The Interaction of Rents and Wages when Labour is Mobile Across Regions and Subject to Matching Frictions

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Abstract

We develop a two-region search-and-matching model where unemployed can choose in which region to search, but where moving to a particular region also implies paying a region-specific rent. We first show that the search approach provides a natural framework that substantially reduces the need to rely on decreasing returns to scale or relocation costs to dampen regional labour relocation. We find that both under Nash bargaining for wages and the sharing rule based on labour productivity a regional rent increase raises negotiated wages in all regions, which is consistent with the anecdotal evidence that rents play an important role in wage negotiations. Moreover, a regional productivity shock pushes rents and house prices in different direction than a regional rent shock, which gives a theoretical foundation for distinguishing regional productivity shocks from regional rent shocks in empirical work.

JEL classification: E24, J31, J42, J61, O18.

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1 Introduction

Interest in issues related to the housing sector has been increasing among macroeconomists since the start of the housing boom in the U.S. in the early 2000s and the interest has only increased when the boom ended with the Great Recession (Piazzesi and Schneider, 2016). This has given rise to numerous theoretical and empirical contributions analysing the relation between house prices, rents, and wages, see for instance Moretti (2011), Henderson (2014), Bracke (2015), Albouy (2016), and many others. Much of this literature has been in the field of spatial economics, where the focus is typically not on business cycles. Recently, however, this has shifted to some extent. For instance, Piazzesi and Schneider (2016) argue that the housing market is essentially a collection of markets that are geographically distinct and also differ along other dimensions. Importantly, they point out that these differences can affect the aggregate effects of shocks that hit the economy if not all agents in the economy are able to bear the effects of such shocks in the same way. This notion makes it interesting to study the relation between regional labour and housing markets - two important features of the spatial economics literature - in a business cycle model.

The relation between rents and wages is not interesting also from a more practical perspective, as changes in rents make it difficult for employers to retain key workers. This is why e.g. civil servants in London are entitled to the so-called "London weighting," a wage allowance to compensate for higher cost of living in London. Similarly, NHS employees in London can get up to 20% higher salary if they live in Inner London. This is not something specific to the U.K. Rising rents have for instance been cited as one of the main drivers of the increase in the living wage in Ireland between 2016 and 2017: "The current housing crisis, and associated increases in rent levels, has been the main driver of the wage rate increasing in 2017," (Living Wage Technical Group, 2017). Moreover, the notion that rising rents can lead to wage pressures does not apply only to wages at the lower end of the wage spectrum. Mr. Simon Coveney, the Irish Minister for Housing, Planning, Community and Local Government, has stated that1

"...rents are spiralling upwards... [ ]. This situation... [ ] ...is threatening our economic recovery by undermining competitiveness, driving up wage demands and mak-

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ing Ireland less attractive investment destination."

If this is indeed the case, then it would indicate that developments in the housing market have much broader consequences for the macroeconomy, and these can be more dire for small open economies. This would particularly be the case for those small open economies that are in a monetary union, where wage adjustment is an important mechanism for adjusting to shocks.

Given the anecdotal evidence that rents affect living wage determination and given that there is some notion that high rents affect wages, it is interesting to examine more formally what is the relation between the two and whether this changes under different circumstances, in particular those related to the labour market. To investigate these issues, we build a theoretical model that consists of two regions with potentially different levels of rents and allow unemployed to look for jobs in both regions.

When considering the move, we allow workers to take into account that moving to a different region involves paying the prevailing rent in that particular region. We use this setup to study how aggregate and local productivity shocks as well as an exogenous rent increase in one region affect regional employment, unemployment, wages, rents, and house prices. Our approach differs from the typical approaches used in the spatial economics literature (see Rosen (1979) and Roback (1982) for the beginnings and Moretti (2011), for the current standard model) in that we consider a more realistic setting of the labour market. Because the focus of spatial economics literature is typically the long run, the models used are often static and labour markets are stylised. A typical assumption is that labour supply to a city/region is infinitely elastic and that wages are determined on spot markets. While unemployment and job prospects are important in determining relocation of workers and consequently house prices and rents empirically, as in De Bruyne and Van Hove (2013), they tend to be absent from formal models.

We adopt a search-and-matching approach to labour markets, which allows us to analyse the relation between rents and wages in the context of regional unemployment, job finding prospects and wage bargaining arrangements. Moreover, we show that the search-and-matching approach provides a micro-founded way to dampen inter-regional labour flows, which allows us to significantly decrease the role of decreasing returns of scale or costs of relocation between regions. The reason is that workers who consider moving to a different region obtain a job only
with some probability. Even when this probability increases after a positive shock to a region, it still implies that a worker may remain unemployed after the move, which dampens relocation of labour. Finally, our model is dynamic and allows us to analyse, in principle, the effects of both temporary and permanent shocks. The model can be thought as a - admittedly very stylised - representation of Ireland, where about 40% of the population lives in the Greater Dublin Area and the remainder elsewhere in Ireland.

We find that the presence of regional wage bargaining typically amplifies the effects of shocks considered, compared to a rule where wages are determined by the marginal product of labour. Spillovers between regions due to labour reallocation play an important role. An exogenous increase in rents in one region spills over to the other region when labour reallocation is elastic, which results in a rent increase in both regions. This happens because a rent increase in one region tends to drive labour to the other region, increasing demand for housing in that region. An increase in rents in both regions also puts an upward pressure on wages, and wages increase by somewhat more in the region where rents have increased. A regional productivity increase attracts workers to the region through higher wages and higher probability of finding a job. However, it also leads to the regional wage increase. The latter, however, is not strong enough - under our calibration - to drive out labour coming to the region.

