Sovereign Default Triggered by Incapability to Repay Debt under a Nonlinear Fiscal Policy Rule

Michinao Okachi*

May 28, 2018

Abstract

Although the conventional default model is founded upon governments’ strategic decisions to default, the model fails to capture key features of default in advanced countries. In this paper, I provide a DSGE model for the analysis of sovereign default triggered by insolvency under a government’s nonlinear fiscal policy rule, and identify the cause of default by comparing with the strategic default model. The quantitative analysis shows that the Greek default is better explained to have been caused by Greek government’s incapability to repay debt because of limited capability to improve its fiscal condition and no incentive for the government to choose to default. In addition to this, the simulation result from the incapability-to-repay default model replicates the economic dynamics around its default well.

Key words: Sovereign Default, Dynamic Stochastic General Equilibrium, Incapability to Repay Debt, Fiscal Limit, Laffer Curve, Strategic Decision to Default

JEL Classification: E32, E44, F34, H63

* Economist, Institute for Monetary and Economic Studies, Bank of Japan (E-mail: michinao.okachi@boj.or.jp)

The author thanks Junko Koeda, Taisuke Nakata, Eiji Okano, Tatsuyoshi Okimoto, Tatsushi Okuda, Tomohiro Tsuruga, Chung Tran, Francesco Zanetti, the participants at the Australian National University seminar, Chuo University seminar, JEA 2017 Spring Meeting and workshop at Hosei University. Views expressed in this paper are those of the author and do not necessarily reflect the official views of the Bank of Japan.
1 Introduction

The Greek virtual default in 2012 awakened the idea that the occurrence of debt crises was not restricted to developing countries. Although we need to use an appropriate model to analyze default in advanced countries, conventional strategic default models, which assume governments’ willingness rather than incapability to repay debt, are designed to be applicable only to emerging economies (Aguiar et al. 2016, Bi 2012 and Daniel 2018). In addition to this, looking at the case of the Greek default, the government had been trying to prevent default by undertaking austerity measures before defaulting and EU nations and international organizations put pressure on the government to weaken the fear of disorderly default. Thus, instead of the government’s strategic decision to default, there is a possibility that its default might have been caused by another reason: the incapability to repay debt. Indeed, Bi (2012) and Bi and Leeper (2012) assume government insolvency in their analysis of Greek default.

In this paper, I propose a Dynamic Stochastic General Equilibrium (DSGE) model for sovereign default triggered by governments’ incapability to repay debts and the model of strategic decision to default is reconciled to be comparable to the model of default by insolvency. Then, I identify the cause of the default in Greece. The incapability-to-repay default model assumes that a government falls to a default state if its debt outstanding exceeds a fiscal limit, which is defined as the discounted summation of future maximum primary surplus. Besides, because of the government’s limited capability for repayment, it follows a more progressive non-linear tax rate rule in order to satisfy intertemporal budget constraints. Meanwhile, following the seminal study of Eaton and Gersovitz (1981), the strategic default model assumes transition to the default state if the default value function exceeds the non-default value function. Also, the fiscal austerity attitude to suppress the debt outstanding is weaker than the incapability-to-repay default. Economic contraction by default originates

1Indeed, the compatibility of the strategic default model to debt crises in advanced countries is not sound. For example, the model reports much lower debt-to-GDP ratio. Although the actual Argentinean debt-to-GDP level was about 37%, Arellano (2008) and Mendoza and Yue (2012) replicate the mean debt-to-GDP ratio to 5.95% and 23% respectively.

2For instance, the government had increased both value added and income taxes, cut salaries for and reduced the number of public employees, decreased pension payments and announced the large scale privatization of public firms. Data clearly indicates that the government’s austerity stance prior to default had increased the tax rate from 38.9% in 2009 to 41.3% in 2010 and 44.0% in 2011.

3In 2011, a national referendum on whether to accept the bailout and austerity measures was cancelled due to external political pressures.
from two channels: the government’s obligation to set a high tax rate as an austerity policy and financial autarky to borrow working capital for the importation of intermediate goods, following Mendoza and Yue (2012).

I find that the Greek default is better explained to have been caused by the government’s incapability to repay debt through a strategic decision by the government from the comparison of comparative statistics of these two models, since the threshold of the default trigger of the former model mapped by the underlying macroeconomic fundamentals is lower than in the latter model. The reason is that the country’s capability to increase its fiscal surplus is limited because the maximum tax revenue derived from the Laffer curve is only slightly larger than steady state government consumption, so the fiscal limit is not high enough to ensure the repayment of high Greek debt. In addition to this, the government had no incentive to choose to default due to the high risk of economic contraction following a decision to default.4

The simulation result of the incapability-to-repay default model matches key features of advanced countries. For instance, the model replicates the high debt-to-GDP ratio observed in advanced countries, and moments of the frequency of default and lower consumption volatility than GDP. Besides, GDP or consumption dynamics around the default capture the magnitude and duration of its contraction. Therefore, from the comparative statistics and simulation results, it appears that the Greek default was caused not by the government’s strategic decision but by its incapability-to-repay debts.

My paper builds upon research on strategic sovereign default, limited capability for debt repayment and fiscal limits. Canonical analyses of the strategic sovereign default have been studied by Aguiar and Gopinath (2006) and Arellano (2008). They calibrate the Argentinean economy to analyze its default in 2001 within the framework of an RBC model. The growing number of sovereign default analyses is constructed upon this strategic default model. Mendoza and Yue (2012) introduce the production sector and assume financial autarky as punishment for a government’s decision to default. Sosa Padilla (2014), Perez (2015), Engler and Steffen (2016) analyze the effects of sovereign default via the financial system. Fernan-

4This result is consistent with prior strategic default literature which finds governments extremely reluctant to choose to default under the assumption of severe economic contraction. Aguiar and Gopinath (2006) report from the analysis of Argentinean default in 2001 that the frequency of default is only 0.02% per quarter under their baseline model without the assumption of bailout. Mendoza and D’Erasmo (2016) show that as the exogenous cost of default increases, default frequency lowers due to the government’s weak incentives to choose to default.

Compared to strategic default, incapability-to-repay default has been less analyzed. As I have introduced, Bi (2012) and Bi and Leeper (2012) model a partial default when the government debt outstanding exceeds a fiscal limit, which is derived from the discounted summation of future maximum fiscal surplus. Daniel and Shiamptanis (2013) assume capital loss on debt by insolvency of debt, deriving fiscal limits from a fiscal reaction function. Without the possibility of default, Ghosh et al. (2012), Tanner (2013), D’Erasmo et al. (2015) and Daniel and Shiamptanis (2015) obtain debt limits to be able to sustain debt outstanding also from fiscal reaction functions. The incapability-to-repay default model in my paper differs from these past studies to assume an explicit default state and consider the transition between non-default and default states. Related to this research, some papers such as Bocola (2016) and Davig et al. (2010, 2011) exogenously assume the distribution of default probability and study the pass-through of sovereign risk on the economy in the non-default state. Each default model of strategic decision and incapability-to-repay debt has been applied without clear identification of its cause. However, as far as I know, this paper is the first to connect these two default models and identify the cause of default quantitatively.

This paper is organized as follows: Section 2 presents default models to analyze both incapability to repay debts and strategic decisions. Section 3 sets functional forms, calibrates parameters and explains the solution method of the model. Section 4 delivers the comparative statics of both types of default and identifies the cause of default. Section 5 provides nonlinear simulation results of incapability-to-repay default. Section 6 concludes.
2 Model

The framework of the model is a DSGE of a small open economy, consisting of four domestic agents: households, final goods firms, intermediate goods firms and a government, and two foreign agents: foreign investors and foreign firms. Households and both types of domestic firms are a continuum of unit measure. Figure A-1 in Appendix A depicts the overall picture of the economy in the non-default state. I explain common parts of the two default models of incapability to repay debt and strategic decision making in sections 2.1 to 2.5. Then, I describe different parts of the models, transition of economic states and tax rate rules, in the section 2.6.

2.1 Government

In the non-default state, the government’s expenditure consists of the reimbursement of government bonds issued in the previous period $B_t$ and government consumption $G_t$, and its revenue consists of tax revenue $T_t$ and newly issued government bonds $B_{t+1}$ priced by foreign investors to $q_t$. Characteristics of government bonds are one period maturity, zero coupon and non-contingent, taking both negative and positive values. The negative value of issuance of government bonds means they are held as assets, yielding interest income from foreign investors with the world risk-free interest rate $r^f$.

First, I assume that government consumption $G_t$ is determined exogenously. Following Gali et al. (2007), government consumption measured as deviation from the steady state criteria $G$ standardized by the steady state GDP\footnote{Gali et al. (2007) standardize steady state output $Y$ due to the closed economy. In my model, GDP is defined as output minus imports.} is:

$$g_t \equiv (G_t - \overline{G})/GDP \quad (1)$$

$g_t$ follows the AR(1) process as:

$$g_t = \rho_g g_{t-1} + \varepsilon_{g,t} \quad (2)$$

where $\varepsilon_{g,t} \sim N(0, \sigma_g^2)$. The government imposes tax only on production of final goods $Y_t$, so
the total tax revenue $T_t$ is defined as:

$$T_t = \tau_t Y_t$$ (3)

where $\tau_t \in [0, 1]$ is the tax rate on final goods production. I explain the tax rate rule in section 2.6. The government issues new government bonds $B_{t+1}$ priced by $q_t$ for foreign investors to satisfy the intratemporal budget constraint:

$$q_t B_{t+1} \geq B_t + G_t - T_t$$ (4)

The government does not issue new bonds exceeding the minimum requirement, so the constraint is held with equality.

If the government falls into a default state, it does not repay any debts to foreign investors. Instead, international organizations (e.g. IMF) accept all defaulted debts and pay a discounted amount to the investors. Then, the organizations provide a bailout $\Phi_t^F$ to satisfy the government budget constraint\(^6\) because the government cannot issue new bonds due to its exclusion from the bond market.

When the government regains the normal state from the default state at the end of period $t$, the organizations impose a requirement on the government to pay $B^r$ amount of debt to foreign investors at the next period $t + 1$.

