Real and Nominal Effects of Monetary Shocks under Time-Varying Disagreement

Vania Esady
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Department of Economics
City, University of London
Motivation

Effects of monetary policy often state-dependent

- Difficult to ascertain the power of monetary policy
- Under recessions/high uncertainty: monetary policy less powerful
- Why? Agents more cautious, so respond more slowly
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Recent heightened uncertainty challenges monetary policymakers

- Uncertainty for economic agents ⇒ uncertainty for central banks
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Recent heightened uncertainty challenges monetary policymakers

- Uncertainty for economic agents $\Rightarrow$ uncertainty for central banks

Large efforts to measure uncertainty and assess monetary transmission under different uncertainties

- Forecast errors, VIX, Economic policy uncertainty index, JLN
- Established literature: Forecaster Disagreement (Dispersion)
Uncertainty and Disagreement

“The literature has convincingly shown that disagreement is not uncertainty. They are conceptually different.”

— Ricardo Reis (June 2018)
ECB Forum in Central Banking, Sintra
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Which uncertainty? Kozeniauskas, Orlik and Veldkamp (JME, 2018):

- Macro uncertainty: $U_t = Std[y_{t+1}|I_t] = \sqrt{E[(y_{t+1} - E(y_{t+1}|I_t))^2|I_t]}$

- Micro uncertainty: $U_t = \int (y_{it} - \bar{y}_t)^2 di$

- Higher-order uncertainty: $U_t = \int (E[y_{t+1}|I_{it}] - \bar{E}(y_{t+1}|I_t))^2) di$
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- **Micro uncertainty:**
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- **Higher-order uncertainty:**
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- Higher-order uncertainty: $U_t = \int (E[y_{t}|I_{it}] - \bar{E}[y_t])^2 di$

Disagreement and uncertainties measures conceptually very distinct but they may/not co-move. Why?
Does Disagreement Among Forecasters Matter?

- When firms imperfectly observe factors that affect optimal prices ...
- ... they attach a less-than-unity weight on their signals...
- ... leading to a sluggish response of prices.
Does Disagreement Among Forecasters Matter?

- When firms have better information on their signals...
- ... prices react faster...
- ... leading to a steeper Phillips curve
Disagreement on Output Matters for Monetary Policymaker

Monetary policy also transmits through expectations

- Expectations, and thus disagreement of expectations, affect forward-looking decisions (e.g.: price-setting)
- Long literature on heterogeneity of inflation expectation:
  - Mankiw, Reis and Wolfers (2003); Falck et al. (2017); Coibion et al. (2018)
- Dual-mandate central bank also cares about output expectation
Monetary policy also transmits through expectations

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Standard New Keynesian Phillips curve:

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa E_t \tilde{y}_t \]
\[ \pi_{t+1} = \beta E_{t+1} \pi_{t+2} + \kappa E_{t+1} \tilde{y}_{t+1} \]
\[ \pi_t = \beta^2 E_{t+1} \pi_{t+2} + \kappa E_t \tilde{y}_t + \kappa E_{t+1} \tilde{y}_{t+1} \]
\[ \pi_t = \kappa E_t \sum_{j=0}^{\infty} \beta^j \tilde{y}_{t+j} \]
This Paper

How does heightened disagreement affect the effectiveness of monetary policy on output and inflation?
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Empirical Methodology:

- Threshold VAR to estimate the heterogeneity of monetary policy transmission under different disagreement regimes
- Disagreement: Survey of Professional Forecasters dispersions
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Main Finding:

In high-disagreement periods, monetary policy has weaker price effects, yet stronger real effects
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Main Finding:

In high-disagreement periods, monetary policy has \textit{weaker} price effects, yet \textit{stronger} real effects

Stylised Rational Inattention Model:

- Dissecting relation of disagreement & uncertainty
- How they affect price-setting behaviours and monetary transmission
Data

Time series data

- Real GDP*, GDP Deflator*, Commodity Price Index*, Fed Funds Rate
  * log first-differenced
- Source: FRED database, for period 1970Q1 - 2015Q3
- Wu-Xia shadow rate: 2009Q1 - 2015Q3 – to account for ELB and QE
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Baseline Recursive Identification

