 0$, $G'' < 0$, and $G(1) = 1$. As shown by Kimball (1995), the assumptions on $G$ imply that the demand for a good is decreasing in its relative price, and that the elasticity of demand for a good increases with its relative price, which in turn implies a higher persistence of aggregate inflation dynamics.
given by

\[ P_t = [(1 - \zeta_p)(P^*_t)G^{-1} - \zeta_p\Pi_{t-1}^\prime(1-\nu)P_{t-1}G^{-1} \left( \frac{\Pi_{t-1}^\prime(1-\nu)P_{t-1}}{P_t} \right)] \]

where \( P^*_t \) is the optimal price in period \( t \).

Final good producers act under perfect competition. They buy the intermediate goods and bundle them in final goods. Their maximization problem reads

\[ \max_{Y_i, \Pi Y_t} P_t Y_t - P_t(i)Y_t(i) \]

\[ s.t. \int_0^1 G(\frac{Y_t(i)}{Y_t}, \lambda_{p,t}) \, di = 1. \]

The price markup, \( \lambda_{p,t} \), is time-varying and subject to markup shocks, \( e_{p,t} \), which follow an AR(1)-process.

**Unions and Labor Packers**

The supply of labor to intermediate good firms is organized by unions and labor packers. Households supply labor to a labor union, which differentiates the labor services and sets wages. Unions act in monopolistic competition with each other and set their wages, \( W_t(i) \), as a markup over the average marginal rate of substitution between consumption and leisure of households. We assume that the wage setting process, in the same way as price setting, is subject to a Calvo type friction, and that unions, which cannot adjust their wages in a given period index their last wage to a weighted average of last periods inflation and steady state inflation. Labor packers buy the labor services from unions, bundle them and provide them to intermediate good firms at the wage \( W_t \). Thus, the maximization problem of labor packers is

\[ \max_{L_t, L_t(i)} W_t L_t - W_t(i)L_t(i) \]

\[ s.t. \int_0^1 G_w(\frac{L_t(i)}{L_t}, \lambda_{w,t}) \, di = 1, \]

where \( L_t \) is labor provided by labor packers to intermediate good firms, \( L_t(i) \) is the labor services sold by unions to labor packers, and \( G_w \) is the labor aggregator on which we make the same assumptions as on \( G \). The wage markup, \( \lambda_{w,t} \), is time-varying and subject to wage markup shocks, \( e_{w,t} \), which follows an AR(1)-process.

Labor unions observe the average marginal rate of substitution between consumption and leisure of all households,

\[ MRS_t = \frac{\nu(C_t - hC_{t-1})^\sigma}{(1 - L_t)} \]
and charge a markup on top of it. Their maximization problem is

\[
\max_{W_t(i)} \sum_{s=0}^{\infty} (\beta \zeta_w) s L_{t+s}(i) \left[ W_t(i) \Pi_{t+1}^w \Pi_{t+1}^{1-\eta_w} - M R S_{t+s} \right] L_{t+s}(i) \quad (12)
\]

\[
s.t. \frac{L_{t+s}(i)}{L_{t+s}} = G'_{t+1} \left( \frac{W_t(i) \Pi_{t+1}^w \Pi_{t+1}^{1-\eta_w}}{W_t(i)} \right). \quad (13)
\]

\(W_t(i)\) is the wage set by union \(i\), and parameter \(\eta_w\) is the degree of wage indexation, and \(\tau_{w,t+1} \equiv \int_0^1 G'_{w} \left( \frac{L_{t+1}(i)}{L_{t+1}} \right) \frac{L_{t+1}(i)}{L_{t+1}} di\). The aggregate wage index is in this case given by

\[
W_t = [(1 - \zeta_w)(W^*_t G'_{w})^{-1} \left( \frac{W^*_w}{W_t} \right) + \zeta_w \Pi_{t-1}^w \Pi(1-\eta_w) W_{t-1} G'_{w} \left( \frac{\Pi_{t-1}^w \Pi(1-\eta_w) W_{t-1} \tau_w}{W_t} \right)] (14)
\]

where \(W^*_t\) is the optimal wage set by labor unions in period \(t\).

**Policy and market clearing**

The model features government spending \(G_t\), which is financed via lump sum taxes. Government spending is exogenous and described by an AR(1) process. The aggregate resource constraint in real terms reads

\[
Y_t = C_t + I_t + G_t + a(U_t)K_{t-1},
\]

where the last term on the right hand side of the equation marks the resource costs of adjusting the utilization of installed capital. Lastly, monetary policy is modelled as a Taylor-Rule on short-term nominal interest rates, and takes account of the occasionally binding nature of the zero lower bound

\[
R_t = \max \left\{ 1, R_{t-1}^\rho \left( \frac{\Pi_t}{\Pi_{t-1}} \right)^{\phi_\pi} \left( \frac{Y_t}{Y_{t-1}} \right)^{\phi_y} \right\}^{(1-\rho)} e^{\rho r_{t,t}}.
\]

\(\rho\) is the interest smoothing parameter, and \(\phi_\pi\), \(\phi_y\), and \(\phi_{dy}\) are the coefficients, which govern the strength of the response of interest rate policy to deviations of inflation, output and output growth from their target level. \(r_{t,t}\) represents a shock to the Taylor rule and follows an AR(1)-process.