### 2.2 Foreign Investors

Foreign investors are risk-neutral and behave perfectly competitively, investing both government bonds and working capital for final goods firms.\(^7\) In terms of investment in government bonds, they can receive the full amount of bonds issued in the previous period $B_t$ and purchase new government bonds $B_{t+1}$ with their price $q_t$ under the non-default state. However, if the government defaults, they receive a discounted amount of repayment $(1 - \delta)B_t$ from the international organizations, where $\delta$ is the haircut rate of the repayment. Thus, the

\(^6\)The amount of bailout $\Phi_t^F$ can be negative if the tax revenue is larger than government consumption.

\(^7\)The investment of working capital is explained later in the section on final goods firms.
expected return from investment of government bonds $\pi^b_t$ is:

$$\pi^b_t = \left( \frac{1}{1 + r_f} \right) \left\{ (1 - P^e_t) + P^e_t (1 - \delta) \right\} B_{t+1} - q_t B_{t+1}$$  (5)

where $P^e_t$ is the probability of default in the next period. From the zero profit condition of perfect competition, the price of government bonds is obtained as:

$$q_t = \frac{1}{1 + r_f} \left\{ (1 - P^e_t) + P^e_t (1 - \delta) \right\}$$  (6)

### 2.3 Households

Households derive utility from consumption $C_t$ and disutility from labor supply $L_t$. They provide labor force to both final and intermediate goods firms, and receive wages $w_t L_t$ from these two firms and profits $\pi^f_t$ and $\pi^m_t$ from each firm respectively. In every period, they solve the utility maximization problem as:

$$\max_{C_t, L_t} E_t \left[ \sum_{i=0}^{\infty} \beta^i u(C_{t+i}, L_{t+i}) \right]$$  (7)

s.t. $C_t \leq w_t L_t + \pi^f_t + \pi^m_t$

where $w_t$ is the wage rate. The utility function $u : \mathbb{R}_+^2 \to \mathbb{R}$ is continuous, twice differential and satisfies $\frac{\partial u}{\partial C} > 0$, $\frac{\partial^2 u}{\partial C^2} < 0$, $\frac{\partial u}{\partial L} < 0$, $\frac{\partial^2 u}{\partial L^2} < 0$ and $\frac{\partial^2 u}{\partial C \partial L} - \left( \frac{\partial^2 u}{\partial C^2} \frac{\partial^2 u}{\partial L^2} \right) > 0$.

Then, the first-order condition of the households’ optimization problem is:

$$w_t = \frac{u_L(C_t, L_t)}{u_C(C_t, L_t)}$$  (8)

### 2.4 Final Goods Firms

Final goods firms produce final goods from intermediate goods $M_t$ and labor force $L^f_t$. Intermediate goods consist of domestic intermediate goods $m^d_t$ and imported intermediate goods $m^*_t$, which are aggregated on a continuum of differentiated imported intermediate goods $m^*_j$ for $j \in [0, 1]$. Final goods firms have to borrow working capital $\kappa_t$ with its interest rate $r^*_t$ in advance in order to import a certain fraction $\theta \in [0, 1]$ of intermediate goods. Then, the
working capital constraint for these imported intermediate goods is:

\[
\frac{\kappa_t}{1 + r^*_t} \geq \int_0^\theta p^*_{j,t} m^*_{j,t} dj
\]  

(9)

where \( p^*_{j,t} \) is the price of differentiated imported intermediate input \( m^*_{j,t} \). These firms do not borrow working capital exceeding the minimum requirement, so the constraint is held with equality. The general price of imported intermediate goods \( p^*_t \) is derived from the cost minimization problem subject to the technology of combining these differentiated imported intermediate inputs. While the interest rate on working capital \( r^*_t \) is equal to the world risk-free interest rate \( r^f \) in the non-default state, the rate \( r^*_t \) in the default state diverges to infinity and final goods firms cannot access working capital. Thus, they do not import intermediate goods which require working capital as long as the default state continues.

The technology of final goods production follows the Cobb-Douglas function as:

\[
Y_t = e^{A_t} \left( M \left( m^d_t, m^*_t \right) \right)^{\alpha_M} \left( L^f_t \right)^{\alpha_L}
\]  

(10)

where parameters \( 0 < \alpha_M, \alpha_L < 1 \) are the shares of intermediate goods and labor inputs respectively, satisfying \( \alpha_M + \alpha_L < 1 \), and TFP follows the AR(1) process as:

\[
A_t = \rho_A A_{t-1} + \varepsilon_{A,t}
\]  

(11)

where \( \varepsilon_A \sim N(0, \sigma^2_A) \). Final goods firms maximize their profit with respect to imported and domestic intermediate inputs \( m^*_t \) and \( m^d_t \) and labor input \( L^f_t \).

In Appendix B, I provide both problems of cost minimization with respect to the importation of intermediate goods and profit maximization, and derive first-order conditions.

---

*I abstract capital input for simplicity in order to reduce an endogenous state variable. This assumption is justified because, according to Mendoza and Yue (2012), capital stock has little effects on the fluctuation of business around crises.*
2.5 Intermediate Goods Firms

Intermediate goods firms produce intermediate goods from labor $L^m_t$ and sell them to final goods firms. The technology of intermediate goods production is:

$$m^d_t = A^f (L^m_t)^\gamma$$

where $A^f$ is an invariant TFP coefficient and $\gamma \in [0, 1]$ is the share of labor inputs for intermediate goods production. The profit maximization problem with respect to labor input for intermediate goods production is:

$$\max_{L^m_t} \pi^m_t = p^m_t A^f (L^m_t)^\gamma - w_t L^m_t$$

Then, the first-order condition is derived as:

$$w_t = \gamma p^m_t A^f (L^m_t)^{\gamma - 1}$$

2.6 Default Scheme

I separately provide the default mechanism and the government’s tax rate rule in both default models of incapability to repay debt and strategic decision. The reason I assume a rule-based tax rate is, in the spirit of Leeper (1991), to incorporate a fiscal feature of a smoothing tax rate over time into more complicated models.\footnote{In addition to the theoretical point of view, a growing number of countries are practically adopting fiscal rules. According to Schaechter et al. (2012) and the Fiscal Rules Dataset of the IMF, the number increased from only five countries in 1990 to 96 countries in 2015.}

2.6.1 Incapability-to-Repay Default

**Tax Rate Rule** The tax rate rule in the incapability-to-repay default model consists of three regimes. First, when the debt outstanding is small enough not to cause default, following Davig et al. (2010) and Bi (2012), the government increases its tax rate linearly as the debt outstanding increases:

$$\tau^r_t = \tau + \kappa (B_t - B)$$

\footnote{In addition to the theoretical point of view, a growing number of countries are practically adopting fiscal rules. According to Schaechter et al. (2012) and the Fiscal Rules Dataset of the IMF, the number increased from only five countries in 1990 to 96 countries in 2015.}
where $\tau$ and $B$ are the criteria of tax rate and debt outstanding for the determination of the tax rate rule, and $\kappa(\geq 0)$ is the elasticity of the debt outstanding to the tax rate.

Second, the government sets its tax rate more progressively when the debt outstanding is not secured by the tax rate corresponding to the linear tax rate rule. From the current budget constraint $[4]$, the intertemporal budget constraint is derived as:

$$B_t \leq \prod_{j=1}^{n} q_{t+j} B_n + \sum_{i=1}^{n} \prod_{j=1}^{i} q_{t+j} (T_{t+i} - G_{t+i})$$  \hspace{1cm} (16)

The government has to satisfy the transversality condition that the debt outstanding does not increase faster than the price of government bonds permanently as:

$$\lim_{n \to \infty} \prod_{j=1}^{n} q_{t+j} B_n = 0$$ \hspace{1cm} (17)

Then, the intertemporal budget constraint with the transversality condition for taking $n \to \infty$ is:

$$B_t \leq \sum_{i=1}^{\infty} \prod_{j=1}^{i} q_{t+j} (T_{t+i} - G_{t+i})$$ \hspace{1cm} (18)

This intertemporal budget constraint indicates that the current debt obligation has to be lower than the summation of future primary surplus discounted by the price of government bonds $q_t$. Because the debt outstanding $B_t$ and government consumption $G_t$ are predetermined and exogenous variables respectively, the government needs to satisfy this constraint with tax revenue $T_t$ through the tax rate $\tau_t$. However, when the debt outstanding is large enough, the constraint is not held by simply setting the tax rate from the linear rule $[15]$, so the government increases the rate further when the debt outstanding hits this constraint.

Because the risk of sovereign default, especially in advanced countries, is extremely low, the right hand side of inequality $[18]$ is simplified not to be discounted by the price of government bonds but by the risk-free interest rate $r_f$. Then, given the current tax rate $\tau_t$, the summation of expected future fiscal surplus discounted by the risk-free interest rate (expected surplus for short hereafter), $S$, is defined as:

$$S(\tau_t, A_t, g_t) \equiv E_t \sum_{i=0}^{\infty} \left( \frac{1}{1 + r_f} \right)^{t+i} (T(\tau_t, A_{t+i}) - G(g_{t+i}))$$ \hspace{1cm} (19)
The threshold at which the debt outstanding exceeds the expected surplus is set to $Z^{int}(A_t, g_t)$. If the debt outstanding is high enough and the tax rate corresponding to the linear tax rule is lower than the tax rate corresponding to the expected surplus $S(\tau_t, A_t, g_t)$, the government sets its tax rate on the curve of expected surplus $\tau^{es}_t$ as:

$$
\tau^{es}_t = S^{-1}(B_t, A_t, g_t)
$$

(20)

Finally, the model assumes an upper tax rate limit for the government to set. To begin with, there is no incentive for the government to set a tax rate exceeding the threshold where the marginal rate of revenue loss from output decline is larger than the rate of revenue increase by raising the tax rate. In other words, tax revenue $T_t$ corresponds the Laffer curve mapped by the tax rate and the government does not set its tax rate higher than the rate at the top of the Laffer curve. In addition to this, the government faces political disturbance $\varrho \in [0, 1]$ that prevents achieving the top of the curve. Then, the maximum tax rate that the government can set, $\tau^{ma}_t$, is:

$$
\tau^{ma}_t(A_t) = (1 - \varrho) \cdot \left( \arg \max_{\tau_t} T(\tau_t, A_t) \right)
$$

(21)

The summation of expected future maximum fiscal surplus discounted by the risk-free interest rate (the maximum expected surplus for short hereafter) $Z^{ma}_t$ is:

$$
Z^{ma}(A_t, g_t) = E_t \sum_{i=0}^{\infty} \left( \frac{1}{1 + r_f^i} \right)^{i+i} \left( T(\tau^{ma}_{t+i}, A_{t+i}) - G(g_{t+i}) \right)
$$

(22)

If the debt outstanding exceeds the maximum expected surplus $Z^{ma}(A_t, g_t)$, the government does not increase its tax rate any further. Therefore, the government’s tax rate rule is summarized as:

---

10This political disturbance is interpreted as policy makers’ lacking the power or will to introduce a severe austerity policy because the tax rate near the top of the Laffer curve is a heavy tax burden for taxpayers.
\[
\tau_t = \begin{cases} 
\tau_t^{lr} & \text{if } B_t \leq Z^{int}(A_t, g_t) \\
\tau_t^{es} & \text{if } Z^{int}(A_t, g_t) < B_t \leq Z^{ma}(A_t, g_t) \\
\tau_t^{ma} & \text{if } B_t > Z^{ma}(A_t, g_t) 
\end{cases}
\]

(23)

Figure A-2 in Appendix A describes the image of this tax rate rule. In the case of \(Z^{int}(A_t, g_t) > Z^{ma}(A_t, g_t)\), the tax rate rule only comprises two regimes of \(\tau_t^{lr}\) and \(\tau_t^{ma}\).