- Real GDP, GDP Deflator, Commodity Price Index, FFR, Disagreement*
- Robust to other ordering schemes
Measuring Disagreement

**Threshold variable:** *Disagreement*

- Survey of Professional Forecaster (SPF) **Dispersion**
- Baseline using Nowcasts of Real GDP Interquartile range normalised using the median
- Interquartile range: outliers do not unfairly influence the measure of disagreement
- Robustness check: 1-year ahead Real GDP forecasts and Nowcasts of Nominal GDP from SPF
Threshold VAR

\[ Y_t = \left[ c_1 + \sum_{j=1}^{p} \gamma^1(L) Y_{t-j} \right] + \left[ c_2 + \sum_{j=1}^{p} \gamma^2(L) Y_{t-j} \right] I(y^*_t > \theta) + U_t \]

where:

- \( Y_t \): a vector of endogenous (stationary) variables
  - Real GDP, GDP Deflator, Commodity Prices, FFR, Dispersion
- \( \gamma^1(L) \) and \( \gamma^2(L) \): lag polynomials with order \( p \)
  - Lag order selection by Akaike information criteria
- \( y^* \in Y \): Threshold variable (Endogeneous threshold)
- \( \theta \): Threshold parameter (estimated) with delay \( d = 1 \)
Threshold VAR

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where:

- \( Y_t \): a vector of endogenous (stationary) variables
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- \( \gamma^1(L) \) and \( \gamma^2(L) \): lag polynomials with order \( p \)
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- \( y^* \in Y \): Threshold variable (Endogeneous threshold)
- \( \theta \): Threshold parameter (estimated) with delay \( d = 1 \)

Estimation of the threshold \( \theta^* \) uses maximum likelihood:

\[ \theta^* = \min_{\theta} \left[ \frac{T_1}{2} \log |\hat{\Sigma}^{(1)}(\theta)| + \frac{T_2}{2} \log |\hat{\Sigma}^{(2)}(\theta)| \right] \]
SPF Disagreement – Real GDP and Regimes

Real GDP Nowcast Disagreement

Real GDP 1-year Ahead Disagreement

Grey shade – NBER dated Recessions Red shade – High Uncertainty

Red line – Estimated Threshold
Impulse Response Functions (IRFs) – Linear VAR

1 SD shock to FFR, 68% bootstrapped CI, Nowcasts of Real GDP
Generalised IRFs – TVAR

1 SD shock to FFR, 68% bootstrapped CI, Nowcasts of Real GDP
1 SD shock to FFR, 68\% bootstrapped CI, 1-year ahead Forecast of Real GDP
SPF Disagreement – Nominal GDP and Regimes

Nominal GDP Nowcast Disagreement

Nominal GDP 1-year Ahead Disagreement

Blue shade – NBER dated Recessions Red shade – High Uncertainty
Red line – Estimated Threshold
Generalised IRFs – TVAR

1 SD shock to FFR, 68% bootstrapped CI, Nowcasts of Nominal GDP
Conclusion So Far

Robust results using different horizons and variable for disagreement

Main results

Under high disagreement, monetary policy is:

- Less powerful controlling prices
- More powerful controlling output

Under low disagreement, monetary policy:

- Reduce inflation significantly
- Little output loss

Different to result from uncertainty literature in assessing monetary transmission
Intuition of the Model

Lower expected profit losses

Feasible attention set region

$K^y$

$K^a$

$K^y_0$

$K^a_0$
Intuition of the Model

Lower demand uncertainty or higher productivity uncertainty

Feasible attention set region
Price-setters face unobserved aggregate demand:

\[ y_t = b_t - c \cdot r_t, \quad \text{where} \quad b_t \sim N(0, \sigma_b^2) \]  

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Full information optimal price:

\[ p_{it}^* = \varphi y_t - a_{it}, \quad \text{where} \quad a_{it} \sim N(0, \sigma_a^2) \]  

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Price-setters face unobserved aggregate demand:

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(1)

