Calibrating an agent-based model with learning gradients:

Extended Abstract

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Abstract

This paper presents new empirical methods for both estimating and validating agent-based financial models. The key issue is the estimation of learning gradients, or the objective surface on which adaptive agents move as they adapt their behavior. These surfaces are estimated using a new data set that allows for the construction of very long volatility time series which are used to assess optimal trading strategies. It is shown that strategies relying on relatively short half-lives thrive, and dominate longer, slower to adjust, strategies. These features are then used to estimate the intensity of choice parameter in an agent-based financial market setting. This novel estimation strategy uses the gradient surface to generate a hypothetical agent population which can then be used to test a simulated model, or update the intensity of choice in an iterative fashion. The paper shows both that this is a useful technical tool for assessing agent-based models, and also emphasizes the importance of short memory behavior in explaining the key dynamic properties of financial markets.

Keywords: Agent-based finance, volatility, learning

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1 Introduction

Agent-based financial markets have recently made great strides in the area of empirical validation. Most attempts to validate models have started with either calibration, or more detailed fitting of basic models to many empirical features of financial and macro-economic data.

Although making great progress in understanding the dynamics of agent-based models, the macro tradition of fitting various aggregate features has some limitations. Certain key parameters can often not be well identified. Also, macro level models can leave open questions about the learning dynamics of agents at the micro level and whether they are consistent with what is happening in real markets. The best solution to both of these problems is to acquire micro (agent level) data on behavior and individual responses to their observed environments. This is often impossible in the world of finance where many data sets are unavailable and/or proprietary.

This paper presents a new approach to this problem by directly estimating learning gradients, or fitness landscapes from the data, and using these to calibrate an agent-based model. These landscapes underlie almost all models with learning dynamics of any kind, but are largely ignored as a possible empirical feature that can be used in model testing and validation. Fitness landscapes are derived from agents’ preferences, and empirical performance for various families of heterogeneous strategies. These ex post performance measures are easily obtained from commonly available financial time series.

2 Volatility forecasting

Since the early paper of Mandelbrot (1963) we have known about the predictability of volatility of stock returns. These features were an important part of the explosion of ARCH/GARCH models.

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1 See recent surveys by Lux & Zwinkels (2018) and Dieci & He (2018).
2 An early example of this is Boswijk, Hommes & Manzan (2007), and a more recent example using slightly different tools is Chiarella, He & Zwinkels (2014).
3 Among these are the critical intensity of choice parameter which often controls learning dynamics. See Hommes & in’t Veld (2017) for an example.
4 Two early examples using data of finer granularity are Goldbaum & Mizrach (2008) with mutual fund fund flow data, and Goldbaum & Zwinkels (2014) using individual foreign exchange forecasts to estimate the structure of traders’ predictions at the micro level. Another solution is the use of laboratory experiments as in Hommes (2011).
5 A standard estimated HAM model with a small number of types implicitly maps data into relative performance measures which are then mapped to fractions of trader types (fundamental/chartist). This is close to what is proposed here, but when many rules are present, there is a much richer mapping available for testing.
in the 1980’s and beyond. In the past decade volatility attention has shifted to models built from realized volatility estimates using higher frequency time series to estimate a lower frequency volatility series.

The importance of this empirical research is that risk (or volatility) is predictable to some extent in financial data. By traditional econometric measures, such as R-squared, it far exceeds expected returns in terms of predictability. However, for some reason it is not a key component in models with adaptive learning agents in finance. This paper is not the first to address volatility in an agent-based setting. However, this is probably the first paper to turn most of its attention to volatility forecasting as the primary agent learning dynamic.

The objective here is to analyze in detail a simple, possibly misspecified, learning model for estimating conditional variances on a relatively short horizon. Agents build forecasts from a simple exponential filter. Let $R_t$ be the return of a risky asset at time $t$, and $r_t = \log(1 + R_t)$. When an explicit time series for volatility (ie. realized volatility) is available, then a simple volatility forecast is given by,

$$\hat{\sigma}^2_{t+1|t} = (1 - \lambda_i)\hat{\sigma}^2_{t|t-1} + \lambda_i \sigma^2_t$$

where $\sigma^2_t$ is the contemporaneous realized volatility measure. This is an exponential filter, and depends on only a single parameter, $\lambda_i$, which controls the forecast gain, or how far into the past the forecast reaches. The forecast is an exponentially weighted average of previous values.

This model provides a useful benchmark for agent-based modeling, because it represents a simple rule of thumb. Also, it can be easily explored by learning agents who are experimenting with different values of $\lambda_i$. It generates one step ahead forecasts which are numerically close to those from a standard GARCH(1,1) model, an important benchmark in empirical volatility modeling. It is also the optimal forecast in the case where the target follows a random walk plus noise.

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6 The volatility modeling area of econometrics is enormous. The pioneering ARCH/GARCH papers are Engle (1982) and Bollerslev (1986). There are now many surveys of the literature including Andersen, Bollerslev, Christoffersen & Diebold (2006).

