Abstract

Following Casas et al. (2017) we construct a small open economy (SOE) DSGE model interacting with the rest of the world (ROW). With this framework we then depart from the standard SOE model of Gali (2015) along the following dimensions: Firstly, we nest two different pricing paradigms: dominant currency pricing (in dollars) alongside producer currency pricing. Secondly, the production function uses not just labour but also capital inputs produced domestically and abroad. Thirdly, international asset markets are incomplete with only riskless bonds being traded, as opposed to the assumption of complete markets. We make two main contributions to this literature: first, we explore the empirical evidence for PCP vs DCP pricing paradigms through a Bayesian estimation likelihood race and a validation comparison with the second moments of the data. Second, we examine the implications of these two paradigms for the conduct of monetary policy using Taylor-type interest rate rules.

JEL Classification: C68, E52, F41

Keywords: Imperfect Exchange Rate Path-Through; Producer and Dominant Currency Pricing; DSGE Model; Small Open Economy.
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1 Introduction

With more than ten years since the great financial crisis of 2007/08, there are still many questions left unanswered and lessons to be learned. An important set of questions relates to the impact of advanced economies’ monetary policy on emerging economies. In particular, what are the size and sign of the spillovers and what are the channels via which domestic monetary policy transmits internationally? The growing literature on this is far from having yet established any consensus, as results tend to point in all directions. This disagreement may possibly be due to differences in data sources, methodology and modelling choices. 1 Moreover, there is still little clarity on the transmission of monetary policy into emerging markets (EM henceforth) given the scarcity of high frequency data and the non-existence of any narrative methods (a la Romer and Romer) to identify the shocks in EM. Recent study by Georgiadis and Jancokova (2017) listed 60 empirical studies of monetary policy shocks in EM and SOE with only 5 papers using HFI and non-using a narrative approach.

In the meantime, in many developing countries the conduct of monetary policy by the advanced economies’ central banks in the aftermath of the financial crisis is seen as worrying action that could lead to a currency war.2 Indeed, such reaction may be justified under a classic Mundell-Fleming understanding, according to which a cut in interest rates (in the US for instance) will make the developing country’s economy less competitive and decrease the demand for their products. However, if one is to allow for price rigidity, as one does in a Mundel-Fleming framework, it is crucial to know in which currency prices are rigid. In this regard, a distinction among three different paradigms has emerged: prices may be set in the producer’s currency (Producer Currency Pricing, PCP henceforth), in the buyer’s currency (Local Currency Pricing, LCP henceforth), or in a dominant currency that is neither the buyer’s nor seller’s (Dominant Currency Pricing, DCP henceforth), often the dollar.

Each of these pricing models will have different implications on both domestic and

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1 See for instance: Bernanke and Gertler (1995), Christiano et al. (1999), Uhlig (2005), Taylor (2013) among others

2 Former Brazilian President Dilma Rousseff stated that advanced economies are creating a “monetary tsunami” by using quantitative easing as a policy instrument. Similarly, leaders from other emerging markets (i.e. Turkey, India and Indonesia etc.) showed their concerns. Furthermore, Brazil finance minister then also warned that such policies may lead to currency wars
international levels. In the basic Mundell-Fleming model, which assumes that prices are rigid in the PCP, a cut to fed's rates will, through different channels, lead to a depreciation of the dollar vis-à-vis the trading partner’s currency, thus resulting in more demand for US domestic products and in an appreciation of the trading balance of the US (in a phenomenon known as expenditure switching). In other words, under PCP assumption, local prices will fluctuate in response to changes in the nominal exchange rates and hence law of one price hold. Therefore, the exchange rate pass-through into prices is 100% (i.e. perfect pass-through).

Despite decades of research in this area, this analysis of the impact of monetary policy is still the main simple way to explain the international propagation of domestic shocks into the international arena. However, another two models emerged as an alternative narrative to the story.

Alternatively, under a LCP regime, prices are sticky in the destination country’s currency. In this case, shocks to the nominal exchange rate will not affect the prices of imported goods and hence in short-run there will be a deviation from the law of one price. Therefore, exchange rate pass-through into imported prices is 0%.

Recent studies, however, suggest that both of these pricing schemes seem to be at odds with the trends seen in international trade transactions. Firms set prices in very few currencies (with the dollar being the most frequently used currency) and do not change prices often (Goldberg and Tille (2010), Gopinath (2015)). These observations have led to the recent emergence of a literature that considers a Dominant Currency Pricing (DCP henceforth) paradigm. Under this regime, prices are anchored in a third currency that is not necessarily that of any of the parties involved in the transaction. In this case, changes in nominal exchange rate will only weakly impact the terms of trade while the main factor in terms of prices and quantity of imported goods will be country’s currency value vis-à-vis the dollar.

Motivated by the recent studies on pricing paradigms and in search of more conclusive evidence on the magnitude and direction of international spillovers, we attempt to contribute to the literature by shedding light on the impact of advanced/large economies monetary policy on small open economies.

More specifically, in this paper, we are working on investigating the impact of the US
monetary policy on its two neighbours Canada and Mexico (in addition to Chile) under two different pricing regimes: producer currency pricing (PCP); and dominant currency pricing (DCP) models.

The choice of these countries is stimulated by their geographical proximity to the US (in case of Canada and Mexico), which makes shocks in the largest economy (i.e. the US) have plausibly sizeable effects on these smaller neighbouring countries, but also by the dissimilarity in economic structures\(^3\) between Mexico and Canada. It is also motivated by the recent signing of the new trade agreement in North America called the United States-Mexico-Canada Agreement (USMCA) that is meant to replace The North American Free Trade Agreement (NAFTA). Furthermore, Chile, on the other hand, represents a good example of an emerging market with a high volume of its exports and imports invoiced in the dollar.\(^4\)

We seek to use a structural DSGE model and historical data in order to estimate the impact of US domestic monetary policy on its trading partners. We do this using a state-of-the-art dynamic stochastic general equilibrium model (NK DSGE model) that capture the impact of US monetary policy on these two countries. We model the US as a closed NK economy that is not affected by shocks from the other, small, countries. Mexico and Canada are in turn modelled as small open economies (SOE) that are affected by shocks from the ROW. The model is estimated for both pricing regimes and the results of the two countries under these pricing models are compared. More details on the choice of methodology and estimation method are mentioned in the third section on the methodology.

Our contribution to the literature is threefold. First, we add to the expanding literature on dominant currency pricing by showing the impactions of DCP in the case of Canada, Mexico, and Chile as well as we construct a comparison between the pricing systems (i.e. PCP and DCP) to see which of them fit the data better. Second, we further the understanding of the transmission of US monetary policy into small open economies (and especially the emerging markets in case of Mexico and Chile) by comparing the spillovers from the US monetary policy across these countries and discuss the similarities and differ-

\(^3\)I.e. in terms of labour market structure, financial market development and integration and overall government policies etc...

\(^4\)see: Cravino (2017)
ences in terms of sign and size of the impact. Third, we examine the implications of these two paradigms for the conduct of monetary policy using welfare-optimized Taylor-type interest rate rules.

The remainder of the paper is as follows. Section 2 briefly discusses the related literature. Section 3 sets out the model. Section 4 describes the estimation method. Section 5 presents and discusses the estimation results. Section 6 conducts the policy analysis using the model estimated for the three countries and the ROW. Finally, a concluding section summarises the paper’s key findings and maps out future research.

2 Literature Review

This paper is related to three strands of literature: that on international monetary policy spillovers, on pricing in open economy models, and on emerging economies. There have been numerous studies in all three areas, the most relevant of which we try to bring together.

The impact of domestic monetary policy on other countries has been a very active area of research in the last few decades. The main questions that it discusses revolve around the following. Does a monetary contraction in the U.S. lead to recessions or expansions in other countries? Does it improve or worsen financial conditions abroad? Does it lead to capital inflows or outflows? Are spillovers different across advanced and emerging economies, or across countries pegging their exchange rate to the dollar and those retaining monetary autonomy? The existing literature suggests that spillover effects can be sizeable, but that there is considerable heterogeneity across countries in the response of macroeconomic variables, asset prices, and financial flows, with no discernible link between effects and country characteristics (see Dedola et al. (2017)).

To address these questions many of the previous studies focused on one or two of these areas. Many studies have focused on investigating the main channels through which shocks are transmitted. Using a VAR model, S. (2001) found that a decrease in the world interest rate to play a major role in the propagation of the shocks while trade balance to play a much less important role. In the meantime, investigating credit channel, Romer and Romer. (1993) and Ramey (1993) found that credit channel to play an insignificant role, while Bernanke and Gertler (1995) find that there is a direct relationship between
monetary policy shocks and credit channel.

Another area of research under this topic is measuring the spillover impact on other countries. This is done through the employment of empirical methods such as VAR, as well as DSGE models. The idea in this type of research is to quantify the size and sign of the impact of the international spillovers coming from domestic policies. An example of this type is Canova (2005) which investigates the impact of US monetary policy on Latin America countries. The paper uses monthly data with VAR and sign restrictions and found that the US monetary policy shocks have a significant impact on the eight Latin American countries that have been studied. A more general paper is that of Ammer et al. (2016) in which it attempts to compute the impact of US monetary policy internationally. Although using back-of-the-envelope calculation, it shows that it is important to consider the various channels that come into play. It shows that without considering all these different channels it is hard to be certain on the sign or size of this impact. In their simple example, they show that it is actually possible that an expansionary US monetary policy can lead to a positive international impact. Finally, Georgiadis (2016) studies the main factors that influence the impact of the US monetary policy internationally. It showed that the impact of monetary policy globally is not the same across countries and that it is influenced by factors such as the exchange rate regime, the degree of openness, trade and financial integration, financial market development, and industry structure. The paper suggested that countries could eventually protect themselves or minimize the impact of US shocks through close trading ties with the US, by improving their domestic financial and labour markets and by adopting a more flexible exchange rate regime. The paper used quarterly data for 61 countries with a global VAR method.

Another dimension to this literature is the differences in terms of effectiveness domestically as well as in terms of global effects between conventional and unconventional monetary policy. While the main believe is that unconventional monetary policy has a larger propagation, a recent paper by Curcuru et al. (2018) investigated this assumption using a new method. That paper used a method to disentangle the long term return on bond "into expected short rate and term premium components" to be able to compare the impact of conventional and unconventional monetary policies on these different
components. The results concluded that in contrast to the common believe, conventional monetary policy has a higher impact.

The second area is related to pricing in open economy models. As explained before, this branch of literature focuses on the importance of which currency prices are sticky in. This literature can be traced back to the seminal model of Mundell-Fleming (Mundell (1963), Fleming (1962), in which they initiated the modelling of the interaction between macroeconomic variables within an open economy. As explained earlier in this model and the literature that follows the maintained assumption is that prices are preset in the producer currency. In a few decades, other studies came to suggest that the first framework fail to explain the trends that are observed in the data. This gave rise to a new paradigm which has been called local pricing producers. This paradigm assumes that prices are rigid in the destination currency. This paradigm can be attributed to papers such as Betts and Devereux (2000) and Devereux and Engel. (2003). Since then there has been a growing body of literature that studies the observation of the frequent use of the dollar as a currency that firms set their prices in. These studies which sometimes called dollar pricing, try to explain how firms make their choice of which currency to use in setting their prices in international trade.

A more recent and important paper in this literature is Casas et al. (2017) which draws on Gita Gopinath’ and co-authors’ work over the past ten years on international trade transactions and firms’ choice of currency. This collective work suggests that what is observed in the international transaction data is that most firms set their prices in a few currencies and that they do not change them frequently. Also, it shows that among these few currencies the dollar remains the most often used currency (insert table showing list of currency used). As argued in the paper, firms invoice in dollars for possibly two reasons: 1. Strategic complementarities in prices 2. Intermediate goods prices Imported intermediate inputs. From these empirical facts, Casas et al. (2017) built a theoretical model to test the impact of this new paradigm that they named the dominant currency paradigm on the transmission of shocks. They compared predictions of the three pricings paradigms (i.e. PCP, LCP and DCP) and showed that under DCP terms of trade is less sensitive to the changes in the bilateral nominal exchange rate and more affected by the

\footnote{This include Gopinath and Rigobon. (2008); Gopinath and Neiman (2014), Gopinath (2015); and Boz et al. (2017)}
changes in relation to the dollar. They also showed that a constant increase in value of the dollar may weaken the global trade as prices of goods will increase globally.

3 Model Description

The model economy is a two-block dynamic general equilibrium model. We consider two cases:

1. A small open economy interacting with the ROW but with no policy strategic interdependence. Then the ROW is modelled as the closed NK model which can be estimated separately.

2. Two large blocs (e.g., the Euro-zone and the US) with strategic policy interdependence. Then an equilibrium of welfare-optimizing rules depend on whether the two blocs coordinate their policy or not.

We first consider the latter case.

3.1 Notation and Dixit-Stiglitz Aggregates

In each bloc, domestically produced and imported goods are consumed with prices denominated in the country’s currency with notation summarized in Table 1.

<table>
<thead>
<tr>
<th></th>
<th>Domestic Production</th>
<th>Imported Good</th>
<th>Aggregate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Home Country Quantity</td>
<td>$C_{H,t}$</td>
<td>$C_{F,t}$</td>
<td>$C_t$</td>
</tr>
<tr>
<td>Home Country Price</td>
<td>$P_{H,t}$</td>
<td>$P_{F,t}$</td>
<td>$P_t$</td>
</tr>
<tr>
<td>Foreign Country Quantity</td>
<td>$C^*_{F,t}$</td>
<td>$C^*_{H,t}$</td>
<td>$C^*_t$</td>
</tr>
<tr>
<td>Foreign Country Price</td>
<td>$P^*_{F,t}$</td>
<td>$P^*_{H,t}$</td>
<td>$P^*_t$</td>
</tr>
</tbody>
</table>

Table 1: Consumption Notations

For the Home country, aggregate Dixit-Stiglitz consumption and price indices are given by

\[
C_t = C^{DS}(w_C, \mu C, C_{H,t}, C_{F,t}) \equiv \left[ w_C^{\frac{1}{\mu C}} C_{H,t}^{\frac{\mu C - 1}{\mu C}} + (1 - w_C) P_{F,t}^{\frac{1}{\mu C}} C_{F,t}^{\frac{\mu C - 1}{\mu C}} \right]^{\frac{1}{1 - \mu C}} \tag{1}
\]

\[
P_t = P^{DS}(w_C, \mu C, C_{H,t}, C_{F,t}) \equiv \left[ w_C P_{H,t}^{1 - \mu C} + (1 - w_C) P_{F,t}^{1 - \mu C} \right]^{\frac{1}{1 - \mu C}} \tag{2}
\]
\[ P_tC_t = P_{H,t}C_{H,t} + P_{F,t}C_{F,t} \]  

(3)

The weight \( w_C \) in the consumption baskets in the home bloc is a measure of ‘home bias’. If \( w_C = 1 \), we have autarky.

Maximising total consumption (1) subject to a given aggregate expenditure \( P_tC_t = P_{H,t}C_{H,t} + P_{F,t}C_{F,t} \) yields

\[ C_{H,t} = w_C \left( \frac{P_{H,t}}{P_t} \right)^{-\mu_C} C_t \]  

(4)

\[ C_{F,t} = (1 - w_C) \left( \frac{P_{F,t}}{P_t} \right)^{-\mu_C} C_t \]  

(5)

For the Foreign country, aggregate Dixit-Stiglitz consumption and price indices are given by

\[ C^*_t = C^{DS}(w^*_C, \mu^*_C, C^*_{F,t}, C^*_{H,t}) \]  

(6)

\[ P^*_t = P^{DS}(w^*_C, \mu^*_C, C^*_{F,t}, C^*_{H,t}) \]  

(7)

\[ P^*_t C^*_t = P_{F,t}^* C_{F,t}^* + P_{H,t}^* C_{H,t}^* \]  

(8)

Then, foreign consumption functions corresponding to (4) and (5) are:

\[ C^*_{F,t} = w^*_C \left( \frac{P_{F,t}^*}{P_t^*} \right)^{-\mu^*_C} C_t^* \]  

(9)

\[ C^*_{H,t} = (1 - w^*_C) \left( \frac{P_{H,t}^*}{P_t^*} \right)^{-\mu^*_C} C_t^* \]  

(10)

Investment is analogous: in each bloc, domestically produced and imported goods are used for investment with prices denominated in the country’s currency with notation summarized in Table 2

<table>
<thead>
<tr>
<th></th>
<th>Domestic Production</th>
<th>Imported Good</th>
<th>Aggregate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Home Country Quantity</td>
<td>( I_{H,t} )</td>
<td>( I_{F,t} )</td>
<td>( I_t )</td>
</tr>
<tr>
<td>Home Country Price</td>
<td>( P_{H,t} )</td>
<td>( P_{F,t} )</td>
<td>( P_t^I )</td>
</tr>
<tr>
<td>Foreign Country Quantity</td>
<td>( I_{F,t}^* )</td>
<td>( I_{H,t}^* )</td>
<td>( I_t^* )</td>
</tr>
<tr>
<td>Foreign Country Price</td>
<td>( P_{F,t}^* )</td>
<td>( P_{H,t}^* )</td>
<td>( P_t^{I*} )</td>
</tr>
</tbody>
</table>

Table 2: Investment Notations
For the Home country, aggregate Dixit-Stiglitz investment and price indices are given by

\[ I_t = I_{DS}(w_I, \mu_I, I_{H,t}, I_{F,t}) \equiv \left[w_I^{1-\mu_I} I_{I,t} + (1 - w_I)\frac{1}{\mu_I} I_{F,t}\right]^{\frac{1}{1-\mu_I}} \] (11)

\[ P_{t} = P_{DS}(w_I, \mu_I, I_{H,t}, I_{F,t}) \equiv \left[w_I P_{H,t}^{1-\mu_I} + (1 - w_I)P_{F,t}^{1-\mu_I}\right]^{\frac{1}{1-\mu_I}} \] (12)

\[ P_{I} = P_{H,t}I_{H,t} + P_{F,t}I_{F,t} \] (13)

The weight \( w_I \) in the consumption baskets in the home bloc is a measure of ‘home bias’ for investment. If \( w_I = 1 \), we have autarky.

