Optimal Monetary Policy Rule with Downward Nominal Wage Rigidity: the Role of Heterogeneity∗

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Abstract
I study the welfare-maximizing monetary policy rule in a sticky price model where workers are heterogeneous in their productivity and wage changes are subject to asymmetric menu costs. When calibrated to the U.S. wage distribution, the model identifies a much larger cost for downward wage adjustments than upward ones, which is known as downward nominal wage rigidity (DNWR). In the calibrated model, DNWR generates a sizable welfare loss through an inefficient cross-sectional allocation of labor as well as the aggregate fluctuations. Consequently, the optimal monetary policy substantially differs from that in a homogeneous agent model. Positive inflation can improve social welfare because it facilitates real wage adjustments in a downturn. Moreover, a simple modification of monetary policy rule can yield higher welfare. The optimal monetary policy rule responds strongly to the output gap as well as to inflation. It is also effective in addressing the asymmetry arising from DNWR to respond more aggressively to a contractionary shock than to an expansionary one.

JEL classification: E24; E31; E52.

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1 Introduction

A heterogeneous agent model has attracted a lot of attention of researchers in recent years. To this end, Heathcote et al. (2009), in their survey, point out that one of the main advantages of taking into account heterogeneity is that the model can answer welfare questions in a realistic situation. In other words, the welfare consequences of a heterogeneous agent model can substantially differ from those of a homogeneous agent model since the aggregate fluctuations do not necessarily have symmetric effects on individual agents. They stress that explorations of a heterogeneous agent model deliver rich policy implications.

In the analysis of monetary policy, the early literature has made a tremendous effort to develop a model that captures the dynamics of macro variables while abstracting heterogeneity among agents in both positive and normative analysis (i.e., Clarida et al. (1999), Christiano et al. (2005)). On the other hand, recent studies find that the transmission mechanism of monetary policy can be altered in the presence of heterogeneity in various dimensions including asset holdings (Kaplan et al. (2018)), demography (Wong (2018)), and firm productivity (Winberry (2016)). Natural subsequent questions are the welfare consequences of heterogeneity and their implications to the optimal monetary policy, which are the main theme of this paper.

In this paper, I investigate the optimal monetary policy rule in a heterogeneous agent New Keynesian model. Specifically, I focus on the heterogeneity of individual workers’ wages and study the consequences of nominal rigidities. In this regard, a growing body of literature using micro data uncovers the heterogeneous behavior in wage setting. (i.e., Barattieri et al. (2014), Grigsby et al. (2018)). Moreover, recent studies suggest that wage rigidity, in particular downward nominal wage rigidity (DNWR), plays a key role to determine the aggregate dynamics after the Great Recession (Daly and Hobijn (2014), Schmitt-Grohé and Uribe (2016)). Built upon these studies, I contribute to the literature by exploring a normative aspect of the heterogeneity of workers’ wages in the presence of nominal rigidities.
For that purpose, I build a sticky price model with heterogeneous workers who are subject to idiosyncratic productivity shocks and therefore have different levels of desired wages. To capture lumpy and asymmetric wage adjustments observed in data, I introduce asymmetric menu costs for wage changes. I numerically solve the model in a dynamic setting by using the modified Krusell and Smith (1998) algorithm developed in my companion paper Mineyama (2018). When calibrated to the U.S. wage distribution, the model identifies a much larger cost for downward wage adjustments than upward ones, which is consistent with DNWR.

The welfare loss in the calibrated model is substantially larger compared to that in a homogeneous agent model. The main reason behind the result is because individual workers have desires to adjust their wages responding to idiosyncratic fluctuations in their productivity as well as the aggregate conditions. Therefore, the lack of wage adjustments due to DNWR leads to a sizable welfare loss through an inefficient cross-sectional allocation of labor, which is abstracted in a homogeneous agent model.

Given such differences in the welfare consequences of DNWR, the optimal monetary policy in the heterogeneous agent model substantially differs from that in a homogeneous agent model. In this regard, one potential policy implication is about the optimal steady-state inflation rate. As Tobin (1972) points out, positive inflation “greases the wheels of the labor market,” i.e., it facilitates real wage declines upon an adverse shock when nominal wages are downwardly rigid. Indeed, I find that in the presence of heterogeneity the optimal steady-state inflation rate can be higher than stated by the previous studies that use a homogeneous agent model.\footnote{Many of the studies that use a homogeneous agent model find that the positive optimal inflation generated by DNWR is quantitatively trivial. For example, Kim and Ruge-Murcia (2018) find that the optimal inflation rate is not significantly different from zero in an estimated New Keynesian model with DNWR. Using a similar New Keynesian model with DNWR, Kim and Ruge-Murcia (2009) report that the optimal inflation rate is about 0.35 percent per year. Kim and Ruge-Murcia (2018) argue that the difference from their earlier study is mainly because of their approximation methods; Kim and Ruge-Murcia (2018) use the third-order perturbation method whereas Kim and Ruge-Murcia (2009)’s approximation is up to the second-order.}

I also investigate the optimal responsiveness to macro economic variables in a
monetary policy rule.\textsuperscript{2} I find that the optimal monetary policy rule responds strongly not only to inflation but also to the output gap. Intuitively, price and wage rigidities generate two distinct markups and therefore the central bank faces a trade-off in closing the two markups simultaneously. In that environment, the responsiveness to the output gap stabilizes the marginal rate of substitution of households, which helps to mitigate the inefficient fluctuations of the wage markup.

Lastly, I find that monetary policy can improve social welfare by implementing an asymmetric policy rule; responding more aggressively to a contractionary shock than to an expansionary one. This is because the distortion arising from DNWR is particularly severe in a downturn when a larger share of workers is constrained by DNWR. Importantly, I find that the asymmetric monetary policy rule yields higher welfare even at a low steady-state inflation rate compared to the calibrated Taylor rule at the optimal steady-state inflation rate.

\textbf{Related literature.} This paper falls into a growing literature on the optimal monetary policy in a heterogeneous agent model. Previous studies mainly focus on the heterogeneity in asset and debt holdings of households (Lippi et al. (2015), Nuño and Thomas (2017)), firm productivity (Adam and Weber (2019)), and firm pricing (Blanco (2018)).\textsuperscript{3} It is also noteworthy that the most of the existing literature on the optimal monetary policy with DNWR uses a homogeneous agent model (Kim and Ruge-Murcia (2009), Carlsson and Westermark (2016)). An exception is Faia and Monacelli (2007), who consider heterogeneity in individual workers’ wages in a stationary environment. This paper extends their framework to accommodate the aggregate dynamics and consider a realistic situation where the central bank adjusts the policy rate so as to stabilize the economy. The setting allows for analyzing a

\textsuperscript{2}I adopt the idea of simple and implementable monetary policy rules suggested by Schmitt-Grohé and Uribe (2007). To be precise, I restrict the attention to a simple monetary policy rule in which the central bank sets the nominal interest rate in response to observable macroeconomic variables and explore the welfare-maximizing responsiveness to each macroeconomic variable.

\textsuperscript{3}Though here I assess the studies that deal with a continuum of agents, another strand of the literature considers two or finite types of agents to investigate the optimal monetary policy. See Aoki (2001), Debortoli and Gálı (2017), for example.
wide range of monetary policy rules whereas Faia and Monacelli (2007) focus on the steady-state inflation rate.\footnote{Moreover, this paper differs from Faia and Monacelli (2007) in the specification of wage and price settings. In this regard, I develop a general environment for wage setting that qualitatively and quantitatively accommodates the heterogeneous behavior among workers observed in micro data. Regarding price setting, though Faia and Monacelli (2007) calibrate the quadratic adjustment cost, the welfare cost of nominal price rigidity in the steady state does not have a micro founded interpretation.}

**Layout.** The remainder of the paper is organized as follows. Section 2 develops the model. Section 3 describes the computation method as well as calibration procedure. Section 4 studies the welfare consequences of the model and Section 5 conducts the optimal monetary policy analysis. Section 6 offers conclusion.

## 2 Model

### 2.1 Stylized model

**Setting.** I start by building a stylized model that embeds DNWR so as to explore its welfare implications. Specifically, I consider a situation where (i) each worker stays in the same job forever, (ii) nominal wages are subject to the absolute DNWR, and (iii) there is no aggregate shocks. Each of these assumptions is relaxed in a quantitative model developed in the next subsection.

Here I focus on the household wage setting problem. There is a continuum of households indexed by $j$ on the unit interval, each of whom supplies a differentiated labor service to the production sector by using the following production technology:

$$l_t(j) = z_t(j)h_t(j),$$

where $l_t(j)$ is the effective unit of labor whereas $h_t(j)$ is hours worked. $z_t(j)$ denotes the worker specific productivity. The aggregate labor supply takes the Dixit-Stiglitz
where $\theta_w$ represents the labor demand elasticity. The user of these labor services minimizes the cost of using a certain amount of composite labor inputs, taking each labor service provider’s wage as given. The first order condition for the cost minimization problem leads to the following individual labor demand function:

$$L_t \equiv \left( \int_0^1 l_t(j) \frac{\theta_w - 1}{\theta_w - 1} \, dj \right)^{\frac{\theta_w}{\theta_w - 1}},$$

(2)

where $w_t(j)$ is the hourly wage rate of worker $j$ and $W_t$ is the aggregate wage index that is defined as

$$W_t \equiv \left( \int_0^1 \frac{w_t(j)}{z_t(j)} \, dj \right)^{\frac{1}{1-\theta_w}}.$$

(4)

**Household problem.** Each household receives utility from consumption $c_t(j)$ and receives disutility from hours worked $h_t(j)$. Expected lifetime-utility is given by

$$\mathbb{E}_t \left[ \sum_{s=0}^{\infty} \beta^s u(c_{t+s}(j), h_{t+s}(j)) \right],$$

