Fiscal policy over the business cycle

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Abstract

This paper emphasizes the role of household heterogeneity for the transmission of shocks to government expenditure in a business cycle model with incomplete markets and nominal rigidities. In particular, I investigate the aggregate and distributional effects of changes in government consumption and fiscal transfers, and evaluate if these effects vary according to the state of the economy. My findings indicate that household heterogeneity generates significant differences in fiscal multipliers and impulse responses of aggregate variables. For both shocks, I find that multipliers are generally higher when the shock hits in a recession. Key driver of these results is the counter-cyclicality of the fraction of borrowing constrained households. This leads to significant differences in the distributions of the marginal propensity to consume across the population, and ultimately to differences in the reactions of aggregate variables.

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1 Introduction

The effectiveness of discretionary fiscal policy is an ongoing and controversial issue in economic research. The Great Recession and the large number of stimulus packages in its aftermath reinforced this debate, shifting the focus to the transmission channels of fiscal policy and how economic conditions affect these channels. The central question in this debate is whether multipliers are state-dependent, or, in a more general sense, whether countercyclical fiscal policy could be an effective tool to mitigate recessions.

Indeed, recent empirical studies (e.g. Auerbach and Gorodnichenko, 2012, 2013; Linnemann and Winkler, 2016) use a variety of different estimation techniques and find substantial nonlinear effects, indicating the existence of countercyclical fiscal multipliers.\(^1\) Theoretical channels that rationalize these findings are rather scarce. In this paper, I address this issue within a quantitative model and propose a mechanism in which incomplete insurance markets and occasionally binding borrowing constraints generate state-dependent responses to fiscal policy shocks. Key driver of this result is the substantial heterogeneity in the marginal propensity to consume (MPC) across households generated by the aforementioned model components. The general framework is based on McKay and Reis (2016) and incorporates nominal rigidities into an incomplete markets model with aggregate uncertainty. Households are heterogeneous with respect to their access to capital markets, their patience, and in their exposure to idiosyncratic shocks. This rich heterogeneity is important to generate realistic wealth and income distributions as well as a non-trivial distribution of households’ MPC.

Differences in MPC across households are supported by empirical evidence. Jappelli and Pistaferri (2014) use survey questions to determine how much of an unexpected positive income shock would be spent on consumption and document significant differences across income groups. In particular, they find a strong negative correlation between the MPC and disposable income. Concerning the relationship between MPC heterogeneity and fiscal policy, Parker et al. (2013) use data from the Consumer Expenditure Survey (CEX) to determine consumption responses of households to the tax rebates that were implemented in the Economic Stimulus Act in February 2008. They find huge discrepancies in the consumption responses across households after the stimulus payments, ranging from 50 to 90 percent of the payments in the quarter of the receipt, where the largest response is found for low-income households. Similar results are found by Johnson et al. (2006) for the 2001 federal income tax rebates and by Misra and Surico (2014) who use the CEX to investigate both the tax rebates from 2001 and 2008. Therefore, I conclude that differences in MPC play a major role when trying to understand the transmission of fiscal policy.

\(^1\)These findings are, however, not uncontroversial. Ramey and Zubairy (2018) use U.S. historical data and find no significant differences in multipliers across the business cycle.
The remainder of the model consists of a firm sector, a fiscal authority and a central bank. The firm sector has the typical New Keynesian structure characterized by a competitive final goods sector but monopolistic competition in the intermediate goods sector. These firms face nominal rigidities in the form of quadratic price adjustment costs and real rigidities in the form of convex capital adjustment costs. This ensures that a meaningful demand channel is operating and the effects of government expenditure shocks are reasonably close to empirical estimates. Aggregate uncertainty takes the form of neutral technology shocks that regularly hit the economy and create cyclical movements that are in line with their empirical counterparts. The fiscal authority collects distortionary income and consumption taxes, and trades nominal bonds with the household sector to finance its own consumption expenditures, fiscal transfers that are paid in a lump-sum fashion, unemployment benefits and their outstanding liabilities. Monetary policy is conducted via a Taylor rule.

I investigate the effects of two different government expenditure shocks, a government consumption shock and a fiscal transfer shock. In the benchmark model, these shocks are deficit-financed with either fiscal transfers or government consumption adjusting to prevent an explosive government debt path. In a later section, I also consider deficit-financed shocks with the distortionary income tax rate adjusting according to the level of government debt. The policy experiment is to simulate changes in government consumption or fiscal transfers at different points in the state space to investigate the different effects for discretionary fiscal policy shocks during recessions and expansions.

My findings indicate that household heterogeneity and in particular the non-trivial MPC distribution generate significant differences in fiscal multipliers and impulse responses of aggregate variables. For government consumption, I find impact multipliers between 0.8 and 0.69 when the shock hits in a recession, while the impact multipliers are between 0.5 and 0.36 when the shock hits during an expansion. In general, lower multipliers correspond to financing the increase in government debt by adjusting the income tax rate. Long-run multipliers range from 0.23 to −0.03 during recessions compared to a range from 0.13 to −0.1 during booms, implying that the tax rule scenario always leads to negative output multipliers after a few years. For fiscal transfer shocks, I find that the signs of impact multipliers differ conditional on the financing scenario. While there is a small positive increase of output when government consumption adjusts to public debt, it is generally negative when the income tax adjusts. In both cases, recession multipliers are larger on impact but decline faster over time than expansion multipliers. So in the long run, the effect of a fiscal transfer shock is more detrimental when the rise occurs in a recession.

The intuition of these results is as follows. In each simulated period of the model’s stochastic steady state, a number of households is borrowing constrained and these
households become virtually hand-to-mouth consumers. However, the share of these households is not fixed and varies considerably over the business cycle. In particular, I find that this share is countercyclical such that during recessions more households are borrowing constrained than during expansions. This implies that MPCs in the lower quintiles of the wealth distribution are considerably higher during recessions which has an immediate impact on aggregate demand. This demand effect caused by fiscal stimulus is therefore significantly stronger when induced during a recession.

The logic behind these differences in multipliers across different financing schemes is straightforward. Consider the government consumption shock. Although downward-adjusting fiscal transfers are a negative income shock for parts of the population, the increase in the real wage through the demand effect can make up for that loss, at least in the short term. If the income tax rate adjusts, the labor supply of the whole workforce is further distorted and leads to a considerably smaller demand effect. In this sense, the benchmark financing scheme only hurts the bottom of the income distribution, while the tax-adjusting scheme hurts the whole population and especially the high-productive. Since the tax rate is only slowly adjusting to the level of public debt, the impact effect of an government consumption shock in this case is not that much affected. After a few periods, however, the demand effect dies out and the economy in the tax-adjusting case is generally in a worse state than the one with the transfer-adjusting scheme.

The consumption response for the government consumption shock is slightly positive during recessions. A decomposition according to the wealth distribution reveals that this positive impact response is driven by the wealth-poorest in the economy. Since their MPC is much higher than the MPC of the wealth-rich, the additional income is spent to a large part on consumption. However, the impulse responses also reveal that the demand effect is so strong that even the wealth-poorest can afford to save a little bit of their additional income. The wealth-rich, instead, lower their consumption and spend their additional income on bonds.

As already stated, the theoretical literature on countercyclical fiscal multipliers is relatively scarce. Shen and Yang (2018) use a simple New Keynesian model with downward nominal wage rigidity to explain the state-dependence of government consumption multipliers. Downward nominal wage rigidity is modeled as an occasionally binding constraint that is only binding in sufficiently severe recessions. In these recessions, a government consumption shock reduces unemployment and prevents the real interest rate to increase as much as in normal times so that the usual crowding-out effect is smaller during recessions. This mechanism generates recession multipliers that are more than twice as large as expansion multipliers. Canzoneri et al. (2016) use a borrower-saver framework with an ad-hoc countercyclical bank intermediation cost that creates a financial accelerator, which is much stronger in recessions than in expan-
sions. They find that multipliers in their recession scenario are about twice as large as in the expansion scenario.

Another paper that is also concerned with state-dependent fiscal multipliers is Sims and Wolff (2018). In this paper, the authors study output and welfare multipliers over the business cycle in an estimated New Keynesian DSGE model. However, for a government consumption shock, they actually find slightly procyclical multipliers. The key difference to the aforementioned papers is the implementation of deep habits and “useful” government consumption, which are both usually employed to get a positive consumption response. The usefulness of government consumption is modeled as an additional term in the households’ utility function that is complementary to private consumption. While Sims and Wolff (2018) do not highlight a particular transmission channel that drives their results, they point out that this complementarity might be responsible for the procyclical of government consumption shocks.

Although these papers incorporate several additional components into New Keynesian DSGE models, all of them rely on the assumption of a representative households or on a distribution of households with only two mass points. Thus, this paper is the first to investigate the issue of state-dependent fiscal multipliers in a model that takes distributional aspects of policy changes serious. Therefore, it also contributes to a growing literature that combines incomplete-markets models with nominal rigidities to simultaneously assess aggregate and distributional consequences of policy shocks.

