FinancialIntegration in a Monetary Union with Heterogeneous Firms and Unemployment *

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Abstract

Financial markets segmentation has pervasive effects in economic stability and welfare. This paper develops a framework for analyzing financial integration in the context of a monetary union. Starting from the basic ingredients of a new Keynesian model with labor and financial frictions, a banking sector as in Gerali et al. (2010) is introduced and extended to allow for an interbank market across countries in the union. In order to capture market segmentation, the cost of borrowing from interbank markets is decreasing in the quality of the bank’s balance sheet. Alternative macroprudential policies are then analyzed in terms of their ability to increase financial integration and promote economic stability and welfare, both at the national and union-wide level.

Keywords: Macroprudential policy, interbank market, financial accelerator, new Keynesian model.

JEL Class.: E24, E32, E44, F45

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1 Introduction

European financial markets are highly segmented. This remains true even after the efforts to overcome the Great Recession and the movement towards a more complete banking union. To what extent financial market segmentation determined the economic outcomes since the Great Recession, and which policies can be used to prevent the strong effects in the future is in the scope of this paper.

Recent research documents this market segmentation for the post-Great Recession period. For example, Gabrieli and Labonne (2018) investigate the source of this segmentation by looking at sovereign risk exposure and credit risk (balance sheet) of banks in GIIPS countries versus non-GIIPS ones. The authors find that bad quality of balance sheets in GIIPS countries difficted their access to interbank markets between 2011-15. The higher lending costs were also harmful for non-GIIPS countries, in particular, those highly exposed to banks in GIIPS countries. This situation was alleviated by the OMT program carried out by the ECB.

These differences in funding costs are visible along other dimensions too. Figure 1 provides a snapshot of the financial situation in the Euro area. The top left panel shows the evolution of MFI loans to non-MFIs by geographic location of the counterpart. It shows the high home bias of loans towards domestic non-MFIs. Panel B reports the different cost of interbank market loans in the EA. The panel displays the average of interbank interest rates by country group, according to the ECB. Notice how the dispersion increased during the crisis and is now practically diluted. Panel C shows the dispersion of banks’ CDS premia also by country group. This standard deviation increased considerably during the crisis and was reduced by the bond purchases implemented by the ECB. Finally, Panel D reports the decline in the volume of interbank lending in the Euro area, in particular since 2008.

This paper argues that the reduction in credit, together with the increase in funding costs have implications in terms of welfare and economic activity. These spillovers do not necessarily remain within the borders of countries with weaker banks, but may affect other countries in the union. These facts have been widely documented empirically, but we lack

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1The ECB in his Financial Integration Indicators publication reports some of the statistics by country group. Countries in the EA are classified into two groups: group A includes “euro area countries that have not experienced a significant deterioration in their credit rating since the end of 2008: Belgium, Germany, Estonia, France, Latvia, Lithuania, Luxembourg, Malta, the Netherlands, Austria, Slovakia and Finland;” whereas group B is defined as “euro area countries that have experienced a significant deterioration in their credit rating since the end of 2008: Ireland, Greece, Spain, Italy, Cyprus, Portugal and Slovenia,” (Financial Integration in Europe, May 2017).
a theoretical framework that can account for all of them, as a policy tool. This paper provides a setup that can account for this interbank market segmentation within a currency union, in an otherwise standard new Keynesian model. The objective is to analyze the role of alternative policy instruments, such as macroprudential policies, in increasing financial market integration and by extension, in macroeconomic stabilization in the context of a currency union.

The model economy builds on standard well-established ingredients of new Keynesian models used for policy analysis. In particular, the model considers two countries within a monetary union with both labor and financial frictions. Labor frictions are introduced to help explain the dynamics of the labor market, both at the intensive and extensive margins. Financial frictions arise in the capital production sector.

In each country, there are households, firms and financial intermediaries. The economy is populated by a large number of identical, infinitely-lived households who consume, deposit at the banks, invest in government bonds, work and pay taxes. Within each household, individuals can be either employed or unemployed. There is search and matching in the labor market, where households supply differenciated labor as in Erceg et al. (2000), and firms post vacancies, and bargain over hours and wages. At the end of every period, households receive profits from firms.

The productive side of the economy consists of three sectors: intermediate labor goods producers, wholesale differentiated goods producers, and retailers. The final good is homogeneous and compounded by the retail producers out of differentiated goods. It can be used for consumption and investment purposes. This sector produces under perfect competition. Wholesale producers transform the homogeneous intermediate labor good, at a real cost $\phi_t$, into differentiated output in a monopolistic competitive market and change prices à-la-Calvo with some probability $\xi_p$. Those firms which cannot change prices, apply steady state inflation. Intermediate labor goods producers sell their output in a competitive market to the wholesale producers. These firms use capital and labor as input factors according to a constant returns-to-scale production function. This technology is affected by an economy-wide productivity shock, $A_t$, common to all firms. These firms post vacancies, hire workers and rent capital from capital goods producers. Firms and workers then bargain over hours and wages.
Capital good producers are modelled following Fernández-Villaverde (2010) setup. Every period, these agents purchase the outstanding capital stock and use it to generate new investment of new capital goods. They operate under perfect competition, and I assume adjustment costs to investment.

