Innovation and Technological Locations of Firms: An Agent-based Approach

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Abstract

We investigate the relationship between location choices of firms in a technology space and innovations at the firm and technological category levels. We estimate firm distribution in a technology space with patent citation overlaps and trace the locational changes of each firm. We find that a firm generates more quality-adjusted patents if it moves less in the technology space it belongs, the average distance to other firms is smaller, and technology is less accumulated around that location. At the technological category level, patent productivity tends to be high with low average moving distance but high standard deviation of it, low average distance between firms, and high entry rate. We then develop an agent-based model that generates dynamic behaviors of firms in a technology space. In the model, a firm sequentially chooses its location to maximize firm value conditional on the location profile of other firms minus the cost of moving. The firm value depends on knowledge spillover and market competition, where those factors are related to technological distances. With agent-based simulation, we analyze the impacts of regulation shifts such as patent protection policy and R&D subsidy.

Keywords: Agent-based model, innovation, technological distance.

JEL classification: O32, C63

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1 Introduction

In this study, we develop an agent-based model to analyze how inter-firm technological relations affect firm-level and aggregate innovations. A firm’s R&D strategy depends on the prospect of the current R&D projects, market conditions such as the number of rivals, and regulations such as intellectual property right policies. Based on the existing literature, firm-level R&D productivity does not stand alone but is significantly related to another firm typically through knowledge spillover. Dating back to the seminal paper by Jaffe (1986), the impact of knowledge spillover, proxied by technological proximity, on R&D has been repeatedly observed. Apart from technological relationship, firms are also related in markets. As Bloom et al. (2013) point out, the level of market competition also affects R&D strategies by reducing the profits stemming from innovation.

To see these impacts from firm relations clearly, we consider virtual technology spaces where firms choose their locations by R&D strategy. Technological relations among firms are described as firm distribution in a technology space. If two firms are close in the technology space, they conduct similar R&D projects. Firms change their locations in the technology space according to the location profiles of the others. We estimate firm locations in a technology space with using patent citation overlaps for relational data and trace the dynamics of firm locations. As presented in the next section, both firm-level and category-level estimations show significant relationship between the amount of quality-adjusted patent applications and moving distances. Moreover, properties of firm distribution in a technology space, such as average distance to others, also matter for innovation.\footnote{Kitahara and Oikawa (2017) investigate the degree of polarization of firm distribution in technology space affects innovation output.}

Then, we build an agent-based model of firms’ location changes in a technology space, which can regenerate the observed movements of firms. The important components of the model are knowledge spillover and market competition. When firms are located with closer distances to one another, they will benefit from knowledge spillover in R&D activities. But, at the same time, firms in a close range tend to produce rival goods and compete in a similar market. There exists a tradeoff, implying the existence of the optimal distance to others. After an theoretical analysis with a simple setting, we simulate the full model to show how firms behave in a technology space and what parameter set generates the observed properties of distributions of firm locations and moving distances. Then we introduce policy parameters such as patent width/length and R&D subsidy into the agent-based model to examine the impacts of innovation policies. Final evaluation of those policies are calculated by combination of the simulated movings of firms and estimated impacts of moving statistics on innovation outputs.

The rest of the paper is organized as follows. In Section 2, we estimate the relationship
between firms’ locational changes in technology spaces and innovation outputs. In section 3, we build a simple model of firms location change and show some theoretical results, which provide overall tendency of firm distribution generated from the model. In section 4, we set up the full agent-based model and simulate it. Section 5 is policy experiments.

2 Observation

In this section, we observe the dynamic behavior of firms in technology spaces. We use the NBER US patent database, which contains patents granted during 1976-2006 at the United States Patent and Trademark Office. To locate firms in technology spaces, we apply the method described in Kitahara and Oikawa (2017). In their paper, technological dissimilarity is measured with using pairwise patent citation overlaps and firm locations in a technology space are estimated through multi-dimensional scaling. One advantage of this method is that we can trace the dynamic changes of firm distributions within technological categories.2

We consider two-digit classification, which is called subcategory, defined by Hall et al. (2001). There are 31 subcategories except for “miscellaneous” subcategories. A firm belongs to a technology space when it applies a successful patent (namely, granted later). The locations of those firms are estimated on a two-dimensional technology space in each year. When firms locate close to each other, their patent citation lists have more common elements, which we interpret that they are technologically close. A firm can belong to multiple technology spaces if it applies patents in multiple subcategories. Since we also computed intertemporal dissimilarity of the same firm, firm location mappings are dynamically connected. Thus, we trace dynamic movements of firms. Figure 1 exemplifies the post-MDS firm location mappings for year of 1980, 1990, and 2000 in subcategory of computer hardware and software. We estimated these mappings for every subcategory (except for biotechnology because of too small number of patents in early years) and every year (1976-2005 for the data constraint).

