Financial Factors and the Natural Rate of Interest Puzzle

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Abstract

This paper develops and estimates a DSGE model for the US economy to study the natural rate of interest and its drivers. The originality of the analysis lies in the assessment of the role financial factors in the fluctuations of the natural rate by including financial intermediaries and unconventional monetary policy in the model. We get three main results. First, the analysis shows that permanent shocks, that capture a secular stagnation effect, are not a critical driver of the natural rate. Second, the persistent low level of the natural rate after the financial crisis finds its roots in the very long-lasting nature of financial shocks, as stated in the debt supercycle theory. Third, we find that the effectiveness of the unconventional monetary policy in stabilizing the natural rate is conditional on the type of shocks. In particular, a credit policy is effective in offsetting financial and supply shocks.

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1 Introduction

The natural rate of interest (sometimes referred to as the equilibrium real rate) has recently been at the forefront of monetary economics debates.\(^1\) This interest rate has become a leading indicator in determining whether monetary policy stance was enough accommodative following the implementation of unconventional policy measures. By comparing the effective real rate with its natural counterpart, monetary policymakers are able to determine whether the stance of monetary policy is too tight or too loose.\(^2\) In a context of nominal interest rates set at zero, the natural rate can also be used as a guide for an interest rate hike and a reduction of central balance sheet when economic conditions have normalized.

In this context, a new debate rage constantly among economists about the driving factors beyond low interest rates. On the one hand, the secular stagnation hypothesis, pioneered by Summers [2015], explains the decline in real rates as a result of structural factors acting on both the supply and demand side of savings. The main consequence of this misallocation on the saving market is a structural decline in aggregate demand and interest rates. On the other hand, Rogoff [2015] and Borio [2017] explains this downward pressure on interest rates through financial factors.\(^2\) According to these authors, the financial nature of the great depression has caused a very long-lasting damage to the economy. Since financial cycles are more persistent than standard business cycles, financial imbalances require more time than expected to clear, and by so interest rates to normalize. In particular, Borio [2017] suggests to incorporate financial factors in the estimation of the natural rate, and expect the latter to be above zero and considerably higher than the secular stagnation hypothesis suggests. Thus, the puzzle regarding the natural interest rate lies in the identification of its driving forces that would validate any of the financial drag or the secular stagnation hypothesis.

Given the current debate on the natural rate in a low interest rate environment, we set up and estimate over the period 1995:1:2017:I a DSGE model for the US economy that allows to examine the role of structural and financial factors as suggested by Borio [2017]. The originality of our approach is threefold. First, we extend the workhorse model of Smets and Wouters [2007] by including credit frictions a la Gertler and Karadi [2011] in order to disentangle standard business cycles from their financial counterpart. Second, we also disentangle the contribution of short run versus long drivers of the natural rates by including and estimating a stochastic trend on the labor-productivity. Third, we estimate the contribution of unconventional monetary policy measures on the natural interest rate. Our framework is then amenable for finding the economic conditions that makes unconventional monetary policy effective in rising the natural rate during the financial crisis episode.

\(^1\)From an historical standpoint, the natural rate concept was first coined by Wicksell [1898]. It determined the level of interest rates that would be necessary to maximize output by clearing the market between saving and investment. More recently, it was redefined by Woodford [2003] in a real business cycle framework as the interest rate that allows an economy to reach its potential, consistent with a stable inflation.

\(^2\)For a concrete illustration of the natural rate as an indicator employed by policymakers, see Yellen [2015]. Beyond being a straightforward indicator, it can also be incorporated as a target in Taylor [1993] rules.

\(^3\)Rogoff [2015] refers to debt supercycles while Borio [2017] to a financial drag. These two theories shares many aspects.
Our analysis is related to a growing body of estimation methods of the natural interest rate, that can be divided in three different strands of methodology. The fist one estimates semi-structural models using the Kalman filter, as introduced by [Laubach and Williams, 2003, 2015], where the natural rate is assumed to depend on the trend growth rate of the economy as well as on unobserved components. They find that the natural interest rate has fallen sharply since the start of the Great Recession and this drop was generalized to most of developed economies. The second methodology group uses pure econometric methods to estimate the natural rate of interest. Using a time-varying parameter vector autoregressive model, Lubik and Matthes [2015] show that the natural rate has recently been above its effective counterpart, which means that the monetary policy is too loose. Yi and Zhang [2016] also find evidence of a structural decline of real interest rates in numerous countries, this decline is due to a reduction in investment demand rather than to a saving rise. Finally, the last strand of methods relies on New Keynesian DSGE models estimated using Bayesian techniques. Justiniano and Primiceri [2010] and Barsky et al. [2014] estimate a standard new keynesian model to measure the natural rate of interest for the US economy. Cúrdia [2015] and Cúrdia et al. [2015] enrich this environment by including the role of forward guidance while Gerali and Neri [2017] disentangle the role of short term versus long term effects on the natural interest rate through the introduction of trends. In addition, Del Negro et al. [2015], Del Negro et al. [2017] interestingly extend the Smets-Wouters model to include a financial accelerator mechanism, and use inflation expectations and the treasury yield as observable in their fit exercise.

Our work is strongly connected to Del Negro et al. [2017] as we both feature trends and financial frictions in our analysis. However, we do not analyze the role of forward guidance as an unconventional policy tool, but rather the central bank’s balance sheet. Our analysis also include a set of three observable financial variables in addition to the seven standard macroeconomic variables of Smets and Wouters [2007]. Our approach is rather complementary to Del Negro et al. [2017], as it allows to measure the role of financial factors and unconventional monetary policy on the fluctuations of the natural rate interest over the sample period.

Our results are rather in line with the findings of the New Keynesian methodology. The main result of the paper suggests that financial factors are an important feature for the measure of natural rate of interest: First, the analysis shows that permanent shocks, that capture the effects of a secular stagnation, are not a critical driver of the natural rate. This suggests that the observed decline in the natural rate is not the result of secular effect à la Summers [2015]. Second, the persistent low level of the natural rate after the financial crisis rather finds its roots in the very long lasting nature of financial shocks, consistent with both Rogoff [2015] and Borio [2017]. Third regarding the scope of unconventional monetary policy, we find that its effectiveness in mitigating the natural rate drop during the financial crisis is very sensitive to the nature of the shocks, in particular it is very effective in dampening the capital quality and the permanent productivity shocks, while the effects on the other shocks is modest. However, the credit policy implemented by the Fed provides very limited stabilizing effects on the natural rate.

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4See Mesonnier and Renne [2007], Hamilton et al. [2015] or Fries et al. [2016] for other estimation exercises on the natural rate through semi-structural models.

5By disabling markups shocks and nominal rigidities, any new keynesian model can characterize the natural dynamics of an economy through the determination of the rate of interest that makes the economy reach its potential.
natural rate following the financial crisis, thus requiring the Fed to implement a more aggressive credit policy to obtain significant results.

The remainder of the paper is organized as follows. In Section 2, we develop a New-Keynesian model à la Smets and Wouters [2007] with financial frictions à la Gertler and Karadi [2011]. Section 3 presents the estimation strategy. Section 4 reports the empirical decomposition of the natural interest rate. Section 5 discusses the role of unconventional monetary policy on rising the natural rate. Finally, Section 6 concludes.

2 The Model

In this section, we develop a New-Keynesian model inspired both by Gertler and Karadi [2011] and Smets and Wouters [2007]. This gives rise to what we could call a Gertler-Karadi-Smets-Wouters model, as we incorporate frictions of Smets and Wouters [2007] in the Gertler and Karadi [2011] framework, which focuses on the role of financial intermediaries. We also show how we derive the underlying flexible price model. This allows us to shed light on the role of the financial sector on the fluctuations of the natural rate of interest.

2.1 Households

There is a continuum of identical households indexed by \( j \in (0,1) \). At each period households supply labor, consume and save. They have two choices to save: either lending their money to the government, or to financial intermediaries that will finance firms. In each household, there are bankers and workers. Each banker manages a financial intermediary and transfers profits to the household. Nevertheless, households can’t lend their money to a financial intermediary owned by one of their members. Members who are workers supply labor and return their salaries to the household they belong to.

Agents can switch between the two occupations over time. There is a fraction \( f \) of agents who are bankers and a probability \( \theta \) that a banker stays banker in the next period. Thus, \( (1-f)\theta \) bankers become workers every period and vice versa, which keeps the relative proportions constant. Exiting bankers give their retained earnings to the household, which will use it as start-up funds for the new banker.