7 Two early examples are in Andersen, Bollerslev, Diebold & Labys (2003) and Barndorff-Nielsen & Shephard (2002).

8 The original Santa Fe Artificial stock market allowed for changing volatility Arthur, Holland, LeBaron, Palmer & Tayler (1997). More recent models with changing volatility include Branch & Evans (2011) and LeBaron (2012). Volatility can cause a sudden increase in selling, as agents all increase their risk assessment simultaneously. A closely related dynamic can occur in models with leverage limits, or VaR targets. See Thurner, Farmer & Geanakoplos (2012) for an example.

9 This important result goes back to Muth (1960). It is relevant in the volatility world, since volatility is extremely
A final aspect of volatility forecasting is its use in dynamic strategy construction. In principle any volatility forecast should yield an estimate of a variance/covariance matrix, and eventually optimal portfolio weights. In all the settings in this paper there is only one risky asset, so the optimal portfolio weights for a myopic constant relative risk aversion investor are given by,

$$
\alpha_t = \frac{E_t[\nu_{t+1}] - r_{f,t+1} + (1/2)\hat{\sigma}_{t+1|t}^2}{\gamma\hat{\sigma}_{t+1|t}^2},
$$

where $\gamma$ is the coefficient of relative risk aversion. $\alpha_t$ is the fraction of wealth to invest in the risky asset. Therefore, the overall portfolio return is given by

$$
R_{p,t+1} = \alpha_t R_{t+1} + (1 - \alpha_t) R_{f,t+1}.
$$

Finally, investor success or failure is measured through a risk adjusted performance measurement. One can assume a linearized utility estimate as in

$$
E_t(U_{t+1}) = E_t(U_{p,t+1}) - (1/2)\gamma\hat{\sigma}_{p,t+1|t}^2,
$$

$$
U = E(U_t) = E(U_{p,t}) - (1/2)\gamma\sigma_p^2
$$

which is also a certainty equivalent return. Time averages using returns and realized variances provide an overall risk adjusted return measure for any given portfolio strategy. This value can often be the input into the reinforcement learning part of any standard agent-based market setup. Finally, it can be indexed for any model in a given forecast family. If one limits the modeling dimension in this case to the variance gain parameter, $\lambda_t$, as in equation, then this gives a set of expost utility measures indexed by $\lambda_t$, $U(\lambda_t)$ representing a learning gradient surface on which adaptive agents should be moving.

10See Campbell & Viceira (2002) for this and many related examples. This formula is not exact, but is a log linear approximation which is standard in the finance literature.
3 Data and initial empirical results

This section presents some initial empirical results indicating that the fitness landscape given by $U(\lambda_i)$, and estimated from U.S. stock return data is consistent with some of the complex behavior generated in simulated agent-based models.

The data used is a long series of intra-day data for the Dow Jones Industrial index provided in the Global Financial Data set (GFD). This series samples the Dow index at hourly frequency back to 1933, providing an extremely accurate method for building a realized volatility series using the high frequency data to estimate the variance for a lower frequency weekly series. The daily close of the Dow along with the dividend yield are also taken from GFD to construct a weekly total return series, inclusive of dividends. The risk free rate is taken from the CRSP 90 day Tbill return series, and is interpolated from monthly to weekly data, as is the U.S. inflation rate which will be used to adjust all returns for inflation. This gives a joined series as,

$$[r_t, \sigma^2_t, r_{f,t}, \pi_t]$$

representing the weekly return, volatility, risk free rate, and inflation rate. Using equation 2 one can construct next period’s portfolio weight. To do this requires equation 1, and assuming that the unconditional sample mean is the best forecast for $E_t(r_{t+1})$.

Now with the dynamic strategy represented in $\alpha_t$, the risk adjusted real return for the strategy can be estimated on the entire sample. It is plotted for varying gain levels in figure 1. Gain levels are mapped into half-lives. A half-life of 30 weeks corresponds to a rule that reduces the weight of a value at $t - 30$ to $(1/2)$ the weight of the current value at $t$. The figure displays risk adjusted returns (certainty equivalence) across a range of gains from 1 to over 200 weeks in half-life, and for three different risk aversion levels, $(3, 4, 5)$. All three display a maximum in the range of 25 to 45 weeks, or less than one year. This is a landscape that will draw volatility forecasts into an intermediate range that does some short term time averaging, but draws agents away from using longer term averages. If the gradient surface were drawing the agents to forecasts putting heavier weight on older data (or equivalently assuming stationarity in the time series), then we should observe a steadily upward sloping fitness landscape, moving learning agents to very large half-
life forecasts. The fact this does not occur appears consistent with some of the behavior observed in models with multiple gain levels operating simultaneously. In other words, sets of agents who are subject to relatively short term swings in variance play a key role in driving market volatility. They are a slightly sophisticated form of noise trader.\footnote{LeBaron (2002) is an early example of this.}

Many agent-based models emphasize the expected return component of agent forecasts. This can be added to the portfolio decision problem by replacing $E_t r_{t+1}$ with some adaptive expectation. Figure 2 does this by allowing agents to forecast both the mean and the variance. In this case the expected return uses a similar exponential filter as used for volatility, along with the same

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**Figure 1: Fitness landscape**

*Description:* Risk and inflation adjusted annual returns for varying volatility forecasts. Forecast memory length is represented as half-life in units of weeks.
Figure 2: Fitness Momentum/Volatility

Description: Risk and inflation adjusted annual returns for varying volatility forecasts, and momentum based return forecast. Forecast memory length is represented as half-life in units of weeks.