In addition, we find that wage bargaining arrangements, whether they are regional or country-wide, make no significant quantitative difference in terms of aggregate outcomes and relocation of population between regions. They do make a difference, however, in the way relocation of workers is achieved. When wage negotiations are country-wide, the strong response of vacancies is crucial to achieve sufficient movement in matching probabilities to compensate for the movement in regional wages.

These findings are interesting also from empirical perspective, as they indicate how one could distinguish between regional productivity shocks and regional shocks to rents. When there is a positive shock to rents in one region (i.e., rents increase), this tends to increase rents in the other region as well. On the other hand, a positive productivity shock in one region increases rents in the region, but lowers them in the other region. The difference in the response of regional rents would in principle allow identification of the two shocks.

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2In this paper, our focus are temporary, but persistent, shocks.
2 Model

We assume the economy consists of two regions \( i \), where \( i \in [1, 2] \). Each region has its own labour market, but the two markets are linked through directed search. Unemployed workers are allowed to decide in which of the regions they will search for work. For simplicity, we assume no disutility of working and we assume that all unemployed workers search for jobs. All workers are members of a single representative household that maximises the following utility function:

\[
\max_{\mathbf{E}_t} \sum_{t=0}^{\infty} \beta^t \left[ \frac{c_{1,t}^{1-\gamma} - 1}{1-\gamma} + \operatorname{pop}_{1,t} \xi_1 \frac{h_{1,t}^{1-\nu_1}}{1-\nu_1} + \operatorname{pop}_{2,t} \xi_2 \frac{h_{2,t}^{1-\nu_2}}{1-\nu_2} \right],
\]

subject to the budget constraint

\[
c_t + k_{t+1} + r_{H_i,t} h_{i,t} = \tilde{w}_t + \tilde{b}_t + (1-\delta)k_t + r_t k_t + \Pi_{1,t} + \Pi_{2,t},
\]

where \( c_t \) is consumption, \( k_t \) is aggregate capital, \( h_{i,t} \) are housing services, \( r_{H_i,t} \) is the rent in region \( i \) and \( \operatorname{pop}_{i,t} \) is population in region \( i \). \( \delta \) is the depreciation rate, \( r_t \) is the return on capital, \( \Pi_{1,t} \) and \( \Pi_{2,t} \) are firm profits and net rental income in regions 1 and 2, respectively (both are assumed to be taken as lump-sum by the household; rents are distributed by the landlords to the members of the household as lump sum). \( \tilde{w}_t \) is the aggregate wage income received by household members working in regions 1 and 2, \( w_{1,t} n_{1,t+1} + w_{2,t} n_{2,t+1} \), and \( \tilde{b}_t \) are the aggregate unemployment benefits, \( b(u_{1,t+1} + u_{2,t+1}) \).

The household as a whole chooses consumption and capital (investment), which leads to the following optimality conditions:

\[
c_t^{-\gamma} = \lambda_t \quad (2)
\]

\[
\lambda_t = \lambda_{t+1} (r_{t+1} + 1 - \delta), \quad (3)
\]

where \( \lambda_t \) is the Lagrange multiplier on the budget constraint and is equal to the marginal utility of consumption. Equation 3 is the standard Euler equation.

We assume that the household decides on aggregate quantities by pooling income of its
members. This guarantees the full insurance, as in Merz (1995) or Andolfatto (1996). Once the aggregate capital is decided, it can be used by firms in both regions:

\[ k_{t-1} = k_{1,t} + k_{2,t}. \]  

(4)

We first discuss labour flows and then provide more details on how we model the rental market.

2.1 Labour flows

Total population of the economy consists of a continuum of workers with the mass of one. The workers can be either employed or unemployed, in both regions. If \( n_{1,t} \) is the number of employed in region 1, and \( n_{2,t} \) is the number of workers employed in region 2, while \( u_{1,t} \) and \( u_{2,t} \) is the number of unemployed workers in regions 1 and 2, respectively, we have:

\[ 1 = n_{1,t} + n_{2,t} + u_{1,t} + u_{2,t}. \]  

(5)

Matching. Each region \( i \) has its own matching function:

\[ M_{i,t} = \phi_i v_{i,t} u_{i,t}^{1-\mu_i}. \]  

(6)

The number of matches in each region, \( M_{i,t} \), is determined by the number of searching workers in the region, \( u_{i,t} \), the number of vacancies posted by firms in the region, \( v_{i,t} \), the efficiency of the matching process, \( \phi_i \), and the elasticity of the matching function, \( \mu_i \). For simplicity we assume that the parameters of the matching functions are identical across regions, i.e., \( \mu_1 = \mu_2 \) and \( \phi_1 = \phi_2 \).