Entrepreneurs purchase capital from capital good producers and rent it to intermediate good firms. They own initial net worth but this is not enough to purchase the capital, so they borrow the difference from financial intermediaries. In this process the return obtained from each unit of new capital is affected by an idiosyncratic shock, which is private information to the entrepreneur. This asymmetry of information leads to a costly state verification problem which is solved with a standard debt contract. The contract establishes a productivity threshold, such that for productivity realizations above this threshold, the entrepreneur has incentives to reveal its true outcome and pays back the loan plus interest. If productivity is below the threshold, the entrepreneur does not return the loan, but the bank seizes all the outcome net of monitoring costs. This financial friction introduces a wedge into marginal products and the cost of inputs. Now, a higher probability of bankruptcy increases the cost of the loan, and vice versa. Risk shocks affect the expected return of entrepreneurs, and therefore the probability of repayment of the loan. This is the source of balance sheet risk for the banks.

Financial intermediaries in this model follow the structure in Gerali et al. (2010), extended to a currency union with interbank market. Banks are structured in three types of branches. The deposit branch gets deposits from households. Households deposit funds across local banks in exchange for a return. Banks act with some monopolistic power and charge a markdown on deposits relative to the policy rate. The loans branch gets funds to lend from the wholesale branch and gives them to the entrepreneurs, under perfect competition. Finally, the wholesale branch is in charge of managing the capital structure of the bank. Bank capital is accumulated via non-distributed profits and net of depreciation. It gets deposits from the deposit branch, which together with the bank capital it owns are transformed into bank loans to the loans branch.

If more funds are needed to lend, the wholesale branch can access the interbank market and borrow, or lend in case of excess funds. In order to capture financial market segmentation, the cost of funds in the interbank market depends on the balance sheet position of the bank.
In particular, for levels of loans above the capital requirement, the borrowing bank will pay a premium over the interbank lending rate. This will force the bank to deleverage, affecting real activity in its country.

The main contribution of this paper is to analyze the open economy dimension within a currency union. Loans to firms and deposits from households are local, that is, no cross-border borrowing or lending is allowed. However, banks in one country will be exposed to the balance sheet situation of the other country in the union, reflecting the empirical facts described above. The model is evaluated in terms of the real effects of financial shocks (on output and unemployment) together with the financial implications. The currency union setup allows for additional channels in the transmission of risk across banks: the model is suitable for the analysis of the exposure of banks to sovereign risk too. Commercial banks holding government bonds in their balance sheets and participating in the interbank market may propagate the effects of domestic shocks across the union.

This setup is then used for policy analysis. Alternative macroprudential policies are considered to study their stabilization properties in response to risk shocks. In particular, countercyclical capital requirements are introduced in order to prevent the build up of too much leverage ex-ante, in contrast with the previous mechanism of penalizing more borrowing ex-post. These two policies are compared in terms of economic stabilization and welfare.

Preliminary results show that in the event of an increase in the riskiness of firms in the periphery, the local economy falls into a recession. The bankruptcy rate increases making it more difficult for local firms to borrow. Unemployment goes up and households reduce their deposits. The downturn affects negatively entrepreneurs’ net worth and banks’ profits. Local banks use the interbank market to borrow the funds they need. Lower bank capital and higher interbank borrowing increase the cost of funds for banks. An increase in wages and inflation raises the union policy rate. This affects negatively the core, despite the inicjal boom experienced by the switch of consumption from local to imported goods in the periphery. This situation is considered the benchmark that reflects the financial market segmentation observed in the data. The next step is to analyze alternative policies to reduce this fragmentation.

The paper is structured as follows. Papers related to this are commented in Section 2. The model is presented in Section 3. Section 4 details the calibration of the model. Section 5 considers alternative macroprudential policies. And Section 6 closes the paper.
2 Literature

To be completed ....

3 The Model

The model represents two countries belonging to a monetary union, core and periphery or size $\varsigma$ and $1 - \varsigma$, respectively. In each country, there are households, final good producers, intermediate goods firms, capital producers, entrepreneurs and banks. The model is an open economy new Keynesian model with financial frictions extended to introduce financial intermediaries. In this setup, banks across countries interact through the interbank market. The baseline setup builds on Auray and de Blas (2013), Fernández-Villaverde (2010), and Gerali et al. (2010).

The economy is populated by a large number of identical, infinitely-lived households who consume, invest in bonds and physical capital, and work. Within each household, individuals can be either employed or unemployed. The productive side of the economy consists of three sectors: one producing the final good in each country; intermediate good firms, and capital goods producers. There are frictions in the labor market in the form of search and matching. This allows the model to display effects on unemployment. Intermediate good firms produce differentiated output in a monopolistic competitive market and change prices à-la-Calvo. The final good is homogeneous and can be used for consumption and investment purposes.

Countries within the monetary union are symmetric, unless otherwise stated. From now on, the description will correspond to the periphery country. When necessary, the corresponding equations for the core economy will be shown. Core variables will be denoted with an asterisk.

3.1 The household

There is a continuum of households in the interval $[0,1]$. We follow Merz (1995) and assume that the household is big, in the sense of providing with some insurance for the risk in labor income. Household preferences are characterized by the lifetime utility function:

$$
\mathcal{H}_t(n_{lt}) = E_t \sum_{l=0}^{\infty} \beta^{l-t} \left( \frac{c_t^{1-\sigma}}{1 - \sigma} - \Psi \int_0^1 n_{lt} \frac{h_{lt}^{1+\psi}}{1 + \psi} dl \right),
$$

(1)
where \( 0 < \beta < 1 \) is a constant discount factor, \( c \) denotes consumption; \( n_l \) is labor supply at firm \( l \), and \( h_l \) is the number of hours that each individual of the household works at the firm.

