Inequality over the Life Cycle, Housing, and the Business Cycle

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Abstract:
We study the redistributive impact of a productivity shock in both the consumption goods and housing sector in a life-cycle model that distinguishes between home owners and renters. Our model is able to replicate the heterogeneity of income and wealth and the qualitative life-cycle behavior of housing and consumption. The model clarifies how the interaction between the business cycle and housing affects inequality. In accordance with the experience of the Great Recession, we find that income inequality increases when house prices fall. Young homeowners with high debt are particularly vulnerable to an economic downturn.
1 Introduction

The effects of the business cycle on the distribution of income and wealth have attracted the interest of many studies starting with the work by Doepke and Schneider (2006a) and Doepke and Schneider (2006b). Inspired by the experience of the oil crisis and inflationary period in the 1970s, these authors consider the effects of inflation on the inequality of income and wealth. More recent work by Heer and Scharrer (2018) in the wake of the Great Recession 2007-2008 has shifted the focus from monetary to fiscal policy, which has helped to alleviate the severe consequences of the economic downturn in the US subsequently. The present paper contributes to this literature. In particular, it emphasizes the housing channel which many economists see as the major source of the Great Recession and main explanation for its perseverance.

During the recent crisis, income inequality has increased, particularly among the young middle- and low-income households and those whose primary asset is housing. Kenworthy and Smeeding (2013) report that income has fallen by 8 percent at the median, while the average loss of the value of owner-occupied housing during the Great Recession amounted to 30%. As a consequence, consumption in the middle part of the income and wealth distribution has fallen during the Great Recession. The importance of housing as primary asset of the middle-class has played a crucial factor during the recent recession. In this paper, we present a model that captures the importance of the housing channel of the transmission from business cycle downturn to inequality. In particular, we consider a disaggregate model that distinguishes between households with positive savings and rule-of-thumb consumers on the one hand and renters and owners of houses on the other hand. We show that, following a negative productivity shock as in the recent Great Recession, the middle class loses, which, however, is only partly reflected in the response of the aggregate measure of the Gini coefficient due to compensatory effects of other income groups, such as rule-of-thumb consumers.

Our model builds on the work by Davis and Heathcote (2005) who develop a multisector growth model with correlated shocks to explain certain facts like, for example,
the higher volatility of residential investment in comparison to business investment over the business cycle. Moreover, Sun and Tsang (2017) include a rental market with nominal rigidities in a standard DSGE model following Iacoviello and Neri (2010) so that they are able to match the stickiness of nominal rents in the data. Previous work on redistributive effects of business cycles has examined the effects of inflation on the distribution of wealth and income. Doepke and Schneider (2006a) and Doepke and Schneider (2006b), in particular, find that the losers from inflation are rich, old households because their bondholdings lose in real value, while young, middle-class households with fixed-rate mortgages are the main winners. Their analysis is inspired by the experience of the 1970s where inflation had increased in the US. While their studies make an explicit effort to construct nominal asset and liability positions in the US, their analysis is only conducted in partial equilibrium, while Heer and Maußner (2012) use a Neo-Keynesian model to explore the distributional effects of monetary policy that is based upon the stochastic real economy OLG model of Ríos-Rull (1996). They find that unanticipated inflation in the wake of a monetary shock decreases inequality of both market and disposable income. In the analysis of Heer and Scharrer (2018), the focus in the analysis of inequality is shifted from monetary to fiscal policy. In the aftermath of the Great Recession, the cyclical component of government consumption spending in the US increased by approximately 3%,\(^1\) while the US government budget deficit even increased to 12% in 2009. Given the importance of government spending in fighting the severe consequences of the Great Recession, the authors consider the redistributive effects of extra government consumption depending on its financing by either debt or taxes. They find that higher government spending increases both income and wealth inequality, while debt-financing may be particularly harmful to those retirees with high accumulated private savings. So far, to the best of our knowledge, there exists no other study that considers the redistributive effects of a productivity shock that considers the transmission channel of housing wealth distinguishing between owner and renters.

\(^1\)See, for example, Section 4.2.2 in Heer (2019).
The paper is structured as follows. In Section 2, we present our large-scale 240-period OLG model with a separate housing sector. Section 3 describes the life-cycle behavior of the landlords and renters and provides a description of their income and wealth inequality. In Section 4, the main results of the paper are described and we analyze the effects of a supply shock on house prices and the distribution of income and wealth. Section 5 concludes. More detailed derivation of the model’s equilibrium conditions are provided in the Appendix.

2 The Model

In the following, we describe our large-scale OLG model with 280 cohorts using a quarterly period length. We introduce various sources of heterogeneity in both the production and household sector. In the production sectors, a consumption good which we call the normal good and houses are produced with the help of labor and capital. In the household sector, we distinguish households by age, individual productivity and consumption behavior. While Ricardian households save and own houses, rule-of-thumb consumers spend their total disposable income on consumption of the two goods, the normal goods and residential services. The latter are rented from the landlords. In addition, we introduce adjustment costs of capital in both production sectors so that we have variable and possibly different asset prices of capital and housing wealth.

2.1 Normal Goods Sector

In the normal goods sector (index = $g$), a representative perfectly competitive firm produces the good $Y_{t,g}$ which can either be used for consumption $C_t$ or investment in business capital $I_t$. The firm faces productivity shocks $Z_{t,g}$ and uses a constant returns to scale technology with capital $K_{t,g}$ and labor $N_{t,g}$ as input factors:

$$Y_{t,g} = Z_{t,g} N_{t,g}^{1-\alpha_g} K_{t,g}^{\alpha_g}. \quad (1)$$
The parameter $\alpha_g$ determines the share of output claimed by capital and the productivity shock follows an AR(1)-process, $Z_{t,g} = \rho_g Z_{t-1,g} + \epsilon_{t,g}$ with $\epsilon_{t,g} \sim N(0, \sigma_g)$. Profit maximization implies the following first order conditions:

\begin{align*}
t_{t,g} &= Z_{t,g} \left(1 - \alpha_g\right) \left(\frac{K_{t,g}}{N_{t,g}}\right)^{\alpha_g}, \\
r_{t,g} &= Z_{t,g} \alpha_g \left(\frac{K_{t,g}}{N_{t,g}}\right)^{\alpha_g - 1},
\end{align*}

where $w_{t,g}$ and $r_{t,g}$ represent the real wage and the interest rate in the normal goods sector.