Maximising total investment (11) subject to a given aggregate expenditure \( P_{I} = P_{H,t}I_{H,t} + P_{F,t}I_{F,t} \) yields

\[ I_{H,t} = w_I \left(\frac{P_{H,t}}{P_{I}}\right)^{\mu_I} I_t \] (14)

\[ I_{F,t} = (1 - w_I) \left(\frac{P_{F,t}}{P_{I}}\right)^{\mu_I} I_t \] (15)

For the Foreign country, aggregate Dixit-Stiglitz investment and price indices are given by

\[ I_t^* = I_{DS}(w_I^*, \mu_I^*, I_{F,t}^*, I_{H,t}^*) \] (16)

\[ P_{t}^* = P_{DS}(w_I^*, \mu_I^*, I_{F,t}^*, I_{H,t}^*) \] (17)

\[ P_{I}^* = P_{F,t}^*I_{F,t}^* + P_{H,t}^*I_{H,t}^* \] (18)

Then, foreign investment functions corresponding to (14) and (15) are:

\[ I_{F,t}^* = w_I^* \left(\frac{P_{F,t}^*}{P_{I}^*}\right)^{\mu_I^*} I_t^* \] (19)

\[ I_{H,t}^* = (1 - w_I^*) \left(\frac{P_{H,t}^*}{P_{I}^*}\right)^{\mu_I^*} I_t^* \] (20)

If \( w_C = w_I = w_C^* = w_I^* = 1 \) we have autarky.
3.2 The Law of One Price, Terms of Trade and Inflation

Let $S_t$ be the nominal exchange rate defined as the cost of one unit of Foreign currency in the Home bloc. We assume the law of one price holds and hence

$$S_t P_{H,t}^* = P_{H,t}$$

$$S_t P_{F,t}^* = P_{F,t}$$ (21)

The terms of trade for the home country are defined as $T_t \equiv \frac{P_{F,t}}{P_{H,t}}$, the price of the imported good relative to the domestic one, and $T_t^* \equiv \frac{P_{H,t}^*}{P_{F,t}^*}$ for the Foreign bloc. Hence from the law of one price

$$T_t \equiv \frac{P_{F,t}}{P_{H,t}} = \frac{S_t P_{F,t}^*}{S_t P_{H,t}^*} = \frac{P_{F,t}^*}{P_{H,t}^*} = \frac{1}{T_t^*}$$ (22)

Now define CPI, domestic and imported inflation rates over the interval $[t-1, t]$ for the Home bloc by

$$\Pi_{t-1,t} = \frac{P_t}{P_{t-1}}$$ (24)

$$\Pi_{H,t-1,t} = \frac{P_{H,t}}{P_{H,t-1}}$$ (25)

$$\Pi_{F,t-1,t} = \frac{P_{F,t}}{P_{F,t-1}}$$ (26)

The foreign counterparts of CPI, domestic and imported inflation rates over the interval $[t-1, t]$ are defined by

$$\Pi_{t-1,t}^* = \frac{P_{t}^*}{P_{t-1}^*}$$ (27)

$$\Pi_{F,t-1,t}^* = \frac{P_{F,t}^*}{P_{F,t-1}^*}$$ (28)

$$\Pi_{H,t-1,t}^* = \frac{P_{H,t}^*}{P_{H,t-1}^*}$$ (29)

Then from (2) and (7) we have

$$\Pi_{t-1,t} = \left[ w_C \left( \Pi_{H,t-1,t} \frac{P_{H,t-1}}{P_{t-1}} \right)^{1-\mu_C} + (1-w_C) \left( \Pi_{F,t-1,t} \frac{P_{F,t-1}}{P_{t-1}} \right)^{1-\mu_C} \right]^{1/1-\mu_C}$$ (30)
\[ \Pi^*_{t-1,t} = \left[w^*_C \left( \Pi^*_{F,t-1,t} P^*_{F,t-1,t} \right)^{1-\mu^*_C} + \left(1 - w^*_C \right) \left( \Pi^*_{H,t-1,t} P^*_{H,t-1,t} \right)^{1-\mu^*_C} \right]^{\frac{1}{1-\mu^*_C}} \]  

(31)

where for the Home bloc:

\[ \frac{T_t}{T_{t-1}} = \frac{\Pi_{F,t-1,t}}{\Pi_{H,t-1,t}} \]  

(32)

\[ \frac{P_t}{P_{H,t}} = \left( w_C + (1 - w_C) T_t^{1-\mu_C} \right)^{\frac{1}{1-\mu_C}} \]  

(33)

\[ \frac{P_t}{P^*_{F,t}} = \left( w_C T_t^{\mu_C-1} + (1 - w_C) \right)^{\frac{1}{1-\mu_C}} \]  

(34)

\[ \frac{P_t^*}{P_{H,t}} = \left( w_I + (1 - w_I) T_t^{1-\mu_I} \right)^{\frac{1}{1-\mu_I}} \]  

(35)

\[ \frac{P_t^*}{P^*_{F,t}} = \left( w_I T_t^{\mu_I-1} + (1 - w_I) \right)^{\frac{1}{1-\mu_I}} \]  

(36)

and for the Foreign bloc:

\[ \frac{T^*_{t-1}}{T^*_t} = \frac{\Pi^*_{H,t-1,t}}{\Pi^*_{F,t-1,t}} \]  

(37)

\[ \frac{P^*_{t}}{P^*_{F,t}} = \left( w^*_C + (1 - w^*_C) (T^*_t)^{1-\mu^*_C} \right)^{\frac{1}{1-\mu^*_C}} \]  

(38)

\[ \frac{P^*_{t}}{P^*_{H,t}} = \left( w^*_C (T^*_t)^{\mu^*_C-1} + (1 - w^*_C) \right)^{\frac{1}{1-\mu^*_C}} \]  

(39)

\[ \frac{P^*_{t}}{P^*_{F,t}} = \left( w^*_I + (1 - w^*_I) (T^*_t)^{1-\mu^*_I} \right)^{\frac{1}{1-\mu^*_I}} \]  

(40)

\[ \frac{P^*_{t}}{P^*_{H,t}} = \left( w^*_I (T^*_t)^{\mu^*_I-1} + (1 - w^*_I) \right)^{\frac{1}{1-\mu^*_I}} \]  

(41)

The real exchange rate is defined as \( RER_t = \frac{S_t P^*_t}{P^*_t} \). Then from (34) and (38) and the law of one price we have

\[ RER_t = \frac{S_t P^*_t}{P_t} = \frac{S_t P^*_t / P^*_{F,t}}{P_t / P^*_{F,t}} = \frac{P_t^* / P^*_{F,t}}{P_t / S_t P^*_{F,t}} = \frac{P_t^* / P^*_{F,t}}{P_t / P^*_{F,t}} \]  

(42)

\[ = \left( w^*_C + (1 - w^*_C) (T^*_t)^{1-\mu^*_C} \right)^{\frac{1}{1-\mu^*_C}} \left( w_C T_t^{\mu_C-1} + (1 - w_C) \right)^{\frac{1}{1-\mu_C}} \]  

(43)
Thus in the symmetric bloc case where \( w_C = w_C^* \) and \( \mu_C = \mu_C^* \), since \( T_t = 1/T_t^* \) the law of one price \((RER_t = 1)\) holds for the aggregate price indices. Otherwise it does not.

This completes the equilibrium for Home variables \( \{T_t, \frac{P^*_t}{P^*_t}, \frac{P_t}{P_t^*}, \Pi_{t-1,t}, \Pi_{F,t-1,t}, C_{H,t}, C_{F,t}, I_{H,t}, I_{F,t}, \} \) given \( C_t \), \( I_t \) and domestic inflation \( \Pi_{H,t-1,t} \), and for the corresponding Foreign variables \( \{T_t^*, \frac{P^*_t}{P^*_t}, \frac{P_t}{P_t^*}, \Pi_{t-1,t}, \Pi_{H,t-1,t}, C_{F,t}, C_{H,t}, I_{F,t}, I_{H,t}, \} \) given \( C_t^* \), \( I_t^* \) and foreign domestic inflation \( \Pi_{F,t-1,t} \).

### 3.3 Households

Households in the H bloc hold both domestic and foreign bonds, but those in the F bloc only hold domestic bonds. Households are divided in those who participate in the financial sector and can lend or borrow to each other. These are Ricardian consumers. The remaining rule-of-thumb consumers are credit-constrained and must consume out of wage income net of tax.

#### 3.3.1 Ricardian Consumers

There are \((1 - \lambda)\) non-credit constrained Ricardian (R) consumers. Households single-period utility is:

\[
U_t^R = U(C_t^R, L_t^R) = \frac{(C_t^R - \chi C_{t-1}) (1-\phi)(1-\sigma) (1 - H_t^R) \phi^{(1-\sigma)} - 1}{1 - \sigma} \tag{44}
\]

The household solves

\[
\max_{C_t^R, L_t^R} \mathbb{E}_t \left[ \sum_{s=0}^{\infty} \beta^s U(C_{t+s}^R, H_{t+s}^R) \right] \tag{45}
\]

subject to a nominal budget constraint given by

\[
P_t^B B_{H,t} + P_t^B S_t B_{F,t} = B_{H,t-1} + S_t B_{F,t-1} + P_t W_t (1 - \tau^w) H_t^R - P_t C_t^R + \Gamma_t \tag{46}
\]

with nominal profits given by \( \Gamma_t \) and a proportional labour tax given by \( \tau^w \). \( B_{H,t} \) and \( B_{F,t} \) are domestic and foreign bonds respectively, bought at nominal prices \( P_t^B \) and \( P_t^B^* \) and denominated in the respective currencies. \( P_t \) is the CPI index that includes an imported component (see (2) above) and \( S_t \) is the nominal exchange rate.
Maximizing (45) subject to the budget constraint we have

\[ P_t^B = \mathbb{E}_t \left[ \frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}} \right] \]  
\[ P_t^{B^*} = \mathbb{E}_t \left[ \frac{\Lambda_{t,t+1} S_{t+1}}{\Pi_{t,t+1} S_t} \right] \]  
\[ \frac{U^R_{H,t}}{U^R_{C,t}} = -\frac{\varrho}{1 - \varrho} \frac{C_t^R - \chi C_{t+1}^R}{1 - H_t^R} = -W_t (1 - \tau_t^w) \]

where \( \Lambda_{t,t+1} \equiv \beta \frac{U_{R,C,t+1}}{U_{C,t}} \).

Nominal return on home bonds is by definition \( R_t = \frac{1}{P^B_t} \), where \( R_t \) is set by the central bank. We assume foreign bonds are subject to a risk premium that depends on the exposure to foreign debt, \( R_t^{*} = \frac{P_t^{B*} \phi \left( \frac{S_{t+1} B_{t+1}^F}{P_{H,t+1} Y_t} \right)}{P_t^{B*} \phi \left( \frac{S_{t+1} B_{t+1}^F}{P_{H,t+1} Y_t} \right)} \). Additionally, we assume \( \phi(0) = 0 \) and \( \phi' < 0 \). A functional form with these properties is

\[ \phi(x) = \exp(-\phi_B x); \phi_B > 0 \]  

Write equation (47) as

\[ 1 = \mathbb{E}_t \left[ \frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}} \right] R_t \]  
\[ 1 = R_t^* \phi \left( \frac{S_{t+1} B_{t+1}^F}{P_{H,t+1} Y_t} \right) \mathbb{E}_t \left[ \frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}} \Pi_t^S \right] \]  

where \( \Pi_t^S \equiv \frac{S_{t+1}}{S_t} \) is the rate of change of the nominal exchange rate over the interval \([t, t + 1]\) (i.e., the depreciation rate). Equations (51) and (52) together give a UIP condition modified to allow for risk.

### 3.3.2 Credit-constrained Households

The remaining \( \lambda \) consumers are credit constrained (C) and have no income from monopolistic retail firms. They must consume out of wage income and their consumption is given by

\[ C_t^C = W_t (1 - \tau_t^w) H_t^C \]
Liquidity constrained consumers now choose \( C_t^C \) and \( L_t^C = 1 - H_t^C \), to maximize an analogous welfare function to (45) subject to (53). The first order conditions are now the same for both types

\[
\frac{U_{H,t}^C}{U_{C,t}^C} = -\frac{\varrho}{1 - \varrho} \frac{C_t^C - H_t^C}{1 - H_t^C} = \frac{U_{H,t}^R}{U_{C,t}^R} = -W_t (1 - \tau_t^w) \tag{54}
\]

### 3.4 Aggregate Consumption and Labour

Total consumption and hours are then

\[
C_t = \lambda C_t^C + (1 - \lambda)C_t^R \tag{55}
\]

\[
H_t = \lambda H_t^C + (1 - \lambda)H_t^R \tag{56}
\]

### 3.5 Export Demand, Trade Balances and Output Equilibrium

Total exports by the Home and Foreign Country are respectively given by

\[
EX_t = C_{H,t}^* + I_{H,t}^* = (1 - w_t^C) \left( \frac{P_{H,t}^*}{P_t} \right)^{-\mu_t^C} C_t^* + (1 - w_t^I) \left( \frac{P_{H,t}^*}{P_t} \right)^{-\mu_t^I} I_t^* \tag{57}
\]

\[
EX_t^* = C_{F,t}^* + I_{F,t}^* = (1 - w_t^C) \left( \frac{P_{F,t}^*}{P_t} \right)^{-\mu_t^C} C_t + (1 - w_t^I) \left( \frac{P_{F,t}^*}{P_t} \right)^{-\mu_t^I} I_t \tag{58}
\]

Note that for the two-bloc model, we no longer allow \( w_t^C \) and \( w_t^I \) to tend to unity.

Nominal trade balance in the Home and Foreign blocs are respectively

\[
P_t TB_t = S_t P_n^{*O} Y^O + P_{H,t} Y_t - P_t C_t - P_t I_t - P_{H,t} G_t
\]

\[
P_t^* TB_t^* = P_{F,t}^* Y_t^* - P_t^* C_t^* - P_t^* I_t^* - P_{F,t}^* G_t^*
\]

Output equilibrium in the two blocs requires

\[
Y_t = C_{H,t} + C_{H,t}^* + I_{H,t} + I_{H,t}^* + G_t \tag{61}
\]
\[ Y^*_t = C^*_t + C^*_t + I^*_t + F^*_t + G^*_t \]  

(62)

Then combining (59)–(62) we have

\[
P_tTB_t = S_t P^*_t Y^O + P_{H,t}(C^*_t + I^*_t) - P_{F,t}(C^*_t + I^*_t)
\]

(63)

\[
P^*_tTB^*_t = P^*_t(C^*_t + I^*_t) - P^*_t(C^*_t + I^*_t)
\]

(64)

denominated in units of H and F currency respectively, where \( IM_t \) and \( IM^*_t \) are imports for the two blocs. In units of the H currency (90) becomes

\[
S_t P^*_t TB^*_t = S_t P^*_t (C^*_t + I^*_t) - S_t P_{H,t}(C^*_t + I^*_t) = P_{F,t}EX^*_t - P_{H,t}EX_t
\]

(65)

3.6 Firms

There are wholesale and retail sectors. The former act in perfect competition producing a homogeneous intermediate good, the latter in monopolistic competition producing differentiated final goods. In addition we now have capital producers.

3.6.1 Wholesale sector

Production technology:

\[
Y^W_t = F(A_t, H_t, K_{t-1}) = (A_t H_t)^\alpha K_{t-1}^{1-\alpha}
\]

(66)

Wholesale firms sell at nominal price \( P^W_t \) to retailers, so profit maximisation implies

\[
F_{H,t} = \frac{Y^W_t}{H_t} \frac{P^W_t}{P_t} = W_t
\]

(67)

\[
F_{K,t} = (1 - \alpha) \frac{Y^W_t}{K_{t-1}} \frac{P^W_t}{P_t} = r^K_t
\]

(68)

where \( P_t \) is price index of final consumption goods.
3.6.2 Capital Producers

Capital producers purchase investment goods from home and foreign retail firms at real price \( \frac{P_I}{P_t} \) selling at real price \( Q_t \) to maximize expected discounted profits

\[
E_t \sum_{k=0}^{\infty} \Lambda_{t,k} \left[ Q_{t+k} (1 - S(I_{t+k}/I_{t+k-1})) I_{t+k} - \frac{P_I}{P_t} I_{t+k} \right]
\]

where total capital accumulates according to

\[
K_t = (1 - \delta) K_{t-1} + (1 - S(X_t)) I_t
\]

(69)

\[
I_t = \left[ \frac{1}{w_I} I_{H,t}^{\mu_I^{-1}} + (1 - w_I) \frac{1}{\mu_I} I_{F,t}^{\mu_I^{-1}} \right] ^{\mu_I}
\]

(70)

This results in the first-order condition

\[
Q_t (1 - S(X_t) - X_t S'(X_t)) + E_t \left[ \Lambda_{t+1} Q_{t+1} S'(X_{t+1}) \frac{I_{t+1}^2}{I_t^2} \right] = \frac{P_I}{P_t}
\]

(71)

where we define

\[
S(X_t) \equiv \phi_X(X_t - X)^2
\]

(72)

Note that in the absence of investment adjustment costs, the relative price of capital, \( Q_t \) will equal \( \frac{P_I}{P_t} \). Finally, we define \( R^K_t \) as gross real return on capital taking into account corporate taxation given by

\[
R^K_t = \frac{r^K_t (1 - \tau^K_t) + (1 - \delta) Q_t}{Q_{t-1}}
\]

(73)

where \( \tau^K_t \) is a tax on corporate profits assumed exogenous in the model.

3.6.3 Retail Sector

Each home retailer \( m \in (0, 1) \) purchases output from the intermediate good sector at price \( P^W_{H,t} \) and converts into a differentiated good sold at price \( P^W_{H,t}(m) \) to households, capital
good producers and governments who use the technology

\[ C_{H,t} = \left( \int_0^1 C_{H,t}(m)^{(\zeta^{-1})} \frac{\zeta}{\zeta-1} dm \right)^{\zeta/(\zeta-1)} \] (74)

to combine into baskets, where \( \zeta \) is the elasticity of substitution. Similarly for \( I_{H,t} \) and \( G_t \).

The price-setting proceeds exactly as for the closed economy NK model: for the SOE we simply replace \( P_t, \Pi_{t-1,t} \) and \( \Pi_{t,t+1} \) with \( P_{H,t}, \Pi_{H,t-1,t} \) and \( \Pi_{H,t,t+1} \) respectively.

### 3.7 Commodity Sector

We introduce a commodity sector (e.g. oil) treating output as an exogenous constant endowment \( Y^O \). Revenues is then driven only by the price of the commodity \( P^O_t \) denominated in foreign currency which is an exogenous process as for the other shock processes in the model. The commodity is entirely exported and the only channel through which commodity production and price effects the model is the trade balance and government budget constraint given below. A tax rate \( \tau^O_t \) applies to this sector.

### 3.8 Financial Intermediation

Efficient financial intermediation within the Home country implies the zero arbitrage condition:

\[
\mathbb{E}_t \left[ \Lambda_{t,t+1} R^K_{t+1} \right] = \mathbb{E}_t \left[ \frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}} \right] R_t = 1 \] (75)

which we take as the equilibrium equation for \( Q_t \).

### 3.9 Central Bank Foreign Assets and Monetary Policy

The nominal interest rate \( R_t \) is a policy variable, typically given in the literature by a standard Taylor-type rule\(^7\) that includes an exchange rate depreciation term:

\[
\log \left( \frac{R_t}{R} \right) = \rho_r \log \left( \frac{R_{t-1}}{R} \right) + (1 - \rho_r) \left[ \theta_a \log \left( \frac{\Pi_{t-1,t}}{\Pi} \right) + \theta_s \log \left( \frac{\Pi_{S,t-1,t}}{\Pi_S} \right) + \theta_y \log \left( \frac{Y_t}{Y} \right) + \theta_{dy} \log \left( \frac{Y_t}{Y_{t-1}} \right) \right] + \epsilon_{M,t} \] (76)

---

\(^7\)In a closed-economy NK model with credit-constrained consumers, Bilbiie (2008) shows that an inversion of the Taylor principle occurs with a sufficient high proportion of such households.
Foreign bond holdings evolves according to home country nominal terms

$$P_t^{B^*} S_t B^*_{F,t} = S_t B^*_{F,t-1} + P_t TB_t \quad (77)$$

Now define $B_{F,t} \equiv \frac{S_t B^*_{F,t}}{P_t}$ to be the stock of foreign bonds in home country consumption units. Then

$$P_t^{B^*} P_t B_{F,t} = \frac{S_t}{S_{t-1}} P_{t-1} B_{F,t-1} + P_t TB_t \Rightarrow$$

$$P_t^{B^*} B_{F,t} = \frac{\Pi^S_{t-1,t}}{\Pi_{t-1,t}} B_{F,t-1} + TB_t \quad (78)$$

Finally a government nominal balanced budget constraint gives

$$P_{H,t} G_t = P_t W_t H_t \tau^w_t + (1 - \alpha) Y^W_t P_{H,t} M C_t \tau^k_t \quad (79)$$

recalling that $MC_t \equiv \frac{P^W_I}{P^H_I}$.

$$\tau^w_t = \frac{P_{H,t} G_t - (1 - \alpha) Y^W_t M C_t \tau^k_t - RER_t P^*_t Y^O_t \tau^O}{W_t H_t} \quad (80)$$

With non-distortionary lump-sum taxes we put $\tau^k_t = \tau^w_t = 0$ and replace (80) with

$$P_{H,t} G_t = T_t \quad (81)$$

which defines the lump-sum taxes necessary to finance government spending with a balanced government budget constraint.

**Monetary Policy in ROW Block**

The ROW nominal interest rate is given by the following Taylor-type rule

$$\log \left( \frac{R^*_t}{R^*} \right) = \rho_r \log \left( \frac{R^*_{t-1}}{R^*} \right) + (1 - \rho^*_r) \left[ \theta^*_r \log \left( \frac{\Pi^*_{t-1,t}}{\Pi^*} \right) + \theta^*_y \log \left( \frac{Y^*_t}{Y^*} \right) + \theta^*_d \log \left( \frac{Y^*_{t-1}}{Y^*_{t-1}} \right) \right] + \epsilon^*_M \quad (82)$$
3.10 Shock processes

The structural shock processes in log-linearised form are assumed to follow AR(1) processes which for the Home country are:

\[
\begin{align*}
\log A_t - \log A &= \rho_A (\log A_{t-1} - \log A) + \epsilon_{A,t} \\
\log G_t - \log G &= \rho_G (\log G_{t-1} - \log G) + \epsilon_{G,t} \\
\log MS_t - \log MS &= \rho_{MS} (\log MS_{t-1} - \log MS) + \epsilon_{MS,t} \\
\log IS_t - \log IS &= \rho_{IS} (\log IS_{t-1} - \log IS) + \epsilon_{IS,t} \\
\log tot_t - \log tot &= \rho_{tot} (\log tot_{t-1} - \log tot) + \epsilon_{tot,t} \\
\log P_t^*O - \log P^O &= \rho_{P^O} (\log P^O_{t-1} - \log P^O) + \epsilon_{P^O,t}
\end{align*}
\]

where \( MS = A = IS = tot = P^O = 1 \) in the steady state (so \( \log MS = \log A = \log IS = \log tot = \log P^O = 0 \)), while the monetary policy shock \( \epsilon_{M,t} \) is assumed to be i.i.d with zero mean. For the terms of trade shock we replace the equilibrium terms of trade variable \( T \) with \( T \, tot_t \) where \( tot_t \) is a temporary exogenous deviation from the equilibrium given by the AR1 process. Noting that for this shock the steady state \( tot = 1 \).