In the default state, the international organizations force the government to set the tax rate to collect the maximum tax revenue without political disturbance \(\varrho\). Thus, the tax rate consists of one regime at the top of the Laffer curve.

**Default Scheme** The government defaults when the debt outstanding exceeds a fiscal limit in the incapability-to-repay default model. The fiscal limit \(b_t^*\) is drawn from the distribution of fiscal limit, \(B_t^*\), defined by the summation of future maximum fiscal surplus discounted by the risk-free interest rate as:

\[
B^*(A_t, g_t) = \sum_{i=0}^{\infty} \left( \frac{1}{1 + r_f} \right)^{t+i} (T(\tau_t^{ma}, A_{t+i}) - G(g_{t+i}))
\]

(24)

I set the state of the economy at the beginning of period \(t\) to \(\xi_t \in \{\xi_{n,t}, \xi_{d,t}\}\), consisting of the non-default state \(\xi_{n,t}\) and default state \(\xi_{d,t}\). As a matter of course, the transition from the non-default state to the default state can occur only under the non-default state of the economy (i.e. \(\xi_t = \xi_{n,t}\)). The default set under the incapability-to-repay default \(\Gamma_I(B_t, \xi_{n,t}, b_t^*)\) is defined as the set of TFP and government consumption such that debt outstanding exceeds the fiscal limit as:

\[
\Gamma_I(B_t, \xi_{n,t}, b_t^*) = \{A_t \in \mathcal{A}, g_t \in \mathcal{G} : B_t > b_t^*\}
\]

(25)

The probability of default in the next period under the incapability-to-repay default \(P_t^{I,e}\) is derived from conditional cumulative probability density such that a combination of exogenous

---

\(^{11}\)It is possible to set the criteria of fiscal limit as the deterministic value of maximum expected surplus \(Z_t^{ma}\). However, as Ghosh et al. (2012) point out, even a slight uncertainty in the primary balance will affect variation in the fiscal limit, so I assume a probabilistic fiscal limit.
state variables, TFP and government consumption, are in the default set:

\[ P_{I}^{e}(A_t, g_t, B_{t+1}, \xi_{n,t+1}, b^*_{t+1}) = \int \int \Gamma_t^{(B_{t+1}, \xi_{n,t+1}, b^*_{t+1})} f^A(A_{t+1}, A_t) f^g(g_t+1, g_t) dA_{t+1} dg_{t+1} \]  

(26)

where \( f^A \) and \( f^g \) are the transition probability function of TFP and government consumption respectively. Similarly, the probability of default under the strategic default model \( P_{S}^{e} \) is derived as shown in the next section 2.6.2. I do not assume that both types of default causes are intertwined, so the probability of default that foreign investors face for their pricing of government bonds is summarized as:

\[ P_{e} = \begin{cases} 
P_{I}^{e} & \text{if the default mechanism is incapability-to-repay} \\
P_{S}^{e} & \text{if the default mechanism is strategic decision} 
\end{cases} \]  

(27)

Then, I define the competitive equilibrium of the incapability-to-repay debt default model. For convenience, I summarize this model's state vector as \( s^I \in \{ A, g, B, \xi, b^* \} \).

**Definition 1**

A recursive equilibrium of the model of incapability-to-repay debt default is defined as given the vector of state variables \( s^I \) and tax rate \( \tau \), a set of government policies \( \{ B'(s^I), G(g), T(s^I) \} \), private allocation \( \{ C(s^I), Y(s^I), L(s^I), M(s^I), m^d(s^I), m^s(s^I), L^d(s^I), L^m(s^I) \} \), factor prices \( \{ w(s^I), r^*(s^I), p^*(s^I), p^m(s^I) \} \), and the price of government bonds \( q(A, g, B', \xi_n, b^*) \) such that:

(a) Government policies satisfy the rules of government consumption \[3] and taxation \[4], and for \( (\xi, \chi) = (\xi_n, \chi_n) \), given \( q(A, g, B', \xi_n, b^*) \), the amount of newly issued government bonds follows the rule \[4\]; (b) Households, final and intermediate goods firms solve their optimization problems respectively; (c) The domestic intermediate goods market clears; (d) The market for labor clears, \( L_t = L^I_t + L^m_t \); (e) The market for final goods clears, \( Y_t - T_t - p^*_t m^*_t = C_t \); (f) for \( \xi = \xi_n \), given the default set \( \Gamma_t^I(B, \xi_n, b^*) \) and the probability of default \( P^*(A, g, B', \xi_n, b^*) \), the price of government bonds \( q(A, g, B', \xi_n, b^*) \) satisfies the foreign investors' zero profit condition \[6\].

The aggregate constraints in the normal and default states are \( Y_t - p^*_t m^*_t - (B_t - q_t B_{t+1}) = C_t + G_t \) and \( Y_t - p^*_t m^*_t + \Phi_t^F = C_t + G_t \) respectively. By substituting the government’s
budget constraints in the normal state held with equality and that in the default state
\[ T_t + \Phi_t^F = G_t \]
for each state of aggregate constraints, the same constraint is derived as is in the market clearing condition for final goods.

If the government falls into a default state, I assume the government has to stay in the default state for exogenous mandatory period of time. Then, nature decides to regain the normal state with exogenous probability \( \vartheta \) after that period.

2.6.2 Strategic Default

Tax Rate Rule In the case of the strategic default model, it is assumed that the government does not default due to its incapability to repay debts, so the government does not need to satisfy the intertemporal budget constraint. Also, I assume there is no upper bound on the tax rate because if the government is allowed to set a low enough tax rate in the non-default state no matter how it accumulates debt, the government does not choose to default at all. Thus, the government sets its tax rate only on the linear tax rate rule as:

\[ \tau_t = \tau_t^{lr} = \tau + \kappa (B_t - \bar{B}) \]  

(28)

Default Scheme The government conducts its default decision to maximize households’ lifetime utility. The government’s optimal decision to repay or default can be formulated as:

\[
V(A_t, g_t, B_t, \xi_{n,t}) = \max_{d_t \in \{0,1\}} \{(1 - d) V_n(A_t, g_t, B_t, \xi_{n,t}) + d V_d(A_t, g_t, \xi_{n,t}) \}
\]  

(29)

where \( V_n \) and \( V_d \) represent the government’s value functions in the non-default and default states respectively. The reason why the default value function depends on the non-default state \( \xi_{n,t} \) is that the government compares two values only in the non-default state at the beginning of period \( t \). The value functions of non-default and default are defined respectively as:

\[
V_n(A_t, g_t, B_t, \xi_{n,t}) = \{u(C_t, L_t) + \beta E_t[V(A_{t+1}, g_{t+1}, B_{t+1}, \xi_{n,t+1})|A_t, g_t, B_t, \xi_{n,t}] \}
\]  

(30)
Following prior literature, the government has a chance to regain the non-default state soon after it defaults, otherwise the model will be intricate due to history dependence. The government follows the rule of its determination of consumption and tax rate, so the choice variable for the government is only the decision to repay or default, \( d_t \).

Similar to the incapability-to-repay default model, the default set and probability of default in the strategic default model are defined respectively as:

\[
\Gamma^S(B_t, \xi_{n,t}) = \{ A_t \in A, g_t \in \mathbb{G} : V_d(A_t, g_t, \xi_{n,t}) > V_n(A_t, g_t, B_t, \xi_{n,t}) \} \quad (32)
\]

\[
P^{S,e}(A_{t+1}, g_{t+1}, B_{t+1}, \xi_{n,t+1}) = \int \int_{\Gamma^S(B_{t+1}, \xi_{n,t+1})} f^A(A_{t+1}, A_t) f^g(g_{t+1}, g_t) dA_{t+1} dg_{t+1} \quad (33)
\]

I define the competitive equilibrium of the strategic default model in Appendix D.

### 3 Calibration, Functional Form and Solution Method

#### 3.1 Calibration and Functional Form

The model is calibrated to the Greek data from 1999Q1 to 2016Q4. First of all, for the quantitative analysis, I specify the utility function to the Greenwood-Hercowitz-Huffman (1988) (GHH thereafter) preference as

\[
u(c_t, L_t) = \left( \frac{c_t}{L_t^{1+\eta}} \right)^{1-\sigma} - 1,
\]

where \( \sigma \) is the degree of risk aversion, setting to unity for the log-utility function, and \( \eta \) is the inverse Frisch elasticity of labor supply, setting to 0.455 from the standard value of the RBC model. The discount factor \( \beta \) is set to 0.99 also from the standard value of the RBC model. I set the intermediate input share in final goods production \( \alpha_M \) to 0.39 to match total intermediate consumption over gross output in the early 2000s from the OECD’s STAN Input-Output data. The labor input share in final goods production \( \alpha_L \) is calculated at 0.42 since the proportion of labor supply depend only on the wage rate. Otherwise, as Mendoza and Yue (2012) point out, if the labor supply is in inverse relation with consumption by other specifications of the utility function, the sharp drop in consumption especially at the time of default may induce an increase of labor supply, which is not observed from empirical analyses.

