A Structural Investigation of Quantitative Easing
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Abstract

Did the quantitative easing measures (QE) conducted by the Fed during the Great Recession induce real effects? We seek to answer this question by estimating and filtering a medium scale DSGE model featuring a banking sector, financial frictions and the zero lower bound (ZLB). The expected duration of the ZLB is modeled to be endogenous.

Using the smoothed series for counterfactual analysis we conclude that from 2009 to 2015 the overall QE measures contributed about 0.5 percent to output. While the effects of the liquidity provision was negligible, both the bond and MBS purchases had positive impact on consumption. Other than MBS purchases, government bond purchases had negative effects on investment and a negative net contribution to GDP. Whereas QE stimulates asset prices in the short-run, the persistent reduction of excess spreads lowers banks’ net worth, the loan supply, and hence real economic activity in the mid-run.

Forecasts suggest that, through the link between banks balance sheet and investment, shutting down the QE program will have a strong recessionary "hangover" effect, leading to a negative net-effect on output in the years after the end of QE.

Keywords: Quantitative Easing, Liquidity Facilities, Zero Lower Bound, Bayesian Estimation
JEL: E63, C63, E58, E32, C62

1 Introduction

In 2008, the financial sector in the US collapsed, causing a sharp drop in asset prices and credit supply as well as what is known today as the Great Recession. As a response the Federal Reserve reduced its Federal Fund Rate to zero and engaged in quantitative easing measures (QE) such as large scale purchases of government debt and Mortgage Backed Securities, and – even before – committed to major liquidity injections into the

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banking sector. The effects of these measures are subject to a controversial debate. While critics’ assessments range from "a drop in the bucket" (Cochrane, 2011) and not having "fulfilled its stated objective of stimulating the economy" (Taylor, 2014), its proponents have firmly stated that "evidence suggests that forward guidance and security purchases [...] did help spur consumption and business spending, lower the unemployment rate, and stave off disinflationary pressures" (Yellen, 2017).

While there is emerging consensus that QE measures had at least some impact on financial market indicators\(^1\), evidence for the effects of quantitative easing on the real economy is still sparse. We seek to fill this gap by employing a DSGE model, which incorporates several QE measures and takes account of the zero lower bound on nominal interest rates (ZLB) as a lens to look at the data. The model is a hybrid of the models by Boehl and Strobel (2019) – which is a variant of Smets and Wouters (2007) – and Gertler and Karadi (2013). It thereby features all the bells and whistles that are necessary to make it suitable for estimation, it has properties that allow to remain at the ZLB for an extended period of time, and includes the financial frictions needed to allow for effects of QE on the real economy. Quantitative easing in our model affects different parts of the balance sheet of the financial sector, capital holdings, bond holdings, and its liabilities. The effects are passed through to the real economy via the supply of loans to non-financial firms. After estimation, we analyse the drivers of macroeconomic dynamics in the aftermath of the Great Recession and quantitatively assess the effects of QE on the US economy.

We find that QE had a relevant impact on the US economy. According to our estimates, in its peak, QE raised aggregate output and labor hours by roughly 0.7%, consumption by 0.6%, and aggregate investment by 3%. The impact on inflation was very small. However, while we find that QE had a stimulating impact, this stimulus was small in comparison to the sharp and persistent drop in output and labor below the pre-crisis trend. Furthermore, our analysis suggests that the stimulating effects of QE are followed by a drag on the real economy once the balance sheet of the central bank shrinks. In our model this drag can be attributed to the persistent decline in excess spreads on capital assets and government bonds, which weakens the balance sheet of financial intermediaries and in turn reduces investment and output in the long run.

In a second step, we decompose the effects of QE into the effects of three separate policy measures: large-scale purchases of capital assets and bonds, as well as emergency liquidity injections into the banking sector. We find that the purchases of capital assets had the strongest effect in stimulating the economy. The purchases of bonds on the other hand, while slightly increasing consumption, crowded out investment and had a negative net-effect on output and labor. The effect of liquidity injections on asset prices and interest rate spreads is found to be sizable, but they die out quickly. Hence their long term effect on the real economy is negligible.

To our knowledge, the analysis of the effects of QE in a non-linearly estimated medium scale DSGE model is a novel contribution to the literature. In order to overcome the challenges associated with the estimation of a model at the zero lower bound, we employ the methods suggested by Boehl (2019) which are implemented in the in the pydsge library.

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The linearized model is solved with the zero-lower bound as an endogenous occasionally binding constraint. The likelihood is evaluated with an ensemble-type Kalman filter (the IPAS filter, which is a hybrid between Particle Filter and Iterative Kalman Filter)\(^2\), and full Bayesian estimation of the parameters it done using the Affine Invariant MCMC Ensemble sampler (Goodman and Weare, 2010). The IPAS is also used to extract shocks and initial states for our quantitative analysis.

The financial sector in our model builds on Gertler and Karadi (2013). The effects of the asset purchases by the central bank are therefore largely similar to the effects in this model: they initially stimulate the economy, but a few years after implementation of the policy measures the economy drops below the steady state. However, given the additional rigidities and our estimated parameter values, these QE effects are somewhat smaller than in the calibrated model. There are several other papers that analyze the effects of QE but in calibrated models. Chen et al. (2012) find that the effects of government bond purchases in a model with segmented bond markets are rather small, whereas Del Negro et al. (2017) find that in response to a shock that raises the convenience yield, liquidity injections can have a sizable effect. Accordingly, our results are more in line with the modest effects of QE implied by Chen et al. (2012). Cúrdia and Woodford (2011) point out, that in a model, in which firms finance investment with long-term debt, the effect of QE, depends on which type of assets assets the central bank buys. Bond purchases are neutral in their framework, whereas purchases of assets from markets which are in distress can alleviate the pressure on the financial sector and stimulate the economy. While government bond purchases do have effects in our framework, we confirm the assertion that purchases of capital assets are more effective and last less heavily on investment in the long run. This is consistent with a different line of thought suggested by Brunnermeier and Koby (2018), who warn of potentially contractionary effects of QE. They argue that the persistently decreased net margins of the financial sector can induce a reduction of real activity. Our quantitative result of a hangover after the initial stimulus of QE is consistent with their theoretical argument. The design of this study is closely related to Boehl and Strobel (2019), where we focus on the effects of forward guidance in a smaller model without a financial sector. We document that the most parameter estimates are largely unaffected by the introduction of a financial sector in Appendix C.

This paper is also linked to empirical work on the effects of QE. Using a Bayesian VAR, Hesse et al. (2017) find that earlier asset purchase programs had stronger effects in the US and in the UK than the later programs, as they had a larger surprise effect on market participants. VAR studies for Japan (Schenkelberg and Watzka, 2013) and the UK (Kapetanios et al., 2012) also find significant effects of QE on the real economy. Thus our finding of significant positive macroeconomic effects are broadly in line with the VAR literature on this topic.

Section 2 presents the model in use. Section 3 includes a brief description of the data, as well as of the solution, filtering and estimation methodology, and presents and discusses the estimation results. Section 4 entails our analysis of the drivers of the US economy, as well as a detailed discussion of the effects of QE measures. Lastly, Section

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\(^2\)The package includes model parser, solution method, high level API to the smoother and estimation as well as tools for quantitative analysis and can be found at https://github.com/gboehl/pydsge.

\(^3\)A standalone implementation of the filter is available at https://github.com/gboehl/econsieve.
concludes.

2 Model

For our analysis, we extend a standard medium scale model as in Smets and Wouters (2007) with separable preferences as in Boehl and Strobel (2019) and financial intermediates as in Gertler and Karadi (2011), which extend loans to nonfinancial firms and hold government bonds. This section gives a brief description of the model.

Time is discrete, and one period in the model represents one quarter. The model features households, banks, intermediate good producers, capital good producers, retailers, labor unions, a fiscal and a monetary authority. Households consume, supply labor, and save in the form of bank deposits. Labor is differentiated by a union with monopoly power that faces nominal rigidities. The firm sector consists of three types of firms. Intermediate good producers employ labor and capital to produce their goods. Each period, after producing their output, they sell their used capital stock to the capital goods producers. The latter repair it, and invest in new capital. At the end of the period they re-sell the capital to the intermediate good producers, which use it for production in the next period. The intermediate good producers finance their purchases of capital with loans from the banks. Intermediate goods are purchased by retailers, which repackage them, and sell them with a markup as final goods to households, the capital producers, and to the government. Banks hold loans and government bonds on the asset side of their balance sheets. On the liability side are deposits and the banks net worth. The government consumes final goods, collects taxes, and issues government bonds. Monetary policy takes the form of a Taylor rule. The model includes habit formation in consumption, convex investment adjustment costs, and nominal rigidities as in Calvo (1983) in both, price and wage setting to enhance the empirical plausibility of the model dynamics.

2.1 Households

There is a continuum of households with a unit mass. As in Gertler and Karadi (2011) a constant fraction $f$ of each household’s members works as banker, whereas the other fraction $(1 - f)$ consists of workers who supply labor to the intermediate good producers. While workers receive their wage income every period, bankers reinvest their gains in asset holdings of the bank over several periods, and contribute to the households income only when exiting the banking sector, bringing home the accumulated profits. To ensure that both fractions of the household face the same consumption stream, perfect consumption insurance within the household is assumed. Households’ expected lifetime utility is as follows

$$U_t = E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{(C_t(k) - C_{t-1})^{-\sigma_e} - 1}{1 - \sigma} - \nu \log(1 - L_t(k)) \right)$$

where parameters $\beta$, $h$, and $\sigma_e$ are, respectively, the discount factor, the degree of external habit formation in consumption, and the coefficient of relative risk aversion. $C_t(k)$ is consumption and $L_t(k)$ are the hours worked. Habit formation is with respect to past aggregate consumption, $C_{t-1}$ The optimizing household $k$ earns nominal wages $W_t$ from
its labor supply \( L_t(k) \) and receives income from one-period zero coupon bonds \( B_t(k) \) that yield an interest rate \( R_t \). The return on deposits is subject to a disturbance term, \( v_{d,t} \), which follows an AR(1) process. This term drives a wedge between the interest rate controlled by the central bank and the required return on assets. Smets and Wouters (2007) label these shocks risk-premium shocks. In our context, this shock to the deposit rate can as well be interpreted as variations in the confidence in the banking system. Households spend their funds on consumption \( C_t(k) \), and save in new deposits. The budget constraint of consumers reads

\[
P_t C_t(k) + \frac{D_t(k)}{v_{d,t} R_t} = D_{t-1}(k) + W_t L_t(k) - T_t + \Psi_t. \tag{1}
\]

Here, \( P_t \) denotes the price level at time and \( T_t \) are lump sum taxes raised by the government to finance government spending, and \( \Psi_t \) are profits of monopolistic firms and banks that accrue to the households.