In this paper, we focus on how firms move in technology spaces and, more importantly, how those movements are related to innovation output. In the rest of this section, we present some typical facts learned from observing firm-level moving in a technology space.

2.1 Trends in Moving Distances

First, we confirm that firms significantly move in a technology space. Figure 2 depicts estimated densities of moving distances for three distinct years, where subcategories are pooled. For reference, the estimated firm location maps lie within an area of 20 × 20 at

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2Stuart and Podolny (1996) also considers citation overlaps to estimate firm locations on a technology space. See Oikawa (2017) for comparison between Stuart and Podolny (1996) and the current method.
widest (scale is meaningless). Second, the distribution of moving distances is right-skewed and the skewness is increasing over time. The modes of those moving distance distributions lie on a relatively short distance range but there are some firms that move much longer than typical firms. This tendency is common across subcategories. Figure 3 illustrate the time-series movement of average moving distances and skewness of moving distances. Clearly, there exists an upward trend in skewness and a downward trend in mean.

We interpret that long moving distance implies that a firm conducts an R&D project which is new for the firm. Thus we can consider the thickness of the tail distribution of moving distance is an indicator that indicates how many firms take a risk in their R&D activities. In this line of interpretation, the time-series property of moving distance distribution implies that the number of risk-taking firms has been getting smaller in the United States.

2.2 Firm Location and Innovation: Firm-level Estimation

We estimate the relationship between moving in technology spaces and innovation outputs, which is measured by citation-weighted patents. Because citation-weighted patents are count data, we use Poisson and negative binomial regressions. The explanatory variables are each firm’s moving distance from the location in the previous period (MoveDist), square of it (MoveDist2). We also take into account the locational property for each firm in the previous year such as the average distance to other firms (AvgDistOthL1), technology accumulation (TechCumL1) which is defined later. We control firm size captured by sales in all estimation and control knowledge stock: firm’s patent stock in the same category (PatStock), firm’s patent stock across categories (AllPatStock), both with the depreciation rate of 15%.3 All

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315% depreciation is conventional and often used (Aghion et al. (2005)). The actual depreciation rate should be different across subcategories. See Li (2012).
estimations include technological category dummies and year dummies. Based on the agent-based model described in Section 4, we define technology accumulation as

$$\text{TechCum}(x) = \sum_{\tau=1}^{t-1} \frac{\beta Z}{\tau} \sum_{x' \in X_\tau} f(\delta_{x,x'} | \sigma Z),$$

where $x$ is a position of a firm, $X_\tau$ is the firm location profile in period $\tau$, $\delta_{x,x'}$ is the Euclidean distance between $x$ and $x'$. $f$ is a Gaussian weight such as

$$f(\delta | \sigma) = \frac{\exp \left\{ -\delta^2 / 2\sigma^2 \right\}}{\sqrt{2\pi\sigma}}.$$

$\beta Z$ is a depreciation rate and $\sigma Z$ is the span of influence of technology accumulation. Here, we use $\beta Z = 0.5$ and $\sigma Z = 1$. Technology accumulation at $x$ is high if many firms have located around $x$ in the past, implying that a large part of technological potential in such an area has been already developed. We expect that technology accumulation is negatively affects the current innovation outputs.

Table 1 summarizes the robust variance estimation results. First, moving distance and innovation has nonmonotonic relationship. Shorter moving distance is better for R&D out-

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AllPatStock and Sales are firm-level variable. The other variables are subcategory x firm-specific variables. PatStock, AllPatStock, and Sales are taken log.
put but such a negative impact of a long journey is restrained as the distance is getting farther probably because a firm would not migrate over long distance if it could not have a good prospect in the new field, e.g., there is a technology boom around the destination. Second, the lagged average distance to others is negatively associated with innovation after controlling knowledge stocks, implying knowledge spillover. If many firms around, R&D efficiency is boosted. Third, the lagged technology accumulation is also negatively associated with innovation. Unlike average distance to others, technology accumulation is based on the history of the location profiles. High technology accumulation implies that there have been more firms around in the past. Hence, such a region in a technology space is somewhat obsolete, or we have eaten all low-hanging fruits around there (Cowen (2011)). The significant negative coefficients implies that such a fishing-out effect matters.