Similarly the ROW exogenous variables are assumed to also be AR1 processes.

\[
\begin{align*}
\log A^*_t - \log A^* &= \rho^*_A (\log A^*_{t-1} - \log A^*) + \epsilon^*_{A,t} \\
\log G^*_t - \log (G^*) &= \rho^*_G (\log G^*_{t-1} - \log (G^*)) + \epsilon^*_{G,t} \\
\log MS^*_t - \log MS^* &= \rho^*_{MS} (\log MS^*_{t-1} - \log MS^*) + \epsilon^*_{MS,t} \\
\log IS^*_t - \log IS^* &= \rho^*_{IS} (\log IS^*_{t-1} - \log IS^*) + \epsilon^*_{IS,t}
\end{align*}
\]

where \( MS^* = IS^* = 1 \) in the steady state (so \( \log MS^* = \log IS^* = 0 \)), while the monetary policy shock \( \epsilon_{M*,t} \) is assumed to be i.i.d with zero mean.

This completes the specification of the two-bloc open-economy model.

3.11 Bloc Size Effects and the SOE

In our representative agent model, all variables such as \( Y_t \) and \( Y^*_t \) are per capita quantities and can differ for example because labour productivity in the steady state \( A \neq A^* \). The
implication up to now is that population sizes are the same in both blocs. We now let the
F bloc have a population \(n\) times that of the H bloc. In the limit as \(n \to \infty\) we get to the
SOE-ROW model.

The output equilibria in two blocs now become

\[
Y_t = C_{H,t} + nC^*_{H,t} + I_{H,t} + nI^*_{H,t} + G_t
\]

\[
= C_{H,t} + I_{H,t} + G_t + EX_t \tag{83}
\]

\[
nY^*_t = nC^*_{F,t} + C_{F,t} + nI^*_{F,t} + I_{F,t} + nG^*_t
\]

\[
= n(C^*_{F,t} + I^*_{F,t} + G^*_t + EX^*_t) \tag{84}
\]

where \textit{per capita} exports by the Home and Foreign Country are respectively given by

\[
EX_t \equiv nC^*_{H,t} + nI^*_{H,t}
\]

\[
= n(1 - w^*_C) \left( \frac{P^*_{H,t}}{P^*_t} \right)^{-\mu^*_C} C^*_t + n(1 - w^*_I) \left( \frac{P^*_{H,t}}{P^*_t} \right)^{-\mu^*_I} I^*_t \tag{85}
\]

\[
EX^*_t \equiv C_{F,t}/n + I_{F,t}/n
\]

\[
= (1 - w_C)/n \left( \frac{P_{F,t}}{P_t} \right)^{-\mu_C} C_t + (1 - w_I)/n \left( \frac{P_{F,t}}{P_t} \right)^{-\mu_I} I_t \tag{86}
\]

Nominal trade balance in the Home and Foreign blocs are respectively

\[
P_t TB_t = S_t P^n_0 Y^O + P_{H,t} Y_t - P_t C_t - P^I_t I_t - P_{H,t} G_t
\]

\[
= S_t P^n_0 Y^O + P_{H,t} Y_t - P_{H,t} C_{H,t} - P_{F,t} C_{F,t} - P_{H,t} I_{H,t} - P_{F,t} I_{F,t} - P_{H,t} G_t \tag{87}
\]

\[
P^*_t TB^*_t = P^*_{F,t} Y^*_t - P^*_t C^*_t - P^*_t I^*_t - P^*_t G^*_t
\]

\[
= P^*_{F,t} Y^*_t - P^*_t C^*_t - P^*_t C^*_t - P^*_t I^*_t - P^*_t I^*_t - P^*_t I^*_t - P^*_t G^*_t \tag{88}
\]

Then combining (83)-(84) and (87)-(88) we have

\[
P_t TB_t = S_t P^n_0 Y^O + P_{H,t} n(C^*_H + I^*_H) - P_{F,t}(C_{F,t} + I_{F,t})
\]

\[
= S_t Y^O P^n_0 + P_{H,t} EX_t - P_{F,t}n EX^*_t \tag{89}
\]

\[
P^*_t TB^*_t = P^*_{F,t}(C_{F,t} + I_{F,t})/n - P^*_t(C^*_H + I^*_H)
\]
denominated in units of H and F currency respectively for the two blocs.

3.12 General n Case

For any $1 \leq n < \infty$ we can set up the model with the output and trade equilibria given by (83) – (96). Then using trade data we can calibrate $w_C^*$, $w_I^*$, $w_C^*$ and $w_I^*$ in the steady state as follows: In the steady state write (96) in terms of observable non-dimensional quantities as

$$
\frac{TB}{Y} = P_H n (1 - w_C^*) \left( \frac{P_H^*}{P^*} \right)^{-\rho_C} C^* \frac{Y}{Y} + P_H n (1 - w_I^*) \left( \frac{P_H^*}{P^*} \right)^{-\rho_I} I^* \frac{Y}{Y}
$$

$$
- P_F (1 - w_C) \left( \frac{P_F}{P} \right)^{-\rho_C} C \frac{Y}{Y} - P_F (1 - w_I) \left( \frac{P_F}{P} \right)^{-\rho_I} I \frac{Y}{Y}
$$

$$
+ \frac{RER \cdot Y^O \cdot P^{\cdot O}}{Y}
$$

(91)

The first term on the rhs of (91) is the share consumption goods exports, excs say. Given trade data for excs and letting $n$ be the relative population size we can then calibrate $w_C^*$ to hit excs. Similarly we can calibrate $w_I^*$ to hit the share of investment goods exports, the second term on the rhs of (96) exis say.

The third and fourth terms on the rhs of (91) are the shares of consumption and investment goods imports. Let these be imcs amd imis respectively and they can be used to calibrate $w_C$ and $w_I$ in a similar fashion.

The last term (exco) is the share commodity revenue with respect to non-oil nominal GDP, we need to calibrate exogenous and fixed commodity output $Y^O$ to hit an extra target of exco. But letting $tb = \frac{TB}{Y}$ we must have from (91) that

$$
tb = \text{excs} + \text{exis} - \text{imcs} - \text{imis} + \text{exco}
$$

(92)

Thus our trade data and trade balance data must be chosen to satisfy (92).

We can also calibrate $A^*/A$ to hit the per capita GDP ratios of the F and H blocs using the correct prices, namely $(\frac{S_y P_F Y^*}{P_H Y^*}) = \left( \frac{P_F Y^*}{P_H Y} \right) = (\frac{T Y^*}{Y^*})$, and $\beta/\beta^*$ to hit tb. Thus given $n$ (the relative population size) we can calibrate six parameters, $w_C^*$, $w_I^*$, $w_C^*$, $w_I^*$, 

\[21\]
A*/A and \( \beta*/\beta \) to hit *six targets* obtained from data: excs, exis, imcs, imis, the GDP ratio and tb.

To set **initial values** for \( w_C^*, w_I^*, w_C, w_I \) in fsolve, we can use an approximation when \( T = 1 \). Then for \( w_C^* \) we can use:

\[
\text{excs} = n(1 - w_C^*) \frac{C^*}{Y^*} = n(1 - w_C^*) \frac{C^* Y^*}{Y}
\]

Then use the steady state value for \( \frac{C^*}{Y} \) for the symmetric case of weights all equal to 0.5, \( Y^*/Y \) is the GDP ratio target and \( n \) the relative population size is a parameter. Similarly for the other three weights.

### 3.13 Calibrating the four Bias Weights, exogenous and fixed commodity output, \( \beta*/\beta, A*/A \)

In the .mod file we set the six target parameters \( \text{tb, excs, exis, imcs, imis, } \frac{Y^*/Y \text{ and exco}}{Y} \), based on our data. The external steady state solves using fsolve the followings for parameters \( w_C^*, w_I^*, w_C, w_I, \beta*/\beta, A*/A \) and \( Y^*/Y \) to hit target trade ratios excs, exis, imcs, imis, \( \text{tb, } \frac{Y^*/Y \text{ and exco}.}{} \)

\[
\begin{align*}
\text{excs} &= \frac{P_H}{P} n (1 - w_C^*) \left( \frac{P_C^*}{P^*} \right)^{\mu_C} \frac{C^*}{Y} \\
\text{exis} &= \frac{P_H}{P} n (1 - w_I^*) \left( \frac{P_I^*}{P^*} \right)^{\mu_I} \frac{I^*}{Y} \\
imcs &= \frac{P_F}{P} (1 - w_C) \left( \frac{P_C}{P} \right)^{-\mu_C} \frac{C}{Y} \\
imis &= \frac{P_F}{P} (1 - w_I) \left( \frac{P_I}{P} \right)^{-\mu_I} \frac{I}{Y} \\
exco &= \frac{\text{RERP} P^O Y^O}{Y} \\
tb &= \frac{TB}{Y} \\
\frac{Y^*/Y}{} &= \frac{T}{Y} \frac{Y^*}{Y}
\end{align*}
\]
3.14 Closed ROW-SOE Special Case

Now consider the ROW-SOE case as \( n \to \infty \) and \( w^*_C \to 1 \) and \( w^*_I \to 1 \). Then for the F-bloc:

\[
EX^*_t = \frac{C_{F,t}}{n} + \frac{I_{F,t}}{n} \to 0 \quad (94)
\]

\[
TB^*_t = P^*_{F,t}EX^*_t - P^*_{H,t}EX_t/n \to 0 \text{ as } n \to \infty \quad (95)
\]

Hence the ROW becomes a closed economy bloc. But \( TB_t \neq 0 \) and is given by

\[
TB_t = \frac{P_{H,t}}{P_t} EX_t - \frac{P_{F,t}}{P_t} n EX^*_t + RER_t P^*_t Y^O
\]

\[
= \frac{P_{H,t}}{P_t} n (1 - w^*_C) \left( \frac{P^*_{H,t}}{P^*_t} \right)^{-\mu^*_C} C^*_t + \frac{P_{H,t}}{P_t} n (1 - w^*_I) \left( \frac{P^*_{H,t}}{P^*_t} \right)^{-\mu^*_I} I^*_t
\]

\[- \frac{P_{F,t}}{P_t} (1 - w_C) \left( \frac{P_{F,t}}{P_t} \right)^{-\mu_C} C_t - \frac{P_{F,t}}{P_t} (1 - w_I) \left( \frac{P_{F,t}}{P_t} \right)^{-\mu_I} I_t
\]

\[+ RER_t P^*_t Y^O \quad (96)\]

Thus as \( n \to \infty \) \( TB^*_t \to 0 \), but at the same time \( w^*_C \to 1 \) and \( w^*_I \to 1 \) with \( n (1 - w^*_C) \) and \( n (1 - w^*_I) \) remaining non-zero. Hence \( TB_t \neq 0 \). From the steady state below (with corrections), now with all prices equal to unity (and therefore the terms of trade also unity), we have a set-up with the ROW a closed economy (as in Chapter 4) and in the home country \( n(1 - w^*_C) \) replaced with \( \frac{excs}{C^*Y} \) and \( n(1 - w^*_I) \) replaced with \( \frac{exis}{I^*Y} \) (see steady state (B)–(B.95)). Then as before in the calibration \( imcs, imis, tb \) and \( Y^*/Y \) are set equal to targets found from data.

3.15 Summary of changes in the Dynamic Calibrated Two-bloc Model

for (Closed ROW-SOE Case)

We replace the following
\[
TB_t^* = \frac{P_{t,t}}{P_t^*} EX_i^* - \frac{P_{H,t}}{P_t^*} EX_t
\]
\[
EX_i^* = \frac{CF_i}{n} + \frac{IF_i}{n}
\]
\[
EX_t = n CH_t^* + n IH_t^*
\]

with the following

\[
TB_t^* = 0
\]
\[
EX_t^* = 0
\]
\[
EX_t = \frac{\text{targexcs}}{C^* / Y} \left( \frac{P_{H,t}}{P_t^*} \right)^{\mu_C} C_t^* + \frac{\text{targexis}}{I^* / Y} \left( \frac{P_{H,t}}{P_t^*} \right)^{\mu_I} I_t^*
\]

3.16 Imperfect Exchange Rate Pass-Through: DCP with LCP Exporters only in H Bloc

We must now distinguish the price setting in domestic and foreign markets. We assume that the F bloc are producer currency prices but in the H bloc the prices of goods sold domestically, \( C_{H,t} + I_{H,t} + G_t \), are set in domestic currency, but those exported, \( C_{H,t}^* + I_{H,t}^* \), are invoiced in foreign currency. This is the Dominant Currency Pricing Case (DCP) with the currency in the F-bloc dominant.

For goods sold domestically the (corrected) inflation dynamics are given by:

\[
1 = \xi (\Pi_{H,t-1,t})^{\zeta - 1} + (1 - \xi) \left( \frac{JJ_{H,t}}{J_{H,t}} \right)^{1 - \zeta}
\]
\[
\Delta_t = \xi (\Pi_{H,t-1,t})^{\zeta} \Delta_{t-1} + (1 - \xi) \left( \frac{JJ_{H,t}}{J_{H,t}} \right)^{-\zeta}
\]
\[
JJ_{H,t} = \frac{\zeta}{\zeta - 1} \frac{P_{H,t}}{P_t} (C_{H,t} + I_{H,t} + G_t) MS_t MC_t + \xi \mathbb{E}_t \left[ \Lambda_{t,t+1} (\Pi_{H,t,t+1})^\zeta JJ_{H,t+1} \right]
\]
\[
J_{H,t} = \frac{P_{H,t}}{P_t} (C_{H,t} + I_{H,t} + G_t) + \xi \mathbb{E}_t \left[ \Lambda_{t,t+1} (\Pi_{H,t,t+1})^\zeta - JJ_{H,t+1} \right]
\]
\[
MC_t = \frac{P_W}{P_{H,t}}
\]

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Denoting the optimal price at time $t$ for exported good $m$ as $P^*_t (m)$ in F currency, the firms allowed to re-optimize prices maximize real (consumption price) expected discounted profits by solving
\[
\max_{P^*_t (m)} \mathbb{E}_t \sum_{k=0}^{\infty} \xi^k \frac{\Delta_t, t+k}{P_{t+k}} (C^*_{t+k} (m) + I^*_{t+k} (m)) \left[ S_{t+k} P^*_t (m) - P_{t+k}^W \right]
\] (97)
subject to the demand schedule which now becomes
\[
C^*_{t+k} (m) + I^*_{t+k} (m) = \left( \frac{P^*_t (m)}{P^*_t (t, t+k)} \right)^{\zeta} (C^*_{t+k} + I^*_{t+k})
\] (98)
Substituting in this demand schedule, taking first-order conditions with respect the new price and rearranging leads to
\[
P^*_t (m) = \frac{\zeta}{\zeta - 1} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} \xi^k \Delta_t, t+k \left( P^*_t (m) \right)^{\zeta} (C^*_{t+k} + I^*_{t+k}) P^W_{t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty} \xi^k \Delta_t, t+k \left( P^*_t (m) \right)^{\zeta} (C^*_{t+k} + I^*_{t+k})}
\] (99)
where the $m$ index is dropped as all firms face the same marginal cost so the right-hand side of the equation is independent of firm size or price history.

Now note that the real (own exported good price) marginal cost for each retailer is given by
\[
MC^*_{t, t} = \frac{P^W_t}{S_t P^*_t (t, t)} = \frac{MC_t P^*_t (t, t)}{S_t P^*_t (t, t)}
\] (100)
We can now write the fraction (99)
\[
\frac{P^*_t (m)}{P^*_{t, t}} = \frac{\zeta}{\zeta - 1} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} \xi^k \Lambda_{t, t+k} \left( \Pi^*_{t, t+k} \right)^{\zeta-1} \left( S_{t+k} P^*_t (m) \right)^{\zeta} (C^*_{t+k} + I^*_{t+k}) MC^*_{t, t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty} \xi^k \Lambda_{t, t+k} \left( \Pi^*_{t, t+k} \right)^{\zeta-1} \left( S_{t+k} P^*_t (m) \right)^{\zeta} (C^*_{t+k} + I^*_{t+k})}
\] (101)
Denoting the numerator and denominator by $JJ^*_t (m)$ and $J^*_t (m)$ respectively, and introducing a mark-up shock $MS_t$ to $MC_t$, we write in recursive form
\[
\frac{P^*_t (m)}{P^*_{t, t}} = \frac{J^*_t (m)}{J^*_{t, t}}
\]
\[
JJ^*_t (m) - \xi \beta \mathbb{E}_t [\Pi^*_{t, t, t+1}^\zeta JJ^*_{t, t+1}] = \frac{1}{1 - \frac{S_t P^*_t (t, t+k) MC^*_{t, t+k}}{P_t (t, t+k)}} U_{t, t} MC^*_{t, t+k} MS_t
\]
Using the aggregate producer price index $P_{H,t}$ and the fact that all resetting firms will choose the same price, by the Law of Large Numbers we can find the evolution of the price index as given by

$$J_{H,t}^* - \xi \beta E_t [\Pi_{H,t,t+1}^* (1-\zeta) J_{H,t+1}^*] = \frac{S_t P_{H,t}^*}{P_{H,t}} \left( C_{H,t+k}^* + I_{H,t+k}^* \right) U_{C,t}(P_{H,t}^*)$$

which can be written in the form required

$$1 = \xi \left( \Pi_{H,t-1,t}^* \right)^{\zeta-1} + (1 - \xi) \left( \frac{P_{H,t}^*}{P_{H,t}} \right)^{1-\zeta}$$

Foreign exporters from the large ROW bloc are PCPers so we have

$$P_{F,t} = S_t P_{F,t}^*$$

As before define the terms of trade for the home bloc (import/export prices in one currency) as $T_t = \frac{P_{F,t}}{P_{H,t}}$. Define the terms of trade for the foreign bloc as $T_t^* = \frac{P_{H,t}}{P_{F,t}^*}$. With PCPers only the law of one price holds and $T_t^* = \frac{S_t P_{H,t}}{S_t P_{F,t}^*}$, $= \frac{P_{H,t}}{P_{F,t}^*} = \frac{1}{T_t}$, but with LCPers this no longer is the case. Now we have that

$$T_t^* = \frac{P_{H,t}}{P_{F,t}^*} = \frac{P_{H,t}^*}{P_{F,t}^*} \frac{P_{H,t}}{P_{H,t}^*} = \frac{1}{P_t^*} \frac{P_{H,t}}{P_{H,t}^*} = \frac{S_t P_{H,t}^*}{P_{F,t}^*} = \frac{T_t^*}{T_t}$$

Hence in (102) we can write $\frac{S_t P_{H,t}^*}{P_{F,t}^*} = \frac{S_t P_{H,t}^*}{P_{H,t}^*} = \frac{T_t^*}{T_t} = \frac{M^* C_t}{M_t C_{H,t}^*}$ or alternatively from 100 we have $T_t^* T_t = \frac{M^* C_t}{M_t C_{H,t}^*}$ which completes the set-up.

Table 3 summarizes the notation used.