Several studies have assessed the transmission of fiscal policy within heterogeneous agent models. As in this paper, Hagedorn et al. (2017) use a model with incomplete markets and nominal rigidities to determine multipliers for government consumption and fiscal transfer shocks. However, their approach differs from mine as they only consider “MIT shocks” and abstract from aggregate uncertainty and potential state-dependent effects. Brinca et al. (2016) use an overlapping-generations model with incomplete markets but flexible prices and find that multipliers depend on wealth inequality and the share of credit-constrained households. In this sense, their mechanism is closely related to mine. However, they abstract from the cyclical component and aggregate risk in general. McKay and Reis (2016) use a model that is particularly close to the one presented in the next section and investigate the effect of automatic stabilizers over the business cycle but do not consider fiscal multipliers.

The remainder of the paper is organized as follows. Section 3.2 presents the model, while Section 3.3 discusses the calibration and the numerical implementation of the

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3See Bayer et al. (2015), Gornemann et al. (2016), Guerrieri and Lorenzoni (2017), Kaplan et al. (2018), and Ravn and Sterk (2016), among others.
4MIT shocks are unexpected shocks that are assumed to only hit once and never again. The shock is conducted at the steady state and leads to a transition path either back to the old steady state or to a new one in a finite number of periods.
model. In Section 3.4, I first provide the cyclical and distributional properties of the model and afterwards study the effects of government consumption and fiscal transfer shocks at different points in the state space. Section 3.5 presents the results for the same two shocks with an alternative financing scheme and Section 3.6 concludes.

2 Model

The structure of the model is to a large part identical to McKay and Reis (2016). There are two types of households that differ along several dimensions, most importantly in their exposure to idiosyncratic risk. The firm sector is characterized by a representative final goods firm and monopolistically competitive intermediate goods producers, who face nominal rigidities in the form of quadratic price adjustment costs. As is done in much of the recent New Keynesian literature, I consider a cashless-limit economy as in Woodford (1998). Monetary policy is characterized by a Taylor rule that governs the nominal interest rate. The fiscal authority has access to several instruments and finances its expenditures by collecting taxes and issuing bonds.

2.1 State space

Let $X$ be the vector of aggregate state variables, where $X_t := (K_t, B_t, R_{t-1}, \zeta_t, \Phi_t)$. $K_t$ is the aggregate capital stock, $B_t$ is government debt, $R_{t-1}$ is the lagged nominal interest rate, $\zeta_t$ is the vector of exogenous aggregate shocks, and $\Phi_t$ is the type distribution of households.

In the model simulations below, there are always two aggregate shocks so that $\zeta$ consists of two elements. When investigating the effects of a government consumption shock $G_t$, $\zeta$ is defined as $\zeta_t := (z_t, G_t)$, where $z_t$ is an aggregate productivity shock. When investigating the consequences of a fiscal transfers shock $Tr_t$, it is defined as $\zeta_t := (z_t, Tr_t)$.

Households are characterized by four idiosyncratic states. Let $H_t \in \{p, i\}$ denote the specific type of a particular household, where $p$ characterizes the household as patient and $i$ as impatient. This property is constant over time such that both the share of (im)patient households is always the same as well as the specific type of a particular household will not change. Furthermore, let $E_t := (s_t, \eta_t, a_t)$ denote the three potentially time-varying idiosyncratic states. $s_t \in \{e, u\}$ is the employment status, where $e$ means that a household is currently employed, while $u$ indicates unemployment. $\eta_t \in \mathcal{E} \subseteq \mathbb{R}^+$ is the idiosyncratic productivity level and $a_t \in \mathcal{A} = [0, \infty)$ is the amount of asset holdings.

One important difference between patient and impatient households is that patient households have access to a complete set of insurance contracts which implies that
they are not exposed to idiosyncratic risk.\footnote{Patient households trade Arrow-Debreu securities among each other to pool idiosyncratic risk. To conserve on notation, I omit the details of these securities from the model description.} Therefore, they are always assumed to be employed, their productivity level is fixed and all impatient households hold the same amount of assets. This is not true for impatient households whose employment status as well as their idiosyncratic productivity are determined by stochastic processes. The type distribution of households is thus given by $\Phi_t := \Phi(H_t, E_t)$.

### 2.2 Patient households

This group of households with mass $1 - \nu$ (and subscript $p$) is assumed to be relatively more patient and, as already mentioned, does not face idiosyncratic shocks. Their preferences over consumption $c_{p,t}$ and hours worked $l_{p,t}$ are given by the following infinite sum of discounted period utilities,

$$U_p = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_p^t \left( \frac{(c_{p,t} - h(l_{p,t}))^{1-\sigma} - 1}{1 - \sigma} \right), \quad (1)$$

where $\beta_p \in (0,1)$ is the specific discount factor, $\sigma$ is the coefficient of relative risk aversion, $h : \mathbb{R}^+ \to \mathbb{R}^+$ is assumed to be strictly increasing in both arguments and twice continuously differentiable, and $\mathbb{E}_0$ is the expectations operator conditional on the time-0 information set.

Patient households receive income from four different sources. Since they are the sole owner of the firms in this economy, they get dividends $d_{p,t}$, they receive returns on bonds at the gross nominal rate $R_{t-1}$, and they earn labor income at the real wage rate $w_t$. Moreover, they also obtain lump-sum transfer payments $Tr_t$ from the government. The labor productivity of patient households is fixed at $\bar{\eta}$. This can be seen as a within-group average, since idiosyncratic shocks are perfectly insured. Thus, patient households face the following budget constraint at period $t$,

$$(1 + \tau_c)c_{p,t} + a_{p,t+1} = \frac{R_{t-1} - \pi_t}{\pi_t} a_{p,t} + (1 - \pi_t)(w_t \bar{\eta} l_{p,t} + d_{p,t}) + Tr_t, \quad (2)$$

where $\tau_c$ is the consumption tax rate, $a_{p,t}$ is the real value of their nominal bond holdings, $\pi_t := P_t / P_{t-1}$ is the inflation rate, and $\tau_t$ is the proportional (potentially time-varying) income tax rate.
2.3 Impatient households

The group of impatient households (with subscript \(i\)) has mass \(\nu\) and the same type of preferences as patient households,

\[
U_i = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_i^t \left( \frac{(c_{i,t} - h(l_{i,t}))^{1-\sigma} - 1}{1 - \sigma} \right),
\]

where \(\beta_i \in (0,1)\) is the specific discount factor. To generate a realistic degree of skewness in the wealth distribution, I follow McKay and Reis (2016) and assume that \(\beta_i \leq \beta_p\). In contrast to patient households, impatient households face two types of uninsurable idiosyncratic risk. On the one hand, impatient households are subject to unemployment risk. The employment status \(s_t\) follows a first-order Markov chain with transitions \(\pi(s_{t+1}|s_t, z_{t+1}, z_t)\) that depend on the aggregate technology level. Therefore, unemployment risk as well as the number of unemployed households varies over the business cycle. On the other hand, impatient households face labor productivity risk that also follows a first-order Markov process which is assumed to be acyclical. Let \(\eta_{i,t}\) denote the stochastic time-varying labor productivity of an impatient household, which evolves according to an autoregressive process,

\[
\log \eta_{i,t} = \rho_c \log \eta_{i,t-1} + \varepsilon_{c,t},
\]

where \(\varepsilon_{c,t} \sim N(0, \sigma_c)\) is the shock term and \(\rho_c \in [0,1)\) measures the persistence of the process.

Conditional on being employed, an impatient household can choose the number of hours to work \(l_{i,t}\) so that individual labor income is given by \(w_t \eta_{i,t} l_{i,t}\). In case of being unemployed, labor income is zero but the household receives unemployment benefits \(b_{i,t}\). I assume that these benefits depend on the current productivity state of the respective household to capture the link between unemployment benefits and previous earnings, which is a reasonable approximation given a sufficiently high persistence of the individual labor productivity.\(^7\)

Impatient households are assumed to not own shares in firms which is in line with the fact that the majority of U.S. households does not directly own any equity (see e.g. Mankiw and Zeldes, 1991). However, impatient households can use risk-free bonds to save. Altogether, this group receives income from three different sources. They get lump-sum transfers \(T_{r,t}\) from the government, they receive returns on bonds at the gross nominal rate \(R_{t-1}\), and they obtain labor income at real wage rate \(w_t\) if they are

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\(^6\)Krusell and Smith (1998) demonstrate that including heterogeneous discount factors significantly helps to match the key features of the U.S. wealth distribution.