### 2.1 Linearized Equilibrium conditions

This subsection briefly presents the linearized equilibrium conditions. A detailed derivation of the linearized equations is discussed in the appendix to Smets and Wouters (2007). Small letters denote the log-deviation of the corresponding variable from its steady state value.

\[
c_t = \frac{h/\gamma}{(1 + h/\gamma)} c_{t-1} + \frac{1}{1 + h/\gamma} E_t[c_{t+1}] - \frac{(1 - h/\gamma)}{(1 + h/\gamma) \sigma_c} (r_t - E_t[\pi_{t+1}] + \psi_{d,t}) \quad (15)
\]
Equation (15) is the aggregated Euler equation for consumption. The presence of habit formation justifies the presence of lagged consumption in the equation.

\[
\bar{\iota}_t = \frac{1}{1 + \beta} \pi_t \bar{\iota}_{t-1} + \frac{\beta}{1 + \beta} \bar{E}_t \pi_{t+1} + \frac{1}{(1 + \beta) \gamma^2 S^\eta \eta_t} \quad (16)
\]

where \( \beta = \beta \gamma (1 - \sigma_c) \). Equation (16) is the linearized first order condition for investment. The dynamics of investment are governed by Tobin’s q. \( S^\eta \) is the steady state value of the second derivative of the investment adjustment cost function.

\[\bar{k}_t = \frac{(1 - \delta)}{\gamma} \bar{k}_{t-1} + (1 - (1 - \delta)/\gamma) \bar{\iota}_t + (1 - (1 - \delta)/\gamma)(1 + \beta) \gamma^2 S^\eta \nu_{t,t} \quad (17)\]

Equation (17) is the accumulation equation of physical capital, while (18) is the arbitrage condition between capital and bond holdings for the household.

\[R_t - E_t[\pi_{t+1}] + \nu_{d,t} = \frac{R_k}{R_k + (1 - \delta)} E_t[\iota_{t+1}] + \frac{(1 - \delta)}{R_k + (1 - \delta)} E_t[\eta_{t+1}] - \eta_t. \quad (18)\]

\[k_t = \frac{1 - \psi}{\psi} r_t^k + \bar{k}_{t-1} \quad (19)\]

\[k_t = w_t - r_t^k + l_t \quad (20)\]

\[y_t = \Phi(\alpha k_t + (1 - \alpha) l_t + z_t) \quad (21)\]

\[y_t = \frac{G}{\bar{Y}} g_t + \frac{C}{\bar{Y}} c_t + \frac{I}{\bar{Y}} i_t + \frac{R_k K}{\bar{Y} \psi} 1 - \psi r_t^k \quad (22)\]

The relation between physical capital and effective capital is given by (19). Here, parameter \( \psi \) is the elasticity of the capital utilization adjustment cost function and normalized to be between zero and one. Condition (20) is derived from cost minimization of intermediate good producer, (21) is the aggregate production function, and (22) is the aggregate resource constraint.

\[\pi_t = \frac{\beta}{1 + \beta} E_t \pi_{t+1} + \frac{1_p \beta}{1 + 1_p \beta} \pi_{t-1} + \frac{(1 - \zeta_p \beta)(1 - \zeta_p)}{(1 + \beta_p \zeta_p)((\Phi - 1) \epsilon_p + 1)} (w_t - z_t + \alpha k_t - \alpha k_t) \quad (23)\]

Equation (23) is the New Keynesian Phillips curve. The last term in parenthesis corresponds to the marginal cost of production. As we employ the Kimball aggregator, the sensitivity of inflation to fluctuations in marginal cost is affected by the market power of firms, represented by the steady state price markup, \( (\Phi - 1) \). Furthermore, the curvature of the Kimball aggregator, \( \epsilon_p \), affects the adjustment of prices to marginal cost, since the higher \( \epsilon_p \), the higher is the degree of strategic complementarity in price setting, and

\[Note, \text{ that in equilibrium, the fixed cost parameter is related to the steady state price markup by a zero profit condition.}\]
dampens the price adjustment to shocks.

\[
    w_t = \frac{1}{1 + \beta \gamma} (w_{t-1} + \pi_{t-1} \pi_{t-1}) + \frac{\beta \gamma}{1 + \beta \gamma} \left[ E_t[w_{t+1} + \pi_{t+1}] - \frac{1 + \lambda_w \beta \gamma}{1 + \beta \gamma} \pi_t \right] 
    + \frac{(1 - \zeta_w \beta \gamma)(1 - \zeta_w)}{(1 + \beta \gamma) \zeta_w (\lambda_w - 1) \epsilon_w + 1} (w_t^h - w_t) 
\]