Households solve the following maximization problem:

\[
\max_{\{C_{jt}, H_{jt}, B_{jt+1}\}} \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left[ c_{t+i} \left( \frac{(C_{jt+i} - h C_{jt+i+1})^{1-\sigma}}{1-\sigma} - \frac{\chi}{1+\varphi} H_{jt+i}^{1+\varphi} \right) \right],
\]

subject to

\[
C_{jt} = w_h^t H_{jt} + \Pi_{jt} + T_{jt} + R_t B_{jt} - B_{jt+1},
\]

where \( \beta \in (0,1) \) is the discount factor, parameters \( \sigma, \varphi > 0 \) shape the utility function of the \( j^{th} \) household associated to risk consumption \( C_{jt} \) and hours worked \( H_{jt} \). The consumption index \( C_{jt} \) is subject to external habits with degree \( h \in [0;1) \) while \( \chi > 0 \) is a shift parameter allowing us to pin down the steady state amount of hours worked. Labor supply is remunerated at real desired wage \( w_h^t \) that will be negotiated by unions, \( \Pi_{jt} \) are dividends from the ownership of firms (both financial and non-financial) that
will serve as start-up funds for the new banker and $T_j$ are lump sum taxes. As we assume that intermediary deposits and government bonds are one period bonds, $R_j t, B_{j t}$ are interests received on bonds held and $B_{j t + 1}$ are bonds acquired. Household consumption preferences are affected by a shock $\varepsilon_t^B$ affecting the intertemporal allocation of consumption following an AR(1) shock process: $\log(\varepsilon_t^B) = \rho_B \log(\varepsilon_{t-1}^B) + \sigma_B \eta_t^B$, with $\eta_t^B \sim N(0,1)$.

Solving the first order conditions, we get the labor supply equation:

$$\varrho_t w_t^h = \chi H_{jt}^e,$$

where $\varrho_t$ is the marginal utility of consumption:

$$\varrho_t = \varepsilon_t^B(C_{jt} - hC_{jt-1})^{-\sigma} - \beta h E_t \left\{ \varepsilon_{t+1}^B(C_{jt+1} - hC_{jt})^{-\sigma} \right\},$$

and $\beta E_t \Lambda_{t,t+1} R_{t+1} = 1$,

with $\Lambda_{t-1,t} = \frac{\varrho_t}{\varrho_{t-1}}$.

2.2 Unions

Households delegate the wage negotiation process to unions. Households provide differentiated labor types, sold by labor unions to perfectly competitive labor packers who assemble them in a CES aggregator and sell the homogenous labor to intermediate firms.\footnote{Labor packers are perfectly competitive and maximize profits, $W_t H_{jt}^d - \int_{0}^{1} W_{jt} H_{jt} d \varepsilon_j$, under their packing technology constraint, $H_t = \int_{0}^{1} (H_{jt} (\varepsilon_{w-1})/\varepsilon_j) d \varepsilon_j (\varepsilon_{w-1})$. Here, $W_t$ is the nominal wage, $H_t^d$ is the labor demand and $\varepsilon_{w} > 1$ is a substitution parameter. The first order condition which determines the optimal demand for the $j^{th}$ labor type is, $H_{jt} = (W_{jt}/W_t)^{-\varepsilon_{w}} H_{jt}^d, \forall j$. Thus the aggregate wage index of all labor types in the economy emerges from the zero-profit condition: $W_{jt} = [\int_{0}^{1} (W_{jt}^{-\varepsilon_{w}}) d \varepsilon_j]^{1/(1-\varepsilon_{w})}$.}

Unions negotiate the real margin between the real desired wage of households $W_{jt}$ and the real marginal product of labor $W_{jt}/P_t$. Using a Calvo wage nominal rigidity device, each period a random fraction $\theta_W$ of unions is unable to re-negotiate a new wage. Assuming that the trade union is able to modify its wage with a probability $(1 - \theta_W)$ the $j^{th}$ union chooses the nominal optimal wage $W_{jt}^*$ to maximize its expected sum of profits:

$$\max_{\{W_t\}} E_t \sum_{s=0}^{\infty} (\beta \theta_W) s \Lambda_{t,t+s} \left[ \frac{W_{jt}^s}{P_t} \prod_{k=1}^{s} \pi_{t+k-1} W_{jt}^s - \varepsilon_{t+s} W_{jt}^s \right] H_{jt+s},$$

subject to the downward sloping demand constraint from labor packers:

$$H_{jt+s} = \left( \frac{W_{jt}^s}{W_{jt}^s \prod_{k=1}^{s} \pi_{t+k-1} W_{jt}^s} \right)^{-\varepsilon_w} H_{jt}^d,$$

where $\varepsilon_{t+s}^W$ is an ad-hoc wage-push shock to the real wage equation which captures exogenous fluctuations in the wage margin negotiated by unions and affects in turn the productivity of the economy. It follows an ARMA process, $\log(\varepsilon_t^W) = \rho_W \log(\varepsilon_{t-1}^W) + \sigma_W \left( \eta_t^W - \eta_{t-1}^W \right), \text{ with } \eta_t^W \sim N(0,1)$, where $\rho_W \in [0,1]$ is the AR term and $\eta_W \in [0,1]$ the MA one. The latter captures high frequency fluctuations in the variations of the wage inflation rate.
2.3 Production sector

The production sector is made of two kinds of firms: intermediate goods firms and retail firms. Intermediate goods firms produce differentiated types of intermediates goods that are bought and packed by retail firms into an homogeneous good sold to households.

2.3.1 Intermediate goods firms

At the end of the period, intermediate goods firms acquire capital $K_{it+1}$ from capital producing firms. They finance this acquisition by issuing claims $S_{it}$ that they sell to financial intermediaries. Firms price each claim at the price of a unit of capital and issue as much claims as they buy capital. We assume no frictions in the process of obtaining funds. Thus, we have the following equality:

$$Q_t K_{it+1} = Q_t S_{it}. \quad (8)$$

For simplicity, we will write $L_{it} = Q_t S_{it}$ the stock of loans in the economy. Intermediate goods firms use this capital to produce in the next period and are able to resell it on the open market. As we don’t consider adjustment costs, the firm’s capital choice problem is always static. Unlike households, bankers have perfect information on firms and no problem enforcing payments. Hence, firms face no capital constraint in obtaining funds but are indirectly subject to the capital constraint faced by financial intermediaries.

The production function reads as follows:

$$Y_{it} = \begin{cases} (U_t K_{it}^E) \alpha (A_t H_{it}^d)^{1-\alpha} - A_t \Phi \bar{Y}_i & \text{if } (U_t K_{it}^E) \alpha (A_t H_{it}^d)^{1-\alpha} > A_t \Phi \bar{Y}_i \\ 0 & \text{otherwise} \end{cases}. \quad (9)$$

Here, $U_t$ is the utilization rate of capital affecting the services of effective capital $K_{it}^E$, $H_{it}^d$ labor demand and where $\alpha \in [0, 1]$ is the effective capital share. As in Gertler and Karadi [2011], we introduce a shock on the stock of physical capital capturing exogenous variations in the quality of capital, $K_{it}^E = \varepsilon_{it}^K K_{it}$, where $\log(\varepsilon_{it}^K) = \rho_K \log(\varepsilon_{t-1}^K) + \sigma_K \eta_{it}$, with $\eta_{it} \sim N(0, 1)$. The parameter $\Phi$ is the fixed cost in production with $\Phi \in [0, 1)$ expressed as a fraction of steady state output $\bar{Y}_i$. In addition, we introduce a time-varying labour-augmenting trend $\gamma_t$ on labor, featuring a stochastic growth rate in the economy. The stochastic trend is determined by:

$$\frac{A_t}{A_{t-1}} = \gamma_t = \bar{\gamma} \varepsilon_t, \quad (10)$$

where $\bar{\gamma} \geq 0$ is the gross growth rate of the economy that is estimated in the fit exercise, the latter is affected by persistent technology shock $\varepsilon_t$ defined by $\log(\varepsilon_t) = \rho_A \log(\varepsilon_{t-1}) + \sigma_A \eta_t^A$, with $\eta_t^A \sim N(0, 1)$. A positive realization of $\eta_t^A$ thus features a permanent increase in the growth rate of the economy through a rise in labor productivity.

Letting $P_{it}^m$ be the real price of intermediate goods, the representative firm maximizes the expected stream of profits under the supply constraint 9 and the funding constraint 8:

$$\max \left\{ S_{it}, K_{it}^E, H_{it}^d \right\} E_t \left\{ \sum_{s=0}^{\infty} P_{it+s}^m Y_{it+s} - w_{it+s} H_{it+s}^d + Q_{t+s} (1 - \delta(U_{t+s})) K_{it+s}^E - Q_{t+s} R_{it+s}^k S_{t+s} \right\} ,$$

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Here, $\delta_t(U_t)$ the time-varying depreciation rate. It is given by $\delta_t(U_t) = \delta_c + \frac{b}{1 + \zeta} U_t^{1+\zeta}$, where $\zeta \geq 0$ is the utilization rate elasticity and $\delta_c \in [0, 1]$ is the depreciation rate parameter of the real business cycle literature.\(^7\)

We assume that the replacement price of used capital is also fixed and at unity to get the following utilization rate, labor demand and rate of return of physical capital:

\[
\delta'(U_t) = \frac{\alpha_t}{K_t} \frac{Y_t + \gamma_t \Phi Y_t}{U_t}, \quad (11)
\]

\[
w_t = \frac{\alpha_t}{H_t} \frac{Y_t + \gamma_t \Phi Y_t}{(1 - \alpha)} , \quad (12)
\]

\[
R_t^k = \left[ \frac{\alpha_t}{K_t} \frac{Y_t + \gamma_t \Phi Y_t}{\varepsilon K_t} + Q_t (1 - \delta(U_t)) \right] \frac{\varepsilon^k}{Q_{t+1}}. \quad (13)
\]

### 2.3.2 Retail firms

A continuum of $f$ differentiated retail firms produce final output according to a CES function: $Y_t = \left[ \int_0^1 Y_{f,t}^{(e - 1)/\epsilon} d\rho \right]^{1/(1 - \epsilon)}$, where $Y_{f,t}$ is output by retailer. From cost minimization by users of final output, we get: $Y_{f,t} = (P_{f,t}/P_t)^{-\epsilon} Y_t$, and $P_t = \left[ \int_0^1 P_{1-t}^{(1-\epsilon)} d\rho \right]^{1/(1 - \epsilon)}$.