\(^7\)Furthermore, I assume that households do not internalize the dependence of unemployment benefits on potential/previous earnings.
employed or unemployment benefits $b_{i,t}^u$. This yields the following period-\(t\) budget constraint,

\[
(1 + \tau_c)c_{i,t} + a_{i,t+1} = \begin{cases} 
R_{i,t-1}a_{i,t} + (1 - \tau_t)w_i\eta_{i,t}l_{i,t} + Tr_t & \text{if employed,} \\
\frac{R_{i,t-1}}{\pi_t}a_{i,t} + (1 - \tau_t)b_{i,t}^u(\eta_{i,t}) + Tr_t & \text{if unemployed.}
\end{cases}
\]

(5)

Moreover, impatient household face the borrowing constraint

\[a_{i,t+1} \geq 0.\]

(6)

### 2.4 Final goods producers

The final consumption good $Y_t$ is produced by a competitive representative firm that combines a continuum of intermediate goods $Y_t(j)$, indexed by $j \in [0, 1]$, taking the input prices $p_t(j)_{j \in [0,1]}$ as given and using the constant returns to scale technology

\[
Y_t = \left( \int_0^1 y_t(j)^{(\theta-1)/\theta}dj \right)^{\theta/(\theta-1)},
\]

(7)

where $\theta > 1$ is the elasticity of substitution between intermediate goods. The maximization problem is then given by

\[
\max_{Y_t, y_t} \Pi_t^F = P_t Y_t - \int_0^1 p_t(j)y_t(j)dj \quad \text{s.t.} \quad (7),
\]

(8)

which yields the demand function for intermediate good $j$,

\[
y_t(j) = \left( \frac{p_t(j)}{P_t} \right)^{-\theta} Y_t,
\]

(9)

and subsequently the price index for the final good

\[
P_t = \left( \int_0^1 p_t(j)^{1-\theta} \right)^{1/(1-\theta)}.
\]

(10)

### 2.5 Intermediate goods producers

Each intermediate good $j$ is produced by a monopolistically competitive firm according to a production function given by

\[
y_t(j) = z_t F(k_t(j), n_t(j)),
\]

(11)
where \( z_t \) is aggregate productivity in period \( t \), and \( k_t(j) \) and \( n_t(j) \) are the effective capital and effective labor inputs of firm \( j \). I assume that \( F: \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+ \) is strictly increasing, twice differentiable in both arguments, exhibits constant returns to scale, and satisfies the Inada conditions. Aggregate productivity follows an AR(1) process, given by

\[
z_t = \bar{z} \left( \frac{z_{t-1}}{\bar{z}} \right)^{\rho_z} \exp(\epsilon_{z,t}), \tag{12}\]

where \( \epsilon_{z,t} \sim \mathcal{N}(0, \sigma_z^2) \) is an exogenous shock term and \( \rho_z \in [0, 1) \) measures the persistence of the shock.

Intermediate producers rent capital and hire labor services in competitive markets at rates \( r^K_t \) and \( w_t \), respectively. Since intermediate goods are imperfect substitutes, each intermediate producer sells its output in a monopolistically competitive market and sets price \( p_t(j) \) for its output. The maximization problem of intermediate goods firms, therefore, consists of choosing price level \( p_t(j) \), and production inputs \( k_t(j) \) and \( n_t(j) \) to maximize profits \( \Pi_{I,t}(j) \) subject to price adjustment costs and the given demand for the respective intermediate output. Formally, the producer of intermediate good \( j \) solves the following problem

\[
\max_{p_t(j), k_t(j), n_t(j)} \Pi_{I,t}(j) = D_{I,t}(j) + \mathbb{E}_{t} \sum_{s=1}^{\infty} \phi_{t,t+s} D_{I,t+s}(j), \tag{13}\]

subject to (9) and (11), where \( \phi_{t,t+1} \) is the stochastic discount factor. Since patient households are the sole owner of all firms, the stochastic discount factor is defined as

\[
\phi_{t,t+1} = \beta \frac{\partial U_p}{\partial c_{p,t+1}} = \beta \left( \frac{c_{p,t+1} - h(l_{p,t+1})}{c_{p,t} - h(l_{p,t})} \right)^{-\sigma}. \tag{14}\]

Dividends of intermediate goods producers are defined as

\[
D_{I,t}(j) := \frac{p_t(j)}{P_t} y_t(j) - w_t n_t(j) - r^K_t k_t(j) - \frac{\varphi_p}{2} \left( \frac{p_t(j)}{\bar{p}_t(j)} - 1 \right)^2 y_t(j) - \kappa, \tag{15}\]

where \( \varphi_p \geq 0 \) determines the magnitude of price adjustment costs, \( \bar{\pi} \) is the steady state inflation rate and \( \kappa \geq 0 \) are fixed costs of production. This form of quadratic price adjustment cost is based on the idea of Rotemberg (1982), while the specific functional form stems from Ireland (1997).
I assume symmetry among firms such that all firms charge the same prices and choose the same amounts of capital and labor services. This yields the following static first-order conditions for the factor prices

\[ r^K_t = z_t \Psi_t \mathcal{F}_k(k_t, n_t), \quad (16) \]
\[ w_t = z_t \Psi_t \mathcal{F}_n(k_t, n_t), \quad (17) \]

where \( \mathcal{F}_x \) is the first derivative of the production function with respect to the input factor in the subscript and \( \Psi_t \) are real marginal costs. The first-order condition for the price level gives the following dynamic equation,

\[ \varphi_p \left( \frac{\pi_t}{\bar{\pi}} - 1 \right) \frac{\pi_t}{\bar{\pi}} = 1 - \theta + \theta \Psi_t + \varphi_p \mathbb{E}_t \left\{ \phi_{t,t+1} \left( \frac{\pi_{t+1}}{\bar{\pi}} - 1 \right) \frac{\pi_{t+1}}{\bar{\pi}} \frac{Y_{t+1}}{Y_t} \right\}, \quad (18) \]

which is also known as the New Keynesian Phillips Curve.

### 2.6 Capital good producers

There is a representative firm that owns the capital stock and rents it to intermediate goods firms at the competitive price \( r^K_t \). It maximizes profits by investing in new capital subject to quadratic adjustment costs. Formally, its maximization problem is given by

\[
\max_{K_{t+1},I_t} \Pi_{C,t} = D_{C,t} + \mathbb{E}_t \sum_{s=1}^{\infty} \phi_{t,t+s} D_{C,t+s} \\
\text{s.t. } K_{t+1} = (1 - \delta) K_t + I_t, \quad (19) \]

where \( I_t \) denotes investment and \( \delta \) is the depreciation rate of capital. Dividends are given by

\[
D_{C,t} := r^K_t K_t - I_t - \frac{\varphi_c}{2} \left( \frac{I_t}{K_t} - \delta \right)^2 K_t, \quad (21) \]

where \( \varphi_c \geq 0 \) determines the magnitude of capital adjustment costs. The first order conditions are given by

\[
q_t = 1 + \varphi_c \left( \frac{I_t}{K_t} - \delta \right), \quad (22) \]
\[
q_t = \mathbb{E} \phi_{t,t+1} \left[ r^K_{t+1} - \frac{\varphi_c}{2} \left( \frac{I_{t+1}}{K_{t+1}} - \delta \right)^2 + \varphi_c \left( \frac{I_{t+1}}{K_{t+1}} - \delta \right) \frac{I_{t+1}}{K_{t+1}} + q_{t+1} (1 - \delta) \right], \quad (23) \]

where \( q_t \) is the Lagrange multiplier for the law of motion of capital.
2.7 Government and central bank

The fiscal authority collects revenues from income and consumption taxes, and issues bonds $B_{t+1}$, to finance its interest payments, government consumption $G_t$, unemployment benefits $Tr^u_t$ and lump-sum fiscal transfers $Tr_t$. The government budget constraint is therefore given by

$$\frac{R_{t-1}}{\pi_t}B_t + G_t + Tr^u_t + Tr_t = B_{t+1} + \tau_t(w_t n_t + D_t) + \tau_c C_t,$$  \hspace{1cm} (24)

where $w_t n_t$ is aggregate labor income and $D_t$ are aggregate dividends.

In the sections below, I analyze the effects of two aggregate fiscal shocks, a government consumption shock and a fiscal transfer shock. In general, I assume that both shocks are deficit-financed. To ensure that government debt remains non-explosive, at least one additional fiscal instrument has to adjust. In the benchmark scenario, I assume that lump-sum transfers respond to government debt when the economy is hit by a government consumption shock, and that government consumption adjusts when the economy is hit by a fiscal transfer shock. The set of equations for the government consumption shock is given by

$$G_t = \bar{G} \left( \frac{G_{t-1}}{G} \right)^{\rho_T} \exp(\varepsilon_{T,t}),$$ \hspace{1cm} (25)

$$Tr_t = \bar{Tr} \left( \frac{B_t}{B} \right)^{\gamma_T},$$ \hspace{1cm} (26)

while for the fiscal transfer shock the set of equations is given by

$$Tr_t = \bar{Tr} \left( \frac{Tr_{t-1}}{Tr} \right)^{\rho_T} \exp(\varepsilon_{T,t}),$$ \hspace{1cm} (27)

$$G_t = \bar{G} \left( \frac{B_t}{B} \right)^{\gamma_T},$$ \hspace{1cm} (28)

where $\varepsilon_{T,t} \sim \mathcal{N}(0, \sigma^2_T)$ is the shock term in each scenario, and $\rho_T \in [0, 1)$ measures the persistence of the respective shock. The parameter $\gamma_T$ determines the adjustment of the respective variable to government debt deviations from its steady state.\footnote{For a better comparison, I assume that the shock persistence, the shock variance and the adjustment speed in the fiscal rule are the same across experiments.}

Unemployment benefits $b^u_{i,t}(\eta_{i,t})$ are given by the replacement rate $\xi \in [0, 1)$ times potential labor income,

$$b^u_{i,t}(\eta_{i,t}) = \xi w_t \eta_{i,t} l_{i,t},$$ \hspace{1cm} (29)
where potential labor income is defined as the labor income of an employed agent with the same idiosyncratic productivity state. Then, total benefits paid to unemployed households amount to

\[ Tr_u = \zeta_w \int \eta_l d\Phi(i, (u, \eta_l, a_l)). \] (30)

The central bank follows a Taylor rule, in which the nominal interest rate \( R_t \) responds to the deviation of current inflation from its steady state and the deviation of the lagged nominal interest rate from its steady state. Formally, the rule is given by

\[ R_t = \bar{R} \left( \frac{\pi_t}{\bar{\pi}} \right)^{\phi_\pi} \left( \frac{R_{t-1}}{\bar{R}} \right)^{\phi_R}, \] (31)

where \( \phi_\pi > 1 \) captures the central bank’s reaction to inflation deviations and \( \phi_R > 0 \) measures the degree of policy inertia.