Table 2 includes cross term between moving distance and the current average distance to others (MoveDist×AvgDistOth), and moving distance and the current technology accumulation (MoveDist×TechCum). Both coefficients are significantly negative. Hence, when a firm moves long distance, shorter average distance to others and smaller technology accumulation are better for innovation output. This result implies that, a firm moving long distance comes to an area currently crowded but relatively new in the history.⁵

⁵We observe that young or small firms, typically entrants, tend to move longer.
### Table 1: Firm location, moving distance, and innovation

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* z scores in parentheses. * p < 0.10, ** p < 0.05, *** p < 0.01
Table 2: Destination matters

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<td>MoveDist × AvgDistOth</td>
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* z scores in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$
2.3 Firm Distribution and Innovation: Subcategory-level Estimation

Table 3 shows the subcategory level panel estimation with subcategory fixed effect and year dummies. The dependent variable is the average citation-weighted patent application within subcategories. Explanatory variables are the average moving distance (AvgMD), average distance among firms in the previous period (AvgDistL1), the average technology accumulation among firms in the previous period. We also consider standard deviation and skewness of moving distances, and the subcategory-level entry rate. The negative impact of moving distance still holds after subcategory-level aggregation. Similarly, firm-level results still hold for average distance and technology accumulation, implying knowledge spillover and the fishing-out effect. About the new variables, standard deviation positively matters for overall innovation at the subcategory level. Even though low average distance is good, no jump in technology space is bad. However, the skewness does not matter. The subcategory-level entry rate is associated with innovation output. It is natural that a growing technology field attracts new entrants.

3 Simple Analytical Model

In this section, we present an analytical framework which becomes the basis of the subsequent agent-based simulations. By looking at the simplified analytical model, we make the basic mechanism of firms’ location changes on a technology space clear.

There are \(n\) firms in a technology space.\(^6\) Firm \(i\) has a position \((x_i, y_i)\) in the technology space. We suppose that the value of each firm depends on the distances to other firms on the technology space in two ways. First, technological proximity implies more knowledge spillover from each other.\(^7\) It positively affects firm value by expecting higher R&D productivity. Second, the distance on the technology space negatively affects profit flows of firms because they tend to produce similar products if they are technologically similar to each other.\(^8\)

We formulate the value of firm \(i\), conditional on the current location profile, as

\[
v_i = \frac{1}{n-1} \left[ w \sum_{j \neq i} f(\delta_{ij} | \sigma_T) - \sum_{j \neq i} f(\delta_{ij} | \sigma_M) \right],
\]

where \(\delta_{ij}\) is technological distance between firms \(i\) and \(j\), \(w > 0\) is the weight on the

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\(^6\)Technology space and industry are not exactly matched with each other.

\(^7\)Technology spillover is a significant factor to determine R&D productivity. cf. Jaffe (1986).

\(^8\)Bloom et al. (2013) consider technology spillover effect and market competition effect.
Table 3: Category level estimation

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$t$ statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$
technology factor, and function $f$ is a Gaussian weighting function,

$$f(\delta|\sigma) = \frac{\exp\left\{-\frac{\delta^2}{2\sigma^2}\right\}}{\sqrt{2\pi\sigma}}. \tag{2}$$

$\sigma_T$ and $\sigma_M$ are the spans of influence of knowledge spillover and market competition, respectively. We call $v_i$ in equation (1) as conditional firm value, meaning that it is the firm value when the current firm locations are fixed. Note that negative $v_i$ does not necessarily imply exit because it turns out to be positive if the firm changes its location appropriately.

Suppose that firms choose their locations sequentially. Firm $i$ maximizes $v_i$ by choosing its position, taking other firms’ locations given. We assume there is no cost of moving (which is relaxed in the agent-based simulation). Firm $i$ increases $x_i$ if and only if

$$\frac{w}{\sigma_T^2} \sum_{j \neq i} (x_j - x_i) e^{-\frac{\delta_{ij}^2}{2\sigma_T^2}} > \frac{1}{\sigma_M^2} \sum_{j \neq i} (x_j - x_i) e^{-\frac{\delta_{ij}^2}{2\sigma_M^2}}. \tag{3}$$

Similarly for $y_i$.

First, we consider a special Case in which $\sigma_T = \sigma_M \equiv \sigma$. When the technology effect and market effect are equally evaluated, there is no substantial tradeoff between them. Condition (3) is now

$$(w - 1) \sum_{j \neq i} (x_j - x_i) e^{-\frac{\delta_{ij}^2}{2\sigma^2}} > 0. \tag{4}$$

If $w > 1$, $x_i$ should be increased when

$$x_i < \sum_{j \neq i} \left[ \frac{\exp\left\{-\frac{\delta_{ij}^2}{2\sigma^2}\right\}}{\sum_{h \neq i} \exp\left\{-\frac{\delta_{ih}^2}{2\sigma^2}\right\}} \right] x_j,$$

or $x_i$ is less than the weighted average of other firms positions. On the contrary, if $w < 1$, then $x_i$ is increased when it is greater than the weighted average of the others’ positions. Hence, $w < 1$ gives firms diverging incentives and the system turns out a divergence equilibrium, i.e., $\delta_{ij} \to \infty$ for any $i, j$. Contrarily, $w > 1$ gives them converging incentives and the system turns out a convergence equilibrium, i.e., $\delta_{ij} = 0$ for all $i, j$. When $w = 1$, they have no incentive to move (any distributions are indifferent).