The role of retail firms is simply to re-package output produced by intermediate firms. As they use one unit of intermediate output to produce one unit of final output, the marginal cost is equal to the intermediate output price $P_{mt}$. Following Gertler and Karadi [2011], we add nominal rigidities as in Christiano et al. [2005]. There is a probability $1 - \theta_P$ that a firm is able to freely adjust its price. Otherwise, it can only index it to the lagged inflation. Retail firms thus choose the optimal reset price $P^*_t$ according to the following maximization problem:

\[
\max_{\{P_t^*\}} E_t \sum_{i=0}^{\infty} (\theta P) t A_t, t+i \left[ \frac{P^*_t}{P_{t+i}} \prod_{k=1}^{i} \xi_{t+k-1}^{\xi_{t+k-1}} - \xi_{t+i}^{\xi_{t+i}} P^m_{t+i} \right] Y_{f,t+i} , \quad (14)
\]

where $\pi_t = P_t/P_{t-1}$ is the rate of inflation from $t - i$ to $t$ and $\xi_{t+i}^{\xi_{t+i}}$ is an ad-hoc cost-push shock to the inflation equation following an AR(1) process which captures exogenous inflation pressures. As for wages, the price-push shock follows an ARMA process, $\log(\xi_{t+i}^{\xi_{t+i}}) = \rho P \log(\xi_{t-1}^{\xi_{t-1}}) + \sigma_P (\eta_{t+i}^{P} - \eta_{t+i}^{P - 1})$, with $\eta_{t+i}^{P} \sim N(0, 1)$, where $\rho_P \in [0, 1)$ is the AR term and $\eta_{t+i}^{P} \in [0, 1)$ the MA one.

The optimal price $P^*_t$ is given by the following sum:

\[
\sum_{i=0}^{\infty} (\theta P) t A_t, t+i \left[ \frac{P^*_t}{P_{t+i}} \prod_{k=1}^{i} \xi_{t+k-1}^{\xi_{t+k-1}} - \xi_{t+i}^{\epsilon P_{t+i}} P^m_{t+i} \right] Y_{f,t+i} = 0. \quad (15)
\]

\(^7\)Assuming that $\delta_t(U)$ is calibrated and with $U = 1$, we compute in the detrended steady state the following parameters: $b = P^m \alpha (1 + \Phi) Y/K$ and $\delta_c = \delta_t(U) - b/ (1 + \zeta)$. 

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2.4 Capital producing firms

We assume that households own capital producing firms and receive any profits. These firms buy capital from intermediate goods firms at the end of period \( t \) and then repair depreciated capital and build new capital. They then sell both the new and refurbished capital. As we showed earlier, the value of a unit of new capital is \( Q_t \). We suppose that there are flow adjustment costs associated with producing new capital. Then, capital producing firms are facing the following maximization problem:

\[
\max_{\{I^s_t\}} E_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s} \left\{ (Q_{t+s} - 1) I^s_{t+s} - \frac{\kappa}{2} \left( \frac{\varepsilon I_{t+s}}{I^s_{t+s-1}} + \frac{I^s_{t+s}}{I^s_{t+s-1} + I_{t+s-1}^s} - \gamma_{t+s} \right)^2 \left( I_{t+s}^n + \bar{I}_{t+s} \right) \right\}
\]

where \( I^s_t \) and \( I_t \) are respectively net and gross capital created, \( \bar{I}_t = I A_t \) is the steady state investment including a trend and \( \delta(U_t) \varepsilon_t^K K_t \) is the quantity of refurbished capital.

We differ from Gertler and Karadi [2011] by including a shock \( \varepsilon_t^l \) as in Smets and Wouters [2002] to captures exogenous variations in the cost of producing physical capital in the economy. The latter shock follows \( \log(\varepsilon_t^l) = \rho_t \log(\varepsilon_{t-1}^l) + \sigma_t (\eta_t - u_t \eta_{t-1}) \), with \( \eta_t \sim N(0, 1) \). Thus, we get the following value for \( Q_t \):

\[
Q_t = 1 + \frac{\kappa}{2} (t_t - \gamma_t)^2 + \kappa \varepsilon_t^l (t_t - \gamma_t) t_t - \beta \varepsilon_t^l E_t \left\{ \Lambda_{t,t+1} \kappa (u_{t+1} - \gamma_{t+1}) I^s_{t+1} + \bar{I}_{t+1} \right\} \frac{I^s_{t+1} + \bar{I}_{t+1}}{I_t^s + I_{t+1}}. \tag{18}
\]

where \( t_t = \varepsilon_t^l (I_{nt} + I A_t) / (I_{nt}^s + I A_t) \).

2.5 Financial intermediaries

As we saw previously, households save by lending their money to financial intermediaries. These financial intermediaries will, in turn, lend this money to non-financial firms. For the sake of simplicity, they represent the whole banking sector and can be thought as universal banks. As financial intermediaries do in the real life, they fund themselves with short term liabilities (one-period bonds offered to households) and invest on a long term horizon, thus engaging in maturity transformation.

A financial intermediary balance sheet can be depicted as:

\[
Q_t S^p_{jt} = N_{jt} + B_{jt+1}, \tag{19}
\]

where \( S^p_{jt} \) is the quantity of financial claims a single intermediary owns on non-financial firms and \( Q_t \) is their relative price. \( N_{jt} \) is the amount of wealth an intermediary \( j \) has at the end of period \( t \) and \( B_{jt+1} \) the deposits obtained from households. Another way to read this equation is to see the left-hand side as the assets of the financial intermediary and the right-hand side as its liabilities, \( N_{jt} \) being its equity capital and \( B_{jt+1} \) its debt. Over time, the financial intermediary’s equity capital evolves as the difference between return earned on financial claims hold \( R^e_{t+1} \) and interests paid to household \( R_t \):

\[
N_{jt+1} = (R^e_{t+1} - R_t) Q_t S^p_{jt} + R_t N_{jt}. \tag{20}
\]
Thus, there is a fixed part in the growth in equity, which is the riskless rate of return. The variable part depends on the risk premium $R_{t+1}^k - R_t$ as well as on the total quantity of assets held by the financial intermediary. As any profit-making entity, the banker will only fund projects that he believes will earn positive return. Applying the stochastic discount $\beta^s \Lambda_{t,t+1}$, we get the following objective function:

$$E_t \beta^s \Lambda_{t,t+1+s} (R^k_{t+1+s} - R_{t+s}) \geq 0, \quad \forall t > 0.$$  \hspace{1cm} (21)

As long as the risk premium is positive, the banker will keep expanding his balance sheet. Mathematically, we get the following objective function:

$$\max_{\{L_{jt}\}} V_{jt} = E_t \sum_{t=0}^{\infty} (1 - \theta) \theta^s \beta^{s+1} \Lambda_{t,t+1+s} [ (R^k_{t+1+s} - R_{t+s}) Q_t S^p_{jt} + R_{t+s} N_{jt+s} ]$$  \hspace{1cm} (22)

To avoid financial intermediaries to grow indefinitely, we introduce a moral hazard problem. At the beginning of each period, bankers can divert a fraction $\lambda \in (0, 1]$ of invested funds back to the household they belong to. In this case, depositors can force them into bankruptcy but will only be able to recover the remaining $1 - \lambda$. Therefore, the following incentive constraint must be respected:

$$V_{jt} \geq \lambda \varepsilon^L_{jt} Q_t S^p_{jt}.$$  \hspace{1cm} (23)

where $\varepsilon^L_{jt}$ is an assets diversion shock that follows an AR(1) shock process: $\log(\varepsilon^L_{jt}) = \rho \log(\varepsilon^L_{j,t-1}) + \sigma L \eta^N_{jt}$, with $\eta^N_{jt} \sim N(0, 1)$. This shock aims at capturing bank capital short-fall during financial crisis episodes, which materializes by high excess returns and lower corporate investment. A positive realization of this shock can be linked to a bank run à la Diamond and Dybvig [1983] where depositors force the intermediary into bankruptcy and recover the remaining fraction of assets $1 - \varepsilon^L_{jt} \lambda$.

Concretely, it means that the expected gain of staying a banker is superior or equal to the gain realized when a banker diverts funds. We can rewrite the objective function in a recursive way:

$$V_{jt} = v_t Q_t S^p_{jt} + \eta_t N_{jt},$$  \hspace{1cm} (24)

with $$v_t = E_t \{ (1 - \theta) \beta \Lambda_{t,t+1} (R^k_{t+1} - R_t) + \beta \Lambda_{t,t+1} \theta x_{t,t+1} + v_{t+1} \},$$  \hspace{1cm} (25)

and $$\eta_t = (1 - \theta) + E_t \{ \beta \Lambda_{t,t+1} \theta z_{t,t+1} + \eta_{t+1} \},$$  \hspace{1cm} (26)

where $x_{t,t+s} \equiv Q_{t+s} S^p_{jt+s} / Q_t S^p_{jt}$ is the gross growth rate in assets between $t$ and $t+s$ and $z_{t,t+s} \equiv N_{jt+s} / N_{jt}$ is the gross growth rate of net worth. The variable $v_t$ is the expected discounted marginal gain to the banker of expanding assets $S^p_{jt}$ by a unit, holding net worth $N_{jt}$ constant, and while $z_t$ is the expected discounted value of having another unit of $N_{jt}$, holding $S^p_{jt}$ constant.