Full information optimal price:

\[ p_{it}^* = \varphi y_t - a_{it}, \quad \text{where} \quad a_{it} \sim N(0, \sigma_a^2) \]  

(2)

To help set optimal prices, each firm \( i \) receives the signals \( s_{it} = \{s_{it}^y, s_{it}^a\} \) on key variables \( y_t \) and \( a_{it} \):

\[ s_{it}^y = y_t + \varepsilon_{it}^y, \quad \varepsilon_{it}^y \sim N(0, \sigma_{\varepsilon_y,t}^2) \]  

(3)

\[ s_{it}^a = a_{it} + \varepsilon_{it}^a, \quad \varepsilon_{it}^a \sim N(0, \sigma_{\varepsilon_a,t}^2) \]  

(4)

The firms choose \( \sigma_{\varepsilon_y,t}^2 \) and \( \sigma_{\varepsilon_a,t}^2 \) subject to information constraint \( K \)
Information Setup

Rational inattention

- Model attention as an information flow
- Model limited attention as a bound on information flow $K$
- Individuals have to decide how to allocate their attention
Rational inattention

- Model attention as an information flow
- Model limited attention as a bound on information flow $K$
- Individuals have to decide how to allocate their attention
- Task: quantify information flow – reduction in entropy

$$I (p^*_t; s_{it}) = H (p^*_t) - H (p^*_t | s_{it}) \leq K$$  \hspace{1cm} (5)
Rational inattention

- Model attention as an information flow
- Model limited attention as a bound on information flow $K$
- Individuals have to decide how to allocate their attention
- Task: quantify information flow – reduction in entropy

$$I(p_{it}^*; s_{it}) = H(p_{it}^*) - H(p_{it}^* | s_{it}) \leq K$$  (5)

$$H(y_t) - H(y_t | s_{it}^{y}) + K_{it}^{y} + H(a_{it}) - H(a_{it} | s_{it}^{a}) \leq K$$  (6)
\[
\frac{1}{2} \log_2 \left( \frac{\sigma^2_y}{\sigma^2_{\varepsilon_y,t}} + 1 \right) + \frac{1}{2} \log_2 \left( \frac{\sigma^2_{a_i}}{\sigma^2_{\varepsilon_{a,t}}} + 1 \right) \leq K
\]
Attention Allocation

\[
\frac{1}{2} \log_2 \left( \frac{\sigma^2_y}{\sigma^2_{\varepsilon_y,t}} + 1 \right) + \frac{1}{2} \log_2 \left( \frac{\sigma^2_{a_i}}{\sigma^2_{\varepsilon_{a,t}}} + 1 \right) \leq K
\]

Implied perceived volatility of tracking noises from attention allocation:

\[
\sigma^2_{\varepsilon_{y,t}} = \frac{1}{2^Y_{K_{it}} - 1} \sigma^2_y \quad \text{disagreement about aggregate condition} \quad (8)
\]

\[
\sigma^2_{\varepsilon_{a,t}} = \frac{1}{2^A_{K_{it}} - 1} \sigma^2_{a_i} \quad (9)
\]

Limited attention \(\rightarrow\) noisy perception of true realisation of key variables
Firm’s profit-loss minimisation problem:

$$\min \{K^y_{it}, K^a_{it}\} \in \mathcal{R}^+ \quad E \left[ (p_{it} - p^*_{it})^2 | s_{it} \right] \quad \text{s.t.} \quad K^y_{it} + K^a_{it} \leq K$$
Firm’s profit-loss minimisation problem:

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\min_{\{K_{it}^y, K_{it}^a\} \in \mathcal{R}^+} \mathbb{E} \left[ (p_{it} - p_{it}^*)^2 | s_{it} \right] \quad \text{s.t.} \quad K_{it}^y + K_{it}^a \leq K
\]

Firm’s best guess of true optimal price, given signal it receives

\[
p_{it} = \mathbb{E} [p_{it}^* | s_{it}] = \varphi \mathbb{E} [y_t | s_{it}^y] - \mathbb{E} [a_{it} | s_{it}] \tag{10}
\]
Optimal Pricing Rule