3.1 Different Spans of Influence 1: $\sigma_T > \sigma_M$

Next, we suppose that the technology factor more widely influence than the market factor, namely $\sigma_T > \sigma_M$. To see what happens in location choices clearly, we start with only two firms at distinct locations: $(x_i, y_i)$ where $i = 1, 2$. Without loss of generality, we assume
\(x_2 > x_1\). Denote \(\delta = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\). Condition (3) implies that \(x_1\) should be increased if
\[
\frac{w}{\sigma_T^3} (x_2 - x_1) e^{-\frac{\delta^2}{2\sigma_T^2}} > \frac{1}{\sigma_M^3} (x_2 - x_1) e^{-\frac{\delta^2}{2\sigma_M^2}},
\]
or
\[
\frac{\sigma_M^3}{\sigma_T^3} > e^{-\frac{\delta^2}{2} \left( \frac{1}{\sigma_M^2} - \frac{1}{\sigma_T^2} \right)} \iff \delta^2 > \frac{2 \ln \frac{\sigma_T^3}{\sigma_M^3}}{\frac{1}{\sigma_M^2} - \frac{1}{\sigma_T^2}} \equiv \delta^*^2. \tag{5}
\]

To have real \(\delta^*\) in equation (5), we need
\[
w < \frac{\sigma_T^3}{\sigma_M^3} \equiv \hat{w}. \tag{6}
\]

If \(w \geq \hat{w}\), then \(x_1\) should be increased and thus firm 1 approaches firm 2 for any current distance, \(\delta\). However, if \(w < \hat{w}\), then \(x_1\) should be increased if and only if \(\delta > \delta^*\). In other words, firm 1 has a converging incentive if it is sufficiently distant from firm 2 and it has a diverging incentive if sufficiently close to firm 2. Hence, \(\delta^*\) is the optimal distance between firm 1 and 2. \(\delta^*\) is decreasing in \(w\) with \(\lim_{w \to 0} \delta^* = \infty\), \(\lim_{w \to \hat{w}} \delta^* = 0\). The case in which \(x_2 < x_1\) is analogous.\(^9\) The same arguments hold for the other axis, \(y\).

The following proposition summarizes the above result and it also holds for the case with \(n\) firms.

**Proposition 1** Suppose \(\sigma_T > \sigma_M\). If \(w \geq \hat{w}\), then there exists a convergence equilibrium. If \(w < \hat{w}\) and \(n - 1\) does not exceed the dimension of the technology space, then there exists an equilibrium with pairwise distance of \(\delta^*\).

Proof is in Appendix. Since a firm enjoys technological benefit (spillover) of similarity even in the region that the competition effect of similarity is insignificant, they tend to conform to each other until the competition effect gets significantly large. There exists unique threshold, \(\delta^*\), if the importance of technology spillover effect, \(w\), is sufficiently small. Dimension matters because, for example, we cannot place more four or more firms with the identical pairwise distance on a two-dimensional plane. On the other hand, if \(w\) is so large that competition does not matter relatively to knowledge spillover effect, they converge to one another.

### 3.2 Different Spans of Influence 2: \(\sigma_T < \sigma_M\)

Next, we assume that \(\sigma_T < \sigma_M\) and \(n = 2\). Assume \(x_2 > x_1\) WLOG. Condition (3) implies that \(x_1\) should be increased if \(\delta < \delta^*\). In other words, with the cutoff distance \(\delta^*\)
\(^9\)If \(x_1 = x_2\), the first order condition for \(x_1\) holds for any \(w\) and \(\delta\) but it is a local max. A bit perturbation leads to divergence of firms as long as \(w < \hat{w}\).
defined in equation (5), firm 1’s marginal return of approaching firm 2 is positive if they are sufficiently close, and it is negative otherwise. This implies that the optimal distance for them is a corner solution, 0 or $\infty$.

Substitute $\delta = 0$ and $\delta = \infty$ into equation (1),

$$v_1(0) = \frac{1}{\sqrt{2\pi}} \left( \frac{w}{\sigma_T} - \frac{1}{\sigma_M} \right), \quad v_1(\infty) = 0.$$  \hspace{1cm} (7)

Then, $v_1(0) > v_1(\infty)$ if and only if

$$w > \frac{\sigma_T}{\sigma_M} \equiv \bar{w}.$$  \hspace{1cm} (8)

**Proposition 2** Suppose $\sigma_T < \sigma_M$. If $w < \bar{w}$, then there exists a divergence equilibrium. If $w > \bar{w}$, then there exists a convergence equilibrium.