We can rewrite the incentive constraint as:

$$\eta_t N_{jt} + v_t L_{jt} \geq \lambda \varepsilon^L_{jt} Q_t S^p_{jt},$$  \hspace{1cm} (27)

and when the constraint is binding, we get:

$$\eta_t N_{jt} + v_t L_{jt} = \lambda \varepsilon^L_{jt} Q_t S^p_{jt}$$

and

$$Q_t S^p_{jt} = \phi_t N_{jt},$$  \hspace{1cm} (28)
where \( \phi_t = \frac{\eta_t}{\lambda e_{L_t}} \) is the leverage ratio of financial intermediaries (or the ratio of privately intermediated assets to equity). Replacing \( Q_t S_{pt} \) we can now rewrite the evolution of the financial intermediary’s net worth:

\[
N_{jt+1} = (R_{t+1}^e - R_t) \phi_t N_{jt} + R_t N_{jt}
\]

\[
N_{jt+1} = [ (R_{t+1}^e - R_t) \phi_t + R_t ] N_{jt}.
\] (29)

Here, we can clearly see that the leverage ratio amplify the impact of the anticipated risk premium of the banker’s net worth. We can also rewrite \( z_{t,t+1} \) and \( x_{t,t+1} \) as:

\[
z_{t,t+1} = \frac{N_{jt+1}}{N_{jt}} = (R_{t+1}^e - R_t) \phi_t + R_t
\] (30)

\[
x_{t,t+1} = \frac{Q_{t+1} S_{pt}^{t+1}}{Q_t S_{pt}^{t}} = \frac{\phi_{t+1} N_{jt+1}}{\phi_t N_{jt}} = \frac{\phi_{t+1}}{\phi_t} z_{t,t+1}.
\] (31)

The leverage ratio \( \phi_t \) does not depend on firm-specific factors. Thus, we can write \( S_{pt} \) as the aggregate quantity of bankers’ assets and \( N_t \) their aggregate capital. Removing \( j \) from the equation, we get:

\[
Q_t S_{pt}^{t} = \phi_t N_t.
\] (32)

Considering \( N_t \) is made of both the net worth of existing financial intermediaries and entering ones, we can split it into two parts, and remembering the fraction \( \theta \) of bankers survive, we find:

\[
N_t = N_{te} + N_{tn},
\] (33)

\[
N_{te} = \theta[(R_{t}^e - R_{t-1}) \phi_{t-1} + R_{t-1}] N_{t-1} e_{t}^{N},
\] (34)

\[
N_{tn} = \omega Q_t S_{t-1},
\] (35)

Parameter \( \omega \in [0, 1) \) is the proportion of funds transferred to entering bankers.

### 2.6 Authorities

#### 2.6.1 Government

We assume that government consumption is a fixed proportion of current output and subject to a government consumption shock: \( G_t = \frac{\xi_t}{\xi_t} Y_t e_{t}^{\xi}. \) Capital evolves according to the following law of motion: \( K_{t+1} = \epsilon_t^K K_t + I_t^n \) and government finances its expenditures thanks to lump sum taxes as well as the excess return earned on financial intermediation:

\[
G_t + \tau \psi_t L_t = T_t + s_t B_{t-1}^g,
\] (36)

where \( B_{t-1}^g \) are bonds sold by the government to households and is thus equal to the amount of assets the central bank intermediates. \( \tau \psi_t L_t \) is the cost of implementing unconventional monetary policy for the government. Unlike financial intermediaries, the government always honors its debt, which means that there is no need for an incentive constraint. However, the central bank suffers an efficiency cost \( \tau \geq 0 \) to perform financial intermediation.
2.6.2 Conventional monetary policy

The central bank follows a simple Taylor [1993] rule to set the interest rate:

$$i_t - \bar{i} = \rho_c (i_{t-1} - \bar{i}) + (1 - \rho_c) \left[ \phi_\pi (\pi_t - \bar{\pi}) + \phi_y (Y_t - \bar{Y_t}) A_{t-1}^{-1} \right] + \varepsilon^R_t,$$

(37)

where $\bar{i}$ is the steady state of the nominal rate $i_t$, $\rho_c \in [0, 1)$ is the smoothing coefficient, $\phi_\pi \geq 1$ is the inflation stance penalizing deviations of inflation from the steady state, $\phi_y$ is the output gap stance penalizing deviations of detrended output from its natural counterpart $\bar{Y_t}$. Parameter $\rho_c$ is a the monetary policy smoothing coefficient, $\phi_\Delta y$ is the growth gap target and $\varepsilon^R_t$ is an exogenous shock to monetary policy that follows an AR(1) shock process: $\varepsilon^R_t = \rho_R \varepsilon^R_{t-1} + \sigma_R \eta^R_t$, with $\eta^R_t \sim N(0, 1)$.

Moreover, the relationship between the nominal and the real interest is modelled through the Fisherian equation:

$$i_t = R_t E_t \{ \pi_{t+1} \}.$$

(38)

2.6.3 Unconventional monetary policy

Using credit policy, the central bank can substitute for financial intermediaries and directly fund firms. Unconventional monetary policy is characterized by $\psi_t$, the fraction of publicly intermediated assets. This variable respond to variations in the credit spread and in the stock of loans as we will explain later. We can then write the combined private and public intermediation as:

$$L_t = Q_t S_t = Q_t S^p_t + Q_t S^g_t,$$

(39)

where $Q_t$ is the relative price of financial claims owned by financial intermediaries or the central bank on non-financial firms and $S^p_t$ and $S^g_t$ are respectively the amount of assets held by the private and the public sector.

The central bank lends money to non financial firms at the market rate $R^k_{t+1}$ and sells bonds paying $R_t$ to households to fund these purchases. We assume that the central bank is willing to fund the fraction $\psi_t$ of intermediated assets: $Q_t S^p = \psi_t L_t$, which allows to rewrite the total leverage of the economy for both privately and publicly intermediated assets as:

$$L_t = \frac{\phi_t}{1 - \psi_t} N_t$$

(40)

This equation shows that the credit policy can ease the constraint on net wealth of intermediary by increasing the total amount of intermediated assets $L_t$ by rising the fraction of asset purchased $\psi_t$.

The central bank follows a countercyclical credit policy rule to decide the share of intermediated assets $\psi_t$ held by the central bank. This rule responds to variations in the credit spread $s_t = E_t \{ R^k_{t+1} \} - R_t$ and in the stock of loans as follows:

$$\psi_t = \rho_u \psi_{t-1} + (1 - \rho_u) [\kappa_s (s_t - \bar{s}) - \phi_L (L_t A_{t-1}^{-1} - \bar{L})] + \varepsilon^\psi_t,$$

(41)

where $\rho_u \in [0, 1)$ is the rule smoothing coefficient, $\kappa_s \geq 0$ is the reaction parameter to excess returns on financial markets, $\phi_L \geq 0$ is the countercyclical response of credit...
policy to credit imbalances measured by the detrended gap between credit $L_t A_t^{-1}$ to its deterministic steady state $L$. Finally $e_t^\psi$ represents a shock to the credit policy following an $AR(1)$ shock process: $e_t^\psi = \rho^\psi e_{t-1}^\psi + \sigma^\psi \eta_t^\psi$, with $\eta_t^\psi \sim N(0, 1)$. The latter is motivated by credit policy shocks which are not directly motivated by spreads and credit gaps.

2.7 Aggregation and market equilibrium

The general equilibrium of the model is set as follows. After (i) aggregating all agents and varieties in the economy, (ii) imposing market clearing for all markets, (iii) substituting the relevant demand functions, we get the general equilibrium conditions of the model.

Output is composed of consumption, investment, government consumption and the efficiency cost paid by the central bank on its financial intermediation activity:

$$Y_t = \Delta^P_t \left[ C_t + I_t + (I_{nt} - 1)^2 (I_{nt} + \bar{I}) + G_t + \tau^\psi L_t \right].$$  \hspace{1cm} (42)

where $\Delta^P_t = \int_0^1 \frac{P_{jt}}{P_t} - 1 d\tilde{j}$ denotes the price dispersion term, which is induced by the assumed nature of price stickiness.

In addition, the labor market clears when the following condition holds:

$$H_t = \Delta^W_t H^d_t,$$  \hspace{1cm} (43)

where the wage dispersion terms is given by $\Delta^W_t = f_0^1 (W_{jt}/W_t)^{-\epsilon_W} d\tilde{j}$.