Whilst the distribution of prices is not required to track the evolution of the aggregate price
index, (110) below implies a loss of output due to dispersion in prices. Using the demand schedules, we can write the price dispersion that gives the average loss in output as

$$\Delta_{H,t} = \frac{1}{M} \sum_{m=1}^{M} \left( \frac{P_{H,t}(m)}{P_{H,t}} \right)^{-\zeta}$$

(106)

$$\Delta_{H,t}^* = \frac{1}{M} \sum_{m=1}^{M} \left( \frac{P_{H,t}(m)}{P_{H,t}} \right)^{-\zeta}$$

(107)

for firms $m = 1, ..., M$. It is not possible to track all $P_t(m)$ but as it is known that a proportion $1 - \xi$ of firms will optimise prices in period $t$, and from the Law of Large Numbers, that the distribution of non-optimised prices will be the same in as the overall distribution. Therefore, price dispersion can be written as a law of motion

$$\Delta_{H,t} = \xi(\Pi_{H,t-1},t) \Delta_{H,t-1} + (1 - \xi) \left( \frac{J_{J_{H,t}}}{J_{H,t}} \right)^{-\zeta} \cdot$$

(108)

$$\Delta_{H,t}^* = \xi(\Pi_{H,t-1},t) \Delta_{H,t-1} + (1 - \xi) \left( \frac{J_{J_{H,t}}}{J_{H,t}} \right)^{-\zeta} \cdot$$

(109)

Using this, aggregate final output is divided between exports $EX_t = C_{H,t}^* + I_{H,t}^*$ and domestic consumption $Y_t - EX_t = C_{H,t} + I_{H,t} + G_t$. Then allowing for dispersion we have

$$Y_t = \left( Y_t^W \frac{EX_t}{Y_t} \right) \left( \frac{1 - EX_t}{\Delta_{H,t}} \right)$$

(110)

Note that as the law of one price does not hold anymore so equation 65 will be considered as following:

$$P_t^* TB_t^* = P_{F,t}(C_{F,t} + I_{F,t}) - P_{H,t}(C_{H,t}^* + I_{H,t}^*)$$

(111)

4 Estimation Method

This section describes our data, calibration approach, and presents a brief discussion of the prior distribution and identification of parameters. We also have summarized the separated estimation of oil price shock and copper price shock. In order to evaluate the performance of the model, we use a combination of calibrated and estimated parameters. We choose to calibrate some of the parameters, mainly because the data set is not rich enough to identify all of them.

For the purpose of estimating the models, the steady state equation are based on a non-zero inflation steady state ($\Pi > 1$). This is done using Dynare for the estimation of our model. In the Appendices the steady state of the model is described.

4.1 Data

To estimate the model, we use quarterly information on seven key variables for Chile, Canada and Mexico: GDP, consumption, investment, consumer price index (CPI), nominal exchange rate,
nominal interest rate, oil price for Canada and Mexico and Copper price for Chile. The sample runs from QII:1996 to QIV:2018 for Chile, QII:1993 to QII:2017 for Canada and Mexico to compute posterior distributions and marginal likelihood values. Quarterly crude Oil price and copper price is obtained from FRED Economic Data. Other quarterly data are from IMF’s International Financial Statistics. Real investment and real consumption obtained by deflating using the consuming price index (CPI) and for real Oil price and Copper price, we have used consuming price index of USA. We compute quarter to quarter output growth, consumption growth and investment growth, nominal exchange rate growth, Copper price and oil price growth as log difference of real series and multiply the growth rates by 100 to convert them into percentages. Inflation rates are defined as log differences of the consuming price index (CPI) and converted into percentages. All variables are seasonally adjusted except the exchange rate.

4.2 Calibrated Parameters

Tables (4.2) and (4.2) summarizes the calibrated values of parameter in our model, at a quarterly frequency for Chile, Mexico and Canada, where we calibrate a set of parameters, and the steady state values for some endogenous variables, which characterise the model economy. Our calibration strategy aims to match, as accurately as possible, the empirical evidence and available data on key statistics of these three economies. Each period is assumed to correspond to a quarter.

As in much of the literature, the depreciation rate of capital, $\delta$, is set at 10 per cent per annum, implying a quarterly value of 0.025. Home discount rate are set at $\beta = 0.99$. This value is consistent with the quarterly estimates of discount factor for Pakistan economy as given in Ahmed et al. (2012). In the studies of Gabriel et al. (2010), Khera (2016) and Anand and Khera (2016) for India and Haider et al. (2012) for Pakistan and Batini et al. (2011) for emerging economies $\beta$ is set at 0.9823, 0.994, 0.994, 0.991 and 0.9881 respectively.

For risk aversion parameter ($\sigma_R$), Tabova (2009) estimates a value of 2.00 for middle-income countries and Gabriel et al. (2010) estimate a value of 1.99. In line with this literature, we assume the value of 2.00 for Ricardian risk aversion.

The substitution elasticity between imported and home goods ($\mu_C$), following previous emerging economy estimates in the literature that range from 1.07 to 2.50 in Castillo et al. (2008) for Peru, 0.6 in Medina et al. (2005) for Chile, 1.45 in Gabriel et al. (2010) for India, 1.15 and 1.20 in Khera (2016) for South Africa and India respectively and 1.50 in Batini et al. (2011) for emerging economies, we calibrate it at a value of 1.50 and following Medina et al. (2005), Chang et al. (2015) and Adler et al. (2016), export elasticity demand, $\mu_C^*$ and $\mu_I^*$, is set to 1.50.

In terms of the elasticity of substitution among different retail varieties, Khera (2016)and Gabriel et al. (2010), estimate it in India equal to 7.12 and 7.02. Thus, we adopt a mean of 7 for ($\zeta$).

Following IMF data based, we set an average of 12%, 21% and 11% for government share of production ($g_y$), in Chile, Canada and Mexico respectively. Also set an average of 11% for Copper taxation rate in Chile and 23% and 51% for Oil taxation rate in Canada and Mexico respectively.

The tarde weights, foreign productivity and foreign discount factor in each model is calibrated to hit the trade targets which are explainched in details in section two.
### Parameter Values

<table>
<thead>
<tr>
<th>Calibrated/Imposed parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Home Discount factor</td>
<td>$\beta$</td>
<td>0.99</td>
</tr>
<tr>
<td>Depreciation rate</td>
<td>$\delta$</td>
<td>0.025</td>
</tr>
<tr>
<td>Risk aversion</td>
<td>$\sigma$</td>
<td>2.00</td>
</tr>
<tr>
<td>Labour share</td>
<td>$\alpha$</td>
<td>0.70</td>
</tr>
<tr>
<td>Hours worked</td>
<td>$H$</td>
<td>1/3</td>
</tr>
<tr>
<td>Preference parameter</td>
<td>$\varrho$</td>
<td>calibrated so $H=1/3$</td>
</tr>
<tr>
<td>Risk premium elasticity (Others)</td>
<td>$\phi_B$</td>
<td>0.001</td>
</tr>
<tr>
<td>Price Substitution elasticity (Home/Foreign goods)</td>
<td>$\zeta$</td>
<td>7.00</td>
</tr>
<tr>
<td>Foreign Substitution elasticity (Export/Foreign goods)</td>
<td>$\mu_C = \mu_I$</td>
<td>1.50</td>
</tr>
<tr>
<td>Capital taxation rate</td>
<td>$\tau_r$</td>
<td>0.00</td>
</tr>
<tr>
<td>Substitution elasticity (Home/Foreign goods)</td>
<td>$\mu_C^* = \mu_I^*$</td>
<td>1.50</td>
</tr>
<tr>
<td>Standard deviation of shocks</td>
<td>$\sigma_i$</td>
<td>1.00</td>
</tr>
<tr>
<td>Government spending in Chile</td>
<td>$g_y$</td>
<td>0.12</td>
</tr>
<tr>
<td>Government spending in Canada</td>
<td>$g_y$</td>
<td>0.21</td>
</tr>
<tr>
<td>Government spending in Mexico</td>
<td>$g_y$</td>
<td>0.11</td>
</tr>
<tr>
<td>Copper taxation rate in Chile</td>
<td>$\tau_o$</td>
<td>0.11</td>
</tr>
<tr>
<td>Oil taxation rate in Canada</td>
<td>$\tau_o$</td>
<td>0.23</td>
</tr>
<tr>
<td>Oil taxation rate in Mexico</td>
<td>$\tau_o$</td>
<td>0.51</td>
</tr>
<tr>
<td>Home Bloc Imported share of Investment in Chile</td>
<td>$1 - w_I$</td>
<td>calibrated so $\text{exis}=0.149$</td>
</tr>
<tr>
<td>Copper output in Chile</td>
<td>$Y^O$</td>
<td>calibrated so $\text{exco}=0.145$</td>
</tr>
<tr>
<td>Home Bloc Imported share of Consumption in Canada</td>
<td>$1 - w_C$</td>
<td>calibrated so $\text{excs}=0.095$</td>
</tr>
<tr>
<td>Home Bloc Imported share of Consumption in Canada</td>
<td>$1 - w_C$</td>
<td>calibrated so $\text{excs}=0.087$</td>
</tr>
<tr>
<td>Oil output in Canada</td>
<td>$Y^O$</td>
<td>calibrated so $\text{exco}=0.057$</td>
</tr>
<tr>
<td>Home Bloc Imported share of Investment in Mexico</td>
<td>$1 - w_I$</td>
<td>calibrated so $\text{exis}=0.170$</td>
</tr>
<tr>
<td>Home Bloc Imported share of Consumption in Mexico</td>
<td>$1 - w_C$</td>
<td>calibrated so $\text{excs}=0.0716$</td>
</tr>
<tr>
<td>Oil output in Mexico</td>
<td>$Y^O$</td>
<td>calibrated so $\text{exco}=0.068$</td>
</tr>
<tr>
<td>Foreign Discount factor in Chile</td>
<td>$\beta^*$</td>
<td>calibrated so $\text{tb}=0.052$</td>
</tr>
<tr>
<td>Foreign Discount factor in Canada</td>
<td>$\beta^*$</td>
<td>calibrated so $\text{tb}=0.019$</td>
</tr>
<tr>
<td>Foreign Discount factor in Mexico</td>
<td>$\beta^*$</td>
<td>calibrated so $\text{tb}=0.031$</td>
</tr>
<tr>
<td>Foreign productivity in Chile</td>
<td>$A^*$</td>
<td>calibrated so $Y^*/Y=5.296$</td>
</tr>
<tr>
<td>Foreign productivity in Canada</td>
<td>$A^*$</td>
<td>calibrated so $Y^*/Y=1.22$</td>
</tr>
<tr>
<td>Foreign productivity in Mexico</td>
<td>$A^*$</td>
<td>calibrated so $Y^*/Y=5.56$</td>
</tr>
</tbody>
</table>

Table 4: Calibrated or Imposed Parameters for Home Bloc (Chile, Canada, Mexico)

### 4.3 Bayesian Estimation

We estimate the model using Bayesian approach in Dynare. This choice is driven by the widely recognised advantages of the Bayesian-Maximum Likelihood methodology, which are as follows. First, prior information about parameters available from empirical studies or previous macroeconomic studies, can be incorporated with the data in the estimation process. Second, it facilitates
## Table 5: Calibrated or Imposed Parameters for Foreign Bloc (US)

<table>
<thead>
<tr>
<th>Calibrated/Imposed parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foreign Discount factor</td>
<td>β</td>
<td>0.99</td>
</tr>
<tr>
<td>Depreciation rate</td>
<td>δ*</td>
<td>0.025</td>
</tr>
<tr>
<td>Hours worked</td>
<td>H*</td>
<td>1/3</td>
</tr>
<tr>
<td>Preference parameter</td>
<td>φ*</td>
<td>calibrated so H*=1/3</td>
</tr>
<tr>
<td>Government spending</td>
<td>g_y*</td>
<td>0.2</td>
</tr>
<tr>
<td>Capital taxation rate</td>
<td>τ*</td>
<td>0.00</td>
</tr>
</tbody>
</table>

representing and taking fuller account of the uncertainties related to models and parameter values. Third, it allows for a formal comparison between different miss-specified models that are not necessarily encapsulated in the marginal likelihood of the model. In addition, there has been a growing trend among central banks to employ Bayesian methods for conducting policy analysis.

In this model which is in fact a two block (ROW and SOE) economy, we need to have the ROW block estimated first and then call the posterior means of estimated parameters for the estimation of small open economy. We estimate directly the non-linear ROW model using the US dataset and the results are in the Appendices. As our aim is the specification of SOE model, so here we concentrate on the SOE estimation.

### 4.4 Estimation of Oil Price Shock and Copper Price Shock

According to the commodity section, the price of oil and copper (P^*_O,t) has an exogenous process in the model, so we have estimated the standard deviation of this shock separately by fitting an AR(1) process and then imposed the estimated coefficient and standard deviation to the model as the level of shock persistence in parameter block and as the standard deviation in shock block respectively:

<table>
<thead>
<tr>
<th>Dependent Variable: Oil</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method: Least Squares</td>
</tr>
<tr>
<td>Variable</td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>Oil(-1)</td>
</tr>
<tr>
<td>S.E. of regression</td>
</tr>
</tbody>
</table>

Table 6: Estimation of Oil Price Shock

<table>
<thead>
<tr>
<th>Dependent Variable: Copper</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method: Least Squares</td>
</tr>
<tr>
<td>Variable</td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>CO(-1)</td>
</tr>
<tr>
<td>S.E. of regression</td>
</tr>
</tbody>
</table>

Table 7: Estimation of Copper Price Shock
4.5 Prior distribution

In order to implement Bayesian estimation, it is first necessary to define prior distributions for the estimated parameters. The choice of priors for the estimated parameters is usually determined by theoretical implications and evidence from previous studies. However, since estimated DSGE models for emerging economies are limited, especially for Iran, we will choose relatively diffuse priors that cover a wide range of parameter values. The use of a diffuse prior reduces the importance of the mean of the prior distribution on the outcome of the estimation. In general, a beta distribution is used as a prior for fractions or probabilities, a gamma distribution is used for parameters in $\mathbb{R}^+$, inverse gamma distributions for variance parameters, and normal distributions are used when more informative priors seem to be necessary.

Table 15, 16 and 17 (columns 1 to 4) lists the prior distribution along with the prior mean and standard deviation of all the estimated parameters. For risk aversion parameter of non-Ricardian household ($\sigma_C$), in line with the literature to calibrate the Ricardian risk aversion parameter, we assume a normal distribution, with the prior mean of 2 and a standard deviation of 0.25.

Following Gabriel et al. (2010), the prior for proportion of RT consumers, we assume beta distribution with mean of 0.5 and standard deviation of 0.1.

To ensure that the consumption habit persistence, $\chi$, is bounded between 0 and 1, we assign a beta distribution, previous studies show mixed evidence regarding its value for developing countries. Castillo et al. (2008) estimate a large value in the range of 0.7 to 0.9 for Peru, whereas smaller values of 0.24 are estimated for India in Khera (2016). So we assign it a mean of 0.5, with a standard deviation of 0.1.

The prior for price indexation parameters are chosen the same as beta distribution with mean of 0.5 and standard deviation of 0.1.

We assume a normal distribution of mean 4 and standard deviation of 1.5 for investment adjustment cost ($\phi_I$) based on the calibrated values in Khera (2016) and Gabriel et al. (2010).

Previous estimates of central bank’s weight on inflation in the Taylor-rule, ranges from 1.27 in Carlstrom and Fuerst (2001) to 2.5 and 1.5 in Gabriel et al. (2010) and Khera (2016) respectively. In order to include these values within the 95 per cent confidence band, we assign a normal prior with a mean of 2.00 and standard deviation of 0.25 to $\theta_{\Pi}$. For the feedback parameter on GDP ($\theta_y$), GDP growth($\theta_d$) and depreciation rate ($\theta_{ds}$), we assume normal distribution of mean 0.10 and standard deviation of 0.05.

The lagged monetary policy coefficient ($\rho_M$), shows mixed results in the previous estimates. Castillo et al. (2008) find it to be 0.4 for Peru, whereas Gabriel et al. (2010) estimate significantly higher values of 0.8 for India. Based on these studies, a beta distribution with mean of 0.75 and standard deviation of 0.1 is assigned to $\rho_M$.

Regarding the priors for the parameters relating to shock processes, we use a beta distribution for the persistence of all shocks, where the mean is set at 0.5 with a standard deviation of 0.10. Given the lack of evidence regarding the sources of business cycle fluctuations, we adopt uninformative gamma distributions for standard deviations of all shocks, with prior mean of 0.10 along with a standard deviation of 0.20.
4.6 Identification Analysis

Fig. 6-11 presents the results of the identification tool provided by Dynare. Parameter identifiability gives an indication of the informativeness of the estimate, and is important for making sensible and meaningful inferences. The sources of identification failure could be from the model structure, or lack of information from the data. All the figures show that all parameters are identified, and are listed from left to right with increasing degree of identification strength.

5 Empirical Results

5.1 Posterior Estimates

Tables 15, 16 and 17 (column 5), states the posterior means of the Bayesian estimation along with the 95% confidence intervals. Overall, the parameter estimates are plausible. The economy has higher values for the volatility of shocks, which is consistent with the literature regarding open economies being more volatile in general. Below, we provide a detailed description of posterior estimates in the model.

Turning to the Calvo price stickiness, there is significant different between the two price regimes. Totally in the models of DCP the Calvo Price Stickiness has a higher estimated value in comparison with the PCP models which is estimated to be more flexible. This remarks the importance of imperfect exchange rate path-through setting.

Looking at price indexation parameters estimates, the estimation indicates that in the Canada model relative to Mexico and Chile the prices are less indexed to last period’s aggregate inflation. The non-Ricardian risk aversion is estimated less in Canada in comparison with the other two countries. The posterior mean estimate for investment adjustment cost in the model of Chile is estimated higher than the two the two countries.

In terms of the policy parameters, there is a high degree of policy inertia, however the estimate for output stabilization, implying that the central bank does not place too much importance on output fluctuations, also it does not react too strongly to exchange rate depreciation which indicate that the central bank is likely to choose a relatively less flexible exchange rate regime. On the other hand, we observe that the central bank responds more strongly to the inflation rate in comparison to GDP. We find a relatively less response of interest rate to movements in output growth as well.

Overall we get high estimated values for the standard deviation of investment, terms of trade and markup relative to other shocks in the economy, which ranges from 2 to 6 and the least value is estimated for government expenditures shock which can say the high role for the government expenditures that usually is get in closed economy models may simply be picking up open economy effects. The high values which are estimated for standard deviation of shocks are due to the data of economy which is highly vulnerable by different shocks. This can be improved upon by modelling the formation of priors in different approach, we can address this issue for example with endogenous priors. Note that higher estimate for Terms of trade shock in the PCP models in comparison to DCP models, suggesting that without imperfect exchange rate path through setting there will be even higher misspecifications in the rest of the models as important channels in the economy are shut down.
5.2 Testing for Convergence of MCMC

The multivariate diagnostics shown in figure 12-17 indicate that the chains converge to similar means and distributions. *Interval* refers to the interval measure, and $m_2$, $m_3$ refer to second and third order multivariate moment measures.\(^8\)

5.3 Model Fit to the Data

Can the model capture the underlying characteristics of the actual data? The model favoured in the space of competing models may still be poor (potentially misspecified) in capturing the important dynamics in the data. To further evaluate the absolute performance of our model against data, it is necessary to compare the model’s implied characteristics with those of the actual data. In this section to further illustrate how the estimated model captures the data statistics and persistence Bayesian estimation and validation of model in particular, we obtain the model-generated moments based on the real world data (i.e. posterior distribution) and compute these model-implied moments by solving the models at the posterior means obtained from estimation. After that we perform a variance decomposition exercise and finally, we analyse the business cycle implications of these features, by comparing the estimated posterior impulse responses to exogenous shocks across all specifications.

**Standard Moment Criteria**

In tables 19, 20 and 21, the results of the models’ second moments are compared with the second moments in the actual data to evaluate the models’ empirical performance for Chile, Canada and Mexico in two case of PCP and DCP. In terms of the standard deviations, the model is able to reproduce acceptable volatility for the main variables of the DSGE model, however relatively high volatility compared to the actual data. Tables 19, 20 and 21 also reports the cross-correlations of the seven observable variables vis-a-vis output. The model does roughly well at capturing the contemporaneous correlations observed in the data. In order to illustrate more how the estimated model captures the data statistics, we plot the autocorrelations up to order 10 of the actual data and those of the endogenous variables generated by the model variants in figures 18, 20 and 19. The following figure 21-25 depicts the mean responses corresponding to a positive one standard deviation monetary policy shock between different price regimes in each country and cross countries. The endogenous variables of interest are the observables in the estimation and each response is for a one period horizon. All impulse responses are computed simulating the vector of DSGE model parameters at the posterior mean values computed from estimation.