In each and every period, the representative household faces a budget constraint of the form

\[
B_{t}^{gov} + D_t + P_t c_t + P_t T_t \leq \int_0^1 W_{l,t} n_{l,t} h_{l,t} d l + (1 - n_t) P_t b_t + R_{t-1}^{d} D_{t-1} + R_{t-1}^{gov} B_{t-1}^{gov} + P_t T_t + P_t, \tag{2}
\]

where \( B_{t}^{gov} \) and \( D_t \) are nominal government bonds and bank deposit holdings carried over to period \( t \); \( P_t \) is the CPI index; \( R_{t}^{gov} \) is the gross nominal interest rate on bonds, and \( R_{t}^{d} \) denotes the gross nominal interest rate on deposits, both between \( t \) and \( t+1 \); \( W_{l,t} \) is the nominal wage paid by firm \( l \); \( n_{l,t} \) is the number of employees working \( h_{l,t} \) hours at firm \( l \); \( b \) denotes the unemployment benefit received if not working; and \( n_t = \int_0^1 n_{l,t} d l \). In this economy, government bonds are in zero net supply, that is, \( B_{t}^{gov} = 0 \) in equilibrium. In addition, the household receives nominal profits, \( P_t \), earned by the firms which he owns: final and intermediate firms and entrepreneurs. Finally, households receive a net transfer from entrepreneurs at the end of the period, \( T_t \)

\[
T_t = (1 - \gamma^e_t) n w_t - w^e,
\]

where \( \gamma^e_t \) refers to the survival rate of entrepreneurs, \( n w_t \) is the entrepreneurs net worth, and \( w^e \) is the initial net worth given to any new born entrepreneur.

The representative household maximizes utility \( (1) \) subject to the budget constraint, by
choosing the paths of $c_t$, $D_t$ and $B_t^{gov}$. The first order conditions are

$$u'(c_t) = \lambda_t,$$

(3)

$$\lambda_t = \beta E_t \lambda_{t+1} \frac{R_t^{gov}}{\pi_{t+1}},$$

(4)

$$\lambda_t = \beta E_t \lambda_{t+1} \frac{R_t^d}{\pi_{t+1}},$$

(5)

$$\frac{B_t^{gov}}{P_t} + \frac{D_t}{P_t} + c_t + T_t = \int_0^1 W_{l,t} n_{l,t} n_{l,t} dl + (1 - n_t) b + \frac{R_{t-1}^d}{\pi_t} D_{t-1} + \frac{R_{t-1}^{gov}}{\pi_t} B_{t-1}^{gov} + T_t + P_t,$$

(6)

where $\lambda_t$ denotes the Lagrange multiplier associated with the budget constraint, and $\pi_t$ is used for local CPI inflation.

Households’ total consumption expenditure is composed of local ($c_{Ht}$) and imported ($c_{Ft}$) goods, such that

$$P_t c_t = P_{Ht} c_{Ht} + P_{Ft} c_{Ft},$$

(7)

where $P_t$ is the consumer price index in the periphery, and $P_{Ht}, P_{Ft}$ denote the price of local and imported goods, respectively, both expressed in the local currency. Consumption goods are combined using a CES function, with elasticity of substitution across goods $\eta_c$, and degree of openness $\gamma$. Along the paper it is assumed that the law of one price holds for local and imported goods. The standard demand equations arise.

3.1.1 Deposit demand decision

As in most of the literature, we assume banks have some kind of market power in the market for deposits. Deposits allocation is local, that is, households can only save in their own country, and there are no cross-border deposits.

Accordingly, we assume a Dixit-Stiglitz framework, where households demand deposit contracts which are a composite of differentiated deposits offered by the banks. Each house-
hold will demand a total of deposits $D_t$ in order to

$$\max_{d_t(j)} \int_0^1 \varepsilon_d^d(j)d_t(j)\,dj$$

subject to

$$\left[ \int_0^1 d_t(j) \frac{\varepsilon_d(j-1)}{\varepsilon_d(j)} \,dj \right]^{\frac{\varepsilon_d}{\varepsilon_d-1}} \leq D_t,$$

with $\varepsilon_d < -1$ being the elasticity of substitution across deposits. The optimal demands from this problem yield

$$d_t(j) = \left( \frac{r_t^d(j)}{r_t^d} \right)^{-\varepsilon_d} D_t,$$

where $r_t^d$ denotes the net nominal interest rate on deposits.

### 3.2 Retail producers

The final good is produced by combining a continuum of differentiated wholesale goods indexed by $j$. This process is described by the following CES function:

$$Y_t = \left( \int_0^1 Y_t(j) \frac{1}{\varepsilon_f} \,dj \right)^{\varepsilon_f},$$

where $\varepsilon_f \in [1, \infty)$ determines the elasticity of substitution between the various inputs. Producers in this sector are assumed to behave competitively, and to determine their demand for each good, $Y_t(j)$, $j \in (0, 1)$ by maximizing the static profit equation

$$\max_{\{Y_t(j)\}_{j \in (0,1)}} P_{H,t} Y_t - \int_0^1 P_{H,t}(j)Y_t(j)\,dj,$$

subject to (11), where $P_{H,t}(j)$ denotes the price of the intermediate good $j$. This yields input demand functions of the form

$$Y_t(j) = \left( \frac{P_{H,t}}{P_{H,t}(j)} \right)^{\frac{\varepsilon_f}{\varepsilon_f-1}} Y_t,$$

and the following aggregate price index for locally produced goods:

$$P_{H,t} = \left( \int_0^1 P_{H,t}(j) \frac{1}{\varepsilon_f} \,dj \right)^{1-\varepsilon_f}. $$
3.3 Wholesale producers

In our model each wholesale firm $j \in (0, 1)$ produces the differentiated wholesale goods, $Y_t(j)$, for the final good sector through a constant returns to scale production function, by use of a homogeneous intermediate labor good purchased at nominal price $P_{H,t}\phi_t$.