Based on this we can define two matching probabilities. \( p_{W_{i,t}} \) is the probability for unemployed to find a job and \( p_{F_{i,t}} \) is the probability that a firm will find a worker:

\[ p_{W_{i,t}} = \phi_i \left( \frac{v_{i,t}}{u_{i,t}} \right)^{\mu_i} \]  

(7)
\[ p_{F,i,t} = \phi_i \left( \frac{v_{i,t}}{u_{i,t}} \right)^{\mu_i - 1}. \] (8)

**Labour flows.** Employment (and therefore unemployment) in each region follows a law of motion. We assume that in the beginning of each period, a fraction \( \rho \) of employed workers in each region loses their jobs. This yields the following laws of motion for each region \( i \):

\[ n_{i,t+1} = (1 - \rho) n_{i,t} + p_{W,i,t} u_{i,t}. \] (9)

We assume that all newly matched workers become productive immediately. The number of workers that are producing in the current period is therefore the sum of all those whose employment relationship has survived the exogenous separation, plus all newly-hired workers.

### 2.2 Value functions

Workers in each region can be either employed and unemployed, and each of these two states has the corresponding asset value. The value of working in region \( i \), \( W_{i,t} \), is defined as:

\[ W_{i,t} = w_{i,t} - h_{i,t} + \beta \lambda t^{t+1} \left[ (1 - \rho) W_{i,t+1} + \rho U_{i,t+1} \right], \] (10)

where \( w_{i,t} \) is the wage in region \( i \) the worker receives when working, \( h_{i,t} \) is the per-period rent cost in region \( i \), and \( U_{i,t+1} \) is the (expected) value of ending being unemployed in the next period in region \( i \) in case the employment relationship ends. The value of being unemployed is determined as follows:

\[ U_{i,t} = b - h_{i,t} + \beta \lambda t^{t+1} \left[ (1 - p_{W,i,t+1}) U_{i,t+1} + p_{W,i,t+1} W_{i,t+1} \right], \] (11)

where \( b \) are unemployment benefits (assumed constant across both regions). Note that being in a region \( i \) implies that one has to pay the rent in that region, regardless of the employment status. This is very different than, for instance, taxes, that are paid only if a worker is employed and as a fraction of wages.

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3Note that the asset values of the worker can be derived from the first order conditions of the household.
Equation 10 consists of the wage income net of rent payments earned in each period, $w_{i,t} - h_{i,t}$, plus the continuation value, which is a discounted value of two states that an employed worker faces in the next period. With probability $p_x$ the employment relationship ends and the worker end up being unemployed, receiving the value of being unemployed in the next period, $U_{i,t+1}$. With probability $1 - p_x$ the employment relationship continues and the worker keeps the value of being employed in the next period, $W_{i,t+1}$. Similarly, equation 11 states that the value of being unemployed consists of unemployment benefits net of rent payments in each period, $b - h_{i,t}$, plus the discounted continuation value for each of the states the worker can be in the next period. With probability $p_{W,i,t+1}$ the unemployed worker finds a job in the next period and receives the value of $W_{i,t+1}$, or remains unemployed with probability $(1 - p_{W,i,t+1})$ and receives the value of being unemployed in the next period.

In the same spirit as for workers, we can define two value functions of the firm. We assume that there is a continuum of firms in each region, and each of them can post a vacancy and employ one worker. The value for a firm in region $i$ of having a worker employed in time $t$, $J_{i,t}$ is:

$$J_{i,t} = MP_{L,i,t} - w_{i,t} + \beta \lambda_{t+1} [(1 - p_x) J_{i,t+1}],$$  \hspace{1cm} (12)

where $MP_{L,i,t}$ is the marginal product of a worker in the benchmark case with Nash bargaining. Where we consider the wage rule for wages, we assume that $MP_{L,i,t}$ is equal to the output per worker, after capital has been paid off.

Firms can enter the labour market by posting a vacancy, $v_{i,t}$. We assume that having a vacancy open entails a per-period flow cost of $\psi$. Paying this cost gives a firm the chance that it will find the worker, which occurs with the probability $p_{F,i,t}$, and the value of this worker for the firm is $J_{i,t+1}$, because it takes one period before workers become productive. If the vacancy remains unfilled, we assume that it expires. At the optimum, the cost of having a vacancy open must equal the expected gain for the firm of finding the worker. Firm will therefore enter the market (post vacancies) until the following condition holds:

$$\psi = p_{F,i,t} \beta \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma} J_{i,t+1}.\hspace{1cm} (13)$$
2.3 Rents and house prices

For simplicity, we assume that rents are collected by a representative landlord, who is a member of the representative household. Rents that are collected by the representative landlord are redistributed as lump-sum to the household, which shares the income. Demand for housing services comes from workers, who have housing in their utility and pay rents to landlords, who in turn provide housing. We assume all workers are renters. The first order condition of each renter with respect to housing in each region is (taking rent as given):

\[ \text{pop}_{i,t} \xi_i H_i^{\nu_i} = c_t - \gamma r_{H,i,t}. \]  

(14)

In the optimum, the marginal utility of housing services demanded in each region has to be equal to the rent paid in the region. The obtained equation is similar to the standard Rosen-Roback model (see Moretti (2011)), where rents are assumed to be exogenously determined and depend on the population in the region.