This formulation of the utility function – separability in consumption and leisure – stands in contrast to the non-separable formulation in Smets and Wouters (2007). As Boehl and Strobel (2019) argue, the non-separable formulation has unfavorable effects on the size of the determinacy region at the ZLB. While the same model with separable preferences can, given an appropriately strong recessionary shock, easily stay at the ZLB for 25+ periods (which are of course fully anticipated on impact), in the model with non-separable preferences larger shocks quickly lead to non-existence of an rational expectation equilibrium (without additional central bank activity).

2.2 Firm sector

The model contains three types of firms. Intermediate goods are produced by perfectly competitive firms, which use capital and labor as inputs for production. Monopolistically competitive retailers buy a continuum of intermediate goods, and assemble them into a final good. Nominal frictions as in Calvo (1983) make the retailers optimization problem dynamic. Additionally, a capital producing sector buys up capital from the intermediate good producer, repairs it, and builds new capital, which it sells to the intermediate good sector again. Investment in new capital is subject to investment adjustment costs.

2.2.1 Intermediate good producers

Intermediate good producers are in monopolistic competition, employ labor and capital services from households, and set their prices as markups over the marginal cost. Firm \( i \) produces according to the Cobb-Douglas Function

\[
Y_{m,t}(i) = e^{z_t} K_t(i)^\alpha \left( \gamma^\gamma L_t(i) \right)^{1-\alpha} - \gamma^\gamma \Phi. \tag{2}
\]

Here, \( Y_{m,t}(i) \) are intermediate goods, \( z_t \) is a TFP-shock, \( K_t(i) \) is the effective capital used in production defined as \( K_t(i) = U_t(i) K_{t-1}(i) \), and parameter \( \alpha \) is the output elasticity with respect to effective capital. \( \gamma^\gamma \) represents the labor-augmenting growth rate in the economy, and \( \Phi \) is the fixed cost of production.

At the end of each period the intermediate good producer sells the capital stock that it used for production to the capital producer which repairs the capital, and purchases the capital stock that it is going to use in the next period from the capital producer.
To finance the purchase of the new capital at the price \( Q_t \) per unit, it issues a claim for each unit of capital it acquires to banks, which trade at the same price. The interest rate the firm has to pay on the loan from the bank is \( R_{k,t} \). Under the assumption that the competitive firms make zero profits, the interest rate on their debt will just equal the realized ex-post return on capital. Furthermore, we assume that the firm incurs costs of capital utilization that are proportional to the amount of capital used, \( a(U_t)K_t \). The period profit function of firm \( i \) is therefore

\[
P_{m,t}(i)Y_t(i) - W_tL_t(i) - R^K_tK_t(i) - aU_t(i)K_t(i) - \alpha(U_t)K_t(i) + (1 - \delta)Q_{t+1}\]

Hence each period the firm determines its optimal capital purchase by maximizing

\[
E_t[\beta A_{t,t+1}(-R_{k,t+1}Q_tK_t(i) + P_{m,t+1}(i)Y_{m,t+1}(i) - W_{t+1}L_{t+1}(i) - a(U_t)K_t(i) + (1 - \delta)Q_{t+1}K_t(i))]
\]

with respect to \( K_t(i) \). As all firms make the same decisions, we can drop the index \( i \). In optimum the ex-post return then is as follows

\[
R_{k,t+1} = \frac{P_{m,t+1}Y_{m,t+1}}{K_t} + (1 - \delta)Q_{t+1}. \tag{3}
\]

Additionally, the choices for optimal labor input and optimal capital utilization yield the first order conditions

\[
W_t = P_{m,t}(1 - \alpha)\frac{Y_{m,t}}{U_t}. \tag{4}
\]

\[
a'(U_t)K_t = \alpha P_{m,t}\frac{Y_{m,t}}{U_t}. \tag{5}
\]

### 2.2.2 Capital good producers

Capital evolves according to the following law of motion

\[
\bar{K}_t = (1 - \delta)\bar{K}_{t-1} + v_{i,t} \left( 1 - S \left( \frac{I_t}{I_{t-1}} \right) \right) I_t, \tag{6}
\]

where \( \delta \) is the depreciation rate and the function \( S() \), indicates a cost of adjusting the level of investment. In steady state it holds that \( S = 0, S' = 0, \) and \( S'' > 0. \) and \( v_{i,t} \) follows an AR(1) process. While Smets and Wouters (2007) interpret \( v_{i,t} \) as an investment specific technology disturbance, Justiniano et al. (2011) stress that this shock can as well be viewed as a reduced-form way of capturing financial frictions, as it drives a wedge between aggregate savings and aggregate investment.

The capital good producer’s role in the model is to isolate the investment decision that becomes dynamic through the introduction of convex investment adjustment costs, which is a necessary feature to generate variation in the price of capital. Capital good producers buy the used capital, restore it and produce new capital goods. Since capital producers buy and sell at the same price, the profit they make is determined by the difference between the quantities sold and bought, i.e. investment. Capital producers bear the
resource costs associated with changes in investment. They choose the optimal amount of investment to maximize

\[ E_0 \sum_{t=0}^{\infty} \beta^t \Delta L_{0,t} \left\{ Q_t \left( 1 - S \left( \frac{I_t(k)}{I_{t-1}(k)} \right) \right) v_{t,t} - 1 \right\} I_t. \]

The first order condition of the capital producer reads

\[ 1 = Q_t v_{t,t} \left( 1 - S \left( \frac{I_t}{I_{t-1}} \right) - S' \left( \frac{I_t}{I_{t-1}} \right) \right) \]

\[ + E_t \left\{ Q_{t+1} v_{t+1,t+1} S' \left( \frac{I_{t+1}}{I_t} \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \right\} \]

2.2.3 Final good producers

Final good producers buy the goods produced by the intermediate good producers and sell them to final good producers. They act under monopolistic competition. Each period, retailers firms face a constant probability of being able to optimally adjust their prices and sell them to final good producers. They act under monopolistic competition. Each period, retailers firms face a constant probability of being able to optimally adjust their prices, \( \zeta_p \). Those firms, which cannot optimally adjust their prices in a given period, index their prices to a weighted average of last periods inflation and steady state inflation. These assumptions give rise to the following maximization problem for firm \( i \)

\[ \max_{P_t(i)} E_t \sum_{s=0}^{\infty} (\beta \zeta_p)^s \frac{\Lambda_{t,s}}{\Pi_{t,s}} \left[ P_t(i) \Pi_{t+1}^p (\Pi_{t+1}^{1-\zeta_p} - MC_{t+s}) \right] Y_{t+s}(i) \]

\[ s.t. \quad Y_{t+s}(i) = G^{-1} \left( \frac{P_t(i) \Pi_{t+1}^p (\Pi_{t+1}^{1-\zeta_p})}{P_{t+s}^p \tau_{t+s}} \right). \]

\( P_t(i) \) is the price set by firm \( i \), \( \Pi_{t,t+s} \) is the accumulated change in the aggregate price level between periods \( t \) and \( t+s \), \( \Lambda_{t,s} \) is the stochastic discount factor of the firm, \( Y_{t+s} \) is the demand by final good firms for intermediate goods. Parameter \( \zeta_p \) is the degree of price indexation. Function \( G \) governs how the relative price of firm \( i \) affects the amounts of goods it can sell. We make the same assumptions on \( G \) as Smets and Wouters (2007).\(^4\) Furthermore, \( \tau_{t+s} \equiv \int_0^1 G' \left( \frac{Y_{t+s+1}(i)}{Y_{t+s+1}} \right) \frac{Y_{t+s+1}(i)}{Y_{t+s}} \right) di. \) The aggregate price index is in this case given by

\[ P_t = \left\{ (1 - \zeta_p)(P_t^*) G' \right\} \left[ \frac{P_t^*}{P_t} \right] + \zeta_p \Pi_{t-1}^{1-\zeta_p} P_{t-1} G' \left( \frac{\Pi_{t-1}^{1-\zeta_p} P_{t-1}}{P_t} \right) \]

where \( P_t^* \) is the optimal price in period \( t \). The price markup set by final goods producers, \( v_{p,t} \), is time-varying and subject to markup shocks, \( v_{p,t} \), which follow an AR(1)-process.