5.4 Bayesian Model Comparisons

First let us go back to Bayes Rule

$$p(\theta|y) \propto p(y|\theta)p(\theta)$$

\(^8\)Testing for convergence of the posterior distribution is notoriously difficult, and Dynare utilizes some indicative statistics, summarized by diagrams, as recommended by Brooks and Gelman (1998). These diagrams are made up of 3 multivariate figures, representing convergence indicators for all parameters considered together and 3 figures for each parameter, representing univariate convergence indicators.
So far we have not needed the unconditional density \( p(y) \) to maximize \( p(\theta|y) \) wrt \( \theta \). This is computed by integrating over the prior distribution to obtain

\[
p(y) = \int_{\Theta} p(y|\theta)p(\theta)d\theta
\]

For a particular model \( i \) from a number of alternatives, say \( m_i \), we can define a density conditional on this model

\[
p(y|m_i) = \int_{\Theta} p(y|\theta, m_i)p(\theta, m_i)d\theta
\]

where \( p(\theta, m_i) \) is the prior for that model. We refer to \( p(y|m_i) \) as the marginal likelihood associated with model \( m_i \).

Bayesian inference also allows a framework for comparing alternative and potentially misspecified models based on their marginal likelihood. For a given model \( m_i \in M \) and common dataset, the latter is obtained by integrating out vector \( \theta \),

\[
p(y|m_i) = \int_{\Theta} p(y|\theta, m_i)p(\theta|m_i)d\theta
\]

where \( p_i(\theta|m_i) \) is the prior density for model \( m_i \), and \( p(y|m_i) \) is the data density for model \( m_i \) given parameter vector \( \theta \). To compare models (say, \( m_i \) and \( m_j \)) we calculate the posterior odds ratio which is the ratio of their posterior model probabilities (or Bayes Factor when the prior odds ratio, \( p(m_i)p(m_j) \), is set to unity):

\[
PO_{i,j} = \frac{p(m_i|m)}{p(m_j|m)} = \frac{p(y|m_i)p(m_i)}{p(y|m_j)p(m_j)} \quad (112)
\]

\[
BF_{i,j} = \frac{p(y|m_i)}{p(y|m_j)} = \frac{\exp(LL(y|m_i))}{\exp(LL(y|m_j))} \quad (113)
\]

defining the log-likelihood

\[
LL(y|m_i) \equiv \log(p(y|m_i))
\]

noting that \( x = \exp(\log x) \). Components (112) and (113) provide a framework for comparing alternative and potentially misspecified models based on their marginal likelihood. Such comparisons are important in the assessment of rival models.

Given Bayes factors we can easily compute the model probabilities \( p_1, p_2, \ldots, p_n \) for \( n \) models. Since \( \sum_{i=1}^{n} p_i = 1 \) we have that

\[
\frac{1}{p_1} = \sum_{i=2}^{n} BF_{i,1}
\]

from which \( p_1 \) is obtained. Then \( p_i = p_1 BF(i, 1) \) gives the remaining model probabilities. We have provided a MATLAB programme which computes these probabilities give the the log-likelihood values from the competing models. The following table provides a formal Bayesian comparison of the benchmark SOE model (Model 1, with the proportion \( \lambda \) of
ROT consumers freely estimated) with a restricted model without ROT consumers (Model 2, with $\lambda = 0$), suing both the first and second stage log-likelihood (preferable, as it pertains to the recovered posterior density).

<table>
<thead>
<tr>
<th></th>
<th>Chile DCP</th>
<th>Chile PCP</th>
</tr>
</thead>
<tbody>
<tr>
<td>LLs (1st stage)</td>
<td>-622.0070</td>
<td>-645.9085</td>
</tr>
<tr>
<td>prob.</td>
<td>1.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>LLs (2nd stage)</td>
<td>-622.5899</td>
<td>-646.2603</td>
</tr>
<tr>
<td>prob.</td>
<td>1.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Table 8: Marginal Log-likelihood Values and Posterior Chile Model

<table>
<thead>
<tr>
<th></th>
<th>Mexico DCP</th>
<th>Mexico PCP</th>
</tr>
</thead>
<tbody>
<tr>
<td>LLs (1st stage)</td>
<td>-728.9487</td>
<td>-782.9983</td>
</tr>
<tr>
<td>prob.</td>
<td>1.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>LLs (2nd stage)</td>
<td>-730.1270</td>
<td>-785.5000</td>
</tr>
<tr>
<td>prob.</td>
<td>1.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Table 9: Marginal Log-likelihood Values and Posterior Mexico Model

<table>
<thead>
<tr>
<th></th>
<th>Canada DCP</th>
<th>Canada PCP</th>
</tr>
</thead>
<tbody>
<tr>
<td>LLs (1st stage)</td>
<td>-728.9487</td>
<td>-782.9983</td>
</tr>
<tr>
<td>prob.</td>
<td>1.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>LLs (2nd stage)</td>
<td>-730.1270</td>
<td>-785.5000</td>
</tr>
<tr>
<td>prob.</td>
<td>1.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Table 10: Marginal Log-likelihood Values and Posterior Canada Model

Our model comparison analysis suggests that setting the imperfect exchange rate path through in the model seems to be significantly relevant and does improve model fit remarkably.

5.5 Variance Decomposition of Business Cycle Fluctuations

What are the driving forces of the observed business cycle fluctuations? what are the impacts of the structural shocks on the main macroeconomic time series? To address these questions this section investigates the contribution of each of the structural shocks to the variance of endogenous variables in the model, i.e. the underlying sources of fluctuations, which explains how important a shock is in business cycle dynamics.

Overall, the results of the variance decomposition are in line with the business cycle literature and VAR studies for emerging economies. The most significant observation in all
models regardless of the producer or dominant currency price regimes is the considerable role of foreign monetary policy shock in explaining the dynamics of the model rather than the other forces even in comparison with the domestic drivers. The second noticeable result is the disturbances from the terms of trade shock which is the most important at explaining the dynamics of endogenous variable in the producer currency price regimes rather than it’s negligible disturbances in the models of dominant currency pricing. It is followed by the price markup, then technology shocks and with relatively smaller contribution by the monetary policy, commodity price, investment and the other foreign shocks.

The variation in total output, is mainly explained by price markup and technology shocks, followed by foreign monetary policy, terms of trade and investment shocks. The role of other shocks are relatively very small. Aggregate consumption follow the same contribution but considerably more share of foreign monetary policy shock. Movements in the imported consumption, investment and inflation, massively explained by the foreign interest rate shock but the domestic ones are mostly explained by markup, investment and technology shocks respectively.

Looking at quantity of exports and imports, relative prices of exports and imports and the Terms of trade, the variations are more explained by foreign monetary shock which in the models of DCP has more contributions than that of PCP regimes but still dominates the other forces. In the PCP models, the decline in the share of foreign monetary shock is instead explained by the terms of trade and markup shocks.

In sum, in the model economy the key contribution of the variation in the economy is for the foreign interest rate which dominates in the variance decomposition matrix and the contrast between the two model of pricing is best explained with the terms of trade shock which almost dominate the VDM matrix in all PCP models.

5.6 Comparison of Results for PCP and DCP

In this sub-section we present impulse responses to a monetary policy shock to contrast the responses under different pricing regimes of producer currency pricing and dominant currency pricing and between the three case study. As the irfs reveals, there are different implications for exchange rate pass through, the terms-of-trade and the volume of export and import under the different currency pricing regimes and across countries.

Figures 1-5 plot the impulse response to a positive one percent exogenous increase in domestic and foreign interest rates. In each plot we contrast the response under the regimes of DCP and PCP in three country of Chile, Canada and Mexico.

Exchange rate and Inflation: Following the monetary shock, domestic interest rates
increase and as the exchange rate appreciates reducing inflationary pressures on the economy. As seen in Figure 1-3 the decrease in inflation in the case of PCP exceeds that of DCP since exchange rate movements have a smaller impact on the domestic prices when export prices are sticky in foreign currency. These results are in line with Casas et al. (2017).

**Terms-of-Trade:** The exchange rate appreciation is associated with almost appreciation of the terms-of-trade in the case of PCP. Distinctively, in the case of DCP the terms-of-trade depreciates but for the case of Canada (figure 3) it depreciates negligibly and remains stable at medium horizons. Looking at figure 5 the deprecation of terms of trade under foreign monetary Policy shock or foreign interest rate increase is quite small considering that for Chile it is more volatile in comparison with the case of Canada and Chile. In sum, one could conclude that in our model unlike Casas et al. (2017) export and import prices are not necessarily stable in the dominant currency and from country to country it differs.

**Exports and Imports:** Under DCP, the relative home currency price of exports and imports fall with the exchange rate appreciation as depicted in Figures 1-3. This in turn generates a significant rise in imports, despite the contractionary effect of monetary policy, and only a modest increase in exports (Figures 1-3) is in contrast with the PCP benchmark that generates a large decrease in exports (from the demand contraction). The increase in imports in the case of DCP is much higher than that under PCP because of export contraction under PCP. So the exchange rate pass-through of non-dominant currencies is negligible and while appreciations have a limited contractionary impact on exports, expenditure switching still occurs through imports, arising from fluctuations in the relative price of imported to domestic goods. In turn, these are driven by movements in a country’s exchange rate relative to the dominant currency. In the case of exports, in contrast to PCP, which associates exchange rate appreciations with decreases in quantities exported, DCP predicts a negligible impact on goods exported to the dominant currency destination.

**Output:** Comparing Figures 1-5, the inflation-output trade off in response to domestic monetary policy worsens under DCP relative to PCP. In line with Casas et al. (2017), we find that the inflation-output trade off in response to a monetary policy shock (under an inflation targeting monetary rule) worsens under DCP relative to PCP. That is, a monetary rate contraction, decreases inflation by much more than it reduces output, as compared to PCP. As depicted in Figure 1-3 the expansionary impact of foreign increase of interest rate on output is muted under DCP relative to PCP. Under DCP there is an expenditure switching effect from imports towards domestic output, while DCP misses out the expansionary impact on exports under PCP.

**Cross Country Comparison:** Among these three economies, with respect to the foreign
Monetary policy shock, the main observation is the different reactions of irfs for Chile in comparison to Canada and Mexico which is not the case for the domestic monetary policy. Regarding the main contributions of Casas et al. (2017), the estimated model of Canada is the only model which is most in line with their findings.

6 Optimal Policy

This section first considers the general optimal policy problem where the policymaker has a number of instruments and sets out to maximize a general discounted welfare criterion subject to the constraints of a DSGE model. If the policymaker is able to commit, the setting of instruments can be conducted in terms of the ex ante optimal policy. If the expected discounted household utility is chosen as the welfare criterion this becomes the well-known Ramsey problem. A problem with such a solution is that it involves a complex rule even for quite simple NK models. Much of the optimal policy literature therefore focuses on simple Taylor-type commitment rules that are optimized so as to come close to mimicking the Ramsey solution and this is the approach of this paper. In the absence of an ability to commit the policymaker must set policy to be time-consistent.

We consider a model as a special case of the following general setup in non-linear form

\[
Z_t = J(Z_{t-1}, X_t, w_t, \epsilon_t) \quad (114)
\]

\[
E_tX_{t+1} = K(Z_t, X_t, w_t) \quad (115)
\]

where \(Z_t, X_t\) are \((n - m) \times 1\) and \(m \times 1\) vectors of backward and forward-looking variables, respectively, \(\epsilon_t\) is a \(\ell \times 1\) i.i.d shock variable and \(w_t\) is an \(r \times 1\) vector of instruments. Under perfect information all variables dated \(t\) or earlier are observed at time \(t\) including shocks.

Now define

\[
y_t = \begin{bmatrix} Z_t \\ X_t \end{bmatrix}
\]

Then, as in Dynare User Guide, chapter 7, (114) and (115) can be written

\[
E_t[f(y_t, y_{t+1}, y_{t-1}, w_t, \epsilon_{t+1})] = 0 \quad (116)
\]

\[
E_t[\epsilon_{t+1}] = 0
\]

\[
E_t[\epsilon_{t+1}'] = \Sigma_{\epsilon}
\]

This is quite general in that \(y_t\) can be enlarged to include lagged and forward-looking variables.

The general problem is to maximize at time \(0\), \(\Omega_0 = E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(y_t, y_{t-1}, w_t) \right]\) subject to (116) given initial values \(Z_0\).
6.1 Optimized Simple Rules

Optimal policy in the form of the Ramsey solution can be expressed as $w_t = f(Z_t, \lambda_{2,t})$. This poses problems for the implementability of policy in terms of complexity and the observability of elements of $Z_t$ (such as the technology process $A_t$, but more importantly $\lambda_{2,t}$). The macroeconomic policy literature therefore often focuses on simple rules, using the Ramsey solution as a benchmark.

The general optimal policy problem seeks an optimized simple rule in which the vector of instruments $w_t$ respond to an observed subset of macroeconomic variables in a prescribed (for example log-linear) fashion. All our rules take the log-linear form

$$\log w_t = D \log y_t$$

where we define $\log w_t \equiv [\log w_{1,t}, \log w_{2,t}, \cdots, \log w_{r,t}]'$ over $r$ instruments, and similarly for $\log y_t$, and the matrix $D$ selects a subset of $y_t$ from which to feedback. Again this is quite general in that $y_t$ can be enlarged to include lagged and forward-looking variables.

The optimized simple rules then defines the inter-temporal welfare time $t$, sets steady-state values for instruments $w_t$, denoted by $w_t$, computes a second-order solution for a particular setting of $w$ and solves the maximization problem at $t = 0$,

$$\max_{w,D} \Omega_0(Z_0, w, D)$$

given initial values $Z_0$. In a purely stochastic problem we put $Z_0 = Z$, the steady state of $Z_t$, maximizing the conditional welfare at the steady state. In a purely deterministic problem there is no exogenous uncertainty and the optimization problem is driven by the need to return from $Z_0$ to its steady state, $Z$. We now examine optimal monetary policy conducted in terms of the Ramsey solution, discretion and either of two simple Taylor interest rate rules used up to now, (76) and (82) which are special cases of (117). To allow for the possibility that $\rho_r = 1$, we re-parameterize the feedback coefficients by setting $\alpha_{\pi} = (1 - \rho_r) \theta_{\pi}$, $\alpha_{y} = (1 - \rho_r) \theta_{y}$ and optimizing with respect to $\rho_r$, $\alpha_{\pi}$ and $\alpha_{y}$.

To proceed we write the inter-temporal welfare at time $t$ as

$$\Omega_t = E_t \left[ \sum_{\tau=0}^{\infty} \beta^\tau U_{t+\tau} \right]$$

where the $U_t = U_t(C_t, H_t)$ is the household’s single-period utility function.

Suppose there is no growth in the steady state. Then (119) can be programmed in Bellman form as

$$\Omega_t = U_t + \beta E_t [\Omega_{t+1}]$$

It is now established that the Ramsey-solution to NK models such as ours sets $\Pi = 1$ in the
steady state. Optimized rules then set \( \Pi = 1 \) and optimize a second-order approximation of the mean of \( \Omega_t \) over \( \rho_r, \theta_\pi \) and \( \theta_{r,y} \). In what follows we focus on the purely stochastic problem (as defined above) and therefore start at the steady state.

### 6.2 ZLB Considerations

We can impose a zero lower bound (ZLB) effect on the nominal interest rate by modifying the discounted loss criterion as follows.\(^9\) Consider first the ZLB constraint on the nominal interest rate. Rather than requiring that the gross rate \( R_t \geq 1 \) for any realization of shocks, we impose the constraint that the mean gross rate should at least be \( k \) standard deviation above the ZLB. Again, for analytical convenience we use discounted averages.

Define \( \bar{R} \equiv E_0 \left[ (1 - \beta) \sum_{t=0}^{\infty} \beta^t R_t \right] \) to be the discounted future average of the nominal interest rate path \( \{ R_t \} \). Our ‘approximate form’ of the ZLB constraint is a requirement that \( \bar{R} \) is at least \( k \) standard deviations above the zero lower bound; i.e., using discounted averages that

\[
\bar{R} \geq k \text{sd}(R_t) = k \sqrt{\bar{R}^2 - (\bar{R})^2} \tag{120}
\]

Squaring both sides of (120) we arrive at

\[
E_0 \left[ (1 - \beta) \sum_{t=0}^{\infty} \beta^t R_t^2 \right] \leq K_r \left[ E_0 \left[ (1 - \beta) \sum_{t=0}^{\infty} \beta^t R_t \right] \right]^2 \tag{121}
\]

where \( K_r = 1 + k^{-2} > 1 \). We can write this as two sufficient constraints

\[
E_0 \left[ (1 - \beta) \sum_{t=0}^{\infty} \beta^t R_t^2 \right] \leq m \\
E_0 \left[ (1 - \beta) \sum_{t=0}^{\infty} \beta^t R_t \right] \geq \sqrt{\frac{m}{K_r}}
\]

which is equivalent to adding \( E_0(1 - \beta)[w_r \sum_{t=0}^{\infty} \beta^t R_t^2 + \mu_r \sum_{t=0}^{\infty} \beta^t R_t] = w_r E_0(1 - \beta) \sum_{t=0}^{\infty} \beta^t(R_t - \frac{\mu_r}{w_r})^2 - \) a constant, where \( w_r, \mu_r > 0 \) are Lagrange multipliers, to the Lagrangian of the optimization problem.

Thus imposing the constraint that the ZLB is hit with only a given low probability per period \( p \) is equivalent to shifting the steady state interest rate to a new higher target \( R_{n,t}^* \equiv \frac{\mu_r}{w_r} \) and lowering the variance about this new steady state by increasing \( w_r \) (see Levine et al. (2008)).

\(^9\)This follow the treatment of the ZLB in Woodford (2003) and Levine et al. (2008)
6.3 Results

In the results we impose the ZLB constraint without increasing the inflation and nominal interest rate in the steady state. The optimized rule is found by optimizing a modified intertemporal welfare

\[ \Omega_{t}^{\text{mod}} = U_t - w_r(R_t - R)^2 + \beta\mathbb{E}_t\left[\Omega_{t+1}^{\text{mod}}\right] \]

By increasing the penalty parameter \(w_r\) the variance of the nominal interest rate is lowered thus decreasing the size of the tail of the distribution and the probability of \(R_t < 1\).

The Taylor-type rules given by (76) and (82) for the chosen SOE and ROW respectively are re-parameterized as:

\[ \log \left( \frac{R_t}{R} \right) = \rho_r \log \left( \frac{R_{t-1}}{R} \right) + \alpha_\pi \log \left( \frac{\Pi_{t-1,t}}{\Pi} \right) + \alpha_s \log \left( \frac{\Pi_{S,t-1,t}}{\Pi_s} \right) + \alpha_y \log \left( \frac{Y_t}{Y} \right) + \alpha_{dy} \log \left( \frac{Y_{t-1}}{Y} \right) + \alpha_{ds} \log \left( \frac{Y_{S,t-1}}{Y_{S,t-1}} \right) + \epsilon_{M,t} \quad (122) \]

\[ \log \left( \frac{R^*_t}{R^*_s} \right) = \rho_r \log \left( \frac{R_{t-1}^*}{R^*_s} \right) + \alpha_\pi^* \log \left( \frac{\Pi_{t-1,t}^*}{\Pi^*} \right) + \alpha_s^* \log \left( \frac{\Pi_{S,t-1,t}^*}{\Pi_s^*} \right) + \alpha_y^* \log \left( \frac{Y_t^*}{Y^*} \right) + \alpha_{dy}^* \log \left( \frac{Y_{t-1}^*}{Y_{t-1}^*} \right) + \epsilon_{M^*,t} \quad (123) \]

Then optimized rules with a ZLB constraint are found by maximizing (6.3) with respect to the feedback parameters \(\rho_r, \alpha_\pi, \alpha_y, \alpha_{dy}\) and \(\alpha_{ds}\) for each of the three SOE and \(\rho_r, \alpha_\pi, \alpha_y\) and \(\alpha_{dy}\) for the ROW, subject to the estimated model (apart from the monetary rule). Tables 11–14 set out the results for Chile, Mexico, Canada and the ROW respectively.

In these Tables the true welfare given by the mean of \(\Omega_t\) in a second-order perturbation approximation as in (119) is reported. Then the consumption equivalent gain from stabilization subject to the ZLB constraint, \(ce\), is reported in consumption equivalent terms. For this first define a consumption equivalent defined as the steady state of the welfare increase from a permanent increase of 1% in consumption, i.e., \(CE_t = U_t(1.01C_t, H_t) - U_t(C_t, H_t)/(1 - \beta) \times 100\%\). Then in the tables we define \(ce = \Omega^{Opt}_t - \Omega^{Est}_t\)/\(CE\) (%) for each country and currency pricing regime.

