Explaining Bond and Equity Premium Puzzles Jointly in Macro-Finance Model*

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January 20, 2018

Abstract

Investors require compensation in the form of a premium for holding risky long-term bonds and equity. The novelty of this paper is to explain bond and equity premium puzzles jointly in a macro-finance model extended with Epstein-Zin preferences and costly firm-entry without compromising the fit of the model to unconditional second moments of the main macroeconomic aggregates. Epstein-Zin preferences help matching nominal term premium by making investors sufficiently risk-averse while costly firm entry generate procyclical dividends as well as asset returns and, hence, a premium on unlevered equity over risk-free bonds. The model is estimated on post-war US macroeconomic and financial data with Generalised Method of Moments using a third-order approximate solution of the model.

*We are grateful to seminar participants at Cardiff Business School, Hungarian Economics Society, Central Bank of Hungary and MMF 2016 London. We are indebted to Alessia Campolmi for her detailed comments on the paper. Special thanks to Gianluca Benigno, Tamas Briglevics, Max Gillman, Henrik Kucsera, Robert Lieli, Patrick Minford, Panayiotis Pourpurides, Eyno Rots, David Staines, Balazs Vilagi and Mike Wickens for comments. We are thankful for the support of the Czech Academy of Sciences (GACR P402/12/097).
Macro-finance Dynamic Stochastic General Equilibrium (DSGE) models aim to jointly match a set of unconditional macroeconomic and finance moments such as the standard deviation of consumption and the mean and variability of bond and equity returns. A number of papers are successful in matching either the mean and variability of the equity or bond premium together with macroeconomic regularities. This paper shows that a workhorse New Keynesian model featuring Epstein-Zin preferences and extended with costly firm entry can jointly match bond and equity premium puzzles without distorting the fit of the model to a set of macroeconomic second moments. Investors of long-term nominal bonds and equities ask for a compensation in the form of risk premium contained in the yields of these assets. To the best of our knowledge, there is no existing model which can jointly match the unconditional mean and variability of bond (nominal) and equity premium without distorting the fit of the model to moments of macroeconomic variables. Firm entry and exit which are mimicked in our model by product creation and destruction has been found important in explaining fluctuations in aggregate output. In a pioneering work Bernard et al. (2010) studied US manufacturing firms and showed that the value of new products account for 33.6% of the overall output over a business cycle horizon (5 years) while product destruction explaining -30% loss in the value of output (both at existing firms). Hence the overall contribution of the extensive margin (product creation and destruction) is quite substantial.

Epstein-Zin preferences facilitate the explanation of bond premium while firm-entry helps to account for the size of equity premium in our model. Epstein-Zin preferences in our model follow the specification in Rudebusch and Swanson (2012) and are introduced into the model to help raise risk-aversion of the representative household who will demand high risk-premium for holding risky nominal bonds. Costly firm entry is introduced into the model following Bilbiie et al. (2007, 2012) where the mass of firms entering the industry in each period are subject to a time-varying sunk entry cost and a time-to-build lag in production. Bilbiie et al. (2012) stress the success of firm entry model in generating procyclical profits which we claim an important feature of the model in matching the large equity premium. The source of uncertainty in our model are temporary technology and dividend shocks.
Next we explain the mechanisms through which the model is able to match the equity and bonds premium puzzles.

The firm-entry part of our model helps to produce a large mean and standard deviation of the equity premium due to the procyclical nature of firm entry. After a positive productivity shock profit opportunities arise and new firms enter the industry. More labour is used to set up new firms which start to produce consumption goods using labour too. Due to the entry of new firms more dividend accumulates to households which they can use to consume more goods. Thus, the unexpected improvement in technology leads to a positive wealth effect which induces households to consume more. Due to the procyclical nature of firm entry consumption has strong comovement with dividends and with asset returns. Procyclical asset returns generate large mean of the equity premium.