\(^4\)That is that \( G' > 0, G'' < 0, \) and \( G(1) = 1. \) As shown by Kimball (1995), the assumptions on \( G \) imply that the the demand for a good is decreasing in its relative price, and that the elasticity of demand for a good increases with its relative price, which in turn implies a higher persistence of aggregate inflation dynamics.
2.2.4 Retailers

Retailers act under perfect competition. They buy the goods from final good producers, bundle them in final goods, and sell them to the public. Their maximization problem reads

\[ \max_{Y_t, Y_t(i)} P_t Y_t - P_t(i) Y_t(i) \tag{10} \]

subject to

\[ \int_0^1 G \left( \frac{Y_t(i)}{Y_t}, \lambda_{p,t} \right) \, di = 1. \tag{11} \]

2.2.5 Unions and Labor Packers

The supply of labor to intermediate good firms is organized by unions and labor packers. Households supply labor to a labor union, which differentiates the labor services and sets wages. Unions act in monopolistic competition with each other and set their wages, \( W_t(i) \), as a markup over the average marginal rate of substitution between consumption and leisure of households. We assume that the wage setting process, in the same way as price setting, is subject to a Calvo type friction, and that unions, which cannot adjust their wages in a given period index their last wage to a weighted average of last periods inflation and steady state inflation. Labor packers buy the labor services from unions, bundle them and provide them to intermediate good firms at the wage \( W_t \). Thus, the maximization problem of labor packers is

\[ \max_{L_t, L_t(i)} W_t L_t - W_t(i) L_t(i) \tag{12} \]

subject to

\[ \int_0^1 G_w \left( \frac{L_t(i)}{L_t}, \lambda_{w,t} \right) \, di = 1, \tag{13} \]

where \( L_t \) is labor provided by labor packers to intermediate good firms, \( L_t(i) \) is the labor services sold by unions to labor packers, and \( G_w \) is the labor aggregator on which we make the same assumptions as on \( G \). The wage markup, \( \lambda_{w,t} \), is time-varying and subject to wage markup shocks, \( \nu_{w,t} \), which follows an AR(1)-process, \( \nu_{w,t} \).

Labor unions observe the average marginal rate of substitution between consumption and leisure of all households,

\[ MRS_t = \frac{\nu(C_t - hC_{t-1})^\sigma}{(1 - L_t)} \tag{14} \]

and charge a markup on top of it. Their maximization problem is

\[ \max_{W_t(i)} \mathbb{E}_t \sum_{s=0}^{\infty} (\beta \xi_w) s A_{t+s} \left( [W_t(i) \Pi_{t+s}^w (\Pi_{t+s}^w - 1)^{1-s} - MRS_{t+s}] \right) L_{t+s}(i) \tag{15} \]

subject to

\[ \frac{L_{t+s}(i)}{L_{t+s}} = G_w^{-1} \left( \frac{W_t(i) \Pi_{t+s}^w (\Pi_{t+s}^w - 1)^{1-s}}{W_{t+s}} \right). \tag{16} \]

\( W_t(i) \) is the wage set by union \( i \), and parameter \( \nu_w \) is the degree of wage indexation, and
\[ \tau^w_{t+s} = \int_0^1 G^w \left( \frac{L_{t+s}^{i.i.d.}}{L_{t+s}} \right) di. \]

The aggregate wage index is in this case given by

\[ W_t = [(1 - \zeta_w)(W^*_t)G^w - 1 \left( \frac{W^*_t}{W_t} \right) + \zeta_w \Pi_{t-1}^{w} \Pi^{w(1-i_w)} W_{t-1} \left( G^w - 1 \left( \frac{\Pi_{t-1}^{w} \Pi^{w(1-i_w)}}{W_t} \right) W_{t-1} \right) \]

where \( W^*_t \) is the optimal wage set by labor unions in period \( t \).

2.3 Banks

Banks finance their operations by creating deposits, \( D_t \), which are held by households, and by their net worth, \( N_t \). They use their funds to extend loans to intermediate good producers for acquiring capital, \( K_t \), and for the purchases of government bonds, \( B_t \) at their market price \( Q^p_t \). To distinguish the the capital assets and bonds that are held by banks from those which are held by the central bank, we denote them with \( K_{b,t} \) and \( B_{b,t} \).

The balance sheet of bank \( j \) is given by

\[ Q_t K_{b,t}(j) + Q^p_t B_{b,t}(j) = N_t(j) + D_t(j). \]

The banks retain the earnings, generated by the return on their assets purchased in the previous period, and add it to their current net worth. Thus, the law of motion for the net worth of a bank is given by

\[ N_t(j) = R_{b,t} Q_{t-1} K_{b,t-1}(j) + R_{b,t} Q^p_{t-1} B_{b,t-1}(j) - v_{d,t} R_{t-1} D_{t-1}(j). \]

Note that while the interest rate on deposits raised in period \( t - 1 \), is determined in the same period, the return of the risky capital assets and risky government bonds purchased in period \( t - 1 \) is determined only after the realization of shocks at the beginning of period \( t \). Substituting the balance sheet into the law of motion for net worth yields

\[ N_t(j) = (R_{b,t} - v_{d,t} R_{t-1}) Q_{t-1} K_{b,t-1}(j) + (R_{b,t} - v_{d,t} R_{t-1}) Q^p_{t-1} B_{b,t-1}(j) + v_{d,t} R_{t-1} N_{t-1}(j). \]

Bankers continue accumulating their net worth, until they exit the business. Each period, each banker faces a lottery, which determines, regardless of the history of the banker, whether he exits his business or stays in the sector. Bankers exit the business with an exogenous probability \( 1 - \theta \), or continue their operations with probability \( \theta \). The draws of this lottery are i.i.d.. When a banker leaves the sector, it adds his terminal wealth \( V_t(j) \) to the wealth of its household. Therefore, bankers seek to maximize the expected discounted terminal value of their wealth

\[ V_t(j) = \max E_t \sum_{i=0}^{\infty} (1 - \theta) \beta^i \beta^{i+1} \Lambda_{t,t+1} N_{t+1}(j) \]

\[ = \max E_t \left[ \beta \Lambda_{t,t+1} (1 - \theta) N_{t+1}(j) + \theta V_{t+1}(j) \right]. \]

As banks operate under perfect competition, in the case of perfect capital markets the risk adjusted return on loans and government bonds would equal the return on deposits. However, bankers face an endogenous limit on the amount of funds that households are willing to supply as deposits. Following Gertler and Karadi (2011), we assume that
bankers can divert a fraction of their assets and transfer it to their respective households. However, if they do so, their depositors will choose to withdraw their remaining funds and force the bank into bankruptcy. To avoid this scenario, households will keep their deposits at a bank only as long as the bank’s continuation value is higher or equal to the amount that the bank can divert. Formally, the incentive constraint of the bank reads

\[ V_t(j) \geq \lambda Q_t K_{b,t}(j) + \lambda_b Q_{b,t}^b(j), \]  

where \( \lambda \), is the fraction of loans that the bank can divert, and \( \lambda_b \) is the fraction of government bonds it can divert. We calibrate \( \lambda_b \) to be smaller than \( \lambda \). This is motivated by the fact, that, in general, the collateral value of government bonds is higher than that of loans.\(^5\) The reason is that loans to private firms are less standardized than government bonds contracts. Additionally, information on the credit-worthiness of the government is publicly available, while the credit-worthiness of private firms is often only known to the bank and the firm, and not easy to assess for depositors, making it easier for banks to divert a fraction of their value.

The initial guess for the form of the value function is

\[ V_t(j) = \nu_{k,t}(j) Q_t K_{b,t}(j) + \nu_{b,t}(j) Q_{b,t}^b(j) + \nu_{n,t} N_t(j), \]  

where \( \nu_{k,t}, \nu_{b,t} \) and \( \nu_{n,t} \) are time varying coefficients. Maximizing (22) with respect to loans and bonds, subject to (21) yields the following first order conditions for loans, bonds, and \( \mu_t(j) \), the Lagrangian multiplier on the incentive constraint

\[ \nu_{k,t}(j) = \lambda \frac{\mu_t(j)}{1 + \mu_t(j)}, \]  

\[ \nu_{b,t}(j) = \lambda_b \frac{\mu_t(j)}{1 + \mu_t(j)}, \]  

\[ \nu_{b,t}(j) Q_t K_{b,t}(j) + \nu_{b,t}(j) Q_{b,t}^b(j) + \nu_{n,t} N_t(j) = \lambda Q_t K_{b,t}(j) + \lambda_b Q_{b,t}^b(j). \]  

Given that the incentive constraint binds\(^6\), a bank’s supply of loans can be written as

\[ Q_t K_{b,t}(j) = \frac{\nu_{b,t}(j) - \lambda_b}{\lambda - \nu_{b,t}(j)} Q_{b,t}^b(j) + \frac{\nu_{n,t}(j)}{\lambda - \nu_{b,t}(j)} N_t(j). \]  

As (27) shows, the supply of loans decreases with an increase in \( \lambda \), which regulates the tightness of the incentive constraint with respect to capital, and increases with an increase in \( \lambda_b \), which makes the holding of bonds more costly in terms of a tighter constraint. Plugging the demand for loans into (23), and combining the result with (24) and (25)

\(^5\)This is in the vein of Meeks et al. (2014), who use the same approach to distinguish between the collateral values of loans and asset backed securities.

\(^6\)The constraint binds in the neighborhood of the steady state. For convenience, we make the assumption that it is binding throughout all experiments.
one can write the terminal value of the banker as a function of its net worth\textsuperscript{7}

\[ V_t(j) = (1 + \mu_t(j)) \nu_{n,t}(j) N_t(j). \]  

(27)

A higher continuation value, \( V_t(j) \) is associated with a higher shadow value of holding an additional marginal unit of assets, or put differently, with a higher shadow value of marginally relaxing the incentive constraint. Defining the stochastic discount factor of the bank to be

\[ \Omega_t(j) \equiv \Lambda_{t-1,t}((1 - \theta) + \theta(1 + \mu_t(j))\nu_{n,t}(j)), \]  

(28)

plugging (28) into the Bellman equation, and using the law of motion for net worth, one can then write the value function as

\[ V_t(j) = E_t[\beta \Omega_{t,t+1}((1 - \theta)N_{t+1}(j) + \theta V_{t+1}(j))] \]

\[ = E_t[\beta \Omega_{t+1}(j)((R_{k,t+1} - v_{d,t} R_t)Q_t K_t(j) + (R_{b,t+1} - v_{d,t} R_t)Q_t^b B_t(j) + v_{d,t} R_t N_{t-1}(j))], \]

and verify the initial guess for the value function as

\[ \nu_{k,t}(j) = \beta E_t[\Omega_{t+1}(j)(R_{k,t+1} - v_{d,t} R_t)], \]  

(29)

\[ \nu_{b,t}(j) = \beta E_t[\Omega_{t+1}(j)(R_{b,t+1} - v_{d,t} R_t)], \]  