---

12The specification of this utility is important to make labor supply depend only on the wage rate. Otherwise, as Mendoza and Yue (2012) point out, if the labor supply is in inverse relation with consumption by other specifications of the utility function, the sharp drop in consumption especially at the time of default may induce an increase of labor supply, which is not observed from empirical analyses.

13This data does not specify the exact period.
share is assumed to be 70% of the summation of labor and capital inputs from most literature. The technology of the combination of domestic intermediate goods and imported intermediate goods follows the CES aggregator as $M\left(m^d_t, m^i_t\right) = \left[\lambda \left(m^d_t\right)^\psi - (1 - \lambda) \left(m^i_t\right)^\psi\right]^{\frac{1}{\psi-1}}$, where $\lambda \in [0, 1]$ is the Armington weight of domestic inputs and $\psi(> 0)$ is the elasticity of substitution across domestic and imported intermediate goods. The former parameter $\lambda$ is set to 0.63 from the share of domestic intermediate goods to output in the OECD’s STAN Input-Output data, and the latter parameter $\psi$ is set to 0.9 from Evers (2015). I assume the technology to combine the continuum of differentiated imported intermediate goods follows Dixit-Stiglitz aggregator as $m^i_t = \left[\int_0^1 \left(m^i_{j,t}\right)^{\frac{1}{\nu-1}} dj + \int_0^\theta \left(m^i_{j,t}\right)^{\frac{1}{\nu-1}} dj\right]^{\frac{\nu}{\nu-1}}$, where $\nu(> 1)$ is the substitution elasticity between differentiated imported intermediate goods, setting 3 from Feenstra et al. (2014). The upper bound of imported inputs with working capital $\theta$ is set to 0.26 from the World Bank data in 2005, and the invariant TFP coefficient for intermediate goods production $A^I$ is set to 0.31 from Mendoza and Yue (2012). The world risk-free interest rate $r_f$ is set to 0.01, which is the standard value of the RBC model. The labor share in the production of intermediate goods $\gamma$ is also set to 0.7. I set the haircut rate $\delta$ to 0.05 from the risk premium of the quarterly long-term interest rate toward Germany in 2012. This is because the haircut rate for the government in the model is not the ex post proportion of debt relief but the risk premium of issuance of new government bonds before default.\footnote{The ex post haircut rate of Greek debt restructuring was about 60% (Zettelmeyer et al. 2013).} Using the HP filter, the persistence and standard deviation of government consumption shock are 0.65 and 0.04 respectively, and those of TFP shock are 0.6 and 0.015 respectively.\footnote{I adopt TFP as GDP per person employed.} I set the steady state level of government consumption criteria over GDP, $\bar{G}$, to 0.385 from 1999 to 2008, which is one year before the government’s debt manipulation was revealed in 2009. The government debt criterion $B$ is set to be 104% of the debt-to-GDP ratio\footnote{I define government consumption as total expenditure minus interest payments.} also from the average value between 1999 to 2008.

Next, I set the criterion of tax rate to reconcile the criterion of the government consumption. The left graph in Figure\footnote{The government debt outstanding is not measured as the net base but as the gross base because of the limited data set. However, the gap between gross and net debt shifted to less than 10% of GDP in Greece before its default, so the measurement difference between net and gross debt does not affect the result much.} shows the long term relation between the debt-to-GDP ratio and tax rate of the Greek economy standardized by the tax rate criterion. The data can be roughly categorized into four regions. Before the debt-to-GDP ratio hit around 100%
from 1984 to 1992, these two variables were positively related. Then, the tax rate elastically shifted to about 100% of debt-to-GDP ratio from 1993 to 2008. However, the debt-to-GDP ratio increased further after the global financial crisis in 2008 and the deterioration of market confidence toward the Greek government due to its fiscal manipulation in 2009. Finally, the government virtually defaulted in 2012, and both debt-to-GDP ratio and tax rate have remained high since then. I set the linear tax rate rule of elasticity of debt-to-GDP ratio to tax rate, $\kappa$, to 0.02 by the OLS estimation of data from 1984 to 1992. The maximum tax rate is defined as the tax rate corresponding to the top of the curve times the political disturbance $\varrho$. The value of $\varrho$ is set to 0.09 based on data that the tax rate in 2011 was about 9% lower than the average tax rate from 2012 to 2016. The right graph of Figure 1 shows the calibrated tax rate rule of the incapability-to-repay default model. The government sets its tax rate on the linear line under the case that the debt-to-GDP ratio on the rule is below the expected surplus until it reaches about 39% of tax rate. Then, if the line exceeds the expected surplus, the government sets its tax rate on the curve. However, the tax rate that the government can impose is limited to about 40.5%. When the government defaults, the tax rate is set to about 44.5% corresponding to the top of the Laffer curve.

Finally, I set the probability of recovering to the non-default state from the default state, $\vartheta$. As the baseline duration of the default state in the incapability-to-repay default model, the government has to be in a mandatory default state for 26 quarters after default, that
is from the time of the Greek default in March 2012 to the termination of bailout program in August 2018. After that, the government can obtain the chance to recover the normal state with exogenous probability $\vartheta$, setting 0.044 from 5.7 years of the average duration of exclusion from international capital markets by default (Dias and Richmond, 2008). However, in the strategic default model, the government has a chance to regain the normal state soon after the default. I summarize all calibrated parameters in Appendix H.

### 3.2 Solution Method

I separately calculate the default models of incapability-to-repay and strategic decision. The former model consists of the following two parts: the expected surplus and fiscal limit, and the discrete state space (DSS). While the expected surplus is obtained deterministically given the initial exogenous state variables of $A_t$ and $g_t$, I calculate the distribution of the fiscal limit from future shocks of these state variables drawn by a Markov Chain Monte Carlo (MCMC) simulation. In order to obtain the DSS of the incapability-to-repay default model, following Coleman (1991) and Davig (2004), I find a fixed point of the decision rule for government bonds by using the monotone map method for all three tax rate cases, $\tau^{fr}_t$, $\tau^{es}_t$ and $\tau^{ma}_t$. Then, I select endogenous variables to satisfy the tax rate rule in (23). Next, for the solution of strategic default model, I follow the literature on strategic sovereign default models using the two-loop algorithm. The outside loop iterates the price of government bonds, and the inside loop is the iteration of government value functions in both non-default and default states. A detailed explanation of the computational algorithm is in Appendix C.

### 4 Comparative Statistics of Cause of Default

#### 4.1 Greek Default

Graph (a) in Figure shows the Laffer curve under different values of TFP, showing the tax rate on the horizontal axis and tax revenue over steady state GDP on the vertical axis. They define partial market access as "the first year in which there are positive net bond and bank transfers to the public or private sector" and full market access as "the first year of positive net bond and bank transfers to the private or public sector greater than 1.0% of GDP." The Greek economic stagnation was exceptionally longer than their finding, so I assume the government had a chance to regain only after the mandatory default duration. Government consumption, which is another exogenous state variable, does not affect tax revenue.
Figure 2: Greek Default

(a) Laffer Curve

(b) Default Probability of Incapability-to-Repay Default

(c) Value Function of Strategic Default

(d) Cause of Default

Note: The level of GDP is the steady state value.

Under any levels of TFP, the tax rate at the highest level of tax revenue is about 44.5% and the maximum tax rate is about 40.5%. The maximum tax revenue that the government can collect is about 40% of GDP ratio and the steady state criterion of government consumption is 38.5% over GDP, so the government can potentially accumulate its fiscal surplus at approximately 1.5% of GDP ratio at steady state every period.

The next graph (b) reports the default probability in incapability-to-repay debt, which shows the cumulative density functions of the fiscal limit, under different values of initial TFP. The criterion of fiscal limit, $b^*_t$, is drawn from this distribution. If TFP is at steady state (the black curve), the probability of default is 10% at about 100% debt-to-GDP ratio,
and the rate increases to 50% and 90% at about 160% and 230% of debt-to-GDP ratios respectively. The probability of default increases (decreases) under low (high) TFP, but the proportion of change is limited because TFP is expected to return to the steady state as time goes on following the AR(1) process as in equation (11).

Graph (c) shows the value function of non-default minus that of default in the strategic default model mapped from the debt-to-GDP ratio, under different values of current TFP. The government does not choose to default strategically if this value is higher than zero, otherwise it is willing to default. According to the graph, the value is always higher than zero in any states of TFP within the range of 0% and 300% debt-to-GDP ratio, despite the gap between the non-default value and the default value narrowing as debt-to-GDP increases. Thus, the government does not choose to default strategically under the case of calibrated parameters.

The last graph (d) describes the areas of cause of default under the combinations of state variables of TFP and debt-to-GDP ratio, corresponding to graphs (b) and (c). The area of incapability-to-repay default represents the boundary of 50% default probability. The area of strategic default does not appear in the graph, and the cause of the Greek default is only attributed by its incapability to repay debts. Besides, the state of the economy one quarter prior to the default is in the area of incapability-to-repay default since debt-to-GDP and TFP were 170.3% and 2.6% below the steady state respectively at that time. Therefore, it would be considered that the Greek default was caused by incapability to repay debt.

<Discussion: Counterfactual Scenario>

Then, under what condition was the Greek government willing to choose to default strategically? From equation (29), if the government has a higher incentive to choose default than the baseline scenario, the default value function may exceed the non-default value function within the legitimate range of debt-to-GDP ratio. Thus, as one example of high default incentive, I examine the case that the government does not take austerity measures in the default state as a counterfactual scenario. The left graph (a) in Figure 3 shows the non-default value function minus default value function of strategic default under the low tax rate in the default state subtracting the value of political disturbance and 5% lower government

21I explain the cases of different values of government consumption of default probability in incapability-to-repay default and value function of strategic default in Appendix F.
22The alternative example is that the non-default value function is lower than baseline due to severe austerity measures in the non-default state.
Figure 3: Greek Default: Counterfactual

Note 1: The level of GDP is the steady state value.
Note 2: The economic state is set to be low government consumption and low tax rate during the default state.

consumption than the steady state. In this case, the default value exceeds the non-default value at around 160% of debt-to-GDP ratio in the steady state TFP. The threshold of debt-to-GDP ratio where the non-default value function falls below the default value function decreases as TFP decreases.