### 2.8 Market clearing and aggregation

The labor market clears when

\[ n_t = \int \eta_l d\Phi(i, (e, \eta_t, a_t)) + (1 - \nu)\bar{\eta}l_{p,t}, \] (32)

at the wage rate given in (17). The market for capital services clears when

\[ K_t = k_t, \] (33)

at the rental rate given in (16). The bonds market clears when

\[ B_{t+1} = \int a_{t+1} d\Phi(i, (s_t, \eta_t, a_t)) + (1 - \nu)a_{p,t+1}. \] (34)

Finally, the goods market clears when

\[ Y_t = C_t + I_t + G_t + \frac{\phi_p}{2} \left( \frac{\pi_t}{\bar{\pi}} - 1 \right)^2 Y_t + \frac{\phi_c}{2} \left( \frac{I_t}{K_t} - \delta \right)^2 K_t + \kappa. \] (35)

Aggregate consumption is defined as the weighted average of individual consumption,

\[ C_t = \int c_t d\Phi(i, (s_t, \eta_t, a_t)) + (1 - \nu)c_{p,t}. \] (36)
Total dividends are the sum of dividends from intermediate and capital goods producers,

\[ D_t = D_{C,t} + D_{I,t}, \quad (37) \]

as well as the weighted average of individual dividends received,

\[ D_t = (1 - v)D_{p,t}, \quad (38) \]

taken into account that only patient households own firm shares.

### 2.9 Equilibrium

**Definition 1.** An equilibrium in this economy is defined as a sequence of aggregate quantities \( \{Y_t, C_t, D_t, l_t, K_t, k_t, I_t, \Psi_t, D_{C,t}, D_{I,t}, d_p\} \) \( t \in \mathbb{N} \), prices \( \{\pi_t, w_t, r^K_t, q_t\} \) \( t \in \mathbb{N} \), distributions \( \Phi_t(H, E) \) \( t \in \mathbb{N} \), decision rules \( \{c_t(H, E), l_t(H, E), a_t+1(H, E)\} \) \( t \in \mathbb{N} \) and government choices \( \{R_t, \tau_t, Tr_t, Tr^u_t, B_t, B_{t+1}\} \) \( t \in \mathbb{N} \) such that

1. patient households maximize (1) subject to (2),
2. impatient households maximize (3) subject to (5) and (6),
3. final goods producers behave optimally according to (9) and (10),
4. intermediate goods producers maximize (13) subject to (9) and (11),
5. capital good producers maximize (19) subject to (20) and (21),
6. fiscal policy respects (24), (29), and (25) and (26), or (27) and (28), respectively,
7. monetary policy follows (31),
8. markets clear according to (32)-(35),
9. aggregation identities (36)-(38) hold,
10. the aggregate law of motion is induced by the exogenous stochastic processes for idiosyncratic and aggregate risk as well as the households’ decision rules.

### 3 Calibration

In this section, I describe how I map the model economy to the data. Since I am particularly interested in the business cycle properties of fiscal policy, the model is calibrated to the U.S. economy over the time period 1970q1 to 2017q4. One model period is a quarter. Table 1 summarizes the parameter choices.
3.1 Households

The patient households’ discount factor $\beta_p$ is set to 0.9925, implying a steady state annual real interest rate of 3 percent. The discount factor of impatient households $\beta_i$ is set to 0.9698 so that the bottom 20 percent in the wealth distribution have a total net worth of zero.

The functional form for the disutility of labor is given by

$$h(l) = \chi \eta l^{1+1/\gamma} \frac{1}{1+1/\gamma},$$  \hspace{1cm} (39)

where $\gamma$ is the Frisch elasticity of labor supply and $\chi$ is a scaling parameter. The inclusion of $\eta$ implies that households only adjust their labor supply to changes in the wage rate per efficiency unit and not to changes in their idiosyncratic productivity.

I choose standard values from the literature. In particular, I set the risk aversion parameter in the utility function $\sigma$ to 2 as, for example, in Kindermann and Krueger (2014). The Frisch elasticity of labor supply $\gamma$ is equal to 0.5 as in Bayer et al. (2015). The disutility of labor $\chi$ is set to 4.159 so that hours worked are 0.5 in steady state. Following McKay and Reis (2016), I choose $\nu$ to be 0.8, implying that patient households make up for 20 percent of the economy.

3.2 Firms

Intermediate goods firms are assumed to produce according to a Cobb-Douglas production function

$$F(k_t, n_t) = k_t^\alpha n_t^{1-\alpha},$$  \hspace{1cm} (40)

where $\alpha \in [0, 1]$ measures the output elasticity of capital. $\alpha$ is then set to 0.33, implying a labor share of income of about 60 percent. The elasticity of substitution between intermediate goods $\theta$ is 11, such that the steady state markup over marginal costs is 10 percent. The depreciation rate $\delta$ is 0.025 to have an annual depreciation on capital equal to 10 percent. Steady state inflation rate $\bar{\pi}$ is set to 1.005 for an annual inflation rate of 2 percent. Fixed cost parameter $\kappa$ is chosen to guarantee a steady state dividend-to-output ratio of 3.8 percent. The parameter that governs the magnitude of the price adjustment costs $\phi_p$ is set to 117. Given the steady state markup, this is equivalent to price changes once every four quarters in a linearized Calvo framework. Finally, the capital adjustment parameter $\phi_c$ is chose to be 15, which is well in the range of values typically used in the literature.
3.3 Government and central bank

The steady state values of three fiscal variables are calibrated based on averages over the sample period considered in this paper. In particular, the share of government consumption in aggregate output is set to 20 percent. The ratio of fiscal transfers (including unemployment benefits) to output is 11 percent, while the annualized government debt to output ratio is 40 percent. The consumption tax rate is set to 11.5 percent, following Altig et al. (2001) so that the steady state of the income tax rate is then set such that the government budget constraint is satisfied. The adjustment parameter in the fiscal rules $\gamma_T$ is set to −0.4, which is in the interval of values that are estimated in the literature. Following Blank and Card (1991), I set the replacement rate $\xi$ to 10 percent.

For the Taylor rule, I pick standard values from the literature. The reaction parameter for inflation deviations $\phi_\pi$ is equal to 1.5 and smoothing parameter $\phi_R$ is set to 0.81.

3.4 Aggregate shocks

I derive the properties of the aggregate technology shock from the utilization-adjusted TFP series from Fernald (2014). In particular, I construct an index from the calculated growth rates for the time period 1970Q1 to 2017Q4 and take the natural logarithm of this index. The series is then detrended by a one-sided HP filter, as suggested by Watson and Watson (1999), with a smoothing value of 1600. The estimated persistence coefficient $\rho_z$ is equal to 0.7533 and the standard deviation of the shock term $\sigma_z$ is equal to 0.009. Finally, I discretize the process by the Rouwenhorst method into a two-state Markov chain. For the fiscal shocks, I follow Fernández-Villaverde et al. (2015) and set $\rho_T$ to 0.8 and $\sigma_T$ to 0.0025.

3.5 Idiosyncratic risk

Households face two types of idiosyncratic risk, countercyclical unemployment risk and, conditional on being employed, acyclical productivity risk.

---

9 See the appendix for the respective data series.
10 Leeper et al. (2010) find a value of −0.23 for government consumption and value of −0.5 for fiscal transfers. I eventually chose −0.4 because this value always ensured boundedness of government debt and convergence of my computations.
11 This method is particularly useful for persistent processes. More details on this method can be found in Kopecky and Suen (2010).
Table 1: Calibrated parameters