Firm’s profit-loss minimisation problem:

$$\min_{\{K^y_{it}, K^a_{it}\} \in \mathcal{R}^+} E \left[ (p_{it} - p^*_{it})^2 | s_{it} \right] \quad \text{s.t.} \quad K^y_{it} + K^a_{it} \leq K$$

Firm’s best guess of true optimal price, given signal it receives

$$p_{it} = E \left[ p^*_{it} | s_{it} \right] = \varphi E \left[ y_t | s^y_{it} \right] - E \left[ a_{it} | s^a_{it} \right]$$ (10)

For a given attention allocation, standard Bayesian updating, pricing rule and noise volatility, the optimal price setting decision is:

$$p_{it} = \varphi \frac{\sigma^2_y}{\sigma^2_y + \sigma^2_{\varepsilon_y,t}} s^y_{it} - \frac{\sigma^2_a}{\sigma^2_a + \sigma^2_{\varepsilon_a,t}} s^a_{it}$$ (11)

$$= \varphi \left( 1 - 2^{-2K^y_{it}} \right) s^y_{it} - \left( 1 - 2^{-2K^a_{it}} \right) s^a_{it}$$ (12)
Optimal Pricing

Firm’s profit-loss minimisation problem:

$$\min_{\{K^y_{it}, K^a_{it}\} \in \mathcal{R}^+} \mathbb{E} \left[ (p_{it} - p^*_{it})^2 | s_{it} \right] \quad \text{s.t.} \quad K^y_{it} + K^a_{it} \leq K$$

where,

$$p_{it} = \varphi \left( 1 - 2^{-2K^y_{it}} \right) s^y_{it} - \left( 1 - 2^{-2K^a_{it}} \right) s^a_{it}$$

$$p^*_{it} = \varphi y_t - a_{it}$$
Optimal Pricing

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where,

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p_{it} = \varphi \left( 1 - 2^{-2K^y_{it}} \right) s^y_{it} - \left( 1 - 2^{-2K^a_{it}} \right) s^a_{it}
\]

\[
p^*_{it} = \varphi y_t - a_{it}
\]

Substituting optimal pricing behaviour into the expected profit loss:

\[
E \left[ (p_{it} - p^*_{it})^2 | s_{it} \right] = \varphi^2 2^{-2K^y_{it}} \sigma^2_y + 2^{-2K^a_{it}} \sigma^2_a
\]

\[
= \varphi^2 2^{-2K^y_{it}} \sigma^2_y + 2^{-2K^a_{it}} \sigma^2_a
\]

- note the independence of fundamental and noise shock
- last equality results from prior variances \( \sigma^2_y = \sigma^2_b \), as monetary policy component is observable
Optimal Attention Allocation

Optimal attention allocation to aggregate demand:

\[ K_{it}^{y*} = \frac{1}{2} \log_2 \left( \frac{\varphi \sigma_b}{\sigma_a} \right) + \frac{1}{2} K \]

The attention paid to aggregate demand is:

- increasing with total attention \( K \)
- increasing uncertainty surrounding demand \( \sigma_b \)
- decreasing in productivity uncertainty \( \sigma_a \)
  - making firms reallocate attention
Optimal Attention Allocation

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Optimal attention allocation to idiosyncratic productivity:

\[ K_{it}^{a*} = \frac{1}{2} \log_2 \left( \frac{\sigma_a}{\varphi \sigma_b} \right) + \frac{1}{2} K \]
As we have now solved for the optimal attention allocation, examine:

- How disagreement of demand conditions $\sigma_{\varepsilon_{y,t}}^2$ responds to changes in
  1. Total attention available $K$
  2. Productivity uncertainty $\sigma_a^2$
  3. Demand uncertainty $\sigma_b^2$

- The prices’ reaction to monetary policy shock in response to changes in
  1. Total attention available $K$
  2. Productivity uncertainty $\sigma_a^2$
  3. Demand uncertainty $\sigma_b^2$. 