(30)

\[ \nu_{n,t}(j) = \beta E_t[\Omega_{t+1}(j)v_{d,t} R_t]. \]  

(31)

To facilitate aggregation of financial variables, we assume that banks share the same structure to the extent that they derive the same respective values from holding loans and bonds, and from raising deposits (i.e., \( \forall j : \nu_{k,t}(j) = \nu_{k,t}, \nu_{b,t}(j) = \nu_{b,t}, \nu_{n,t}(j) = \nu_{n,t} \)). Furthermore, we assume that all banks have the same ratio of capital assets to government bonds, \( \varsigma_t \equiv Q_t K_{b,t} / Q_{b,t} B_{b,t} \), in their portfolio. As an implication, the leverage ratio of banks does not depend on the conditions that are specific to individual institutes, and all banks share the same weighted leverage ratio\textsuperscript{8}

\[ \phi_t \equiv \frac{\nu_{n,t}(1 + \varsigma_t)}{(1 + \frac{\varsigma_t}{\varsigma_t})} = \frac{Q_t K_{b,t} + Q_t^b B_{b,t}}{N_t}. \]  

(32)

Note that the lower divertability of government bonds relative to capital assets, allows the bank to increase its leverage ratio, compared to a scenario in which banks only hold capital assets. The aggregate balance sheet constraint reads

\[ Q_t K_{b,t} + Q_t^b B_{b,t} = D_t + N_t. \]  

(33)

The net worth of the fraction of bankers that survive period \( t - 1 \) and continue operating in the banking sector, \( \theta \), can be written as

\[ N_{ot} = \theta \left[ R_{k,t} Q_{t-1} K_{b,t-1} + R_{b,t} Q_{t-1}^b B_{b,t-1} - v_{d,t} R_{t-1} D_{t-1} \right]. \]  

(34)

A fraction \((1 - \theta)\) of bankers leaves the business. There is a continuum of bankers, and

\textsuperscript{7}Detailed derivations are delegated to the appendix.

\textsuperscript{8}Details are delegated to the appendix.
the draws out of the lottery, which determines whether a banker stays in business or exits the sector, are iid. Hence, by the law of large numbers, it follows that the share of assets that leaves the sector is a fraction \((1 - \theta)\) of the total assets. At the same time, new bankers enter the sector. New bankers are endowed with “start-up funding” by their households. The initial endowment of the new bankers is proportionate to the assets that leave the sector. The net worth of the new bankers, \(N_{n,t}\), can be written as

\[
N_{n,t} = \omega \left[ Q_{t-1} K_{b,t-1} + Q_{t-1}^b B_{b,t-1} \right],
\]

where \(\omega\) is calibrated to ensure that the size of the banking sector is independent of the turnover of bankers. Aggregate net worth, \(N_t\), is then the sum of the net worth of old and new bankers

\[
N_t = N_{ot} + N_{n,t}.
\]

2.4 Policy and market clearing

As policy actors we have the central bank as conductor of interest rate policy and QE, and the government sector in our model.

2.4.1 Monetary Policy

The central bank in this model has various policies at its disposal. The control of the short term nominal interest rate, large-scale purchases of government bond and private assets and liquidity injections to the banking sector.

The short-term nominal interest rates follows a Taylor rule that takes account of the occasionally binding nature of the zero lower bound

\[
R_t = \max \left\{ 1, R_{t-1}^\rho \left( R \left( \frac{\Pi_t}{P_t} \right) \phi_\pi \left( \frac{Y_t}{Y_t} \right) \phi_y \left( \frac{Y_t}{Y_{t-1}} \right) \phi_{dy} \right)^{(1-\rho)} e^{v_{rt}} \right\},
\]

\(\rho\) is the interest smoothing parameter, and \(\phi_\pi\), \(\phi_y\), and \(\phi_{dy}\) are the coefficients, which govern the strength of the response of interest rate policy to deviations of inflation, output and output growth from their target level. \(v_{rt}\) represents a shock to the Taylor rule and follows an AR(1)-process.

Additionally, the central bank We model central bank purchases of capital assets and government bonds as exogenous. The share of central bank capital assets and government bonds of the economy-wide stock of capital and bonds can be written as

\[
X_{K,t} = \frac{K_t - K_{b,t}}{K_t},
\]

\[
X_{B,t} = \frac{B_t - B_{b,t}}{B_t}.
\]

We model these purchases as exogenous and following AR(1) processes. Lastly, the central can lend to banks in the form of emergency liquidity facilities, which are modelled as well as exogenous and following an AR(1) process. We assume that they are lent at a zero interest rate and as in Gertler and Kiyotaki (2010) they relax the incentive constraint of commercial banks. The extended incentive constraint of commercial banks then reads
\[ V_t \geq \lambda Q_t K_{b,t} + \lambda_b Q_t^b B_{b,t} - \lambda_{cbl} CBL_t, \]  

where \( CBL_t \) are central bank liquidity injections, and \( \lambda_{cbl} \) governs the degree to which the liquidity injections relax the constraint. As a consequence central bank liquidity affects the supply of bank loans to firms

\[ Q_t K_{b,t} = \frac{\nu_{b,t} - \lambda_b}{\lambda - \nu_{b,t}} Q_t^b B_{b,t} + \frac{\lambda_{cbl}}{\lambda - \nu_{b,t}} CBL_t + \frac{\nu_{n,t}}{\lambda - \nu_{b,t}} N_t. \]  

Throughout all our exercises we assume that the central bank purchases as sets from all banks equally and provides the same amount liquidity to all banks.

### 2.4.2 Fiscal policy and market clearing

Government spending, \( G_t \), is exogenous and follows an AR(1) process

\[ G_t = G_0 g_t, \quad \text{and} \quad g_t = \rho g_{t-1} + \epsilon^g_t, \]

where \( G_0 \) is the steady state government consumption, \( \rho \) is the autocorrelation of government consumption, and \( \epsilon^g_t \) is a shock to government spending. The government finances its expenditures, by issuing government bonds, which are bought by banks and the central bank, and by raising lump sum taxes, \( T_t \). Taxes follow a simple feedback rule, such that they are sensitive to the level of debt

\[ T_t = T + \kappa_b (B_{t-1} - B), \]

where \( T \) and \( B \) are the steady state levels of tax revenue and government debt, respectively. \( \kappa_b \) is set to ensure that the real value of debt grows a rate smaller than the gross real rate on government debt. As shown by Bohn (1998), this rule is a sufficient condition to guarantee the solvency of the government. To allow for the calibration of a realistic average maturity of government debt, bonds are modeled as consols with geometrically decaying coupon payments, as in Woodford (1998) and Woodford (2001). A bond issued in period \( t \) at the price of \( Q_t^b \), pays out a coupon of \( r_c \) in period \( t + 1 \), a coupon of \( \rho_c r_c \) in period \( t + 2 \), a coupon of \( \rho^2_c r_c \) in \( t + 2 \), and so on. Setting the decay factor \( \rho_c \) equal to zero captures the case of a one-period bond in which the entire payoff of the bond is due in period \( t + 1 \). Setting \( \rho_c = 1 \) delivers the case of a perpetual bond. The average maturity of a bond of this type is \( 1/(1 - \beta \rho_c) \). For investors, this payoff structure is equivalent to receiving the coupon \( r_c \) and a fraction, \( \rho_c \), of a similarly structured bond in period \( t + 1 \). The beginning-of-period debt of the government can thus be summarized as \( (r_c + \rho_c Q_t^b)B_{t-1} \).

\[ R_{b,t} = \frac{r_c + \rho_c Q_t^b}{Q_t^b} \]

The flow budget constraint of the government reads

\[ G_t + R_{b,t} Q_{t-1}^b B_{t-1} = Q_t^b B_t + T_t \]
Finally, the aggregate resource constraint in real terms reads
\[ Y_t = C_t + I_t + G_t + a(U_t)K_{t-1}, \]
where the last term on the right hand side of the equation marks the resource costs of adjusting the utilization of installed capital.

2.5 Linearized Equilibrium conditions

This subsection briefly presents the linearized equilibrium conditions. A detailed derivation of the linearized equations is discussed in the appendix to Smets and Wouters (2007). Small letters denote the log-deviation of the corresponding variable from its steady state value.

2.5.1 Non-financial part of the economy

Equation (44) is the aggregated Euler equation for consumption. The presence of habit formation justifies the presence of lagged consumption in the equation.

\[
\dot{c}_t = \frac{h/\gamma}{(1 + h/\gamma)} c_{t-1} + \frac{1}{1 + h/\gamma} E_t[c_{t+1}] - \frac{(1 - h/\gamma)}{(1 + h/\gamma)\sigma_c} (r_t - E_t[\pi_{t+1}] + \nu_{d,t}) \tag{44}
\]

Equation (44) is the aggregated Euler equation for consumption. The presence of habit formation justifies the presence of lagged consumption in the equation.

\[
i_t = \frac{1}{1 + \beta} [\ddot{c}_{t-1}] + \frac{\beta}{1 + \beta} E_t[i_{t+1}] + \frac{1}{(1 + \beta)\gamma^2 S''} q_t^k \tag{45}
\]

where \( \beta = \beta \gamma^{1-\sigma_c} \) Equation (45) is the linearized first order condition for investment. The dynamics of investment are governed by Tobin’s q. \( S'' \) is the steady state value of the second derivative of the investment adjustment cost function. The accumulation equation of physical capital reads

\[
\overline{k}_t = (1 - \delta)/\gamma \overline{k}_{t-1} + (1 - (1 - \delta)/\gamma) \dot{i}_t + (1 - (1 - \delta)/\gamma)(1 + \beta)\gamma^2 S'' \nu_{i,t}. \tag{46}
\]

The marginal cost of the firms and the marginal product of capital are given by (47) and (48), while (49) is the equation for the return on capital.

\[
mc_t = w_t - z_t + \alpha(l_t - k_t) \tag{47}
\]

\[
mpk_t = w_t - k_t + l_t \tag{48}
\]

\[
R_k r_{k,t} = MC \ast MPK(mc_t + y_t - k_{t-1}) + (1 - \delta)q_t - R_k q_{t-1}. \tag{49}
\]