Wholesale producers are monopolistically competitive, and therefore set prices for the good they produce. We follow Calvo (1983) in assuming that firms set their prices for a stochastic number of periods. In each and every period, a firm either gets the chance to adjust its price (an event occurring with probability $1 - \xi_p$) or it does not. When the firm does not reset its price, it just applies steady state inflation, $\pi^*_H$, to the price it charged in the last period such that $P_{H,t}(j) = \pi^*_H P_{H,t-1}(j)$. When it gets a chance to do it, firm $j$ resets its price, $\tilde{P}_{H,t}(j)$, in period $t$ in order to maximize the expected discounted profit flow this new price will generate. In period $t$, the profit is given by $\Pi(\tilde{P}_{H,t}(j))$. In period $t + 1$, either the firm resets its price, such that it will get $\Pi(\tilde{P}_{H,t+1}(j))$ with probability $1 - \xi_p$, or it does not and its $t + 1$ profit will be $\Pi(\pi^*_H \tilde{P}_{H,t}(j))$ with probability $\xi_p$. Likewise in $t + 2$. The expected profit flow generated by setting $\tilde{P}_{H,t}(j)$ in period $t$ is obtained from

$$\max_{\tilde{P}_{H,t}(j)} E_t \sum_{\tau=0}^{\infty} \beta^\tau \Lambda_{t+\tau}(\xi_p)^T \Pi(\pi^*_H \tilde{P}_{H,t}(j)),$$

subject to the total demand it faces

$$Y_t(j) = \left( \frac{P_{H,t}}{P_{H,t}(j)} \right)^{\frac{\varepsilon_f}{\varepsilon_f - 1}} Y_t,$$

where $\Pi(\pi^*_H \tilde{P}_{H,t}(j)) = \left( \pi^*_H \tilde{P}_{H,t}(j) - P_{H,t+\tau}\phi_{t+\tau} \right) Y_{t+\tau}(j)$ and $\Lambda_{t+\tau}$ is an appropriate discount factor related to the way a household values future, as opposed to current consumption, such that

$$\Lambda_{t+\tau} \propto \left( \frac{\lambda_{t+\tau}}{\lambda_t} \right).$$
This leads to the price setting equation

\[
\frac{1}{\lambda_f} \bar{P}_{H,t}(j) E_t \sum_{\tau=0}^{\infty} (\beta \pi_H^{\tau} \xi_p)\lambda_{t+\tau} \left( \frac{\pi_H^{\tau} \bar{P}_{H,t}(j)}{P_{H,t+\tau}} \right) Y_{t+\tau} = \tag{13}
\]

\[
E_t \sum_{\tau=0}^{\infty} (\beta \xi_p)\lambda_{t+\tau} \left( \frac{\pi_H^{\tau} \bar{P}_{H,t}(j)}{P_{H,t+\tau}} \right)^{\frac{1}{1-\epsilon_f}} P_{H,t+\tau} \phi_{t+\tau} Y_{t+\tau},
\]

which shows that all firms which reset their price in period \( t \) set it at the same level (\( \bar{P}_{H,t}(j) = \bar{P}_{H,t} \), for all \( j \in (0,1) \)).

Given equation (12), the price index comprises surviving contracts and newly set prices. Given that in each and every period a price contract has probability \( 1 - \xi_p \) of ending, the probability that a contract signed in period \( t-s \) survives until period \( t \), and ends at the end of period \( t \) is given by \( (1 - \xi_p) \xi_s^s \). Therefore, the aggregate price level may be expressed as the average of all surviving contracts, namely

\[
P_{H,t} = \left( \sum_{s=0}^{\infty} (1 - \xi_p) \xi_s^s \left( \pi_H^{s} \bar{P}_{H,t-s} \right)^{\frac{1}{1-\epsilon_f}} \right)^{1-\epsilon_f}.
\]

which can be expressed recursively as

\[
P_{H,t} = \left( (1 - \xi_p) \bar{P}_{H,t}^{\frac{1}{1-\epsilon_f}} + \xi_p \left( \pi_H^{\epsilon_f} P_{H,t-1} \right)^{\frac{1}{1-\epsilon_f}} \right)^{1-\epsilon_f}. \tag{14}
\]

### 3.4 Intermediate labor good producers

In the model, there is a continuum of intermediate labor goods, \( y_{l,t} \), who sell their output in a competitive market to the wholesale producer. These firms use capital and labor as input factors according to the following constant returns-to-scale production function:

\[
y_{l,t} = A_l k_{l,t-1}^{\alpha} \left( n_{l,t} h_{l,t} \right)^{1-\alpha} \text{ with } \alpha \in (0,1), \tag{15}
\]

where \( k_{l,t-1} \) is the physical capital input used by firm \( l \) in the production process; \( n_{l,t} \) denotes the number of members of the household who work \( h_{l,t} \) hours at firm \( l \), so we can define total hours worked as \( N_{l,t} = n_{l,t} h_{l,t} \). The variable \( A_l \) is an exogenous stationary stochastic technology shock common to all firms, whose properties will be defined later.

We assume that each firm \( l \) operates under perfect competition in the input market for
capital, and pays $r^K_t$ per unit rented to households. Besides, hours worked are chosen by the firm and are paid a wage $W_t$ determined in a bargaining process to be explained later. Following Mortensen and Pissarides (1999) we assume these firms can hire at most one worker, otherwise their production would be zero. Firms post vacancies in the labor market at time $t$ to hire workers in the same period. This implies a flow cost of vacancy posting:

$$C(v_{l,t}, n_{l,t}) = \chi (1+\varepsilon_c) \left( \frac{v_{l,t}}{n_{l,t-1}} - 1 \right)^{1+\varepsilon_c} n_{l,t-1} - r^K_t k_{l,t} + E_t \Lambda_{t,t+1} J_{t,t+1}(n_{l,t}),$$

subject to the production function given by equation (15), and

$$n_{l,t} = (1 - \rho) n_{l,t-1} + m_{l,t},$$

where $\rho$ is the fraction of the firm-worker matches broken each period, assumed to be constant; and $m_{l,t}$ is the number of matches at firm $l$.