### 2.2 Capital Goods Sector

Following Heer and Scharrer (2018), we introduce a separate capital goods sector (index = $c$) that produces the good $I_t$ using the normal good as an input. In this sector, the price of capital $q_t$ is endogenous due to adjustment cost so that we are able to study the impacts of changes of capital prices on individual wealth. For this reason, we assume a representative perfectly competitive firm that faces the following demand for newly installed capital:

\[ I_t = K_{t+1} - (1 - \delta_k) K_t, \]

where the variables $\delta_k$ and $K_t$ denote the depreciation rate and the stock of aggregate capital. The production of investment goods is described by the technology

\[ I_t = \left(\frac{a_{1,c}}{1 - \zeta} \left(\frac{I_{t,g}}{K_t}\right)^{1 - \zeta} + a_{2,c}\right) K_t. \]

The variable $I_{t,g}$ denotes the demand for normal goods as input factors and the parameter $\zeta_c$ is the elasticity of the investment-to-capital ratio $I_t/K_t$ with respect to the price of capital goods $q_t$. A capital goods producer sells the investment good and rents capital at the rate $r_{t,c}$ so that profits $\Omega_{t,c}$ amount to

\[ \Omega_{t,c} = q_t \left(\frac{a_{1,c}}{1 - \zeta} \left(\frac{I_{t,g}}{K_t}\right)^{1 - \zeta} + a_{2,c}\right) K_t - I_{t,g} - r_{t,c} K_t. \]
The respective first order conditions are given by:

\[
q_t = \frac{1}{a_{1,c}} \left( \frac{I_{t,g}}{K_t} \right)^{\zeta_c}, \tag{7}
\]

\[
r_{t,c} = q_t \left( a_{1,c} \zeta_c \left( \frac{I_{t,g}}{K_t} \right)^{1-\zeta_c} + a_{2,c} \right). \tag{8}
\]

In equilibrium, profits are always equal to zero, \( \Omega_{t,c} = 0 \). In addition, we set the parameters \( a_{1,c} \) and \( a_{2,c} \) in equation (5) so that adjustment costs play no role in the steady state of our model.

2.3 Housing Sector

**Residential Structures.** In the housing sector, a representative perfectly competitive firm produces residential structures \( Y_{t,h} \) (index = h) with capital \( K_{t,h} \) and labor \( N_{t,h} \) as input factors and sells these products to real estate developers. The production technology is:

\[
Y_{t,h} = Z_{t,h} N_{t,h}^{1-\alpha_h} K_{t,h}^{-\alpha_h}. \tag{9}
\]

The constant \( \alpha_h \) controls the capital income share and the productivity shock \( Z_{t,h} \) also follows an AR(1)-process, \( Z_{t,h} = \rho_h Z_{t-1,h} + \epsilon_{t,h} \) with \( \epsilon_{t,h} \sim N(0, \sigma_h) \). The corresponding first order conditions are represented by:

\[
w_{t,h} = p_{t,h} Z_{t,h} \left( 1 - \alpha_h \right) \left( \frac{K_{t,h}}{N_{t,h}} \right)^{\alpha_h}, \tag{10}
\]

\[
r_{t,h} = p_{t,h} Z_{t,h} \alpha_h \left( \frac{K_{t,h}}{N_{t,h}} \right)^{\alpha_h-1}. \tag{11}
\]

The variables \( p_{t,h}, w_{t,h}, \) and \( r_{t,h} \) describe the real price of residential structures, the real wage, and the interest rate in the housing sector, respectively.

**Real Estate Developers.** In our model, real estate developers (index = e) produce new houses using residential structures as inputs. The production of new houses \( I_{t,h} \) is sold at the price \( p_{t,e} \) and can be occupied in the current period. The corresponding demand for new installed houses evolves according to

\[
I_{t,h} = H_t - (1 - \delta_h) H_{t-1}, \tag{12}
\]
where $H_t$ denotes the aggregate housing stock which depreciates at the rate $\delta_h$. Moreover, we assume that real estate developers face adjustment costs similar to the producers in the capital goods sector. In particular, they combine residential structures $Y_{t,h}$ with the existing housing stock $X_t = (1 - \delta_h) H_{t-1}$ at the beginning of period $t$ with the production technology represented by

$$I_{t,h} = \left( \frac{a_{1,e}}{1 - \zeta_e} \left( \frac{Y_{t,h}}{X_t} \right)^{1 - \zeta_e} + a_{2,e} \right) X_t. \quad (13)$$

and profits $\Omega_{t,e}$ are equal to

$$\Omega_{t,e} = p_{t,e} \left( \frac{a_{1,e}}{1 - \zeta_e} \left( \frac{Y_{t,h}}{X_t} \right)^{1 - \zeta_e} \right) X_t - p_{t,h} Y_{t,h} - r_{t,e} X_t. \quad (14)$$

The parameters $a_{1,e}$ and $a_{2,e}$ are exogenously given and are set so that adjustment costs play no role in the steady state. Furthermore, the variable $\zeta_e$ controls the price elasticity of house prices with respect to a change of the ratio of residential structures to the current housing stock and $r_{t,e}$ denotes a lending fee. The respective first order conditions are given by:

$$p_{t,e} = \frac{p_{t,h}}{a_{1,e}} \left( \frac{Y_{t,h}}{X_t} \right)^{\zeta_e}, \quad (15)$$

$$r_{t,e} = p_{t,e} \left( \frac{a_{1,e} \zeta_e}{1 - \zeta_e} \left( \frac{Y_{t,h}}{X_t} \right)^{1 - \zeta_e} + a_{2,e} \right). \quad (16)$$

Again, profits are always equal to zero in equilibrium, $\Omega_{t,e} = 0$.