The right graph (b) depicts the causes of default under this case. The white area shows that the government defaults neither in incapability-to-repay nor as a strategic decision. However, the government’s default is triggered by its strategic decision when the combination of relatively high debt-to-GDP ratio and low TFP (yellow area) occurs, while incapability-to-repay default is caused when both values of debt-to-GDP ratio and TFP are high enough (light blue area). Finally, default is caused in both triggers in the grey area. Therefore, the Greek government would have defaulted with lower debt-to-GDP ratio than the calibrated case if the government had a higher incentive to choose to default such as a low tax rate during the default state.
5 Simulation Results

In the previous section, I explain that the Greek default was considered to be caused not by the government’s strategic decision but its incapability to repay debt. I evaluate the compatibility of an incapability-to-repay default model to the Greek default in this section. I generate stochastic TFP and government consumption processes 5000 times for 500 periods and trace the movement of variables. The initial state of government is non-default, owing 80% of debt-to-GDP ratio, and the initial values of TFP and government consumption are both in the steady state, but I discard the first 100 periods in order to eliminate the effect of these initial conditions.

Table 1: Simulation Results

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Debt-to-GDP</td>
<td>105.9%</td>
<td>108.5%</td>
</tr>
<tr>
<td>Default Frequency</td>
<td>0.54%</td>
<td>0.20%</td>
</tr>
<tr>
<td>Average Bond Spread</td>
<td>0.50%</td>
<td>1.44%</td>
</tr>
<tr>
<td>Std. Cons / Std. GDP</td>
<td>0.68</td>
<td>0.86</td>
</tr>
</tbody>
</table>

Correlation of GDP with
- Bond Spread                  -0.02    -0.42
- Tax Rate                     -0.15    -0.49
- Imported Intermediate Goods  0.90     0.97
- Consumption                  0.98     0.90
- Domestic Intermediate Goods  0.98     0.98
- Labor                        0.54     0.98

Correlation of Bond Spread with
- Tax Rate                     0.37     0.84
- Imported Intermediate Goods  0.05    -0.57
- Consumption                  0.11    -0.56
- Domestic Intermediate Goods  -0.02    -0.57
- Labor                        -0.55    -0.57

Table 1 reports long-term moments of key variables, showing the average debt-to-GDP ratio, frequency of default, average bond spread, proportion of standard deviation of consumption over GDP and the correlation of GDP and bond spread with key endogenous variables in both data of the actual Greek economy and the simulation result. I will explain the properties of the first three sections. The data on average debt-to-GDP ratio is taken from 1984 to 2016. The simulation result excludes the sample below 40% of debt-to-GDP ratio because the actual minimum debt-to-GDP ratio was 40% after 1984. In terms of default
frequency; the data reports the extremely long-term average that the government defaulted four times after 1832\textsuperscript{23}, so the value should be regarded as merely a reference because the economic structure of the calibrated period from 1999 to 2016 would be significantly different from the 19th and 20th centuries. The bond spread in data is the 10-year government bond premium toward the yields of German bonds standardized on a quarterly basis, while that in simulation represents the reciprocal of the government bond price minus the world risk-free interest rate.

First, the simulation result of average debt-to-GDP ratio (108.5%) shows a high enough value, taking a similar ratio with data (105.9%). This is because the government debt criteria to equalize steady state government consumption and tax revenue is set to 104% of debt-to-GDP ratio, and the threshold to default is about 170% as I will show later. This is the advantage of using an incapability-to-repay default model for advanced countries, while the strategic default model usually reports low values. Second, quarterly default frequencies of data and simulation are 0.54% (once in 47 years) and 0.20% (once in 125 years) respectively. Although it seems that the simulation result for default frequency has a low value, it would be regarded as within the legitimate range because of the fact that the Greek default was the first among advanced countries since WWII. Third, the result for average bond spread in my simulation is higher than the data, reporting 0.50% in data and 1.44% in simulation. This is because, in terms of the simulation, the cumulative distribution function of probability of default is fat-tailed (graph b in Figure\textsuperscript{2}), so even though the debt-to-GDP ratio is relatively low, foreign investors take the possibility of default into account in their pricing of government bonds. In terms of data, because Greek government bonds had been regarded as almost as safe as German government bonds, which are the safest bonds in the Euro countries, the bond premium had been suppressed to be significantly low. Fourth, both results of the proportion of standard deviation of consumption over that of GDP are lower than unity, reporting 0.68 in data and 0.86 in simulation. This result captures one of the characteristics of advanced economies, as Aguiar and Gopinath (2007) investigate, while most strategic default models report this value as larger than unity for the analysis of defaults in emerging economies because of government’s procyclical policies.

The model also replicates correlations of GDP and bond spread with the main variables well. The relations between GDP and bond spread, and GDP and tax rate are almost

\textsuperscript{23}This is the year that Greece was acknowledged as an independent nation.
irrelevant in the data, but these two show weak negative relations in simulation because low GDP leads to high debt outstanding, and that increases the bond spread and tax rate. The values of correlation of GDP with imported intermediate goods, consumption, domestic intermediate goods and labor are high in both the data and the simulation. The correlation between labor input and GDP in the data appears relatively low because of the rigidity of the Greek labor market. Finally, the correlations of bond spread with the main variables appear very low in the data because the risk premium of government bonds had been regarded as almost zero until the country’s problem became apparent. However, in the simulation, the bond spread reflects high debt outstanding and high probability of default, so its relation with tax rates is positive and those with imported intermediate goods, consumption, domestic intermediate goods and labor are negative.

Figure 4: Simulation Result 1

(a) Debt-to-GDP ratio

(b) Bond Spread

Note 1: The horizontal axis represents years before the default in 2012Q1.
Note 2: The spotted blue curves show the average of one standard deviations from means.

Figure 4 depicts the transitions in debt-to-GDP ratio and bond spread from four years to one quarter prior to the default. I set the year zero to the first quarter in 2012, the time of the Greek default. The black curves with plus marks show the data, and green curves indicate averages of simulation results, taking one standard deviation bands from the average on spotted blue curves. First, in terms of the debt-to-GDP ratio (graph a), the pace at which debt increased in simulation was slightly slower than that in the data because the government’s decision to issue new government bonds was mild.\footnote{See Appendix E for the decision rule around issuance of new government bonds.} However, both ratios at
one quarter before default are almost same at about 170% of debt-to-GDP. Graph (b) reports the transition of bond spread, showing upward trends in both the data and the simulation. While the bond spread in simulation is about 5% four years before default due to fat-tailed default probability, that in the data is almost zero at that time. Also, another difference is that the simulation shows a gradual increase as the time of default is approaching, but the data shows a rapid increase with high fluctuation.

Next, Figure 5 depicts transitions of the main endogenous variables from three years before to four years after the default. I simulate not only the baseline 6.5 years mandatory default duration scenario (green curves) but also the alternative two years mandatory default duration scenario since the government regains bond market access two years after default in 2014. Graph (a) shows the transition of tax rates. Both simulation and data keep increasing until default, but the pace of increase in the simulation is slower than in the data. After the default, the tax rate in data shifted at a high level with high fluctuation despite seasonal adjustment. Also, the simulation result of the baseline scenario reached a high level at the top of the Laffer curve within the four years range of the time span in the graph, but the rate of the alternative scenario begins to decline two years after the default at a slow pace.

The amount of imports (graph b) shows that both data and simulation had been declining before default, but the rate in the simulation plunges at the time of default. Although the proportion of importation of intermediate goods requiring working capital is 26%, the magnitude of decline is higher than this proportion because of low demand for final goods production. The data also describes the high contraction around the default, but it hits the bottom several years after default.

Graph (c) shows GDP movement. Because of the high tax rate and contraction of imported intermediate goods, GDP in the simulation declines significantly to about 15% lower than the pre-default level at the time of default and shifts about the same level until it has passed the mandatory default duration. On the other hand, the GDP level in the data had been decreasing from two years prior to default and the decline continues one year after default, contracting by more than 20%. Although the magnitude of GDP contraction is within the one standard deviation level at the time of default, the default effect on GDP in data is stricter than the simulation result.

The transition of consumption (graph d) appears similar to GDP, but the decline in consumption due to default is harder than that of GDP in simulation. This is because the
Figure 5: Simulation Results 2

(a) Tax Rate

(b) Imported Intermediate Goods

(c) GDP

(d) Consumption

(e) Domestic Intermediate Goods

(f) Labor

Note 1: Year zero is set to 2012Q1.
Note 2: The data on domestic intermediate goods shows annual basis.
decline in GDP is the weighted average of the decline in consumption and tax revenue from the market clearing condition of final goods and the tax revenue decline is milder than the GDP decline due to the increase in tax rate, so the consumption decline is larger than the GDP decline. The compatibility of the simulation result to data is sound because the data is in the range of one standard deviation most of the time.

Finally, the declines in both inputs, domestic intermediate goods and labor (graphs e and f), in the simulation are lower than in the data. Although domestic intermediate goods and labor in simulation fall about 13% and 16% respectively, those in the data deceased more than 20%. This implies that the reason why the contraction of GDP in the simulation is milder than in the data is because even though the model assumes foreign debt default, the economic contraction mechanism driven by domestic inputs would be also important to capture the default damage on the economy.25

Overall, the model generally accommodates features of the highly nonlinear dynamics of the actual default in Greece, capturing the magnitude of default contraction and its stickiness, although it slightly underestimates the effect of domestic inputs. As an alternative scenario, I show the case of two years mandatory default period because most sovereign default literature refers to the default duration as exclusion from the bond market. However, in the Greek case, the data suggests that regaining market access did not bring economic recovery.