Proof is in Appendix.10

The resulting equilibrium for $\sigma_T > \sigma_M$ and $\sigma_T < \sigma_M$ are drastically different. This simple analytical model tells us basic structure of location choice in a technology space. One important takeaway is that the case with $\sigma_T > \sigma_M$ seems relevant because the observation illustrated in the previous section suggests that firms are neither diverging nor converging over time. When it comes to reality, we abstract many aspects in the current theoretical model. First, we did not consider the cost of moving in a technology space. Moreover, firms are heterogeneous in, e.g., size and R&D efficiency that may affect spans of influences and weight of the technology factor. Thus, the optimal distance, $\delta^*$, should be different among firms in a more realistic circumstance. To see what happens under richer settings, let us move on to the agent-based model.

### 4 Agent-based Simulation

4.1 Setup

Here we present an agent-based model of location changes of firms in a technological space. Each agent is a firm who sequentially optimizes its location, taking the location profile of other firms as given.

To simplify the notation in the following formulation, let us define the following function:

$$F(x|X, \sigma) \equiv \sum_{x' \in X} f(\delta_{x,x'}|\sigma).$$  \hspace{1cm} (9)

10Notice that Propositions 1 and 2 do not eliminate asymmetric equilibrium in which condition (3) or (16) holds with equality for all firms, associated with negative and positive $\lambda$’s. We can show that there does not exist an asymmetric equilibrium for $n = 3$. 

13
Firm’s conditional value function is based on equation (1):

\[
v(x_{it}|X_{-i,t},X^t) = \frac{1}{n-1} \left[ \alpha(x_{it}|X^t)w(x_{it}|X^t)F(x_{it}|X_{-i,t},\sigma_T) - F(x_{it}|X_{-i,t},\sigma_M) \right],
\]

where \( x_{it} \) is firm \( i \)’s location in period \( t \), \( X_t \) is the location profile of all firms in period \( t \), \( X_{-i,t} \equiv X_t \setminus \{x_{it}\} \), and \( X^t \equiv \{X_1,X_2,...,X_{t-1}\} \). \( \alpha(x_{it}|X_{-i,t}) \in [0,1] \) is patent risk of infringement in that they obtain zero technological benefit with probability of \( 1 - \alpha \). The weight on technological factor, \( w \), is location- and history-dependent. We specify

\[
w(x|X^t) \equiv \bar{w} \exp\left\{ -Z(x|X^t) \right\}, \quad \bar{w} > 1,
\]
\[
Z(x|X^t) \equiv \sum_{\tau=1}^{t-1} \beta_{\alpha}^{t-\tau} F(x|X_{\tau},\sigma_{\alpha}), \quad \beta_{\alpha} \in (0,1).
\]

The idea behind this formulation is as follows. Each technological position originally has fixed potential to be developed (\( \bar{w} \)). When the potential is not much developed, technology spillover contributes to sophisticate the technology at the current location. As number of firms in the neighborhood of the location increases, the potential becomes revealed and the technology is getting obsolete or belonging to public knowledge base. Then, the relative importance of market pressure rises. We call \( Z(x|X^t) \) as technology accumulation at location \( x \).

Technology accumulation also affects patent risk. We formulate

\[
\alpha(x|X^t) = \exp\left\{ -\sum_{\tau=1}^{t-1} \beta_{\alpha}^{t-\tau} F(x|X_{\tau},\sigma_{\alpha}) \right\}.
\]

Similarly to the technology weight above, the patent risk is increasing as more firms conduct research and development in the neighborhood on the current technology space. It is worth noting that the parameters \( \beta_{\alpha} \) and \( \sigma_{\alpha} \) are policy parameters such as patent length and patent width, respectively.

We consider sequential location choice of a firm where the opportunity of move is at random. Each firm chooses its new location \( x_{it} \) to maximize the value (10) minus the cost of moving:

\[
c_f + c(\delta(x_{i,t-1},x_{it})),
\]

where \( c_f \geq 0 \) is the fixed cost. Variable costs satisfy \( c(0) = 0 \) and \( c'(\cdot) > 0 \).
4.2 Simulation Results

Figure 4 shows the simulation result of the standard setting without patent risk. The number of firms is 10 and they are homogeneous except locations. $\sigma_T > \sigma_M$ and there is no fixed cost of moving. We run 100 periods and at each period a randomly chosen firm choose a location by comparing the marginal return from conditional firm value and marginal cost of moving, taking the current location profile as given.

TBW.

5 Policy Experiments

TBW.

