Long-term business relationships, bargaining and monetary policy

Mirko Abbritti\textsuperscript{a}, Asier Aguilera-Bravo\textsuperscript{b}, Tommaso Trani\textsuperscript{a}

\textsuperscript{a}Universidad de Navarra
\textsuperscript{b}Universidad Pública de Navarra

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Abstract

A growing empirical literature documents the importance of long-term contracts and bargaining for price rigidity and firms’ dynamics. This paper introduces long-term business-to-business (B2B) relationships and price bargaining into a standard monetary DSGE model. The model is based on two assumptions: first, both wholesale and retail producers need to spend resources to form new business relationships. Second, once a B2B relationship is formed, the price is set in a bilateral bargaining between firms. The model provides a rigorous framework to study the effect of long-term business relationships and bargaining on monetary policy and business cycle dynamics. It shows that these contracts reduce both the allocative role of intermediate prices and the real effects of monetary policy shocks. We also find that the model does a good job in replicating the second moments and cross-correlations of the data, and that it improves over the benchmark New Keynesian model in explaining some of them.

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1. INTRODUCTION

The typical business environment often differs dramatically from the standard Dixit-Stiglitz monopolistic competition framework usually adopted in modern DSGE monetary models. As evidenced by recent empirical research, most firms engage mainly in long-term relationships with their customers, and most of their customers are other firms (see e.g. Blinder et al. (1998) for the US, Fabiani et al. (2006) for the Euro Area and Apel et al. (2005) for Sweden). Most of these long-term relationships are governed by implicit or explicit contracts\textsuperscript{[1]} and these contracts last on average between one and two years. Therefore, negotiations of prices and quantities are the rule rather than the exception. In

\textsuperscript{[1]} Henceforth, we will use the terms “contracts” or “relationships” interchangeably.

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\textit{Email addresses:} mabbritti@unav.es (Mirko Abbritti), asier.aguilera@unavarra.es (Asier Aguilera-Bravo), ttrani@unav.es (Tommaso Trani)
fact, in surveys firms report that the main reason they wish to keep prices stable is that they are concerned about losing customers relationships. For instance, Fabiani et al. (2006) find, on the basis of surveys conducted by nine Eurosystem national central banks, that the existence of implicit and explicit contracts with customers is the most important explanation for rigid prices. Zbaracki et al. (2004) find that customer communications and price negotiation costs account for almost 75% of the total price adjustment cost and are 20 times bigger than the size of the menu costs.

Motivated by this evidence, in this paper we introduce business-to-business (B2B) long-term relationships and price bargaining into a standard monetary DSGE model. In the model there are two types of firms, upstream producers (wholesalers) and downstream producers (retailers). Wholesalers produce intermediate goods, which are transformed by retailers into final goods and sold to households. The intermediate goods market is characterized by search and matching frictions à la Mortensen & Pissarides (1994). Both wholesalers and retailers need to spend time and resources to match and form long-term relationships with other firms. Once a business relationship is formed, the price is bargained between wholesalers and retailers according to a standard Nash bargaining protocol. An important feature of our model is that business relationships are also endogenously destroyed. Therefore, the stock of B2B contracts changes not only through endogenous creation, but also through the endogenous destruction of inefficient matches. Lastly, the presence of quadratic customer communications and price negotiation costs introduce nominal stickiness in intermediate prices and gives a role to monetary policy. The model provides a rigorous framework to study the effect of long-term contracts and bargaining on monetary policy and business cycle dynamics.

The model is calibrated to capture the main features of the US product market. We consider three shocks: a shock to the TFP of wholesalers, a shock to the TFP of retailers and a monetary policy shock. While the calibration of the monetary shock is standard, the two TFP shocks are calibrated to match the volatility of output and the cross-correlation of output with intermediate prices. We compare the performances of the model with the ones of a benchmark two-sector New Keynesian (NK) model with sticky intermediate prices.

We find that the model does a good job in replicating the second moments and cross-correlations of the data, and that it improves over the benchmark NK model in explaining some of them. In particular, introducing B2B long-term relationships helps to improve the volatility of investment, employment, intermediate prices and core inflation as well as the cross-correlation of intermediate prices, core inflation and consumption with output. It also provides better estimates for the cross-correlation of intermediate prices and final-price inflation with core inflation.

Recent research has started to investigate the importance of long-term relationships between firms and customers for price and business cycle dynamics. The vast majority of these papers, however, focus on retail firms to consumers relationships, and do not allow for bilateral negotiations between
the parties. These are important distinctions, because the business environment in B2B transactions is very different from the one in business-to-consumer (B2C) transactions.

To the best of our knowledge, only three papers analyze the implications of B2B relationships and bargaining for price and business cycle dynamics. Drozd & Nosal (2012) introduce dynamic frictions of building market shares into an international real business cycle model and show that the model can account for several pricing puzzles of international macroeconomics. Mathä & Pierrard (2011) introduce two-sided search and matching between wholesalers and retailers into the standard RBC model to study the effect of long-term relationships on business cycle dynamics. Abbritti & Trani (2017) study incomplete pass-through and the allocative power of intermediate goods prices in a model with product market frictions and bargaining over intermediate prices and quantities.

Our paper differs from Drozd & Nosal (2012), Mathä & Pierrard (2011) and Abbritti & Trani (2017) in two main aspects: First, we endogenize the match destruction margin. This is modeled by assuming that the productivity of each match is match-specific, and that inefficient matches are destroyed. Second, we allow for price adjustment costs in the bargaining problem between wholesalers and retailers. These costs, which are meant to capture customer communications and price negotiation costs, introduce nominal price stickiness and give a role to monetary policy. We show in the following that endogenous match destructions and sticky prices potentially play an important role in B2B and pricing dynamics.

The structure of the paper is as follows. Section 2 describes the theoretical B2B model. In Section 3 the calibration strategy is explained and the steady state is analyzed in Section 4. Section 5 includes a monetary policy exercise and a comparison with the benchmark NK model. Sections 6 and 7 discuss the role of technology shocks and a sensitivity analysis of the quantitative results, respectively. Section 8 concludes.

2. THE MODEL

2.1. Firms and Product Market

The product market is composed by two different types of firms, wholesalers and retailers, and follows the search and matching structure developed by Mortensen & Pissarides (1994). In order to sell their products, wholesale producers need to establish long-term customer relationships with retailers. Once both types of firms meet they bargain over the intermediate price at which retailers buy intermediate goods from the wholesalers. The productivity of firms is match-specific and has both an aggregate component and an idiosyncratic one, which we denote as $a_t(i)$. Following Krause & Lubik (2007), $a_t(i)$ is a productivity draw from a time-invariant distribution with c.d.f. $F(a_t(i))$ and

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We assume that the aggregate number of business to business (B2B) relationships $T_t$, follows the law of motion $T_{t+1} = (1 - \delta_{t+1}) (T_t + m_t)$ where $m_t$, the number of new B2B relationships at time $t$, is a constant returns to scale function of the search effort of retailers $V_t$ (purchase managers) and the search effort of wholesalers $S_t$ (advertising and marketing):

$$m_t = \hat{m} S_t^\xi V_t^{1-\xi}$$

The separation rate is defined as $\delta_t = \delta_x + (1 - \delta_x) F(\tilde{a}_t(i))$, where $\tilde{a}_t(i)$ is an endogenously determined productivity threshold below which matches are not profitable and hence terminated.

### 2.1.1. Wholesalers

There is a continuum of wholesale producers with unit mass. Each wholesaler $j$ maximizes the expected present value of future profits

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta_{0,t} \left\{ \left( \frac{P_t(j)}{P_t} - c_t^W(j) \right) T_t(j) - (r_t + \delta_k) K_t(j) - w_t N_t(j) - \gamma W S_t(j) \right\},$$

subject to the production function

$$Y_t^W(j) = q T_t(j) = A_t^W K_t(j)^{\alpha} N_t(j)^{1-\alpha}$$

with $q$ being the quantity produced per match, and the law of motion of the customer base

$$T_t(j) = (1 - \delta_t(j)) (T_{t-1}(j) + S_{t-1}(j) \mu W (\theta_{t-1})).$$

The term $\beta_{t,t+1} = \beta(C_{t+1}/C_t)^{-\sigma}$ denotes the household’s stochastic discount factor. Intuitively, the wholesaler chooses how much search effort, $S_t$, he will execute to find new buyers for his product. Think of this as the firm choosing the number of sales managers it is going to hire. Each unit of effort will provide him with an average of $\mu W (\theta_t) = \tilde{m} \theta_t^{1-\xi}$ retailers at the end of the period, where $\theta_t = V_t/S_t$ is the product market tightness, and the cost of this effort is captured by the last term of the objective function above. The assumption of linear search costs serves a twofold purpose. On one hand it allows us to single out the effects and implications of the endogenous separation rate, which highlights its importance. On the other hand it reduces the technical difficulties that would arise in the process of obtaining the idiosyncratic productivity threshold $\tilde{a}_t(i)$. This is in line with previous literature. For example, Mathä & Pierrard (2011) analyze the implications of having linear and quadratic search costs and find that, as expected, the latter produce hump-shaped dynamics for all variables and increase the persistence of output. However, the fact that they work with an

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3See Gourio & Rudanko (2014).
exogenous separation rate simplifies their analysis.