6 Conclusion

This paper provides a DSGE model for the analysis of incapability to repay debt default, compares with the strategic default model and quantifies default effects on the economy. The trigger of default in the incapability-to-repay model is set that the government’s debt outstanding exceeds the fiscal limit derived from the discounted summation of future primary surplus, while the trigger in the strategic default model is the government’s decision based on comparison between the default and non-default value functions. From the calibration of the Greek default in 2012, I show that the country’s default is attributed to its insolvency. This is because the government had limited capability to increase its fiscal surplus and did not have an incentive to choose to default by itself due to severe economic contraction. The

25For example, Acharya et al. (2014) emphasize the effect of domestic financial contraction in the European debt crisis.
simulation result of compatibility of the incapability-to-repay default model to the Greek default is generally sound, matching important moments such as debt-to-GDP ratio, default frequency and the proportion of standard deviation of consumption over that of GDP. Also, transitions in the main endogenous variables capture their movements around the default.
References


Appendix A. Diagrams

Figure A-1. The Sketch of My Model

Figure A-2. Tax Rate Rule in the Case of Incapability to Default
Appendix B. Optimization Problems of Final Goods Firms

Final goods firms solve the two-stage budgeting method: the profit maximization problem and the cost minimization problem with respect to the importation of intermediate goods. First, the profit maximization problem is defined as:

$$\max_{m^*_t, m^d_t, L^f_t} \pi^f_t = Y_t(1 - \tau_t) - p^*_t m^*_t - p^m_t m^d_t - w_t L^f_t$$  \hspace{1cm} (34)$$

subject to the production function (10) and the combined technology of domestic and imported intermediate goods defined in the section 3.1. Then, the first-order conditions of the profit maximization problem with respect to imported intermediate goods $m^*_t$, domestic intermediate goods $m^d_t$ and labor force $L^f_t$ are:

$$e^{A_M} (M (m^d_t, m^*_t))^{\alpha - \frac{\nu - 1}{\nu}} (1 - \lambda) (m^*_t)^{\frac{1}{\nu}} (L^f_t)^{\alpha L} (1 - \tau_t) = p^*_t$$  \hspace{1cm} (35)$$

$$e^{A_M} (M (m^d_t, m^*_t))^{\alpha - \frac{\nu - 1}{\nu}} \lambda (m^d_t)^{\frac{1}{\nu}} (L^f_t)^{\alpha L} (1 - \tau_t) = p^m_t$$  \hspace{1cm} (36)$$

$$e^{A_L} (M (m^d_t, m^*_t))^{\alpha L - 1} (1 - \tau_t) = w_t$$  \hspace{1cm} (37)$$

Next, by substituting the working capital constraint (9) into the cost function of imported intermediate goods, the cost minimization problem with respect to imported intermediate goods is defined as:

$$\min_{m^*_j} \int_0^1 p^*_j m^*_j dj + (1 + r^*_t) \int_0^\theta p^*_j m^*_j dj$$  \hspace{1cm} (38)$$

subject to the Dixit-Stiglitz aggregator of imported intermediate goods in the section 3.1.

From the first-order conditions with respect to $m^*_j$, for $j \in [0, \theta]$, which requires working capital, and for $j \in [\theta, 1]$, which does not require it, the shadow price of combining imported intermediate goods with the minimum cost is derived as:

$$p^*_t = \left( \int_0^1 (p^*_j)^{1-v} dj + \int_0^\theta ((1 + r^*) p^*_j)^{1-v} dj \right)^{\frac{1}{1-v}}$$  \hspace{1cm} (39)$$

which can be interpreted as the aggregate price of imported intermediate goods.
Appendix C. Computational Algorithm

1-1. Expected Surplus and Fiscal Limit

1. Discretize the state space of tax rate ($\tau$), TFP ($A$) and government consumption in deviation from the steady state normalized by GDP ($g$). Take 101 grid points uniformly for the tax rate between 0.15 and 0.60. Tauchen’s method (1986) is applied to obtain the state space of TFP and government consumption, taking 25 grid points uniformly by setting the center points to zero.

2. Calculate endogenous variables ($C_t, Y_t, L_t, m^d_t, m^s_t, L^f_t, L^m_t, T_t, G_t, w_t, p^m_t, p^t_t$) for all discretized points using equations (1)(3)(8)(10)(12)(14)(35)-(37)(39) and market clearing conditions for labor and final goods.

3. Calculate the expected surplus and fiscal limit separately.

(a) Expected Surplus: Aggregate the expected future fiscal surplus discounted by the risk-free interest rate $E_t \sum_{i=0}^{200} \left( \frac{1}{1+r_f} \right)^{t+i} (T_{t+i} - G_{t+i})$ given the corresponding tax rate on each grid point.

(b) Fiscal Limit: Find the tax rate that maximizes the tax revenue in each state variable ($A_t, g_t$). Then, draw the future shocks of these two variables ($A_{t+i}, g_{t+i}$) given the initial state ($A_t, g_t$) and aggregate the discounted future maximum fiscal surplus for 200 periods for 5000 times in each initial state of ($A_t, g_t$). Finally, calculate the cumulative density function of default probability.

1-2. Discrete State Space of Incapability-to-Repay Default

1. Discretize the state space of TFP ($A_t$), government consumption ($g_t$) and government debt obligation to repay ($B_t$), taking the same number of grid points for TFP and government consumption to the previous section of expected surplus and fiscal limit and 101 grid points for the government debt $B_t$ between -0.5 and 3.0 times the steady state level of GDP. Then, make an initial guess for the issuance of government bonds $f^b_0$.

This is because if the tax rate is close to one, the first-order conditions of final goods firms will not be obtained due to their negative profits after tax. Also, if the tax rate is too low, the fiscal limit will be negative because of low tax revenues.
2. Given the tax rate rules, $\tau_{t}^{lr}$, $\tau_{t}^{ss}$ and $\tau_{t}^{ma}$, evaluate the probability of default $P^{l,e}$ outside grid points of conjectured issuance of new government bond $f_{0}^{b}$ by using the piecewise linear interpolation, and compute endogenous variables.

3. Update the guess of government bond issuance $f_{1}^{b}$ from the old decision rule $f_{0}^{b}$ from the government intratemporal budget constraint \((4)\) and the pricing equation of government bond \((6)\).

4. Accept the decision rule $f_{1}^{b}$ if the difference between the updated and old decision rules is small enough (i.e. $\sup \| f_{1}^{b} - f_{0}^{b} \| < \epsilon$)\(^{27}\) Otherwise, go back to procedure 2 with the updated decision rule on issuance of government bonds.

5. Finally, select the unified the tax rule $\tau_{t}$, the decision rule for government bonds $f_{1}^{b}$ and endogenous variables from three state spaces under different tax rules, $\tau_{t}^{lr}$, $\tau_{t}^{ss}$ and $\tau_{t}^{ma}$, from equation \((23)\).

2. Discrete State Space of Strategic Default

1. Discretize the state space of TFP ($A_{t}$), government consumption ($g_{t}$) and tax rate ($\tau_{t}$)\(^{28}\). Set the tax rate ($\tau_{t}$), corresponding to the tax rate rule \((28)\) between the range of -0.5 and 3 times of government bonds of the steady state GDP. Take the same number of grid points for TFP, government consumption and tax rate as in the prior sections.

2. Make initial guesses for the non-default and default value functions $V_{n,0}$ and $V_{d,0}$ respectively and the government bond price $q_{0}$, setting $1/(1 + r^{f})$.

3. The government’s value function iteration.

   (a) Given $q_{0}$, calculate the private allocations and utilities in both government states of non-default and default for each state space. Then, obtain the tax rate in the next period and the expected value functions, taking a piecewise linear interpolation on the tax rate.

\(^{27}\)To ensure the local uniqueness of the solution, I perturb the policy function $f^{b}$ and check its convergence.

\(^{28}\)The tax rate is mapped only from debt outstanding, so it is indifferent to take the tax rate or government bonds for a variable to discretize.
(b) Calculate updated value functions $V_{n,1}$ and $V_{d,1}$ from the expected value functions and utilities following equations (30) and (31) respectively.

(c) If the difference between current and updated government value functions in each non-default and default case is small enough (i.e. \( \sup \{|V_{n,1} - V_{n,0}|, |V_{d,1} - V_{d,0}|\} < \varepsilon \)), stop the iteration. Otherwise, go back to procedure (a) with the updated government value functions.

4. Calculate the default set $\Gamma^S$ and the probability of default $P^{S,e}_t$, taking the piecewise linear interpolation on the tax rate. Then, derive the price of government bond $q_1$.

5. If the difference between current and updated government bond prices is small enough (i.e. \( \sup |q_1 - q_0| < \varepsilon \)), stop the iteration. Otherwise, go back to procedure 3 with the updated government bond prices.
Appendix D. Competitive Equilibrium of Strategic Default Model

I summarize the state vector of the strategic default model as $s^S \in \{A, g, B, \xi\}$, and the competitive equilibrium is defined as follows:

**Definition 2**

A recursive equilibrium of the model of strategic default is defined as the vector of state variables $s^S$, tax rate $\tau$ and value functions $V_n$ and $V_d$, a set of government policies $\{d(s^S), B'(s^S), G(g), T(s^S)\}$, private allocation $\{C(s^S), Y(s^S), L(s^S), M(s^S), m^d(s^S), m^*(s^S), L^f(s^S), L^m(s^S)\}$, factor prices $\{w(s^S), r^*(s^S), p^*(s^S), p^m(s^S)\}$, and the price of government bond $q(A, g, B', \xi_n')$ such that:

(a) Government policies satisfy the rules of government consumption (1) and taxation (3), for $(\xi, \chi) = (\xi_n, \chi_n)$, given $q(A, g, B', \xi_n')$, the amount of newly issued government bonds follows rule (4) and the government solves its optimization problem (29). (b) Households, final and intermediate goods firms solve their optimization problems respectively. (c) The markets for domestic intermediate goods, labor and final goods clear. (d) for $\xi = \xi_n$, given the default set $\Gamma^S(B, \xi_n)$ and the probability of default $P^e(A, g, B', \xi_n')$, the price of government bonds $q(A, g, B', \xi_n')$ satisfies the foreign investors’ zero profit condition (6).
Appendix E. Additional Explanation of Comparative Statistics

<1. Cause of Default under Different Values of Government Consumption>

In section 4.1., I explain two graphs of the probability of default of the incapability-to-repay default model and the non-default value function minus default value function of the strategic default model under different values of TFP, fixing government consumption at the steady state. In this appendix, I show these two graphs under different values of government consumption fixing TFP at the steady state.

The high (low) value of government consumption increases (decreases) the default probability of incapability-to-repay default and decreases (increases) the difference of value function of strategic default, and their deviations are not much different from the TFP cases. Thus, in graph (b), the value is always higher than zero for any government consumption, so the cause of default is only attributed to the government’s incapability to repay debts.