Landlords hold all housing stock in both regions. They rent it to workers, can trade with housing, but otherwise derive no utility from it. We assume housing stock is fixed and equal to 1 in each region. Landlords do not invest in housing and housing stock does not depreciate. They maximise profits from renting and housing re-sales:

\[ \Pi_{r,t} = P_{H,i,t} H_i t - P_{H,i,t} H_{i,t-1} + r_{H,i,t} H_i t, \]  

(15)

where \( \Pi_{r,t} \) is profit from renting housing, which is distributed as lump-sum to the representative household. The optimality condition of representative landlords gives the equation for house prices in each region:

\[ P_{H,i,t} = r_{H,i,t} + \beta \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma} P_{H,i,t+1}. \]  

(16)

Note that an increase in the population in the region increases the price of housing through the assumption. Housing stock is a slow-moving variable and our assumption should be viewed as a first approximation. The assumption could be relaxed by introducing investment in housing, but for this to be realistic, housing adjustment costs would have to be high. This would not significantly alter the results reported here.
the increase in rent.

2.4 Directed search

We allow unemployed workers to decide in which of the two regional labour markets they will search. Unemployed will try to move from the region where the value of being unemployed is lower to the region where this value is higher. In equilibrium, unemployed workers will stop moving between both regions when they will be indifferent between the two values. This will be the case when

$$U_{1,t} = U_{2,t}.$$  \hspace{1cm} (17)

Equation (17) implicitly determines the number of searching workers in each region $i$.

2.5 Production and determination of region size

Production in each region takes into account land, labour and capital:

$$y_{i,t} = \exp(z_t)\exp(z_{i,t})a_{i,t}^{1-\alpha_i-\zeta_i}k_{i,t}^{\alpha_i}n_{i,t}^{\zeta_i},$$  \hspace{1cm} (18)

where $a_{i,t}$ is land (assumed fixed) in each region, $\alpha_i$ and $\zeta_i$ are shares of capital and labour, respectively, $z_t$ is the (logarithm of) aggregate productivity shock and $z_{i,t}$ is the (logarithm of) region-specific productivity shock. We assume that logarithms of shocks evolve as AR(1) processes:

$$z_t = \rho z_{t-1} + e_{z,t}$$  \hspace{1cm} (19)

$$z_{i,t} = \rho_i z_{i,t-1} + e_{z_{i,t}}$$  \hspace{1cm} (20)

where $\rho$ and $\rho_i$ measure the persistence of shocks, and where $e_{z,t}$ and $e_{z_{i,t}}$ are aggregate and region-specific innovations in productivity.
Determinacy of region sizes. Production functions in the model are constant returns to scale in terms of all production factors, but decreasing returns to scale in terms of capital and labour. The reason for this assumption is that with fully symmetric regions (in terms of calibration), perfectly mobile capital, vacancies, and labour subject only to search frictions in the short run (but fully mobile in the long run), sizes of regions are not determined. Including the fixed factor allows us to determine the size of regions (our calibration is symmetric, so that both regions are of equal size in the steady state). However, we calibrate the production function so that the fixed factor plays no quantitatively meaningful role (i.e., we set $\alpha_i + \zeta_i$ close to 1). The reason why can do this is because search frictions on the labour market act as a cost that substantially slows down labour relocation between regions and substantially increases the region where the model is still stable. We discuss this issue in more detail in the Results section.

2.6 Wage determination

Wage determination is an important feature of our analysis. Labour mobility across regions affects the relative labour supply in each region and can therefore affect wage bargaining. In typical spatial economics models, the assumption is almost always that wages are set at location-specific level (e.g., a city, region), see e.g. Moretti (2011). This implies that wage bargaining is sufficiently decentralised so that regional conditions matter for wage determination. Our benchmark case keeps this assumption, but we also investigate a case where wages are set on a country-wide level.

In the benchmark case, we assume that wages in each region are determined using Nash bargaining, as is standard in the labour literature. Importantly, Nash bargaining takes into account labour market conditions through probabilities that unemployed will find a job in a particular region (regional labour market tightness). As an alternative, we consider a wage rule that is based on the productivity of the worker only, i.e., does not take into account labour market conditions when setting wages.

5Same level of output, employment, consumption, etc. can be achieved by any allocation of labour, capital, vacancies, etc. across the regions.
Nash bargaining. With Nash bargaining, workers and firms in each region bargain over the surplus that is created when the match is formed. Formally, they maximise the following expression:

$$\max_{w_{i,t}} (W_{i,t} - U_{i,t})^{1-\eta_i} J_{i,t}^{\eta_i},$$

where $\eta_i$ is the bargaining power of firms in region $i$. This results in the following expression, which implicitly determines wages in each region:

$$(1 - \eta_i)(W_{i,t} - U_{i,t}) = \eta_i J_{i,t}.$$  

Sharing rule. An alternative bargaining setup is based on the wage norm, for instance as suggested by Hall (2005). An interesting wage norm for our question is one that does not directly depend on matching probabilities or other region-specific labour market characteristics. Because in our framework both capital and labour are mobile across regions, we consider the wage norm similar to the one used by den Haan and Kaltenbrunner (2009), where wages depend only on the capital-to-labour ratio in both regions. Their rule assumes that capital is paid its market price, $r_t k_{i,t}$, and the remaining surplus is split between workers and firms in a fixed proportion. Because we have (slightly) decreasing returns to scale, we do not require an additional parameter to determine profits of firms. We therefore assume that workers are paid their marginal product and anything that remains is the profit of the firm. This gives us the following expression for the wage (in per-capita terms):

$$w_{i,t} = z_t z_{i,t} \zeta_i \alpha_{i,t}^{(1-\alpha_i - \zeta_i)} k_{i,t}^{\alpha_i} n_{i,t}^{\zeta_i - 1},$$

Note that because the movement of capital between regions is not subject to adjustment costs,
this rule yields (almost) the same capital-to-labour ratio and therefore (almost) the same wage in both regions as long as shocks are symmetric or are not shocks to productivity in the region $i$ only.