The general pattern from the previous figure is repeated here, suggesting an optimal gain level with a relatively short half life.

Finally, figure 3 estimates the variance forecast accuracy, measured as mean squared forecast error (MSE), across the set of exponential forecasts indexed by $\lambda_i$. It again shows an optimal forecast at a relatively short half-life. This should not be too surprising since we know that many dynamic variance forecasts beat a naive unconditional (long half-life) variance forecast. However, it is interesting that this relatively atheoretic measure of forecast accuracy comes close to the same
**Figure 3: Variance forecast accuracy**

*Description:* Mean squared forecast error (MSE) for 1 week ahead realized variance forecast measured for different adaptive forecasts at indicated half-life.

half-life range as the two utility based measures.

## 4 Agent-based calibration/simulation

Can the results in figure 3 yield insights as to the underlying data generating process behind it? Given what is known about volatility dynamics, it is likely that a long memory process, or even a mixed frequency process might be able to generate this figure\(^{12}\). The objective here will be to use these results to estimate and calibrate an agent-based model, where learning is taking place on this landscape. Two novel methods will be used in calibrating the model. Both will address the problem of getting good estimates to learning rates, and the intensity of choice.

Many agent-based models use some form of a discrete choice mechanism to map a given fitness, or utility objective into agent population dynamics. For example, in this case we will have,

\[
Z_{i,t} = \frac{e^{\beta U_{i,t}}}{\sum_{j=1}^{J} e^{\beta U_{j,t}}},
\]

\(^{12}\)See Cont (2001), Corsi (2009), and LeBaron (2001) for examples of long memory and its connection to mixed frequency models.
where $z_{i,t}$ gives the fraction of type $i$ at $t$, $U_{i,t}$ is the fitness of type $i$ (out of a set of $J$ types), and $\beta$ is the intensity of choice parameter that critically controls the mapping from fitness units to population dynamics. It is often one of the key parameters in models of this type.

First, fitness landscapes will be constructed from the Dow data, and then used in the simulation to guide agent rule selection. Agent strategy selection will not use the internal series generated by the agent-based model, but will follow along the empirical gradients from the actual market data. These same fitness landscapes taken from the simulation can be compared with the original Dow landscapes as an internal consistency check. Also, several parameters can be adjusted to bring this into as close a match as possible, the most important of these would be the intensity of choice parameter, $\beta$.

Second, using the discrete choice framework, and a given $\beta$, one can convert the empirical fitness landscape into an agent population. This specifies the specific fraction of agents of each type. Loading this into an agent-based model can again give another consistency check as this will generate its own landscape from the simulation. Here the $\beta$ estimation loop can be quite direct in that for a given value of $\beta$ we get a prescribed agent population, and then, from the simulation, a landscape which can be matched to the empirical landscape. Iteration on $\beta$ can then give a best fit model. A discretized version of the landscape could form the objective in a simulated method of moments estimation.

In the paper a new agent-based model is developed which brings together rule selection and structure from LeBaron (2012). These rules emphasize the learning gain parameter families $(\lambda_i)$ that are critical to understanding figure [1]. These rule structures are combined with the faster price determination system used in much of the evolutionary finance literature, Evstigneev, Hens & Schenk-Hoppe (2006), and inspired by Blume & Easley (1990).[13] This speed improvement is critical for many of the iterative estimation procedures previously discussed.

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13A good example of a close technology is in Palczewski, Schenk-Hoppe & Wang (2016). Another technology that may be necessary for efficiently exploring the parameter space is the machine learning surrogates methodology introduced in Lamperti, Roventini & Sani (2018).
5 Conclusions

This paper presents evidence on the structure of fitness landscapes in financial time series. This is done using a new long data set with both returns, and a high quality volatility estimate drawn from a large set of intra-day data. This latter series allows for a greater concentration on the dynamics of risk, and its importance to agent behaviors, including risk avoidance through dynamic portfolio choice. This aspect of the project draws attention to a possible new set of stylized features which map directly into dynamic strategy choice in a wide class of agent-based financial market models. They also emphasize how the empirical dominance of relatively short memory volatility forecasts may form the evolutionary core of many market instabilities.

The empirical results are then connected to a new agent-based model. Estimation and calibration is done concentrating on the fitness levels to assess some parameters which have often been difficult to estimate in the empirical agent-based literature, specifically the intensity of choice. It also emphasizes the class of agent-based models based on heterogeneous gain levels that seem well suited to understanding the fitness levels observed across various gain levels (or forecast memory).
References