<table>
<thead>
<tr>
<th>Param</th>
<th>Explanation</th>
<th>Value</th>
<th>Target/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>β_p</td>
<td>Discount factor of patient HHs</td>
<td>0.9925</td>
<td>Annual real interest rate of 3%</td>
</tr>
<tr>
<td>β_i</td>
<td>Discount factor of impatient HHs</td>
<td>0.9698</td>
<td>Bottom 20% has zero net worth</td>
</tr>
<tr>
<td>σ</td>
<td>Risk aversion parameter</td>
<td>2.0</td>
<td>Kindermann and Krueger (2014)</td>
</tr>
<tr>
<td>γ</td>
<td>Frisch elasticity of labor supply</td>
<td>0.5</td>
<td>Bayer et al. (2015)</td>
</tr>
<tr>
<td>χ</td>
<td>Disutility of labor</td>
<td>4.159</td>
<td>Steady state labor supply of 0.5</td>
</tr>
<tr>
<td>v</td>
<td>Share of impatient HHs</td>
<td>0.8</td>
<td>McKay and Reis (2016)</td>
</tr>
<tr>
<td>η</td>
<td>Productivity of patient HHs</td>
<td>9.2230</td>
<td>62.9% of total earnings (SCF avg.)</td>
</tr>
<tr>
<td>α</td>
<td>Capital share of income</td>
<td>0.33</td>
<td>60% labor share of income</td>
</tr>
<tr>
<td>δ</td>
<td>Depreciation rate of capital</td>
<td>0.025</td>
<td>Annual depreciation of 10%</td>
</tr>
<tr>
<td>θ</td>
<td>Price elasticity of demand</td>
<td>11.0</td>
<td>Steady state mark up of 10%</td>
</tr>
<tr>
<td>ϕ_p</td>
<td>Rotemberg adj. cost coefficient</td>
<td>117.0</td>
<td>approx. 75% of firms adjusting</td>
</tr>
<tr>
<td>ϕ_c</td>
<td>Capital adjustment costs</td>
<td>15.0</td>
<td>Literature</td>
</tr>
<tr>
<td>κ</td>
<td>Fixed production cost</td>
<td>0.2592</td>
<td>Steady state dividends of 2.5%</td>
</tr>
<tr>
<td>π</td>
<td>SS inflation rate</td>
<td>1.005</td>
<td>Annual inflation rate of 2%</td>
</tr>
<tr>
<td>φ_R</td>
<td>Inflation coefficient in Taylor rule</td>
<td>1.5</td>
<td>Standard value</td>
</tr>
<tr>
<td>γ_T</td>
<td>Fiscal rule</td>
<td>−0.4</td>
<td>see Text</td>
</tr>
<tr>
<td>B/Y</td>
<td>SS gov. debt-GDP-ratio (annually)</td>
<td>0.40</td>
<td>Data</td>
</tr>
<tr>
<td>G/Y</td>
<td>SS gov. spending-GDP-ratio</td>
<td>0.20</td>
<td>Data</td>
</tr>
<tr>
<td>Tr/Y</td>
<td>SS transfer-GDP-ratio</td>
<td>0.11</td>
<td>Data</td>
</tr>
<tr>
<td>τ</td>
<td>Income tax rate</td>
<td>0.3532</td>
<td>Budget balance</td>
</tr>
<tr>
<td>τ_c</td>
<td>Consumption tax rate</td>
<td>0.115</td>
<td>Altig et al. (2001)</td>
</tr>
<tr>
<td>ξ</td>
<td>Replacement rate</td>
<td>0.10</td>
<td>Blank and Card (1991)</td>
</tr>
<tr>
<td>ρ_e</td>
<td>Persistence of TFP shock term</td>
<td>0.0090</td>
<td>Fernald (2014)</td>
</tr>
<tr>
<td>σ_e</td>
<td>SD of TFP shock term</td>
<td>0.0983</td>
<td>Fernald (2014)</td>
</tr>
<tr>
<td>σ_e</td>
<td>Persistence of labor prod. shock</td>
<td>0.1476</td>
<td>Heathcote et al. (2004)</td>
</tr>
<tr>
<td>σ_T</td>
<td>SD of labor prod. shock</td>
<td>0.8000</td>
<td>Heathcote et al. (2004)</td>
</tr>
<tr>
<td>σ_T</td>
<td>Persistence of fiscal shock</td>
<td>0.0025</td>
<td>Fernández-Villaverde et al. (2015)</td>
</tr>
<tr>
<td>σ_T</td>
<td>SD of fiscal shock</td>
<td>0.0025</td>
<td>Fernández-Villaverde et al. (2015)</td>
</tr>
</tbody>
</table>

3.5.1 Unemployment risk

For idiosyncratic unemployment risk, I follow the setup of Krusell and Smith (1998, 1999), where unemployment shocks and the aggregate technology shock are correlated. It is assumed that the idiosyncratic shocks each satisfy a law of large numbers so that individual risk averages out and aggregate shocks are the only source of aggregate uncertainty. This implies that the unemployment rate is only a function of the aggregate technology level. Let $u_g$ denote the unemployment rate in the good technology state and $u_b$ in the bad technology state.
Furthermore, let $\pi_{z,z'}^{s,s'} := \pi(s_{t+1}|s_t, z_{t+1}, z_t)$ denote the joint probability of transition from state $(z,s)$ today to $(z',s')$ tomorrow. These transition probabilities then have to satisfy the following two restrictions

\[
\begin{align*}
\pi_{z,u}^{z,u'} + \pi_{z,e}^{z,e'} &= \pi_{e,u}^{z,u'} + \pi_{e,e}^{z,e'} = \pi_{z,z'}^{z,z'}, \\
u_z \frac{\pi_{z,u}^{z,u'}}{\pi_{z,z'}} + (1 - \nu_z) \frac{\pi_{e,u}^{z,u'}}{\pi_{z,z'}} &= u_{z'},
\end{align*}
\]  

(41)  

(42)

for all $(z,z')$, where $\pi_{z,z'}^{z,z'}$ are the marginal probabilities of transition from $z$ to $z'$. To uniquely pin down the four $2 \times 2$ joint transition matrices, I follow Krusell and Smith (1998, 1999) and impose the following assumptions:

(i) the average duration of an unemployment spell is 1.5 quarters in the good technology state and 2.5 quarters in the bad state,

(ii) the unemployment rate is 4 percent in the good technology state and 10 percent in the bad state,

(iii) $\frac{\pi_{g,u}^{g,u}}{\pi_{g,g}} = 1.25$ and $\frac{\pi_{b,u}^{b,u}}{\pi_{b,b}} = 0.75$.

Using the fact that $\pi_{z,u}^{z,u'} + \pi_{z,e}^{z,e'} = 1$ for all $(z,z')$, one can use (i) to find $\pi_{u,u}^{g,b} / \pi_{g,b} = 3/4$ and $\pi_{u,u}^{b,b} / \pi_{b,b} = 1/4$. Using (42) for the pairs $(g,b)$ and $(b,g)$, one further obtains $\pi_{u,u}^{g,b} / \pi_{g,b} = 1/60$ and $\pi_{u,u}^{b,g} / \pi_{b,g} = 7/96$. The remaining parameters can be obtained with the property that all row probabilities in the transition matrices must add up to one.

Unemployment risk is therefore given by the following four $2 \times 2$ matrices,

\[
\begin{pmatrix}
0.6000 & 0.4000 \\
0.0445 & 0.9555
\end{pmatrix},
\]  

(43)

for the transition $(z,z') = (z_b,z_b)$, where the first row is the individual state of unemployment and the second row is the state of employment,
for the transition \((z_g, z_g)\),

\[
\begin{pmatrix}
0.7500 & 0.2500 \\
0.0729 & 0.9271
\end{pmatrix}
\]

(45)

for the transition \((z_g, z_b)\),

\[
\begin{pmatrix}
0.2500 & 0.7500 \\
0.0167 & 0.9833
\end{pmatrix}
\]

(46)

for the transition \((z_b, z_g)\).\(^{12}\)

### 3.5.2 Labor productivity risk

To determine the parameters of the idiosyncratic labor productivity shock \((\rho_e, \sigma_e)\), I make use of the estimates of Heathcote et al. (2004), who use annual data from the Panel Study of Income Dynamics (PSID). For the period 1967 to 1996, their estimates are \(\rho^a_e = 0.9426\) and \(\sigma^a_e = 0.1476\), where the superscript indicates annual values.

I follow Krueger et al. (2016) in their procedure to transform annual values into their quarterly counterparts. This implies that \(\rho_e = (\rho^a_e)^{0.25} = 0.9853\) to ensure that the annual persistence of productivity risk does not change. Similarly, I impose that the quarterly variance of risk is the same as the annual variance, which is achieved by the following condition,

\[
\frac{(\sigma^a_e)^2}{1 - \rho^a_e} = \frac{\sigma^2_e}{1 - \rho_e}, \quad (47)
\]

yielding \(\sigma_e = 0.1476\). The shock is then discretized into a four-state Markov chain by the Rouwenhorst method.

The production level of patient households, on the other hand, is fixed as this group is assumed to have access to insurance markets that prevent them from productivity risk. I set this value to 9.223 which then yields a steady state earnings share of 62.9 percent for the top quintile of the earnings distribution which is determined by taking the average of various waves of the Survey of Consumer Finance (SCF).

### 3.6 Numerical Implementation

The model is solved using a solution method that combines a Krusell and Smith (1998)-type algorithm with a time iteration procedure. See Appendix B for more details.

\(^{12}\)The numbers are rounded to four digits.
### Table 2: Business cycle statistics

<table>
<thead>
<tr>
<th></th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>1.00</td>
<td>0.88</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.68</td>
<td>0.98</td>
</tr>
<tr>
<td>Investment</td>
<td>2.81</td>
<td>0.95</td>
</tr>
<tr>
<td>Wage rate</td>
<td>0.77</td>
<td>0.99</td>
</tr>
<tr>
<td>Labor supply</td>
<td>0.23</td>
<td>0.99</td>
</tr>
<tr>
<td>Gov. debt</td>
<td>5.29</td>
<td>−0.34</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.04</td>
<td>−0.31</td>
</tr>
<tr>
<td>Nom. interest rate</td>
<td>0.52</td>
<td>−0.25</td>
</tr>
<tr>
<td># Constrained HHs</td>
<td>0.30</td>
<td>−0.36</td>
</tr>
</tbody>
</table>

*Notes: Model with TFP shocks only. For a more detailed data description, see Appendix.*

## 4 Results

In this section, I present the results of the baseline model given the parametrization presented in the previous section. I start by showing the cyclical and distributional properties of the model and compare them to the corresponding moments retrieved from data. Afterwards, I analyze the effects of the two government expenditure shocks in different states of the business cycle. In particular, I present impulse response functions (IRF) of aggregate variables, calculate fiscal multipliers and show the distributional effects of these shocks.