2.7 Calibration

We calibrate the model using a set of standard values from the literature whenever available. In the calibration, we harmonise the model with Nash bargaining and the model with the sharing rule so as to obtain the same steady state for all real variables in both models. The reason for doing so is that this greatly facilitates the comparison between the two setups. The downside of this is that the profits of entrepreneurs, who post vacancies, can be relatively small and very volatile, which results in large fluctuations in vacancies. This is particularly evident in the model with Nash bargaining, where wages tend to react somewhat less and therefore amplify the response of profits. The values and sources (where available) for the calibration are reported in Table 1.

We calibrate the population of the economy to have a mass of one, distributed equally across the two regions. This can be thought as a - admittedly very stylised - representation of Ireland, where about 40% of the population lives in the Greater Dublin Area and the remainder elsewhere in Ireland.

3 Results

Before we turn to the analysis of the interaction of rents and wages, we briefly discuss the role of search frictions in ensuring the determinacy of region sizes in the model.

3.1 Role of matching frictions and determinacy of region sizes in the model

In principle, a model of two (or more) fully symmetric regions with constant returns to scale in production, matching, and fully free movement of labour and capital, suffers from the fact that

\[ \text{If production function were constant returns to scale, the capital-to-labour ratio and therefore the wage would be identical in both regions.} \]
### Table 1. Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target/Source</th>
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<tbody>
<tr>
<td><strong>Utility function</strong></td>
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<td>Inv. of int. el. of sub., $\gamma$</td>
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<tr>
<td><strong>Production</strong></td>
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<tr>
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<td>Keeping production close to CRS</td>
</tr>
<tr>
<td>Labour share, $\zeta_i$</td>
<td>0.699</td>
<td>Keeping production close to CRS</td>
</tr>
<tr>
<td>Depreciation, $\delta_i$</td>
<td>0.025</td>
<td>10% annually</td>
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<tr>
<td><strong>Matching function</strong></td>
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<tr>
<td>Elast. w.r.t. unempl., $\mu_i$</td>
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<td>Petrongolo and Pissarides (2001)</td>
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<tr>
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<td>$p^W = 0.6$, den Haan et al. (2000)</td>
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<tr>
<td><strong>Other labour-related</strong></td>
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<td>Breakup rate, $\delta_x$</td>
<td>0.0315</td>
<td>Unemp. rate 5%</td>
</tr>
<tr>
<td>Bargaining power, $\eta_i$</td>
<td>0.1</td>
<td>Used to equalise wages across both models</td>
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<tr>
<td>Vacancy posting cost, $\psi_i$</td>
<td>0.0092</td>
<td>$p^F = 0.7$, den Haan et al. (2000)</td>
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<td>Replacement ratio, $rr$</td>
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<td>Adjusted to equalise steady states</td>
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<td>Responsiveness to population, $\phi_N$</td>
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<td>Responsiveness to house prices, $\phi_{Ph}$</td>
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<tr>
<td><strong>Persistence of shocks</strong></td>
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<td>Aggregate productivity, $\rho$</td>
<td>0.9</td>
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<tr>
<td>Regional productivity, $\rho_i$</td>
<td>0.9</td>
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</table>

the region size is not determined. Even if a symmetric steady state is imposed, a tiny positive shock in one region completely empties the other region.\(^7\) There are two standard solutions to this problem: Use a fixed factor or impose a cost for relocation.

When a fixed factor is used (e.g., land), it has to play a sufficiently strong role in production to prevent that shocks specific to one region cause implausibly strong relocation of labour (in terms of production functions used in this model, $1 - \alpha_i - \zeta_i$ has to be relatively large). Moreover, with decreasing returns in mobile factors the size of the region plays a role, and the role of the region’s size becomes more important the more important the fixed factor is, which may be an undesirable feature.

The advantage of using search and matching frictions is that they provide a micro-founded

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\(^7\)When perturbation methods are used (Adjemian et al., 2018), which is what we do, the solution is approximated around a steady state. If the decreasing returns (or relocation costs) are not sufficiently large, then a small shock can result in the outflow of the entire population (or more) from a region. This is an artefact of the solution method, which does not take into account the occasionally binding constraint that the population of a region is limited. The issue would not arise if one used global methods.
framework where search frictions act as a cost that significantly slows down labour relocation. As a result, one can significantly reduce $1 - \alpha_i - \zeta_i$ and still obtain economically plausible relocation magnitudes. We demonstrate this by comparing the search-and-matching model with a standard RBC model of two regions with decreasing returns to scale, but no search frictions.

First note that there is no unemployment in a standard RBC model and the condition that determines relocation is that wages (after rents) have to be equal in both regions, $w_{1,t} = w_{2,t}$.\footnote{As explained above, in a search-and-matching model the relocation condition is $U_{1,t} = U_{2,t}$.} This implies that in a regional RBC model, if there are no relocation costs, decreasing returns to scale for mobile factors are the only way to regulate the magnitude of relocation. This implies that - in the absence of other costs - the decreasing returns to scale in mobile factors have to be relatively large.