\( P_{It}(j) \) denotes the price of the intermediate good, that is decided after the successful match in a bilateral bargain with the retailers. The term \( c^W_t = \frac{\phi^W}{2} (P_{It}(j)/P_{It-1}(j) - \pi_t)^2 \) captures quadratic price adjustment costs. We assume that this cost, which is intended to capture price negotiation and communication costs, is proportional to the number of B2B relationships \( T_t \).

The wholesaler also decides how much capital, \( K_t(j) \), and labor, \( N_t(j) \), he is going to rent. \( A^W_t \) is an AR(1) TFP shock and, for simplicity, we normalize \( q = 1 \). The real rate of interest is \( r_t \), the depreciation rate of capital is \( \delta_k \) and the real wage is denoted by \( w_t = W_t/P_t \).

From the first-order necessary conditions we get that both the capital-labor ratio

\[
\frac{K_t(j)}{N_t(j)} = \frac{\alpha}{1 - \alpha} \frac{w_t}{r_t + \delta_k}
\]

and the marginal cost

\[
m_{ct} = (A^W_t)^{-1} \left( \frac{w_t}{1 - \alpha} \right)^{1-\alpha} \left( \frac{r_t + \delta_k}{\alpha} \right)^{\alpha} \tag{1}
\]

are equal across wholesalers. This is because \textit{ex-ante} all the wholesale producers are identical since the match-specific productivity draws are not realized until the matches occur and intermediate prices are bargained.

Further combinations of the FOCs give us the following expression:

\[
J^W_t(j) = \frac{P_{It}(j)}{P_t} - c^W_t(j) - m_{ct} + E_t \beta_{t+1,t+1} (1 - \delta_{t+1}(j)) J^W_{t+1}(j) \tag{2}
\]

This equation captures the expected value (across matches) of a B2B relationship for wholesaler \( j \). This depends positively on the intermediate price that the retailer pays him and negatively on the marginal cost of production. The last term, \( E_t \beta_{t+1,t+1} (1 - \delta_{t+1}(j)) J^W_{t+1} \), captures the expected continuation value of a match. This brings dynamic effects into the model coming from the fact that in the next period only a fraction equal to \( (1 - \delta_{t+1}(j)) \) of the matches survives and both wholesalers and retailers benefit from them.

The optimal amount of search is chosen to equate the expected marginal cost and the marginal benefit of a new business relationship:

\[
\frac{\gamma^W_t}{\mu^W(t)} = E_t \beta_{t+1,t+1} (1 - \delta_{t+1}(j)) J^W_{t+1}(j) \tag{3}
\]

This equation makes it clear that the search effort is executed in one period but it does not pay off until the next period and only if the match resulting from it is not destroyed.
2.1.2. Retailers

There is a continuum of retail producers with unit mass that buy the intermediate goods from wholesalers and sell it to consumers in perfectly competitive markets. Each retailer draws a match-specific productivity from a time-invariant distribution with c.d.f. \( F(a) \) and p.d.f. \( f(a) \). We assume that the draw of productivity takes place after intermediate price bargaining. This timing assumption simplifies considerably the bargaining problem and the solution of the model because it implies that the bargained price is identical for every match. The total production of retailer \( i \) is given by

\[
A_t^R T_t(i) \int_{\tilde{a}}^{\infty} a \frac{f(a)}{1 - F(\tilde{a})} da = A_t^R T_t(i) H(\tilde{a}_t(i))
\]

where \( A_t^R \) is an AR(1) TFP shock, \( T_t(i) \) is the number of productive or functional matches and \( H(\tilde{a}_t(i)) \) is the conditional expectation of the idiosyncratic shock \( E[a | a \geq \tilde{a}_t(i)] \). The productivity threshold \( \tilde{a}_t(i) \) is endogenously determined such that below it matches are not profitable and hence destroyed. In a similar way to the case of the wholesale producers, the number of B2B relationships of retailer \( i \) follows a law of motion that depends on the current-period separation rate and the previous-period number of functional matches and search effort exercised, \( V_{t-1}(i) \)

\[
T_t(i) = (1 - \delta_t(i)) (T_{t-1}(i) + V_{t-1}(i) \mu_R(\theta_{t-1}))
\]

where \( \mu_R(\theta_t) = \bar{\mu} \theta_t^{-\xi} \) is the average number of wholesalers attracted in the current period per unit of effort.

Retailers maximize the expected present value of profits before the realization of the idiosyncratic shock \( a \), i.e. based on the expected output \( E_a Y_t^R(i) = A_t^R T_t(i) H(\tilde{a}_t(i)) \). Specifically, every retailer \( i \) maximizes:

\[
\mathbb{E}_t \sum_{i=0}^{\infty} \beta_{t,t+1} \left\{ A_t^R T_t(i) H(\tilde{a}_t(i)) - \left( \frac{P_{t+1}(i)}{P_t} + c_t^R(i) \right) T_t(i) - \gamma R V_t(i) \right\}
\]

subject to the law of motion of the customer base. Retailers also face a cost of changing the bargained price, which is defined as \( c_t^R = \frac{2a_t}{2} (P_{t+1}(i)/P_{t+1} - \pi_t)^2 \) and it is also proportional to the number of B2B relationships \( T_t(i) \). The last term of the equation captures the cost of search effort.

At the beginning of each period the retailer chooses the level of production and the search effort. The intermediate price \( P_{tt} \) is decided after the successful match in a bilateral bargaining between retailers and wholesalers.

From the first-order necessary conditions we get the expected value (across matches) of a customer relationship for the retailer

\[
J_t^R(i) = A_t^R H(\tilde{a}_t(i)) - \left( \frac{P_{t+1}(i)}{P_t} + c_t^R(i) \right) + \mathbb{E}_t \beta_{t,t+1} (1 - \delta_t(i)) J_{t+1}^R(i) \tag{4}
\]
The value of a match depends positively on its production and negatively on the marginal cost, which is the relative price the retailer has to pay to the wholesaler. Similar to the case of the wholesaler, the last term in the equation connects the value of the matches in two subsequent periods bringing the dynamic effects into the model. Although (most) variables are connected in general equilibrium, we can notice a *ceteris paribus* effect of the threshold on the value of the matches. In particular, a higher threshold implies a higher average value of the matches because the previously least productive matches are destroyed, leaving operative those with higher productivity.

In equilibrium, the expected cost of a new match in a given period is equal the marginal benefit that will be realized in the subsequent periods:

$$\frac{\gamma_R}{\mu_R (\theta_t)} = \mathbb{E}_t \beta_{t,t+1} (1 - \delta_{t+1} (i)) J_{t+1}^R (i)$$  \hspace{1cm} (5)

2.1.3. *Endogenous separation*

We assume that a successful match is endogenously destroyed whenever the realization of the idiosyncratic shock does not make it profitable for at least one of the parties. Since prices are determined before the realization of $a_t$, the value of a B2B relationship for a wholesaler, $J^W_t (j)$, does not depend on $a_t$. Let us define by $J^R_t (a_t)$ the marginal value for the retailer of a match with idiosyncratic productivity $a_t$. The threshold $\tilde{a}_t$ is endogenously determined as solution of $J^R_t (\tilde{a}_t) = 0$. Combining this equation with the first-order conditions of the retailer the critical threshold below which matches are terminated is implicitly defined as:

$$\tilde{a}_t (i) = (A^R_t)^{-1} \left( \frac{P_t (i)}{\bar{P}_t} + \epsilon_t^R (i) - \frac{\gamma_R}{\mu_R (\theta_t)} \right)$$  \hspace{1cm} (6)

The threshold $\tilde{a}_t$ is increasing on the relative intermediate price and on the cost of changing prices because the higher these are the more profitable the match has to be to allow the retailer to pay for them. On the other hand a higher TFP shock of the retailers makes them more productive and can compensate for a lower idiosyncratic draw.