(5)

where $\beta$ is the subjective discount factor. Household $j$’s period budget constraint is given by

$$c_t(j) + \frac{b_t(j)}{P_t} \leq \frac{w_t(j)}{P_t} h_t(j) + R_{t-1} \frac{b_{t-1}(j)}{P_t} + \frac{\tau_t(j)}{P_t} + \Phi_t(j),$$

(6)

where $b_t(j)$ is the amount of nominal bond holdings, $\tau_t(j)$ is the lump-sum transfer, and $\Phi_t(j)$ is the share of producer’s real profits distributed to household $j$. $P_t$ is the aggregate price index and $R_t$ is the gross nominal interest rate. In addition, I assume
that nominal wages are subject to absolute DNWR, that is,
\[ w_t(j) \geq w_{t-1}(j). \quad (7) \]

Each household \( j \) maximizes expected lifetime utility (5) by choosing consumption \( c_t(j) \), nominal bond holdings \( b_t(j) \), and nominal wages \( w_t(j) \) subject to the budget constraint (6), individual labor demand (3), and the DNWR constraint (7).\(^5\) The first order condition for \( w_t(j) \) takes the following form:
\[
\psi_t(j) = \left( \frac{w_t(j)}{P_t} - \mu_w z_t(j) \frac{u_{h,t}(j)}{u_{c,t}(j)} \right) \left( u_{c,t}(j) \frac{\theta_w h_t(j)}{w_t(j)} \right) + \beta_t \mathbb{E}_t[\psi_{t+1}(j)], \quad (8)
\]
where \( u_{c,t}(j) = \frac{\partial u(c_t(j), h_t(j))}{\partial c_t(j)} \) and \( u_{h,t}(j) = \frac{\partial u(c_t(j), h_t(j))}{\partial h_t(j)} \). \( \mu_w \equiv \theta_w / (\theta_w - 1) \) is the steady-state wage markup that arises from the workers’ monopolistic power over their labor service. \( mrs_t(j) \) is the marginal rate of substitution (MRS) of hours worked for consumption. \( \psi_t(j) \) denotes the Lagrange multiplier for the DNWR constraint, which represents the shadow value of relaxing the DNWR constraint by one unit. The complementary slackness conditions for the DNWR constraint (7) are given by
\[
\psi_t(j) \geq 0, \quad (9)
\]
\[
\psi_t(j)(w_t(j) - w_{t-1}(j)) = 0. \quad (10)
\]

Notice that if it were not for the DNWR constraint \( \psi_t(j) = 0 \) would hold for all
\(^5\) Notice that wages are a workers’ choice variable because of their monopolistic power over their labor service. The setting implies that the amount of labor in equilibrium is determined by labor demand of firms rather than households’ desire to supply labor. One can take the situation that wages are posted by firms along with the corresponding labor demand while workers choose the most preferable wage among the available ones. To this end, the literature provides ample evidence on firms’ reluctance to offer nominal wage cuts (e.g., Bewley (1999), Chemin and Kurmann (2014)).
In that case, the optimality conditions are reduced to

\[ \frac{w^f_t(j)}{P_t^f} = \mu_w z_t(j) \text{mrs}_t^f(j), \]  \hspace{1cm} (11)

where \( x_t^f \) denotes the variable \( x_t \) under flexible prices and wages. Equation (11) suggests that the flexible wages are determined by idiosyncratic labor productivity and the MRS as well as the steady-state wage markup.

In the presence of DNWR, on the other hand, rearranging the optimality conditions (8), (9), and (10) yields the following wage rule:

\[ \frac{w_t(j)}{P_t} = \max \left\{ \frac{w_{t-1}(j)}{P_{t-1}}, \frac{1}{\Pi_t^P} \right\}, \]  \hspace{1cm} (12)

where \( \frac{w^d_{t}(j)}{P_t} = \mu_w z_t(j) \text{mrs}_t(j) - \beta_t \mathbb{E}_t[\psi_{t+1}(j)] \left( u_{c,t}(j) \frac{\theta_w h_t(j)}{w_t(j)} \right)^{-1}. \)  \hspace{1cm} (13)

\( \Pi_t^P \equiv P_t/P_{t-1} \) is the gross inflation rate. \( w^d_t(j) \) denotes the desired wages, which are the wages chosen when DNWR does not bind in the current period. According to (12), the wages chosen by workers take a max function with the first element corresponding to the case where DNWR does not bind in the current period and the second element to the case where it binds.

**Welfare implications of DNWR.** Deviations from the equilibrium condition under flexible prices and wages (11) are the sources of the welfare losses due to nominal rigidities. The wage rule (12) and (13) indeed suggests that wages can deviate from the flexible wages both upwardly and downwardly. The upward deviations are straightforward to see in (12). When DNWR binds in the current period, the actual wages tend to be higher than the flexible wages that reflects the MRS and labor productivity.\(^6\) On the other hand, downward deviations occur due to a forward looking effect of DNWR. To this end, (13) suggests that the desired wages are weakly lower than

\(^6\)Strictly speaking, the actual wages can be lower than the flexible ones even when DNWR binds if the forward looking effect is strong enough.
the flexible wages due to the second term in the RHS.\footnote{Notice that the complementary slackness condition (9) guarantees $\psi_t(j)$ is non-negative.} Intuitively, a household internalizes the possibilities that the DNWR constraint might bind in the future periods and therefore desires to hold a buffer to prevent the future constraints from binding. This means that the actual wages can be lower than the flexible wages in the presence of DNWR. The implication is consistent with the finding by Elsby (2009). It is also noteworthy that positive inflation mitigates these effects of DNWR by ensuring a room for real wage declines upon an adverse shock.

2.2 Quantitative model

In this subsection, I develop a quantitative model to match with data by generalizing the stylized model in the previous subsection. The economy consists of households, monopolistically competitive firms, and a central bank. Households supply differentiated labor service and earn wages with wage rigidity, and the make saving-consumption decision. Firms produce differentiated goods and set prices under the staggered contract \textit{`a la} Calvo (1983). The central bank follows an interest rate feedback rule of Taylor (1993). I consider two sources of aggregate fluctuations: risk premium shocks and technology shocks.

2.2.1 Households

Wage rigidity. I assume that nominal wage changes are subject to asymmetric menu costs. The setting nests the absolute DNWR in the previous subsection when the menu cost for downward wage changes is infinitely large while that for upward wage changes is zero. In addition, I introduce job changes. Specifically, I assume that job-changers, workers who switch their jobs, are free from the menu costs for wage changes. This assumption reflects the empirical findings by previous studies that wage changes are quite frequent when workers change their jobs (i.e., Barattieri et al. (2014), Grigsby et al. (2018)). For simplicity, I assume that job changes occur
randomly with a probability \( s \in (0, 1) \). I also allow for a random fraction \( \alpha \in (0, 1) \) of job-stayers is free from menu costs to capture small wage changes observed in data.\(^8\) In sum, the wage adjustment costs for each worker are given below.

\[
m_t(j) = \begin{cases} 
  m_+ \mathbf{1}\{w_t(j) > w_{t-1}(j)\} + m_- \mathbf{1}\{w_t(j) < w_{t-1}(j)\} & \text{w.pr. } (1-s)(1-\alpha) \\
  0 & \text{w.pr. } (1-s)\alpha \\
  0 & \text{w.pr. } s 
\end{cases} 
\]

\( \ldots \) job-stayers with menu costs

\( \ldots \) job-stayers without menu costs

\( \ldots \) job-changers,

(14)

where \( m_+ \) and \( m_- \) are the menu costs for positive and negative wage changes.

**Labor productivity.** Labor productivity \( z_t(j) \) consists of the non-stationary component \( z_{1,t}(j) \) and the stationary one \( z_{2,t}(j) \),

\[
\ln(z_t(j)) = \ln(z_{1,t}(j)) + \ln(z_{2,t}(j)),
\]

(15)

I consider different processes of labor productivity for job-stayers and job-changers to match the wage change patterns of each group of workers observed in data. For job-stayers, I assume that a labor productivity shock hits infrequently with a probability \((1-\gamma) \in (0,1) \). I also consider a deterministic trend growth rate \( \mu_{st}^z \). That is,

\[
\begin{align*}
\ln(z_{1,t}^st(j)) &= \mu_{st}^z + \ln(z_{1,t-1}(j)), \\
\ln(z_{2,t}^st(j)) &= \begin{cases} 
  \rho_{st}^z \ln(z_{2,t-1}(j)) + \epsilon_{zt}^st(j), \epsilon_{zt}(j) \sim i.i.d. N(0, \sigma_{zt}^2) & \text{w.pr. } 1-\gamma \\
  \ln(z_{2,t-1}(j)) & \text{w.pr. } \gamma.
\end{cases}
\end{align*}
\]

(16)

(17)

For job-changers, I assume that labor productivity follows a random walk with a trend growth rate \( \mu_{ch}^z \). Notice that \( \mu_{ch}^z \) can be different from \( \mu_{st}^z \) and I define the unconditional growth rate among workers \( g \equiv (\mu_{st}^z + s\mu_{ch}^z)/(1+s) \). To capture

\(^8\)This specification is often used in the price setting literature (i.e., Nakamura and Steinsson (2010), Vavra (2013)).
larger variance of job-changers’ wage changes observed in data, I assume that a labor productivity shock is drawn from a uniform distribution with a support $U_{z}^{ch} > 0$. The process is given as follows.

\[
\ln(z_{1,t}^{ch}(j)) = \mu_{z}^{ch} + \ln(z_{1,t-1}(j)),
\]

\[
\ln(z_{2,t}^{ch}(j)) = \ln(z_{2,t-1}^{ch}(j)) + \epsilon_{z,t}^{ch}(j), \quad \epsilon_{z,t}^{ch}(j) \sim i.i.d.U[-U_{z}^{ch},U_{z}^{ch}].
\]

**Household problem.** Here I specify the functional form of household utility. I assume that the period utility takes the following form:

\[
u(c_{t}(j), h_{t}(j)) = \ln c_{t}(j) - h_{t}(j)^{1+1/\eta} \frac{1}{1 + 1/\eta},
\]

where $\eta$ is the Frisch labor supply elasticity. Noting that the log utility of consumption is consistent with balanced growth.