### 4.1 Cyclical properties

Table 2 compares second moments generated by the model and the counterparts from U.S. data (based on HP-filtered series with smoothing parameter 1600) for the period 1970q1 to 2017q4. Column 2 to 4 report the model-generated moments (without fiscal shocks), namely the relative standard deviation with respect to output, the contemporaneous correlation with output and the first-order autocorrelation of eight aggregate variables. Columns 5 to 7 report the respective moments from the data. Note that none of these values is targeted in the calibration exercise. However, the model replicates the empirical moments quite well. It is consistent with the large variability of investment and public debt to output as well as the relatively lower volatility of consumption, the wage rate and inflation. Labor supply and the nominal interest rate are somewhat off as these variables are not volatile enough in the model.

The model also produces negative output correlations for government debt, inflation and the nominal interest rate although the data series either show acyclical be-
behavior or a positive correlation with output. The model-produced autocorrelations are mostly in line with the data, so that I conclude that the model does a good job when it comes to producing reasonable cyclical variations. Of course, one could improve the fit even more by incorporating some of these moments into the calibration procedure. This, however, would come at the cost of significantly higher computational time.

Another variable of interest for the following analysis is the number of constrained households. To the best of my knowledge, there is no (quarterly) measure for this variable in the data so that I only report the model-generated moments. Most importantly, Table 2 shows a negative correlation between output and the share of borrowing constrained households, implying that this share is higher when the economy is in a recession. The fact that households with a binding borrowing constraint have a higher MPC than households that hold positive wealth is one of the crucial results of this paper as this is key mechanism that generates the state-dependent multipliers in this setting.

### 4.2 Distributional properties

The distributional properties of the model are summarized in Table 3. In particular, this table reports selected statistics for the earnings and wealth distribution of the United States, drawn from the SCF, and the respective model statistics. Since the model is calibrated for the period between 1970q1 to 2017q4, I include two waves of the SCF.

Two of these values were part of the calibration strategy, namely the earnings share of the top quintile and the wealth share of the bottom quintile. All other moments of the empirical cross-sectional earnings and wealth distributions were not targeted and emerge endogenously from the model.

In general, the model is able to match the empirical distributions considerably well. Since the model does not allow negative earnings or negative wealth, one would need further ingredients or further assumptions to exactly match the lowest quantiles of both distributions. However, since both model values are close to their empirical counterparts, I refrain from those.

Overall, the model is able to generate the skewed earnings and wealth distribution of the United States and therefore, depicts a realistic environment for the question at hand.

### 4.3 Computational experiment

After having provided that the model economy delivers a good description of both the cyclical and distributional properties of the United States data, I now proceed to implement the main analysis of the paper. In this experiment, I simulate government
### Table 3: Income and wealth distributions: Data vs. model

<table>
<thead>
<tr>
<th></th>
<th>Earnings</th>
<th></th>
<th></th>
<th>Wealth</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1</td>
<td>2.7</td>
<td>−0.4</td>
<td>−0.1</td>
<td>0.0</td>
<td>−0.4</td>
</tr>
<tr>
<td>Q2</td>
<td>3.9</td>
<td>3.2</td>
<td>4.2</td>
<td>1.8</td>
<td>1.7</td>
</tr>
<tr>
<td>Q3</td>
<td>10.7</td>
<td>12.5</td>
<td>11.7</td>
<td>6.1</td>
<td>5.7</td>
</tr>
<tr>
<td>Q4</td>
<td>19.8</td>
<td>23.3</td>
<td>20.8</td>
<td>15.7</td>
<td>13.4</td>
</tr>
<tr>
<td>Q5</td>
<td>62.9</td>
<td>61.4</td>
<td>63.5</td>
<td>76.4</td>
<td>79.6</td>
</tr>
<tr>
<td>Gini</td>
<td>0.61</td>
<td>0.63</td>
<td>0.64</td>
<td>0.75</td>
<td>0.78</td>
</tr>
</tbody>
</table>

*Notes:* Data is taken from the Diaz-Giménez et al. (1997) and Diaz-Giménez et al. (2011). The model-generated moments refer to the steady state of the model.

consumption shocks as well as fiscal transfer shocks in different states of the business cycle.

The implementation is as follows. First, I simulate the model for 5600 periods without any fiscal shock so that the technology shock is the only source of aggregate uncertainty. The first 500 periods are the burn-in phase, which ensures that the stochastic steady state of the model is reached. The following 5100 periods are then used to determine an average expansion and an average recession. For that purpose, I use aggregate output as the determinant and construct an output distribution for the simulated economy. Then, I choose the midpoint of the 20th percentile for the average recession and the midpoint of the 80th percentile for the average expansion. Equipped with these points, I simulate the model again until I reach them and increase the respective variable by one percent.

The IRFs in the next sections are then relating the simulation with the respective shock to the counterfactual simulation, in which no fiscal shock occurred. Formally, the IRF for variable $x_t$ is given by

$$IRF(x_t) = \frac{x_t}{x_t^*} - 1,$$

where the asterisk denotes the value for $x_t$ in counterfactual simulation.

In the analysis below, the impact period of recession scenario is characterized by an aggregate output level that is 3.35 percent smaller than in the expansionary scenario. The capital stock is 1.91 percent smaller, while the wage rate is 2.59 percent lower. The share of households at the borrowing constraint is 35.6% when the shocks hit in a recession compared to 19.7% in the expansion. The technology state is in the high state in the expansionary scenario, whereas it is in the low state at the beginning of the recessionary scenario.
4.4 Impulse responses

4.4.1 Government consumption shock

Figure 1 reports the IRFs to a one percent increase in government consumption. The solid line depicts the recession scenario, whereas the dashed line displays the expansion scenario. Starting with the recession scenario, the increase in government consumption induces a demand effect on impact that leads to rises in output, consumption, hours worked and the real wage. This demand effect is caused by price adjustment costs that create a wedge between the optimal price and the actual price that is set by intermediate firms so that there is a rightward shift in the labor demand curve. This then leads to an increase in production and subsequently to an increase in the real wage. The magnitude of this demand effect translates into a positive output response of about 0.15 percent.

Inflation rises by about 0.03 percent on impact which leads to a strong reaction of the nominal interest rate that follows a Taylor rule. The increase in the real interest rate then leads to a strong decline of investment and consequently in capital. Thus, the small crowding-in effect of consumption is contrasted with a strong crowding-out effect of investment. The number of constrained households drops to zero. This shows that the demand effect is especially important for the wealth-poorest households who can now afford to save at least some of their additional labor income.

The initial demand effect is rather short-lived and dies out after about one year. Afterwards output, consumption, hours worked and the real wage go below zero, implying that these variables are now lower than in the scenario without the shock occurring. This is basically because the strong crowding-out of investment causes the capital stock to decline significantly. The number of constrained households begins to increase, and this is because of two reasons. First, the real wage decreases quickly and therefore labor income. On the other hand, since the shock is deficit-financed with fiscal transfers adjusting downwards, low-income households eventually face a considerable negative income shock so that they start to dissave.

In the expansion scenario, an increase in public consumption also induces a demand effect on impact, although it is not as pronounced as in the recession. Here, output increases by about 0.10 percent while the consumption response is effectively zero. The increase in inflation is significantly smaller in this state so that both nominal and real interest rate do not respond as strongly as in the recessionary state. Thus, the crowding-out of investment is not as severe and the capital stock is reduced less markedly. The fraction of constrained households drops to zero as well but since the overall number of households during the expansion state is smaller than in the recessionary state, the effect on consumption turns out to be smaller as well.
Figure 1: IRFs to one percent increase in government consumption

- Output
- Consumption
- Hours worked
- Real Wage
- Inflation
- Nominal rate
- Investment
- Capital
- % Constrained
- Transfers
- Gov. debt
- Gov. spending
4.4.2 Fiscal transfer shock

Figure 2 displays the IRFs to a one percent increase in fiscal transfers. In the recession scenario (blue solid lines), the increase in transfers induces a small demand effect, resulting in a 0.01 percent increase in output. While hours worked and the real wage rise in a similar magnitude, the response of private consumption of 0.08 percent is much more pronounced. This results is driven by the considerable amount of households at or close to the borrowing constraint, which have a particular high MPC and, therefore, spend a large part of their additional income on consumption. However, as can be seen in Figure 2, the number of constrained households goes to zero, which implies that the additional income induces the wealth-poor to save as well. Inflation and subsequently the nominal interest rate rise on impact, accompanied by a substantial crowding-out of investment.