Figure 1 shows the comparison of two models, the benchmark search and matching model with Nash bargaining that we will use in the remainder of the paper, and the standard RBC model of two regions. We set returns to the fixed factor, $1 - \alpha_i - \zeta_i$, to 0.1 in the RBC model, and to 0.001 in the search and matching model. Figure[1] plots the response to a positive productivity shock in region 1 only. For illustrative purposes, the size of the shock is 0.0007, i.e., 10-times less than in a typical RBC model. This tiny shock, in combination with a relatively strong decreasing returns, still causes about 7% of population to relocate from region 2 to region 1. On the other hand, the search and matching model causes less than 1% of population to relocate, which, while still high, is nevertheless by almost an order of magnitude less than in the RBC model. Note that this is the case despite the fact that we set the importance of the fixed factor in the production to be 100 times smaller in the search and matching model compared to the RBC model. In addition, the relation between wages and labour supply in the search model is not as rigid as in the RBC model, and variables such as job finding probability play a role in wage negotiations.
Figure 1. Comparison of a search-and-matching model with the RBC model

Notes: All variables are reported as percent deviations from their initial pre-shock values. The exceptions are aggregate and regional employment and unemployment, which are reported in percent of total population. Probabilities are reported as percentages.
3.2 Responses to shocks

We report results for two models, one with Nash bargaining and the other with the sharing rule for wages. To facilitate comparison, we calibrate both models so that they have identical initial steady state, and then investigate the effects of three shocks:

- Simultaneous productivity increase in both regions
- Productivity increase in one region only
- Exogenous rent increase in one region only

We solve the model using perturbation methods, see Adjemian et al. (2018).

3.2.1 Simultaneous productivity increase in both regions

The effects of a productivity increase across both regions are shown in Figures 2 and 3. An increase in productivity means that more can be produced with existing capital and labour, but also that it is profitable to employ new workers in order to benefit from the productivity increase. Employment increases and unemployment falls in both models. The model with Nash bargaining, however, leads to stronger employment and output response. Without sticky wages, the increase in firm surplus in the model with the sharing rule is the same as the increase in wages and return on capital. With Nash bargaining, the increase is somewhat larger because wages respond by less (see also Trigari (2009)). The stronger increase in firm surplus (profits) induces firms to post more vacancies, which leads to higher probability of finding a job for workers, higher employment and lower unemployment rate (which drops by about 25% lower in region 1). Because employment is higher with Nash bargaining, the benefits of the productivity shock can be exploited better, which is why output is higher. Note that the stronger responses of the model with Nash bargaining are in part the consequence of our calibration of both models. To achieve better comparability, we had to start from the same initial steady state. This resulted in small profits of entrepreneurs and very high responsiveness of vacancies when wages do not

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9 All shocks are temporary, but persistent.
10 All results are first-order approximations.
11 In order to achieve comparability of both models, we had to reduce the bargaining power of workers in the model with Nash bargaining.
adjust fully in accordance with the marginal product of labour (this is one of the solutions for
the so-called Shimer puzzle (Shimer, 2005), see also Hagedorn and Manovskii, 2008).

Note that because regions are symmetric and the shock is the same in both, there is no
reallocation of population. The change in rents is due to the change of house prices, which is in
turn due to the aggregate consumption. This is the same across the country and house prices
increase by the same amount in both regions (see bottom row of Figure 3). Overall, because
of the symmetry of regions and shocks, two regions behave as one (larger) region. In this sense
aggregate shocks are less interesting and we proceed by focussing on region-specific shocks.
Figure 2. Simultaneous productivity increase in both regions: Main variables

Notes: All variables are reported as percent deviations from their initial pre-shock values. The exceptions are aggregate and regional employment and unemployment, which are reported in percent of total population. Probabilities are reported as percentages.
Figure 3. Simultaneous productivity increase in both regions: Auxiliary variables

Notes: All variables are reported as percent deviations from their initial pre-shock values. The exceptions are aggregate and regional employment and unemployment, which are reported in percent of total population. Probabilities are reported as percentages.
3.2.2 Regional productivity increase

A regional productivity increase is modelled in the same way as the aggregate productivity increase, but we assume that the productivity increase occurs only in region 1. One can think of this shock as an arrival of a large firm with advanced technology in region 1. The results are shown in Figures 4 and 5. The key difference from the results in the previous section is that now firm profits increase only in region 1, which means that only firms in region 1 post new vacancies. Unemployed move to region 1, where jobs are available (causing a brief increase in unemployment), and this increases rents in region 1. Rents in region 2 decrease. We assumed that housing supply is relatively inelastic, which results in a substantial increase in rents. House prices (Figure 5) reflect the movements in rents and increase in region 1 and decrease in region 2.

Wages in region 1 increase and decrease in region 2. The increase in wages in region 1 is stronger than in the case when the productivity increase was symmetric across both regions. This happens because of the strong increase in rents in region 1, which dampen the reallocation of labour from region 2 to region 1. Because workers move from region 2 to region 1, rents in region 2 fall. This makes it - together with the strong rent increase in region 1 - somewhat more attractive to stay in region 2 and labour supply in region 1 increases by less than it would if rents had not moved in the opposite direction. The reason why this happens in both models is that the decision to move to the other region depends (among other things) on the relative rent. The fact that wages in region 1 increase even in the model with the sharing rule is a direct consequence of labour not relocating in sufficient quantities, which drives up the marginal product of labour and hence wages.