Regarding an aggregate shock, I consider exogenous fluctuations in risk premium on the nominal interest rate. The risk premium $Q_t$ follows an AR(1) process as below.

\[
\ln(Q_{t}) = \rho_{q} \ln(Q_{t-1}) + \epsilon_{q,t}, \quad \epsilon_{q,t} \sim i.i.d.N(0, \sigma_{q}^{2}).
\]

A positive innovation $\epsilon_{q,t}$ is a contractionary demand shock, which causes households to lose their desire to consume by borrowing from the future periods.

Reflecting the generalizations above, the period budget constraint is modified to:

\[
c_{t}(j) + m_{t}(j)C_{t} + \frac{b_{t}(j)}{P_{t}} \leq \frac{w_{t}(j)}{P_{t}} h_{t}(j) + Q_{t-1} R_{t-1} \frac{b_{t-1}(j)}{P_{t}} + \frac{\tau_{t}(j)}{P_{t}} + \Phi_{t}(j),
\]

where I specify the menu costs of wage adjustments as real resource costs that are proportional to the aggregate consumption. I also assume that each household has access to complete contingent claim markets which guarantees that consumption is identical across households, i.e., $c_{t}(j) = C_{t}$. The assumption is extensively used in
the literature (i.e., Erceg et al. (2000)) and allows one to focus on the heterogeneity of individual wages and the resulting allocation of labor.

The first order conditions for consumption and nominal bond holdings yield the consumption Euler equation:

$$\beta \mathbb{E}_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-1} \frac{Q_t R_t}{\Pi_{t+1}} \right] = 1. \quad (23)$$

The individual wage setting problem can be written in the following recursive representation:

$$V \left( \frac{w_{t-1}(j)}{P_{t-1}}, z_t(j) \right) = \max_{w_t(j)} \left[ h_t(j)^{1+1/\eta} + C \left( \frac{w_t(j)}{P_t} h_t(j) - m_t(j) C_t \right) \right]$$

$$+ \mathbb{E}_t \left[ V \left( \frac{w_{t+1}(j)}{P_t}, z_{t+1}(j) \right) \right], \quad (24)$$

subject to production technology (1), labor demand (3), exogenous process of labor productivity (16)-(19), given all the aggregate variables. Notice that the wage setting problem is separable from consumption choice conditional on the marginal utility of wealth due to The additive separability of utility in consumption and labor.

### 2.2.2 Firms

There is a continuum of monopolistically competitive firms indexed by $i$ on the unit interval, each of which produces a differentiated good. Firm $i$ uses composite labor inputs $l_t(i)$ with a linear production technology:

$$y_t(i) = A_l l_t(i), \quad (25)$$

where

$$l_t(i) = \left( \int_0^1 l_t(i, j) \frac{a_{\eta-1}}{\eta} dj \right)^{\frac{\eta}{\eta-1}}. \quad (26)$$
$l(i, j)$ is the labor service supplied by household $j$ and used in firm $i$. Productivity $A_i$ is common across firms and follows an AR(1) process:

$$\ln(A_t) = \rho_a \ln(A_{t-1}) + \epsilon_{a,t}, \epsilon_{a,t} \sim i.i.d. N(0, \sigma_a^2).$$  \hspace{1cm} (27)

I assume that firms are a price taker in the factor market. The first order condition of the cost minimization problem to determine the labor inputs $l_t(i)$ takes the following form:

$$MC_t = \frac{W_t}{A_t P_t},$$  \hspace{1cm} (28)

where $MC_t$ is the real marginal cost of producing one unit of output. Notice that firm index $i$ is dropped in 28 because marginal cost is identical across firms.

I define the aggregate output $Y_t$ as the CES aggregator of individual outputs:

$$Y_t \equiv \left( \int_0^1 y_t(i)^{\theta_p^{-1}} \frac{\theta_p}{1-\theta_p} \, di \right)^{\frac{\theta_p}{1-\theta_p}},$$  \hspace{1cm} (29)

where $\theta_p$ is the elasticity of substitution across goods. Each firm faces the following individual demand curve:

$$y_t(i) = \left( \frac{p_t(i)}{P_t} \right)^{-\theta_p} Y_t,$$  \hspace{1cm} (30)

where the price index is defined as

$$P_t \equiv \left( \int_0^1 p_t(i)^{1-\theta_p} \, di \right)^{\frac{1}{1-\theta_p}}.$$  \hspace{1cm} (31)

Firms set their prices under the staggered contract à la Calvo (1983). In each period, a fraction $\xi \in (0, 1)$ of firms keeps their prices unchanged whereas the remaining fraction $(1 - \xi)$ of firms resets their prices. The reset price $P_t^*$ maximizes the expected
real profits:
\[
\mathbb{E}_t \left[ \sum_{s=0}^{\infty} \xi^s \beta^s \left( \frac{C_{t+s}}{C_t} \right)^{-1} \left( \frac{P^*_{t+s}}{P_{t+s}} - \frac{y_{t+s}}{y_{t+s}} - MC_{t+s}y_{t+s} \right) \right],
\]  
(32)
subject to the individual good demand:
\[
y_{t+s} = \left( \frac{P^*_{t+s}}{P_{t+s}} \right)^{-\theta_p} Y_{t+s},
\]  
(33)
where \( \Lambda_{t,t+s} \) is the stochastic discount factor between time \( t \) and \( t + s \) and \( \Phi_{t+s|t} \) is the period real profit at time \( t + s \) of the firms that reset their prices at time \( t \). The first order condition is derived as below.
\[
\frac{P^*_{t+s}}{P_t} = \frac{\Omega_{1,t}}{\Omega_{2,t}},
\]  
(34)
where
\[
\Omega_{1,t} = \mu_p MC_t C_t^{-1} Y_t + \xi \beta \mathbb{E}_t \left[ (\Pi_{t+1}^p)^{\theta_p} \Omega_{1,t+1} \right],
\]  
(35)
\[
\Omega_{2,t} = C_t^{-1} Y_t + \xi \beta \mathbb{E}_t \left[ (\Pi_{t+1}^p)^{\theta_p-1} \Omega_{2,t+1} \right],
\]  
(36)
where \( \mu_p \equiv \theta_p / (\theta_p - 1) \) is the steady-state price markup that arises from the firms’ monopolistic power over their products. The price index can be rearranged to the following equation.\(^9\)
\[
1 = (1 - \xi) \left( \frac{P^*_{t+s}}{P_t} \right)^{1-\theta_p} + \xi (\Pi_t^p)^{\theta_p-1}.
\]  
(38)
By integrating individual production function over firms, the aggregate production

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\(^9\)Taking the first order approximation of (34)-(38) around the zero-inflation steady state yields the well-known linearized New Keynesian Phillips Curve,
\[
\pi_t^p = \beta \mathbb{E}_t [\pi_{t+1}^p] + \kappa \hat{m}_t
\]  
(37)
where \( \kappa \equiv \frac{(1-\xi)(1-\beta \xi)}{\xi} \), \( \pi_t^p = \ln \Pi_t^p \), and variables with hats denote the log-deviations from the steady-state values.
is given by

\[ Y_t = \frac{A_t L_t}{D_t}, \quad (39) \]

where

\[ L_t \equiv \int_0^1 l_t(i) di, \quad (40) \]

and

\[ D_t \equiv \int_0^1 \left( \frac{p_t(i)}{P_t} \right)^{-\theta_p} di. \quad (41) \]

The relative price dispersion \( D_t \) evolves according to the following recursive formula.

\[ D_t = \xi \Pi_t^{\theta_p} D_{t-1} + (1 - \xi) \left( \frac{P^*_t}{P_t} \right)^{-\theta_p}. \quad (42) \]

Goods market clearing condition is given by

\[ Y_t = C_t \left( 1 + \int_0^1 m_t(j) dj \right). \quad (43) \]

### 2.2.3 Central bank

The central bank follows an interest rate feedback rule. As the benchmark case, I consider the Taylor (1993) rule where the central bank sets the gross nominal interest rate \( R_t \) to stabilize the gross inflation rate \( \Pi_t \) around its target rate \( \Pi^* \) and the output gap \( Y_t/Y^*_t \).

\[ R_t = R^* \left( \frac{\Pi_t}{\Pi^*} \right)^{\phi_\pi} \left( \frac{Y_t}{Y^*_t} \right)^{\phi_y}, \quad (44) \]

where \( \phi_\pi \) and \( \phi_y \) are the responsiveness to inflation and the output gap, \( R^* = \Pi^* g/\beta \) is the steady-state nominal interest rate, and \( Y^*_t \) is the output under flexible prices and wages. In this specification, the target inflation rate \( \Pi^* \) is achieved in the deterministic steady state where all the exogenous shocks are muted.\(^\text{10}\)

\(^{10}\)Recent studies find that the deterministic steady state and the stochastic mean, or the risky steady state, do not necessarily coincide with each other in a non-linear environment (i.e., Coeurdacier et al. (2011) and Nakata and Schmidt (2016)). However, I find that the difference of the
2.2.4 Equilibrium

A recursive competitive equilibrium consists of a household’s policy function for individual real wages $\tilde{w}_t(j) = h(\tilde{w}_{t-1}(j), z_t(j); g_{t-1}, D_{t-1}, A_t, Q_t)$, a policy function for a set of aggregate jump variables $X_t \equiv \{Y_t, L_t, C_t, \Pi^p_t, R_t, D_t\} = f(g_{t-1}, D_{t-1}, A_t, Q_t)$, and a law of motion $\Gamma$ for cross-sectional density of individual real wages $g_t$, given exogenous processes $\{z_t(j)\}_{j \in (0,1)}$ and $\{A_t, Q_t\}$, such that
(i) a household’s policy function $h$ solves the individual wage setting problem;
(ii) an aggregate policy function $f$ satisfies the aggregate optimality conditions;
(iii) markets clear;
(iv) a law of motion $\Gamma$ is generated by $g$, that is, the cross-sectional density $g$ satisfies a recursive rule:

$$g_t = \Gamma(g_{t-1}, D_{t-1}, A_t, Q_t).$$

2.2.5 Social welfare

I define the social welfare as the unconditional expectation of average household utility:

$$SW \equiv \mathbb{E} \left[ \ln(C_t) - \int_0^1 \frac{h_t(j)^{1+1/\eta}}{1 + 1/\eta} dj \right].$$

In what follows, I consider the consumption equivalent welfare losses from the flexible price and wage economy. Notice that the flexible price and wage economy is still inefficient due to monopolistic distortions of households and firms. In other words, the welfare losses measure the size of the distortion arising from nominal rigidities.

\[\text{inflation rates in these steady states is quite small in my model setting as long as the steady-state inflation remains moderate.}\]
3 Quantitative analysis

3.1 Numerical method

I solve the model numerically by using the modified Krusell-Smith algorithm that is developed by my companion paper Mineyama (2018). The details of the equilibrium computation are provided in Online Appendix A.