Recall, disagreement is a function of exogenous fundamental uncertainty and related to endogenous decision of attention allocation $K_{it}$

$$\sigma_{\varepsilon_{y,t}}^2 = \frac{1}{2^{2K_{it}^*}} \sigma_b^2$$
Recall, disagreement is a function of exogenous fundamental uncertainty and related to endogenous decision of attention allocation $K_{it}^*$

$$\sigma_{\varepsilon y,t}^2 = \frac{1}{2^{2K_{it}^*} - 1} \sigma_b^2$$

Differentiating it with respect to $K$, $\sigma_a^2$ and $\sigma_b^2$ results in:

$$\frac{d\sigma^2_{\varepsilon y,t}}{dK} = -\sigma_b^2 \ln(2) 2^{2K_{it}^y} \left( \frac{1}{2^{2K_{it}^y} - 1} \right)^2 < 0 \quad (13)$$

$$\frac{d\sigma^2_{\varepsilon y,t}}{d\sigma_a^2} = \frac{1}{2} \frac{\sigma_b^2}{\sigma_a^2} 2^{2K_{it}^y} \left( \frac{1}{2^{2K_{it}^y} - 1} \right)^2 > 0 \quad (14)$$

$$\frac{d\sigma^2_{\varepsilon y,t}}{d\sigma_b^2} = -2 + 2^{2K_{it}^y} \geq 0 \quad (15)$$
Recall, disagreement is a function of exogenous fundamental uncertainty and related to endogenous decision of attention allocation $K_{it}^*$

$$
\sigma_{\epsilon_y,t}^2 = \frac{1}{2^{2K_{it}^y} - 1} \sigma_b^2
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Differentiating it with respect to $K$, $\sigma_a^2$ and $\sigma_b^2$ results in:

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\frac{d\sigma_{\epsilon_y,t}^2}{dK} = -\sigma_b^2 \ln(2) 2^{2K_{it}^y} \left( \frac{1}{2^{2K_{it}^y} - 1} \right)^2 < 0
\quad (13)
\]

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\frac{d\sigma_{\epsilon_y,t}^2}{d\sigma_a^2} = \frac{1}{2} \frac{\sigma_b^2}{\sigma_a^2} 2^{2K_{it}^y} \left( \frac{1}{2^{2K_{it}^y} - 1} \right)^2 > 0
\quad (14)
\]

\[
\frac{d\sigma_{\epsilon_y,t}^2}{d\sigma_b^2} = \frac{-2 + 2^{2K_{it}^y}}{2(2^{2K_{it}^y} - 1)^2} \geq 0
\quad (15)
\]
Comparative Statistics: Price Setting

How does prices respond to monetary shocks under different conditions?

Combining the equation on true optimal price $p_{it}$ and $s_{yt}$ where,

$s_{yt} = y_{t} + \varepsilon_{yt}$

$p_{it} = b_{t} - c_{rt} + \varepsilon_{yt}$:

\[
\frac{dp_{it}}{dr_{t}} = \frac{dp_{it}}{ds_{yt}} \cdot \frac{ds_{yt}}{dr_{t}}
\]

\[
\frac{dp_{it}}{dr_{t}} = \left(1 - 2 - 2K_{yt}\right) \cdot (-c_{rt}) < 0 \quad (16)
\]

\[
\frac{dp_{it}}{dr_{t}} = -c_{rt} \left(1 - \sigma_{a} \sigma_{b} \phi_{2} - K_{c}\right)
\]

Taking the second-order comparative statics eq (17) w.r.t. $K$, $\sigma_{a}^{2}$ and $\sigma_{b}^{2}$:

\[
\frac{d^{2}p_{it}}{dr_{t} dK} = -\ln(2) \sigma_{a} \phi \sigma_{b}^{2} - K_{c} < 0 \quad (18)
\]

\[
\frac{d^{2}p_{it}}{dr_{t} \sigma_{a}^{2}} = \frac{1}{\phi} \sigma_{b}^{2} - K_{c} > 0 \quad (19)
\]

\[
\frac{d^{2}p_{it}}{dr_{t} d\sigma_{b}^{2}} = -\sigma_{a} \phi \frac{1}{\sigma_{b}^{2} - K_{c}} < 0 \quad (20)
\]
Comparative Statistics: Price Setting

How does prices respond to monetary shocks under different conditions?