Figure 6: Greek Default under Different Values of Government Consumption

(a) Default Probability of Incapability-to-Repay Default

(b) Value Function of Strategic Default

Note 1: The level of GDP is the value under the steady state.
Note 2: "High" and "Low" represent 5% higher and lower than the steady state respectively.
Next, I explain the decision rules of key variables in the incapability-to-repay default model. I assume that the government does not fall into the default state (i.e. the maturity base of current debt obligation $B_t$ is always lower than the stochastic fiscal limit $b_t^*$). First, Figure 7 reports the relationship between the new issuance of government bonds $B_{t+1}$, the probability of default $P_t^e$ and the price of government bonds $q_t$ mapped by maturity base debt standardized by steady state GDP, fixing TFP and government consumption at the steady state.

Figure 7: Decision Rules of Bond Related Variables

(a) Government Bond: Level

(b) Government Bond: Difference

(c) Probability of Default

(d) Price of Government Bond

Note: Debt in the horizontal axes represent the current amount of repayment of government bonds except for the maturity base debt amount in graph (c).
Graph (a) represents the decision rule of new issuance of government bonds. The red curve marked by diamonds and pink curve indicate the maturity base \((B_{t+1})\) and discount by its price base \((q_t B_{t+1})\) respectively, and the black dotted line is the 45 degree line. If the red curve is above this line, the government has to repay more debt in the next period than the current period. Similarly, if the pink curve is above the line, the government acquires more than the amount of repayments. The decision rules of government bonds show that both maturity and discount base curves are almost on the 45 degree line under the low debt-to-GDP ratio. However, as the debt outstanding increases, the government has to repay more debt in the next period than the current period because of the decline in the price of government bonds. In order to grasp the process of the increased issuance of government bonds well, I take the difference of both curves from the 45 degree line in graph (b). The discounted base of new issuance of government bonds (the pink curve) is slightly lower than zero under the low debt-to-GDP ratio because the tax is not imposed on the GDP level of final goods production but the output level, so tax revenue slightly exceeds government consumption. The curve falls at about 125% of debt-to-GDP ratio because the tax rate shifts from the linear tax rule to the curve of the expected surplus (the right graph of Figure 1) and tax revenue increases slightly. Even though the debt-to-GDP ratio increases, the government cannot increase its tax rate to exceed the maximum tax rate \(\tau_{ma}^t\), so the discounted base of newly issued government bonds shifts horizontally at about 1.5% of GDP lower than the debt repayment. However, the difference between the maturity base of newly issued government bonds and the amount of repayment is expanding as the debt-to-GDP ratio increases due to the declining price of government bonds. For instance, the repayment amount in the next period increases by about 10% of the debt-to-GDP ratio at the 200% level.

Next, graph (c) shows the probability of default. The blue curve and the red curve marked by diamonds are mapped from the repayment base debt \((B_t)\) and the maturity base debt \((B_{t+1})\) respectively. The dotted green and light blue curves show the criteria of probability of default based on the fiscal limit under low and high levels of TFP respectively from graph (b) in Figure 2. The criterion whether the government will fall into default or not in the next period depends on maturity base government bonds, and these bonds are mapped from repayment base debt corresponding to the decision rule in graph (a). Higher repayment debt outstanding induces higher issuance of government bonds, so increases the probability of default. For example, the probability of default is 80% when the maturity base bond is
about 210% of debt-to-GDP ratio, which corresponds to 200% of repayment-based debt.

Graph (d) depicts the price of government bonds following the foreign investors’ bond pricing equation (6). If the debt-to-GDP ratio is less than about 70% of the debt-to-GDP ratio, the bond price is relatively constant at about 0.99 because of the low probability of default. Then, the price of government bonds begins to decline as the debt-to-GDP ratio increases, and the decline continues until 230% debt-to-GDP ratio, reaching almost 100% probability of default. After exceeding 230% debt-to-GDP ratio, the price converges to about 0.94, which is almost same as the discount rate by the risk-free interest rate (1/1.01) times the repayment proportion at the time of default (1 − 0.05).

Therefore, as the graphs in Figure [7] show, variables of the amount of new issuance of government bonds, probability of default and price of government bonds are closely related to each other. If the debt repayment outstanding is high, the probability of default increases. Then, the high probability of default leads to the low price of government bonds, and the low price of government bonds induces a high amount of issuance of new government bonds. Thus, a high debt outstanding in the current period arouses a higher debt outstanding in the next period, and this cycle eventually causes default unless high TFP or low government consumption suppress the issuance of debt.

Next, I show the decision rules of other endogenous variables in Figure [8] showing the steady state and low TFP in both default and normal states. Obviously, the amount of debt outstanding is irrelevant to any variables under the default state. First of all, the tax rate (graph a) in the normal state is a function of only the debt outstanding regardless of TFP values until it hits the curve of expected surplus, so the rate increases linearly as the value of debt-to-GDP increases. The threshold of hitting the curve in the low TFP case is lower than the steady state TFP; the government sets its rate on the curve when the linear rule hits the curve. Then, because the government cannot increase the tax rate to exceed the maximum tax rate $\tau^m$, the rate converges to about 40.5% in any values of TFP. The tax rate in the default state is set to about 44.5% at the top of the Laffer curve. Second, default effects on imported intermediate goods (graph b) are relatively large. When the debt-to-GDP ratio is 100%, imported goods in the default state are 35% lower than in the normal state under steady state TFP. This large decline in importation of intermediate goods and the high tax rate are the main causes of economic contraction by default. From these two paths, GDP and consumption (graphs c and d) decline about 15% and 20% respectively by shifting to
Figure 8: Decision Rules of Main Variables

(a) Tax Rate  (b) Imported Intermediate Goods

(c) GDP  (d) Consumption

(g) Domestic Intermediate Goods  (h) Labor

the default state at 100% of the debt-to-GDP ratio. Finally, domestic intermediate goods and labor (graphs e and f) are decreasing against the debt-to-GDP ratio because demand for them is lowered by the increased tax rate. In terms of labor, as explained in section of 3.1, the specification of utility function to the GHH preference creates the relation where shifting to the default state lowers labor input. Otherwise, if labor supply is a function of wages and consumption, a decrease in consumption due to default may increase the labor supply.
Appendix F. Counterfactual Simulation

In this appendix, I explain counterfactual experiments changing several key parameters in the policy stance or economic structure, reporting the average debt-to-GDP ratio, default frequency, average bond spread, proportion of standard deviation of consumption over that of GDP, fiscal limit and GDP decline by default in Table 2.

Table 2: Counterfactual Simulation

<table>
<thead>
<tr>
<th></th>
<th>Average D/GDP</th>
<th>Default Freq.</th>
<th>Average Spread</th>
<th>Std. Cons/ Std. GDP</th>
<th>Fiscal Limit</th>
<th>GDP Decline by default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>103.8%</td>
<td>0.53%</td>
<td>0.50%</td>
<td>0.68</td>
<td>172%</td>
<td>-18.8%</td>
</tr>
<tr>
<td>Base</td>
<td>108.5%</td>
<td>0.20%</td>
<td>1.44%</td>
<td>0.86</td>
<td>171%</td>
<td>-15.2%</td>
</tr>
<tr>
<td>(1) Tax rate criterion ((r_f): 0.385)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-1%:</td>
<td>113.8%</td>
<td>0.35%</td>
<td>2.79%</td>
<td>0.85</td>
<td>146%</td>
<td>-15.3%</td>
</tr>
<tr>
<td>+1%:</td>
<td>92.8%</td>
<td>0.06%</td>
<td>0.39%</td>
<td>0.84</td>
<td>195%</td>
<td>-14.5%</td>
</tr>
<tr>
<td>(2) Government consumption criterion ((G/GDP): 0.385)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-1%:</td>
<td>88.7%</td>
<td>0.05%</td>
<td>0.27%</td>
<td>0.94</td>
<td>209%</td>
<td>-14.8%</td>
</tr>
<tr>
<td>+1%:</td>
<td>107.7%</td>
<td>0.36%</td>
<td>3.08%</td>
<td>0.84</td>
<td>132%</td>
<td>-15.6%</td>
</tr>
<tr>
<td>(3) The world risk-free interest rate ((r_f): 0.01)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.007:</td>
<td>94.2%</td>
<td>0.10%</td>
<td>0.68%</td>
<td>0.86</td>
<td>217%</td>
<td>-15.0%</td>
</tr>
<tr>
<td>0.013:</td>
<td>104.9%</td>
<td>0.26%</td>
<td>1.98%</td>
<td>0.88</td>
<td>138%</td>
<td>-16.0%</td>
</tr>
<tr>
<td>(4) haircut rate ((\delta): 0.05)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.00:</td>
<td>80.3%</td>
<td>0.07%</td>
<td>0.00%</td>
<td>0.88</td>
<td>170%</td>
<td>-15.4%</td>
</tr>
<tr>
<td>0.10:</td>
<td>128.1%</td>
<td>0.11%</td>
<td>5.19%</td>
<td>0.83</td>
<td>172%</td>
<td>-16.5%</td>
</tr>
</tbody>
</table>

Section (1) shows results that the tax rate criterion changes 1% from the baseline in the normal state. In the case of a 1% low tax rate (the upper row), tax revenue is decreased and the government needs to issue more bonds than in the baseline case. Thus, the values of the average debt-to-GDP ratio, default frequency and average bond spread increase. The fiscal limit decreases because the low tax rate criterion lowers the tax revenue so also lowers the maximum fiscal surplus. The proportion of standard deviation of consumption to GDP and the GDP decline due to default do not change much from the baseline result. The case of a higher tax rate (the lower row) reports generally the opposite result to the low tax rate case. A one percent increase in the tax rate declines the default frequency from 0.20% to 0.06% per quarter though the GDP level in the non-default state decreases.

Section (2) reports policies that change the criteria of government consumption. Lower
level government consumption contributes to stabilizing the economy, decreasing the average debt-to-GDP ratio, default frequency, average bond spread and GDP decline by default and increasing the fiscal limit. This section and section (1) emphasize the importance that even a slight change to be the austerity stance can significantly reduce the default frequency and increase fiscal limits.

Section (3) provides results by changing the world risk-free interest rate. When the interest rate is low, the government can take future fiscal surplus into account more, so both expected surplus and fiscal limits become higher than the baseline result. Thus, the frequency of default and average bond spread are lowered from the baseline scenario.