### 2.4 Financial Intermediary

For simplicity, we assume that a financial intermediary decides about the aggregate stocks of capital $K_{t,g}$ and $K_{t,h}$ in the normal goods and in the housing sector such that both sectors yield the same interest rates, $r_{t,g} = r_{t,h}$. Moreover, the financial intermediary lends the stock of aggregate capital $K_t = K_{t,g} + K_{t,h}$ to capital goods producers and pays the composite return $r_t$ back to households:

$$r_t = (r_{g,t} + r_{c,t} - \delta_k q_k) \frac{K_{g,t}}{K_t} + (r_{h,t} + r_{c,t} - \delta_k q_k) \frac{K_{h,t}}{K_t}. \quad (17)$$
2.5 Household Sector

Demography. Every period, a new cohort of constant size at age \( s = 1 \) (corresponding to a real life age of 21) enters the economy. Households live at most \( T = 70 \) years and each \( s \)-year old household faces a probability \( \phi_s \) of surviving up to age \( s + 1 \). The number of living agents \( \psi_s \) at age \( s \) evolves according to the following formula:

\[
\psi_s = \phi_{s-1} \psi_{s-1}.
\] (18)

All households supply labor in the first \( T_w \) periods and are retirees in the remaining \( T_r = T - T_w \) periods of their life. Moreover, we assume that every newborn generation at age \( s = 0 \) consists of \( v_{RoT} \psi_0 \) rule-of-thumb (RoT) consumers that have no access to financial markets and have to rent houses over the life-cycle. The exogenous parameter \( v_{RoT} \) denotes the share of rule-of-thumb-consumers. In contrast, Ricardian households are modeled as home owners and landlords who build up wealth over their life-cycle. Their respective mass is given by \((1 - v_{RoT}) \psi_0\) and these households do not change their consumer type over the life-cycle. Furthermore, households also differ by their idiosyncratic productivity level \( e^s_j \) that depends on the productivity type \( j \in \{1, 2, 3\} \) and age \( s \). For simplicity, we also also assume that the share \( v_j \) of the productivity type \( j \) remains constant in each cohort and that a household does not change its productivity type over the life-cycle.

Ricardian Households (Landlords). A Ricardian household at age \( s \) in period \( t \) with productivity type \( j \in \{1, 2, 3\} \) derives (dis)utility from consumption \( c_{t,j}^s \), service flows from housing \( d_{t,j}^s \), and labor supply \( n_{t,j}^s \). In addition, he derives utility from leaving a bequest \( b_{t,j}^T \) at the end of his life. More specifically, the Ricardian household maximizes the following discounted expected lifetime utility \( U_{t,j} \) at age \( s = 1 \):

\[
U_{t,j} = E_t \sum_{s=1}^{T} \beta^{s-1} \left( \prod_{x=1}^{s} \phi_{x-1} \right) u \left( c_{t+s-1,j}^s, d_{t+s-1,j}^s, n_{t+s-1,j}^s \right) + \beta^{T-1} \left( \prod_{x=1}^{T} \phi_{x-1} \right) u_{beq} \left( b_{t+T-1,j}^T \right),
\] (19)
where the instantenous utility function is given by

\[ u(c^s_{t,j}, d^s_{t,j}, n^s_{t,j}) = \left[ \theta (c^s_{t,j})^\sigma + (1 - \theta) (d^s_{t,j})^\sigma \right]^{\frac{1-\eta}{1-\eta}} - \gamma_0 \frac{(n^s_{t,j})^{1+1/\gamma}}{1+1/\gamma} \]  

(21)

and the bequest motive is modeled as a warm glow,

\[ u_{beg}(b^T_{t,j}) = \nu_b \frac{(b^T_{t,j})^{1-m_b} - 1}{1 - \eta_b} \]  

(22)

These preferences feature a constant elasticity of substitution \( \sigma_p \) between consumption \( c^s_{t,j} \) and service flows from housing \( d^s_{t,j} \) with \( \sigma = (\sigma_p - 1) / 1 \). Moreover, the intertemporal elasticity of substitution is denoted by \( 1/\eta \) and \( \gamma \) represents the Frisch elasticity of labor supply \( n^s_{t,j} \), where the parameter \( \gamma_0 \) controls the corresponding labor supply in the steady state of our model. The parameters \( \nu_b \) and \( \eta_b \) are exogenously given and \( b^T_{t,j} \) represents a bequest of the oldest household in the form of capital wealth.

Households at age \( s = 1 \) are born without assets and accumulate capital and housing wealth over their live cycle. In particular, we assume that housing wealth at the beginning of period \( t \) consists of depreciated owner-occupied housing wealth \( (1 - \delta_h) d^s_{t-1,j} \) and additional housing wealth \( (1 - \delta_h) x^s_{t,j} \) that was rented out to rule-of-thumb consumers at the price \( p_{t-1,r} \) in the previous period. Their capital \( k^s_{t} \) earns the composite real interest rate \( r_t \) and depreciates at the rate \( \delta_k \). The net labor income of workers depends on the real wage \( w_{t} \), the age-specific productivity \( e^*_j \) of productivity type \( j \), and the contribution rate \( \tau^p_t \) for the PAYG system, where pensions \( pens_t^s \) are only paid to retired agents. In addition, the households receive lump-sum transfers \( tr_t \) from the government. Furthermore, we follow Doepke and Schneider (2006c) and assume that the oldest household only rents houses before he
and the respective budget constraints of households at age \( s \leq T - 1 \) are given by

\[
e_{t,j}^s = (1 - \tau_t^p) w_t e_{t,j}^s n_{t,j}^s + (q_t + r_t) k_{t,j}^s + pens_{t,j}^s + tr_t
+ [(1 - \delta_h) p_{t,e} + r_{t,e}] \left( d_{t-1,j}^{s-1} + x_{t-1,j}^{s-1} \right) + p_{t,r} x_{t,j}^s
- q_t k_{t+1,j}^{s+1} - p_{t,e} \left( d_{t,j}^s + x_{t,j}^s \right),
\]