Note that employment increase in region 1 is stronger than the unemployment decrease in region 2 (and the opposite holds for unemployment), so that aggregate employment increases and unemployment decreases. This is much more pronounced in the model with Nash wage bargaining. The reason is similar as for the aggregate productivity shock, but somewhat more involved. Regional productivity shock increases profits in region 1, resulting in a high number of posted vacancies. Unemployed, who are able to move between regions, seize the opportunity and move to the region where there are vacancies. This means that there are both vacancies
and unemployed in the same region, which prevents the congestion of the matching process. In this way some of the benefits of the productivity increase in region 1 are shared with region 2. Because with the sharing rule the reaction of vacancies is less vigorous, there is more congestion and therefore lower aggregate employment effects.
Notes: All variables are reported as percent deviations from their initial pre-shock values. The exceptions are aggregate and regional employment and unemployment, which are reported in percent of total population. Probabilities are reported as percentages.
Figure 5. Productivity increase in region 1: Auxiliary variables

Notes: All variables are reported as percent deviations from their initial pre-shock values. The exceptions are aggregate and regional employment and unemployment, which are reported in percent of total population. Probabilities are reported as percentages.
3.2.3 Regional rent increase

This section investigates the effects of an exogenous increase of rents in region 1. This shock can be viewed as a removal of rent regulation or an increase in the local monopoly power of landlords, for instance due to more congestion on the main commuting roads. The results are shown in Figures 6 and 7.

The immediate effect of a rent increase in region 1 is that it becomes less attractive to live there and people try to move away. This has two immediate consequences. First, the number of workers searching for work in region 1 drops, while it increases in region 2, resulting in a short increase in unemployment there. Second, rents in region 2 increase as workers move in the region. Note that productivity (and everything else) is the same in both regions, so employers in region 1 try to compensate the outflow of labour to region 2 by offering higher wages (somewhat less so under the Nash bargaining than under the sharing rule). Because the probability of finding a job in region 1 increases due to the outflow of workers, the value of being unemployed in region 1 increases. This value serves as the outside option in wage bargaining in region 2, where wages also increase, although by slightly less than in region 1.

There is an initial wage decrease in both regions, which is the consequence of temporarily higher aggregate unemployment (see bottom row of Figure 6). In both models, the rent increase does not change the aggregate employment by much. However, it does induce substantial reallocation of the population. Everything else equal, the relocation of searching workers to region 2 improves the probability that firms in region 2 find workers. Firms in region 2 realise this and post more vacancies than they would otherwise, which leads to more matches in region 2 (which compensates the reduction of matches in region 1). Therefore, aggregate unemployment decreases somewhat in the medium run.
Figure 6. Exogenous rent increase in region 1: Main variables

Notes: All variables are reported as percent deviations from their initial pre-shock values. The exceptions are aggregate and regional employment and unemployment, which are reported in percent of total population. Probabilities are reported as percentages.
Figure 7. Exogenous rent increase in region 1: Auxiliary variables

Notes: All variables are reported as percent deviations from their initial pre-shock values. The exceptions are aggregate and regional employment and unemployment, which are reported in percent of total population. Probabilities are reported as percentages.
3.3 Country-wide vs. regional wage bargaining

Most spatial economics models assume that wage bargaining takes place at region, commuting-zone, or even city-specific level. While this may be the case for some professions (e.g., civil servants in London), it may not be the case for all. In particular, one would expect that countries where wage bargaining is centralised or industry-specific, wages would depend less on the conditions specific to a particular location. This section investigates this by assuming that wage bargaining is economy-wide and not region-specific as above.

To analyse this, we have to modify the value functions in the benchmark model with Nash bargaining. If wage bargaining is country-wide, then the values of being employed, unemployed, and the value of having a worker are based on country-wide averages. To implement this, we proceed as follows. First, we assume that regional values of being employed and unemployed depend on a common wage, \( \tilde{w}_t \). Second, we assume that the job finding probability used in the country-wide Nash bargaining depends on the weighted job finding probability, \( \tilde{p}_W,t = (p_{W,1,t}u_{1,t} + p_{W,2,t}u_{2,t})/(u_{1,t} + u_{2,t}) \). Third, we assume that the value of having a worker for the firm that bargains is the country-wide weighted average value of having a worker, where the weights are employment shares, \( J_A,t = (J_{1,t}n_{1,t} + J_{2,t}n_{2,t})/(n_{1,t} + n_{2,t}) \). Finally, we assume that the rent relevant for bargaining is the average rent, \( \tilde{h}_t = h_{1,t}pop_{1,t} + h_{2,t}pop_{2,t} \). These definitions give us the following set of aggregate value functions on which wage bargaining is based (we denote aggregate value functions with the subscript \( A \)):

\[
W_{A,t} = \tilde{w}_t - \tilde{h}_t + \frac{\lambda_{t+1}}{\lambda_t} [(1 - \rho_x)W_{A,t+1} + \rho_xU_{A,t+1}],
\]

(24)

\[
U_{A,t} = b - \tilde{h}_t + \frac{\lambda_{t+1}}{\lambda_t} [(1 - \tilde{p}_{W,t+1})U_{i,t+1} + \tilde{p}_{W,t+1}W_{i,t+1}],
\]

(25)

the Nash bargaining condition is

\[
(1 - \eta)(W_{A,t} - U_{A,t}) = \eta J_{A,t}.
\]

(26)

Note that in the wage bargaining itself, the aggregate rent cancels out, as it is present both in \( U_{A,t} \) and in \( W_{A,t} \). However, the rent differential is important for individuals’ relocation decision.
The rent differential, therefore, still plays a role in wage bargaining through the reallocation of labour (directed search condition remains the same as before, i.e., based on individual region-specific values of being unemployed, see equation 11) and therefore by changing population weights in equations for the aggregate average job finding probability, rent, and the value of having a worker.