When solving the aggregate parts of the model, I derive the first order approximation of the aggregate conditions around the non-zero steady-state inflation rate $\Pi^*$. However, the system is still non-linear because the aggregate dynamics depend on the level of the steady-state inflation rate and the non-linear fluctuations of real wages governed by the evolution of the cross-sectional wage distribution. In addition, when computing the menu cost $\int_0^1 m_t(j) dj$ in (43), I approximate the fraction of wage changes to its stochastic mean at each steady-state inflation rate. The approximation is largely due to the computation burden to simultaneously compute the fraction and the other variables’ dynamics. However, the approximation errors are considered to be quantitatively small because the changes in the fraction of workers who pay menu costs has up to the second order effects as Nakamura et al. (2018) argue.

Once I solve for equilibrium, I conduct stochastic simulations to evaluate welfare loses. Specifically, I approximate the expectation operator in (45) by taking the mean of simulated series.

3.2 Calibration

I calibrate the model to U.S. macro and micro data. The time frequency is quarterly. Fixed parameters are listed in Panel (A) of Table 1. For utility, the subjective discount factor $\beta$ is set to 0.995 and the Frisch labor supply elasticity $\eta$ is to 1. I choose the elasticity of substitution across individual goods $\theta_p$ equal to 7 following Coibion et

\footnote{This approximation method is often used in previous studies such as Coibion et al. (2012) and Ascari et al. (2018).}
al. (2012). The value implies the steady-state price markup is around 17 percent, which is broadly consistent with the empirical estimates in the literature such as Basu and Fernald (1997). I use the same value for the elasticity of substitution across individual labor service $\theta_w$. The value of the degree of price stickiness $\xi$ is set to 0.66 according to the frequency of price changes reported by Nakamura and Steinsson (2008). The trend productivity growth rate $g$ is set equal to 1.5 percent per year. The value corresponds to the mean growth rate of real GDP per capita during the past few decades. For monetary policy rule, I follow Fernández-Villaverde et al. (2015) to set the responsiveness to inflation $\phi_\pi$ equal to 1.50 and that to the output gap $\phi_y$ to 0.25 as the benchmark case. Regarding exogenous processes, the persistence and the standard deviation of innovations are set to $\rho_z = 0.90$ and $\sigma_z = 0.0020$ for productivity and $\rho_q = 0.85$ and $\sigma_q = 0.0035$ for risk premium. These values reflect the variance and persistence of real GDP during the low inflation period after the Greenspan Era.

Built upon the rapid development of the literature on wage rigidity using micro data, I calibrate the parameters of the cross-sectional wage distribution to match the moments of the empirical distribution. Specifically, I choose the values of a set of parameters $\hat{\Theta} = \{s, m_+, m_-, \alpha, \gamma, \mu_{st}, \rho_{st}, \sigma_{st}, \mu_{st}, U_{st}\}$ to minimize the distance between the target and model moments:

$$\hat{\Theta} = \text{argmin}_{\Theta} : (d - m(\Theta))W_{\Theta}(d - m(\Theta))',$$  \hspace{1cm} (46)

where $d$ is the vector of target moments in data, $m(\Theta)$ is the vector that collects the corresponding model moments, and $W_{\Theta}$ is a weighting matrix. For target moments, I use the moments of individual workers’ wage changes reported by Grigsby et al. (2018). They report detailed facts regarding wage adjustments such as the size and frequency of wage changes for both job-stayers and job-changers using administrative

\footnote{To generate the model moments, I solve for the stationary equilibrium of the model due to the computation burden of deriving the recursive competitive equilibrium repeatedly.}
### Table 1: Calibration

#### Panel (A): Fixed parameters

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subjective discount factor</td>
<td>$\beta$</td>
<td>0.995</td>
</tr>
<tr>
<td>Frisch labor supply elasticity</td>
<td>$\eta$</td>
<td>1.00</td>
</tr>
<tr>
<td>Labor demand elasticity</td>
<td>$\theta_w$</td>
<td>7.00</td>
</tr>
<tr>
<td>Goods demand elasticity</td>
<td>$\theta_p$</td>
<td>7.00</td>
</tr>
<tr>
<td>Price stickiness</td>
<td>$\xi$</td>
<td>0.66</td>
</tr>
<tr>
<td>Trend productivity growth (percent per year)</td>
<td>$g$</td>
<td>1.50</td>
</tr>
<tr>
<td>Responsiveness to inflation</td>
<td>$\phi_\pi$</td>
<td>1.50</td>
</tr>
<tr>
<td>Responsiveness to the output gap</td>
<td>$\phi_y$</td>
<td>0.25</td>
</tr>
<tr>
<td>Persistence of technology</td>
<td>$\rho_z$</td>
<td>0.90</td>
</tr>
<tr>
<td>S.D. of innovations to technology</td>
<td>$\sigma_z$</td>
<td>0.0020</td>
</tr>
<tr>
<td>Persistence of risk premium</td>
<td>$\rho_q$</td>
<td>0.85</td>
</tr>
<tr>
<td>S.D. of innovations to risk premium</td>
<td>$\sigma_q$</td>
<td>0.0035</td>
</tr>
</tbody>
</table>

#### Panel (B): Calibrated parameters

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prob. of job changes</td>
<td>$s$</td>
<td>0.048</td>
</tr>
<tr>
<td>For job-stayers</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Menu cost for positive wage changes</td>
<td>$m_+$</td>
<td>0.046</td>
</tr>
<tr>
<td>Menu cost for negative wage changes</td>
<td>$m_-$</td>
<td>0.194</td>
</tr>
<tr>
<td>Prob. of not subject to menu cost</td>
<td>$\alpha$</td>
<td>0.010</td>
</tr>
<tr>
<td>Prob. of not receiving labor productivity shock</td>
<td>$\gamma$</td>
<td>0.846</td>
</tr>
<tr>
<td>Trend growth of labor productivity</td>
<td>$\mu_{st}^z$</td>
<td>0.003</td>
</tr>
<tr>
<td>Persistence of labor productivity</td>
<td>$\rho_{st}^z$</td>
<td>0.676</td>
</tr>
<tr>
<td>S.D. of innovations to labor productivity</td>
<td>$\sigma_{st}^z$</td>
<td>0.117</td>
</tr>
<tr>
<td>For job-changers</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trend growth of labor productivity</td>
<td>$\mu_{ch}^z$</td>
<td>0.114</td>
</tr>
<tr>
<td>Support of innovations to labor productivity</td>
<td>$U_{ch}^z$</td>
<td>2.293</td>
</tr>
</tbody>
</table>

payroll data in the U.S. It is worth noting their administrative data records per-period pay rate without measurement errors.\textsuperscript{13} Specifically, I focus on (i) probability, (ii) median size, and (iii) mean size of positive and negative wage changes as well as (iv) median, (v) mean, and (vi) standard deviation of unconditional wage changes. I use these moments for different job status (job-stayers, job-changers, and all work-

\textsuperscript{13}Their data records administrative measures of hourly wage for hourly paid workers. For salaried workers, it contains the employees contracted earnings per pay period such as weekly and monthly. More discussion about measurement issues of wages is provided in Online Appendix C.
<table>
<thead>
<tr>
<th>Moment</th>
<th>Quarterly changes</th>
<th>Yearly changes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td>All workers</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Probability of positive wage changes</td>
<td>0.218</td>
<td>0.196</td>
</tr>
<tr>
<td>Probability of negative wage changes</td>
<td>0.041</td>
<td>0.027</td>
</tr>
<tr>
<td>Median size of positive wage changes</td>
<td>0.040</td>
<td>0.057</td>
</tr>
<tr>
<td>Median size of negative wage changes</td>
<td>–0.127</td>
<td>–0.148</td>
</tr>
<tr>
<td>Mean size of positive wage changes</td>
<td>0.091</td>
<td>0.082</td>
</tr>
<tr>
<td>Mean size of negative wage changes</td>
<td>–0.154</td>
<td>–0.173</td>
</tr>
<tr>
<td>Median of unconditional wage changes</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Mean of unconditional wage changes</td>
<td>0.014</td>
<td>0.012</td>
</tr>
<tr>
<td>S.D. of unconditional wage changes</td>
<td>0.081</td>
<td>0.060</td>
</tr>
<tr>
<td>Job-stayers</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Probability of positive wage changes</td>
<td>0.185</td>
<td>0.176</td>
</tr>
<tr>
<td>Probability of negative wage changes</td>
<td>0.009</td>
<td>0.008</td>
</tr>
<tr>
<td>Median size of positive wage changes</td>
<td>0.033</td>
<td>0.055</td>
</tr>
<tr>
<td>Median size of negative wage changes</td>
<td>–0.077</td>
<td>–0.100</td>
</tr>
<tr>
<td>Mean size of positive wage changes</td>
<td>0.057</td>
<td>0.060</td>
</tr>
<tr>
<td>Mean size of negative wage changes</td>
<td>–0.087</td>
<td>–0.096</td>
</tr>
<tr>
<td>Median of unconditional wage changes</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Mean of unconditional wage changes</td>
<td>0.010</td>
<td>0.010</td>
</tr>
<tr>
<td>S.D. of unconditional wage changes</td>
<td>0.037</td>
<td>0.028</td>
</tr>
<tr>
<td>Job-changers</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Probability of positive wage changes</td>
<td>0.527</td>
<td>0.589</td>
</tr>
<tr>
<td>Probability of negative wage changes</td>
<td>0.376</td>
<td>0.403</td>
</tr>
<tr>
<td>Median size of positive wage changes</td>
<td>0.177</td>
<td>0.212</td>
</tr>
<tr>
<td>Median size of negative wage changes</td>
<td>–0.143</td>
<td>–0.206</td>
</tr>
<tr>
<td>Mean size of positive wage changes</td>
<td>0.246</td>
<td>0.214</td>
</tr>
<tr>
<td>Mean size of negative wage changes</td>
<td>–0.172</td>
<td>–0.204</td>
</tr>
<tr>
<td>Median of unconditional wage changes</td>
<td>0.022</td>
<td>0.039</td>
</tr>
<tr>
<td>Mean of unconditional wage changes</td>
<td>0.065</td>
<td>0.044</td>
</tr>
<tr>
<td>S.D. of unconditional wage changes</td>
<td>0.270</td>
<td>0.245</td>
</tr>
</tbody>
</table>