and the respective budget constraints of households at age \( s = T \) are

\[
e_{t,j}^s = (1 - \tau_t^p) w_t e_{t,j}^s n_{t,j}^s + (q_t + r_t) k_{t,j}^s + pens_{t,j}^s + tr_t - b_{t+1,j}^{s+1}
+ [(1 - \delta_h) p_{t,e} + r_{t,e}] \left( d_{t-1,j}^{s-1} + x_{t-1,j}^{s-1} \right) - p_{t,r} d_{t,j}^s,
\]

with \( b_{t,j}^T = q_t k_{t+1,j}^{s+1}, \; b_{t,j}^0 = 0, \; d_{t-1}^0 = 0, \; x_{t-1}^0 = 0, \; n_{t,j}^s = 0 \) for \( s > T_w \), and \( pens_{t,j}^s = 0 \) for \( s \leq T_w \). Utility maximization implies the following first order conditions:

\[
\frac{\partial u(.)}{\partial t_{t,j}^s} = \lambda_{t,j}^s, \; \text{for} \; s \leq T, \tag{25}
\]

\[
\frac{\partial u(.)}{\partial n_{t,j}^s} = -\lambda_{t,j}^s (1 - \tau_t^p) w_t e^s, \; \text{for} \; s \leq T_w, \tag{26}
\]

\[
\frac{\partial u(.)}{\partial d_{t,j}^s} = \lambda_{t,j}^s p_{t,e} - \beta \phi_{s+1} \lambda_{t+1,j}^{s+1} [ (1 - \delta_h) p_{t+1,e} + r_{t+1,e} ], \; \text{for} \; s \leq T - 1, \tag{27}
\]

\[
\lambda_{t,j}^s = \beta \phi_{s+1} \lambda_{t+1,j}^{s+1} \frac{(1 - \delta_h) p_{t+1,e} + r_{t+1,e}}{p_{t,e} - p_{t,r}}, \; \text{for} \; s \leq T - 1, \tag{28}
\]

\[
\lambda_{t,j}^s = \beta \phi_{s+1} \lambda_{t+1,j}^{s+1} \frac{q_{t+1} + r_{t+1}}{q_t}, \; \text{for} \; s \leq T - 1, \tag{29}
\]

\[
\frac{\partial u(.)}{\partial d_{t,j}^T} = \lambda_{t,j}^T p_{t,r}, \; \text{for} \; s = T, \tag{30}
\]

\[
\frac{\partial u_{bseq}}{\partial b_{t,j}^T} = \lambda_{t,j}^T, \; \text{for} \; s = T. \tag{31}
\]

The no arbitrage conditions (28) and (29) for households at age \( s \leq T - 1 \) always hold in the steady state and in our simulations. However, a household is always indifferent between capital and (additional) housing assets because of certainty equivalence resulting from a first order approximation around the steady state. For that reason, we introduce age-specific housing shares \( \chi_{t+1,h}^{s+1} \) as an approximation for the age-specific portfolio choice and assume that households only possess additional housing

\[\text{years.}\]

As a consequence, the households only leave bequests in the form of capital \( k_{t+1}^{T+1} \).
assets after an age of \( o_t \) years (\( \chi_{t,h}^s = 0 \) and \( x_t^s = 0 \) for age \( s \leq o_t \)) when their stock of capital \( k_t^s \) becomes positive in the steady state. The variable \( a_{t+1,j}^{s+1} \) denotes the total individual wealth expenditures for additional housing or capital assets for the age \( s+1 \),

\[
a_{t+1,j}^{s+1} = q_t k_{t+1,j}^{s+1} + (p_{t,e} - p_{t,r}) x_t^s,
\]

so that the aggregate wealth expenditures \( A_{t+1} \) for period \( t+1 \) of households at age \( s \in \{o_t, ..., T-1\} \) years are equal to

\[
A_{t+1} = (1 - \nu_{RoT}) \sum_{j \in J} \sum_{s=0}^{T-1} \nu_j \psi_s a_{t+1,j}^{s+1}.
\]

The resulting age-specific housing shares \( \chi_{t+1,h}^{s+1} \) depend on the aggregate housing demand of renters \( H_{t,r} \) and amount to

\[
\chi_{t+1,h}^{s+1} = \frac{(p_{t,e} - p_{t,r}) H_{t,r}}{A_{t+1}}
\]

with

\[
x_t^s = \frac{\chi_{t+1,h}^{s+1} a_{t+1,j}^{s+1}}{(p_{t,e} - p_{t,r})},
\]

\[
k_{t+1,j}^{s+1} = \frac{(1 - \chi_{t+1,h}^{s+1}) a_{t+1,j}^{s+1}}{q_t}.
\]

**Rule-of-thumb Consumers (Renters).** An \( s \)-year old rule-of-thumb consumer with productivity type \( j \in \{1,2,3\} \) rents housing services and does not save. He maximizes his instantaneous utility with respect to consumption \( e_{t,j}^{s,RoT} \), service flows from housing \( d_{t,j}^{s,RoT} \), and labor supply \( n_{t,j}^{s,RoT} \) in every period \( t \),

\[
u(.) = \left[ \frac{\theta \left( e_{t,j}^{s,RoT} \right)^\sigma + (1 - \theta) \left( d_{t,j}^{s,RoT} \right)^\sigma}{1 - \eta} \right]^{\frac{1-\eta}{\sigma}} - 1 - \gamma_0 \left( n_{t,j}^{s,RoT} \right)^{1+1/\gamma}
\]

subject to the budget constraint

\[
e_{t,j}^{s,RoT} + p_{t,r} d_{t,j}^{s,RoT} = (1 - \tau_t^p) w_t e_{t,j}^{s,RoT} + tr_t + pens_{t,j}^{s,RoT},
\]
where, again, $pens_{s,RoT}^t = 0$ for age $s \leq T_w$. The resulting first order conditions are represented by

$$\frac{\partial u(\cdot)}{\partial c_{s,RoT}^t} = \lambda_{s,RoT}^t, \quad (38)$$

$$\frac{\partial u(\cdot)}{\partial n_{s,RoT}^t} = -\lambda_{s,RoT}^t (1 - \tau_p^t) we^s, \quad (39)$$

$$\frac{\partial u(\cdot)}{\partial d_{s,RoT}^t} = \lambda_{s,RoT}^t p_{t,r}. \quad (40)$$