(24)

Equation (24) is the Wage Phillips curve. \(w_t^h\) is the wage that would prevail in the absence of market power by unions. Therefore, \((w_t - w_t^h)\) is the wage markup. Analogous to equation (23), the terms \(\lambda_w\) and \(\epsilon_w\) represent the steady state wage markup and the curvature of the Kimball aggregator for labor services. The efficient wage equals the average marginal rate of substitution between consumption and leisure of all households.

\[
    w_t^h = \frac{\sigma_c}{(1 - h)} (c_t - h c_{t-1}) + \frac{L}{1 - L} l_t 
\]

(25)

Equation (26) is the linearized Taylor rule in terms of the net interest rate set by the central bank.

\[
    r_t = \max \{ 0, \rho r_{t-1} + (1 - \rho)(\phi \pi_t + \phi_g \tilde{y}_t + \phi_d \tilde{y} - \tilde{y}_{t-1}) + v_r \} 
\]

(26)

where \(\tilde{y}_t\) is the output gap. When the economy is away from the zero lower bound, the stochastic process \(\epsilon_r\) represents a regular interest rate shock. When the nominal interest rate is at zero, \(v_r\) serves as a forward guidance shock as is explained in section 4. Finally, the stochastic drivers of our model are the following seven processes:

\[
    v_{d,t} = \rho_d v_{d,t-1} + \epsilon_{d,t}^d, \\
    z_t = \rho_z z_{t-1} + \epsilon_{z,t}^z, \\
    g_t = \rho_g g_{t-1} + \epsilon_{g,t}^g, \\
    v_{r,t} = \rho_r v_{r,t-1} + \epsilon_{r,t}^r, \\
    v_{i,t} = \rho_i v_{i,t-1} + \epsilon_{i,t}^i, \\
    v_{p,t} = \rho_p v_{p,t-1} + \epsilon_{p,t}^p, \\
    v_{w,t} = \rho_w v_{w,t-1} + \epsilon_{w,t}^w, \\
\]

(27) - (33)

where \(\epsilon_{k,t}^k \sim \mathcal{N}(0, \sigma_k^2)\) for all \(k = \{d, z, g, r, i, p, w\}\).

3 Data and Estimation

We make use of the methodology presented in Boehl (2019) which is implemented in the pydsge package written in Python.\(^5\) In a nutshell, the paper cited above shows how

\(^5\)Python can provide speed benchmarks that are en-par with compiled languages such as Fortran while comprising the advantages of a high-level programming language. We want to promote free and open software and advocate the avoidance of proprietary languages. This in particular counts for Matlab.
to first cast the (linear) model in the following form

\[
N \begin{bmatrix} x_t \\ \tilde{v}_{t-1} \end{bmatrix} + c \max \left\{ b \begin{bmatrix} x_t \\ \tilde{v}_{t-1} \end{bmatrix}, \tilde{v} \right\} = E_t \begin{bmatrix} x_{t+1} \\ \tilde{v}_t \end{bmatrix},
\]

(34)

where \( \tilde{v} \) contains all the (latent) state variables and \( x \) all forward looking variables (variables can be part of both). This way of writing the system has the advantage that the rational expectations solution of the state \( \tilde{v} \) depending on \( \tilde{v}_{t-1} \) of the system in period \( t+s \) assuming \( k \) periods at the ZLB and a transition of \( l \) periods towards it can be expressed in closed form as

\[
L_s(l, k; \tilde{v}_{t-1}) = N^{\max\{s-l,0\}} (N + cb)^{\min\{l,s\}} S(l, k; \tilde{v}_{t-1})
\]

\[
+ (I - N)^{-1} (I - N^{\max\{s-l,0\}}) c \tilde{r},
\]

(35) (36)

where

\[
S(l, k; \tilde{v}_{t-1}) = \left\{ \begin{bmatrix} x_t \\ \tilde{v}_{t-1} \end{bmatrix} : QN^k (N + cb)^l \begin{bmatrix} x_t \\ \tilde{v}_{t-1} \end{bmatrix} = -Q(I - N)^{-1} (I - N^k)c \tilde{r} \right\}
\]

(37)

and \( Q = \begin{bmatrix} I & -\Omega \end{bmatrix} \) for \( x_t = \Omega \tilde{v}_{t-1} \), which represents the linear rational expectations solution of the unconstrained system as given by (Blanchard and Kahn, 1980). This comes in handy for computational reasons since it can be avoided to simulate the whole path to check if \((l,k)\) is an rational expectations equilibrium. The detailed equilibrium conditions and how to find such equilibrium are given in the paper. The resulting transition function

\[
\tilde{v}_t = f_L(\tilde{v}_{t-1})
\]

(38)

is linear for the region where the ZLB does not bind and (increasingly) nonlinear when it binds. The durations of how long the ZLB binds are determined endogenously and hence this corresponds to the concept of rational expectations where agents assume that there will be no further shocks in the future. This assumption only results in an accurate system if shocks are moderate since it ignores the Jensen inequality. We argue that the approximation error is limited, especially compared to observation errors and errors related to model misspecification.\(^6\) This approximation is further necessary to solve a large scale model accurately within reasonable computation time. For the model presented here, the implementation in \textit{pydsge} is able to find the solution of about 80,000 particles per second (given one core).