After the very short-lived stimulation of the economy, output, hours worked and the real wage begin to decrease and fall below the level of the counterfactual. This drop is even more amplified by the gradual decrease of government consumption, which adjusts downwards to keep government debt on a non-explosive path. Aggregate consumption declines as well, albeit at a slower pace, and drops below zero after one year. The significant drop in investment leads to a strong reduction of the capital stock. After about two years, this downward trend is reversed and the net capital response approaches zero. The share of constrained households begins to rise after two quarters and finally exceeds the one of the counterfactual scenario.

In the expansion scenario, the demand effect on impact is substantially smaller with output rising by only 0.005 percent. The increase in private consumption is still relatively large in comparison to output but significantly smaller than in the recession scenario. However, also the crowding-out of investment is less pronounced, implying that the reduction in the capital stock is smaller. This in turn leads to slightly higher net responses of output, wages and labor supply after about five quarters. The share of borrowing constrained households does not go to zero on impact which can be explained by the considerably weaker demand effect so that for some households, the income shock through the transfer increase is not sufficient to afford assets.

To summarize, the transfer shock induces a small demand effect which is larger when the economy is in a recession. The differences are mainly driven by the different number of credit-constrained households, which amplifies the response of basically every aggregate variable. After the initial boost, the transfer shock leads in both scenarios to a contraction compared to the counterfactual, which ultimately questions the effectiveness of fiscal transfers to stimulate the economy.
Figure 2: IRFs to one percent increase in fiscal transfers

[Diagram showing various economic indicators over time, including Output, Consumption, Hours worked, Real Wage, Inflation, Nominal rate, Investment, Capital, % Constrained, Gov. consumption, Gov. debt, and Transfers.]
Table 4: Benchmark multipliers

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4.5 Multipliers

The quantitative effects of fiscal shocks are generally evaluated in the form of multipliers. Following Mountford and Uhlig (2009), I report present-value multipliers which incorporate the entire path of responses and discount them properly.\(^{13}\) The present-value of additional output over a \(k\)-horizon that is generated by a change in the present-value of the respective fiscal instrument \(FI\) is given by

\[
m_t^k = \frac{\sum_{j=0}^{k} \beta^j \left( \frac{Y_j}{Y_j^*} - 1 \right) Y_j^*}{\sum_{j=0}^{k} \beta^j \left( \frac{FI_j}{FI_j^*} - 1 \right) FI_j^*}, \quad FI \in \{G, Tr\},
\]

where the asterisk indicates the path of the respective variable in the counterfactual situation, in which no shock occurred. The impact multiplier is then the present-value multiplier with \(k = 0\). Table 4 reports the results for the benchmark.

For government consumption, the impact multipliers for both scenarios are modest but clearly below one.\(^{14}\) There are considerable differences in the magnitudes. While the recession multiplier amounts to 0.8, the multiplier of the expansion scenario is only 0.5. After one year, both multipliers almost halved and further decrease as time goes by. Four years after the impact, however, multipliers are still positive.

For fiscal transfers, the impact multipliers are much smaller as could already be seen in the IRFs. Both are slightly positive on impact but turn negative after four quarters. Moreover, the recession multipliers contract at a faster pace than the expansion multipliers.

To sum up, multipliers differ significantly across fiscal instruments and the state of the economy. While both instruments are more effective during recessions, multipliers

\(^{13}\)Since households in the model economy have different discount factors, I take a weighted average, resulting in \(\hat{\beta} = 0.9743\).

\(^{14}\)This is not a surprising result as the textbook New Keynesian model with a representative agent is also generally not able to generate multipliers above unity without further assumptions, see e.g. Gali, López-Salido, and Vallés (2007).
always stay below unity. In terms of stimulating the economy, government consumption seems to be the better choice.

4.6 Distributional effects

Figure 3 shows the consumption response on impact to both fiscal shocks for each quintile of the wealth distribution. The blue bars represent the respective responses during a recession, the red bars stand for the responses during an expansion.

For both shocks, the amount of individual wealth has significant influence on the consumption response on impact. While wealth-poor households raise their level of consumption after a government consumption shock, it falls for the wealth-rich. This is because the bottom quintile solely consists of households at the borrowing constraint which are characterized by a high MPC out of additional income. Since the share of borrowing constrained households is countercyclical, the MPC of the bottom quintile is lower during expansions so that the impact response of consumption is lower in this scenario.

A similar pattern is displayed in the right panel of Figure 3. When facing a fiscal transfer shock, all households increase their consumption. The heterogeneity in the size of the increase is again attributed to the heterogeneity in the MPC. Moreover, since the transfer shock does not trigger a substantial demand effect, the relative increase in the income of the wealth-rich is basically zero. And given that their level of consumption is already relatively high, a substantial increase is simply not feasible.
5 Alternative fiscal rules

In the baseline model, the government consumption shock is deficit-financed with fiscal transfers adjusting to the level of government debt, whereas the fiscal transfer shock is deficit-financed with government consumption adjusting. In this section, I investigate the effects of these shocks and the differences to the baseline scenarios when the income tax rate adjusts to the fiscal deficit. In particular, the fiscal rule for both shocks is now

\[ \tau_t = \bar{\tau} \left( \frac{B_t - 1}{B} \right)^{\gamma_T}, \]

where the \( \gamma_T \) again determines the speed of adjustment. \( \gamma_T \) is set to 0.4, equivalent to the baseline case. I present IRFs to the two shocks, the respective multipliers and the distributional effects in the following.

5.1 Impulse responses

5.1.1 Gov. consumption shock

Figure 4 reports the IRFs to a government consumption shock, where the income tax rate instead of fiscal transfers adjusts to public debt. As in the baseline scenario, the increase in government consumption leads to an initial demand effect that raises output, hours worked and the real wage. When the shock hits in a recession, output rises by about 0.13 percent which is slightly below the initial response in the baseline. The same applies to hours worked and the wage rate. Following the fiscal shock, inflation rises by about 0.03 percent, leading to a strong response of the nominal interest rate. Consumption, however, does not increase on impact but slightly decreases, which is in contrast to the baseline scenario. The shock also leads to a crowding-out of investment, leading to a strong decline in the capital stock. The share of households at the borrowing constraints goes to zero as the initial demand effect generates a substantial increase in income that induces even the wealth-poor to save.

The demand effect vanishes as quickly as in the baseline but the more pronounced decline of capital leads to a more severe drop in output. This is because now the income tax rate rises as a reaction to the increase in the fiscal deficit, leading to a much stronger crowding-out of labor supply and private consumption.

When the shock hits during an expansion, a similar but somewhat weaker initial demand effect is generated. Output rises by about 0.06 percent, which is considerably lower than in the recession scenario, but also compared to the rise of about 0.1 percent in the baseline expansion. The same applies for the responses of consumption, hours worked and the real wage. Since the real interest rate does not increase as much as in
Figure 4: IRFs to one percent increase in government consumption (Tax rule)
the recessionary state, the crowding-out of investment is less severe, and thus, capital declines by a smaller amount. The number of borrowing constrained households falls to zero, indicating that the increase in income is sufficiently large so that even the wealth-poor begin to save.

As in the recession scenario, the demand effect dies out after four quarters caused by the substantial reduction of the capital stock. The responses of output, consumption, hours worked and the real wage are lower as in the counterfactual situation but slightly higher in comparison to the recession scenario. This is due to two factors. First, the less pronounced decline in capital, and second, since government debt does not increase as much, the income tax rate rises also by a smaller amount.

To sum up, the effects of the government spending shock with a different fiscal rule differ only quantitatively with respect to the benchmark rule. The initial demand effect is considerably larger when the shock hits during a recession but, on the other hand, smaller compared to the benchmark recession. The stronger crowding-out of capital in the recession scenario is further amplified by a rise in the income tax rate, exacerbating the drop of output once the demand effect is vanished.

5.1.2 Fiscal transfer shock

Figure 5 shows the IRFs to a one percent increase in fiscal transfers with the alternative fiscal rule. When the shock hits in a recession, the increase in transfers instantaneously translates into a reduction in demand as hours worked and output fall on impact. Consumption, however, increases by about 0.01%. The share of constrained households significantly declines, although not as much as in the baseline scenario. The fall in demand leads to a fall in prices, wages and the nominal interest rate. The crowding-out of investment causes a steady decline of the capital stock, which in turn leads to a much more severe decrease in output compared to the baseline.

After the initial increase in consumption, there is also a steady decline as transfers are reduced. The number of constrained households then grows rapidly and exceeds the counterfactual by almost 20 percent.

When the shock hits in an expansion, the responses do not differ much. The initial drops in output, hours worked and the real wage are more pronounced while consumption also decreases on impact. This further decrease in demand can be explained by the smaller share of households at the borrowing constraint during expansions, implying smaller MPCs at the bottom of the wealth distribution. The reduction of this share is also considerably smaller in expansions.

The crowding-out of investment on impact is stronger but approaches the counterfactual level faster after about one year. On the other hand, government debt increases less then during a recession. Thus, the income tax rate does not rise as much, and the
Figure 5: IRFs to one percent increase in fiscal transfers (Tax rule)

- Output
- Consumption
- Hours worked
- Real Wage
- Inflation
- Nominal rate
- Investment
- Capital
- % Constrained
- Tax rate
- Gov. debt
- Transfers
negative effects on labor supply and output are not as strong so that the recovery to counterfactual levels is faster to some degree.