### 2.6 Social Security & Government

The social security authority collects contributions from workers to finance its pension payments to the retired agents. For simplification, we assume that individual pension benefits $pens_{s,j}^t$, which depend on past labour earnings and the replacement ratio $\kappa$, are constant over time,

$$pens_j = \kappa \sum_{s=1}^{T_w} we_{s,j}^s, \quad (41)$$

$$pens_{RoT} = \kappa \sum_{s=0}^{T_w} we_{s,j}^s. \quad (42)$$

Thus, the corresponding budget of the PAYG system is given by

$$\sum_{j \in J} \sum_{s=T_w+1}^{T} \nu_j \psi_s (\nu_{RoT} pens_{RoT}^s + (1 - \nu_{RoT}) pens_j^s) = \tau_p^t we^t N_t. \quad (43)$$

Furthermore, we assume that all bequests are collected by the government and transferred as lump-sums to the household sector. This implies

$$Beq_{t} = \sum_{j \in J} (1 - \nu_{RoT}) \left\{ \sum_{s=1}^{T+1} \nu_j \psi_{s-1} (1 - \phi_s) (q_t + r_t) k_{s,j}^t + \right\}$$

$$\sum_{s=1}^{T} \nu_j \psi_{s-1} (1 - \phi_s) \left( h_{s-1,j}^t + x_{s-1,j}^t \right) \left[ (1 - \delta_h) p_{t,e} + r_{t,e} \right]. \quad (44)$$

with

$$tr_t = \frac{Beq_{t}}{\sum_{j \in J} \sum_{s=0}^{T-1} \nu_j \psi_s}. \quad (46)$$
2.7 Aggregate Equilibrium Conditions

In equilibrium, aggregate variables are equal to the sum of the individual variables:

\[ N_t = \sum_{j \in J} \sum_{s=1}^{T_w} \nu_j \psi_s e_j^s \left[ (1 - \nu_{RoT}) n_{t,j}^s + \nu_{RoT} n_{t,j}^{s,RoT} \right], \]  
(47)

\[ C_t = \sum_{j \in J} \sum_{s=1}^{T} \nu_j \psi_s \left[ (1 - \nu_{RoT}) c_{t,j}^s + \nu_{RoT} c_{t,j}^{s,RoT} \right], \]  
(48)

\[ H_t = \sum_{j \in J} \sum_{s=1}^{T} \nu_j \psi_s \left[ (1 - \nu_{RoT}) d_{t,j}^s + \nu_{RoT} d_{t,j}^{s,RoT} \right], \]  
(49)

\[ K_{t+1} = (1 - \nu_{RoT}) \sum_{j \in J} \sum_{s=1}^{T} \nu_j \psi_s k_{t+1,j}^s, \]  
(50)

where the normal goods and the housing sector clear,

\[ Y_{t,g} = C_t + I_{t,g}, \]  
\[ Y_{t,h} = H_t - (1 - \delta_h) H_{t-1}. \]  

Moreover, in accordance with Iacoviello and Neri (2010), we define the gross domestic product as the sum of consumption and investment by their steady-state nominal shares, 

\[ gdp_t = Y_{t,g} + p_t Y_{t,h}. \]  

2.8 Calibration

Most of our parameters are standard in the RBC/DSGE literature and are summarized in Table 1. Moreover, the age-specific survival probabilities \( \psi_s \) stem from Arias (2014) and describe the year 2010. The age-specific component \( \bar{e}_s \) is taken from Hansen (1993). As a consequence, the model replicates the cross-section age distribution of earnings of the U.S. economy. With regard to the idiosyncratic component \( z_j \), we follow Huggett (1996) and choose a log-normal distribution of earnings for the youngest households with a variance equal to \( \sigma_z^2 = 3.60 \). This variance is chosen so that the Gini coefficients of labor income, gross market income, and wealth are close to the empirical values reported by Budría Rodriguez et al. (2002) that we describe in the next section.
\[
\begin{array}{cccc}
T^w = 43 & T^r = 27 & \alpha_g = 0.36 & \alpha_h = 0.20 \\
\delta_k = 0.02 & \zeta = 0.80 & \zeta_h = 0.10 & \beta = 0.995 \\
\gamma = 0.3 & \sigma = 0 & \theta = 0.927 & \nu_h = 10 \\
\rho_g = 0.81 & \rho_h = 0.81 & \kappa = 0.39 & \alpha_l = 10 \\
\end{array}
\]

Table 1: Annual Parameterization of the OLG model.

3 Steady State

In this section, we first describe the life-cycle behavior of Ricardian and rule-of-thumb households in steady state. We take a particular look at the accumulation of housing assets and distinguish between homeowners and renters. In addition, we present the inequality measures of income and wealth and discuss how well our model replicates the data for the US.

![Figure 1: Steady State Profiles - Ricardians (Homeowners)](image)

Fig. 1 describes the steady-state behavior of the homeowners for the three productivity types \( j \in \{1, 2, 3\} \) which are the Ricardian households in our model. In the
first row, the consumption behavior of the Ricardian households is displayed. In accordance with empirical observations provided by Fernández-Villaverde and Krueger (2007) we also find that the consumption-age profiles is hump-shaped over the life cycle. Since housing and consumption goods are substitutes, the profiles of both the normal good $c_j$ and the housing services $d_j$ are similar to each other.

The labor-supply-age profile of the Ricardian households is presented in the second row of Fig. 1. Notice that the labor supply of the most efficient workers with productivity $j = 3$ is below that of the less efficient workers. This effect stems from our calibration of the intertemporal elasticity of substitution with $\eta = 2$. For $\eta < 1$, the relative labor supply of the more productive workers increases, see Chetty (2006). Over the working life, the labor supply stays almost constant during the first 10 years and falls monotonously thereafter. This pattern is mainly driven by both the increasing wealth effect as agents build up savings for retirement and the hump-shaped efficiency profile over the life cycle (not displayed). The age efficiency increases until the age of 52 and falls thereafter, while wealth is increasing monotonously from age 20 until age 65 when the households enter retirement. The wealth-age profile is displayed in the third row of Fig. 1 and has the typical shape prevailing in large-scale OLG models as in Auerbach and Kotlikoff (1987).