2.1.4. *Bargaining*

After wholesalers and retailers are matched, intermediate prices are determined through a Nash bargaining scheme between them. Precisely, for each match $v$, intermediate goods prices are determined as the outcome of the following bargaining scheme:

$$\max_{P_t} SU_t = \left[ (J^W_t (v))^\eta (J^R_t (v))^{1-\eta} \right]$$

where $\eta$ is the bargaining power of wholesalers.

\footnote{See appendix for the proof of existence and uniqueness of the threshold.}
We assume that prices are determined before the productivity draw of the retailers. Hence, the bargaining problem is the same across matches and the intermediate price will be unique. Dropping the subscript $v$, maximization gives:

$$\varphi_t \left[ \eta J^R_t - (1 - \eta) \left( 1 - \frac{\partial H(\tilde{a}_t)}{\partial \tilde{a}_t} \right) J^W_t \right] = (1 - \eta) \tau_t^R J^W_t + \eta \tau_t^W J^R_t$$

(7)

where

$$\tau_t^W = \phi_W (\pi_{It} - 1) \pi_{It} - E_t \beta_{t+1} \left[ (1 - \delta_{t+1}(j)) \phi_W + (1 - \delta_x) f(\tilde{a}_{t+1})(A_{t+1}^R)^{-1} J_{t+1}^W \phi_R \right] (\pi_{It+1} - 1) \pi_{It+1}$$

and

$$\tau_t^R = \left( 1 - \frac{\partial H(\tilde{a}_t)}{\partial \tilde{a}_t} \right) \left\{ \phi_R (\pi_{It} - 1) \pi_{It} - E_t \beta_{t+1} \left[ (1 - \delta_{t+1}) \left( 1 - \frac{\partial H(\tilde{a}_{t+1})}{\partial \tilde{a}_{t+1}} \right) \right] + (1 - \delta_x) f(\tilde{a}_{t+1}(i))(A_{t+1}^R)^{-1} J_{t+1}^R \phi_R \right\} (\pi_{It+1} - 1) \pi_{It+1}$$

capture the marginal cost of changing the intermediate price, and

$$\varphi_t = \frac{P_{It}}{P_t}$$

is defined as the intermediate relative price. Notice that if prices were flexible and the separation rate exogenous we would have

$$\tau_t^W = \tau_t^R = \frac{\partial H_t}{\partial \tilde{a}_t} = 0$$

and equation (7) would collapse to the standard solution by which each party gets a share of the surplus equal to their bargaining power.

2.2. Households

There is a representative household in the economy and his total lifetime utility is given by:

$$E \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\sigma}}{1 - \sigma} - \lambda \frac{N_t^{1+\nu}}{1+\nu} \right\}$$

which depends positively on consumption, $c_t$, and negatively on labor, $N_t$. The household faces a sequence of flow budget constraints which denoted in real terms can be written as:

$$c_t + \frac{b_{t+1}}{R_t} \pi_{t+1} + I_t = w_t N_t + b_t + (r_t + \delta_k) K_t + d_t$$

(8)

$$K_{t+1} = (1 - \delta_k) K_t + \left\{ 1 - \frac{\phi_I}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right\} I_t$$

(9)

where $b_t$ denote purchases of bonds, $R_t$ is the nominal interest rate on bonds, $w_t$ is the real wage and
$d_t$ are the dividends net of lump sum taxes.

From the first-order necessary conditions we obtain the standard Euler Equation, the labor supply and the no arbitrage condition on the assets:

\[ c_t^{-\sigma} = \beta c_{t+1}^{-\sigma} R_t \pi_{t+1}^{-1} \]  
\[ w_t = \kappa N_t^\nu c_t^\sigma \]  
\[ Q_t = \beta \left[ c_{t+1}^{-\sigma} (r_{t+1} + \delta_k) + Q_{t+1} (1 - \delta_k) \right] \]  
\[ 1 = Q_t \left[ 1 - \frac{\phi_I}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 - \phi_I \left( \frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} \right] + \beta E_t c_{t+1}^{-\sigma} Q_{t+1} \phi_I \left( \frac{I_{t+1}}{I_t} - 1 \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \]  

Where $Q_t$ denotes Tobin’s Q. These equations will determine the level of consumption, the demand for bonds and physical capital and the supply of labor.

### 2.3. Aggregate Constraints and Prices

To close the model we need to aggregate the quantities and the markets to clear. The total output in the economy is the result of adding up the production of every match whose productivity draw was above the threshold:

\[ Y_t^R = A_t^R T_t H (\tilde{a}_t) \]

and

\[ T_t = A_t^W K_t^\alpha N_t^{1-\alpha} \]

And finally notice that output can be either consumed, invested in physical capital or used to pay the cost of changing bargained prices and/or search efforts.

\[ Y_t = c_t + I_t + \phi (\pi_{It} - 1)^2 T_t + \gamma_R V_t + \gamma_W S_t \]

where $\phi = \phi_R + \phi_W$.

From the definition of the relative price, $\varphi_t = P_{It}/P_t$, we are able to establish the relationship between Consumer Price Index and Producer Price Index inflations:

\[ \frac{\pi_{It}}{\pi_t} = \frac{\varphi_t}{\varphi_{t-1}} \]

### 2.4. Monetary Policy

The monetary policy is described by a simple Taylor-type rule where the nominal interest rate set by the monetary authority depends on core inflation, output and the previous-period nominal interest
rate:  
\[ \frac{R_t}{R} = \exp(-z_t) \left[ \left( \frac{P_{It}}{P_{It-1}} \right)^{\phi_R} \left( \frac{Y_t}{Y} \right)^{\phi_{\pi}} \right]^{1-\phi_R} \left( \frac{R_{t-1}}{R} \right)^{\phi_R} \]

where \( \phi_R, \phi_{\pi}, \text{ and } \phi_Y \) are the relative weights on the previous period interest rate, current core (intermediate price) inflation and output, respectively, and \( z_t \) is an AR(1) shock.

**3. CALIBRATION**

We calibrate the model at the quarterly frequency, so we set \( \beta \) to match a standard annualized interest rate of 4%. Therefore, the discount factor equals 0.99. We use standard values also for the share of capital in income and the rate of capital depreciation. These are, respectively, \( \alpha = 0.33 \) and \( \delta_k = 0.025 \).

Our calibration of the search and matching with bargaining follows largely the strategy developed by Abbritti & Trani [2017]. This is based on survey interviews to business managers from various sectors of the U.S. economy and on survey data on employment in sales-related activities. Given the average opinion of business managers, the most reasonable duration of firm-to-firm relationships is 12 months. This sets a target for the quarterly separation rate, which here has an exogenous component as well as an endogenous one. In other words, we calibrate both components so that \( \delta = 0.20 \). We do not have priors on the importance of the exogenous component relative to the endogenous component. In this sense, we could attribute to each component 50% of the overall destruction rate. However, the labor search literature has assumed that the exogenous component explains the most of the separation rate. For example, in Krause & Lubik [2007], the exogenous component is 3/4 of the overall separation rate. Hence, we set \( \delta_x = 0.66 \delta \), which in turn implies that the endogenous component is \( \delta_n = F(\tilde{a}) = 0.34 \delta / [1 - 0.66 \delta] \). In one of our robustness exercises we study the sensitivity of our results to this choice, considering both a lower and a larger \( \delta_x \), and they basically remain unchanged. By assumption, \( F(\tilde{a}) \) is a lognormal distribution. We normalize its mean, so that \( \mu_{LN} = 1 \), and set its volatility \( \sigma_{LN} \) to 0.175. Consequently, \( \tilde{a} \) is equal to 0.78.