The first order conditions are given by

$$r^K_t = \phi_t \alpha A_t k_{l,t}^{\alpha - 1} (n_{l,t} h_{l,t})^{1-\alpha},$$

$$\chi z_{l,t}^{\varepsilon_c} = \zeta_t q (\theta_t),$$

$$\zeta_{l,t} = \phi_t m p l_{l,t} h_{l,t} - \frac{W_t}{P_{H,t}} h_{l,t} + E_t \Lambda_{t,t+1} \left[ \frac{\varepsilon_c}{1 + \varepsilon_c} \chi z_{l,t+1}^{1+\varepsilon_c} + (1 - \rho) \zeta_{t+1} \right],$$

where I have defined $z = \frac{v_{l,t}}{n_{l,t-1}}$, and $\zeta_{l,t}$ is the multiplier associated to constraint (16).

Notice that from (??), and given constant returns to scale in production, all firms have the same capital-labor ratio $k_{t,l}/n_{l,t} h_{l,t} = k_t/n_h$ for all $l$, what implies that the marginal product of labor is also the same for all firms, that is, $mpl_{l,t} = m p l = (1 - \alpha) k_t^{\alpha} (n_{l,t} h_{l,t})^{-\alpha}$.\footnote{I assume that new hires become effective at time $t$, as in Blanchard and Gali (2010). In using this notation, I follow the line of the literature that allows for some workers and vacancies to find matches immediately, that is, without spending a full period unemployed. This implies that shocks can be adjusted both by unemployment and by hours per worker, improving empirically the performance of the model along the extensive and intensive margins of labor market in response to shocks at business cycle frequencies. Otherwise, in a quarterly model from the empirical point of view, it is unnatural to assume that it takes at least one period to become productive in a new job.}
After substituting in the job creation condition \((??)\), we get the Euler equation for vacancies
\[
\frac{x^\epsilon_c}{q(\theta_t)} = \phi_t \text{mpl}_t h_{l,t} - \frac{w_{l,t}}{P_{H_l}/P_t} h_{l,t} + E_t \Lambda_{t+1} \left[ \frac{\epsilon_c}{1 + \epsilon_c} x^{\epsilon_c}_{l,t+1} + (1 - \rho) \frac{x^{\epsilon_c}_{l,t+1}}{q(\theta_{t+1})} \right],
\]
which states that the cost of posting a vacancy to hire an additional worker equals the marginal benefit that the additional worker brings into the firm, i.e. the marginal revenue product net of wage payments, plus the continuation value of the job and the savings for the firm for not having to post another vacancy at \(t + 1\).

### 3.5 Labor market

As mentioned above, we assume that intermediate labor goods firms bargain with workers in the labor market to determine the nominal wage, \(W_{l,t}\), and hours worked, \(h_{l,t}\).

Job formation is assumed to involve a matching process where the matching function is given by
\[
m_t = \sigma_m \left( u_t^s \right)^\theta v_t^{1-\theta},
\]
where \(v_t\) is the number of vacancies in the job market; \(u_t^s\) is the number of effective work seekers in period \(t\); \(\sigma_m > 0\) is the scale parameter of the matching function; and, \(\theta \in (0, 1)\) is the elasticity of the unemployed searchers in the matching function. Let us denote \(\theta_t = v_t/u_t^s\) the degree of tightness in the labor market. As usual in this setup, the probability that an unemployed worker finds a job is given by
\[
s_t = \frac{m_t}{u_t^s} = \sigma_m \theta_t^{1-\theta} \equiv s(\theta_t),
\]
and the probability of a vacancy being filled by
\[
q_t = \frac{m_t}{v_t} = \sigma_m \theta_t^{-\theta} \equiv q(\theta_t).
\]
Notice that using this notation, we have that \(s(\theta_t) = \theta_t q(\theta_t)\).

In each country, hours, \(h_{l,t}\), are bargained to match the total surplus of both workers and
firms. Let us define the worker’s surplus in consumption units, $S_{w,l,t} \equiv \frac{\partial H_t}{\partial n_{l,t}}$, as follows:

$$S_{w,l,t} = \frac{w_{l,t} P_{H,t}}{P_t} h_{l,t} - b - \Psi \frac{h_{l,t}^{1+\psi}}{\lambda_t (1 + \psi)} + (1 - \rho) E_t (1 - s(\theta_{t+1})) \Lambda_{t,t+1} S_{w,l,t+1},$$  \hspace{1cm} (24)$$

where $s(\theta_t)$ denotes the probability of being matched with firm $L$ at time $t$. Equation (24) states that the worker’s surplus equals wage payments at the firm plus continuation value, minus disutility of working, and in case of being unemployed, the benefits and expected surplus of being matched with any firm at $t + 1$. Similarly, the firm’s surplus, $S_{f,l,t} \equiv \zeta_{l,t}$, is given by

$$S_{f,l,t} = \phi_t mpl_t h_{l,t} - \frac{w_{l,t} P_{H,t}}{P_t} h_{l,t} + E_t \Lambda_{t,t+1} (1 - \rho) S_{f,l,t+1}$$  \hspace{1cm} (25)$$

Given this, hours are chosen as follows:

$$\max_{h_{l,t},w_{l,t}} \left( S_{w,l,t} + S_{f,l,t} \right).$$  \hspace{1cm} (26)$$

The first order condition with respect to $h_{l,t}$ yields

$$\phi_t mpl_t = \Psi \frac{h_{l,t}^{\psi}}{\lambda_t},$$  \hspace{1cm} (27)$$

implying that $h_{l,t} = h_t$ for all $l$.