Combining the equation on true optimal price $p_{it}$ and $s_{it}^y$

where, $s_{it}^y = y_t + \varepsilon_{it}^y = b_t - cr_t + \varepsilon_{it}^y$

$$\frac{dp_{it}}{dr_t} = \frac{dp_{it}}{ds_{it}^y} \cdot \frac{ds_{it}^y}{dr_t} = \left(1 - 2^{-2K_{it}^y}\right) \cdot (-c) < 0$$  \hspace{1cm} (16)

$$\frac{dp_{it}}{dr_t} = -c \left(1 - \frac{\sigma_a}{\sigma_b \varphi} 2^{-K}\right)$$  \hspace{1cm} (17)
Comparative Statistics: Price Setting

How does prices respond to monetary shocks under different conditions?

Combining the equation on true optimal price $p_{it}$ and $s_{it}^y$

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\frac{dp_{it}}{dr_t} = -c \left(1 - \frac{\sigma_a}{\sigma_b \varphi} 2^{-K} \right) \quad (17)
$$

Taking the second-order comparative statics eq (17) w.r.t. $K$, $\sigma_a^2$ and $\sigma_b^2$:

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\frac{d^2p_{it}}{dr_t dK} = -\ln(2) \frac{\sigma_a}{\varphi \sigma_b} 2^{-K} c < 0 \quad (18)
$$

$$
\frac{d^2p_{it}}{dr_t \sigma_a} = \frac{1}{\varphi \sigma_b} 2^{-K} c > 0 \quad (19)
$$

$$
\frac{d^2p_{it}}{dr_t d\sigma_b} = -\frac{\sigma_a}{\varphi} \frac{1}{\sigma_b^2} 2^{-K} c < 0 \quad (20)
$$
The RI model offers three explanations for the empirical findings:

1. Information processing capacity $K$ of firms could be lower $\rightarrow$ firms reduce attention to aggregate conditions (and others)

2. Higher uncertainties in state variables other than aggregate conditions: $\uparrow \sigma_a \rightarrow$ firms to re-allocate attention away from aggregate conditions

3. *Decrease* in aggregate demand uncertainty have potential impact to make prices more sticky $\rightarrow$ firms reduce attention allocated to monitoring aggregate conditions
Conclusion

Main results

- Under high disagreement, monetary policy is:
  - Less powerful controlling prices
  - More powerful controlling output

- Under low disagreement, monetary policy:
  - Reduce inflation significantly
  - Little output loss

- Mechanism of uncertainty & disagreement for MTM differ

Policy implications:

- Improved CB communications help agents form expectations
- Reducing disagreement $\rightarrow$ lower sacrifice ratio
Thank you!
• More difficult than a standard IRF, due to the non-linearities
• Use Monte-Carlo integration to compute the conditional response for:
  ▶ variable $y$, shock size $\delta$, history $\Omega_{t-1}$ and horizon $h = 0, 1, \ldots, H$
• Then average out over each regime’s set of random histories $\Omega^r$, to get the unconditional responses for each regime
Generalised Impulse Response Function Algorithm

IRF measures the difference between two conditional expectations of the realisation of $y_{t+h}$, $h = 0, 1, \ldots, H$, where the first expectation is conditional on the information set available at date $t - 1$.

$$I_y(h, \delta, \Omega_{t-1}) = \mathbb{E}(y_{t+h} | \omega_{it} = \delta, \Omega_{t-1}) - \mathbb{E}(y_{t+h} | \Omega_{t-1})$$

Algorithm

1. Pick a history $\Omega_{t-1}$ and choose structural shock of magnitude $\delta$
2. Given $\Omega_{t-1}$, simulate two time paths. For example:
   - First time path: $\delta = 1$ – with structural shock
   - Second time path: $\delta = 0$ – without structural shock
3. Subtract the second from first time path. The difference is the estimate of GIRF, conditional on $\Omega_{t-1}$. However, it's a noisy estimate.
4. To eliminate the random variation in the GIRF, repeat steps 2 and 3 many times and average the resulting impulse response estimates.