The model is able to replicate the negative correlation between consumption growth and inflation which Piazzesi and Schneider (2006) as well as Rudebusch and Swanson (2012) consider as the most important ingredient to produce the large term premium on nominal bonds. To see why the negative correlation between consumption growth and inflation generates positive term premium we consider a negative innovation to technology that depresses consumption and leads to an unexpected rise in inflation. High inflation is associated with low real yield at the time of low consumption growth so investors demand a premium to hold the bonds. Hence, the majority of the term premium is a compensation for inflation risks in line with estimates using affine latent factor models on US data (see e.g. Kim and Wright (2005)). Recently, Kliem and Meyer-Gohde (2017) estimate a DSGE model on US data and claim that a major part of the term premium is a compensation for real risks. Further, we show that the negative unconditional correlation between consumption growth and inflation is reproduced by the entry model even when the output gap coefficient in the Taylor rule is small unlike previous papers where this negative correlation is only possible when the output gap coefficient is large (see, e.g. Rudebusch and Swanson (2012)).

The paper is connected to the literature on macro-finance as well as firm-entry. First, we explore how our work is related to macro-finance papers which explain either equity or bond premium puzzles. Some recent examples on the equity side include Li and Palomino (2014) and Menna and Tirelli (2014). The former paper uses Epstein-Zin preferences while the latter

\footnote{Menna and Tirelli (2016) explain the real bond premium puzzle (but not the nominal)
makes use of consumption habits and limited asset market participation to match the equity premium. Both previous papers employ a New Keynesian model with price and wage rigidities. Regarding the bond premium puzzle Rudebusch and Swanson (2012) use a New Keynesian model to match the mean and variability of the nominal term premium joint with second moments of several macroeconomic variables. Swanson (2014) manages to match the size and variability of the equity premium and the slope of the term structure of defaultable and non-defaultable bonds but not the bond premium. Hsu, Li and Palomino (2015) who match real and nominal term premium jointly in a New Keynesian model with both price and wage rigidity, habits and permanent technology shocks. Hördahl et al. (2008) explain the slope of the real yield curve.

Second, we show how our paper is related to the firm-entry literature. Colciago and Etro (2010a,b) use firm-entry in various market structures to explain procyclical profits and countercyclical markups. In particular, Colciago and Etro (2010a) induce strategic interactions among firms through quantity competition a la Cournot while Colciago and Etro (2010b) consider competition in prices a la Bertrand. Lewis and Winkler (2017) demonstrate that it is a difficult task to explain consumption crowding-in puzzle jointly with the entry of new firm in response to government spending shocks. Kaszab and Marsal (2013) explores how the above model without firm entry but including richer fiscal setup in the form of income taxation affect nominal term premium. The MNB working paper version of this paper also explores the relevance of income taxation in explaining the bond premium puzzle. In our paper firm exit is exogenous. Cavallari (2015) introduces endogenous exit into an endogenous entry model which is found to describe US data better compared to a model where the exit rate and the number of firms are fixed.

Some recent models include heterogenous agents to explain finance puzzles. Lansing (2015) show that capital income redistribution shocks contribute to a large premium on unlevered equity in a heterogenous agent model. Horvath et al. (2016) use a model with Ricardian and non-Ricardian households and show that the redistribution of income is endogenous from non-Ricardians to Ricardians with respect to monetary policy shocks and the model produces large and volatile equity returns. None of the above papers can explain bond and equity premium puzzles jointly.

The paper proceeds as follows. Section two contains the model used in

and equity premium puzzle together.
the paper with special focus on the description of firm-entry. Section three provides a short account of how bonds and equities are priced in the paper. Section four explains the estimation strategy. Section five summarises the main results. Finally we conclude.

1 The model

1.1 The intermediary firm’s problem

In this section we explain how firm entry is introduced into the basic New Keynesian model. Our short description of the production sector borrows from Bilbiie et al. (2007) who feature a two-sector RBC model with price rigidity. Labour is the only factor of production. In one sector labour is used to produce consumption goods. The other sector requires labour effort to set up new firms. We start with the description of the latter one.