According to the evidence on sales-related activities, wholesalers’ search \( S \) is 9% of intermediate goods output. Since in this model the volume of trade between firms coincides with the number of matches (i.e., there is only an extensive margin of trade), this means that wholesalers’ search is close to 9% of GDP. This target allows us to determine both the search cost parameter and the matching efficiency. Therefore, assuming \( \eta = \xi = 0.5 \), we obtain \( \gamma = 0.5752 \) and \( \tilde{m} = 2.8658 \). The main justification for a conservative parametrization of the bargaining power \( \eta \) and elasticity of matching \( \xi \) is that there is no useful evidence for choosing them, so, setting them to 0.5, we can better relate our results to the endogenous separation of the matches and the other new features of the model.\(^5\)

\(^5\)In a model that abstracts from nominal price rigidity and endogenous destruction, Abbritti & Trani [2017] show...
We then set the time spent producing goods $N$ to 0.2, which implies that working time represents 20% of the total available time (see Mathé & Pierrard (2011)). Together with the elasticity of labor supply, this pins down the labor disutility $\kappa$. We choose a labor elasticity larger than 1, which is consistent with macroeconomic estimates (restated recently by Peterman (2016)). However, since the debate over the elasticity of labor supply is still open, we choose an elasticity equal to 1.6 by setting $\nu = 0.625$. In their model, Krause & Lubik (2007) introduce one-sided price rigidity and calibrate its parameter to a value of 40. Since in our model (B2B) there is two-sided price rigidity, we equally distribute the price rigidity between both sides and set the parameters governing the degree of price rigidities $\phi_W = \phi_R = 20$. These are intermediate values among all those used in the NK literature.

Lastly, we describe our strategy for calibrating the monetary policy and the shocks. For comparability, we choose the parameters of the monetary policy rule to calibrate the benchmark NK model in a standard way and, then, use the same values in our search and matching framework. We assume that the strength of the reaction of the Central Bank to PPI inflation is $\phi_{\pi I} = 1.5$ and to output growth is $\phi_Y = 0.125$. The persistence of the interest rates is $\phi_R = 0.85$. The standard deviation of monetary policy shocks is set to 0.15%, which is consistent with Christiano et al. (2014). As far as the TFP shocks are concerned, we assume that their persistence is a standard 0.9 and choose their volatility to match some moments of the data. Specifically, we target the volatility of GDP and the contemporaneous correlation of the intermediate good price with GDP. As a result, our model with B2B relationships implies $\sigma_{AW} = 0.9165\%$ and $\sigma_{AR} = 0.2585\%$. Conditional on these choices, we control the relative volatility of investment using the parameter $\phi_I$ which is set equal to 0.158 in our model.

Even though the main goal of this paper is not to perfectly replicate the data we can show that it does a fairly good job in replicating some key second moments statistics of the data. To assess the quantitative validity of our model, we show in the following table the relative standard deviation and the contemporaneous correlation with output and core inflation generated by it and we compare them not only with the data but also with those generated by the B2B model without endogenous separation rate. The data are collected from FRED covering the period from 1975Q1 to 2015Q2.

As we can see in Table 1 by calibration, both models match the standard deviation of GDP and the relative volatilities of investment and the intermediate price with the GDP. However, the fact that the B2B model with endogenous separation rate provides a better match of the relative volatility of the intermediate price suggests that allowing firms to decide whether they want to continue with a business relationship has important effects on price dynamics. This is a remarkable result, especially since the novelty of this model is the structure of the product market that determines the intermediate price.

---

that one can choose $\eta$ to approximate the volatility of the PPI in the data, with little consequence for the other moments. See Section 6.2 for the sensitivity of the present model to $\eta$.

*Actually, the presence of the retailer’s TFP shock is sufficient to match the sign of the contemporaneous correlation of the intermediate good price with GDP observed in the data.
Table 1: Second Moments

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>B2B (I)</th>
<th>B2B (II)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volatility GDP</td>
<td>1.39</td>
<td>1.39</td>
<td>1.39</td>
</tr>
<tr>
<td>(\text{Vol}(x)/\text{Vol}(\text{GDP}))</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Investment</td>
<td>3.49</td>
<td>3.49</td>
<td>3.49</td>
</tr>
<tr>
<td>Employment</td>
<td>0.96</td>
<td>0.78</td>
<td>1.13</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.82</td>
<td>0.41</td>
<td>0.49</td>
</tr>
<tr>
<td>Wages</td>
<td>0.42</td>
<td>0.83</td>
<td>1.14</td>
</tr>
<tr>
<td>Interm. Price</td>
<td>0.40</td>
<td>0.41</td>
<td>0.64</td>
</tr>
<tr>
<td>CPI Inflation</td>
<td>0.20</td>
<td>0.55</td>
<td>0.88</td>
</tr>
<tr>
<td>PPI Inflation</td>
<td>0.33</td>
<td>0.31</td>
<td>0.33</td>
</tr>
<tr>
<td>Corr(x, GDP)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Investment</td>
<td>0.89</td>
<td>0.97</td>
<td>0.97</td>
</tr>
<tr>
<td>Employment</td>
<td>0.85</td>
<td>0.59</td>
<td>0.58</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.89</td>
<td>0.85</td>
<td>0.82</td>
</tr>
<tr>
<td>Wages</td>
<td>0.85</td>
<td>0.76</td>
<td>0.71</td>
</tr>
<tr>
<td>Interm. Price</td>
<td>0.22</td>
<td>0.22</td>
<td>0.38</td>
</tr>
<tr>
<td>CPI Inflation</td>
<td>0.43</td>
<td>-0.09</td>
<td>-0.30</td>
</tr>
<tr>
<td>PPI Inflation</td>
<td>0.25</td>
<td>0.26</td>
<td>0.25</td>
</tr>
<tr>
<td>Corr(x, PPI Infl.)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interm. Price</td>
<td>0.28</td>
<td>0.20</td>
<td>0.15</td>
</tr>
<tr>
<td>CPI Inflation</td>
<td>0.25</td>
<td>0.40</td>
<td>0.19</td>
</tr>
</tbody>
</table>

Notes: B2B (I) is the basic model specification (endogenous separation rate). B2B (II) is the basic model specification with exogenous separation rate (computed by setting to zero the variance of the match-specific productivity).

The next section elaborates more on the contribution and importance of the endogenous separation rate of matches and how it shapes the transmission of monetary policy.

4. MONETARY POLICY TRANSMISSION

The repeated nature of the interactions between wholesalers and retailers points toward an important issue: bargained intermediate prices may not be allocative, in the sense that they may not affect the final production of firms. For example, if the real intermediate price decreases, wholesale firms may decide not to adjust production if they expect buyers to compensate them in the future for the reduced profits incurred in the current period. This issue is especially relevant if one considers the recent empirical evidence, which suggests that nominal price stickiness arises mainly at the intermediate rather than at the retail level. In fact, as first shown by [Barro (1977)](Barro1977), the real effects of monetary policy when prices are sticky crucially depend on prices being allocative.\(^7\)

In the B2B model intermediate prices still play an allocative role through two different channels. On one hand, they have a direct effect on the separation threshold \(\tilde{a}\). *Ceteris paribus*, a lower relative intermediate price increases the value of the marginal match for the retailer and thus decreases the

\(^7\)See also [Abbriitti & Trani (2017)](Abbriitti2017) for a discussion.
threshold $\hat{a}$. Consequently, both the separation rate $\delta$ and the average productivity of surviving matches $H(\hat{a})$ decrease. Through this mechanism, intermediate good prices have a direct allocative role on the number of B2B relationships and final output. Therefore, in the presence of sticky prices monetary policy can affect directly the number of endogenous separations, the average productivity of surviving relationships and final and intermediate output.

On the other hand, the intermediate price affects the value of a B2B relationship to wholesalers and retailers and thus the incentives of firms to engage in costly search activities. Notice, however, that this effect is opposite for firms in the two sides of the market: while a decrease in the relative intermediate price induces wholesalers to decrease their search effort, it also increases the search effort of retailers. The two effects thus tend to cancel out. The overall effect on the formation of new matches depends on the initial product market tightness, on the presence of search externalities and on the separation rate.