<table>
<thead>
<tr>
<th>(w_r)</th>
<th>(\rho_r)</th>
<th>(\alpha_\pi)</th>
<th>(\alpha_y)</th>
<th>(\alpha_{dy})</th>
<th>(\alpha_{ds})</th>
<th>SD((R_n))</th>
<th>Pr(ZLB)</th>
<th>(\Omega)</th>
<th>(CE)</th>
<th>(ce(%))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Est (PCP)</td>
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<td>0.39</td>
<td>0.0045</td>
<td>-51.8139</td>
<td>117.69</td>
</tr>
<tr>
<td>Opt (DCP)</td>
<td>250</td>
<td>1.0000</td>
<td>0.0963</td>
<td>0.0000</td>
<td>0.0251</td>
<td>0.0000</td>
<td>0.32</td>
<td>0.0007</td>
<td>-52.5326</td>
<td>107.10</td>
</tr>
</tbody>
</table>

Table 11: Estimated and Optimized Rule with a ZLB for Chile
Two main results stand out from these results. First, the welfare gains from optimal stabilization subject to the ZLB constraint in our estimated model are small and of the order of the costs of the business cycle found by in the seminal study by Lucas (1987). But these are subject to the ZLB constraint in which we have not allowed a shift in the steady state inflation and nominal interest rate. Indeed taking account the need to respect this constraint which is not respected in the estimated models leads to a negative “gain”. Allowing for an increase in the steady state inflation rate and introducing other frictions (such as wage stickiness, financial frictions) will increase the these gains from optimized rules.

Second, the optimized rules in all countries and for both PCP and DCP currency pricing regimes take the form of an optimized monetary policy close to a price-level rule. Indeed in one case, Canada assuming PCP, the optimized policy is exactly a price level rule. In all cases optimization rules out exchange-rate targeting. The benefits of price-level targeting versus inflation targeting have been studied in the literature now for some time. (See, for example, Svensson (1999), Schmitt-Grohe and Uribe (2000), Vestin (2006), Gaspar et al. (2010), Giannoni (2014), Deak et al. (2019)). These papers examine the good determinacy/stability and robustness properties of price-level targeting. Holden (2016) shows these benefits extend to a ZLB setting. Our paper shows that these results

### Table 12: Estimated and Optimized Rule with a ZLB for Mexico

<table>
<thead>
<tr>
<th>$w_r$</th>
<th>$\rho_r$</th>
<th>$\alpha_\pi$</th>
<th>$\alpha_\pi$</th>
<th>$\alpha_\pi$</th>
<th>$\alpha_\pi$</th>
<th>SD$(R_n)$</th>
<th>Pr(ZLB)</th>
<th>$\Omega$</th>
<th>CE</th>
<th>ce(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Est (PCP)</td>
<td>n.a.</td>
<td>0.7395</td>
<td>2.0540</td>
<td>-0.0221</td>
<td>0.1024</td>
<td>0.0601</td>
<td>0.86</td>
<td>0.1190</td>
<td>-88.2426</td>
<td>188.82</td>
</tr>
<tr>
<td>Est (DCP)</td>
<td>n.a.</td>
<td>0.7610</td>
<td>2.0017</td>
<td>-0.0417</td>
<td>0.1282</td>
<td>0.0662</td>
<td>0.66</td>
<td>0.0618</td>
<td>-84.4402</td>
<td>175.41</td>
</tr>
<tr>
<td>Opt (PCP)</td>
<td>500</td>
<td>1.0000</td>
<td>0.0503</td>
<td>0.0000</td>
<td>0.0149</td>
<td>0.0000</td>
<td>0.34</td>
<td>0.0014</td>
<td>-81.2144</td>
<td>188.82</td>
</tr>
<tr>
<td>Opt (DCP)</td>
<td>500</td>
<td>1.0000</td>
<td>0.0586</td>
<td>0.0000</td>
<td>0.0257</td>
<td>0.0000</td>
<td>0.24</td>
<td>0.0001</td>
<td>-79.2816</td>
<td>175.41</td>
</tr>
</tbody>
</table>

### Table 13: Estimated and Optimized Rule with a ZLB for Canada

<table>
<thead>
<tr>
<th>$w_r$</th>
<th>$\rho_r$</th>
<th>$\alpha_\pi$</th>
<th>$\alpha_\pi$</th>
<th>$\alpha_\pi$</th>
<th>$\alpha_\pi$</th>
<th>SD$(R_n)$</th>
<th>Pr(ZLB)</th>
<th>$\Omega(w_r)$</th>
<th>CE</th>
<th>ce(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Est (PCP)</td>
<td>n.a.</td>
<td>0.8611</td>
<td>1.1402</td>
<td>-0.0178</td>
<td>0.0713</td>
<td>-0.0198</td>
<td>0.86</td>
<td>0.1190</td>
<td>-64.5516</td>
<td>94.07</td>
</tr>
<tr>
<td>Est (DCP)</td>
<td>n.a.</td>
<td>0.8713</td>
<td>1.7251</td>
<td>0.0222</td>
<td>0.1227</td>
<td>0.0137</td>
<td>0.57</td>
<td>0.0375</td>
<td>-93.1547</td>
<td>181.53</td>
</tr>
<tr>
<td>Opt (PCP)</td>
<td>50</td>
<td>1.0000</td>
<td>0.0650</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.19</td>
<td>0.0001</td>
<td>-65.2872</td>
<td>94.07</td>
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<tr>
<td>Opt (DCP)</td>
<td>50</td>
<td>1.0000</td>
<td>0.0663</td>
<td>0.0000</td>
<td>0.0251</td>
<td>0.0000</td>
<td>0.26</td>
<td>0.0001</td>
<td>-92.0126</td>
<td>181.53</td>
</tr>
</tbody>
</table>

### Table 14: Estimated and Optimized Rule with a ZLB for ROW

<table>
<thead>
<tr>
<th>$w_r$</th>
<th>$\rho_r$</th>
<th>$\alpha_\pi$</th>
<th>$\alpha_\pi$</th>
<th>$\alpha_\pi$</th>
<th>$\alpha_\pi$</th>
<th>SD$(R_n)$</th>
<th>Pr(ZLB)</th>
<th>$\Omega(w_r)$</th>
<th>CE</th>
<th>ce(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Est</td>
<td>n.a.</td>
<td>0.7842</td>
<td>2.5572</td>
<td>0.0051</td>
<td>0.1869</td>
<td>0.45</td>
<td>0.0119</td>
<td>-97.0050</td>
<td>154.58</td>
<td>0</td>
</tr>
<tr>
<td>Opt</td>
<td>100</td>
<td>1.0000</td>
<td>0.1832</td>
<td>0.0000</td>
<td>0.0670</td>
<td>0.22</td>
<td>0.0001</td>
<td>-104.6030</td>
<td>154.58</td>
<td>-0.049</td>
</tr>
</tbody>
</table>
for a closed economy carry over to an open-economy setting. The intuition for the benefits of price-targeting is as follows: faced with an unexpected temporary rise in inflation, price-level stabilization commits the policymaker to bring inflation below the target in subsequent periods. In contrast, with inflation targeting, the drift in the price level is accepted.

7 Conclusions

In this study, we aim to present an analysis of the empirical relevance of imperfect exchange rate path-through in the context of domestic and foreign monetary policy. For this, we build a comprehensive two-country DSGE model with limited asset market participation which interacts with the rest of the world (ROW) in a framework of new Keynesian model in two price regimes of PCP and DCP. We also estimate our proposed models for different economies, the ROW model on US data and SOE model on Canada, Mexico and Chile data respectively using Bayesian estimation techniques. This provides not only a better understanding about the consequences of shocks that generate fluctuations in the exchange rate on different small open economies but also an interesting international comparison cross countries as well. Our findings can be summarized as following:

First, the striking result is that in a likelihood race DCP beats PCP for all countries with a huge marginal Likelihood differences. So our model comparison analysis suggests that setting the imperfect exchange rate path through in the model seems to be significantly relevant and does improve model fit remarkably.

Second, by analysing the variance decomposition of business cycle fluctuations, the dominant role of foreign monetary shock in overall fluctuations are seen which is followed by the markup and technology shocks. Another observation across all variables is the significant contribution of the terms of trade shocks, which does seem to matter for the variability observed in the PCP models and is remarkably dampened in the DCP models. This is due to the fact that when export and import prices are set in the foreign currency then the terms of trade will be less affected by any consequences of shocks that generate fluctuations in the exchange rate or currency movements.

Third, by the IRFs, we noticed that there are different implications for exchange rate pass through, the terms-of-trade and the volume of export and import under the different currency pricing regimes and across countries. Following the domestic monetary shock, the variation in inflation in the case of PCP exceeds that of DCP since exchange rate movements have a smaller impact on the domestic prices when export prices are sticky in foreign currency. Regarding terms of trade, the exchange rate movement is associated with almost movement of the terms-of-trade in the case of PCP, but in the case of DCP the movement of terms-of-trade is inverse and only for the case of Canada it is almost
negligible and remains stable at medium horizons, so the export and import prices are not necessarily stable in the dominant currency and from country to country it differs. Under DCP relative to PCP, the relative home currency price of exports and imports significantly change with the exchange rate movements. This in turn generates a significant change in imports, and only a modest change in exports which is in contrast with the PCP benchmark that generates a large variation in exports. So the exchange rate pass-through of non-dominant currencies is negligible and while appreciations have a limited contractionary impact on exports, expenditure switching still occurs through imports, arising from fluctuations in the relative price of imported to domestic goods. In turn, these are driven by movements in a country’s exchange rate relative to the dominant currency. We also find that the inflation-output trade off in response to a monetary policy shock worsens under DCP relative to PCP. That is, a monetary policy contraction, decreases inflation by much more than it reduces output, as compared to PCP. Concerning the expansionary impact of foreign interest rate on output, it is muted under DCP relative to PCP. Under DCP there is an expenditure switching effect from imports towards domestic output, while DCP misses out the expansionary impact on exports under PCP. Finally removing DCP setting from export sector affect the variations of export, trade balance and export, import prices. Hence, we find that there are important propagation channels active in the emerging economies which taking into account these features is essential for any policy-related study.

Taken together, these empirical findings imply that a weakening of emerging market currencies relative to the dominant (dollar) currency following, say, a monetary policy expansion in the former or a decline in commodity prices, will be associated with a decline in SOE trade (exports plus imports) relative to PCP and a weakening of dominant currencies (dollar) relative to the domestic currency following, a foreign monetary policy expansion in the ROW or a decline in foreign commodity prices, will be associated with an increase not only on imports but also on export in DCP model which lead to the increase of trade in the SOE relative to PCP. By a cross-country comparison of posterior estimated IRFs between our three case study, one can conclude that almost all the models are in line with the main contributions of Casas et al. (2017) except for the stability property of the terms of trade in the DCP economy which is only held in the model of Canada. The second is the different behaviour of IRFs for Chile in comparison with Canada and Mexico to the foreign monetary policy shock which is not held for the domestic monetary policy.

Finally our policy analysis shows that the good determinacy/stability and robustness properties of price-level targeting found in the literature for the close economy extends to an open-economy setting.

Future work will extend the study to a SOE interacting with large economies represented as the US and the Euro-Zone. We will then extend the study of this paper along the following
dimensions: Firstly, we will nest three rather than two different pricing paradigms: local currency pricing and dominant currency pricing (in dollars) alongside producer currency pricing. Secondly, the production function will use capital but also and intermediate inputs produced domestically and abroad. Thirdly, we will allow for strategic complementarity in pricing using a Kimball aggregator as in Kimball (1995) and Dotsey and King (2005) that gives rise to variable mark-ups, as opposed to constant mark-ups. Fourthly we will introduce wage stickiness and capacity utilization thus making our model an open-economy counterpart of Smets and Wouters (2007). Fifthly, our current model overestimates the variances observed in the data. This suggests that using “endogenous priors” as in Del Negro and Schorfheide (2008) instead of imposed ones will improve our model fit. Finally we will extend the policy analysis to other targeting rules including the nominal wage (as in Levine et al. (2008)).

References


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<th>Output</th>
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<th>Interest Rate</th>
<th>Real Exchange Rate</th>
<th>Terms of Trade</th>
<th>Export Quantity</th>
<th>Relative Export Price</th>
<th>Import Quantity</th>
<th>Relative Import Price</th>
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<tr>
<td>Terms of Trade</td>
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</tbody>
</table>

Figure 1: Impulse Response to a Domestic (upper) and foreign (Lower) Monetary policy shock- Chile Case
Figure 2: Impulse Response to a Domestic (upper) and foreign (Lower) Monetary policy shock- Mexico Case
Figure 3: Impulse Response to a Domestic (upper) and foreign (Lower) Monetary policy shock- Canada Case
Figure 4: Impulse Response to a Domestic (upper) and foreign (Lower) Monetary policy shock- PCP Comparison
Figure 5: Impulse Response to a Domestic (upper) and foreign (Lower) Monetary policy shock- DCP Comparison
A Summary of Dynamic Two-bloc Model. Producer Currency Pricing Case

Home country bloc

Production, capital law of motion, capital and labour demand

\[ Y_t^W = (A_t H_t)^{\alpha} K_{t-1}^{1-\alpha} \]  
\[ Y_t = \frac{Y_t^W}{\Delta_t} \]  
\[ K_t = (1 - \delta)K_{t-1} + (1 - S(X_t))I_t \]  
\[ r_t^K = (1 - \alpha) \frac{Y_t^W}{K_{t-1}} MC_t \frac{P_{H,t}}{P_t} \]  
\[ W_t = \alpha \frac{Y_t^W}{H_t} MC_t \frac{P_{H,t}}{P_t} \]  
\[ R_t^K = \frac{r_t^K (1 - \tau_k) + (1 - \delta)Q_t}{Q_{t-1}} \]  

Price Setting

\[ 1 = \xi (\Pi_{H,t-1,t})^{\zeta-1} + (1 - \xi) \left( \frac{JJ_t}{J_t} \right)^{1-\zeta} \]  
\[ \Delta_t = \xi (\Pi_{H,t-1,t})^{\zeta} \Delta_{t-1} + (1 - \xi) \left( \frac{JJ_t}{J_t} \right)^{-\zeta} \]  
\[ JJ_t = \frac{\zeta}{\zeta - 1} \frac{P_{H,t}}{P_t} Y_t MS_t MC_t + \xi E_t \left[ \Lambda_{t,t+1} (\Pi_{H,t,t+1})^{\zeta} JJ_{t+1} \right] \]  
\[ J_t = \frac{P_{H,t}}{P_t} Y_t + \xi E_t \left[ \Lambda_{t,t+1} (\Pi_{H,t,t+1})^{\zeta} J_{t+1} \right] \]  
\[ E_t \left[ \Lambda_{t,t+1} R_{t+1}^K \right] = E_t \left[ \Lambda_{t,t+1} \frac{J_{t+1}}{\Pi_{t,t+1}} \right] R_t = 1 \]  
\[ Q_t (1 - S(X_t) - X_t S'(X_t)) + E_t \left[ \Lambda_{t,t+1} Q_{t+1} S'(X_{t+1}) \frac{T_{t+1}^2}{T_t^2} \right] = \frac{P_t}{P_t} \]  
\[ S(X_t) \equiv \phi_X (X_t - X)^2 \]  
\[ S'(X_{t+1}) \equiv 2 * \phi_X (X_t - X) \]

Household labour supply, Euler equation

\[ U_{C,t}^R = (1 - \varrho) \left( C_{t}^R - \chi C_{t-1}^R \right)^{(1-\varrho)(1-\sigma)-1} (1 - H_t^R)^{\varrho(1-\sigma)} \]  
\[ \Lambda_{t,t+1} \equiv \beta \frac{U_{C,t+1}^R}{U_{C,t}^R} \]
\[
\frac{\varrho}{1 - \varrho} C_t^C - \chi C_t^C = W_t (1 - \tau_t^w)
\] (A.17)

\[
\frac{\varrho}{1 - \varrho} C_t^R - \chi C_t^R = W_t (1 - \tau_t^w)
\] (A.18)

\[
H_t = \lambda H_t^C + (1 - \lambda) H_t^R
\] (A.19)

\[
1 = E_t \left[ \frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}} \right] R_t
\] (A.20)

**Household consumption and investment**

\[
C_t^R = \frac{1}{1 - \lambda} C_t - \frac{\lambda}{1 - \lambda} C_t^C
\] (A.21)

\[
C_t^C = W_t H_t^C
\] (A.22)

\[
C_t = \frac{C_{H,t}}{w_C} \left( \frac{P_{H,t}}{P_t} \right)^{-\mu_C}
\] (A.23)

\[
C_{F,t} = (1 - w_C) \left( \frac{P_{F,t}}{P_t} \right)^{-\mu_C} C_t
\] (A.24)

\[
C_{H,t} = Y_t - I_{H,t} - EX_t - G_t
\] (A.25)

\[
I_{H,t} = w_I \left( \frac{P_{H,t}}{P_t} \right)^{-\mu_I} I_t
\] (A.26)

\[
I_{F,t} = (1 - w_I) \left( \frac{P_{F,t}}{P_t} \right)^{-\mu_I} I_t
\] (A.27)

**Interest rate**

\[
\log \left( \frac{R_t}{R} \right) = \rho_r \log \left( \frac{R_{t-1}}{R} \right) + (1 - \rho_r) \left( \theta_{y} \log \left( \frac{\Pi_{t-1,t}}{\Pi} \right) + \theta_s \log \left( \frac{\Pi_{S,t-1,t}}{\Pi_S} \right) \right) + \epsilon_{M,t}
\] (A.28)

**UIP, net foreign assets evolution, terms of trade, price ratios, trade balance**

\[
\frac{T_t}{T_{t-1}} = \frac{\Pi_{F,t-1,t}}{\Pi_{H,t-1,t}}
\] (A.29)

\[
\frac{P_{F,t}}{P_{H,t}} = \left( w_C + (1 - w_C) T_t^{1-\mu_C} \right)^{-\mu_C}
\] (A.30)

\[
\frac{P_{t}}{P_{F,t}} = \left( w_C T_t^{\mu_C-1} + (1 - w_C) \right)^{\frac{1}{1-\mu_C}}
\] (A.31)

\[
\frac{P_{I,t}}{P_{H,t}} = \left( w_I T_t^{\mu_I-1} + (1 - w_I) \right)^{\frac{1}{1-\mu_I}}
\] (A.32)

\[
\frac{P_{I,t}}{P_{F,t}} = \left( w_I T_t^{\mu_I-1} + (1 - w_I) \right)^{\frac{1}{1-\mu_I}}
\] (A.33)

\[
\frac{P_{I,t}}{P_t} = \frac{P_{I,t}}{P_{F,t}} \frac{P_{F,t}}{P_t}
\] (A.34)

55
\[
\Pi_{t-1,t} = \left[ w_C \left( \Pi_{H,t-1,t} \frac{P_{H,t-1}}{P_{t-1}} \right)^{1-\mu_C} + (1 - w_C) \left( \Pi_{F,t-1,t} \frac{P_{F,t-1}}{P_{t-1}} \right)^{1-\mu_C} \right] \frac{1}{\mu_C} \tag{A.35}
\]

\[
TB_t = P_t^{*O} RER_t Y^O + \frac{P_{H,t}}{P_t} Y_t - C_t - \frac{P_t^I}{P_t} I_t - \frac{P_{H,t}}{P_t} G_t \tag{A.36}
\]

\[
\tau^w_t = \frac{P_{H,t}^* G_t - r_t K_t^{*k} I_{t-1} - P_t^{*O} RER_t Y^O \tau^O}{W_t H_t} \tag{A.37}
\]

**General Case**

\[
EX_t = nC_{H,t}^* + nI_{H,t}^* \tag{A.38}
\]

\[
Differences
\]

\[
\Pi_{F,t+1} = \Pi_{F,t+1}^* \Pi_{F,t+1}^S \tag{A.39}
\]

\[
\Pi_{H,t+1} = \Pi_{H,t+1}^* \Pi_{H,t+1}^S \tag{A.40}
\]

\[
\phi(x) = \exp(-\phi_B \frac{B_{F,t}^*}{P_{F,t}^*} Y_t) \tag{A.41}
\]

\[
1 = R_t^* \phi \left( \frac{S_B^* B_{F,t}^*}{P_{F,t}^* Y_t} \right) \tag{A.42}
\]

\[
P_t^{B*} B_{F,t} = \frac{\Pi_{F,t+1}^*}{\Pi_{H,t+1}^*} B_{H,F,t-1} + TB_t \tag{A.43}
\]

\[
RER_t = \left( \frac{w_C^* (1 - w_C^*) (T^*)^{1-\mu_C}}{1-\mu_C} \right) = \frac{P_{F,t}^*}{P_{F,t}^*} \tag{A.44}
\]