Bayesian filtering – i.e. the extraction of the most likely distribution of states in each period given the series of observables – is done using the IPAS filter that is also

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\(^6\)It is hard to test how much this approximation error is since an accurate global solution of a model with large state is extremely difficult. To get an idea one could use the simple New Keynesian model (insert citation) and compare the global solution with the perfect foresight solution. It turns out that the approximation error can be large, but only for deviations of GDP that are one magnitude higher than those observed during the great recession.
presented in Boehl (2019). IPAS stands for iterative path adjusted Transpose Ensemble RTS-Smoother and is based on particle filter methodology. In conjunction with the assumption that the states at each point in time are approximately Gaussian distributed, the concept of statistical linearization can be used to efficiently filter large-scale nonlinear systems with only a few hundred particles within milliseconds. The method of iterative path adjustment then guarantees that the nonlinearity of the transition function is fully respected. IPAS is implemented in the econsieve package.

For the Bayesian estimation the Affine Invariant MCMC Ensemble sampler is used as proposed by Goodman and Weare (2010). This relatively new method hast the advantage that the sampling can be efficiently parallelized: instead of having a small number of chains that are each dependent on its own state as in the Metropolis algorithm, the ensemble sampler uses a large number of chains that communicate after each iteration and exchange states and regions of the parameter space with high likelihood. The method has been extensively applied in particular in the field of astrophysics. Prior to the MCMC sampler, the posterior mode maximization is done using the simplex method in Nelder and Mead (1965).

We for estimation – and later for the filtering of shock series – we use a dataset of seven observables. Our observables are real GDP growth, real consumption growth, investment growth, labor hours, the log change of the GDP deflator, real wage growth, and the federal funds rate. The details of the construction of these variables as well as data sources are provided in Appendix A. We employ a sample from 1998Q1 to 2018Q2 in our quantitative analysis. This is shorter than in Gust et al. (2017), Linde et al. (2017) or Fratto and Uhlig (2014). The reason for that is that we want the estimation to capture idiosyncrasies of the episode where the zero lower bound was binding, in particular the slope of the Phillips Curve and the persistence of endogenous and exogenous variables. Hence, using a longer sample would have born the risk of misspecification. On the other hand, using a shorter sample yields less data points for the estimation procedure and might let priors dominate the posterior distribution for parameters that are not well identified. We opt for the shorter sample but will keep the last critique point in the back of our minds. We assume an observation error of $\sqrt{1}$ of each time series standard deviation apart from the federal funds rate, where we only $\sqrt{0.01}$ of the standard deviation take as measurement error. We opt to make use of observation errors not only because we think they are a realistic feature that enriches the model, but also because this allows more leeway for the smoother to find more probable shocks. The smoothed observables are depicted in Figure 1 and 2. The orange line depicts the data series, the blue line, depicts the filtered and smoothed series. As the figure shows our IPAS-filtered series matches the data series of the observables very well although our choice for the observation error is quite broad.

We fix several parameter prior to estimating the others. In line with Smets and Wouters (2007), we set the depreciation rate $\delta = 0.025$, the steady state government share in GDP $G/Y = 0.18$, and the curvature parameters of the Kimball aggregator for prices and wages $\epsilon_p = \epsilon_w = 10$. The steady state markup in the labor market is set to $\lambda_w = 1.1$. The steady state of $L$ is fixed at a conventional 0.33. Furthermore, we fix the values of two parameters, which have been virtually unchanged in several estimation runs $\alpha = 0.19$ and $h = 0.8$. Lastly, we set the target inflation rate at 0.5 percent quarterly.

For the common trend growth rate and the mean of the observation equation for labor hours we employ the respective means of the pre-crisis decade, 1998-2008. These
Figure 1: Observables with IPAS-filtered series. 200 draws from posterior distribution, the same shock series are used for the counterfactual analyses.

at 0.344 percent for the quarterly growth rate and 6.5415 for the mean of our index for labor hours. Assuming a high trend of output implies a relatively lower trend of the percentage steady state deviation during the great repossessions. Likewise, assuming a very low trend of output will imply that output recovered quickly after the financial crises. It is clear that from the probabilistic perspective of the model, a recession is far more likely if it is short than if it is long. Hence, an estimation of this trend parameter will always prefer a very low trend. But this would not be in line with common the common understanding that the recession was both very strong, quite long, and followed by a period of little to no growth. In order to account for that feature, we cannot leave this decision to the estimation but calibrate this parameters. The reasoning motivates our calibration of the mean for the observable labor hours.