The wage set by firms satisfies the following sharing rule:

$$\eta S_{f,l,t} = (1 - \eta) S_{w,l,t},$$  \hspace{1cm} (28)$$

where $\eta$ is the bargaining power of the workers. This sharing rule implies that workers obtain a fraction of the total surplus equal to their bargaining power. With Nash bargaining wages maximize a weighted average of the joint surplus.

From the optimization, we obtain

$$\frac{W_t}{P_{H,t}} h_t = \eta \phi_t mpl_t h_t + (1 - \eta) b + (1 - \eta) \Psi \frac{h_t^{1+\psi}}{\lambda_t (1 + \psi)} + \eta (1 - \rho) E_t \Lambda_{t,t+1} \left[ \frac{\varepsilon_c}{\varepsilon_c + 1} q(\theta_{t+1}) + s(\theta_{t+1}) \right] \frac{\chi z_{l,t+1}}{q(\theta_{t+1})},$$  \hspace{1cm} (29)$$
3.6 Capital goods producers

In each country, capital is owned and produced by agents who operate in a perfectly competitive market. New capital, $x_{t+1}$, is produced using the following technology:

$$x_{t+1} = x_t + \left(1 - S\left[\frac{i_t}{i_{t-1}}\right]\right)i_t,$$

(30)

where $S\left[\frac{i_t}{i_{t-1}}\right]$ denotes adjustment costs, such that $S'[\cdot] > 0; S''[\cdot] > 0; S[1] = 0$; and $S'[1] = 0$.

Capital units are worth $P_{H,t}q_t$ currency units, where $q_t$ denotes the relative price of capital. In the production process, capitalists purchase the remaining capital from last period, and invest. Capital good producers therefore maximize

$$E_t \sum_{t=0}^{\infty} \beta^t \frac{\lambda_{t+1}}{\lambda_t} [P_{H,t}q_t x_{t+1} - P_{H,t}q_t x_t - P_{H,t} i_t]$$

(31)

subject to (30), where $\delta \in [0,1]$ is the depreciation rate of physical capital.

Optimal behavior implies

$$q_t \left(1 - S\left[\frac{i_t}{i_{t-1}}\right]\right) - S'\left[\frac{i_t}{i_{t-1}}\right] + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} S'\left[\frac{i_{t+1}}{i_t}\right] \left[\frac{i_{t+1}}{i_t}\right]^2 = 1,$$

(32)

and the law of motion of capital is expressed by

$$k_t = (1 - \delta)k_{t-1} + \left(1 - S\left[\frac{i_t}{i_{t-1}}\right]\right)i_t,$$

(33)

where $x_t = (1 - \delta)k_{t-1}$.

3.7 Entrepreneurs

Entrepreneurs are in charge of transforming installed capital into inputs for use by intermediate good producers. Entrepreneurs buy new capital, $k_t$, from capital good producers at a price $q_t$, to be used at $t + 1$. Their output is then rented to intermediate good producers at a cost $r_{t+1}^k$ per unit of capital rented at time $t + 1$.

At the end of the period, entrepreneurs repurchase the old non-depreciated capital, $q_{t+1} (1 - \delta)$. Thus, the ex-post average return of the entrepreneur per unit of investment
between $t$ and $t+1$, $R^k_{t+1}$, measured in units of consumption goods today is given by

$$R^k_{t+1} = \Pi_{H,t+1} \frac{r^k_{t+1} + q_{t+1} (1 - \delta)}{q_t}.$$  \hspace{1cm} (34)

Entrepreneurs’ technology is affected by an idiosyncratic shock, $\omega_{t+1}$ assumed to be log-normally distributed with a cumulative distribution function $F(\omega, \sigma_{\omega t})$ with parameters $\mu_{\omega t}$ and $\sigma_{\omega t}$ such that $E_t \omega_{t+1} = 1$ for all $t$. Besides, we assume that dispersion follows

$$\tilde{\sigma}_{\omega t} = \rho_{\sigma_{\omega}} \tilde{\sigma}_{\omega,t-1} + \eta_{\sigma_{\omega}} \epsilon_{\sigma_{\omega},t} \text{ where } \epsilon_{\sigma_{\omega},t} \sim N(0,1).$$  \hspace{1cm} (35)

where $\rho_{\sigma_{\omega}}$ is the persistence coefficient that takes values within $[0,1]$ and $\eta_{\sigma_{\omega}}$ is the volatility of the shock. The shock $\epsilon_{\sigma_{\omega},t} \sim N(0,1)$ is revealed at the end of the period, just before the investment decisions for $t+1$ are taken.

Thus, per unit of capital purchased from capitalists, the entrepreneur gets as return, on average,

$$E[\omega_{t+1}] R^k_{t+1} P_{H,t} q_t.$$  \hspace{1cm} (36)

Entrepreneurs finance this investment with their own internal funds and loans from banks. Internal funds are composed by the end-of-period net worth, $nw_t$; while external funds consist of loans borrowed from financial intermediaries, $B^l_t$. Their balance sheet in nominal terms is given by

$$B^l_t = P_{H,t} q_t k_t - P_t nw_t.$$  \hspace{1cm} (37)

The realization of $\omega_{t+1}$ is private information to entrepreneurs, and the contract with financial intermediaries is signed before it is known. This private information leads to possible moral hazard problems that is solved via a standard debt contract.