References


A Proofs

For convenience of exposition, we define a new variable:

$$
\lambda_{ij} \equiv \frac{w \sigma_M^3}{\sigma_T^3} \exp \left( -\frac{\delta_{ij}^2}{2\sigma_T^2} \right) - \exp \left( -\frac{\delta_{ij}^2}{2\sigma_M^2} \right).
$$

Using this notation, the first-order condition implies that $x_i$ should be increased when

$$
\sum_{j \neq i} (x_j - x_i) \lambda_{ij} > 0.
$$

Proof of Proposition 1  Suppose $w \geq \hat{w}$. From $\sigma_T > \sigma_M$, $\lambda_{ij} \geq 0$ for any pair of firms, regardless of distances. Then, we can rewrite the condition (16) as

$$
x_i < \sum_{j \neq i} \frac{\lambda_{ij}}{\sum_{h \neq i} \lambda_{ih}} x_j.
$$

Therefore, $x_i$ is increased if it is less than the weighted average of other firm positions on the $x$-axis, implying a converging incentive. Hence, there exists a convergence equilibrium when $w \geq \hat{w}$. 

16
Next, suppose \( w < \hat{w} \). In this case, \( \delta^* \) is well-defined and

\[
\lambda_{ij} > 0 \quad \Leftrightarrow \quad \delta_{ij} > \delta^*.
\]

Consider a situation in which \( \delta_{ij} > \delta^* \) for all \( i, j \). Then, The condition (17) holds and firm \( i \) has a converging incentive. On the other hand, when \( \delta_{ij} < \delta^* \) for all \( i, j \), condition (17) is replaced with

\[
x_i > \sum_{j \neq i} \frac{|\lambda_{ij}|}{\sum_{h \neq i} |\lambda_{ih}|} x_j.
\]

Therefore, \( x_i \) is increased if it is greater than the weighted average of other firm positions on the \( x \)-axis, implying a diverging incentive.

These results imply that there exists an equilibrium with \( \delta_{ij} = \delta^* \) for all pairs of firms. However, it is impossible to locate firms with the identical pairwise distance if the number of firms is greater than the number of dimensions. \( \blacksquare \)

**Proof of Proposition 3** Suppose \( w < \hat{w} \), yielding \( \lambda_{ij} < 0 \) for any pair of \( i, j \), regardless of distances. Then we can rewrite inequality (16) to

\[
x_i > \sum_{j \neq i} \frac{|\lambda_{ij}|}{\sum_{h \neq i} |\lambda_{ih}|} x_j.
\]

Therefore, \( x_i \) is increased if it is greater than the weighted average of other firm positions on the \( x \)-axis, implying a diverging incentive.

Next, suppose \( w \geq \hat{w} \), implying that \( \delta^* \) is well defined. When all pair of firms have \( \delta_{ij} > \delta^* \), again \( \lambda_{ij} < 0 \) for all pairs. The situation is the same as the above and firms have diverging incentives.

If \( \delta_{ij} < \delta^* \) for any pair of firms, then we can rewrite inequality (16) to

\[
x_i < \sum_{j \neq i} \frac{\lambda_{ij}}{\sum_{h \neq i} \lambda_{ih}} x_j.
\]

Therefore, \( x_i \) is increased if it is less than the weighted average of other firm positions on the \( x \)-axis, implying a converging incentive.

Since we have two corner solutions under \( w \geq \hat{w} \), compare \( \delta_{ij} \to \infty \) for all \( i, j \) and \( \delta_{ij} = 0 \) for all \( i, j \). The conditional firm value in the limit is the same as equations in (7).

Thus, there is no incentive to deviate from the convergence equilibrium if \( w > \hat{w} \). On the other hand, there is no incentive to deviate from the divergence equilibrium if \( w < \hat{w} \). Since \( \hat{w} < \bar{w} \) under \( \sigma_T < \sigma_M \), we have the statement in the proposition. \( \blacksquare \)
Asymmetric Equilibria? Here we consider possibility to have two or more distinct strictly positive distances as equilibrium. Suppose \( n = 3 \). For the loss of generality, \((x_1, y_1) = (0, 0), (x_2, y_2) = (a, 0), (x_3, y_3) = (b, c)\). So \( \delta_{12} = a, \delta_{13} = \sqrt{b^2 + c^2} \), and \( \delta_{23} = \sqrt{(b-a)^2 + c^2} \). (Equilibrium just specifies the “shape” of mappings. No exact positions, no rotation dependence). We assume that there exist interior solutions for all firms.