From the law of large numbers, the following relations for the evolution of the price and wage levels emerge:

$$P_t = \left[ (1 - \theta^p)(P_t^*)^{1-\epsilon^p} + \theta^p (\pi_t^p P_{t-1}^{1-\epsilon^p}) \right]^{1/(1-\epsilon^p)}.$$

$$W_t = \left[ (1 - \theta^w)(W_t^*)^{1-\epsilon^w} + \theta^w (\pi_t^w W_{t-1}^{1-\epsilon^w}) \right]^{1/(1-\epsilon^w)}.$$  \hspace{1cm} (44)

2.8 The natural allocation

Even though the natural rate of interest is not directly observable, it is thus possible to compute it from any New Keynesian model by developing a parallel version of the model with no nominal rigidities as shown in Woodford [2003]. In our setup, these rigidities are driven by Calvo devices for both prices and wages and their respective markups shocks. In absence of these nominal rigidities, the economy reaches its full employment level with stable inflation. This equilibrium constitutes a first best allocation that monetary policy seeks to reach by adjusting the nominal rate in normal times, and possibly assets purchase programs during large recessions.

As Barsky et al. [2014] and Del Negro et al. [2017], we can compute the real natural rate $\tilde{R}_t$ from the Euler equation from the natural allocation:

$$\beta E_t \hat{A}_{t+1} \tilde{R}_t = 1,$$  \hspace{1cm} (46)

For comparison purposes with the nominal rate, the real natural rate is taken in nominal terms by including the expected inflation:

$$\tilde{i}_t = \tilde{R}_t E_t \{ \pi_{t+1} \}.$$  \hspace{1cm} (47)
The interest of our framework is to be able to measure the role of unconventional monetary policy on the natural rate of interest. In particular, the credit policy materialized by $\psi_t$ affects natural allocation by relaxing the constraint on the net worth of financial intermediaries:

$$\tilde{L}_t = \tilde{\phi}_t \tilde{N}_t + \psi_t \tilde{L}_t$$

(48)

Since privately intermediated funds are constrained by intermediary net worth, the credit policy increases the amount of privately intermediated funds, which is expected to generate a rise in both economic activity and the natural rate of interest. During recessions with binding zero lower bound, the gap between the nominal rate and the natural gap is large as monetary policy cannot provide the appropriate stimulus with its conventional instrument. The introduction of a second instrument will both affect the nominal and natural rate and thus optimally reduce the gap between the two rates. If the credit policy stimulus is high enough, characterized by a natural rate above zero, monetary can optimally leave the zero lower bound by a rate hike.

### 3 Estimation strategy

The model is estimated using Bayesian methods and quarterly data for the US economy. We estimate the structural parameters and the sequence of shocks following the seminal contributions of Smets and Wouters [2007] and An and Schorfheide [2007]. In a nutshell, a Bayesian approach can be followed by combining the likelihood function with prior distributions for the parameters of the model to form the posterior density function. The posterior distributions are drawn through the Metropolis-Hastings sampling method. In the following fit exercise, we solve the model using a linear approximation to the policy function, and employ the Kalman filter to form the likelihood function. For a detailed description, we refer the reader to the original papers.

#### 3.1 Data and measurement equations

The Bayesian estimation relies on US quarterly data over the sample period 1995:I to 2017:I. Therefore, each observable variable is composed of 89 observations. The dataset includes 10 times series: output (GDPC96), consumption (PCEC), investment (FPI), hours worked (PRS85006023), inflation (GDPDEF), federal funds rate (FEDFUNDS), wages (COMPRNFB), risk premium, stock of loans and the share of assets intermediated by the central bank. The stock of loans refers to the total credit to non-financial sector while the risk premium refers to the series built by Gilchrist and Zakrajšek [2012]. In order to estimate unconventional monetary policy parameters and shocks, we constructed a series representing assets purchases of the Fed. We added federal debt and mortgage-backed securities held by the central bank before dividing it by the total stock of bonds in the US. Overall, our sample includes three more (financial) series compared to the seminal contribution of Smets and Wouters [2007].

Concerning the transformation of the series, the point is to map non-stationary data to a stationary model. The variables are made stationary in three steps. First, they are divided by the working age population (LNS10000000). Second, they are divided by
the GDP deflator price index. Third, they are taken in logs and we use a first difference filtering to obtain growth rates. Following Chang et al. [2002], who underline the limited coverage of the nonfarm business sector compared to GDP, we multiply the index of average hours for the nonfarm business sector (all persons) by civilian employment (CE16OV). The corresponding vector of observable is given by:

\[
\Theta_t^{ob} = 100 \times [ \Delta \log Y_t^{ob}, \Delta \log C_t^{ob}, \Delta \log I_t^{ob}, \log \pi_t^{ob}, \\
\Delta \log w_t^{ob}, \dot{i}_t^{ob}, \Delta \log L_t^{ob}, s_t^{ob}, \Delta \psi_t^{ob} ]',
\]

where \( Y_t^{ob}, C_t^{ob}, I_t^{ob}, L_t^{ob} \) are respectively the real per capita production, consumption, investment and corporate loans; while \( H_t^{ob} \) is the hours worked index, \( \pi_t^{ob} \) the deflator growth rate, \( w_t^{ob} \) the real wage growth rate, \( \dot{i}_t^{ob} \) the quarterly nominal rate, \( s_t^{ob} \) the risk premium and \( \psi_t^{ob} \) the share of intermediated assets held by the central bank.

Regarding the model, the introduction of a stochastic trend makes our endogenous variables non-stationary in steady state. However, the solution method used here implies a local approximation around a fixed point, thus requiring us to rewrite the model in a detrended fashion. The corresponding measurement equations are given by:

\[
\Theta_t = 100 \times [ \log \left( \gamma_t Y_{t-1} \right), \log \left( \gamma_t C_{t-1} \right), \log \left( \gamma_t I_{t-1} \right), \dot{H} + \log \left( \frac{H_t}{H} \right) \log(\pi_t), \\
\log \left( \gamma_t \frac{L_t}{L_{t-1}} \right), R_t, \log \left( \gamma_t \frac{L_t}{L_{t-1}} \right) s_t, \Delta \psi_t ]',
\]

where the bar above the variables’ names denote the steady state value of the corresponding variable, while \( \dot{H} \) denotes a shift parameter in the observable mean of hours worked as in Smets and Wouters [2007]. We capture the information contained in the mean of the sample through the steady state of our measurement equations which are different from zero.

### 3.2 Calibration and prior distribution

Table 1 summarizes our calibration and Table 2 displays the steady state moments of the model. We set \( L=1/3 \) for the steady state share of hours worked per day; \( \bar{U} \), the utilization rate is normalized one; \( \delta (\bar{U})=0.025 \), the depreciation rate of physical capital; \( \alpha=0.33 \), the capital share in the technology of firms; \( G/Y=0.20 \), the ratio of public spending to GDP; and \( \epsilon_P = \epsilon_W = 10 \) as in Smets and Wouters [2007] for the elasticity of substitution between goods and labor types. Regarding parameters related to the financial intermediaries, we borrow calibration from Gertler and Karadi [2011] with \( \omega = 0.0022 \) the start-up funds to new bankers and \( \theta = 0.972 \) the survival rate of bankers in the next period.

The rest of the parameters are estimated using Bayesian methods. Table 4 and Figure 1 report the prior (and posterior) distributions of the parameters for the US economy. Most of our prior distributions are either relatively uninformative or consistent with previous works involving Bayesian estimations such as Smets and Wouters [2007].

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8 Stock and endogenous variables which exhibits a trend are detrended using \( \tilde{Y}_t = Y_t / A_t \), while the marginal utility of consumption is detrended via \( \tilde{\psi}_t = \theta A_t \).

9 The posterior distribution combines the likelihood function with prior information. To calculate the posterior distribution to evaluate the marginal likelihood of the model, the Metropolis-Hastings algo-
For ARMA terms, $\sigma$, $h$, $\varphi$, $\kappa$, $\theta_P$, $\xi_P$, $\theta_W$, $\xi_W$, $\Phi$, $\phi_\pi$, $\rho_c$, $\dot{H}$, $(\bar{\pi} - 1) \times 100$, $(\beta^{-1} - 1) \times 100$ and $(\gamma - 1) \times 100$, our priors are directly borrowed from Smets and Wouters [2007]. For the standard deviation of shocks, we impose an inverse gamma distribution of type 2 as in Christiano et al. [2014] with prior inputs closed to Smets and Wouters [2007] with mean 0.1 and standard deviation 0.5. In addition, we use the same policy weight on output as Christiano et al. [2014]. For the credit policy rule, we use the same prior information as for the Taylor rule for $\rho_u$ and $\phi_L$, while for the stance on the risk premium $\phi_\pi$ we impose a positive support with a gamma distribution to make the rule countercyclical, with mean 10 and standard deviation 2.5 to have a rule in the ballpark of the simulations of Gertler and Karadi [2011]. In addition, for the fraction of diverted assets $\lambda$, we use an uninformative prior with beta distribution of mean 0.5 and standard deviation 0.1 that includes the calibration of 0.38 of Gertler and Karadi [2011]. Finally, for the capital utilization elasticity $\zeta$, we employ a normal distribution with mean 5 and and standard deviation of 2.