Table (1) shows our priors for the estimated parameters as well as the posterior means, standard errors and the 95% credible set. Our priors are closely aligned with the ones used by Smets and Wouters (2007). However our posterior estimates differ significantly, what is not surprising, given the different choices of sample and methodology. Compared to Linde et al. (2017), who also estimate the model at the ZLB, we find a similarly flat Phillips curve, which is mainly determined by the Calvo parameter $\zeta_p = 0.758$ and the gross price markup $\Phi_p = 1.803$. Also the parameter for price indexation, $i_p = 0.247$, is closely in line with their estimates. Our assumption of $\lambda_w$ and our estimate of $\zeta_w = 0.630$ imply a relatively steep wage Phillips curve, and the wage indexation parameter $i_w =$
0.452 is somewhat smaller than in Linde et al. (2017). This result stands in particular in contrast to the estimates of Gust et al. (2017), who obtain an wage adjustment cost parameter of 4420 in a Rotemberg setting, which implies a virtually inexistente wage Phillips curve. Our estimates for the feedback coefficients in the Taylor rule as well as the interest rate smoothing parameter, $\rho = 0.79$, are commonly found values. The persistence parameter of the risk premium shock has a posterior mean of $\rho_{\text{u}} = 0.761$, which is comparable to, though slightly lower than what Linde et al. (2017) and Fratto and Uhlig (2014) find.

Crucially, for the monetary policy shock we estimate a persistence parameter of $\rho_{\text{r}} = 0.688$. This is significantly higher than the estimates by Smets and Wouters (2007) and Linde et al. (2017). The reason is that they employ a longer sample reaching back until 1966. When we extend our sample, we end up similarly low estimates as well. We discuss the discrepancy of the estimates of the short and long sample in section 5. Note, that in our context a relatively high $\rho_{\text{r}}$ is crucial to generate meaningful economic effects of forward guidance, as at the zero lower bound interest rate shocks only alter the expected path of the interest rate going forward. All else equal, the more persistent the forward guidance shock, the longer the expected extension of the zero lower bound and the stronger the effects on economic activity.\footnote{Another way of introducing forward guidance is chosen by Gust et al. (2017), who introduce interest}
details on the estimation procedure in Appendix B.

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Table 1: Estimation results

4 Decomposition of the Great Recession

At the core of this paper we decompose the dynamics before, during and after the great recession, much similar to Fratto and Uhlig (2014) with the crucial difference that we take the constraint imposed by the ZLB serious. Which shocks were mainly responsible for the weak economic development? We answer this question by drawing 200 parameter sets from the posterior distribution (more precisely: from the converged Markov chains). For each of these draws we use the IPA smoother to filter out the most probable series rate smoothing with respect to the notational rate instead of the realized rate.
of shock innovations given the distribution of observables and the initial distribution of states. We then switch off certain shocks in order to measure their contribution. The results of this exercise are documented in Figures 3 to 5 which shows the medians over the draws. When possible in terms of clear display we also add 68% intervals. Note that, due to the nonlinearity of the model, the order in which we switch off shocks is not irrelevant. For reasons that will become clear in a second we start with the risk premium shock.

4.1 The role of risk-premium shocks

Our findings suggest that the risk-premium shock, that we borrow from Smets and Wouters (2007), adequately capture the effect of risks that emanated at that time from the distressed financial sector and spilled over to the macroeconomy. Hence, they can be viewed as a shorthand for what other authors have modelled in the form of financial frictions coupled with capital quality shocks (see, e.g., Gertler and Karadi (2011b), Gertler and Karadi (2013)), risk shocks (see, Christiano et al. (2014)), liquidity shocks (see Del Negro et al. (2017)) or other shocks. Let us now turn to the question of why the economy only slowly recovered and why it stayed at the ZLB for a prolonged period of time.

(show figure of interest rate spreads and compare to progression of $u_t$)

From the figures it is clear that output, consumption and investment are almost entirely driven by the risk premium shock together with the investment specific shock and
the conduct of monetary policy. Risk premium shocks are the main and very dominant
drivers of the post-2008 US-dynamics. Due to these shocks, inflation rates have been
lowered and the pressure on the labor market and real economic activity was severe.

Recall that the economy was hit by several recessionary risk premium shocks. Absent
the recessionary pressure of these shocks, output and its components, as well as labor
and wages would have experienced far stronger growth after 2011. The influence of these
shocks on the inflation rate, was similarly pronounced. Switching off these shocks reduces
the zero lower bound duration by three years and allows for an exit in 2012 even while
leaving monetary policy proactive. Without further risk premium shocks, labor hours
would have recovered to its pre-crisis level in 2012, and wages would not have fallen
at all. Consumption and investment would have experienced a strong boom starting in
2011. This underlines the dominant role for financial factors for the US economy after
the great recession.