The relation between physical capital and effective capital is given by (50). Here, parameter \( \psi \) is the elasticity of the capital utilization adjustment cost function and normalized to be between zero and one.

\[
k_t = \frac{1 - \psi}{\psi} r_t^k + \overline{k}_{t-1} \tag{50}
\]
(51) is the aggregate production function, and (52) is the aggregate resource constraint.

\[ y_t = \Phi(\alpha k_t + (1 - \alpha)l_t + z_t) \]  

(51)

\[ y_t = G \frac{Y_t}{g_t} + C \frac{e_t}{\psi} + I \frac{\psi}{\psi} - K \frac{1 - \psi}{\psi} r_t \]  

(52)

\[ \pi_t = \frac{\beta}{1 + \beta} E_t \pi_{t+1} + \frac{1}{1 + \beta} \pi_{t-1} + \frac{(1 - \psi)(1 - \lambda)}{(1 + \beta)(1 + \psi)} (w_t - z_t + \alpha l_t - \alpha k_t) \]  

(53)

Equation (53) is the New Keynesian Phillips curve. The last term in parenthesis corresponds to the marginal cost of production. As we employ the Kimball aggregator, the sensitivity of inflation to fluctuations in marginal cost is affected by the market power of firms, represented by the steady state price markup, \((\Phi - 1)\).\(^9\) Furthermore, the curvature of the Kimball aggregator, \(\epsilon_p\), affects the adjustment of prices to marginal cost, since the higher \(\epsilon_p\), the higher is the degree of strategic complementarity in price setting, and dampens the price adjustment to shocks.

\[ w_t = \frac{1}{1 + \beta} (w_{t-1} + w_t \pi_{t-1}) + \frac{\beta}{1 + \beta} \frac{E_t [w_{t+1} + \pi_{t+1} - 1 + w_t \beta \gamma \pi_t]}{(1 - \psi)(1 + \psi)} (w_t^h - w_t) \]  

(54)

Equation (54) is the Wage Phillips curve. \(w_t^h\) is the wage that would prevail in the absence of market power by unions. Therefore, \((w_t - w_t^h)\) is the wage markup. Analogous to equation (53), the terms \(\lambda_w\) and \(\epsilon_w\) represent the steady state wage markup and the curvature of the Kimball aggregator for labor services. The efficient wage in (55), equals the average marginal rate of substitution between consumption and leisure of all households.

\[ w_t^h = \frac{\sigma_c}{(1 - h)} (c_t - h c_{t-1}) + \frac{L}{1 - L} l_t \]  

(55)

The relationship between the real and the nominal interest rate is given by the Fisher equation.

\[ r_r = r_t - E_t [\pi_{t+1}] \].  

(56)

\(^9\)Note, that in equilibrium, the fixed cost parameter is related to the steady state price markup by a zero profit condition.
2.5.2 The financial sector

The financial sector features the linearized law of motion for net worth

\[ n_t N = \frac{\theta}{\gamma} [R_k K_r t + (R_k - R) K(q_t - 1 + k_{t-1}) + R_b Q^b B_r b t + (R_b - R) Q^b B(q^b_{t-1} + b_{t-1})] \]

\[ + \frac{\theta}{\gamma} (R N n_{t-1} - R D (r r_{t-1} + v d_{t-1})) + n_{n,t} N_n \]

(57)

and the linearized equation for the net worth that bankers are endowed with that newly enter the business.

\[ n_{n,t} N_n = \frac{\omega}{\gamma} [K(q_t - 1 + k_{t-1}) + B(q^b_{t-1} + b_{t-1})] \]

(58)

The linearized first order conditions for the banking sector yield the shadow value of holding capital assets as a function of the spread between the loan rate and the rate on deposits, (59) and the shadow value of holding bonds as function of the spread between the bond return and the deposit rate, (60). The shadow value of another unit of net worth depends on the rate on deposits (61). Equation (62) shows that in optimum these shadow values are in balance.

\[ \nu_{k,t} \nu_k = \frac{\Omega}{\gamma} [(R_k - R) \tilde{\Omega}_{t+1} + r_{k,t+1} R_k - (r r_t + v d_t) R], \] \(\text{for} \) (59)

\[ \nu_{b,t} \nu_b = \frac{\Omega}{\gamma} [(R_b - R) \tilde{\Omega}_{t+1} + r_{b,t+1} R_b - (r r_t + v d_t) R], \] \(\text{for} \) (60)

\[ \nu_{n,t} = \tilde{\Omega}_{t+1} + r r_t + v d_t, \]

(61)

\[ \nu_{k,t} = \nu_{b,t}, \]

(62)

The linearized stochastic discount factor of the banker is

\[ \tilde{\Omega}_t \Omega = \theta \nu_n (1 + \mu) (\tilde{\Omega}_{t+1} + r r_t + v d_t + \nu_{k,t} \nu_k) \frac{\nu_k}{\lambda_k - \nu_k} - (r r_{t-1} + v d_{t-1})\Omega. \]

(63)

Furthermore, the loan supply of the financial intermediaries to non-financial firms reads

\[ (q + k_{b,t} - \nu_{k,t}) K = \frac{(\nu_b - \lambda_b)(q^b_{t-1} + b_{t-1}) + \nu_{b,t} \nu_b}{\lambda_b - \nu_b} Q^b B + \frac{\nu_n N \nu_n N}{\lambda_k - \nu_k} + \frac{\lambda_{CB L}}{\lambda_k - \nu_k} c_{b t} \]

(64)

2.5.3 Policy and exogenous processes

The fiscal sector can be summarized by the linearized budget constraint, which already entails the tax rule (65), and the linearized return on long-term bonds, (66).

\[ \frac{R_b Q^b B}{\gamma} (r_{b,t} + q^b_{t-1} + b t - 1) * G g_t - \kappa b b_{t-1} B = Q^b B(q^b_{t} + b_t), \] \(\text{for} \) (65)

\[ R_b (r_{b} + q^b_{t-1}) = \rho c q^b_t. \]

(66)
Equation (67) is the linearized Taylor rule in terms of the net interest rate set by the central bank.

\[ r_t = \max \{0, \rho r_{t-1} + (1 - \rho)(\phi_x \pi_t + \phi_y \tilde{y}_t + \phi_{dy}(\tilde{y}_t - \tilde{y}_{t-1})) + v_{rt}\}, \tag{67} \]

where \( \tilde{y}_t \) is the output gap. When the economy is away from the zero lower bound, the stochastic process \( e_r \) represents a regular interest rate shock. When the nominal interest rate is at zero, \( v_{rt} \) serves as a forward guidance shock as is explained in section 4. The asset purchases by the central bank can be written as

\[ x_{k,t} = k_t - k_{b,t}, \]
\[ x_{b,t} = b_t - b_{b,t}. \]

Finally, the stochastic drivers of our model are the following ten processes:

\[ v_{d,t} = \rho_d v_{d,t-1} + \epsilon^d_t, \tag{68} \]
\[ z_t = \rho_z z_{t-1} + \epsilon^z_t, \tag{69} \]
\[ g_t = \rho_g g_{t-1} + \epsilon^g_t, \tag{70} \]
\[ v_{r,t} = \rho_v v_{r,t-1} + \epsilon^v_t, \tag{71} \]
\[ v_{i,t} = \rho_i v_{i,t-1} + \epsilon^i_t, \tag{72} \]
\[ v_{p,t} = \rho_p v_{p,t-1} + \epsilon^p_t, \tag{73} \]
\[ v_{w,t} = \rho_w v_{w,t-1} + \epsilon^w_t, \tag{74} \]
\[ x_{k,t} = \rho_x k_{k,t-1} + \epsilon^k_t, \tag{75} \]
\[ x_{b,t} = \rho_x b_{b,t-1} + \epsilon^b_t, \tag{76} \]
\[ x_{CBL,t} = \rho_{CBL} x_{CBL,t-1} + \epsilon^{CBL}_t. \tag{77} \]

where the last three processes are the unconventional policy tools employed by the central bank. It holds that \( \epsilon^k_t \overset{iid}{\sim} N(0, \sigma^2_k) \) for all \( k = \{d, z, g, r, i, p, w, k, b, CBL\} \).

3 Estimation and Methodology

This section first gives a very brief idea of the solution, filtering and estimation methodology. We then describe the data and finally present and discuss our parameter estimates.

3.1 Solution, filtering and estimation technique

We make use of the methodology presented in Boehl (2019) which is implemented in the pydsge package written in Python.\(^{10}\) In a nutshell, the paper cited above shows how

\(^{10}\)Python can provide speed benchmarks that are en-par with compiled languages such as Fortran while comprising the advantages of a high-level programming language. We want to promote free and open software and advocate the avoidance of proprietary languages. This in particular counts for Matlab. Open source alternatives already provide by far more efficient and more flexible environments. With the sole reason for sticking within the proprietary sphere are log-in costs in terms of expertise and specialized software libraries. If institutions would donate funds payed for licenses instead to open-source projects, theses specialized libraries could be reimplemented within very short time.
to first cast the (linear) model in the following form

$$N \left[ \begin{array}{c} x_t \\ v_{t-1} \end{array} \right] + c \max \left\{ b \left[ \begin{array}{c} x_t \\ v_{t-1} \end{array} \right], \bar{r} \right\} = E_t \left[ \begin{array}{c} x_{t+1} \\ v_t \end{array} \right], \quad (78)$$

where $v_t$ contains all the (latent) state variables and $x_t$ all forward looking variables (variables can be part of both). This way of writing the system has the advantage that the rational expectations solution of the state $v_t$ depending on $v_{t-1}$ of the system in period $t+s$ assuming $k$ periods at the ZLB and a transition of $l$ periods towards it can be expressed in closed form as

$$L_s(l, k; v_{t-1}) = N^{\max\{s-l,0\}} (N + cb)^{\min\{l,s\}} S(l, k, v_{t-1}) \quad (79)$$

$$+ (I - N)^{-1}(I - N^{\max\{s-l,0\}})c\bar{r}, \quad (80)$$

where

$$S(l, k, v_{t-1}) = \left\{ \left[ \begin{array}{c} x_t \\ v_{t-1} \end{array} \right] : Q N^k (N + cb)^l \left[ \begin{array}{c} x_t \\ v_{t-1} \end{array} \right] = -Q(I - N)^{-1}(I - N^k)c\bar{r} \right\} \quad (81)$$

and $Q = [I - \Omega]$ for $x_t = \Omega v_{t-1}$, which represents the linear rational expectations solution of the unconstrained system as given by (Blanchard and Kahn, 1980). This comes in handy for computational reasons since it can be avoided to simulate the whole path to check if $(l,k)$ is a rational expectations equilibrium. The detailed equilibrium conditions and how to find such equilibrium are given in the paper. The resulting transition function

$$v_t = f_L(v_{t-1}) \quad (82)$$

is linear for the region where the ZLB does not bind and (increasingly) nonlinear when it binds. The durations of how long the ZLB binds are determined endogenously and hence this corresponds to the concept of rational expectations where agents assume that there will be no further shocks in the future. This assumption only results in an accurate system if shocks are moderate since it ignores the Jensen inequality. We argue that the approximation error is limited, especially compared to observation errors and errors related to model misspecification.\textsuperscript{11} This approximation is further necessary to solve a large scale model accurately within reasonable computation time. For the model presented here, the implementation in \texttt{pydsge} is able to find the solution of about 80,000 particles per second (given one core).