**Figure 1:** Comparison of the IRFs of the B2B Model With and Without Endogenous Separation Rate to a Monetary Policy Shock

To gauge the quantitative size of these two channels, Figure 1 compares the effects of an expansionary monetary policy shock between our model with endogenous separation rate and the same model but with exogenous separation rate. In the B2B model, the reduction of the nominal interest rate
stimulates the economy increasing the levels of consumption and investment, and therefore aggregate demand. As a result prices increase. However, since price rigidity occurs in the intermediate level, the final price increases more than the intermediate price and hence the relative intermediate price goes down. This leads to a decrease in the separation threshold which reduces the separation rate and increases the number of matches. In other words, to satisfy the increase in aggregate demand firms increase their production adjusting through the endogenous separation margin, i.e. they keep alive matches with lower productivity. Nevertheless, wholesalers and retailers know that the shock is short-lived and, anticipating the need to reduce their stock of B2B relationships in the future, both reduce their search effort. A completely different pattern is observed when adjustments through the separation rate are no longer available. In the model with exogenous separation rate, a reduction in the nominal interest rate also leads to a decrease of the relative intermediate price. Consequently, the value of a match for a wholesaler decreases, whereas that of the retailer increases. Accordingly, wholesalers will search less and retailers more. Notice that in the case with endogenous separation rate retailers searched less but this was because they could adjust through the separation margin, which in the exogenous separation case is not available anymore. In the end the opposite effects in searching effort cancel out and there is no change at all neither in production nor on the number of B2B relationships. Actually, there is no change in any other real variable except search effort for that matter.

4.1. Comparison With the New Keynesian Model

4.1.1. The New Keynesian Model

To validate the importance of our contribution, we compare the results of our product market frictions (B2B) model with the ones of a benchmark New Keynesian (NK) model with monopolistic competition. To make the models comparable, we assume that also in the benchmark model there are two sectors of production, wholesalers and retailers. Wholesalers are monopolistically competitive and face quadratic price adjustment costs. Retailers combine the varieties of the intermediate goods in a single bundle and sell it to households.

Specifically, in the benchmark NK model retailers operate under perfect competition and flexible prices. Their production function is $y_{rt} = A^{R}_{t} y_{It}$, where

$$y_{It} = \left[ \int_{0}^{1} y_{It}(j) \frac{\varepsilon_{NK}-1}{\varepsilon_{NK}} dj \right]^{\frac{\varepsilon_{NK}}{\varepsilon_{NK}-1}}$$

is a bundle of intermediate varieties bought from different wholesalers. The optimal demand of each variety $j$ is

$$y_{It}(j) = \left( \frac{P_{It}(j)}{A^{R}_{t} P_{t}} \right)^{-\varepsilon_{NK}} y_{It}$$

14
Each wholesaler \( j \) operates under monopolistic competition and faces quadratic adjustment costs

\[
C^p_j (\pi) = \frac{\psi_p}{2} \left( \frac{P_{1t}(j)}{P_{lt-1}(j)} - \pi \right)^2
\]

Notice that this cost function is identical to one faced by wholesalers and retailers in the B2B model. Wholesaler \( j \) maximizes the expected present value of future profits

\[
E_0 \sum_{t=0}^{\infty} \beta_{0,t} \left\{ \left( \frac{P_{1t}(j)}{P_{lt}} - c^p_t (j) \right) y_{lt} (j) - (r^k_t + \delta_k) K_t(j) - w_t N_t(j) \right\}
\]

subject to the production function

\[
y_{lt}(j) = A_t K^\alpha_t N^1_{\alpha - 1}(j)
\]

From the wholesaler’s maximization problem we obtain the following FOCs:

\[
mc_t = \frac{w_t}{(1-\alpha)} \left( \frac{P_{lt}}{P_t} \right)^{1-\alpha} \left( \frac{r^k_t + \delta_k}{\alpha} \right)^\alpha
\]

\[
w_t = \frac{1}{\alpha} K_t(j) N_t(j)
\]

\[
\frac{P_{1t}(j)}{P_t} = \frac{\varepsilon_{NK}}{\varepsilon_{NK} - 1} \left( mc_t + c^p_t (j) - \frac{\tau_{Pt}(j)}{\varepsilon_{NK}} \right)
\]

where

\[
\tau_{Pt}(j) = \psi_p (\pi_{It}(j) - \pi) \pi_{lt}(j) - \beta_{lt+1} \frac{y_{lt+1}(j)}{y_{lt}(j)} \{ \psi_p (\pi_{lt+1}(j) - \pi) \pi_{lt+1}(j) \}
\]

captures the marginal costs of changing prices. The first two equations capture the marginal costs and the capital labor ratio. Equation (16) is instead a version of the Phillips curve relating present and future inflation rates to marginal costs. In fact, aggregating across firms and log-linearizing around steady state one can rewrite equation (16) as:

\[
\hat{\pi}_{lt} = \beta \hat{\pi}_{lt+1} + \frac{(\varepsilon_{NK} - 1)}{\psi_p} \left( \hat{mc}_t - \hat{A}_t^R \right)
\]

where variables with hats denote log deviations from steady state.

4.1.2. Results

In order to compare the real effects of a monetary policy shock between both models we have calibrated the common parameters to the same value. To calibrate the degree of price rigidity in the NK model we have followed Krause & Lubik (2007) and set \( \psi_p = 40 \). The rest of parameters that are specific to the NK model take standard values in the literature. Through this approach we hope to infer the differences of the real effects of monetary policy that are not due to differences in adjustment costs or in different elasticities of output to factors of production. Figure 2 compares the effect of a monetary policy shock in the B2B model with the ones in the benchmark NK model. Results are qualitatively similar but quantitatively rather different. The main fact that stands out is
that the effects of monetary policy shocks are larger in the NK model than in the B2B model. This is consistent with the idea of a lower allocative role of intermediate prices in B2B relationships.

5. THE ROLE OF PRODUCTIVITY SHOCKS

Once we distinguish between sectors of production, it is interesting to study how the effects of TFP shocks that affect primarily the production of wholesalers differ from the ones that affect the production of retailers. For simplicity, we assume that the two shocks are uncorrelated. Adding a degree of cross-correlation between the two shocks does not change the main results.\(^8\)

Let us consider first a TFP shock to the wholesalers’ production function, shown in Figure 3. A positive shock makes the wholesalers more productive increasing total production \((Y_t)\). The increase in \(T_t\) comes from two different sources. On one hand, the reduction of wholesalers’ marginal costs increases the total value of each match and induces both wholesalers and retailers to increase their search efforts, which results in the creation of a higher number of new matches. On the other hand, the threshold of the idiosyncratic shock, \(\tilde{a}_t\) goes down, which reduces the endogenous destruction rate.

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\(^8\)See section 6.1 for a sensitivity analysis.
Figure 3: Impulse Responses to a 1 Standard Deviation Shock to Wholesalers TFP

of matches, $\delta_t$, as well as the average productivity of the matches. The high persistence of output is driven by the high persistence of the number of B2B relationships ($T_t$), which is mainly caused by search and investment adjustment costs. On impact, the shock reduces the intermediate relative price, which is accompanied by a lower intermediate price inflation and a higher final price inflation.

As we can see in Figure 4, a positive TFP shock to the retailers’ production function has, to some extent, a similar effect as a shock to the TFP of wholesalers. The search effort and the number of B2B relationships both increase, exhibiting high persistence. At the same time the match-specific productivity threshold declines and this reduces the endogenous destruction rate of matches on impact. There is however one important difference between the two shocks: while the relative intermediate price is negatively correlated with output following both a monetary policy shock and a wholesalers’ TFP shock, it is positively correlated with output after a retailer’s TFP shock. For this reason, the introduction of the retailer’s TFP shock is crucial for the ability of our model to match the cross-correlation of intermediate good prices with output, which is slightly positive in the data. As previously explained in Section 3, we use this fact to provide a reasonable calibration of the volatility of the TFP shock to retailers and show that the combination of these three shocks allows the B2B model to explain fairly well the second moments of the data.
6. SENSITIVITY OF QUANTITATIVE RESULTS

To validate the strength of our results we conduct different robustness exercises, which are presented in Table 2.

6.1. Correlation Between Productivity Shocks

The first exercise studies the effect of the relationship between the two TFP shocks on the second moments of the model. In particular, we want to test whether adding a degree of cross-correlation between the two shocks changes the main results. The first column of Table 2 reproduces the second moments of the baseline calibration, which assumes that both shocks are uncorrelated, whereas the second column shows the new moments when we set the cross-correlation equal to 0.5. One of the most significant differences is the increase in the volatility of output, which somewhat reduces the relative standard deviations of the rest of the variables. The other (and only) worth-mentioning change comes from the contemporaneous correlation of the intermediate price and GDP, which almost doubles. Still, changes are neither qualitative nor quantitatively important. Therefore, we conclude that our results are robust to including some degree of cross-correlation between TFP shocks.
Table 2: Sensitivity to correlation between TFP shocks and bargaining power.