We distinguish two types of assets, capital and housing. As pointed out in the previous section, we assume that, in accordance with empirical evidence, households only become landlords starting in real-life age 30 (corresponding to age $s = 9$ in the model). Therefore, housing assets coincide with the consumption of the residential good $d_j$ during age $s = 1, \ldots, 10$. Since the consumption of housing goods exceeds total wealth, the households initially holds a negative capital stock. In old age, the holding of housing asset decreases to zero as we assume the households to only leave bequests in the form of capital $k_{t+1}^c$. Our housing-wealth profile is in good accordance with empirical evidence provided by Zhao (2018) who, in his Fig. 2, provides estimates of the US housing wealth-age profile for those aged 50 and above.

\[\text{In future work, we are planning to incorporate a banking sector that provides a mortgage loan to Ricardian households into our model.}\]
In the bottom row of Fig. 1, the gross market income (GMI) is displayed. As a very prominent variable for international comparison of Gini measures, we use the general definition of this variable and include all kinds of income such as pensions and rental income in its measurement. Since both labor income (because of the age-efficiency profile) and interest income are hump shaped over the (working) life, the gross market income is also hump-shaped. Since pensions are below the labor income (with a replacement rate amounting to 50% in the US economy), gross market income drops significantly in the first year of retirement at age 65. Notice further that gross market income of the old retired workers at age 82 and 80 is below the initial gross market income at age 20 for the productivity types $j = 2$ and $j = 3$. This behavior of gross market income is even more pronounced in the case of rule-of-thumb consumers who do not build up savings. Therefore, the income-poorest households in our model are both the very old households and the very young households with efficiency type $j = 1$.

![Figure 2](image_url)

**Figure 2:** Steady State Profiles - Rule of Thumb Consumers (Renters)

Fig. 2 describes the steady-state behavior of the rule-of-thumb consumers for the three productivity types $j \in \{1, 2, 3\}$. As these households do not save, they also do not accumulate assets in the form of either capital or housing. Therefore, they have to rent housing from the landlords and their rental expenditures as displayed by the red line in the top row of Fig. 2 also equals their consumption of the residential
services. Again, the profile of normal and residential consumption is hump-shaped as the hourly wage profile, but drops as the rule-of-thumb consumers enter retirement since they do not use savings to smooth consumption over time. In addition, labor supply is rather flat than hump-shaped. Different from the Ricardian households, wealth does not increase and hence, labor supply does not significantly decrease with age. Gross market income basically mirrors the efficiency-age profile over working life and drops to approximately 30-50% (relative to peak earnings) during retirement depending on the productivity type \( j \in \{1, 2, 3\} \).

The heterogeneity with regard to individual productivity and consumption type (Ricardian versus rule-of-thumb consumer) results in inequality of income and wealth among the households. Our model is able to replicate the heterogeneity of income and wealth distribution with the possible exception of the top 1% of the distribution.\(^4\) The Gini coefficients of income, wealth, and consumption amount to 0.513 (gross income before taxes), 0.681 (gross labor income), and 0.765 (wealth). Our inequality measures for incomes and wealth are very close to the empirical values as reported by Budría Rodriguez et al. (2002) who find Gini coefficients of (gross) income, (gross) labor income and wealth that are equal to 0.553, 0.611 and 0.803, respectively. Our Gini coefficient for consumption amounts to 0.516 and exceeds the value 0.29 for the year 2003 estimated by Krueger and Perri (2006). As the main reason, we neglect idiosyncratic income risk of the households in our model. Iacoviello and Pavan (2013) demonstrate that a model with overlapping generations and heterogeneous agents facing idiosyncratic and aggregate risks can explain this lower value.

\(^4\)In our model, we neglect the top 1% of the earners. In addition, we do not model entrepreneurs. We preferred to keep the model as simple as possible noticing that the negligence of the top 1% of the earners does not significantly affect the redistribution effects from the middle to the end which is the focus of our research.
4 Results

In this section, we analyze the effects of a supply shock shocks in the production sector of the normal good and in the housing sector in turn.

4.1 Supply Side Shock in the Normal Goods Production Sector

Fig. 3 presents the impulse responses of output, consumption of normal goods and residential services, investment, capital, labor, house price and rents, the real rates of return, output in the goods sector, output in the normal goods sector, capital prices and wages (from top left to bottom right). Following a 1% increase of total factor productivity \( Z_{t,g} \) in the normal goods sector, both wages and returns increase in the normal goods sector. Since both labor and capital are mobile, the factor prices increase in the housing sector, too. Therefore, income increases and the households spend more on consumption of both the normal good and residential services. As a consequence, house prices and rents rise and investment in both sectors respond by an immediate increases of 1.25% (normal goods sector) and 0.5% (housing sector), respectively. The increase of labor productivity due to higher capital in the housing sector is ceteris paribus even stronger than the rise in the productivity in the normal goods sector so that labor gets reallocated from the normal goods sector to the housing sector. In the aggregate, labor supply falls as the income effect is more pronounced than the substitution effect. Notice that the prices of both assets in the economy, \( q_t \) and \( p_{t,h} \) increase almost equally by 1%.
Fig. 3: I.R. - Aggregate Variables (TFP-Shock in Goods Sector, r.d.)

Fig. 4 displays the impulse responses of the Gini coefficients for gross market income $GMI$ and wealth, $q_t k_{t,j}^\alpha + (1 - \delta_h) p_{t,e} (d_{t-1,j}^{e-1} + x_{t-1,j}^{e-1})$. Following an increase of the productivity in the normal goods sector, the income distribution becomes more unequal and the Gini coefficient increases by 0.065%, while the wealth distribution is distributed more equally, even though the drop in the Gini coefficient is quantitatively also very small and only amounts to 0.08%. In order to understand the redistribution that takes place among the individual households, we study the behavior of the individuals’ income and wealth in period 2 when the shock occurs.