**Shock processes**

\[
\log A_t - \log A = \rho_A (\log A_{t-1} - \log A) + \epsilon_{A,t} \tag{A.45}
\]

\[
\log G_t - \log G = \rho_G (\log G_{t-1} - \log G) + \epsilon_{G,t} \tag{A.46}
\]

\[
\log MS_t - \log MS = \rho_{MS} (\log MS_{t-1} - \log MS) + \epsilon_{MS,t} \tag{A.47}
\]

\[
\log IS_t - \log IS = \rho_{IS} (\log IS_{t-1} - \log IS) + \epsilon_{IS,t} \tag{A.48}
\]

\[
\log tol_t - \log tol = \rho_{tol} (\log tol_{t-1} - \log tol) + \epsilon_{tol,t} \tag{A.49}
\]

\[
\log P_t^{*O} - \log P^{*O} = \rho_{P^{*O}} (\log P_t^{*O} - \log P^{*O}) + \epsilon_{P^{*O},t} \tag{A.50}
\]

**Foreign country bloc**

**Production, capital law of motion, capital and labour demand**

\[
Y_t^W = (A_t^* H_t^*)^{\alpha^*} K_t^{*1-\alpha^*} \tag{A.51}
\]
\[ Y_t^* = \frac{Y_t^* W}{\Delta_t^*} \]  
\[ K_t^* = (1 - \delta^*) K_{t-1}^* + (1 - S^*(X_t^*)) I_t^* \]  
\[ r_t^{*K} = (1 - \alpha^*) \frac{Y_t^{*W}}{K_{t-1}^*} MC_t^* \frac{P_{F,t}^*}{P_t^*} \]  
\[ W_t^* = \alpha^* \frac{Y_t^{*W}}{H_t^*} MC_t^* \frac{P_{F,t}^*}{P_t^*} \]  
\[ R_t^{*K} = \frac{r_t^{*K} (1 - \tau_t^*) + (1 - \delta^*) Q_t^*}{Q_{t-1}^*} \]  

**Price Setting**

\[ 1 = \xi^* (\Pi_{F,t-1,t}^*)^{\zeta^*} + (1 - \xi^*) \left( \frac{JJ_t^*}{J_t^*} \right)^{1-\zeta^*} \]  
\[ \Delta_t^* = \xi^* (\Pi_{F,t-1,t}^*)^{\zeta^*} \Delta_{t-1}^* + (1 - \xi^*) \left( \frac{JJ_t^*}{J_t^*} \right)^{-\zeta^*} \]  
\[ JJ_t^* = \frac{\zeta^*}{\xi^* - 1} \frac{P_{F,t}^*}{P_t^*} Y_t^* M S_t^* M C_t^* + \xi^* \mathbb{E}_t \left[ \Lambda_{t,t+1}^* (\Pi_{F,t,t+1}^*)^{\zeta^*} JJ_{t+1}^* \right] \]  
\[ J_t^* = \frac{P_{F,t}^*}{P_t^*} Y_t^* + \xi^* \mathbb{E}_t \left[ \Lambda_{t,t+1}^* \frac{\left( \Pi_{F,t,t+1}^* \right)^{\zeta^*}}{\Pi_{t,t+1}^*} J_{t+1}^* \right] \]  
\[ \mathbb{E}_t \left[ \Lambda_{t,t+1}^* R_{t+1}^* \right] = \mathbb{E}_t \left[ \frac{\Lambda_{t,t+1}^*}{\Pi_{t,t+1}^*} \right] R_t^* = 1 \]  
\[ Q_t^* (1 - S^*(X_t^*)) - X_t^* S''(X_t^*) + E_t \left[ \Lambda_{t,t+1} Q_{t+1}^* S'(X_{t+1}^*) \left( \frac{I_t^*}{I_{t+1}^*} \right)^2 \right] = \frac{P_{t}^*}{P_{t+1}^*} \]  
\[ S^*(X_t^*) = \phi_X (X_t^* - X^*)^2 \]  
\[ S''(X_{t+1}^*) = 2 \phi_X^* (X_t^* - X^*) \]  

**Household labour supply, Euler equation**

\[ U_{C,t}^* R = (1 - \theta^*) \left( C_t^* R - \chi^* C_{t-1}^* R \right)^{(1-\sigma^*)} \left( 1 - H_t^* R \right)^{\sigma^*(1-\sigma^*)} \]  
\[ \Lambda_{t,t+1}^* \equiv \beta^* \frac{U_{C,t}^* R}{U_{C,t}^* R} \]  
\[ 1 = \mathbb{E}_t \left[ \frac{\Lambda_{t,t+1}^*}{\Pi_{t,t+1}^*} \right] R_t^* \]  
\[ \frac{\theta^*}{1 - \theta^*} C_{t}^* C - \chi^* C_{t-1}^* C = W_t^* (1 - \tau_t^*) \]  
\[ \frac{\theta^*}{1 - \theta^*} C_{t}^* R - \chi^* C_{t-1}^* R = W_t^* (1 - \tau_t^*) \]  
\[ H_t^* = \lambda^* H_t C^* + (1 - \lambda^*) H_t R^* \]
Household consumption and investment

\[ C_t^s = \frac{1}{1 - \lambda^s} C_t^s - \frac{\lambda^s}{1 - \lambda^s} C_t^s C \]  
\[ C_t^s C = W_t^s H_t^s \]  
\[ C_{F,t}^s = \frac{C_{F,t}}{w_C} \left( \frac{P_{F,t}}{P_t} \right)^{-\mu_C} \]  
\[ C_{F,t}^s = Y_t^s - I_{F,t}^s - EX_t^s - G_t^s \]  
\[ C_{H,t}^s = (1 - w_C^s) \left( \frac{P_{H,t}^s}{P_t} \right)^{-\mu_C} C_t^s \]  
\[ I_{F,t}^s = (w_1^s) \left( \frac{P_{F,t}^s}{P_t} \right)^{-\mu_I^s} I_t^s \]  
\[ I_{H,t}^s = (1 - w_1^s) \left( \frac{P_{H,t}^s}{P_t} \right)^{-\mu_I^s} I_t^s \]  

Interest rate

\[ \log \left( \frac{R_t^s}{R_t^{s-1}} \right) = \rho_t^s \log \left( \frac{R_t^{s-1}}{R_t^{s-2}} \right) + (1 - \rho_t^s) \left( \theta_x^s \log \left( \frac{\Pi_{t-1}^s}{\Pi_t^s} \right) + \theta_y^s \log \left( \frac{Y_t^s}{Y_t^{s-1}} \right) + \theta_{dy}^s \log \left( \frac{Y_t^s}{Y_t^{s-1}} \right) \right) + \epsilon_{M,t}^s \]  

UIP, net foreign assets evolution, terms of trade, price ratios, trade balance

\[ T_t^* = \frac{1}{T_t} \]  
\[ \frac{P_t^s}{P_{F,t}^s} = \left( w_C^s + (1 - w_C^s) T_t^{s-1} - \mu_C^s \right) \left( 1 - \mu_C^s \right) \]  
\[ \frac{P_t^s}{P_{H,t}^s} = \left( w_C^s T_t^{s-1} + (1 - w_C^s) \right) \left( 1 - \mu_C^s \right) \]  
\[ \frac{P_{t+1}^s}{P_{F,t}^s} = \left( w_1^s T_t^{s-1} + (1 - w_1^s) \right) \left( 1 - \mu_I^s \right) \]  
\[ \frac{P_{t+1}^s}{P_{H,t}^s} = \left( w_1^s T_t^{s-1} + (1 - w_1^s) \right) \left( 1 - \mu_I^s \right) \]  
\[ \Pi_{t-1,t}^* = \left[ w_C^s \left( \frac{P_{F,t-1}^s}{P_{t-1}^s} \right)^{1 - \mu_C^s} + (1 - w_C^s) \left( \frac{P_{H,t-1}^s}{P_{t-1}^s} \right)^{1 - \mu_C^s} \right]^{1 - \mu_C^s} \]  
\[ T_t^{w*} = \frac{P_{F,t}^s G_t^s}{T_t^s} \]  
\[ TB_t = \frac{P_{F,t}^s Y_t^s - C_t^s}{T_t} - \frac{P_{t+1}^s}{P_t^s} I_t^s - \frac{P_{t+1}^s}{P_t^s} G_t^s \]  

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General Case \( EX_t^* = IF_t/n + CF_t/n \)

Shock processes

\[
\begin{align*}
\log A_t^* - \log A^* &= \rho_A^*(\log A_{t-1}^* - \log A^*) + \epsilon_{A,t}^* \\
\log G_t^* - \log(G^*) &= \rho_G^*(\log G_{t-1}^* - \log(G^*)) + \epsilon_{G,t}^* \\
\log MS_t^* - \log MS^* &= \rho_{MS}^*(\log MS_{t-1}^* - \log MS^*) + \epsilon_{MS,t}^* \\
\log IS_t^* - \log IS^* &= \rho_{IS}^*(\log IS_{t-1}^* - \log IS^*) + \epsilon_{IS,t}^*
\end{align*}
\]

(A.88) \hspace{1cm} (A.89) \hspace{1cm} (A.90) \hspace{1cm} (A.91)

**General Case**

\[
\begin{align*}
excs_t &= \frac{PH_{t,t}}{P_t} n (1 - w_C) \left( \frac{PH_{t,t}}{P_t} \right)^{-\mu_C^*} C_t^* Y_t \\
\text{exis}_t &= \frac{PH_{t,t}}{P_t} n (1 - w_I) \left( \frac{PH_{t,t}}{P_t} \right)^{-\mu_I^*} I_t^* Y_t \\
imcs_t &= \frac{PF_{t,t}}{P_t} (1 - w_C) \left( \frac{PF_{t,t}}{P_t} \right)^{-\mu_C} C_t Y_t \\
imis_t &= \frac{PF_{t,t}}{P_t} (1 - w_I) \left( \frac{PF_{t,t}}{P_t} \right)^{-\mu_I} I_t Y_t \\
exco &= \frac{Y^O RER_t P_t^{\mu_O}}{Y_t} \\
TB_t = \frac{Y_{t}}{Y_t} &= \text{exco}_t + \text{excs}_t + \text{exis}_t - \text{imcs}_t - \text{imis}_t \\
Y^*byY &= \frac{PF_{t,t}}{P_t} \frac{Y_t^*}{Y_t} \frac{P_t}{PH_{t,t}} \\
\end{align*}
\]

(A.92) \hspace{1cm} (A.93) \hspace{1cm} (A.94) \hspace{1cm} (A.95) \hspace{1cm} (A.96) \hspace{1cm} (A.97)

**B Deterministic Zero-Growth Steady State-Calibrated**

The external steady state solves using fsolve the following for preference parameter \( \varrho \) and \( \varrho^* \) to target a steady state labour supply \( H = \bar{H} \) and \( H^* = \bar{H} \).

\[
\begin{align*}
\frac{\varrho}{1 - \varrho} \frac{C^R (1 - \chi)}{1 - H^R} &= W (1 - \tau^w) \\
\frac{\varrho^*}{1 - \varrho^*} \frac{C^R (1 - \chi^*)}{1 - H^{*R}} &= W^* (1 - \tau^{*w})
\end{align*}
\]

In a non-zero-net inflation steady state we have recursively for:

**Home Bloc:**
\[ A = 1 \]  
\[ MS = 1 \]  
\[ S(X) = 0 \]  
\[ S'(X) = 0 \]  
\[ \Lambda = \beta \]  
\[ H = H' \]  
\[ H^C = \frac{1 - \rho}{1 - \chi \rho} \]  
\[ H^R = \frac{H - \lambda H^C}{1 - \lambda} \]  
\[ \frac{P}{P_H} = 1 \]  
\[ \frac{P}{P_F} = 1 \]  
\[ \frac{P}{P} = 1 \]  
\[ \frac{P}{P} = 1 \]  
\[ Q = 1 \]  
\[ \Pi_H = \Pi \]  
\[ \Pi_F = \Pi \]  
\[ \Pi = \Pi \]  
\[ R^K = \frac{R}{\Pi} \]  
\[ \frac{J}{J} = \left( \frac{1 - \xi (\Pi)^{\zeta - 1}}{1 - \xi} \right)^{\frac{1}{1 - \zeta}} \]  
\[ MC = \frac{J}{J} \frac{J}{\zeta} \left( 1 - \xi (\Pi)^{\zeta + 1} \right) \]  
\[ \Delta = \frac{(1 - \xi) \left( \left( \frac{J}{J} \right)^{-\zeta} \right)}{1 - \xi (\Pi)^{\zeta + 1}} \]  
\[ KY^W = \frac{MC^{PH}}{R^K Q - (1 - \delta) Q} \]  
\[ Y^W = KY^W \frac{P_H}{P} \]  

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\[ W = \frac{Y^W}{H} \quad \text{MC} \quad \text{(B.24)} \]
\[ K = Y^W KY^-W \quad \text{(B.25)} \]
\[ Y = \frac{Y^W}{\Delta} \quad \text{(B.26)} \]
\[ I = \delta K \quad \text{(B.27)} \]
\[ G = g_y Y \quad \text{(B.28)} \]
\[ \bar{G} = g_y Y \quad \text{(B.29)} \]
\[ r^K = (1 - \alpha) \frac{Y^W}{K} MC \left( 1 - \tau^k \right) \quad \text{(B.30)} \]
\[ \tau^w = \frac{G - (1 - \alpha)YMCT^k - YO RER P^*O \tau^O}{WH} \quad \text{(B.31)} \]
\[ C^C = H^C (1 - \tau_w) W \quad \text{(B.32)} \]
\[ J = \frac{\zeta T Y^W}{1 - \xi \beta (\Pi_H)^\xi} \quad \text{(B.33)} \]
\[ J = \frac{f_p p_y}{1 - \xi \beta (\Pi)^\xi - 1} \quad \text{(B.34)} \]

**Foreign Bloc:**

\[ A^* = 1 \quad \text{(B.35)} \]
\[ MS^* = 1 \quad \text{(B.36)} \]
\[ S^*(X) = 0 \quad \text{(B.37)} \]
\[ S''(X) = 0 \quad \text{(B.38)} \]
\[ \Lambda^* = \beta^* \quad \text{(B.39)} \]
\[ H^* = \bar{H}^* \quad \text{(B.40)} \]
\[ H^*C = \frac{1 - \varrho^*}{1 - \chi^* \varrho^*} \quad \text{(B.41)} \]
\[ H^*R = \frac{H^* - \lambda^* H^*C}{1 - \lambda^*} \quad \text{(B.42)} \]
\[ T^* = \frac{1}{T^*} \quad \text{(B.43)} \]
\[ \frac{P^*}{P^*_F} = 1 \quad \text{(B.44)} \]
\[ \frac{P^*}{P^*_H} = 1 \quad \text{(B.45)} \]
\[ \frac{P^*I}{P^*_F} = 1 \quad \text{(B.46)} \]
\[ \frac{P^{*l}}{P^*} = 1 \] (B.47)
\[ \frac{P^{*l}}{P^*} = 1 \] (B.48)
\[ Q^* = 1 \] (B.49)
\[ RER = 1 \] (B.50)
\[ \Pi_H^* = \bar{\Pi} \] (B.51)
\[ \Pi_F^* = \bar{\Pi} \] (B.52)
\[ \Pi^* = \bar{\Pi} \] (B.53)
\[ R^* = \frac{\Pi^*}{\beta^*} \] (B.54)
\[ R^{*K} = \frac{R^*}{\Pi^*} \] (B.55)

**Home Bloc:**
\[ \Pi_S = 1 \] (B.56)
\[ \phi = \frac{\Pi}{\Pi_S} = \frac{\beta^*}{\beta} \text{ (using } R^* = \frac{\Pi^*}{\beta^*} \text{)} \] (B.57)
\[ p_B^* = \frac{1}{R^*} \phi \] (B.58)
\[ B_F = -\frac{Y \log(\phi)}{\phi_B} \geq 0 \text{ iff } \beta \geq \beta^* \] (B.59)
\[ T_B = \left( \frac{1}{\phi R^*} - \frac{\Pi^*}{\Pi} \right) B_F \] (B.60)
\[ C = Y^O RER P^{*O} + Y - I - G - T_B \] (B.61)
\[ C^R = \frac{1 - \lambda}{1 - \lambda} (C - \lambda) C^C \] (B.62)
\[ U^R_C = (1 - \varrho) (C^R - \chi C^R)^{(1 - \varrho)(1 - \sigma) - 1} (1 - H^R)^{\varrho(1 - \sigma)} \] (B.63)

**Foreign Bloc:**
\[ \frac{JJ^*}{J^*} = \left( \frac{1 - \xi^* (\Pi^* F)^{\xi^* - 1}}{1 - \xi^*} \right) \] (B.64)
\[ MC^* = \frac{JJ^*}{J^*} \frac{1 - \xi^* (\Pi^* F)^{\xi^*}}{1 - \xi^* \beta (\Pi^* F)^{\xi^* - 1}} \] (B.65)
\[ \Delta^* = \frac{(1 - \xi^*) \left( \frac{JJ^*}{J^*} \right)^{\xi^*}}{1 - \xi (\Pi^* F)^{\xi^*}} \] (B.66)
\[ KY^{*W} = \frac{MC^* (1 - \alpha^*)}{R^K Q^* - (1 - \delta^*) Q^*} \] (B.67)
\[ Y^{*W} = KY^{*W} \frac{1 - \alpha^*}{H^*} \] (B.68)
\[ W^* = \alpha \frac{Y^* W}{H^*} MC^* \quad (B.71) \]
\[ K^* = Y^* W \frac{KY^*}{H^*} \quad (B.72) \]
\[ Y^* = \frac{Y^* W}{\Delta^*} \quad (B.73) \]
\[ I^* = \delta^* K^* \quad (B.74) \]
\[ G^* = g_y Y^* \quad (B.75) \]
\[ G^* = g_y Y^* \quad (B.76) \]
\[ r^* K = (1 - \alpha^*) \frac{Y^*}{K^*} MC^* \left( 1 - \tau^* k \right) \quad (B.77) \]
\[ \tau^* w = G^* - (1 - \alpha^*) Y^* MC^* \tau^* \quad (B.78) \]
\[ C^* C = H^* (1 - \tau^* w) W^* \quad (B.79) \]
\[ J J^* = \frac{\xi^*}{1 - \tau^* C^*} Y^* MC^* \quad (B.80) \]
\[ J^* = \frac{P_F}{1 - \beta^* (\Pi F)^{C^*}} \quad (B.81) \]
\[ EX = \frac{\alpha}{n} \quad (B.82) \]

**General Case**

\[ EX^* = (\text{targexc} + \text{targimcs}) Y/n \]

**General Case**

\[ TB^* = EX^* - EX/n \]

\[ C^* = Y^* - I^* - G^* - TB^* \quad (B.83) \]
\[ C^* R = \frac{1}{1 - \lambda^*} \left( C^* - \lambda^* \right) C^* C \quad (B.84) \]
\[ U^* C = \left( 1 - \varphi^* \right) \left( C^* R - \lambda^* C^* \right)^{(1 - \varphi^*)/\left(1 - \sigma^*\right)} \left( 1 - H^* R \right)^{\varphi^*/\left(1 - \sigma^*\right)} \quad (B.85) \]
\[ I_F = (1 - w_I) I \quad (B.86) \]
\[ C_F = (1 - w_C) C \quad (B.87) \]
\[ I_H = (w_I) I \quad (B.88) \]
\[ C_H = (w_C) C \quad (B.89) \]
\[ I_F^* = w_I^* I^* \quad (B.90) \]
\[ C_F^* = w_C^* C^* \quad (B.91) \]
\[ I_H^* = (1 - w_I^*) I^* \quad (B.92) \]
\[ C_H^* = (1 - w_C^*) C^* \quad (B.93) \]

**General Case**

\[ \text{excs} = n \left( 1 - w_C^* \right) \frac{C^*}{Y} \]

**General Case**

\[ \text{exis} = n \left( 1 - w_I^* \right) \frac{I^*}{Y} \]
imcs = \( (1 - w_C) \frac{C}{Y} \) \hspace{1cm} (B.94)

imis = \( (1 - w_I) \frac{I}{Y} \) \hspace{1cm} (B.95)

\[\text{exco} = \frac{Y^O \cdot RER \cdot P^O}{Y} \] \hspace{1cm} (B.96)

\[tb \equiv \frac{TB}{Y} = \text{exco} + \text{exc} + \text{exis} - \text{imcs} - \text{imis} \] \hspace{1cm} (B.97)

In this calibrated version of the model, the external steady state also solves using `fsolve` the following for \( w_C, w_I, w_C^*, \beta^*, Y^O \) and \( A^* \) to target \( \text{excs} = \text{targexcs}, \text{exis} = \text{targexis}, \text{imcs} = \text{targimcs}, \text{imis} = \text{targimis}, \text{exco} = \text{targexco}, \text{tb} \equiv \frac{TB}{Y} = \text{targtb} \) and \( \frac{Y^*}{Y} = \text{targY*byY} \)

\[\text{excs} = \text{targexcs} \] \hspace{1cm} (B.98)

\[\text{exis} = \text{targexis} \] \hspace{1cm} (B.99)

\[\text{imcs} = \text{targimcs} \] \hspace{1cm} (B.100)

\[\text{imis} = \text{targimis} \] \hspace{1cm} (B.101)

\[\text{exco} = \text{targexco} \] \hspace{1cm} (B.102)

\[tb = \text{targtb} \] \hspace{1cm} (B.103)

\[\frac{Y^*}{Y} = \text{targY*byY} \] \hspace{1cm} (B.104)

Note that targets have been imposed in (B.82) and (B) which makes the steady state above recursive. Note also that we introduce a new variable `CheckTB` into the code verifies that `CheckTB = 0`.