Notes: The data moments are those reported by Grigsby et al. (2018). They use administrative payroll data from a large payroll processing company in the U.S. The sample period is from 2008 to 2016. The model moments are obtained from the stationary equilibrium. The trend productivity growth and the steady-state inflation rate are set consistently with the average wage growth in the dataset of Grigsby et al. (2018).

ers) and different time frequencies (quarterly and yearly). The total number of the target moments is 54. Regarding the model parameters, I set the probability of job changes $s$ according to Grigsby et al. (2018).$^{14}$ Consequently, the number of model

$^{14}$Their data covers a subset of U.S. firms and is agnostic about worker flows that involve a firm
parameters to estimate is 9. For the weighting matrix $W$, I consider a diagonal matrix and weight the moments for job-stayers, job-changers, and all workers with their unconditional fractions in the data, i.e., $1/(1+s)$, $s/(1+s)$, and 1, respectively.

The calibrated parameter values are listed in Panel (B) of Table 1 whereas the target and model moments are reported in Table 2. Several points are noteworthy in the data of Grigsby et al. (2018). First, negative wage changes are quite rare for job-stayers whereas job-changers experience wage cuts much more often. Second, the size of wage changes tend to be larger for job-changers than for job-stayers. Third, the mean of wage changes is higher for job-changers than for job-stayers. Reflecting these features in the data, the calibrated parameters suggest: (1) the menu cost is much larger for negative wage changes than for positive ones; (2) the probability of not subject to the menu cost is quite low; (3) the standard deviation of labor productivity shocks is larger for job-changers than for job-stayers; (4) the trend growth rate of labor productivity is higher for job-changers than for job-stayers.

3.3 Model validity

To check the validity of the model, Table 3 reports the untargeted moments in the data and the corresponding model moments. Panel (A) compares the cross-sectional moments under high and low wage growth periods. As Grigsby et al. (2018) emphasize, the data suggests state-dependency of wage changes; there are less positive changes and slightly more negative ones during the low wage growth periods compared with the high wage growth periods. As a consequence, the fraction of workers with wage freezes becomes larger at a low wage growth rate. The model is successful in replicating these data patterns. Intuitively, this is because under a low wage growth
Table 3: Untargeted moments

Panel (A): cross-sectional moments

<table>
<thead>
<tr>
<th>Moment</th>
<th>Low wage growth</th>
<th>High wage growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability of positive wage changes</td>
<td>0.177 0.145</td>
<td>0.235 0.239</td>
</tr>
<tr>
<td>Probability of negative wage changes</td>
<td>0.051 0.034</td>
<td>0.039 0.023</td>
</tr>
<tr>
<td>Median size of wage changes</td>
<td>0.033 0.046</td>
<td>0.033 0.061</td>
</tr>
<tr>
<td>Mean size of wage changes</td>
<td>0.030 0.040</td>
<td>0.060 0.060</td>
</tr>
<tr>
<td>Median of unconditional wage changes</td>
<td>0.000 0.000</td>
<td>0.000 0.000</td>
</tr>
<tr>
<td>Mean of unconditional wage changes</td>
<td>0.007 0.007</td>
<td>0.016 0.016</td>
</tr>
<tr>
<td>S.D. of unconditional wage changes</td>
<td>0.081 0.059</td>
<td>0.082 0.063</td>
</tr>
</tbody>
</table>

Notes: The listed moments are those of quarterly wage changes. The data moments are those reported by Grigsby et al. (2018). Low wage growth period in the data is from March 2009 to December 2010 whereas high wage growth period is from January 2012 to December 2016.

Panel (B): Time-series moments

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
<th>Data</th>
<th>Model</th>
<th>Data</th>
<th>Model</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$ (%)</td>
<td>1.10</td>
<td>1.01</td>
<td>0.87</td>
<td>0.84</td>
<td>-0.01</td>
<td>0.03</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>$\rho_Y$</td>
<td>1.33</td>
<td>1.05</td>
<td>0.94</td>
<td>0.86</td>
<td>0.08</td>
<td>-0.02</td>
<td>0.85</td>
<td>0.91</td>
</tr>
<tr>
<td>$\rho_C$</td>
<td>0.95</td>
<td>1.01</td>
<td>0.88</td>
<td>0.84</td>
<td>-0.07</td>
<td>0.03</td>
<td>0.90</td>
<td>1.00</td>
</tr>
<tr>
<td>$\rho_{\pi^p}$</td>
<td>0.25</td>
<td>0.35</td>
<td>0.57</td>
<td>0.87</td>
<td>0.45</td>
<td>0.24</td>
<td>0.30</td>
<td>0.66</td>
</tr>
<tr>
<td>$\rho_{\pi^w}$</td>
<td>0.73</td>
<td>0.50</td>
<td>0.06</td>
<td>0.49</td>
<td>0.46</td>
<td>0.95</td>
<td>0.08</td>
<td>0.63</td>
</tr>
<tr>
<td>$\rho_{W/P}$</td>
<td>1.13</td>
<td>0.60</td>
<td>0.81</td>
<td>0.87</td>
<td>0.53</td>
<td>0.19</td>
<td>0.06</td>
<td>0.86</td>
</tr>
<tr>
<td>$\rho_i$</td>
<td>0.55</td>
<td>0.72</td>
<td>0.94</td>
<td>0.86</td>
<td>0.10</td>
<td>0.11</td>
<td>0.46</td>
<td>0.83</td>
</tr>
</tbody>
</table>

Notes: The standard deviation $\sigma$, first-order autocorrelation $\rho$, skewness $Sk$, and correlation with output $\rho_Y$ are reported. In the data series, $Y$ is the real GDP, $H$ is the total hours worked, $C$ is the real personal consumption expenditure, $\pi^p$ is the GDP deflator, $\pi^w$ is the compensation per hour in the non-farm business sector, $W/P$ is computed from the compensation per hour and the GDP deflator, and $i$ is the effective federal funds rate. $Y$, $H$, $C$, and $W/P$ are taken as log and detrended by the HP-filter. $\pi^p$, $\pi^w$, and $i$ are the quarterly rate. The sample spans from 1987Q4 to 2008Q4. The start point of the sample corresponds to the Greenspan Era, when inflation is relatively stable. The end point of the sample is determined to exclude the ZLB periods. For computing the moments of the model, the steady-state inflation rate is set equal to the mean of the inflation rates in the sample periods.

rate less workers are willing to pay the menu cost to adjust their wages upwardly. Though the opposite mechanism works for negative wage changes, because the calibrated menu cost for negative wage changes is much larger than that for positive ones a larger fraction of workers keep their wages unchanged.

Panel (B) compares the moments of the time-series moments in the data and those of the simulated series in the model. The model does fairly well in matching
the time-series moments of the data in a number of dimensions including: (1) low
standard deviation of price inflation and wage growth relative to that of output and
hours worked; (2) positive skewness of price inflation, wage growth, and real wage;
(3) comovements among variables. In the model, since wages are downward rigid, the
effects of an adverse shock are absorbed by the changes of the amount of labor rather
than wage adjustments. The asymmetric responses in the labor market transmit to
those of inflation through marginal cost. More discussion is provided by Mineyama
(2018).

4 Welfare implications

In this section, I study the welfare consequences of the model with heterogeneous
workers developed in the previous section.