Fig. 5 shows the disaggregate immediate impulse responses of the different age, productivity and consumption types in the economy. In the top row, it is evident that the young and old Ricardian households gain the least among the Ricardian households, while those close to and into the early years of retirement display the highest increase of gross market income. While the former have increases of 0.3-1.0%, the latter experience an increase of gross market income by approximately 1.5%. The strong response of the income among the middle-aged Ricardian households is caused by the increase of asset returns. The wealth-richest households also display the high-
est income increase. Since these households are also among the income-richest, the concentration of gross market income rises.\(^5\) In the case of the rule-of-thumb consumers (left-hand side panel in the top row), the increase of the gross market income among the workers is approximately equal to the increase of the wage and amounts to 1\%, while the income of the retirees increases by a smaller amount. The reason that the income of the RoT-retirees increases at all, even though the pensions are constant, stems from the increase in government transfers.

![Gini Coefficients (TFP-Shock in Goods Sector, r.d.)](image)

**Figure 4:** I.R. - Gini Coefficients (TFP-Shock in Goods Sector, r.d.)

In the bottom row of Fig. 5, the impulse responses of the individual wealth levels are displayed. Of course, the rule-of-thumb consumers hold no wealth so that their response is zero (right-hand side of the bottom row). The 20-year old households are born without assets so that their response among the Ricardian households is also zero. The households aged 21-30, however, react stronger to the increase of factor and asset prices than the older Ricardian households. At the beginning of life, savings are more interest-elastic than during older age.

\(^5\)In accordance with empirical evidence, income and wealth are positively correlated in our model, but not perfectly, though.
Figure 5: I.R. - Cross Section (period 2, TFP-Shock in Goods Sector, r.d.)
4.2 Supply Side Shock in the Normal Goods Production Sector

Fig. 6 presents the impulse response of a total factor productivity shock in the housing sector. As $Z_{t,h}$ increase by 1%, wages in the housing sector and, hence, in the whole economy increase. Since the housing sector is relatively small in relation to the aggregate economy, wages only increase by 0.04%. As a consequence of higher productivity in the housing sector, house prices decline by 1%, too. Households respond by substituting the consumption of normal goods by consumption of residential services. While the former falls by 0.17%, the latter rises gradually over time and increases by 0.2% after 8 quarters (=2 years). The gradual response of housing is a consequence of the adjustment costs in the housing sector.

As productivity in the housing sector rises relative to that in the normal goods sector, labor is shifted from the production of $Y_{t,g}$ to that of $Y_{t,h}$. Therefore, the marginal product of capital even falls in the normal goods sector so that investment in this sector and the price of capital decline. As displayed in the bottom left panel

\textbf{Figure 6:} I.R. - Aggregate Variables (TFP-Shock in Housing Sector, r.d.)
of Fig. 6, the rate of return initially even drops by 0.2% and 0.5% for the two assets. The effect on total GDP is approximately 0.1% and smaller than the original total factor productivity shock in the housing sector which reflects the size of the housing sector relative to the rest of the economy. The response of aggregate labor is positive and also equal to 0.1% as wages rises and asset prices and, hence, wealth declines.

According to Fig. 7, the concentration of both gross market income and wealth increases after a supply side shock in the housing sector. The response of the income distribution is basically driven by the response of the low-productivity workers with $j = 1$ among both the Ricardian and the rule-of-thumb consumers as depicted in Fig. 8. As the price of residential services falls significantly, the income effect associated with the higher purchasing power reduces the labor supply of the low-productivity workers more profoundly than that of the high-productivity workers. In addition, government transfers in the form of bequests fall. Since government transfers constitute a relatively high share of income among the low-productivity workers and the rule-of-thumb consumers, the distribution of gross market income becomes more unequal.
The wealth distribution also becomes more unequal because the young households have the highest percentage drop in their asset value among the Ricardian households. This effect is caused by the difference in the responses of the two asset prices in period 2 when the shock hits the economy. House prices decrease more strongly than capital prices. Since the very young Ricardian households hold negative wealth but already own houses, the value of their wealth drops more significantly than that of the older Ricardian households.

5 Conclusion

We have analyzed the distributional and general equilibrium effects of a supply shock in the consumption goods and housing sector. In particular, we used a 70-period overlapping generations model to study the individual behavior of households that differ in age, home-ownership, productivity and consumption type. The model, therefore, is sufficiently comprehensive to describe the income and wealth distribution in the US economy while also allowing for the explicit specification of
rule-of-thumb consumers and renters in addition to traditional Ricardian households with owner-occupied houses. As our main results, we find that, following a supply shock, income inequality increases, while house prices fall.

We consider our study a first step in a detailed analysis of the redistributive effects of the Great Recession 2007-2008. In the next step, we are planning to incorporate a financial sector that provides financing of the housing expenditures with the help of mortgages. In particular, we stipulate that, in our model, the analysis of mortgage debt and an additional shock in the form of collateral devaluation provides fruitful new insights into the study of the redistribution that took place during and after the Great Recession. Our results indicate that our present framework of a large-scale OLG model provides a useful tool to model the strong income and wealth redistribution from the middle to the end that took place during 2007-2008.
References