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_{W,R} = 0$</th>
<th>$\sigma_{W,R} = 0.5$</th>
<th>$\eta = 0.15$</th>
<th>$\eta = 0.5$</th>
<th>$\eta = 0.85$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Volatility GDP</strong></td>
<td>1.39</td>
<td>1.57</td>
<td>1.39</td>
<td>1.39</td>
<td>1.51</td>
</tr>
<tr>
<td><strong>Vol(x)/Vol(GDP)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Investment</td>
<td>3.49</td>
<td>3.45</td>
<td>3.59</td>
<td>3.49</td>
<td>5.34</td>
</tr>
<tr>
<td>Employment</td>
<td>0.78</td>
<td>0.72</td>
<td>1.13</td>
<td>0.78</td>
<td>0.96</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.41</td>
<td>0.39</td>
<td>0.56</td>
<td>0.41</td>
<td>0.52</td>
</tr>
<tr>
<td>Wages</td>
<td>0.83</td>
<td>0.78</td>
<td>1.22</td>
<td>0.83</td>
<td>1.05</td>
</tr>
<tr>
<td>Interm. Price</td>
<td>0.41</td>
<td>0.33</td>
<td>0.86</td>
<td>0.41</td>
<td>0.26</td>
</tr>
<tr>
<td>CPI Inflation</td>
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<td>0.46</td>
<td>1.03</td>
<td>0.55</td>
<td>0.30</td>
</tr>
<tr>
<td>PPI Inflation</td>
<td>0.31</td>
<td>0.27</td>
<td>0.42</td>
<td>0.31</td>
<td>0.18</td>
</tr>
<tr>
<td><strong>Corr(x, GDP)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Investment</td>
<td>0.97</td>
<td>0.97</td>
<td>0.95</td>
<td>0.97</td>
<td>0.86</td>
</tr>
<tr>
<td>Employment</td>
<td>0.59</td>
<td>0.60</td>
<td>0.59</td>
<td>0.59</td>
<td>0.66</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.85</td>
<td>0.86</td>
<td>0.77</td>
<td>0.85</td>
<td>0.88</td>
</tr>
<tr>
<td>Wages</td>
<td>0.76</td>
<td>0.77</td>
<td>0.70</td>
<td>0.76</td>
<td>0.81</td>
</tr>
<tr>
<td>Interm. Price</td>
<td>0.22</td>
<td>0.40</td>
<td>0.34</td>
<td>0.22</td>
<td>0.11</td>
</tr>
<tr>
<td>CPI Inflation</td>
<td>-0.09</td>
<td>-0.07</td>
<td>-0.34</td>
<td>-0.09</td>
<td>0.18</td>
</tr>
<tr>
<td>PPI Inflation</td>
<td>0.26</td>
<td>0.25</td>
<td>0.37</td>
<td>0.26</td>
<td>0.45</td>
</tr>
<tr>
<td><strong>Corr(x, PPI Infl.)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interm. Price</td>
<td>0.20</td>
<td>0.15</td>
<td>0.52</td>
<td>0.20</td>
<td>-0.06</td>
</tr>
<tr>
<td>CPI Inflation</td>
<td>0.40</td>
<td>0.46</td>
<td>-0.21</td>
<td>0.40</td>
<td>0.61</td>
</tr>
</tbody>
</table>

Notes: The first and fourth columns are the standard calibration of the B2B model.

6.2. Bargaining Power

Table 2 also analyzes the seconds moments of the model for different values of the bargaining power of the wholesalers $\eta$. This parameter is essential to the outcome of the bargaining problem since it determines how the total surplus of a match is split between wholesalers and retailers. In our calibration we have followed a rather cautious approach and have set it equal to 0.5. Remember that we have already mentioned that there is no condition under which the equilibrium allocation is socially efficient. In other words, there is no particular values of $\eta$ for which wholesalers and retailers internalize the search externalities. In this exercise we see how the model behaves when the equilibrium is not symmetric and the allocation is not socially efficient. In particular, we analyze how the main variables react to the extreme values of $\eta$ of 0.15 and 0.85.

The first thing we notice is that although the volatility of output does not change if $\eta$ is lower than 0.5 it increases significantly for higher values. This exercise also reveals that the relative standard deviation of investment, employment, consumption and wages seems to be U-shaped, with a minimum around $\eta = 0.5$. This relation is not symmetric given that when wholesalers hold most of the bargaining power the volatility of output and the relative standard deviation of investment increase more dramatically, which could be explained by the fact that most of the volatility of the TFP shocks comes from the wholesalers side. However, this nonlinear relationship does not seem to hold for the

---

9The standard deviation of the wholesalers account for almost 80% of the total standard deviation of TFP shocks.
relative standard deviation of the intermediate price and both inflation rates, which decreases as the bargaining power of wholesalers increases. By looking at the relative volatility and cross-correlation of CPI inflation one might be tempted to set $\eta = 0.85$ to bring the model closer to the data. However, doing so would come at the expense of a higher volatility of output and investment, and, more importantly, a lower volatility of the intermediate price, setting them further apart from their real counterparts.

6.3. Matching the Data With the New Keynesian Model

In Section 4.1 we compared the real effects of a monetary policy shock between our model and a benchmark New Keynesian model with monopolistic competition. We showed that, for the same calibration, the quantitative effects of an expansionary monetary policy shock were higher in the NK model, which confirmed a lower allocative role of intermediate prices when these are bargained. To convince the reader that the predominance of the B2B over the NK model\textsuperscript{10} does not come from an advantageous calibration, we show in Table 3 the second moments of both models calibrated with the same strategy\textsuperscript{11}.

Since in terms of matching the moments of investment, employment, consumption and wages no model clearly outperforms the other, and taking into account that the main novelty of our model comes from the intermediate price determination we will focus our analysis on the latter. In particular, the most relevant result is that regardless of the calibration strategy followed, the NK model cannot explain more than half of the relative standard deviation of the intermediate price. Both models do a good job in matching the relative standard deviation of core inflation but they fail in explaining the relative standard deviation of CPI inflation. Actually, in terms of cross-correlation between GDP and core inflation, the NK model estimate has the opposite sign of the data. Furthermore, the cross-correlations between core inflation and the intermediate price and CPI inflation are much worse in the NK than in the B2B model. Therefore, this exercise validates the results of our model and allows us to conclude that long-term relationships between firms and price bargaining should be taken into account in the design and implementation of monetary policy.

\textsuperscript{10}At least for explaining pricing dynamics and the real effects of monetary policy.

\textsuperscript{11}The volatility of the TFP shocks of both models are calibrated to match the volatility of GDP and the contemporaneous correlation of the intermediate good price with GDP. Also, the investment adjustment cost of each model is calibrated to replicate the relative volatility of investment.
<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>B2B</th>
<th>NK (I)</th>
<th>NK (II)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volatility GDP</td>
<td>1.39</td>
<td>1.39</td>
<td>1.46</td>
<td>1.39</td>
</tr>
<tr>
<td>Vol(x)/Vol(GDP)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Investment</td>
<td>3.49</td>
<td>3.49</td>
<td>5.83</td>
<td>3.49</td>
</tr>
<tr>
<td>Employment</td>
<td>0.96</td>
<td>0.78</td>
<td>0.59</td>
<td>0.56</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.82</td>
<td>0.41</td>
<td>0.60</td>
<td>0.82</td>
</tr>
<tr>
<td>Wages</td>
<td>0.42</td>
<td>0.83</td>
<td>0.85</td>
<td>0.91</td>
</tr>
<tr>
<td>Intern. Price</td>
<td>0.40</td>
<td>0.41</td>
<td>0.22</td>
<td>0.21</td>
</tr>
<tr>
<td>CPI Inflation</td>
<td>0.20</td>
<td>0.55</td>
<td>0.33</td>
<td>0.38</td>
</tr>
<tr>
<td>PPI Inflation</td>
<td>0.33</td>
<td>0.31</td>
<td>0.29</td>
<td>0.34</td>
</tr>
</tbody>
</table>

| Corr(x, GDP)          |      |     |        |         |
| Investment            | 0.89 | 0.97| 0.94   | 0.87    |
| Employment            | 0.85 | 0.59| 0.60   | 0.14    |
| Consumption           | 0.89 | 0.85| 0.95   | 0.98    |
| Wages                 | 0.85 | 0.76| 0.93   | 0.94    |
| Intern. Price         | 0.22 | 0.22| 0.30   | 0.22    |
| CPI Inflation         | 0.43 | -0.09| 0.11 | -0.07   |
| PPI Inflation         | 0.25 | 0.26| 0.18   | -0.04   |

| Corr(x, PPI Infl.)    |      |     |        |         |
| Intern. Price         | 0.28 | 0.20| 0.09   | 0.04    |
| CPI Inflation         | 0.25 | 0.40| 0.84   | 0.90    |

Notes: NK (I) is the basic model specification with the same parameter values as the B2B model. NK (II) is the same model specification but calibrated to match the data, following the same calibration strategy as with the B2B model.