To sum up, the negative impact effects of output and hours worked are in stark contrast to the baseline scenario. There is no positive demand effect anymore because households anticipate that the tax rate will increase in the near future. Therefore, the negative effects after about one year are much more severe with this alternative fiscal rule.

5.2 Multipliers

Multipliers are calculated as in the baseline scenario and can be found in Table 5. For the government consumption shock, the impact multiplier in the recession is again considerably larger than in the expansion. On the other hand, both impact multipliers are somewhat smaller with this alternative fiscal rule. The cumulated multipliers decline relatively fast. It turns negative after three years in the expansion scenario and after four years when the shock hits in a recession.

In contrast to the baseline, the impact multipliers for the fiscal transfer shock are slightly negative. Although the multiplier for the recession scenario is somewhat larger than for the expansion, the speed of contraction is faster so that the cumulated multipliers after three years are almost equal.

To conclude, the multipliers that arise with the alternative fiscal rule are considerably smaller than in the baseline but still differ across the state of the economy.

5.3 Distributional effects

Figure 6 displays the consumption responses on impact for both shocks conditional on the quintile of the wealth distribution. For the government consumption shock, the responses are similar to the baseline. The wealth-poor increase their consumption level while the wealth-rich lower it. However, the magnitude of the wealth-poor’s increase is considerably lower than in the baseline, while the reduction of the wealth-

<table>
<thead>
<tr>
<th>Impact Multiplier</th>
<th>Gov. consumption</th>
<th>Fiscal Transfers</th>
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<td></td>
<td>Recession</td>
<td>Expansion</td>
</tr>
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<td>Cumul. Multipliers</td>
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<tr>
<td>4 quarters</td>
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<td>12 quarters</td>
<td>0.06</td>
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</tr>
<tr>
<td>16 quarters</td>
<td>−0.03</td>
<td>−0.10</td>
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</table>
The reason is that households anticipate the future tax burden which has a negative effect across the distribution.

For the fiscal transfer shock, the impact responses differ quite substantially from the baseline, as it is no longer the case that all households raise their consumption. Instead, due to the contractionary nature of this shock, the wealth-rich respond by strongly decreasing their consumption. The high MPC of the wealth-poor, however, leads them to increase their consumption expenditures. This is also because the increase in fiscal transfers is much higher than the drop in their labor income.

### 6 Conclusion

Recent empirical studies indicate the presence of countercyclical fiscal multipliers. This paper contributes to this literature by addressing this issue within a quantitative model and proposing a mechanism that naturally generates state-dependent responses to fiscal shocks. The main driver is a non-trivial MPC distribution that is induced by incomplete insurance markets and occasionally binding borrowing constraints. The share of constrained households in this framework is countercyclical, implying that the concentration at the bottom of the wealth distribution rises during recessions. These borrowing-constrained households are characterized by a higher MPC than wealthier households so that a positive effect on their disposable income leads to a larger increase in personal consumption. Therefore, the higher share of constrained households during recessions serves as an amplification of the demand effect that is caused by fiscal shocks.

I investigate two types of government expenditure shocks, namely the traditional government consumption shock as well as a fiscal transfer shock, and control for dif-
different financing schemes. Although the size of fiscal multipliers across shocks and financing varies, the existence of countercyclical multipliers persists.
## A Data Appendix

### Table 6: Data sources

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<th>Series title</th>
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<td>Personal Consumption Expenditures</td>
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<td>DIVIDEND</td>
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B Computational Appendix

Solution Method

The solution procedure can be divided into an inner and an outer problem. I start with the inner problem, which consists of the patient households’ problem, the impatient households’ problem and the simulation.

Patient Households’ Problem.

The state space of this problem consists of six dimensions, current capital stock \( K_t \), current government debt \( B_t \), the lagged nominal interest rate \( R_{t-1} \), government consumption \( G_t \), aggregate technology \( z_t \) and an auxiliary state variable for lagged impatient households’ consumption \( c_{i,t-1} \). This can be summarized by the following vector of state variables,

\[
S_t = \{K_t, B_t, R_{t-1}, G_t, z_t, c_{i,t-1}\}. \tag{51}
\]

I choose equidistant grids in the dimensions of all endogenous state variables. The grids are chosen to be symmetric around the respective steady state (i.e. in the Aiyagari-version of the model) and the bounds are chosen such that the simulated values of each variable are always inside. The resulting space is the hypercube \( \hat{S}_t \). I choose seven grid points in each endogenous dimension.

Given the state space, I solve the problem via a Time Iteration algorithm, which amounts to finding the equilibrium functions for \( K_{t+1} \), \( \pi_t \) and \( w_t \) that are given by

\[
K_{t+1} = f_1(S_t),
\]
\[
\pi_t = f_2(S_t),
\]
\[
w_t = f_3(S_t),
\]

where \( f = (f_1, f_2, f_3) \) and \( f_i : \mathbb{R}^5 \times [z_b, z_g] \rightarrow \mathbb{R} \) for \( i \in \{1, 2, 3\} \).

To approximate \( f \), I set initial function values on every grid point in the state space \( \hat{S} \) and iterate on the set of dynamic first-order conditions that patient households are facing. In particular, I proceed as follows:

1. Guess \( K_{t+1}, \pi_t, w_t \) on every point in \( \hat{S}_t \), and guess a vector of coefficients \( \alpha^0 = (\alpha_1(z_t), \ldots, \alpha_6(z_t)) \).

\textsuperscript{15}This refers to the solution for the government consumption shock with transfers adjusting to public debt. Adjusting the problem for the fiscal transfer shock is straightforward.

\textsuperscript{16}\( z_t \) is the only exogenous state variable here, since the process for TFP is discretized to a Markov chain.
2. Given states and guesses, it is straightforward to calculate the nominal interest rate from (31), investment from (20), fiscal transfers from (26), as well as the patient households’ effective labor supply

\[ l_{p,t} = \left( \frac{1 - \tau_t}{\chi} \right) \gamma, \]

impatient households’ effective labor supply

\[ n_{i,t} = (1 - u_z) \left( \frac{1 - \tau_t}{\chi} w_t \right) \gamma \sum_j p^*_j \eta_j, \]

total effective labor supply

\[ n_t = v n_{i,t} + (1 - v) \bar{\eta} \left( \frac{1 - \tau_t}{\chi} \right) \gamma, \]

and unemployment benefits

\[ T_r^u = u_z \xi w_t^{1-\gamma} \left( \frac{1 - \tau_t}{\chi} \right) \gamma \sum_j p^*_j \eta_j, \]

where \( u_z \) is the share of unemployed households and \( p^* \) is the invariant distribution of the idiosyncratic productivity shock.

3. Now, aggregate output can be calculated from (11), the shadow price of capital from (22), aggregate consumption from (??), next period’s government debt from (24), as well as the rental rate of capital

\[ r^K_t = \frac{w_t n_t}{K_t} \frac{a}{1 - \alpha}, \]

dividends

\[ D_t = z_t K_t^\alpha n_t^{1-\alpha} - w_t n_t - I_t - \frac{\phi_p}{2} \left( \frac{\tau_t}{\alpha} - 1 \right) y_t - \frac{\phi_c}{2} \left( \frac{I_t}{K_t} - \delta \right)^2 K_t - \kappa, \]

and finally real marginal costs

\[ \Psi_t = \frac{w_t^{1 - \alpha} (r^K_t)^\alpha}{z} \frac{1}{(1 - \alpha)^{1-\alpha} a^\alpha}. \]

4. To determine the patient households’ consumption, it is necessary to know impatient consumption. Since this depends on the wealth distribution of this group, I
apply a Krusell and Smith (1998)-type approach and use a log-linear function that
maps from state space $S$ into the real numbers to forecast impatient consumption,

$$c_{i,t} = \exp(\alpha_1(z_t) + \alpha_2(z_t) \log K_t + \alpha_3(z_t) \log B_t + \alpha_4(z_t) \log R_{t-1} +$$

$$\alpha_5(z_t) \log G_t + \alpha_6(z_t) \log c_{i,t-1}),$$

52

so that patient consumption is given by

$$c_{p,t} = \frac{C_t - c_{i,t}}{1 - \nu}.$$
**Simulation**

The simulation procedure works as follows:

1. Generate and fix a time series of length $T$ for the aggregate shocks. Set an initial distribution of assets across $N$ impatient households. For each agent, generate and fix time series of length $T$ for the idiosyncratic shocks.

2. Use the policy functions obtained from the two optimization problems to simulate the economy $T$ periods forward.

**Outer Problem**

The outer problem is then to run the regressions (52) with the time series obtained from the last simulation step. Use the set of estimated coefficients $\hat{\alpha} = (\hat{\alpha}_1(z_t), \ldots, \hat{\alpha}_6(z_t))$ to compute a set of coefficients for the next iteration,

$$
\alpha^{j+1} = b\alpha^j + (1 - b)\hat{\alpha},
$$

where $b \in [0, 1)$ is an updating parameter. If $\alpha^{j+1}$ and $\alpha^j$ are sufficiently close to each other, then the outer problem is solved and the model converged. If not, use $\alpha^{j+1}$ to solve the inner problem.
Equilibrium Forecasting Rules

Table 7: Laws of motion for impatient consumption

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References