### 3.7.1 Costly state verification problem

As in Bernanke, Gertler and Gilchrist (1999), we consider a costly state verification (CSV) problem: entrepreneurs observe their outcome for free, but financial intermediaries need to pay a cost, proportional to the gross payoff of the entrepreneur’s capital.

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3We use the notation $\tilde{x}_t$ to refer to the log-linearized version of variable $x_t$ and $\pi$ for the steady state value of the same variable.
At the moment of the debt contract agreement there is aggregate uncertainty because \( R_{t+1} \) is not known yet. The entrepreneur decides on the amount of capital he wants to purchase, that is, his expenditures for period \( t \), \( P_{Ht}q_tk_t \), and therefore the amount of external funds that he needs, \( B_t \).

The contract will establish a state-contingent non-default repayment \( R_{t+1}^l \) (dependent on the ex-post realization of \( R_{t+1}^k \)) that the entrepreneur promises to pay to the financial intermediary in case he succeeds in his investment project. The standard debt contract also specifies a state-contingent threshold value of the idiosyncratic shock \( \varpi_{t+1} \) (dependent on the ex-post realization of \( R_{t+1}^k \)), below which the entrepreneur defaults. The threshold is determined by the following condition:

\[
R_{t+1}^l B_t = \varpi_{t+1} R_{t+1}^k P_{Ht}q_t k_t. \tag{38}
\]

Both \( R_{t+1}^l \) and \( \varpi_{t+1} \) are chosen to maximize the entrepreneur’s return and such that it is worth for the financial intermediary to enter into the contract, that is,

\[
[1 - F(\varpi_{t+1}, \sigma_{\omega,t})] R_{t+1}^l B_t + (1 - \mu) \int_0^{\varpi_{t+1}} \omega dF(\omega, \sigma_{\omega,t}) R_{t+1}^k P_{Ht}q_t k_t = R_t^b B_t, \tag{39}
\]

which states that the financial intermediary must be at least indifferent between lending to entrepreneurs or getting the safe interest rate on loanable assets.

Finally, the average net wealth is

\[
nw_t = \gamma e \frac{1}{n_t} \left\{ [1 - \mu G(\varpi_t, \sigma_{\omega,t})] R_t^k \frac{q_{t-1}k_{t-1}}{P_{Ht-1}/P_{t-1}} - R_{t-1}^b \frac{B_{t-1}^b}{P_{t-1}} \right\} + w^e. \tag{40}
\]

The standard debt contract is solved by maximizing the entrepreneur’s expected returns

\[
\int_{\varpi_{t+1}}^{\infty} \omega dF(\omega, \sigma_{\omega,t}) R_{t+1}^k P_{Ht}q_t k_t - [1 - F(\varpi_{t+1}, \sigma_{\omega,t})] \varpi_{t+1} R_{t+1}^k P_{Ht}q_t k_t, \tag{41}
\]

subject to the participation constraint of the financial intermediary, equation (39).

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4 We briefly describe the case of aggregate uncertainty. A detailed explanation can be found in de Blas and Malmierca (2019).

5 Average net wealth equals the wealth of the individual entrepreneur since it can be shown that all the entrepreneurs get the same leverage ratio.
3.8 The banking sector

Banks in each country are structured into specialized units: deposits, loans and wholesale (see Gerali et al., 2010). The deposits unit takes deposits from households and passes them to the wholesale unit. The loans unit gets funds from the wholesale branch and lends them to entrepreneurs in need for financing. Finally, the wholesale unit is in charge of managing the capital structure of the bank. In this sense, it accumulates bank capital, and transforms deposits into loans.

In addition, wholesale units of banks in a given country can access the interbank market for funds. Interbank liquidity is exchanged at the interbank rate plus a spread depending on the leverage of each bank. The more leveraged banks will pay a higher rate for borrowing from the interbank market. The central bank acts as the clearing house of interbank loans, and eventually may inject additional funds.

Notice that the retail deposit and loan activities involve only local markets, while the wholesale unit is involved in to cross-border lending.

3.8.1 The deposits unit

The task of the deposit unit is to get funds from households and transfer them to the wholesale unit. Every period, the deposit branch promises a gross interest rate \( R_d^t(j) = 1 + r_d^t(j) \) to households on their deposit contracts. These interest payments are obtained from the wholesale unit that pays the union-wide policy rate, \( R_t \). Managing the interest rate on deposits involves some adjustment costs, as follows:

\[
\max_{r_t^d(j)} \sum_{t=0}^{\infty} \Lambda_{t+\tau} \left\{ r_t^d(j) d_t(j) - \frac{\kappa_d}{2} \left( \frac{r_t^d(j)}{r_{t-1}^d(j)} - 1 \right)^2 r_t^d D_t \right\}
\]

subject to equation (10). The symmetric equilibrium, \( r_t^d(j) = r_t^d(k) \) for \( \forall j \neq k \), in which \( D_t(j) = d_t(j) = d_t \) yields the following optimal condition:

\[
\varepsilon_d \left( \frac{r_t}{r_t^d} \right) + 1 - \varepsilon_d + \kappa_d \left( \frac{r_t^d}{r_{t-1}^d} - 1 \right) \left( \frac{r_t^d}{r_{t-1}^d} \right) = \beta E_t \Lambda_{t+1} \kappa_d \left( \frac{r_{t+1}^d}{r_t^d} - 1 \right) \left( \frac{r_{t+1}^d}{r_t^d} \right)^2 \frac{d_{t+1}}{d_t}.
\]

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3.8.2 The loans unit

In this model the loans unit of financial intermediaries receive loanable funds from the wholesale unit and lends them to entrepreneurs.