### 7. CONCLUSIONS

This paper has introduced search and matching frictions and bargaining between firms into an otherwise standard monetary DSGE model. We demonstrate that, for reasonable calibrations, the long-term nature of the contracts between firms reduces the allocative role of intermediate good prices and the real effects of monetary policy shocks. Also, we show that the model does a good job in replicating the second moments and cross-correlations of US product market and business cycle data.
8. REFERENCES


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Economic Inquiry, 54, 100–120.


9. APPENDIX

A. BARGAINING PROBLEM

Intermediate prices are determined through a Nash bargaining scheme between the retailer and the wholesaler. Precisely, for each match \( v \), intermediate goods prices are determined as the outcome of the following bargaining scheme

\[
\max_{P_t} SU_t = \left[ (J^W_t(v))^\eta (J^R_t(v))^{1-\eta} \right]
\]

where \( \eta \) is the bargaining power of retailers.

Recall the endogenous separation rate:

\[
\tilde{a}_t(i) = (A^R_t)^{-1} \left[ \frac{P_{it}(i)}{P_t} + \frac{\phi_R}{2} \left( \frac{P_{it}(i)}{P_{it-1}(i)} - 1 \right)^2 - \mathbb{E}_t \beta_{t,t+1} (1 - \delta_{t+1}(i)) J^R_{t+1}(i) \right]
\]

(17)

Necessary derivations for the bargaining problem:

\[
\frac{\partial \tilde{a}_t(i)}{\partial P_{it}} = (A^R_t)^{-1} \left[ \frac{1}{P_t} + \phi_R \left( \frac{P_{it}(i)}{P_{it-1}(i)} - 1 \right) \frac{1}{P_{it-1}(i)} - \mathbb{E}_t \beta_{t,t+1} \left( \frac{\partial \delta_{t+1}(i)}{\partial P_{it}} J^R_{t+1}(i) + (1 - \delta_{t+1}(i)) \frac{\partial J^R_{t+1}(i)}{\partial P_{it}} \right) \right]
\]

Since

\[
\frac{\partial \delta_{t+1}(i)}{\partial P_{it}} = \frac{\partial \delta_{t+1}(i)}{\partial \tilde{a}_{t+1}(i)} \frac{\partial \tilde{a}_{t+1}(i)}{\partial P_{it}}
\]

\[
= (1 - \delta_x) f(\tilde{a}_{t+1}(i)) (A^R_{t+1})^{-1} \phi_R \left( \frac{P_{it+1}(i)}{P_{it}(i)} - 1 \right) \left( \frac{P_{it+1}(i)}{P_{it}(i)^2} \right)
\]

\[
= -(1 - \delta_x) f(\tilde{a}_{t+1}(i)) (A^R_{t+1})^{-1} \phi_R \left( \frac{P_{it+1}(i)}{P_{it}(i)^2} - 1 \right) \left( \frac{P_{it+1}(i)}{P_{it}(i)^2} \right)
\]

and

\[
\frac{\partial J^R_{t+1}(i)}{\partial P_{it}} = A^R_{t+1} \frac{\partial H(\tilde{a}_{t+1}(i))}{\partial \tilde{a}_{t+1}(i)} \frac{\partial \tilde{a}_{t+1}(i)}{\partial P_{it}} + \phi_R \left( \frac{P_{it+1}(i)}{P_{it}(i)} - 1 \right) \left( \frac{P_{it+1}(i)}{P_{it}(i)^2} \right)
\]

\[
= A^R_{t+1} \frac{\partial H(\tilde{a}_{t+1}(i))}{\partial \tilde{a}_{t+1}(i)} (A^R_{t+1})^{-1} \phi_R \left( \frac{P_{it+1}(i)}{P_{it}(i)} - 1 \right) \left( \frac{P_{it+1}(i)}{P_{it}(i)^2} \right) + \phi_R \left( \frac{P_{it+1}(i)}{P_{it}(i)^2} - 1 \right) \left( \frac{P_{it+1}(i)}{P_{it}(i)^2} \right)
\]

\[
= \left( 1 - \frac{\partial H(\tilde{a}_{t+1}(i))}{\partial \tilde{a}_{t+1}(i)} \right) \phi_R \left( \frac{P_{it+1}(i)}{P_{it}(i)} - 1 \right) \left( \frac{P_{it+1}(i)}{P_{it}(i)^2} \right)
\]
we obtain

\[
\frac{\partial \tilde{a}_t(i)}{\partial P_{tt}} = (A_t^R)^{-1} \left\{ \frac{1}{P_t} + \phi_R \left( \frac{P_{tt}(i)}{P_{tt-1}(i)} - 1 \right) \frac{1}{P_{tt-1}(i)} \right. \\
- \left. \mathbb{E}_t \beta_{t,t+1} \left[ (1 - \delta_x f (\tilde{a}_{t+1}(i)) (A_{t+1}^R)^{-1} \phi_R \left( \frac{P_{tt+1}(i)}{P_{tt+1}(i)} - 1 \right) \left( \frac{P_{tt+1}(i)}{P_{tt+1}(i)} \right) J_{t+1}^R(i) \right] \right) \\
+ (1 - \delta_{t+1}(i)) \left[ 1 - \frac{\partial H (\tilde{a}_{t+1}(i))}{\partial \tilde{a}_{t+1}(i)} \right] \phi_R \left( \frac{P_{tt+1}(i)}{P_{tt+1}(i)} - 1 \right) \left( \frac{P_{tt+1}(i)}{P_{tt+1}(i)} \right) \right\} \\
\]

Rearranging terms

\[
\frac{\partial \tilde{a}_t(i)}{\partial P_{tt}} = (A_t^R)^{-1} \left\{ \frac{1}{P_t} + \phi_R \left( \frac{P_{tt}(i)}{P_{tt-1}(i)} - 1 \right) \frac{1}{P_{tt-1}(i)} \right. \\
- \left. \mathbb{E}_t \beta_{t,t+1} \left[ (1 - \delta_{t+1}(i)) \left( 1 - \frac{\partial H (\tilde{a}_{t+1}(i))}{\partial \tilde{a}_{t+1}(i)} \right) \right] \phi_R \left( \frac{P_{tt+1}(i)}{P_{tt+1}(i)} - 1 \right) \left( \frac{P_{tt+1}(i)}{P_{tt+1}(i)} \right) \right) \\
+ (1 - \delta_x f (\tilde{a}_{t+1}(i)) (A_{t+1}^R)^{-1} J_{t+1}^R(i) ) \phi_R \left( \frac{P_{tt+1}(i)}{P_{tt+1}(i)} - 1 \right) \left( \frac{P_{tt+1}(i)}{P_{tt+1}(i)} \right) \right\} \\
\]  \hspace{1cm} (18)

Recall

\[
J_{t}^W(j) = \frac{P_{tt}(j)}{P_t} - \frac{\phi_W}{2} \left( \frac{P_{tt}(j)}{P_{tt-1}(j)} - 1 \right) ^2 - \mathbb{E}_t \beta_{t,t+1} (1 - \delta_{t+1}(j)) J_{t+1}^W(j) \\
\]

Differentiating with respect to $P_{tt}$

\[
\frac{\partial J_{t}^W(j)}{\partial P_{tt}} = \frac{1}{P_t} - \frac{\phi_W}{2} \left( \frac{P_{tt}(j)}{P_{tt-1}(j)} - 1 \right) + \mathbb{E}_t \beta_{t,t+1} \left[ - \frac{\partial \delta_{t+1}(j)}{\partial P_{tt}} J_{t+1}^W(j) + (1 - \delta_{t+1}(j)) \frac{\partial J_{t+1}^W(j)}{\partial P_{tt}} \right] \\
\]

Since

\[
\frac{\partial J_{t+1}^W(j)}{\partial P_{tt}} = \phi_W \left( \frac{P_{tt+1}(j)}{P_{tt}(j)} - 1 \right) \left( \frac{P_{tt+1}(j)}{P_{tt+1}(j)} \right) \\
\]