Bayesian filtering – i.e. the extraction of the most likely distribution of states in each period given the series of observables – is done using the IPAS filter that is also presented in Boehl (2019). IPAS stands for \textit{iterative path adjusted Transpose Ensemble RTS-Smoother} and is based on particle filter methodology. In conjunction with the assumption that the states at each point in time are approximately Gaussian distributed,

\textsuperscript{11}It is hard to test how much this approximation error is since an accurate global solution of a model with large state is extremely difficult. To get an idea one could use the simple New Keynesian model (insert citation) and compare the global solution with the perfect foresight solution. It turns out that the approximation error can be large, but only for deviations of GDP that are one magnitude higher than those observed during the great recession.
the concept of statistical linearization can be used to efficiently filter large-scale nonlinear systems with only a few hundred particles within milliseconds. The method of iterative path adjustment then guarantees that the nonlinearity of the transition function is fully respected. IPAS is implemented in the econsieve package.

For the Bayesian estimation the Affine Invariant MCMC Ensemble sampler is used as proposed by Goodman and Weare (2010). This relatively new method has the advantage that the sampling can be efficiently parallelized: instead of having a small number of chains that are each dependent on its own state as in the Metropolis algorithm, the ensemble sampler uses a large number of chains that communicate after each iteration and exchange states and regions of the parameter space with high likelihood. The method has been extensively applied in particular in the field of astrophysics. Prior to the MCMC sampler, the posterior mode maximization is done using the simplex method in Nelder and Mead (1965).

For the estimation – and later for the filtering of shock series – we use time series of 10 observables. Our observables are real GDP growth, real consumption growth, investment growth, labor hours, the log change of the GDP deflator, real wage growth, the federal funds rate, the central bank holdings of government bonds, its holdings of mortgage back securities, and aggregated liquidity facilities by the central banks. The details of the construction of these variables as well as data sources are provided in Appendix A. We employ a sample from 1998Q1 to 2018Q2 in our quantitative analysis. This is in line with

Figure 1: Observables with IPAS-filtered Series. 200 draws from posterior distribution, the same shock series are used for the counterfactual analyses.
the sample chosen in Boehl and Strobel (2019) but shorter than in Gust et al. (2017), Linde et al. (2017) or Fratto and Uhlig (2014). The reason for that is that we want the estimation to capture idiosyncrasies of the episode where QE was effective, in particular the slope of the Phillips Curve and the persistence of endogenous and exogenous variables. Using a longer sample hence would have borne the risk of misspecification. On the other hand, using a shorter sample yields less data points for the estimation procedure and might let priors dominate the posterior distribution for parameters that are not well identified. We opt for the shorter sample but will keep the last critique point in the back
of our minds.

We assume an observation error of \( \sqrt{1} \) of each time series standard deviation apart from the federal funds rate, where we only \( \sqrt{0.01} \) of the standard deviation take as measurement error. We opt to make use of observation errors not only because we think they are a realistic feature that enriches the model, but also because this allows more leeway for the smoother to find more probable shocks. The smoothed observables are depicted in Figure 1 and 2. The orange line depicts the data series while the blue line depicts the filtered and smoothed series. As the figure shows our IPAS-filtered series matches the data series of the observables very well although our choice for the observation error is quite broad.

3.2 Parameter estimates

We fix several parameters in our estimation. This in particular concerns the mean values of most of our observables. For the observables given in growth rates, these means govern the slope of the steady state compared to the model variables. Assuming a high trend of output implies a relatively lower trend of the percentage steady state deviation during the great repossession. Likewise, assuming a very low trend of output will imply that output recovered quickly after the financial crises. It is clear that from the probabilistic perspective of the model, a recession is far more likely if it is short than if it is long. Hence, an estimation of this trend parameter will always prefer a very low trend. But this would not be in line with common the common understanding that the recession was both very strong, quite long, and followed by a period of little to no growth. In order to account for that feature, we can not leave this decision to the estimation but must calibrate these parameters. We however rechecked robustness of our results to other values of the observable means and – surprisingly – found almost no effects.

The target rate of inflation is set to 0.5, which corresponds to an inflation target of about 2% annually. The mean of the observed labor series and of the common growth trend are set to their average values of the pre-crisis decade, i.e. 6.5415 and 0.344, respectively. Prior to the crisis central bank holdings of government bonds relative to GDP did not move much. Here we set the mean of this observable to 5.65 percent of GDP. Holdings of Private Assets and the Liquidity Measures are near zero before the crisis, which motivates our choice for the means of these observables.

Additionally to the means of observables, we fix the steady state markup in the labor market, \( \lambda_w \) to 1.1 and the curvature parameters of the Kimball aggregator for both, labor and goods markets, \( \epsilon \), to 10. As the value of psi does not seem to matter much for our results, we fix it at 0.79. Lastly, from pre-studies we observe that the values of \( \alpha \) and \( h \) are close to invariant across the estimations that we have conducted. Thereby we set \( \alpha = 0.19 \) and \( h = 0.72 \), which corresponds on the one hand to our results and on the other hand are close to conventional values for these parameters.

Table 1 shows our choice of priors and our posterior estimates. Except for the uniform and the inverse gamma distribution, the parameters of the priors correspond to the mean and standard deviation of the distributions. The priors for the parameter that pertain to the real economy are chosen in line with Smets and Wouters (2007). In addition, we estimate the steady state leverage of financial intermediaries, \( LEV \), their survival rate \( \theta \), the sensitivity of the incentive constraint to liquidity injections \( \lambda_{CBL} \), the feedback coefficient for government debt in the tax rule, \( \kappa_b \) and the shock processes for the QE measures. We set the mean of the priors for \( LEV = 3 \) and \( \theta = 0.95 \). The latter
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Table 1: Estimation results

corresponds to an expected time horizon of the bankers of 10 years. The mean for κₜ is set to 0.1. For λ_{cbl} we set a wide uniform prior. For the persistence parameters of the shock processes of the policy measures, we apply the same priors as for the other AR(1) processes. Lastly, we choose the priors for the standard deviations of the QE shocks...
such as to allow for movements in the policy measures on a magnitude such as we see
them in the respective observables. The reader can find further technical details on the
estimation procedure in Appendix B.

Our estimates are largely in line with common findings in the literature. $tpr_{beta}$ is the
time preference rate and in its posterior mean close to the estimate by Smets and Wouters
(2007). We estimate $\sigma_c$ close to log utility. The mean of the investment adjustment cost
parameter, $phiss$ is found to be rather high in our sample. It is noteworthy that the
price Phillips Curve is estimated to be very flat with an estimated Calvo parameter $\zeta = 0.805$, a price indexation parameter of $i_p$ and a fixed cost parameter, $\Phi_p = 1.305$
that in our model matches the steady state markup of firms, and which enters the Phillips
curve due to the use of the Kimball aggregator. The values for the wage Phillips curve
parameters are roughly in line with the estimates by Smets and Wouters (2007), but
imply a steeper wage Phillips Curve than estimated in Linde et al. (2017) who estimate
a Smets and Wouters type model on the US with a binding zero lower bound. The
values are also higher than the corresponding estimates in Boehl and Strobel (2019),
where we use virtually the same methodology. The estimated feedback coefficients as
well as the interest rate smoothing parameter, $\rho$, in the Taylor rule also are in the range
of conventionally found values.

Our estimations show that the data favors small values for our parameters pertaining
to the financial sector, $LEV$ and $\theta$. It can be suspected that this is the fact because
these value govern the sensitivity of the economy with respect to the QE measures,
whereas the model seems to prefer smaller effects. In particular, the posterior mean for
the steady state leverage, $LEV = 1.802$ is below our expectations and our prior mean.
This value is far smaller than the leverage ratio of banks but rather conform to what
Gertler and Karadi (2011) reason to be a plausible value for non-financial corporations.
The posterior mean of $\theta$ implies an average time-horizon of ten quarters for the banker.
For the sensitivity of the incentive constraint to liquidity injections, $\lambda_{CBL} = 2.694$, the
posterior mean is at the lower end of our prior. The estimated persistence parameters
of the shock processes imply that technology shocks and government spending shocks
are less persistent than the often-found values close to a unit root (see, e.g. Smets and
Wouters, 2007, Linde et al., 2017). On the other hand, our value for the persistence
of the monetary shock, $\rho_r$, is relatively high. We find that this difference is due to the
estimation on a shorter sample. For an estimation of the model on a sample starting
in 1966, we again find a relatively small value for $\rho_r$. At the zero lower bound, higher
values of $\rho_t$ imply a stronger ability of the central bank to change the expected path of
future interest rate, and hence stronger effects of forward guidance. The persistence
of the QE shocks is rather high for the purchases and rather low for the liquidity injections,
which conforms to a first eyeballing of the observed series.