I compare the baseline model with a homogeneous agent model with asymmetric
wage adjustment cost. In this regard, the asymmetric wage adjustment cost is often
used to approximate DNWR in the literature such as Kim and Ruge-Marucia (2009,
2018) and Aruoba et al. (2017). Specifically, the aggregate nominal wage growth
\( \Pi_w^t \equiv W_t/W_{t-1} \) is governed by the following wage Phillips curve:

\[
\Psi'_w(\Pi_w^t)\Pi_w^t = \beta E [\Psi'_w(\Pi_{w,+1}^t)\Pi_{w,+1}^t] + \theta_w \left( \frac{H_t^{1/\eta}}{C_t} - \mu_w \frac{W_t}{P_t} \right) \frac{H_t}{C_t},
\]

(47)

where

\[
\Psi_w(\Pi_w^t) \equiv \phi_w \left( \frac{\exp(-\psi_w(\Pi_w^t - 1)) + \psi_w(\Pi_w^t - 1) - 1}{\psi^2_w} \right).
\]

(48)

\( \Psi_w(\Pi_w^t) \) is the wage adjustment cost. The parameter \( \psi_w \) governs the degree of asymmetry whereas \( \phi_w \) determines the overall degree of wage rigidity. The derivation of
(47) is presented in Online Appendix B. I calibrate \( \psi_w \) and \( \phi_w \) according to the estimated parameter values of Kim and Ruge-Murcia (2018).\(^{17}\) Other parts of the

\(^{17}\)I find that the main results here are robust to alternative parameter values estimated by Kim
Table 4: Welfare losses under calibrated Taylor rule

<table>
<thead>
<tr>
<th></th>
<th>(1) with wage rigidity</th>
<th>(2) w/o wage rigidity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heterogeneous workers (baseline)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Welfare loss (CE, %):</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Pi^* = 0.0%$</td>
<td>$-0.72$</td>
<td>$-0.14$</td>
</tr>
<tr>
<td>$\Pi^* = 2.4%$</td>
<td>$-0.63$</td>
<td>$-0.41$</td>
</tr>
<tr>
<td>Homogeneous workers</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Homogeneous agent model</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Welfare loss (CE, %):</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Pi^* = 0.0%$</td>
<td>$-0.13$</td>
<td></td>
</tr>
<tr>
<td>$\Pi^* = 2.4%$</td>
<td>$-0.32$</td>
<td></td>
</tr>
</tbody>
</table>

Notes: All the models embed nominal price rigidity and the central bank follow the Taylor rule (44) with $\phi_\pi = 1.50$ and $\phi_y = 0.25$. The steady-state inflation rate $\Pi^* = 2.4\%$ corresponds to the mean of the inflation rate after the Greenspan Era. Welfare losses are in the consumption equivalent (CE) loss from the economy under flexible prices and wages. The steady-state inflation rate is in the annual rate.

The homogeneous agent model is identical to those of the baseline model.

Table 4 compares the welfare losses in different models; the baseline model with heterogeneous workers, the homogeneous agent model with asymmetric wage adjustment cost, and the homogeneous agent model under flexible wages. The table displays the welfare losses at different levels of the steady-state inflation rates $\Pi^*$. The first thing to note is that the welfare losses of the baseline model with heterogeneous workers in column (1) are substantially larger than those of the homogeneous agent model in column (2) regardless of the level of the steady-state inflation rate. Intuitively, the welfare losses due to imperfect wage adjustment depend on the distance between the desired level of wages and the actual wages. In this regard, in the presence of an individual labor productivity shock, workers face fluctuations in their desired wages even the aggregate wage is stable. Therefore, the lack of wage adjustment due to nominal wage rigidity is larger in the presence of heterogeneity. It is also noteworthy that, in homogeneous agent models in column (2) and (3), adding wage rigidity to a sticky price setting does not bring about significant increases of welfare loss, consistently with the findings in previous studies (i.e., Schmitt-Grohé and Uribe (2005), Galí and Monacelli (2016)).


5 Monetary policy analysis

The welfare analysis in the previous subsection suggests that the presence of the heterogeneity of individual workers' wages leads to a substantial increase of welfare loss arising from nominal rigidities. In this section, I investigate the consequences of these welfare differences for the optimal monetary policy.

5.1 Optimal steady-state inflation

As the optimal monetary policy analysis, I start by investigating the optimal level of the steady-state inflation rate, or the inflation rate targeted by the central bank, while fixing the calibrated Taylor rule. Figure 1 displays welfare losses under different levels of the steady-state inflation rate. For this numerical analysis, I change the steady-state inflation rate in the monetary policy rule \( \Pi^* \) from 0 to 5 percent per year with an interval of 0.2 percent to solve the model and compute welfare losses at each inflation rate. The figure indicates that the optimal steady-state inflation rate \( \Pi^* \) is around 2 percent per year. Intuitively, the model setting implies that positive inflation benefits the economy because workers are less suffered DNWR at a higher inflation rate. On the other hand, higher inflation brings about larger distortion through nominal price rigidity because relative prices become more dispersed at a higher inflation rate. The optimal rate is determined as a consequence of the trade-off between the benefits and costs of inflation.

The figure also shows the case of the homogeneous agent model with asymmetric wage adjustment cost. The result is striking; the social welfare in the homogeneous agent model is monotonically decreasing in the steady-state inflation rate as long as the rate is in positive territory. Indeed, the result is consistent with previous studies which find the optimal inflation rate in a model with DNWR is close to zero. In

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18 Strictly speaking, I find that the optimal steady-state inflation is negative in the homogeneous agent model whereas Kim and Ruge-Marucia (2009, 2018) find the rate is non-negative. A key difference to bring about these consequences is that I take into account trend growth in productivity whereas Kim and Ruge-Marucia (2009, 2018) omit it. In this regard, with a positive growth rate in

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25
other words, the benefits of positive inflation that it facilitates real wage adjustments in the presence of DNWR are not quantitatively significant without heterogeneity. Intuitively, the benefits of having the opportunity of wage adjustments are increasing in the distance between the actual wages and the desired ones. In this regard, idiosyncratic fluctuations of labor productivity generate substantial variations in the desired wages even if the aggregate fluctuations are relatively small. Consequently, the benefits of positive inflation are much larger in the presence of idiosyncratic shocks.

To see the mechanism to determine the optimal steady-state inflation rate in more detail, Figure 2 displays selected moments under different levels of the steady-state inflation rate. Several points are noteworthy. First, in Panel (A), as is investigated in the previous section, the fraction of workers with zero-wage-changes declines as the steady-state inflation rate rises. This implies that more workers choose to pay a menu cost to adjust their wages at a higher inflation rate. Second, in Panel (B), productivity, real wages also have a upward trend. Therefore, a negative steady-state inflation rate can bring the nominal wage growth close to zero to minimize the wage adjustment cost. Related issues are studied by Amano et al. (2009).
Figure 2: Selected moments under different levels of $\Pi^*$

Panel (A): Fraction of wage changes

Notes: The fraction of each type of wage changes is computed at different steady-state inflation rates $\Pi^*$ from 0 to 5 percent per year with an interval of 0.2 percent.

The mean level of consumption $C_t$ is hump-shaped; it increase at low inflation rates whereas it starts to decrease when the inflation rate exceeds some point. There are two offsetting effects behind this result. On the one hand, the distortion arising from nominal price rigidity increases in the steady-state inflation rate. More specifically, the increased relative price dispersion $V_t$ and the increased average markup, represented in the inverse of real wage $W_t/P_t$, generate misallocation of individual goods production and therefore reduce consumption. On the other hand, the misallocation in the labor market is lessened as the inflation rate rises because less workers suffer DNWR at a higher inflation rate. Third, the standard deviation of consumption is decreasing in the inflation rate whereas that for the inflation rate is increasing. At a higher inflation rate, wage changes become more frequent, which makes marginal cost more flexible. In addition, price settings become more aggressive at a higher inflation rate, because otherwise their relative prices deviate from their optimal levels due to inflation. Both effects contribute to the increased standard deviation of the
Figure 2: Selected moments under different levels of $\Pi^*$ (cont.)

Panel (B): Aggregate time-series moments

Notes: The stochastic mean and the standard deviation are computed for selected variables at different steady-state inflation rates $\Pi^*$ from 0 to 5 percent per year with an interval of 0.2 percent.

inflation rate. On the other hand, since price variables are adjusted upon exogenous disturbances, real quantities do not need to fluctuate a lot in equilibrium. These dependency of the standard deviations on the steady-state inflation rate is another source of generating the hump-shaped form of welfare losses.
5.2 Optimal monetary policy rule

The analysis so far searches for the welfare-maximizing inflation rate while fixing the form of the Taylor rule. On the other hand, in this subsection, I investigate whether modifications of the monetary policy rule can improve social welfare.

I consider two simple ways of modifying the monetary policy rule. First, adopting the approach of Schmitt-Grohé and Uribe (2007), I choose the responsiveness to inflation and the output gap in the Taylor rule \( \{\phi_{\pi}, \phi_{y}\} \) to maximize the social welfare. Given the computational burden, I restrict the parameter space in \( \phi_{\pi} \in [0.0, 3.0] \) and \( \phi_{y}/4 \in [0.0, 3.0] \) with an interval of 0.25.\(^{19}\) Second, I allow for asymmetric responses to upward and downward deviations of each variable in the monetary policy rule. Specifically, I consider the following rule:

\[
R_t = R^* \left( \frac{\Pi_t^p}{\Pi^*} \right)^{\phi_{\pi,t}} \left( \frac{Y_t}{Y_t^f} \right)^{\phi_{y,t}},
\]

where \( \phi_{\pi,t} = \phi_{\pi}^+ 1\{\Pi_t^p \geq \Pi^*\} + \phi_{\pi}^- 1\{\Pi_t^p < \Pi^*\} \), and \( \phi_{y,t} = \phi_{y}^+ 1\{Y_t \geq Y_t^f\} + \phi_{y}^- 1\{Y_t < Y_t^f\} \).

I choose a set of parameters \( \{\phi_{\pi}, \phi_{\pi}^+, \phi_{y}^+, \phi_{y}^-\} \) to maximize the social welfare. To restrict the parameter space, I assume that the mean responsiveness is equal to that of the baseline calibration, i.e., \( (\phi_{\pi}^+ + \phi_{\pi}^-)/2 = 1.50 \) and \( (\phi_{y}^+ + \phi_{y}^-)/2 = 0.25 \). By the same token, I fix the steady-state inflation in the modified rules at zero. I label the first case as "optimized symmetric Taylor rule," whereas the second case as "optimized asymmetric Taylor rule."