### 3.3 Posterior distribution

In addition to the prior distributions, Table 4 reports the estimation results that summarize the means and the 5th and 95th percentiles of the posterior distributions. Figure 1 offers a graphical view of priors and posteriors distributions. According to Figure 1, the data were fairly informative, as their posterior distributions did not stay very close to their priors, except for the wage indexation parameter. We thus investigate the possible sources of non-identification for this parameter using methods developed by Iskrev [2010]. Using identification patterns from moments information matrix, we find that the least identified parameter is the steady state inflation parameter. The brute force search method indicates that there no colinearity link between the steady state inflation with any parameter, thus suggesting the lack of identification has another source than the colinearity. However we find a strong cofounding between $\xi_W$ and the MA term on wage $u_W$ thus suggesting that the latter absorbs most of the information on wages to the detriment of the wage indexation. Using a sensitivity analysis, we find that the lack of identification of steady state inflation is due to its limited effect on the likelihood function. Overall, these identification methods show that sufficient and necessary conditions for local identification are fulfilled by our model.

While our estimates of the standard parameters are in line with the business cycle literature for the US economy as in Smets and Wouters [2007] or Christiano et al. [2014], several observations are worth making regarding the means of the posterior distributions of structural parameters. In particular, we find that the consumption habit is much smaller while the labor disutility coefficient is higher, thus suggesting that consumption and labor supply were more volatile over the sample period considered here. In addition, we find that the fraction of diverted assets is higher than in the original calibration of Gertler and Karadi [2011]. Regarding the steady states ratios reported in Table 2, which are dependent on the estimated value of structural parameters, we find

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rithm is employed. We compute the posterior moments of the parameters using a total generated sample of 100,000, discarding the first 10,000, and based on eight parallel chains. The scale factor was set in order to deliver acceptance rates close to 24%. Convergence was assessed by means of the multivariate convergence statisticstaken from Brooks and Gelman [1998]. We estimate the model using the dynare package Adjemian et al. [2011].
that they are consistent with their observable counterpart. The only sizable difference lies in the corporate credit spread that is too low in the model. The motivation for this gap is due to the financial contract of Gertler and Karadi [2011]. In its current form, it cannot provide a high spread with a low riskless rate and gives rise to a trade-off between these two variables. The estimation exercise favors a low interest rate to the detriment of the spread.

4 Path and Drivers of the Natural Rate of Interest

Using our estimated model, this section presents the estimate of the natural rate of interest for the US economy and shows its decomposition in terms of the contribution of the shocks.

4.1 Time path of the natural rate

Figure 2 reports in the first row the time-path of the natural rate of interest and the ex-ante real interest rate from Euler equations as in Equation 4 and Equation 46. The second row reports these rates taken in nominal terms as in Equation 38 and Equation 47 to highlight the stance of monetary policy. Finally, the last row reports deviations of both the natural and effective output from their respective steady states. A positive (negative) wedge between these outputs implies an imbalance on goods markets by the over (under)-utilization of capital and labor resources available.

Consistently with estimated DSGE models such as Barsky et al. [2014] or Curdia [2015], we observe a sharp decline after the Great Financial Crisis followed by a prolonged period of negative natural rate. This situation has been at the heart of debates about the slow recovery following the crisis and could be termed the natural rate of interest puzzle. Our estimation period allows us to show that, even though the NRI hit the ZLB several times, it was still negative at the beginning of 2017 and shows no real sign of recovery. Contrary to the expectations of Borio [2017], the inclusion of financial factors as a driving force of the natural rate has not pushed it above zero after the financial crisis. By comparing production gaps with the natural rate, we see that our estimates are consistent with the theoretical definition of Woodford [2003], as periods characterized by a negative output gap notably after 2007 are accompanied by a nominal rate well above its natural counterpart, thus showing that monetary policy is too tight. These results are very close to the findings of Barsky et al. [2014] in terms of output gap dynamics.

Turning to monetary policy analysis, this supports the decision of the Federal Reserve to be very cautious about normalization of short term rates. However, if we only refer to the natural rate of interest, there is no ground for raising the federal funds target several times since 2015, as the interest rate gap (i.e. the difference between the real short term interest rate and its natural counterpart) is still positive, which means that monetary policymakers should anticipate disinflation in the future and a widening output gap.

Figure 3 reports the comparison of our estimate of the natural rate with the contributions of Curdia [2015] and Holston et al. [2016]. Curdia [2015] employs a DSGE
model with forward guidance but no financial frictions while Holston et al. [2016] use a semi-structural model with no financial variables. Our estimates are rather close to the findings of Curdia [2015] in terms of fluctuations of the natural rate. Our forecasting exercise also provides some similar findings, as the natural rate returns at the same rate to its steady state. The extension to financial factors seems to have two implications with respect to Curdia [2015]. First, large drops of the natural rate following a recession seems to take more time in our setup, probably because the reversal of the cycle is slower when financial shocks matter. Second, after the financial crisis, our estimate of the natural rate is higher than Curdia [2015], thus validating the argument of Borio [2017] stating that financial factors would drive the natural rate to an higher level. However, this contribution of financial factors is not enough to push the rate above zero.

4.2 Historical decomposition

Figure 4 reports the historical decomposition of deviations of the NRI from its steady state. Shocks are stacked by groups (supply, demand, financial, markups and policy) for clarity purpose which allows to measure the contribution of each type of shock to the variations of the interest rate over the sample period.

Overall, we find that there are three main driving forces of the natural rate over the sample period, namely supply, demand and financial shocks. Not surprisingly, markups shocks have no implication on the natural rate as they are removed from natural block in addition to the Taylor rule shock. More notably, the unconventional monetary policy seems to have no role too on the natural rate, thus questioning the interest of adopting unconventional measure to boost aggregate demand. The contributions of this policy appears to be modestly positive after 2014.

Our framework allows to separate trend effects, illustrated here by permanent supply shocks, from short run fluctuations incurred by financial and demand shocks. The validation of a secular stagnation hypothesis of Summers [2015], exemplified here by strong negative contributions of negative supply shocks, is not observed in our simulations of the natural rate. The financial crisis has certainly been triggered by a sizeable negative supply shocks, however the main driver of the slump remains the group of financial shocks. This simulation neither supports fully the financial drag hypothesis of Borio [2017], as financial shocks do not contribute to rise the natural after the financial crisis, as the opposite situation is observed here. The debt supercycle of Rogoff [2015] is rather favored by our model, as we observe that the financial crisis has triggered a negative long-lasting financial shocks that drives the natural rate down.

4.3 Driving forces

Figure 5 reports the forecast error variance decomposition for the natural rate. Five different time horizons are considered, ranging from one quarter (Q1) to ten years.
(Q40) along with the unconditional forecast error variance decomposition (Q∞). In each case, the variance is decomposed into four main components related to supply shocks, demand shocks, financial shocks, and policy shocks.

Overall, our results are very similar to the historical decomposition exercise, i.e. markups and policy shocks have no implication on the natural rate. The main driver remains demand shocks, while the permanent supply shock only explains 15% on the variance of the natural rate. The time horizon matters here only for financial shocks, as in the very short run, the latter have very modest contributions to the variations of the natural rate. However, increasing the time horizon magnifies the contribution of financial shocks up to 35% in the unconditional horizon. This result is consistent with the expected pattern of financial cycles characterized by long-lasting fluctuations. To conclude, the omission of financial factors is not important to explain the short run fluctuations of the interest rate, however in the long run this omission may become critical.

5 The role of unconventional monetary policy

Another original aspect of the estimated model is to include the central bank balance sheet as an unconventional monetary policy instrument. In this section, we examine how credit policy may affect the natural interest rate. We first analyze how the estimated rule, in terms of parameters stance and policy shocks, affects the natural rate. Secondly we replace the credit policy rule by an optimal one by solving a Ramsey allocation problem. We then analyze how the adoption of an optimal credit rule affects the diffusion of our financial shocks.

5.1 A breakdown of the credit policy rule

To evaluate how an average credit policy shock propagates in the economy, we first report the simulated Bayesian system responses of the main macroeconomic variables following a standard credit policy shock in Figure 6. The impulse response functions (IRFs) and their 90% highest posterior density intervals are obtained in a standard way when parameters are drawn from the mean posterior distribution, as reported in Figure 1.

In this model, we assume that the purpose of unconventional monetary policy is to boost aggregate demand. Unlike conventional monetary policy, the scope of a credit policy is wider as it can affect both the natural and sticky price parts of the model. This has strong implications in reviving the economy following an adverse shock as the credit policy can reduce the gap between the nominal and the natural rate, thus making monetary policy more effective.

A credit policy shock reported in Figure 6 lowers the leverage of banks, which reduces the spread and overall leads to successfully boost the total supply of loans in the economy. This in turn affects positively investment, triggering an higher labour supply and ultimately spurring output. As the economy is picking up, real wages will start to grow. Interestingly, the return on capital increases in the first period. This can be explained by a lag between the stimulus of the central bank and the reaction of the...
financial sector. Unconventional monetary policy has a direct effect on firms, that are encouraged to invest more. However the credit supply reacts sluggishly, creating a disequilibrium on credit market cleared by a spike of the rate of return on capital. Hence, the spread first increases before dropping very quickly as explained earlier. Regarding inflation, the effect is rather limited as pointed out already by Gertler and Karadi [2011].