Section (4) shows the case that the haircut rate changes 0.05 points from the baseline scenario of 0.05. When the rate is null, foreign investors regard government bonds as fully warranted by international organizations and the price of government bonds is equivalent to the risk-free value regardless of the probability of default, so the average bond spread is zero. The average debt-to-GDP ratio and default frequency decrease from the baseline case because the government can suppress the issuance of bonds due to the price increase.
Appendix G. Argentinean Default

I conclude that the cause of the Greek default is better explained by Greece’s incapability-to-repay debt. However, another question arises: is the Greek case unique with respect to the cause of default? In order to answer this question, I apply the default models of incapability-to-repay debt and strategic decision to the Argentinean default in 2001 because this case has been frequently used for the analysis of sovereign default, especially among strategic sovereign default models. I calibrate the economy in Appendix H.

Figure 9: Argentinean Default

(a) Laffer Curve

(b) Strategic Default

(c) Cause of Default

The conclusion first, the Argentinean default is regarded to have happened due to the government’s strategic decision. The fundamental differences between Argentina and Greece are potential fiscal surplus and incentives to choose to default. First, graph (a) in Figure 9 shows the Laffer curve in Argentina, and derives the maximum fiscal surplus. The tax
revenue over GDP at the top of the Laffer curve is about 32%, which is about 10% larger than government consumption at the steady state if there was no political disturbance causing the Argentinean government to raise the tax rate. Bi et al. (2014) subtract 33% of the discounted summation of future fiscal surplus by assuming the Argentinean political risk factor from the International Country Risk Guide’s (ICRG’s) index in order to replicate the low fiscal limit of emerging economies. However, even though I set this value on the political disturbance $\varrho$ in my model as in graph (a), the tax revenue over GDP is 27% and the fiscal limit is still too high to reconcile the Argentinean actual data of debt-to-GDP ratio.

Second, graph (b) shows the non-default value function minus the default value function under different values of TFP. Under steady state TFP, the default value function exceeds the non-default value function at about 70% of debt-to-GDP ratio. As the value of TFP decreases, the threshold where the non-default value function falls below the default value function decreases. Corresponding to the result, I depict the area of cause of default in graph (c) depending on TFP and debt-to-GDP ratio under steady state government consumption. The area of incapability-to-repay default does not appear in the graph within the range of 0% and 100% debt-to-GDP ratio because the fiscal limit is considered to be much higher than 100% debt-to-GDP ratio. When the Argentinean government defaulted, TFP was about 3.5% lower than the steady state and the debt-to-GDP ratio was about 60%, and this economic condition satisfies default by the government’s strategic decision.

29 If the risk free interest rate is assumed to 2%, the potential fiscal limit is about 500% of GDP ratio.
30 I set the tax rate in the default state as 5% lower than the steady state tax rate criterion because its rate in the default state (the average of 2002 and 2003) was about 5% lower than that in the non-default state (the average from 1998 to 2001).
Appendix H. Calibration

Table 3 summarizes the calibrated parameters of the Greek economy in section 3.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$: Degree of risk aversion</td>
<td>1</td>
<td>log-utility</td>
</tr>
<tr>
<td>$\eta$: Inverse Frisch elasticity of labor supply</td>
<td>0.455</td>
<td>Standard Value</td>
</tr>
<tr>
<td>$\beta$: Subjective discount factor of households</td>
<td>0.99</td>
<td>Standard Value</td>
</tr>
<tr>
<td>$\alpha_M$: Intermediate input share in final goods production</td>
<td>0.39</td>
<td>OECD</td>
</tr>
<tr>
<td>$\alpha_L$: Labor input share in output of final goods</td>
<td>0.42</td>
<td>Labor Share (70%)</td>
</tr>
<tr>
<td>$\lambda$: Armington weight of domestic inputs</td>
<td>0.63</td>
<td>OECD</td>
</tr>
<tr>
<td>$\psi$: Substitution elasticity across intermediate goods</td>
<td>0.9</td>
<td>Evers(2015)</td>
</tr>
<tr>
<td>$\nu$: Substitution elasticity within intermediate goods</td>
<td>3</td>
<td>Feenstra et al. (2014)</td>
</tr>
<tr>
<td>$\theta$: Imported goods with working capital</td>
<td>0.26</td>
<td>World Bank</td>
</tr>
<tr>
<td>$A_I$: Invariant TFP coefficient for int. goods production</td>
<td>0.31</td>
<td>Mendoza and Yue (2012)</td>
</tr>
<tr>
<td>$r_f$: World risk-free interest rate</td>
<td>0.01</td>
<td>Standard Value</td>
</tr>
<tr>
<td>$\gamma$: Labor input share in int. goods production</td>
<td>0.7</td>
<td>Standard Value</td>
</tr>
<tr>
<td>$\delta$: Haircut rate</td>
<td>0.05</td>
<td>Bond Premium in 2012</td>
</tr>
<tr>
<td>$\rho_g$: Persistence of gov’t consumption shock</td>
<td>0.65</td>
<td>IMF</td>
</tr>
<tr>
<td>$\sigma_g$: Standard deviation of gov’t consumption shock</td>
<td>0.04</td>
<td>IMF</td>
</tr>
<tr>
<td>$\rho_A$: Persistence of TFP shock</td>
<td>0.6</td>
<td>OECD</td>
</tr>
<tr>
<td>$\sigma_A$: Standard deviation of TFP shock</td>
<td>0.015</td>
<td>OECD</td>
</tr>
<tr>
<td>$\bar{G}$: Government consumption criterion</td>
<td>0.0162</td>
<td>IMF ($\bar{G}/GDP = 0.385$)</td>
</tr>
<tr>
<td>$\bar{B}$: Government debt criterion</td>
<td>0.0508</td>
<td>IMF ($\bar{B}/GDP = 1.04$)</td>
</tr>
<tr>
<td>$\bar{\tau}$: Tax rate criterion</td>
<td>0.385</td>
<td>= Gov’t Consumption Criterion</td>
</tr>
<tr>
<td>$\kappa$: Elasticity of tax rate</td>
<td>0.02</td>
<td>OLS</td>
</tr>
<tr>
<td>$\varphi$: Political disturbance</td>
<td>0.09</td>
<td>IMF</td>
</tr>
<tr>
<td>$\vartheta$: Probability of recovery</td>
<td>0.044</td>
<td>Dias and Richmond (2008)</td>
</tr>
</tbody>
</table>

Besides, I explain the calibration of Argentinean economy that I analyze in Appendix G. I apply the same parameters as the Greek parameters that I abbreviate in Table 4. Most of the remaining parameters are based on Mendoza and Yue (2012) such as the intermediate input share in final goods output and the Armington weight of domestic inputs. The risk-free interest rate is set to 2% from the average quarterly money market rate between 1993Q4 and 2001Q3. Thus, I set the subjective discount factor of households to 0.98. Average government consumption over GDP is set to 21.9% also from data between 1993Q4 and 2001Q3, and I set the tax rate criterion to same value as the government consumption criterion. The government debt criterion is set at 33.5 % of the debt-to-GDP ratio from
the average between 1993 and 2000. As I explain in the previous Appendix, the political disturbance is set to 0.33.

Table 4: Calibration of Argentinean Economy

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$: Subjective discount factor of households</td>
<td>0.98</td>
<td>$1 - r_f$</td>
</tr>
<tr>
<td>$\alpha_M$: Intermediate input share in final goods output</td>
<td>0.43</td>
<td>Mendoza and Yue (2012)</td>
</tr>
<tr>
<td>$\alpha_L$: Labor input share in output of final goods</td>
<td>0.40</td>
<td>Mendoza and Yue (2012)</td>
</tr>
<tr>
<td>$\psi$: Substitution elasticity across intermediate goods</td>
<td>2.86</td>
<td>Mendoza and Yue (2012)</td>
</tr>
<tr>
<td>$\nu$: Substitution elasticity within intermediate goods</td>
<td>2.44</td>
<td>Mendoza and Yue (2012)</td>
</tr>
<tr>
<td>$\lambda$: Armington weight of domestic inputs</td>
<td>0.62</td>
<td>Mendoza and Yue (2012)</td>
</tr>
<tr>
<td>$r_f^I$: World risk-free interest rate</td>
<td>0.02</td>
<td>Money Market Rate</td>
</tr>
<tr>
<td>$G$: Government consumption criterion</td>
<td>0.0106</td>
<td>IADB ($G/GDP = 0.219$)</td>
</tr>
<tr>
<td>$\overline{B}$: Government debt criterion</td>
<td>0.0163</td>
<td>IMF ($\overline{B}/GDP = 0.335$)</td>
</tr>
<tr>
<td>$\overline{\tau}$: Tax rate criterion</td>
<td>0.219</td>
<td>= Gov’t Consumption Criterion</td>
</tr>
<tr>
<td>$\varphi$: Political disturbance</td>
<td>0.33</td>
<td>Bi et al. (2014)</td>
</tr>
</tbody>
</table>
Appendix I. Data

<Greece>

- Public Debt-to-GDP:
  - For debt criteria: the measurement is annual and the data source is OECD.
  - For simulation graph: the measurement is quarterly and the data source is Eurostat.

- Government consumption over GDP: The measurement is quarterly and seasonally adjusted. The source is IMF.

- Government tax revenue over GDP: The measurement is quarterly and seasonally adjusted. The source is IMF.

- GDP: The data is seasonally adjusted quarterly real gross domestic product of the expenditure approach. The source is OECD.

- Households’ Consumption: The measurement is quarterly, real and seasonally adjusted. The source is OECD.

- Imported Intermediate Goods: The period is early 2000s. The source is OECD’s STAN Input-Output data.

- Imports of goods and services: The measurement is quarterly, real and seasonally adjusted. The source is OECD.

- Domestic Intermediate Goods: The measurement is annual and real. The source is ECB.

- Labor: The data is total employment, and the source is OECD.

- Bond Spread: The data is quarterly interest rate on 10-year Greek government bonds minus that of German government bonds. The source is OECD.

- TFP: The data is seasonally adjusted quarterly GDP per person employed, and the source is OECD.
<Argentina>

- Money market rate: The measurement is quarterly, and the source is IMF.
- Public Debt-to-GDP: The measurement is quarterly and seasonally adjusted. The source is IMF.
- Government consumption over GDP: The measurement is quarterly, and the source is Inter-American Development Bank.