4 Empirical Analysis

We start this section with a general decomposition of the dynamics during the sample
period. We then zoom in specifically on the different measures of quantitative easing.
Lastly, we compare the magnitude of the effects of QE with those of interest rate and
projection policies. The methodology for each of these exercises is similar: we start by
drawing 200 parameter sets from the posterior distribution (more precisely: from the
converged Markov chains). For each of these draws we use the IPA smoother to filter
out the most probable series of shock innovations given the distribution of observables and the initial distribution of states. See Figures 1 and 2 for a comparison of time series of observables simulated using IPAS-filtered states and the actual observables. For the decompositions and counterfactuals below we then switch one or several of these series or parts thereof and show the median over these draws. When possible in terms of clear display we also add 68% intervals.

4.1 A decomposition of the dynamics

We begin this section by discussing the decomposition of the business cycle dynamics in the Great Recession and its aftermath. We identify variations in the risk premium to be the dominant driver of most of the observables. As the shock series depicted in Figure 3 to 5 convey, the sharp drop in output and its components, and well as in labor hours, inflation and wages was triggered by a sharp increase in the risk premium on safe assets. Variations in the risk premium dominate the other shocks in explaining the macroeconomic dynamics in the years following the financial crisis. This finding coincides with results by Boehl and Strobel (2019) and Gust et al. (2017). This increase in the risk premium forces the nominal interest rate endogenously to the zero lower bound. While it sharply spiked at the end of 2008, coinciding with the collapse of Lehman Brothers, the risk premium stays persistently high until the end of the sample. It is easy to see that without this driver, the Great Recession would at best have been mild and the nominal rate would not have reached the zero lower bound.

As one can see, the effects of the different monetary policy shocks (interest rate shocks - pink, aggregated QE shocks - grey) had an effect on the real economy, but it was not very large compared to the effects of non-policy shocks and, in particular, the risk premium shock. We analyse the consequences of the convetional and unconventional monetary policy shocks in more detail in the next subsections. Next to the risk-premium shock, also of relevance for output was the investment specific technology shock, shown in yellow, which caused additional pressure on output, investment and labor hours during the drop in 2009. For the inflation dynamics, only the wage markup and the price markup shocks had notable impact next to the exogenous part of the risk premium. The wage markup shock also is an important driver of wages in the post-crisis time. The government spending process is mostly negative throughout the sample, with the exception of the years around 2010, which captures the American Recovery and Reinvestment Act (ARRA), which is well-captured by our smoothened series. The effect of government spending at the zero lower bound is very small for output, however the fiscal contraction crowds in investment at the end of the zero lower bound period. Around the onset of the ZLB and the ARRA program, it also manages to crowd in consumption. The measured technology shocks are quite small and hardly matter for business cycle dynamics. We next turn to the effects of the QE measures on the US economy.

4.2 The Effects of the QE Measures

In this section we focus on the contribution of the QE measures to the dynamics of the US economy and discuss the diverse effects of the different single policy measures in isolation. As was shown in the foregoing section, the contribution of QE to business cycle dynamics was rather small compared to the other drivers. However, viewing the effects of QE in isolation reveals that they were of relevance, and that the different tools affected the real economy differently.
Figure 3: Decomposition of the time series. Note: medians over 200 simulations using IPAS-filtered shock innovations, each for one draw from the posterior distribution of the parameters.
Figure 4: Decomposition of the time series. Note: medians over 200 simulations using IPAS-filtered shock innovations, each for one draw from the posterior distribution of the parameters.
Figures 6 and 7 show the total effect of unconventional monetary policy measures decomposed in the effects of the each tools separately. Green marks the effect of MBS purchases, which in our model are equivalent to purchases of private assets, yellow are the effects of government bond purchases, and blue are the effects of the liquidity injections. Liquidity injections took place at the onset of the crisis. As one can see, they had significantly increased the price of capital assets, \( q \), as well as of bonds, \( q^b \), and lowered the excess return on assets (\( prem \) in the lower right panel of the figure is the excess return on capital). The panels for \( \nu_k \) and \( \nu_b \) show that the decrease in the excess returns on the banks assets decreased their respective shadow values for the bank. Additionally, one can see that the relaxation of the financial constraint of the banking sector that is triggered by liquidity injections is associated with an economy-wide portfolio shift from bond holdings to capital, triggering an increase in investment that peaked at 1%. The increase of investment in turn caused an increase in the demand for labor associated with an increase of labor hours and wages, the latter corresponding to the increase in the marginal product of labor due to an increased capital stock. This resulted in a small increase in consumption. The overall increase of output peaked at 0.25%. Given that the peak injections reached 8% of GDP, the peak multiplier for liquidity injections is only at roughly 0.031. Additionally the emergency liquidity facilities were relatively short-lived, and had themselves no strong long-lasting effects on the real economy. This contrasts with the findings by Del Negro et al. (2017) who find large effects by liquidity injections.
Figure 6: Decomposition of the contribution of QE measures into each of the measures. Note: medians over 200 simulations using IPAS-filtered shock innovations, each for one draw from the posterior distribution of the parameters.
Figure 7: Decomposition of the contribution of QE measures into each of the measures. Note: medians over 200 simulations using IPAS-filtered shock innovations, each for one draw from the posterior distribution of the parameters.
One of the potential reasons for this difference is that the persistence of liquidity injections in their model is tied to the very high persistence of the liquidity shock.

The largest effects on the non-financial sector were caused by the central bank purchases of capital assets. The Fed started its purchase program of private securities in the last quarter of 2008, raising its holdings up to roughly 9.7% of GDP in 2015, when it also slowly started to reduce its position. The securities purchases consisted largely of mortgage backed securities (MBS), but included the smaller and more short-lived program of the commercial paper funding facility (CPFF) as well. The effects of private security purchases were in many ways qualitatively similar to the effects of liquidity injections. Figure 8 depicts the theoretical impulse response of selected variables given the estimates for the parameters to an increase in capital purchases by 11% of GDP. The purchases lowered the interest rate spread for capital assets and raised the price of capital, q. By arbitrage they had the same effect on the prices and spreads of bonds as well, just to a lesser extent\textsuperscript{12}. After the initial capital gains, the reduced excess premium on holding assets, reduced the shadow value of banks for holding assets on their balance sheet. Likewise, the effect on the net worth of banks was ambiguous. Initially, the increase in asset prices raised net worth, but after a while, the reduction in the accumulation of new net worth due to the decrease in the interest rate spreads prevails. Additionally, an increase in the central bank holdings of capital is associated with a decline in the holdings of capital assets by financial intermediaries, which further reduced their profits. As in the theoretical impulse responses, the overall effect of capital purchase on the real economy in the US were largely positive. They boosted aggregate investment up to 3.5% and had a lasting positive impact on the accumulation of productive capital. As a consequence, we see in Figure 6 that capital purchases contributed the lion’s share to the stimulus of output, which was raised by up to 0.75%. Again, as with liquidity injections, the capital purchases caused an increase in the demand for labor, increasing labor hours and real wages simultaneously. Additionally, they had the effect of increasing inflation. However, this effect was very small. Nonetheless, it signals that despite increasing the productive capacity, the overall effect of this form of QE acted like a demand shock to the US economy. Furthermore, the increase in inflation reduced the real rate on deposits ($r\tau$ in the figure) at the ZLB, supporting the increase in households’ consumption expenditures. While purchases on capital assets supported the economy for a prolonged time, these effects died out and turned negative at the end of the sample. The reason is that the slow reduction of private security holdings by the Fed is associated with an end of the stimulating effects, while the persistently lower spreads on assets held by the banking sector weaken the sector’s balance sheet and reduce the supply of loans.

Lastly, the effects of government bond purchases played out differently. The Fed reduced its bond holdings during the crisis when it shifted its balance sheet towards liquidity facilities. In 2010 it embarked in large-scale purchases of long-term government bonds, which it later partially neutralized by selling of short-term bonds, the ‘Operation Twist’. Overall the bond holdings of the Fed reached close to 14% of GDP.\textsuperscript{13} On the financial variables, bond purchases had effects that were similar to the ones of capital

\textsuperscript{12}The comovement of asset prices and spreads in a model with financial frictions is a known finding. We therefore abstain from showing the second set of assets and spreads due to space constraints.

\textsuperscript{13}We anchor our mean of the observable of government bonds at 5.65% of GDP. The plotted series...
Figure 8: Impulse response functions wrt. a shock of MBS purchases. Confidence intervals are over 200 draws from the parameter posterior.

purchases. However, the variables’ reaction to bond purchases is smaller. As can be seen in the theoretical impulse responses in Figure 9, the response of asset prices is smaller and dies out more quickly. The reaction of net worth is dominated by the reduction of bank’s profits that stems from the persistent decline in spreads as well as from a reduction of their holdings of bonds, which mirrors the expansion of bond holdings on the Fed’s

in Figure 7 shows what we account for as the exogenous increases, decreases of bond holdings from this level.
Figure 9: Impulse response functions wrt. a shock of government bond purchases. Confidence intervals are over 200 draws from the parameter posterior.

balance sheet. The lower estimated persistence of the bond-related QE shock compared to the capital-related QE shock ($\rho_{x,b} = 0.863$ vs. $\rho_{x,k} = 0.921$) contributes to the quicker dying out of the support of asset prices. The more detrimental effects on the financial sector are associated with negative consequences for the loan supply and crowding-out of investment. Therefore the positive contribution of bond purchases to investment and output quickly disappear after the Fed starts expanding its position. As a consequence, investment, output and labor were raised in the years until 2013 and thereafter reduced by bond purchases. Quantitatively, the reduction was more pronounced. On wages and
inflation, the initial positive push persisted until the end of the sample.