Regarding the natural part of the model, the reaction is slightly different with a more pronounced liquidity effect at first through a larger decline of the natural rate than the real one. This is followed by the expected demand effect, even though potential output rises less than its observable counterpart despite a similar increase in investment. This outcome is mainly due to the perfect adjustment of prices and wages in the natural model, making the overall adjustment of the economy faster than in the new keynesian counterpart. In accordance with Gertler and Karadi [2011], we finally find that the central bank exits very slowly from quantitative easing. The estimated persistence of a credit policy shock even seems higher than in the theoretical framework of Gertler and Karadi [2011], as the central bank only clears its balance sheet after more than 100 quarters.

Rather than looking at the diffusion of a credit policy shock, the model can be used to analyze the stabilizing properties of the credit policy rule for different values of the policy parameters. To do so, we report in Figure 7 the time-path of the natural rate of interest by feeding the model with only one shock at a time to isolate its contribution. For each shock, the simulation is performed using three different scenarios: the thick blue line is what we observe in data and comes from the estimation of the model, the dotted green line is what would have happened if no unconventional monetary policy were implemented and the dotted red line corresponds to a scenario where the central bank would have reacted more aggressively to variations in the credit spread and the stock of loans. In the latter scenario, we multiply estimated parameters $\kappa$, $\psi$, and $\phi_L$ by 20 to match the values of the experiment of Gertler and Karadi [2011].

The first striking pattern is that the estimated paths of the shocks are very close to the ones simulated in the case where there would have been no intervention of the central bank. Nevertheless, the aggressive scenario clearly creates a wedge with the other scenarios. Under this aggressive policy experiment, the path of the natural rate is strongly affected for very specific types of shocks, namely the permanent supply, the preference, the capital quality and the assets diversion shocks. In particular, it does a very good job at mitigating the effects of capital quality shocks on the natural rate of interest, reducing the volatility around the steady state. Since this shock is one of the main drivers of the natural rate over the sample period, we thus deduct that an aggressive credit policy rule would be effective in stabilizing the variations of the natural rate, in particular during large financial shocks. Regarding the second most important shock, the fluctuations induced by the permanent supply shock are also well offset by this policy, in particular during the great financial crisis. For the assets diversion shock, the results are mixed as the size of the fluctuations remains the same but the aggressive policy surprisingly shifts the effects of this shock on the natural rate. Finally, for all other types of shocks, the implications of balance sheet expansions are rather limited.
5.2 Optimal credit policy rule

In the following, we analyze how the would natural rate of interest behave in alternative credit policy rule than the one defined in Gertler and Karadi [2011]. To this end, we compute the paths that the Ramsey policy maker chooses for the model variables in order to maximize household utility, subject to the decision rules of households and firms. The following exercise does not include the zero lower bound in the optimization decision, thus limiting the analysis here to a situation where economic conditions and the nominal rate normalize as in Quint and Rabanal [2017]. Figure 8 and Figure 9 reports the impulse response functions following two financial shocks (capital quality and assets diversion).

Overall, we find that the optimal value of assets intermediated by the central bank $\psi_t$ under a Ramsey allocation is 12.30% in steady state, which should be interpreted as an average of the Fed assets holdings over the cycles. Figure 8 reports the response of the economy following a negative capital quality shock. We find that it is optimal for the central bank to have a very aggressive credit policy that avoids the deflation trap, thus making both the inflation and the nominal rates to be very stable. The large asset purchase program avoids the credit crunch and boosts investment, thus dampening the contraction of output. The optimal credit policy takes times to be implemented, thus making the natural rate fall and the output gap widen in the first period. Similar results are found for the assets diversion shock in Figure 9, with improved stabilization properties compared to the capital shock, as all variables goes back quickly to steady state compared to the estimated rule.

6 Conclusion

The unobservable nature of the natural interest rate makes its measurement very sensitive to the assumptions of the theoretical model. In this paper, we evaluated the role of financial frictions and unconventional monetary policy on the natural rate of interest. We find that financial factors are a non-negligible driver of the natural rate in the long run, which have implications for monetary policymaking. The analysis shows that permanent shocks, that mimic the effects of a secular stagnation, are not a critical driver of the natural rate. Conversely, financial shocks exhibit long and persistent cycles which favor the financial drag theory. Consistently with Rogoff [2015] thesis, we find that negative financial shocks resulting from the last financial crisis still affects negatively the economy through a debt supercycle effect. Regarding the scope of unconventional monetary policy, we find that its effectiveness in rising the natural rate is very sensitive to the nature of the shocks, in particular it is very effective in dampening the capital quality and the permanent productivity shocks while the effects of the other is modest.
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A. Gerali and S. Neri. Natural rates across the atlantic. 2017. 3


A. Justiniano and G. E. Primiceri. Measuring the equilibrium real interest rate. 2010. 3


K. Rogoff. Debt supercycle, not secular stagnation, 2015. 2, 3, 17, 20


Figure 1: Prior and posterior distributions of structural parameters for the USA (excluding shocks).
<table>
<thead>
<tr>
<th>Variable</th>
<th>Interpretation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>Fraction of capital that can be diverted</td>
<td>0.381</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Proportional transfer to the new bankers</td>
<td>0.002</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Survival rate of the bankers</td>
<td>0.972</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Effective capital share</td>
<td>0.330</td>
</tr>
<tr>
<td>$\bar{U}$</td>
<td>Steady state capital utilization rate</td>
<td>1.000</td>
</tr>
<tr>
<td>$\delta(\bar{U})$</td>
<td>Steady state depreciation rate</td>
<td>0.025</td>
</tr>
<tr>
<td>$\epsilon_{P}, \epsilon_{W}$</td>
<td>Elasticity of substitution</td>
<td>10</td>
</tr>
<tr>
<td>$\frac{\bar{C}}{\bar{Y}}$</td>
<td>Steady state proportion of government expenditures</td>
<td>0.200</td>
</tr>
</tbody>
</table>

**Table 1**
Calibrated parameters.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Interpretation</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C/Y$</td>
<td>Ratio of consumption to GDP</td>
<td>0.55</td>
<td>0.59</td>
</tr>
<tr>
<td>$I/Y$</td>
<td>Ratio of investment to GDP</td>
<td>0.24</td>
<td>0.24</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Equity-to-debt ratio</td>
<td>4.4016</td>
<td>1.3–4.7</td>
</tr>
<tr>
<td>$100 \times (\bar{R}^k - \bar{R})$</td>
<td>Corporate spread</td>
<td>0.6841</td>
<td>1.0105</td>
</tr>
<tr>
<td>$100 \times (\bar{R} - 1)$</td>
<td>Nominal rate</td>
<td>0.9169</td>
<td>0.6463</td>
</tr>
<tr>
<td>$100 \times (\pi - 1)$</td>
<td>Inflation rate</td>
<td>0.4871</td>
<td>0.4674</td>
</tr>
<tr>
<td>$100 \times E[\Delta \log (Y_t)]$</td>
<td>Average growth rate</td>
<td>0.2819</td>
<td>0.3000</td>
</tr>
</tbody>
</table>

**Notes:** All sample averages are computed over the period 1995:I-2017:I or taken from Christiano et al. [2014].

**Table 2**
Steady state ratios (empirical ratios are computed using data between 1990 to 2017).
### SHOCK PROCESS $\mathcal{AR}(1)$