6 Appendix

A.1 Mathematical Derivations

- The equations (10) and (11) imply

\[
\begin{align*}
  w_{t,h} &= Z_{t,h} p_{t,h} (1 - \alpha_h) \left( \frac{r_{t,h}}{Z_{t,h} p_{t,h}} \right)^{\alpha_h}, \\
  w_{t,h} &= Z_{t,h}^{1 - \frac{\alpha_h}{\alpha_h - 1} \left( 1 - \alpha_h \right) p_{t,h}^{\frac{\alpha_h}{\alpha_h - 1} \left( 1 - \alpha_h \right) \left( \frac{r_{t,h}}{\alpha_h} \right)^{\alpha_h}}} \alpha_h, \\
  w_{t,h} &= Z_{t,h}^{\frac{\alpha_h - 1 - \alpha_h}{\alpha_h - 1} \left( 1 - \alpha_h \right) p_{t,h}^{\frac{\alpha_h}{1 - \alpha_h} \left( 1 - \alpha_h \right) \left( \frac{r_{t,h}}{\alpha_h} \right)^{\alpha_h}}} \alpha_h, \\
  w_{t,h} &= Z_{t,h}^{\frac{1}{\alpha_h - 1} \left( 1 - \alpha_h \right) p_{t,h}^{\frac{1}{\alpha_h - 1} \left( 1 - \alpha_h \right) \left( \frac{r_{t,h}}{\alpha_h} \right)^{\alpha_h}}} \alpha_h, \\
  \frac{1}{p_{t,h}} &= Z_{t,h}^{\frac{1 - \alpha_h}{w_{t,h}} \left( \frac{r_{t,h}}{\alpha_h} \right)^{\alpha_h},} \\
  p_{t,h} &= Z_{t,h}^{-1} \left( \frac{1 - \alpha_h}{w_{t,h}} \right)^{\alpha_h - 1} \left( \frac{r_{t,h}}{\alpha_h} \right)^{\alpha_h}, \\
  p_{t,h} &= \frac{1}{Z_{t,h} \left( \frac{w_{t,h}}{1 - \alpha_h} \right)^{1 - \alpha_h} \left( \frac{r_{t,h}}{\alpha_h} \right)^{\alpha_h}.}
\end{align*}
\]

- Moreover, \( w_{t,h} = w_{t,g} \) and \( r_{t,h} = r_{t,g} \) with (2) and (3) yield

\[
\begin{align*}
  p_{t,h} &= \frac{1}{Z_{t,h} \left( \frac{Z_{t,g} \left( 1 - \alpha_g \right) \left( k_{t,g} \right)^{\alpha_g}}{1 - \alpha_h} \right)^{1 - \alpha_h} \left( \frac{Z_{t,g} \alpha_g \left( k_{t,g} \right)^{\alpha_g - 1}}{\alpha_h} \right)^{\alpha_h}}, \\
  p_{t,h} &= \frac{Z_{t,g}}{Z_{t,h} \left( \frac{1 - \alpha_g}{1 - \alpha_h} \right)^{1 - \alpha_h} \left( \frac{\alpha_g \left( k_{t,g} \right)^{\alpha_g - 1}}{\alpha_h} \right)} \alpha_h \left( k_{t,g} \right)^{(\alpha_g - 1)\alpha_h}, \\
  p_{t,h} &= \frac{Z_{t,g}}{Z_{t,h} \left( \frac{1 - \alpha_g}{1 - \alpha_h} \right)^{1 - \alpha_h} \left( \frac{\alpha_g \left( k_{t,g} \right)^{\alpha_g - \alpha_g \alpha_h}}{\alpha_h} \right)} \alpha_h \left( k_{t,g} \right)^{\alpha_g \alpha_h - \alpha_h}, \\
  p_{t,h} &= \frac{Z_{t,g}}{Z_{t,h} \left( \frac{1 - \alpha_g}{1 - \alpha_h} \right)^{1 - \alpha_h} \left( \frac{\alpha_g \alpha_h}{\alpha_h} \right)} \alpha_h \left( k_{t,g} \right)^{\alpha_g - \alpha_h}, \\
  \end{align*}
\]

where

\[
\frac{k_{t,g}}{N_{t,g}}
\]
The Lagrangian Function of Ricardians is given by:

\[ L = E_t \sum_{s=0}^{T-1} x(s) u(c_{t+s}^s, d_{t+s}^s, n_{t+s}^s) - \sum_{s=0}^{T-2} x(s) \lambda_{t+s}^s \left( q_{t+s} k_{t+s+1}^s \right) \]  

(A.1.2)

Marginal utility by bequests:

\[ \frac{\partial \lambda_{t+s}^s}{\partial q_{t+s}} = \partial u(\cdot) / \partial c_{t+s}^s = \left[ \phi \left( c_{t+s}^s \right)^\sigma + (1 - \phi) \left( d_{t+s}^s \right)^\sigma \right] \frac{1}{\sigma - 1} \phi \left( c_{t+s}^s \right)^{\sigma - 1}, \]  

(A.1.3)

\[ \frac{\partial \lambda_{t+s}^s}{\partial d_{t+s}^s} = \partial u(\cdot) / \partial d_{t+s}^s = \left[ \phi \left( c_{t+s}^s \right)^\sigma + (1 - \phi) \left( d_{t+s}^s \right)^\sigma \right] \frac{1}{\sigma - 1} (1 - \phi) \left( d_{t+s}^s \right)^{\sigma - 1}, \]  

(A.1.4)

\[ \frac{\partial \lambda_{t+s}^s}{\partial n_{t+s}^s} = -\gamma_0 \left( n_{t+s}^s \right)^{1/\gamma}. \]  

(A.1.5)

Marginal utility for \( \sigma = 0 \):

\[ \frac{\partial \lambda_{t+s}^s}{\partial c_{t+s}^s} = \left[ \phi \left( c_{t+s}^s \right)^\theta + (1 - \phi) \left( d_{t+s}^s \right)^\theta \right] \frac{1}{1 - \eta} \phi \left( c_{t+s}^s \right)^{\theta - 1}, \]  

(A.1.6)

\[ \frac{\partial \lambda_{t+s}^s}{\partial d_{t+s}^s} = \left[ \phi \left( c_{t+s}^s \right)^\theta + (1 - \phi) \left( d_{t+s}^s \right)^\theta \right] \frac{1}{1 - \eta} \frac{1 - \phi}{d_{t+s}^s}, \]  

(A.1.7)

\[ \frac{\partial \lambda_{t+s}^s}{\partial n_{t+s}^s} = -\gamma_0 \left( n_{t+s}^s \right)^{1/\gamma}. \]  

(A.1.8)

Marginal utility by bequests:

\[ \frac{\partial b_t(\cdot)}{\partial k_t^{s+1}} = \nu_t \left( b_t^{s+1} \right)^{-\eta_t} q_t. \]  

(A.1.9)

\[ ^6\text{For convenience, we have dropped the index } j. \]