Overall, the stimulating effects of asset purchases of both kinds are followed by a hangover period. The implications for a future in which the Fed unwinds its position of asset holdings and shrinks its balance sheet are depicted in Figures 10 and 11. Here we show the overall impact of all three QE tools. For the periods after 2018Q2 it becomes a projection. Therefore the bands around the median until this quarter mark the credible set associated with our estimation, whereas they become forecast errors after 2018Q2. Our forecasts clearly show that the negative long-term consequences of QE have the potential to be of the same size as the stimulus effects in the short-run. In particular, investment is projected to fall by more than the peak increase due to QE. Additionally the slump in investment is more persistent than the stimulus caused by QE. In this scenario, the trough for banks’ net worth is projected to be reached in 2019, when the excess spreads on capital assets return to their initial level. The trough for aggregate output, investment, and labor hours follows briefly thereafter. In summary, the detrimental effects of QE that are projected match or exceed the positive effects in their size. Exceptions are consumption and inflation. However, for both variables the responses are rather weak. Our result of a contractionary effect of QE in the long-run is in line with arguments made by Brunnermeier and Koby (2018) that there exists a reversal rate at which accommodative monetary policy reverses its effect and becomes contractionary. In their setting the reversal becomes more likely, the longer QE lasts. As in our model, this is due to the mismatch of the rather short-lived capital gains and the persistent decline of excess spreads.

Nonetheless, as a note of caution, this result of a looming hangover effect should be taken with a grain of salt. The way in which the financial sector is modelled is crucial for the effects of QE, which reach the real economy via the supply of loans. In the model structure by Gertler and Kiyotaki (2010), Gertler and Karadi (2011) and Gertler and Karadi (2013) that we adopt, the possibility of a hangover is hardwired into the model as the decrease in the profitability of banks is more persistent than the increase in asset prices. Given our parameter estimates, the hangover is quite pronounced. Possibly, this result will not survive in an analysis in which QE affects the financial sector through other channels. For instance, the model does not incorporate the possibility of defaults by non-financial agents on their obligations to banks. In this example, lower interest rates on loans would help to prevent defaults and thereby stabilize the banking sector. At the current step of our analysis, we consider this to be a preliminary result.

4.3 Comparing the effects of QE and interest rate policy

Figures 12 to 13 show the effects of interest rate policy in yellow and the combined QE measures in blue. The joint effect of all monetary policy tools on the real economy is non negligible in its size. The Fed Funds Rate hit the zero lower bound in the last quarter of 2008. Interest rate shocks were mostly expansionary before, but also during the ZLB period, and turned contractionary at the end of the sample. At the onset of the crisis, when reaching the zero lower bound, the Fed decreased its interest rate quicker than implied by the Taylor rule. This can also be interpreted by the Fed following the Taylor rule but not accounting for interest rate smoothing.

During the ZLB period, these shocks capture changes in the expectations of interest rate policy. Hence, we interpret these shocks as triggering expectations by the forward guidance announcement of the Federal Open Market Committee, which started a policy
Figure 10: Forecast of net effects of total QE measures (hangover diagram). Note: medians over the difference of 200 simulations with and without QE-shocks using IPAS-filtered shock innovations, each for one draw from the posterior distribution of the parameters.
of announcing the future path of federal fund rate as expected by the Fed. Throughout
the ZLB period, forward guidance shocks also were slightly expansionary, though at a
considerably lower level. This suggests a rather dovish stance of the Fed on the implied
duration of the zero lower bound. The graphs reveal that while the traditional interest
rate shocks had an effect prior to the ZLB period, the forward guidance shocks at the
zero lower bound hardly had an impact on the real economy. However, the effects of
expansionary interest rate shocks prior to the crisis persisted for a long time for several
variables. For instance, whereas the increase of investment by interest rate shocks died
out at the onset of the ZLB, the aftereffects on the capital stock decreased only slowly
over time. As a consequence, effects on consumption and wages lasted for a long time.
At the time of the exit, the interest rate shocks are restrictive signalling that rates were
raised quicker than expected after the exit from the ZLB. This is perceived as a strong
expansionary shock. This role of interest rate policy for the US economy is broadly in
line with the results of Boehl and Strobel (2019), in which we discuss the role of forward
guidance in more detail.

Let us finally turn to a direct comparison of the overall effects of QE with the effects
of interest rate policy by the Fed. During the ZLB, QE proved to be more consequential
for the dynamics of the real economy as well as for financial variables. The stimulus for
output and consumption provided by QE at the ZLB is only slightly smaller than the
Figure 12: Decomposition of dynamics (cumulative QE vs interest rate policy)
magnitude of the effects of interest rate shocks in the pre-crisis decade. Starting with the end of the ZLB, interest policy resumes to be of consequence for the macroeconomy. As the interest rate shocks are restrictive, they reinforce the recessionary effects that come with the hangover following expansionary QE, which were described in the foregoing section. Our results contradict Greenlaw et al. (2018), which make the argument that forward guidance shocks were a more effective policy tool than quantitative easing programs implemented by the Fed. To the contrary we find that at the ZLB quantitative easing has far stronger effects than forward guidance.

5 Conclusion

In this paper, we develop a medium scale DSGE model with a financial sector and use it to analyse the effects of several quantitative easing measures. For this purpose, we solve the model with an occasionally binding zero lower bound, and filter and estimate the model non-linearly. The methods we employ enable us to make statements about the effects of QE in the US during the Great Recession and in the subsequent years, in which the zero lower bound on the nominal interest rate was binding. In modelling the real economy and the financial sector we stay close to popular workhorse models to ensure comparability with the existing literature.

We find that the effects of quantitative easing are relatively small in comparison to the effects of non-policy shocks, in particular, exogenous variations in the risk premium.
Nonetheless, the effects are economically meaningful. We find that the most effective tool for stimulating real activity in the short-run are central bank purchases of claims on the private capital stock. According to our analysis, central bank liquidity injections had the smallest and most short-lived effect on the US economy. Government bond purchases, while lowering the real rate on safe deposits did not support economic growth but rather led to a decline in investment and GDP.

We document that the overall stimulating effects of QE are followed by an implied economic slump that we dub the hangover effect of QE. In our model this is due to the declining profitability of the financial sector due to the decrease in excess spreads on assets purchased by the central bank. As a consequence the weakened financial sector contracts its loan supply in the long run. This mechanism is closely related to the argument of the reversal rate made by Brunnermeier and Koby (2018).

Other sceptics of QE have argued that the uncertainty surrounding the unwinding QE created uncertainty and contributed to the slow recovery of the US economy following the Great Recession (see, e.g. Taylor, 2014). This and other aspects of QE are not captured in our analysis. Investigating how the effects of QE in the US as implied by nonlinearly estimated model hinge on the incorporated financial sector is a promising route for future research.
Appendix A  Data

Our measurement equation contains ten variables:

- GDP: \( \ln(\text{GDP}/\text{GDPCTPI}/\text{CNP16OV}) \times 100 \)
- CONS: \( \ln(\text{PCEC}/\text{GDPCTPI}/\text{CNP16OV}) \times 100 \)
- INV: \( \ln(\text{FPI}/\text{GDPCTPI}/\text{CNP16OV}) \times 100 \)
- LAB: \( \ln((\text{AWHNONAG} \times \text{CE16OV})/\text{CNP16OV}) \)
- INFL: \( \text{GDPCTPI}/4 \)
- WAGE: \( \ln(\text{COMPNB}/\text{A006RD3Q086SBEA} \times 100) \)
- FFR: \( \text{FEDFUNDS}/4 \)
- CB Bonds: \( \text{TREAST}/\text{GDP} \)
- CB Loans: \( (\text{MBST} + \text{WCPFF})/\text{GDP} \)
- CB Liquidity: \( (\text{WACBS} + \text{FEDDT} + \text{TERAUCT} + \text{OTHLT})/\text{GDP} \)

For GDP, CONS, INV and WAGE we use the log changes in our measurement equations.

Data sources:

- GDP: GDP - Gross Domestic Product, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate, FRED
- GDPCTPI: Gross Domestic Product: Chain-type Price Index, Index 2009=100, Quarterly, Seasonally Adjusted, FRED
- PCEC: Personal Consumption Expenditures, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate, FRED
- FPI: Fixed Private Investment, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate, FRED
- AWHNONAG: Average Weekly Hours of Production and Nonsupervisory Employees: Total private, Hours, Quarterly, Seasonally Adjusted, FRED
- CE16OV: Civilian Employment Level, Index Q12009=100, FRED
- CNP16OV_NBD20090101: Civilian Noninstitutional Population, Index Q1 2009=100, Quarterly, Seasonally Adjusted, FRED
- COMPNB, Nonfarm Business Sector: Compensation Per Hour, Index 2009=100, Quarterly, Seasonally Adjusted, FRED
- FEDFUNDS: Effective Federal Funds Rate, Percent, FRED
- TREAST: U.S. Treasury securities held by the Federal Reserve: All Maturities, Millions of Dollars, Quarterly, Not Seasonally Adjusted, FRED
Appendix B  Details on the estimation

Figures B.14 to B.19 show the traces and final distribution after burn in of 100 chains running 4000 iterations. 3000 of these are discarded as burn-in.
Figure B.14: Traces of 100 chains (means, 66% and 95% confidence intervals).
Figure B.15: Traces of 100 chains (means, 66% and 95% confidence intervals).
Figure B.16: Traces of 100 chains (means, 66% and 95% confidence intervals).
Figure B.17: Traces of 100 chains (means, 66% and 95% confidence intervals).
Figure B.18: Traces of 100 chains (means, 66% and 95% confidence intervals).
Figure B.19: Traces of 100 chains (means, 66% and 95% confidence intervals).
Appendix C  Comparison of Parameter Estimates to a model without financial sector

(TBD)

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Table C.2: Estimation results from Boehl and Strobel (2019) (same model without financial frictions).
References


Gertler, M., Karadi, P., January 2013. QE 1 vs. 2 vs. 3 . . . : A Framework for Analyzing Large-Scale Asset Purchases as a Monetary Policy Tool. International Journal of Central Banking 9 (1), 5–53.