<table>
<thead>
<tr>
<th></th>
<th>Prior distributions</th>
<th>Posterior distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Shape Mean Std.</td>
<td>Mean [5%-95%]</td>
</tr>
<tr>
<td>Productivity sd $\sigma_A$</td>
<td>$\mathcal{IG}$ 0.1 0.5</td>
<td>0.63 [0.55:0.71]</td>
</tr>
<tr>
<td>Preference sd $\sigma_B$</td>
<td>$\mathcal{IG}$ 0.1 0.5</td>
<td>1.84 [1.4:2.29]</td>
</tr>
<tr>
<td>Spending sd $\sigma_C$</td>
<td>$\mathcal{IG}$ 0.1 0.5</td>
<td>2.38 [2.04:2.7]</td>
</tr>
<tr>
<td>Investment sd $\sigma_I$</td>
<td>$\mathcal{IG}$ 0.1 0.5</td>
<td>2.37 [2.62:3.13]</td>
</tr>
<tr>
<td>Prices sd $\sigma_P$</td>
<td>$\mathcal{IG}$ 0.1 0.5</td>
<td>0.07 [0.04:0.1]</td>
</tr>
<tr>
<td>Wage sd $\sigma_W$</td>
<td>$\mathcal{IG}$ 0.1 0.5</td>
<td>1.66 [1.42:1.9]</td>
</tr>
<tr>
<td>Conventional MP sd $\sigma_K$</td>
<td>$\mathcal{IG}$ 0.1 0.5</td>
<td>0.12 [0.1:0.13]</td>
</tr>
<tr>
<td>Capital quality sd $\sigma_C$</td>
<td>$\mathcal{IG}$ 0.1 0.5</td>
<td>0.14 [0.1:0.18]</td>
</tr>
<tr>
<td>Assets diversion sd $\sigma_L$</td>
<td>$\mathcal{IG}$ 0.1 0.5</td>
<td>1.29 [0.1:0.157]</td>
</tr>
<tr>
<td>Unconventional MP $\sigma_U$</td>
<td>$\mathcal{IG}$ 0.1 0.5</td>
<td>0.25 [0.21:0.28]</td>
</tr>
<tr>
<td>Productivity AR $\rho_A$</td>
<td>$\mathcal{B}$ 0.5 0.2</td>
<td>0.19 [0.09:0.3]</td>
</tr>
<tr>
<td>Preference AR $\rho_B$</td>
<td>$\mathcal{B}$ 0.5 0.2</td>
<td>0.96 [0.95:0.98]</td>
</tr>
<tr>
<td>Spending AR $\rho_C$</td>
<td>$\mathcal{B}$ 0.5 0.2</td>
<td>0.97 [0.95:0.99]</td>
</tr>
<tr>
<td>Investment AR $\rho_I$</td>
<td>$\mathcal{B}$ 0.5 0.2</td>
<td>0.62 [0.47:0.77]</td>
</tr>
<tr>
<td>Prices AR $\rho_P$</td>
<td>$\mathcal{B}$ 0.5 0.2</td>
<td>0.95 [0.91:0.98]</td>
</tr>
<tr>
<td>Wage AR $\rho_W$</td>
<td>$\mathcal{B}$ 0.5 0.2</td>
<td>0.35 [0.11:0.58]</td>
</tr>
<tr>
<td>Conventional MP AR $\rho_K$</td>
<td>$\mathcal{B}$ 0.5 0.2</td>
<td>0.41 [0.3:0.52]</td>
</tr>
<tr>
<td>Capital quality AR $\rho_C$</td>
<td>$\mathcal{B}$ 0.5 0.2</td>
<td>0.96 [0.93:0.99]</td>
</tr>
<tr>
<td>Assets diversion AR $\rho_L$</td>
<td>$\mathcal{B}$ 0.5 0.2</td>
<td>0.86 [0.82:0.89]</td>
</tr>
<tr>
<td>Unconventional MP AR $\rho_U$</td>
<td>$\mathcal{B}$ 0.5 0.2</td>
<td>0.3 [0.16:0.44]</td>
</tr>
<tr>
<td>Price MA $\omega_P$</td>
<td>$\mathcal{B}$ 0.5 0.2</td>
<td>0.38 [0.19:0.57]</td>
</tr>
<tr>
<td>Wage MA $\omega_W$</td>
<td>$\mathcal{B}$ 0.5 0.2</td>
<td>0.77 [0.67:0.93]</td>
</tr>
</tbody>
</table>

Marginal log-likelihood -793.36

**Notes:** The column entitled “Shape” indicates the prior distributions using the following acronyms: $N$ describes a normal distribution, $G$ a Gamma one, $B$ a Beta one, and $IG$ a Inverse Gamma, type 2.

### Table 3

**Prior and posterior distributions of structural parameters and shock processes.**

<table>
<thead>
<tr>
<th>Structural Parameters</th>
<th>Prior distributions Shape Mean Std.</th>
<th>Posterior distribution Mean [5%-95%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk-consumption $\sigma$</td>
<td>$\mathcal{N}$ 1 0.35</td>
<td>1.30 [1.03:1.56]</td>
</tr>
<tr>
<td>Consumption habits $k$</td>
<td>$\mathcal{B}$ 0.7 0.1</td>
<td>0.22 [0.15:0.29]</td>
</tr>
<tr>
<td>Labor disutility $\varphi$</td>
<td>$\mathcal{N}$ 2 0.75</td>
<td>3.45 [2.67:4.22]</td>
</tr>
<tr>
<td>Investment cost $\zeta$</td>
<td>$\mathcal{N}$ 4 1.50</td>
<td>3.75 [2.01:5.40]</td>
</tr>
<tr>
<td>Utilization elasticity $\zeta$</td>
<td>$\mathcal{N}$ 5 2</td>
<td>7.68 [5.19:10.1]</td>
</tr>
<tr>
<td>Price stickiness $\theta_P$</td>
<td>$\mathcal{B}$ 0.5 0.1</td>
<td>0.79 [0.74:0.85]</td>
</tr>
<tr>
<td>Price indexation $\xi_P$</td>
<td>$\mathcal{B}$ 0.5 0.15</td>
<td>0.36 [0.15:0.57]</td>
</tr>
<tr>
<td>Wage stickiness $\theta_W$</td>
<td>$\mathcal{B}$ 0.5 0.1</td>
<td>0.88 [0.85:0.92]</td>
</tr>
<tr>
<td>Wage indexation $\xi_W$</td>
<td>$\mathcal{B}$ 0.5 0.15</td>
<td>0.51 [0.26:0.76]</td>
</tr>
<tr>
<td>CMP - smoothing $\rho_C$</td>
<td>$\mathcal{B}$ 0.75 0.1</td>
<td>0.82 [0.79:0.86]</td>
</tr>
<tr>
<td>CMP - inflation $\phi_\pi$</td>
<td>$\mathcal{N}$ 1.5 0.25</td>
<td>2.03 [1.74:2.32]</td>
</tr>
<tr>
<td>CMP - output gap $\phi_y$</td>
<td>$\mathcal{N}$ 0.25 0.1</td>
<td>0.21 [0.14:0.28]</td>
</tr>
<tr>
<td>UMP - smoothing $\rho_u$</td>
<td>$\mathcal{B}$ 0.75 0.1</td>
<td>0.98 [0.97:1.00]</td>
</tr>
<tr>
<td>UMP - spread gap $\phi_\sigma$</td>
<td>$\mathcal{G}$ 10 2.5</td>
<td>7.79 [4.36:11.09]</td>
</tr>
<tr>
<td>UMP - credit gap $\phi_L$</td>
<td>$\mathcal{N}$ 0.25 0.15</td>
<td>0.31 [0.14:0.49]</td>
</tr>
<tr>
<td>Fixed cost $\Phi$</td>
<td>$\mathcal{N}$ 0.25 0.05</td>
<td>0.08 [0.01:0.15]</td>
</tr>
<tr>
<td>Fraction of diverted assets $\lambda$</td>
<td>$\mathcal{N}$ 0.5 0.1</td>
<td>0.82 [0.75:0.90]</td>
</tr>
<tr>
<td>Discount factor $(\beta^{-1} - 1) \times 100$</td>
<td>$\mathcal{G}$ 0.25 0.1</td>
<td>0.10 [0.04:0.16]</td>
</tr>
<tr>
<td>Inflation rate $(\pi - 1) \times 100$</td>
<td>$\mathcal{N}$ 0.6 0.1</td>
<td>0.51 [0.37:0.65]</td>
</tr>
<tr>
<td>Growth rate $(\gamma - 1) \times 100$</td>
<td>$\mathcal{N}$ 0.4 0.1</td>
<td>0.27 [0.17:0.37]</td>
</tr>
<tr>
<td>Labor shift $\bar{H}$</td>
<td>$\mathcal{N}$ 0 2</td>
<td>4.10 [2.06:6.09]</td>
</tr>
</tbody>
</table>

Marginal log-likelihood -793.36

**Notes:** The column entitled “Shape” indicates the prior distributions using the following acronyms: $N$ describes a normal distribution, $G$ a Gamma one, $B$ a Beta one, and $IG$ a Inverse Gamma, type 2.

### Table 4

**Prior and posterior distributions of structural parameters and shock processes.**
Figure 2: A comparison of real rate, nominal rate and production gap with their natural counterpart between 1995:1-2017:1

Annual Real Rates

Annual Nominal Rates

Annual Production Gaps

Figure 3: A comparison of natural rate estimates with Curdia [2015] and Holston et al. [2016]
Figure 4: Historical Decomposition of the NRI, USA.

Quarterly real natural rate $\tilde{R}_t - \tilde{R}$

Supply shocks ($\eta_{A}^t$)
Demand shocks ($\eta_{I}^t + \eta_{G}^t + \eta_{B}^t$)
Financial shocks ($\eta_{K}^t + \eta_{L}^t$)
Markup shocks ($\eta_{P}^t + \eta_{W}^t$)
Policy shocks ($\eta_{R}^t + \eta_{\psi}^t$)

Notes: The solid line depicts the time path of the ratio of the deviation from the steady state, while the bars depict the contribution of the shocks in the corresponding point deviation (at the mean of the estimated parameters). Supply shock is the permanent TFP shock, demand shocks include investment, government spending and preference shocks, financial shocks are capital quality and shocks to the ability of banks to attract funds, markup shocks include inflation and wages shocks and policy shocks are both conventional and unconventional monetary policy shocks.

Figure 5: Variance forecast of the NRI, USA.

Real Natural Rate $\tilde{R}_t$

Supply shocks ($\eta_{A}^t$)
Demand shocks ($\eta_{I}^t + \eta_{G}^t + \eta_{B}^t$)
Financial shocks ($\eta_{K}^t + \eta_{L}^t$)
Markup shocks ($\eta_{P}^t + \eta_{W}^t$)
Policy shocks ($\eta_{R}^t + \eta_{\psi}^t$)
Figure 6: Bayesian IRF of an unconventional monetary policy shock
Figure 7: Sensitivity analysis of the NRI, USA.
Figure 8: System responses to a negative capital quality shock under estimated and ramsey-optimal credit policy rules

Figure 9: System responses to a negative assets diversion shock under estimated and ramsey-optimal credit policy rules