C Summary of Dynamic Two-bloc Model. Changes for Dominant Currency Pricing

Previous (PCP)

\[JJ_t \] \hspace{1cm} (C.1)

\[J_t \] \hspace{1cm} (C.2)

\[Y_t \] \hspace{1cm} (C.3)

\[T^*_t \] \hspace{1cm} (C.4)

\[\Pi^{H}_{t,t+1} \] \hspace{1cm} (C.5)

\[JJ_t = \frac{\zeta}{\zeta - 1} P_h M S_t M C_t + \xi E_t \left[ \frac{M_{t+1} (\Pi_{H,t,t+1})^\zeta}{\Pi_{H,t,t+1}} \right] \] \hspace{1cm} (C.6)
\[ J_t = \frac{P_{H,t}}{P_t} Y_t + \xi \mathbb{E}_t \left[ \Lambda_{t,t+1} (\Pi_{H,t,t+1})^{\zeta-1} \right] \]  
\[ Y_t = \frac{Y_t^W}{\Delta_t} \]  
\[ T_t^* = \frac{1}{T_t} \]  
\[ \Pi_{t,t+1}^H = \Pi_{t,t+1}^H \Pi_{t,t+1}^S \]  

**New (DCP)**

\[ JJ_t \]  
\[ J_t \]  
\[ Y_t \]  
\[ T_t^* = \frac{MC_t}{MC_{H,t}^*} \]  

**Added equations (DCP)**

\[ MC_{H,t}^* \]  
\[ \Delta_{H,t}^* \]  
\[ JJ_{H,t}^* \]  
\[ J_{H,t}^* \]  
\[ S_t P_{H,t}^* \]  
\[ P_t \]  
\[ \Pi_{t,t+1}^H \]  

\[ 1 = \xi (\Pi_{H,t-1,t}^H)_{\zeta=1} + (1 - \xi) \left( \frac{P_{H,t}^*}{P_{H,t}} \right)^{1-\zeta} \]  
\[ \Delta_{H,t}^* = \xi (\Pi_{H,t-1,t}^H)_{\zeta=1} \Delta_{H,t-1}^* + (1 - \xi) \left( \frac{JJ_{H,t}^*}{JJ_{H,t}} \right)^{-\zeta} \]  
\[ JJ_{H,t}^* - \xi \beta \mathbb{E}_t [\Pi_{H,t,t+1}^H] J_{H,t,t+1} = \frac{1}{1 - \xi} \frac{S_t P_{H,t}^*}{P_t} (EX_t) U_{C,t} MC_{H,t}^* MS_t \]  
\[ J_{H,t}^* - \xi \beta \mathbb{E}_t [\Pi_{H,t,t+1}^H]_{\zeta=1} J_{H,t,t+1} = \frac{S_t P_{H,t}^*}{P_t} (EX_t) U_{C,t} \]
\[ MC^*_H,t = \frac{MC_{t \frac{P_H,t}{P_t}}}{S_{\frac{P_H,t}{P_t}}} \]  

(C.29)

**Previous SSmodel (PCP)**

\[ JJ = \frac{\zeta P_H Y} {1 - \xi \beta (\Pi_H)^{\zeta}} \]  

(C.30)

\[ J = \frac{P_H Y} {1 - \xi \beta (\Pi_H)^{\zeta-1}} \]  

(C.31)

\[ Y = Y^W \]  

(C.32)

**NOW (DCP)**

"\( EX \) is replaced by (targexcs + targexis)"

\[ Y_t = \left( \frac{(targexcs + targexis)}{\Delta_{H,t}^*} + \frac{(1 - (targexcs + targexis))}{\Delta_t} \right) Y_t^W \]  

(C.33)

\[ JJ = \frac{\zeta P_H (Y - EX)MC}{1 - \xi \beta (\Pi_H)^{\zeta}} \]  

(C.34)

\[ J = \frac{P_H (Y - EX)} {1 - \xi \beta (\Pi_H)^{\zeta-1}} \]  

(C.35)

\[ JJ^*_H = \frac{\zeta S_{\frac{P_H}{S^*}} EXMC^*_H}{1 - \xi \beta (\Pi_H)^{\zeta}} \]  

(C.36)

\[ J^*_H = \frac{S_{\frac{P_H}{S^*}} EX}{1 - \xi \beta (\Pi_H)^{\zeta-1}} \]  

(C.37)

\[ MC^*_H = \frac{JJ^*_H}{J^*_H} \log \left( \frac{1 - \xi}{1 - \xi \beta (\Pi_H)^{\zeta}} \right) \]  

(C.38)

\[ \Delta^*_H = \frac{1 - \xi \beta (\Pi_H)^{\zeta}}{1 - \xi (\Pi_H)^{\zeta}} \]  

(C.39)

\[ \frac{SP^*_H}{P} = \frac{MC_{\frac{P_H}{P}}}{MC^*_H} \]  

(C.40)

**D  Summary of changes in the Dynamic Calibrated Two-bloc Model for (Closed ROW-SOE Case)**

We replace the following
\[ TB_t^* = \frac{P_{H, t}^*}{P_t^*} EX_t^* - \frac{P_{H, d, t}^*}{P_t^*} \frac{EX_t}{n} \]

\[ EX_t^* = \frac{CF_t}{n} + IF_t \]

\[ EX_t = n CH_t^* + n IH_t^* \]

with the following

\[ TB_t^* = 0 \]

\[ EX_t^* = 0 \]

\[ EX_t = \frac{\text{targexcs}}{C_t^*/Y} \left( \frac{P_{H, d, t}^*}{P_t^*} \right)^{\mu C_t^*} + \frac{\text{targexis}}{I_t^*/Y} \left( \frac{P_{H, d, t}^*}{P_{I, t}^*} \right)^{\mu I_t^*} I_t^* \]

### E Summary of changes in the Steady State Calibrated Two-bloc Model (Closed ROW-SOE Case)

We replace the following

\[ TB^* = EX^* - \frac{EX}{n} \]

\[ EX^* = (\text{targimcs} + \text{targimis})Y/n \]

with the following

\[ TB^* = 0 \]

\[ EX^* = 0 \]
### ESTIMATION RESULTS

Log data density is -646.075350.

#### parameters

<table>
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<tr>
<th></th>
<th>prior mean</th>
<th>post. mean</th>
<th>90% HPD interval</th>
<th>prior</th>
<th>pstdev</th>
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<td>0.5326</td>
<td>0.3106</td>
<td>0.7378</td>
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### Estimation Results

Log data density is -622.081548.

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**Standard Deviation of Shocks**

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ESTIMATION RESULTS

Log data density is -783.049782.

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I Mexico-DCP

ESTIMATION RESULTS

Log data density is -729.033611.

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ESTIMATION RESULTS

Log data density is -546.384112.

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Standard deviation of shocks

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Identification

Figure 6: Identification Strength of Prior Mean, Chile- PCP case

Figure 7: Identification Strength of Prior Mean, Chile- DCP case
Identification strength with asymptotic Information matrix (log-scale)

Figure 8: Identification Strength of Prior Mean, Canada- PCP case

Figure 9: Identification Strength of Prior Mean, Canada- DCP case
Figure 10: Identification Strength of Prior Mean, Mexico- PCP case

Figure 11: Identification Strength of Prior Mean, Mexico- DCP case
Figure 12: Multivariate Convergence Check, Chile- PCP case, 100,000 draws

Figure 13: Multivariate Convergence Check, Chile- DCP case, 100,000 draws
Figure 14: Multivariate Convergence Check, Mexico- PCP case, 100,000 draws

Figure 15: Multivariate Convergence Check, Mexico- DCP case, 100,000 draws
Figure 16: Multivariate Convergence Check, Canada- PCP case, 100,000 draws

Figure 17: Multivariate Convergence Check, Canada- DCP case, 100,000 draws
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Table 15: Estimated Parameter Values and Estimated standard deviation of shocks
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Table 16: Estimated Parameter Values and Estimated standard deviation of shocks

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<table>
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<th>Parameter</th>
<th>Symbol</th>
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<th>90% HPD Interval</th>
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<td>$\beta$</td>
<td>0.5, 0.10</td>
<td>0.8432, 0.7270, 0.9852</td>
</tr>
<tr>
<td><strong>Government shock persistence</strong></td>
<td>$\rho_{G}$</td>
<td>$\beta$</td>
<td>0.5, 0.10</td>
<td>0.4947, 0.1631, 0.8176</td>
</tr>
<tr>
<td><strong>Monetary Policy shock persistence</strong></td>
<td>$\rho_{M}$</td>
<td>$\beta$</td>
<td>0.7, 0.10</td>
<td>0.6859, 0.6107, 0.7525</td>
</tr>
<tr>
<td><strong>Share of non-Ricardian consumers</strong></td>
<td>$\lambda$</td>
<td>$N$</td>
<td>0.50, 0.05</td>
<td>0.4929, 0.4182, 0.5617</td>
</tr>
<tr>
<td><strong>Consumption habit formation</strong></td>
<td>$\chi$</td>
<td>$\beta$</td>
<td>0.50, 0.05</td>
<td>0.5676, 0.4841, 0.6523</td>
</tr>
<tr>
<td><strong>Calvo price stickiness</strong></td>
<td>$\xi$</td>
<td>$\beta$</td>
<td>0.50, 0.05</td>
<td>0.4903, 0.3700, 0.6140</td>
</tr>
<tr>
<td><strong>Price index</strong></td>
<td>$\gamma$</td>
<td>$\beta$</td>
<td>0.50, 0.10</td>
<td>0.3924, 0.1638, 0.6238</td>
</tr>
<tr>
<td><strong>Elasticity of Investment adjustment cost</strong></td>
<td>$\phi_{I}$</td>
<td>$N$</td>
<td>4.00, 1.50</td>
<td>4.9859, 3.2083, 6.5189</td>
</tr>
<tr>
<td><strong>Non-Ricardian risk aversion</strong></td>
<td>$\sigma_{c}$</td>
<td>$N$</td>
<td>2.00, 0.25</td>
<td>2.0064, 1.6608, 2.3435</td>
</tr>
<tr>
<td><strong>Feedback from inflation</strong></td>
<td>$\theta_{\pi}$</td>
<td>$N$</td>
<td>2.00, 0.25</td>
<td>1.9770, 1.6970, 2.2371</td>
</tr>
<tr>
<td><strong>Feedback from output</strong></td>
<td>$\theta_{y}$</td>
<td>$N$</td>
<td>0.10, 0.05</td>
<td>0.0119, 0.0244, 0.1493</td>
</tr>
<tr>
<td><strong>Feedback from output growth</strong></td>
<td>$\theta_{\delta y}$</td>
<td>$N$</td>
<td>0.10, 0.05</td>
<td>0.0372, -0.0003, 0.0802</td>
</tr>
<tr>
<td><strong>Feedback from exchange rate depreciation</strong></td>
<td>$\theta_{\delta s}$</td>
<td>$N$</td>
<td>0.10, 0.05</td>
<td>0.0372, -0.0003, 0.0802</td>
</tr>
</tbody>
</table>

Table 17: Estimated Parameter Values and Estimated standard deviation of shocks

82
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior</th>
<th>Posterior</th>
<th>90% HPD Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated Parameter Values</td>
<td></td>
<td>Symbole</td>
<td>Dist. (Mean, Std Dev)</td>
</tr>
<tr>
<td>Technology shock</td>
<td>$\epsilon_A$</td>
<td>$IG$</td>
<td>0.10, 2.00</td>
</tr>
<tr>
<td>Foreign Technology shock</td>
<td>$\epsilon_M$</td>
<td>$IG$</td>
<td>0.10, 2.00</td>
</tr>
<tr>
<td>Investment shock</td>
<td>$\epsilon_IS$</td>
<td>$IG$</td>
<td>0.10, 2.00</td>
</tr>
<tr>
<td>Government shock</td>
<td>$\epsilon_G$</td>
<td>$IG$</td>
<td>0.10, 2.00</td>
</tr>
<tr>
<td>Monetary policy shock</td>
<td>$\epsilon_M$</td>
<td>$IG$</td>
<td>0.10, 2.00</td>
</tr>
<tr>
<td>Technology shock persistence</td>
<td>$\rho_A$</td>
<td>$\beta$</td>
<td>0.50, 0.10</td>
</tr>
<tr>
<td>Markup shock persistence</td>
<td>$\rho_MS$</td>
<td>$\beta$</td>
<td>0.50, 0.10</td>
</tr>
<tr>
<td>Investment shock persistence</td>
<td>$\rho_IS$</td>
<td>$\beta$</td>
<td>0.50, 0.10</td>
</tr>
<tr>
<td>Government shock persistence</td>
<td>$\rho_G$</td>
<td>$\beta$</td>
<td>0.75, 0.10</td>
</tr>
<tr>
<td>Monetary Policy shock persistence</td>
<td>$\rho_M$</td>
<td>$\beta$</td>
<td>0.50, 0.05</td>
</tr>
<tr>
<td>Share of non-Ricardian consumers</td>
<td>$\lambda$</td>
<td>$N$</td>
<td>0.50, 0.05</td>
</tr>
<tr>
<td>Consumption habit formation</td>
<td>$\chi$</td>
<td>$\beta$</td>
<td>0.70, 0.05</td>
</tr>
<tr>
<td>Labour Share</td>
<td>$\alpha$</td>
<td>$\beta$</td>
<td>0.70, 0.05</td>
</tr>
<tr>
<td>Calvo price stickiness</td>
<td>$\xi$</td>
<td>$\beta$</td>
<td>0.50, 0.05</td>
</tr>
<tr>
<td>Elasticity of demand</td>
<td>$\zeta$</td>
<td>$N$</td>
<td>6.00, 2.50</td>
</tr>
<tr>
<td>Price index</td>
<td>$\gamma$</td>
<td>$\beta$</td>
<td>0.50, 0.10</td>
</tr>
<tr>
<td>Elasticity of Investment adjustment cost</td>
<td>$\phi$</td>
<td>$N$</td>
<td>2.00, 1.50</td>
</tr>
<tr>
<td>Non-Ricardian risk aversion</td>
<td>$\sigma_c$</td>
<td>$N$</td>
<td>2.00, 0.25</td>
</tr>
<tr>
<td>Feedback from inflation</td>
<td>$\theta_x$</td>
<td>$N$</td>
<td>2.00, 0.25</td>
</tr>
<tr>
<td>Feedback from output</td>
<td>$\theta_y$</td>
<td>$N$</td>
<td>0.125, 0.05</td>
</tr>
<tr>
<td>Feedback from output growth</td>
<td>$\theta_{dy}$</td>
<td>$N$</td>
<td>0.125, 0.05</td>
</tr>
</tbody>
</table>

Table 18: Estimated Parameter Values and Estimated standard deviation of shocks

<table>
<thead>
<tr>
<th>Output</th>
<th>Inflation</th>
<th>Interest rate</th>
<th>Consumption</th>
<th>Investment</th>
<th>Exchange Rate</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>1.0158</td>
<td>0.6529</td>
<td>0.4782</td>
<td>1.1696</td>
<td>3.9193</td>
<td>4.8731</td>
</tr>
<tr>
<td>PCP Model</td>
<td>2.4716</td>
<td>1.4294</td>
<td>0.5567</td>
<td>1.5722</td>
<td>2.4395</td>
<td>9.4127</td>
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<tr>
<td>DCP Model</td>
<td>2.1327</td>
<td>1.0109</td>
<td>0.4168</td>
<td>1.7919</td>
<td>2.4958</td>
<td>9.3697</td>
</tr>
</tbody>
</table>

Table 19: Selected Second Moments of the Chile Model
Figure 18: Autocorrelations of Observables in the Actual Data and in the Estimated Models- Chile

Figure 19: Autocorrelations of Observables in the Actual Data and in the Estimated Models- Mexico
<table>
<thead>
<tr>
<th>Output</th>
<th>Inflation</th>
<th>Interest rate</th>
<th>Consumption</th>
<th>Investment</th>
<th>Exchange Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>1.3589</td>
<td>2.3253</td>
<td>2.8062</td>
<td>1.3957</td>
<td>4.1803</td>
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<tr>
<td>PCP Model</td>
<td>2.1216</td>
<td>1.1111</td>
<td>0.9020</td>
<td>1.8763</td>
<td>4.4690</td>
</tr>
<tr>
<td>DCP Model</td>
<td>1.6656</td>
<td>0.8475</td>
<td>0.7117</td>
<td>1.9192</td>
<td>3.6329</td>
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<table>
<thead>
<tr>
<th>Standard Deviation</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Cross-correlation with Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
</tr>
<tr>
<td>PCP Model</td>
</tr>
<tr>
<td>DCP Model</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Autocorrelations (Order=1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
</tr>
<tr>
<td>PCP Model</td>
</tr>
<tr>
<td>DCP Model</td>
</tr>
</tbody>
</table>

Table 20: Selected Second Moments of the Mexico Model

![Graphs showing autocorrelations of various economic variables](image)

Figure 20: Autocorrelations of Observables in the Actual Data and in the Estimated Models- Canada
### Table 21: Selected Second Moments of the Canada Model

<table>
<thead>
<tr>
<th></th>
<th>Output</th>
<th>Inflation</th>
<th>Interest rate</th>
<th>Consumption</th>
<th>Investment</th>
<th>Exchange Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Data</strong></td>
<td>0.6196</td>
<td>0.3108</td>
<td>0.4913</td>
<td>0.4790</td>
<td>1.9898</td>
<td>3.3746</td>
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<tr>
<td><strong>PCP Model</strong></td>
<td>3.3992</td>
<td>1.9245</td>
<td>0.6767</td>
<td>0.7143</td>
<td>2.7612</td>
<td>14.1550</td>
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<tr>
<td><strong>DCP Model</strong></td>
<td>1.2473</td>
<td>1.1721</td>
<td>0.4691</td>
<td>1.1959</td>
<td>2.5801</td>
<td>10.6585</td>
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</tbody>
</table>

**Standard Deviation**

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>PCP Model</th>
<th>DCP Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cross-correlation with Output</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td><strong>PCP Model</strong></td>
<td>-0.1341</td>
<td>0.8285</td>
<td>-0.0881</td>
</tr>
<tr>
<td><strong>DCP Model</strong></td>
<td>0.17449</td>
<td>0.2029</td>
<td>-0.1561</td>
</tr>
</tbody>
</table>

**Autocorrelations (Order=1)**

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>PCP Model</th>
<th>DCP Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.5044</td>
<td>-0.3809</td>
<td>0.3490</td>
</tr>
<tr>
<td><strong>PCP Model</strong></td>
<td>-0.0026</td>
<td>0.9471</td>
<td>-0.0593</td>
</tr>
<tr>
<td><strong>DCP Model</strong></td>
<td>0.1916</td>
<td>0.4399</td>
<td>0.8357</td>
</tr>
</tbody>
</table>

---

**Figure 21: IRFs- Monetary Policy shock- Chile**
Figure 22: IRFs- Monetary Policy shock- Canada

Figure 23: IRFs- Monetary Policy shock- Mexico
Figure 24: IRFs- Monetary Policy shock- PCP Case

Figure 25: IRFs- Monetary Policy shock- DCP Case