The first-order conditions about \( \{x_1, x_2, x_3, y_1, y_2, y_3\} \) are depending on each other. We use the following three conditions (about \( x_1, x_2, y_1 \)):

\[
\frac{1}{\omega} \left[ ae - \frac{a^2}{2\sigma_T^2} + be - \frac{b^2 + c^2}{2\sigma_T^2} \right] = ae - \frac{a^2}{2\sigma_M^2} + be - \frac{b^2 + c^2}{2\sigma_M^2},
\]

(21)

\[
\frac{1}{\omega} \left[ -ae - \frac{a^2}{2\sigma_T^2} + (b-a)e - \frac{(b-a)^2 + c^2}{2\sigma_T^2} \right] = -ae - \frac{a^2}{2\sigma_M^2} + (b-a)e - \frac{(b-a)^2 + c^2}{2\sigma_M^2},
\]

(22)

\[
\frac{1}{\omega} \left[ ce - \frac{b^2 + c^2}{2\sigma_T^2} \right] = ce - \frac{b^2 + c^2}{2\sigma_M^2},
\]

(23)

where

\[
\omega = \frac{\sigma_T^3}{w\sigma_M^3}.
\]

(24)

From (23), as long as \( c \neq 0 \),

\[
b^2 + c^2 = \frac{2\ln \omega}{\frac{1}{\sigma_M^2} - \frac{1}{\sigma_T^2}} \quad (\therefore \delta_{13}^2 = \delta^*^2).
\]

(25)

Note that

\[
e^{-\frac{\sigma_T^2}{2\sigma^2}} = \omega e^{-\frac{\sigma_T^2}{2\sigma_M^2}}.
\]

(26)

Then, from (21), we have

\[
ae - \frac{a^2}{2\sigma_T^2} = \omega ae - \frac{a^2}{2\sigma_M^2}.
\]

(27)

Hence, if \( a \neq 0 \),

\[
a^2 = \delta^*^2.
\]

Finally, from (22) and (26),

\[
(b-a)e - \frac{(b-a)^2 + c^2}{2\sigma_T^2} = \omega (b-a)e - \frac{(b-a)^2 + c^2}{2\sigma_M^2},
\]

(28)

which implies \((b-a)^2 + c^2 = \delta^*^2\) if \( a \neq b \).

Therefore, if an equilibrium mapping has distinct locations of firms, their pair-wise distance must be \( \delta^* \).
B Extension of the Analytical Model

B.1 Model Extension 1: Group Size Matters

For the number of firms within groups to matter, we introduce agglomeration and congestion effects. The more firms in a group, the more spillover and competition effects within the group.

Instead of equation (1), let us redefine the objective function such that

\[
v_i = w \left[ \sum_{j \neq i}^{n} f_T(\delta_{ij})^{\frac{1}{\alpha}} \right]^\alpha - \left[ \sum_{j \neq i}^{n} f_M(\delta_{ij})^{\frac{1}{\beta}} \right]^\beta,
\]

(29)

where \( \alpha, \beta \geq 1 \) for \( k = T, M \). Large \( \alpha \) or \( \beta \) implies strong complementarity within group, that is, a high agglomeration effect or a high congestion effect, respectively.

To see how it works, consider there are two groups: \( n_1 + 1 \) and \( n_2 \) \( (n_1 + n_2 = n - 1) \), and the distance between groups is \( \delta \). Then for firm \( i \) in group 1,

\[
v_i = w \left[ n_1^{\alpha} f_T(0) + n_2^{\alpha} f_T(\delta) \right] - \left[ n_1^{\beta} f_M(0) + n_2^{\beta} f_M(\delta) \right]
\]

(30)

The incentive to deviate from the current position depends on the number of firms in each group. If a firm considers the technological factor is more important, it is less attractive to deviate from the current group when \( n_1 \) is larger.

The first-order condition in a general case is

\[
w \left[ \sum_{j \neq i}^{n} f_T(\delta_{ij})^{\frac{1}{\alpha}} \right]^{\alpha-1} \left[ \sum_{j \neq i}^{n} \frac{x_i - x_j}{\delta_{ij}} f_T'(\delta_{ij}) f_T(\delta_{ij})^{\frac{1}{\alpha}-1} \right] > \left[ \sum_{j \neq i}^{n} f_M(\delta_{ij})^{\frac{1}{\beta}} \right]^{\beta-1} \left[ \sum_{j \neq i}^{n} \frac{x_i - x_j}{\delta_{ij}} f_M'(\delta_{ij}) f_M(\delta_{ij})^{\frac{1}{\beta}-1} \right] \Rightarrow x_i \uparrow
\]

(31)

Two Firms Equation (31) is simple if \( n = 2 \). Let \( \delta_{12} = \delta \).

\[
w(x_1 - x_2) f_T'(\delta) > (x_1 - x_2) f_M'(\delta) \Rightarrow x_i \uparrow,
\]