Table 5 compares the welfare losses under the optimized monetary policy rules in each specification. Several points are noteworthy in the table. First, in the symmetric Taylor rule in column (2), the optimal responsiveness to both inflation and the output gap is higher than the calibrated parameter values. The result is in a sharp contrast to a convention wisdom in the literature that price stability, or strong responsiveness

\(^{19}\)A similar range for each parameter is employed by Schmitt-Grohé and Uribe (2007).
Table 5: Welfare losses under optimized monetary policy rule

<table>
<thead>
<tr>
<th>Parameter values:</th>
<th>(1) Optimized steady-state inflation rate</th>
<th>(2) Optimized symmetric Taylor rule</th>
<th>(3) Optimized asymmetric Taylor rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>Π*</td>
<td>1.8</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>φₚ</td>
<td>1.50</td>
<td>3.00</td>
<td>–</td>
</tr>
<tr>
<td>φᵧ</td>
<td>0.25</td>
<td>0.50</td>
<td>–</td>
</tr>
<tr>
<td>(φₚ⁺, φ₋ₚ)</td>
<td>–</td>
<td>–</td>
<td>(1.25, 1.75)</td>
</tr>
<tr>
<td>(φᵧ⁺, φ₋ᵧ)</td>
<td>–</td>
<td>–</td>
<td>(0.125, 0.375)</td>
</tr>
<tr>
<td>Welfare loss (CE, %)</td>
<td>-0.58</td>
<td>-0.62</td>
<td>-0.45</td>
</tr>
</tbody>
</table>

Notes: Parameter values in italic indicate optimized ones whereas the other parameters are fixed. Π* is in the annualized percent. In column (2), the parameter space is restricted to φₚ, φᵧ/4 ∈ [0.0, 3.0] with an interval of 0.25. In column (3), the parameter space is set to φₚ⁺, φ₋ₚ, φᵧ⁺/4, φ₋ᵧ/4 ∈ [0.0, 3.0] with an interval of 0.25 such that (φₚ⁺ + φ₋ₚ)/2 = 1.50 and (φᵧ⁺ + φ₋ᵧ)/2 = 0.25.

only to inflation, is close to optimal even in the presence of wage rigidity (i.e., Schmitt-Grohé and Uribe (2005), Walsh. (2014)). Intuitively, price and wage rigidities generate two distinct markups and therefore the central bank faces a trade-off in closing the two markups simultaneously. In that environment, the responsiveness to the output gap stabilizes the marginal rate of substitution of households, which helps to mitigate the inefficient fluctuations of the wage markup. In this regard, since the distortion arising from wage markup is enlarged in the presence of heterogeneity, the responsiveness to the output gap is more important than in a homogeneous agent model used in the previous studies. Second, once allowing for asymmetric responses in column (3), stronger responsiveness to downward deviations of inflation and the output gap than to upward ones is preferred. This is because the distortion arising from DNWR is particularly severe in a downturn when a larger share of workers is constrained by DNWR. Therefore, monetary policy can reduce the welfare losses by reacting aggressively in a downturn. Third, when comparing the resulting welfare losses across the specifications, the welfare loss under the optimized symmetric Taylor rule in column (2) is fairly close to that in the case of the optimized steady-state inflation rate in column (1). Moreover, the asymmetric monetary policy rule in column (3)
yields the highest welfare in the three specifications.

6 Conclusion

In this paper, I build a sticky price model with heterogeneous workers whose wage changes are subject to asymmetric menu costs. Consistently with the hypothesis of DNWR, the model identifies a much larger cost for downward wage adjustments than upward ones when calibrated to the U.S. wage distribution. I find that the heterogeneity among individual workers’ wages enlarges the welfare losses arising from DNWR by generating a sizable welfare loss through an inefficient cross-sectional allocation of labor. The model has sharp implications to the conduct of monetary policy as well as the steady-state inflation rate.
References


Daly, Mary C. and Bart Hobijn, “Downward Nominal Wage Rigidity Bend the Phillips Curve,” Journal of Money, Credit and Banking, 2014, 46 (S2), 51–93.


A Computation

This section presents an equilibrium computation method that is developed by Mineyama (2018).

A.1 Modified Krusell-Smith algorithm

Approximated equilibrium. To deal with the infinite dimensionality of the cross-sectional distribution, Krusell and Smith (1998) propose an approximated equilibrium where each agent perceives the evolution of aggregate state variables as being a function of a small number of moments of the cross-sectional distribution. Adopting their insights, I assume that the aggregate endogenous state variable, aggregate real wages $\bar{W}_t$, follows an aggregate law of motion (ALM):

$$\bar{W}_t = \Gamma(\bar{W}_{t-1}, D_{t-1}, Q_t, A_t).$$  \hfill (A.1)

where $D_{t-1}$ is the relative price dispersion, $Q_t$ is exogenous risk premium, and $A_t$ is exogenous productivity.

Specification of the ALM. To parameterize the ALM $\Gamma$, I conjecture that it is
approximated by a log-linear form:

$$\log(\bar{W}^s) = B_{0s} + B_{1s} \log(\bar{W}_{-1}) + B_{2s} \log(D_{-1}) \text{, for } (Q_t, A_t) = (Q^s, A^s) \quad (A.2)$$

where the superscript $s$ denotes the exogenous state of the economy. It should be noted that, even though the ALM takes a simple functional form in terms of the state variables in the previous period, it can capture rich non-linear dynamics because of the semi-parametric specification of the coefficients $B = \{B_{0s}, B_{1s}, ..., B_{ks}\}_{s=1}^S$. In other words, it accommodates different impacts of the past state variables $B^s$ across exogenous states $s$.

The log-linear specification of own lagged values is widely used in the literature to approximate the ALM (e.g., Krusell and Smith (1998), Nakamura and Steinsson (2010)). For robustness check, I include the higher order moments and the higher order polynomials in the ALM. I also introduce a jump variable such as output and interest rate as a regressor, which is a strategy to increase the forecasting power of the ALM proposed in the literature (e.g., Vavra (2013)). However, I find that the resulting equilibrium dynamics are numerically close to the baseline specification.

Outline of algorithm. I sketch the outline of the equilibrium computation. The algorithm takes the following steps. First, I formulate a guess for the aggregate law of motion (ALM) of the state variable, that is, aggregate real wages. Second, given the guess for the ALM, I solve for aggregate jump variables as an equilibrium outcome of the aggregate part of the economy. Notice that it is a New Keynesian system that includes the consumption Euler equation (23), the NKPC (34)-(38), and the Taylor rule (44). Importantly, this step is feasible because the aggregate part is independent of individual workers’ behavior once conditional on aggregate real wages. Third, given all the aggregate variables, I derive an individual workers’ real wages as a solution to the individual wage setting problem in (24). Finally, I numerically integrate individual wages to recover the aggregate wages and update the guess for the ALM.
Algorithm. The algorithm takes the following steps in each iteration \( m = 1, 2, 3 \ldots \)

1. (Initial guess) Each agent uses the ALM with the coefficients \( B^{(m)} \) to forecast the current period aggregate state variable \( \bar{W} \).

2. (Aggregate problem) Given the aggregate state variables \( \{Q, A, \bar{W}\} \), the policy function for aggregate jump variables \( f^{(m)} \) is obtained by solving an aggregate New Keynesian system, i.e., the Euler equation, the NKPC, and the Taylor rule, along with the production function and the market clearing condition. Notice that, given \( \bar{W} \), the aggregate part of the economy does not depend on the cross-sectional distribution.

3. (Individual problem) Given all the aggregate variables, i.e., the aggregate state variables \( \{Q, A, \bar{W}\} \) and the aggregate jump variables implied by \( f^{(m)} \), individual households solve their wage setting problem to derive their policy function \( h^{(m)} \).

4. (Stochastic aggregation) Given the aggregate policy function \( f^{(m)} \) and the individual policy function \( h^{(m)} \), the model economy is simulated with \( N \) households for \( T \) periods. The initial distribution is set at the one in the stationary equilibrium, and the initial \( T_0 \) periods are discarded. The simulation delivers the series of aggregate variables \( \{X^{(m)}_t\}_{t=T_0+1}^T \). I set \( N = 10,000 \), \( T = 21,000 \), and \( T_0 = 1,000 \).\(^{20}\)

5. (Update) Using the simulated variables \( \{X^{(m)}_t\}_{t=T_0+1}^T \), the suggested coefficients \( \hat{B} \) are obtained by running the OLS of the ALM. Then, the coefficients \( B^{(m+1)} \) are updated according to the rule:

\[
B^{(m+1)} = \lambda \hat{B} + (1 - \lambda) B^{(m)} \tag{A.3}
\]

where \( \lambda \) is the weight for updating. I set \( \lambda \) at 0.1.

\(^{20}\)I confirm that the computation results do not change even if I further increase \( N \) or \( T \).
6. Repeat from step 1 to step 5 until convergence criterion for the coefficients $B$ are attained.

**Convergence criterion of the ALM.** I use two convergence criterion for the ALM coefficients $B$. First, I repeat iterations until the maximum quadratic distance between the original and updated coefficients becomes smaller than $10^{-4}$. This is a standard criteria used in the literature.

In addition, to guarantee the accuracy of the ALM, I use the Den Haan (2010) statistics as an additional convergence criteria. The statistics measures the maximum distance between the aggregate state variables computed according to the ALM, $\tilde{W}_{alm}^t$, and those derived from equilibrium conditions in the simulation, $\tilde{W}_{sim}^t$:

$$DH(B) = \sup_{t \in [T_0+1,T]} |\log(\tilde{W}_{sim}^t) - \log(\tilde{W}_{alm}^t)|.$$ (A.4)

Den Haan (2010) proposes to use the statistics rather than $R^2$ to check the accuracy of the ALM, because $R^2$ only measures the average error in the one-period ahead forecast. I repeat iterations until $DH(B)$ becomes smaller than $10^{-2}$. The critical value indicates that the cumulative error of agents’ prediction of the aggregate real wage is smaller than 1% over 20,000 periods. The accuracy is in the same order as the one used in the literature.\(^{21}\)

**B Model comparison**

This section presents the homogeneous agent model with asymmetric wage adjustment cost, with which I compare the results of the baseline model with heterogeneous workers in Section 4. The specification largely follows that of Kim and Ruge-Murcia (2018).