4.2 Investment, technology and government spending

Next to the predominant risk premium shocks, the second most important shocks
are the investment-specific shocks. They drive about half of the variation in investment
and contribute about 1.5% of the loss in total GDP. Note that in a model with financial
frictions, investment can be closely tied to financial market dynamics or the behavior
of the banking sector. In comparison, neither the technology shocks nor government spending had notable impact on any of the variables.\(^5\)

![Figure 5: Decomposition of dynamics](image)

4.3 Markup shocks and the Phillips curve

Let us now turn to the markup shocks. As to be expected from the modestly high estimates of \(\zeta_p\) and \(\zeta_w\), the movements of inflation and wages are not well-explained by the fundamental shocks. Let us have a look at Figure 6 where we plot the decomposition of inflation and wages together with the respective markup shocks. It quickly becomes clear that the lion share of inflation is explained by price markup shocks and wage markup shock. However, the major drops of inflation in 2009 and again starting 2011 are as well due to the large risk premia, which depressed demand and hereby lead to a dip in prices, and also led to a decrease in labor supply which weighted on the wage level. Apart from these three inputs, only monetary policy contributed to these time series, which we will investigate separately in the next subsection.

\(^5\)The fact that the series of technology shock starts quite negatively stems from the fact that we allow the filter to rather painlessly choose the initial states by allowing for a large initial state variance. The low initial technology level only partly compensates for the fact that we impose a rather low growth rate.
4.4 Effects of monetary policy during the Zero-lower bound

While the measurement of monetary policy innovations is straightforward during normal times, it calls for an explanation during the ZLB period. When the constraint is binding, monetary policy shocks can still have an impact via the expectations channel, taken as a promise to keep rates lower for longer. This is what we will interpret as forward guidance (FG) in this paper. How then is FG measured? Our FG estimate stems from the most probable distribution of shocks given the full time series of observables. Hence, in order to measure a “FG shock”, we do not only need to observe that rates are zero but also that output and inflation increase. From the FG-Puzzle we learn that such increase must be more than just moderate even for a cautious FG shock. If we however have a look at Inflation, we do not see much reaction to anything. Likewise, output and consumption remain at relatively low level. A measure of strong FG shocks hence would require to also measure even worse shocks to the risk-premium, which are even more unlikely.

We start by conducting a counterfactual analysis, in which we switch off the monetary policy shocks after 2007Q1. We chose this year since the decomposition above suggests heavy expansionary movements in the policy rate that are a response to the 2007 real estate crisis. Figure 7 plots the difference in variables with and without monetary policy innovation and suggests that this drop in interest rates prevented much bad peaking at about 1.5% of GDP. It was however unanticipated since – for the first time – it constituted
a reaction to financial variables rather than to real and nominal ones.

Following the initial crisis, economic conditions remained weak and the Fed kept interest rates at the zero lower bound until 2015Q4. Right at the beginning of the zero lower bound period in 2009Q1 the Federal Open Market Committee started to direct expectations of market participants towards a protracted period of low interest rates, stating in December 2008 that "[economic conditions] are likely to warrant exceptionally low levels of federal funds rates for some time". In the statements to later meetings, the qualifier "for some time" was changed to "for an extended period of time" in March 2009, to "at least through mid-2013" in August 2011, to "at least through late 2014" in January 2012 and to "at least through mid 2015" in September 2012. Our simulations suggest that forward guidance on the path of the nominal short-term interest rate played a small but non-negligible role for stimulating the economy and contributed about 0.5% of GDP from 2010 until 2015.

With respect to the interest rate, we see that even without exogenously prolonging the duration of the interest rate, the zero lower bound on nominal interest rates would have been binding until 2015. That is, in the absence of the stimulus to the economy provided by forward guidance, the economic development would have been sufficiently weak to force the interest rate to zero via the Taylor rule.

A third finding in Figure 7 is striking. While the drop in rates was ahead of time compared to what the Taylor-Rule would have suggested and forward guidance (carefully) contributed to keeping expectations on the expected exit rate down, to the end of our sample our simulations suggest that rates where raised too early and to strongly in 2015. Note however that, even though the median during these time is below zero, the confidence bands do not suggest that the hypothesis of raising the rates at the point in time which was suggested by the Taylor rule can be rejected.

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9Statement to the FOMC Meeting, December 16th 2008.
Lastly, Figure 8 compares the expected simulated ZLB-durations with and without monetary policy shocks. This figure provides two takeaways: first, after rates initially hit the ZLB, agents expected to remain there for a relatively long time (slightly more than a year in median). The following periods where consolidating but in 2011-2012 the expected number of periods increased again. Second, this increase was due to economic conditions and not on how the Fed managed expectations. Overall, Forward guidance did not have much impact on the expected number of periods at the ZLB.

4.5 The impact of hitting the ZLB

As the last of our counterfactual exercises, we repeat the experiment by Gust et al. (2017) and compare the smoothened series with what would have happened if the ZLB just would not exist. This is not only fun but also allows to study the costs that are associated with it. It further is an legit assessment on how relevant it is to actually model the ZLB and take it serious during the estimation procedure.