(32)

which is independent of \( \alpha \) and \( \beta \). Hence, the results for \( n = 2 \) is exactly the same as in the previous section if we set a Gaussian function for \( f \)’s.
Three Firms  Assume $n = 3$ and $f$'s follow Gaussian. (31) becomes

$$
\frac{1}{\omega} \left[ \sum_{j \neq 1} e^{\frac{s_{i1}^2}{2\sigma_M^2}} \right]^{a-1} \left[ (x_2 - x_1) e^{\frac{s_{i1}^2}{2\sigma_T^2}} + (x_3 - x_1) e^{\frac{s_{i1}^2}{2\sigma_M^2}} \right] > \left[ \sum_{j \neq 1} e^{\frac{s_{i1}^2}{2\sigma_M^2}} \right]^{b-1} \left[ (x_2 - x_1) e^{\frac{s_{i1}^2}{2\sigma_M^2}} + (x_3 - x_1) e^{\frac{s_{i1}^2}{2\sigma_M^2}} \right] \Rightarrow x_1 \uparrow \quad (33)
$$

Now suppose that $\delta_{ij} = \delta$ for any pair of firms. Then, the above condition becomes

$$
\frac{1}{\omega} 2^{n-\beta} e^{\frac{\delta^2}{2\sigma_T^2}} [(x_2 - x_1) + (x_3 - x_1)] > e^{\frac{\delta^2}{2\sigma_M^2}} [(x_2 - x_1) + (x_3 - x_1)] \Rightarrow x_1 \uparrow. \quad (34)
$$

If $x_1 < \frac{x_2 + x_3}{2}$, firm 1 increases $x_1$ (approaching the mid point among $x_2$ and $x_3$) if and only if $\delta > \delta^*(\alpha, \beta)$, where

$$
\delta^*(n, \alpha, \beta)^2 = \frac{2[\ln \omega - (\alpha - \beta) \ln(n - 1)]}{\sigma_M^2 - \frac{1}{\sigma_T^2}}.
$$

(35)

On the other hand, if $x_1 > \frac{x_2 + x_3}{2}$, firm 1 decreases $x_1$ if and only if $\delta > \delta^*(\alpha, \beta)$ (approaching the others). Hence, $\delta$ is stable around $\delta^*$ if $\delta^*(\alpha, \beta)$ is well-defined.

To have well-defined $\delta^*(\alpha, \beta)$, we need

$$
\frac{\sigma_T^2}{w\sigma_M^2} > (n - 1)^{a-\beta} \quad \text{if} \quad \sigma_T^2 > \sigma_M^2, \text{vice versa.} \quad (36)
$$

When $\delta^*(n - 1, \alpha, \beta)$ is not well-defined, $\delta \to 0$ or $\delta \to \infty$. Compare the benefits for those corner solutions.

**Proposition 3** Fix $\sigma_T, \sigma_M, w$. Assume $\alpha > \beta$. There exists $N(\sigma_T, \sigma_M, w)$ finite such that $\alpha > \beta$ implies an convergent equilibrium for $n \geq N$.

**Grouping Equilibrium** Suppose that $\delta^*(n - 1, \alpha, \beta)$ is well-defined. Consider a situation in which firm 1 and 2 are at the same position and firm 3 is distant from them by $\delta$. Assume $x_3 < x_1 = x_2$.

Firm 3 has an incentive to get closer to the others if

$$
e^{\frac{\delta^2}{\sigma_M^2} \left( \frac{1}{\sigma_M^2} - \frac{1}{\sigma_T^2} \right)} > \omega^{\beta - \alpha}.
$$

(37)

Thus, the optimal distance for firm 3 to group 1&2 is $\delta^*(2, \alpha, \beta)$. 

20
On the other hand, for firm 1 (and 2),

$$e^{\delta} \left( \frac{\frac{1}{\sigma^2}}{\frac{1}{\sigma^2}} \right) > \omega \left[ 1 + e^{-\frac{\delta^2}{2\sigma^2}} \right]^{\beta-1} \left[ 1 + e^{-\frac{\delta^2}{2\alpha}} \right]^\alpha \Omega^{\alpha-1}. \quad (38)$$

When $\delta = \delta^*(2, \alpha, \beta)$,

$$2^{\beta-\alpha} > \frac{1 + e^{-\frac{\delta^2}{2\sigma^2} M}}{1 + e^{-\frac{\delta^2}{2\beta}}}^{\alpha-1} = \frac{1 + (\omega 2^{\beta-\alpha})^{-\frac{1}{\alpha}}}{1 + (\omega 2^{\beta-\alpha})^{-\alpha}} \Rightarrow x_1 \uparrow \quad (39)$$

Special case: Assume $\sigma_T > \sigma_M, \alpha > 1$ and $\beta = 1$. In this case, the above condition tells us

$$\omega < 2^{1-\alpha} \Rightarrow x_1 \uparrow \quad (40)$$

"2" in the equation is the group size. So, this example suggests that a larger group size attracts firms more if $\alpha > \beta$. 

21