The results of a productivity increase in region 1 when wage bargaining is country-wide are shown in Figures 8 and 9. We only show results for region-specific shocks (aggregate shocks, as in the previous case, are completely symmetric across regions). Note that now both responses in the figures come from the same type of model (with Nash bargaining), where the Nash bargaining model from the previous section serves as the benchmark against which we compare the model with country-wide Nash bargaining. The main difference is the response of wages (Figure 8). With country-wide wage bargaining, wages increase in both regions by the same amount. This causes profits of entrepreneurs in region 1 to increase by more than when wage bargaining is regional, and profits in region 2 by less. As a result, the number of vacancies increases more strongly in region 1 (and decreases more strongly in region 2) then when wage bargaining is regional. This means that, instead by the difference in wages, relocation of unemployed is now driven by the job finding probabilities. The end result with country-wide wage bargaining is therefore similar as with regional wage bargaining, but the way it is achieved differs. Moreover, it does not prevent or mitigate relocation, which can be important if relocation is costly.
Notes: All variables are reported as percent deviations from their initial pre-shock values. The exceptions are aggregate and regional employment and unemployment, which are reported in percent of total population. Probabilities are reported as percentages.
Notes: All variables are reported as percent deviations from their initial pre-shock values. The exceptions are aggregate and regional employment and unemployment, which are reported in percent of total population. Probabilities are reported as percentages.
4 Empirical Evidence

While the aim of this paper is not empirical, it is nevertheless useful to investigate whether some of the relations between rents and wages found in the model have some empirical validity. To do this, we use data from the Survey of Income and Living Conditions (SILC) matched with regional data on rents from the website www.daft.ie, and estimate the extent to which regional variations in the Dublin-rural rent gap explain wages in Dublin.

More formally, we estimate a wage equation augmented by the Dublin-rural rent gap:

\[
\ln W_i = \alpha + \beta_1 X_i + \beta_2 \text{RentGap}_i + \mu_i
\]  

where \( X \) is a vector of wage determining covariates including experience and its square, the individual’s education level, age dummy variables taking the value one if the individual is a member of a union, female, married and zero otherwise, as well as a time trend. The results of this specification, without the rent gap, are displayed in the first column of table 2. The results are fairly standard. Next, we include the rent gap (lngap), which is the difference in rents between Dublin residents and rest-of-Ireland residents. The second column of table 2 shows that this is positive and statistically significant, indicating that there is a positive correlation between the gap in rents and wages in Dublin. However, there is no guarantee that wages in Dublin are not simultaneously influenced by rents and the results in the second column suffer from endogeneity bias. To address this, in lack of better instruments, we instrument the (log) rent gap with its past values and re-run the wage equation. The results are reported in the third column of table 2. The results survive and are consistent with the notion that when rents in Dublin increase by more than elsewhere in Ireland, wages tend to increase as well. While this does not constitute a proof and substantially more work has to be done, it could be considered as a rough initial indication that a regional rent increase exerts upward pressure on wages. In this sense, the findings here lend some support to the anecdotal evidence reported in the introduction.

4.1 Oaxaca Blinder Decomposition

[TBC]

32
We construct a regional dynamic stochastic general equilibrium model with search-and matching frictions on the labour market, where regional rents play a role in workers’ decision to move between regions. Workers’ decision to move affects rents and house prices in both regions.

We first show that search and matching frictions can act as a micro-founded device to moderate inter-regional labour flows. Unlike in a standard frictionless model, moving to a different region does not automatically imply that the worker will find a job in that region. While the probability of finding a job increases after a positive regional shock, it is not equal to one. Workers take into account that finding a job occurs only with some probability, which dampens inter-regional labour flows. Because of this, there is less need to rely on decreasing returns to scale or relocation costs.

We use this setting to study the effects of a common productivity shock in both regions and two region-specific shocks, namely a regional productivity increase and a regional rent increase. Our main findings are that a rent increase in one region spills over to the other region. Moreover, an increase in rents exerts upward pressure on wages in both regions, but more so in the region where the shock occurs. Regional productivity shocks increase rents in the region where they occur, but decrease rents in other regions. These two findings are interesting also from an empirical perspective, as they indicate that shocks related to rents only (but not productivity) can be separated from regional productivity shocks based on the direction of the regional house price responses. Moreover, we find that wage bargaining arrangements, whether they are regional or country-wide, make no significant quantitative difference in terms of aggregate outcomes and relocation of population between regions. They do make a difference, however, in the way relocation labour is achieved. When wage negotiations are country-wide, the response of vacancies is crucial to achieve sufficient movement in matching probabilities to compensate for the movement in regional wages.
References


Table 2. Estimates of the wage equation

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Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1