Figure 9 plots the net difference between the world with and without ZLB. We first observe that in the absence of the GDP would have 1.2% higher in 2010/2011 and about 2% higher in 2014-15, which is an immense loss in overall output and larger than the number of 1% documented by Gust et al. (2017). This surplus in output would have been accompanied by a likewise increase in labor of 3% max. The gain in inflation would have been moderate but noticeable, leveling at about 0.4% annually. In order to prevent this disaster a leeway in interest rates of roughly 3% would have been necessary.

![Figure 9: Costs of ZLB (linear vs ZLB)](image-url)
In the years after the Great Recession the risk of a double-dip recession was widely discussed in the media and considered by policy makers. Our experiment suggests that the fact that the ZLB was binding did obscure the observability of variables but in fact, the driving forces behind the prolonged recession were actually two recessions. This also illustrates that the increase in the expected duration due to forward guidance was not driven by a few large shocks which generated expectations of a long duration, but several smaller shocks, which kept continuously delaying the expected exit from the zero lower bound.

5 Discussion

After having discussed our main results in the previous section, this section can be understood as sort of a chill-out section where we present some smaller results that we hope are still of use for the scientific community. Mainly we will focus on two artefacts: We discuss in how far taking the ZLB serious changes the estimation results, and we seek to find idiosyncrasies in the short time sample that are absent in the long sample.

5.1 Estimation results when ignoring the ZLB

In Table 2 we document the results from estimating the linear model with the same software stack as described above, but with using the standard linear Kalman filter instead of the IPAS. While some parameters are just a miss, we are surprised that others are relatively accurate. In particular, the parameters for price (wage) indexing are quite close to their linear counterparts. Same holds for the Calvo parameters $\zeta_p$ and $\zeta_w$ and the parameters governing the Taylor rule.

The fit of the autocorrelation coefficients is still pretty good, though not as amazing as with the above mentioned. However, the problem lies with the standard deviations of the shock terms at the end of the table. Standard deviations are the input into the transition noise covariance of the filter. If noise is overestimated, fit will get worse. If the ratio in two standard deviations is overstated, so will be the final shock decomposition.

We conclude this small exercise that for the model in hand, the linear estimation – with the limits mentioned above – actually provides a pretty good approximation to the nonlinearly estimated distribution of parameters. However, there exists no proof that a similar good result will be obtained for a different model or a different time series of observables.

5.2 Estimation results when considering the whole sample

In our benchmark estimation, we employ a relatively short sample that starts in 1998.Q1 and ends in 2018Q2. In this section, we compare the estimates obtained in the benchmark estimation with parameter values obtained from an estimation on a longer sample that starts in 1966.Q1. Little surprising, the estimates differ in several aspects. A first key difference is the lower estimate for $\rho_r$. In the longer sample, monetary policy shocks are estimated to have a smaller persistence and therefore are less consequential. This underlines the importance of using the short sample for the analysis of forward

\footnote{see, e.g., Elwell (2012)}
Table 2: Estimation results linear solution

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guidance, which has larger effects for larger values of $\rho_r$. Interestingly, we find that in the longer sample, the Phillips curve is even somewhat flatter than in the short sample. The same holds for the wage Phillips curve. The risk premium process is similarly persistent in both samples, but in the crisis sample, the shock becomes more volatile, contributing more to the dynamics of the economy. The technology shock moves closer a random walk, whereas government spending shocks are estimated to be rather short-lived in the long sample. **To be completed**

5.3 The choice of preferences and determinacy at the ZLB

We depart from the canonical framework by Smets and Wouters (2007) insofar as we choose another functional form for households’ utility. In particular, we exchange a function in which there are complementaries in consumption and leisure against a function that is separable in the two argument. We decided to do so, since we discovered
Table 3: Estimation results long sample

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in a series of trial runs that in this model for a variety of calibrations, the latter utility function doubles the time that the model can spend at the zero lower bound after appropriately large shocks without running into indeterminacy. What is the intuition for why the type of utility function matters for determinacy at the zero lower bound? Consider a large recessionary demand shock that pushes the economy to the zero lower bound. In this case, the constraint that prevents nominal rates to fall further, implies high real interest rates, which exacerbate the fall in consumption. In the case of a separable utility function the fall in consumption induces an outward shift of the labor supply schedule (wealth effect on the labor supply). The increased willingness to work has the potential to dampen the fall in economic activity. In the case of complementarities in consumption and labor, however, the fall in consumption could be accompanied by a contracting labor supply. This concurrent fall of consumption and labor can then worsen the amplify the fall in economic activity, which in the worst case scenario can not be stopped by the
outlook of a recovery after the initial demand shock dies out, and the perspective of a return to a normal economy. In this downward spiral, there is no rational expectation equilibrium. While the model with non-separable preferences still works rather well for most calibrations and plausibly large shocks, we prefer to err on the side of caution in order to avoid indeterminacy areas that would rule out areas in the parameter space and punctuate the posterior in our estimation.\footnote{As a side note, we discovered that the issue of indeterminacy becomes more severe, in an extension with financial frictions (see, (Boehl and Strobel, 2019))}

6 Conclusion

In this paper, we analyze the dynamics of the US economy in the Great Recession and at the zero lower bound. We decompose these dynamics into the contributions of the different driving forces and find that the recession and the slow recovery was mostly due to high risk premiums.