We obtain

\[
\frac{\partial J_{t}^W(j)}{\partial P_{tt}} = \frac{1}{P_t} - \frac{\phi_W}{2} \left( \frac{P_{tt}(j)}{P_{tt-1}(j)} - 1 \right) \frac{1}{P_{tt-1}(j)} \\
+ \mathbb{E}_t \beta_{t,t+1} \left[ (1 - \delta_x f (\tilde{a}_{t+1}(j)) (A_{t+1}^R)^{-1} \phi_R \left( \frac{P_{tt+1}(j)}{P_{tt+1}(j)} - 1 \right) \left( \frac{P_{tt+1}(j)}{P_{tt+1}(j)} \right) J_{t+1}^W(j) \right] \\
+ (1 - \delta_{t+1}(j)) \phi_W \left( \frac{P_{tt+1}(j)}{P_{tt}(j)} - 1 \right) \left( \frac{P_{tt+1}(j)}{P_{tt+1}(j)} \right) \right] \\
\]
Rearranging terms

\[
\frac{\partial J_t^W(j)}{\partial P_{t_i}} = \frac{1}{P_t} - \phi_W \left( \frac{P_{t+1}(j)}{P_{t-1}(j)} - 1 \right) \frac{1}{P_{t-1}(j)} + \mathbb{E}_i \beta_{t,t+1} (1 - \delta_t(i)) P_t \phi_W \\
+ (1 - \delta_x) f (\tilde{a}_{t+1}(i)) (A_{t+1}^R)^{-1} J_t^W(j) \phi_R \left( \frac{P_{t+1}(j)}{P_{t}(j)} - 1 \right) \left( \frac{P_{t+1}(j)}{P_{t}(j)^2} \right) 
\]

Also recall

\[
J_t^R(i) = A_t^R H (\tilde{a}_t(i)) - \left[ \frac{P_{t+1}(i)}{P_t} + \frac{\phi_R}{2} \left( \frac{P_{t+1}(i)}{P_{t-1}(i)} - 1 \right)^2 \right] + \mathbb{E}_i \beta_{t,t+1} (1 - \delta_t(i)) J_{t+1}^R(i) 
\]

Differentiating with respect to \( P_{t_i} \)

\[
\frac{\partial J_t^R(i)}{\partial P_{t_i}} = A_t^R \frac{\partial H (\tilde{a}_t(i))}{\partial \tilde{a}_t(i)} \frac{\partial \tilde{a}_t(i)}{\partial P_{t_i}} - \left[ \frac{1}{P_t} + \frac{\phi_R}{2} \left( \frac{P_{t+1}(i)}{P_{t-1}(i)} - 1 \right) - \mathbb{E}_i \beta_{t,t+1} \left( 1 - \delta_t(i) \right) \left( 1 - \frac{\partial H (\tilde{a}_{t+1}(i))}{\partial \tilde{a}_{t+1}(i)} \right) \right] \\
+ (1 - \delta_x) f (\tilde{a}_{t+1}(i)) (A_{t+1}^R)^{-1} J_t^R(i) \phi_R \left( \frac{P_{t+1}(i)}{P_{t}(i)} - 1 \right) \left( \frac{P_{t+1}(i)}{P_{t}(i)^2} \right) - \left[ \frac{1}{P_t} + \frac{\phi_R}{P_{t-1}(i)} \left( \frac{P_{t+1}(i)}{P_{t-1}(i)} - 1 \right) \right] \\
+ \mathbb{E}_i \beta_{t,t+1} \left[ 1 - \delta_t(i) \right] (A_{t+1}^R)^{-1} \phi_R \left( \frac{P_{t+1}(j)}{P_{t}(j)} - 1 \right) \left( \frac{P_{t+1}(j)}{P_{t}(j)^2} \right) \\
+ (1 - \delta_t(i)) \left( 1 - \frac{\partial H (\tilde{a}_{t+1}(i))}{\partial \tilde{a}_{t+1}(i)} \right) \phi_R \left( \frac{P_{t+1}(j)}{P_{t}(j)} - 1 \right) \left( \frac{P_{t+1}(j)}{P_{t}(j)^2} \right) 
\]

Rearranging terms

\[
\frac{\partial J_t^R(i)}{\partial P_{t_i}} = - \left( 1 - \frac{\partial H (\tilde{a}_t(i))}{\partial \tilde{a}_t(i)} \right) \left\{ \frac{1}{P_t} + \frac{\phi_R}{P_{t-1}(i)} \left( \frac{P_{t+1}(i)}{P_{t-1}(i)} - 1 \right) - \mathbb{E}_i \beta_{t,t+1} \left( 1 - \delta_t(i) \right) \left( 1 - \frac{\partial H (\tilde{a}_{t+1}(i))}{\partial \tilde{a}_{t+1}(i)} \right) \right\} \\
+ (1 - \delta_x) f (\tilde{a}_{t+1}(i)) (A_{t+1}^R)^{-1} J_t^R(i) \phi_R \left( \frac{P_{t+1}(i)}{P_{t}(i)} - 1 \right) \left( \frac{P_{t+1}(i)}{P_{t}(i)^2} \right) 
\]

(20)
From the bargaining problem:

$$
\eta J^R_t = \left( \frac{1}{P_t} - \phi_W \left( \frac{P_{t+1}^{(i)}}{P_{t+1-i}^{(j)}} - 1 \right) \frac{1}{\pi_{t-i+1}^{(j)}} \right)
+ \mathbb{E}_t \beta_{t+1} \left[ ((1 - \delta_{t+1}) \phi_W \right.
+ (1 - \delta_x) f (\tilde{a}_{t+1}^{(i)}) \left( A_{t+1}^{R} \right)^{-1} J_{t+1}^W (j) \phi_R \left. \right)
\left( \frac{P_{t+1}^{(j)}}{P_{t+1}^{(j)}} - 1 \right) \frac{1}{P_{t+1}^{(j)}}
\right) = (1 - \eta) J^W_t \left( (1 - \frac{\partial H(\tilde{a}_t)}{\partial a_t}) \frac{1}{P_t} + \frac{\phi_R}{P_{t-i+1}^{(i)}} \left( \frac{P_{t-i+1}^{(i)}}{P_{t-i+1}^{(i)}} - 1 \right) \right)
+ (1 - \delta_x) f (\tilde{a}_{t+1}^{(i)}) \left( A_{t+1}^{R} \right)^{-1} J_{t+1}^R \phi_R \left( \frac{P_{t+1}^{(i)}}{P_{t+1}^{(i)}} - 1 \right) \frac{1}{P_{t+1}^{(i)}}
$$

$$
\eta J_t^R (\varphi_t - \tau_t^W) = (1 - \eta) J_t^W \left( (1 - \frac{\partial H(\tilde{a}_t)}{\partial a_t}) \varphi_t + \tau_t^R \right)
\left( \eta J_t^R - (1 - \eta) \left( 1 - \frac{\partial H(\tilde{a}_t)}{\partial a_t} \right) J_t^W \right) \varphi_t = (1 - \eta) \tau_t^R J_t^W + \eta \tau_t^W J_t^R
$$

where

$$
\tau_t^W = \phi_W (\pi_t - 1) \pi_t - \mathbb{E}_t \beta_{t+1} \left[ (1 - \delta_{t+1}) \phi_W + (1 - \delta_x) f (\tilde{a}_{t+1}^{(i)}) \left( A_{t+1}^{R} \right)^{-1} J_{t+1}^W \phi_R \right] (\pi_{t+1} - 1) \pi_{t+1}
$$

$$
\tau_t^R = \left( 1 - \frac{\partial H(\tilde{a}_t)}{\partial a_t} \right) \left( \phi_R (\pi_t - 1) \pi_t - \mathbb{E}_t \beta_{t+1} \left[ (1 - \delta_{t+1}) \left( 1 - \frac{\partial H(\tilde{a}_{t+1}^{(i)})}{\partial a_{t+1}^{(i)}} \right) \right.\right.
\left. \left. + (1 - \delta_x) f (\tilde{a}_{t+1}^{(i)}) \left( A_{t+1}^{R} \right)^{-1} J_{t+1}^R \phi_R \right] (\pi_{t+1} - 1) \pi_{t+1} \right)
$$

$$
\varphi_t = \frac{P_t}{P_t}
$$