Loan units operate in a perfectly competitive market. Their objective function is given by

\[
\left\{ [1 - F(\varpi_{t+1}, \sigma_{\omega, t})] R_{t+1}^l B_t^l + (1 - \mu) \int_{0}^{\varpi_{t+1}} \omega d F(\omega, \sigma_{\omega, t}) R_{t+1}^k P_{H,t} q_t k_t - R_t^b B_t^b \right\},
\]

which shows expected returns in case of a successful project, plus revenues in case of default, minus the costs in terms of deposits for the financial intermediary.

The problem is to maximize profits (44) subject to (38). The optimality condition from the loans unit problem is

\[
E_t \frac{R_{t+1}^k}{R_t^k} [1 - F(\varpi_{t+1}, \sigma_{\omega, t})] = E_t \left( \frac{1 - F(\varpi_{t+1}, \sigma_{\omega, t})}{1 - F(\varpi_{t+1}, \sigma_{\omega, t}) - \mu c \varpi_{t+1} F_{\omega}(\varpi_{t+1}, \sigma_{\omega, t})} \right),
\]

\[
\frac{R_{t+1}^k}{R_t^b} [G(\varpi_{t+1}, \sigma_{\omega, t}) - \mu c G(\varpi_{t+1}, \sigma_{\omega, t})] = \frac{q_t k_t - n \omega_t}{P_{H,t}/P_t},
\]

\[
R_t = \pi_{H,t+1} \frac{r_{t+1} + q_t (1 - \delta)}{q_t}.
\]

3.8.3 The wholesale unit

Finally, the wholesale department of financial intermediaries is in charge of managing the capital structure of the bank. This department operates in competitive markets. As in Gerali et al. (2010), we assume that banks accumulate capital, \( K_t^b \). In addition, they can get funds either from the deposit unit, \( D_t \), or from the interbank market, \( B_t^{IB} \). All those funds are transformed into loans and supplied to the loans unit, \( B_t^l \). Notice that any excess of bank capital plus deposits above loans is lent in the interbank market, and vice versa.

We assume that there are adjustment costs in the management of bank capital, which are proportional to the ratio of bank capital to loans. The balance sheet structure of each bank is then

\[
B_t^b + B_t^{IB} = \xi_t \left( K_t^b + D_t \right),
\]

where \( \xi_t \) is the shock to the quality of capital, \( K_t^b \) denotes bank capital, and \( B_t^{IB} \) is the
amount of loans from the interbank market obtained.

The law of motion of bank capital is

$$\pi_t K^b_t = (1 - \delta^b)K^b_{t-1} + \mathcal{P}^b_{t-1}, \quad (49)$$

where $\delta^b$ is the depreciation rate of bank capital, and $\mathcal{P}^b$ denotes bank’s profits defined as

$$\mathcal{P}^b_t = [\Gamma (\varpi_{t+1}, \sigma_{\varpi}) - \mu_r G (\varpi_{t+1}, \sigma_{\varpi})] R^b_t + R^d_t D_t + R^{IB}_t B^{IB}_t -$$

$$\frac{\kappa_d^2}{2} \left( \frac{r^d_t(j)}{r^d_{t-1}(j)} - 1 \right)^2 r^d_t D_t - \frac{\kappa^b}{2} \left( \frac{K^b_t}{B^b_t} - \nu^b \right)^2 K^b_t. \quad (50)$$

The problem is maximize profits

$$E_t \sum_{t=0}^{\infty} \Lambda_{t+\tau} \left\{ r^b_t B^b_t + r^{IB}_t B^{IB}_t - r^d_t D_t - \frac{\kappa^b}{2} \left( \frac{K^b_t}{B^b_t} - \nu^b \right)^2 K^b_t \right\}, \quad (51)$$

subject to the balance sheet (48).

The optimality condition provides the pricing of loans to the loans unit

$$r^b_t = r^d_t - \kappa^b \left( \frac{K^b_t}{B^b_t} - \nu^b \right) \left( \frac{K^b_t}{B^b_t} \right)^2. \quad (52)$$

3.9 The interbank market

Financial intermediaries in the monetary union have access to an interbank market of loans. This activity is carried out by the wholesale departments of financial intermediaries. This market faces demands and supplies of funds to lend.

We assume that the cost of borrowing from this market equals the monetary policy rate, set by the central bank, plus a spread or premium. This premium is increasing in the level of indebtedness of the bank with respect to its assets, that is, its leverage ratio. To reflect the fact that in a currency union interbank loans are made mostly to and from banks within the union, we also assume that the cost of financing depends on the riskiness of the banks in the monetary union, given that lending banks will be exposed to this higher risk.
3.10 The monetary authority

The central bank is assumed to set interest rates according to a standard Taylor rule, as follows:

\[
\frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\gamma_R} \left( \left( \frac{\pi_t^{CU}}{\pi} \right)^{\gamma_\pi} \left( \frac{Y_t^{CU}}{Y} \right)^{\gamma_y} \right)^{(1-\gamma_R)} \exp(\sigma_mm_t). \tag{53}
\]

where the superscript \( CU \) denotes variables at the currency union level; \( \gamma_R \in [0,1] \) reflects interest rate smoothing; \( \gamma_\pi \) and \( \gamma_y \) captures the response of the nominal interest rate to deviations of inflation and output from their steady state values, respectively. Notice that the central bank responds with monetary policy to the economic conditions in the monetary union.

3.11 Market clearing

[TO BE COMPLETED]

4 Calibration

[TO BE COMPLETED]

5 Macroprudential policies

6 Conclusions
References