\(^{21}\) In this regard, Den Haan (2010) compares several computation algorithms to solve a heterogeneous agent model, and find that the Krusell-Smith algorithm, which is the most accurate one, gives around 0.2% as the Den Haan (2010) statistics over 10,000 periods.
Household problem. I consider the following household problem.

\[
\max_{w_t(j), c_t(j), b_t(j)} : \mathbb{E}_t \left[ \sum_{s=0}^{\infty} \beta^s \left( \ln c_t(j) - \frac{h_t(j)^{1+1/\eta}}{1+1/\eta} \right) \right],
\]

s.t. \( c_t(j) + \Psi_w(\Pi_t^w)C_t + \frac{b_t(j)}{P_t} \leq \frac{w_t(j)}{P_t} h_t(j) + R_{t-1} \frac{b_{t-1}(j)}{P_t} + \tau_t(j) + \Phi_t(j), \) \( \text{(A.6)} \)

\( l_t(j) = z_t(j) h_t(j), \) \( \text{(A.7)} \)

\( l_t(j) = \left( \frac{w_t(j)}{z_t(j)} \right)^{-\theta_w} L_t, \)

where \( \Psi_w(\Pi_t^w) \equiv \phi_w \left( \exp(-\psi_w(\Pi_t^w - 1)) + \psi_w(\Pi_t^w - 1) - 1 \right). \) \( \text{(A.9)} \)

\( \Psi_w(\Pi_t^w) \) is the adjustment cost of nominal wages, which I assume is proportional to the aggregate consumption. The notations of the other variables follow those in the baseline model. The first order condition for \( w_t(j) \) takes the following form:

\[
\begin{align*}
- h_t(j)^{1/\eta} \frac{\partial h_t(j)}{\partial w_t(j)} + \lambda_t(j) \left\{ \left( \frac{1}{P_t} h_t(j) + \frac{w_t(j)}{P_t} \frac{\partial h_t(j)}{\partial w_t(j)} \right) - \Psi_w'(\Pi_t^w(j)) \frac{1}{w_{t-1}(j)} C_t \right\} \\
- \beta \mathbb{E}_t \left[ \lambda_{t+1}(j) \Psi_w'(\Pi_{t+1}^w(j)) \left( - \frac{w_{t+1}(j)}{w_t(j)^2} \right) C_{t+1} \right] = 0 \quad \text{(A.10)}
\end{align*}
\]

where \( \lambda_t(j) = 1/c_t(j) \) is the Lagrangian multiplier of the budget constraint (A.6). Equation (A.7) and (A.6) yield

\[
\frac{\partial h_t(j)}{\partial w_t(j)} = \frac{\partial h_t(j)}{\partial l_t(j)} \frac{\partial l_t(j)}{\partial w_t(j)} = \frac{1}{z_t(j)} \left( -\theta_w \frac{l_t(j)}{w_t(j)} \right) \]

\( = -\theta_w \frac{h_t(j)}{w_t(j)} \) \( \text{(A.11)} \)
Using (A.11), (A.10) is rearranged to

\[ \Psi'_w(\Pi^w_t(j))\Pi^w_t(j) = \beta\mathbb{E}\left[\Psi'_w(\Pi^w_{t+1}(j))\Pi^w_{t+1}(j)\right] + \theta_w \left(\frac{h_t(j)^{1/\eta}}{c_t(j)} - \mu_w \frac{w_t(j)}{P_t}\right) \frac{h_t(j)}{c_t(j)}. \]

(A.12)

In the symmetric equilibrium, where workers are homogeneous, (A.12) leads to the wage Phillips curve in the main article.

**Calibration.** For the parameter \( \phi_w \) and \( \psi_w \), I use the estimated values of Kim and Ruge-Murcia (2018). They report \( \phi_w = 33.85 \) and \( \psi_w = 602.48 \) when they estimate the model with normally distributed shocks using U.S. data from 1964Q2 to 2015Q4 (Table 1 of Kim and Ruge-Murcia (2018)). Since I use different value for the elasticity of substitution among labor service \( \theta_w \), I adjust \( \phi_w \) so that the slope of the linearized wage Phillips curve corresponds to that in their paper.\(^{22}\)

## C  Connection to micro evidence on wage rigidity

There exists a rapidly growing literature on wage rigidity using micro data. Though the model setting and calibration in this paper relies on a particular data set used by Grigsby et al. (2018), I assess the validity of them in the light of other micro evidence based on different data set. Table A.1 compares key moments of wage changes reported by previous studies. Several points are noteworthy. First, previous studies including Daly and Hobijn (2014) and Fallick et al. (2016) report that household survey data such as the Current Population Survey (CPS) tends to display frequent wage changes. However, it should be noted that self-reported survey data might contain considerable measurement errors. In the presence of measurement errors, the reported wages might look fluctuating even if the true wages remain unchanged. To address the issue, Barattieri et al. (2014) and Basu and House (2016) employ an econometric

\(^{22}\)I calibrate \( \theta_w = 7 \) whereas Kim and Ruge-Murcia (2018) use \( \theta_w = 3.5 \) (the steady-state wage markup is 40%). The adjusted parameter value is \( \phi_w = 33.85 * 7/3.5 = 67.70 \).
Table A.1: Cross-sectional moments of wage adjustments in different data sets

<table>
<thead>
<tr>
<th>Paper; Categories</th>
<th>Fraction of wage changes (percent)</th>
<th>Data description</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Positive</td>
<td>Negative</td>
<td>Non-zero</td>
</tr>
<tr>
<td>Quarterly wage changes:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Barattieri et al. (2014);</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hourly workers</td>
<td>n.a.</td>
<td>n.a.</td>
<td>21.1-26.6</td>
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<td>Basu and House (2016);</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Salaried workers</td>
<td>n.a.</td>
<td>n.a.</td>
<td>20.9</td>
</tr>
<tr>
<td>Yearly wage changes:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Author’s calculation based on Daly and Hobijn (2014);</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hourly and salaried</td>
<td>n.a.</td>
<td>n.a.</td>
<td>88.0</td>
</tr>
<tr>
<td>Hourly and salaried</td>
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<td>n.a.</td>
<td>84.0</td>
</tr>
<tr>
<td>Author’s calculation based on Elsby et al. (2016);</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hourly workers</td>
<td>55.0-82.6</td>
<td>11.2-25.5</td>
<td>80.5-93.8</td>
</tr>
<tr>
<td>Salaried workers</td>
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<td>21.5-37.1</td>
<td>85.1-91.1</td>
</tr>
<tr>
<td>Fallick et al. (2016);</td>
<td></td>
<td></td>
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<tr>
<td>Jobs</td>
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<td>n.a.</td>
<td>79.5-88.8</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hourly and salaried</td>
<td>72.5</td>
<td>20.0</td>
<td>92.5</td>
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<tr>
<td>Hourly and salaried</td>
<td>59.0</td>
<td>25.0</td>
<td>84.0</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Hourly and salaried</td>
<td>46.1-69.8</td>
<td>20.4-33.1</td>
<td>77.1-90.5</td>
</tr>
</tbody>
</table>

Notes: The data source of each paper is as follows; Barattieri et al. (2014) and Basu and House (2016): the Survey of Income and Program Participation (SIPP). Daly and Hobijn (2014) and Elsby et al. (2016): the Current Population Survey (CPS). Fallick et al. (2016): the Employment Cost Index (ECI), Kurmann and McEntarfer (2018) and Jardim et al. (2019): the Longitudinal Employer Household Dynamics (LEHD). The unit of observation in the ECI is jobs whereas that for the other data is workers. Barattieri et al. (2014) and Basu and House (2016) compute the unconditional probabilities of job-stayers and job-changers whereas the other studies focus on job-stayers. Barattieri et al. (2014) report a range of estimates depending on the assumptions imposed to adjust measurement errors. For Elsby et al. (2016) and Fallick et al. (2016), each range indicates the upper and lower bound of the estimates during sample periods.

technique to adjust measurement errors. After adjustments, both studies’ estimates of the fraction of wage changes are consistent with those by Grigsby et al. (2018), which I use as the calibration target. Second, although I adopt the strategy of Grigsby et al. (2018) to focus on the earnings per pay period as a measure of wages, previous studies suggest that once benefits are included total compensation displays frequent changes with a considerable fraction of negative changes. For example, Kurmann
and McEntarfer (2018) use administrative data in Washington state in the U.S. to report that there are substantial small decreases in hourly compensation. In this regard, Grigsby et al. (2018), who compare base pay and bonus in administrative payroll data, find that base pay accounts for a large portion of total compensation on average though there are substantial heterogeneity among workers. Reflecting the finding, Kurmann and McEntarfer (2018) argue that, although total compensation changes frequently due to adjustments of benefits, the rigidity of base pay may still be a relevant constraint given the small portion of benefits out of total compensation.\footnote{Another important issue is the cyclicality of benefits. In this regard, Lebow et al. (2003), computing changes of wages and benefits separately in the individual data of the Employment Cost Index (ECI), document that, although benefits change more frequently than wages, changes of benefits are not systematically related to wage changes. Based on these observations, they conclude that the hypothesis that benefits are used to offset nominal wage rigidities is not supported in the data. Moreover, Gu and Prasad (2018) find that benefits come to more rigid over time, due to the increases of quasi-fixed components such as health insurance and defined contributions (IRAs, 401k, etc.). They report that the increased rigidity of benefits made the total real compensation countercyclical especially after the Great Recession.} Third, the counter-cyclicality of the fraction of non-zero wage changes is observed in a wide class of data. For example, Daly and Hobijn (2014) and Kurmann and McEntarfer (2018) compare the frequency of wage changes before and after the Great Recession and find this point.