The long duration of the zero lower bound from 2009 to 2015 was, accordingly, a reaction to the anaemic economic growth and not a policy of active deviations from the Taylor-rule implied interest rate. Hence, the main effect of forward guidance was not to prolong the actual duration of the zero lower bound period, but to keep expectations of economic agents about the duration of the zero lower bound high. The anticipation of a long period of low interest rate stimulated aggregate demand and thus prevented a deeper recession. The dynamics of inflation were largely unaffected by interest rate policy. This is due to the flatness of the Phillips curve implied by our estimation. We find the flatness of the Phillips Curve to be a major concern.

Apart from risk premium shocks, the nominal series of inflation and wages are almost entirely driven by markup shocks, hence are detached from movements in the real economy. This inability to explain inflation dynamics in the last two decades poses a serious challenge for this class of models. Furthermore, we find that the presence of the zero lower bound imposed high costs of the economy. We find that the cost of the ZLB in terms of output reached up to 3% of GDP. Thus, ignoring the implications of the ZLB in the analysis of post-crisis dynamics major economic costs. Additionally, we argue that endogenously accounting for the ZLB has notable impact on estimation results and there is no easy way around this. Lastly, we asses that the choice of the sample matters for our estimation results and recommend using smaller samples in analyzing the dynamics in the Great Recession and thereafter, if one wants to accurately capture the structural parameters during this period.

For our analysis we use a quasi-analytical solution method for occasionally binding constraints in linearized difference equations together with a novel Bayesian filter as described by Boehl (2019). The method proofs to be numerically robust and the computational implementation of this method is very fast and can hence be used efficiently in conjunction with nonlinear filtering techniques to identify hidden states and to estimate economic models with large state spaces. Going forward, it is a fruitful endeavor to zoom in on the financial frictions underlying the movements risk premia in the economy and to analyze the effects of unconventional monetary policy measures that have been employed by the Fed in the last years. We follow this route in Boehl and Strobel (2019).
Appendix A  Data

Our measurement equation contains seven variables:

- GDP: $\ln(\frac{\text{GDP}}{\text{GDPCTPI}/\text{CNP16OV}}) \times 100$
- CONS: $\ln(\frac{\text{PCEC}}{\text{GDPCTPI}/\text{CNP16OV}}) \times 100$
- INV: $\ln(\frac{\text{FPI}}{\text{GDPCTPI}/\text{CNP16OV}}) \times 100$
- LAB: $\ln(\frac{(\text{AWHNONAG} \times \text{CE16OV})}{\text{CNP16OV}})$
- INFL: $\frac{\text{GDPCTPI}}{4}$
- WAGE: $\ln(\frac{\text{COMPNFB}}{\text{A006RD3Q086SBEA} \times 100})$
- FFR: $\frac{\text{FEDFUNDS}}{4}$

For GDP, CONS, INV and WAGE we use the log changes in our measurement equations. Data sources:

- GDP: GDP - Gross Domestic Product, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate, FRED
- GDPCTPI: Gross Domestic Product: Chain-type Price Index, Index 2009=100, Quarterly, Seasonally Adjusted, FRED
- PCEC: Personal Consumption Expenditures, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate, FRED
- FPI: Fixed Private Investment, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate, FRED
- AWHNONAG: Average Weekly Hours of Production and Nonsupervisory Employees: Total private, Hours, Quarterly, Seasonally Adjusted, FRED
- CE16OV: Civilian Employment Level, Index Q12009=100, FRED
- CNP16OV_NBD20090101: Civilian Noninstitutional Population, Index Q1 2009=100, Quarterly, Seasonally Adjusted, FRED
- COMPNFB, Nonfarm Business Sector: Compensation Per Hour, Index 2009=100, Quarterly, Seasonally Adjusted, FRED
- FEDFUNDS: Effective Federal Funds Rate, Percent, FRED

Appendix B  Details on the estimation

Figures B.10 to B.14 show the traces and final distribution after burn in of 100 chains running 8000 iterations. 7000 of these are discarded as burn-in.
Figure B.10: Traces of 100 chains (means, 66% and 95% confidence intervals).
Figure B.11: Traces of 100 chains (means, 66% and 95% confidence intervals).
Figure B.12: Traces of 100 chains (means, 66% and 95% confidence intervals).
Figure B.13: Traces of 100 chains (means, 66% and 95% confidence intervals).
Figure B.14: Traces of 100 chains (means, 66% and 95% confidence intervals).
References


Gertler, M., Karadi, P., 2013. QE 1 vs. 2 vs. 3 . . . : A Framework for Analyzing Large-Scale Asset Purchases as a Monetary Policy Tool. International Journal of Central Banking 9, 5–53.