There is a mass of firms. Firm $\omega$ employs labour ($l_t(\omega)$) in order to produce output ($y_t(\omega)$) using a constant-return-to-scale technology: $y_t(\omega) = Z_t l_t(\omega)$ where $Z_t$ is a stationary productivity shock:

$$
\log Z_t = \rho_Z \log Z_{t-1} + \varepsilon^Z_t,
$$

where $\varepsilon^Z_t$ is an independently and identically distributed (iid) stochastic technology disturbance with mean zero and variance $\sigma^2_Z$. The unit cost of production in units of consumption good $C_t$ is $w_t/Z_t$ where $w_t \equiv W_t/P_t$ is the real wage. There is also a mass of prospective entrants. Firms pay an entry cost of $f_E$ effective labour units, equal to $w_t f_E / Z_t$. Each period firms correctly anticipate their future profits and the probability $\delta$ of the exit-inducing shock. The model features a time-to-build lag in the sense that firms entering at time $t$ start to produce one period later. Therefore, the number of firms producing at period $t$, $N_t$, is described by:

$$
N_t = (1 - \delta)(N_{t-1} + N_{E,t-1})
$$

where $N_E$ stands for new entrants and both new entrants and incumbents survive with probability $1 - \delta$.

The real profits of firm $\omega$ at time $t$ (transferred back to households in the form of dividends, $d_t(\omega)$) can be expressed as:

$$
d_t(\omega) = \rho_t(\omega)y_t^D(\omega) - w_t l_t(\omega) - pac_t(\omega)\rho_t(\omega)y_t^D(\omega)
$$
where \( p_t(\omega) \equiv p_t(\omega)/P_t \) is the real price of firm \( \omega \), \( y_t^p(\omega) \) is the demand schedule coming from the cost-minimisation problem \( (y_t^p(\omega) = (p_t(\omega)/P_t)^{-\theta}[C_t + G_t + PAC_t]) \). Lower-case letters denote firm-specific variables while upper-case ones stand for the aggregate.

Adjusting prices is costly. Hence, nominal rigidity is introduced in the form of price adjustment costs that can be described with a quadratic function as in Rotemberg (1982):

\[
PAC_t(\omega) = \frac{\phi_P}{2} \left[ \frac{p_t(\omega)}{p_{t-1}(\omega)} - 1 \right]^2
\]

where \( \phi_P \) measures how strong price adjustment costs are.

The real value of firm \( \omega \) in units of consumption at time \( t \), denoted as \( v_t^{firm}(\omega) \) can be expressed as the sum of present and discounted future dividends:

\[
v_t^{firm}(\omega) = E_t \sum_{j=0}^{\infty} \lambda_{t+j}d_{t+j}(\omega)
\]

where \( \lambda_t \) is the marginal utility of consumption used to discount future profits. Firms face a death shock occurring with probability \( \delta \in (0, 1) \) in each period.

### 1.2 The household’s problem

The representative household maximises the continuation value of its utility \( (V) \):

\[
V_t = \begin{cases} 
U(C_t, L_t) + \beta \left[ E_tV_{t+1}^{1-\alpha} \right]^{\frac{\alpha}{1-\alpha}} & \text{if } U(C_t, L_t) \geq 0 \\
U(C_t, L_t) - \beta \left[ E_t(-V_{t+1})^{1-\alpha} \right]^{\frac{1}{1-\alpha}} & \text{if } U(C_t, L_t) < 0
\end{cases}
\]

with respect to its flow budget constraint. \( \beta \in (0, 1) \) is the subjective discount factor. Utility \( (U) \) at period \( t \) is derived from consumption \( (C_t) \) and leisure \( (1 - L_t) \). As the time frame is normalised to one leisure time \( (1 - L_t) \) is what we are left with after spending some time working \( (L_t) \). The recursive functional form in equation (2) is called Epstein-Zin preferences and is the same as the one used by Rudebusch and Swanson (2012). The period utility
which is additively separable in consumption and labour is given by\(^2\):

\[
U(C_t, L_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \chi_0 \frac{L_t^{1+\varphi}}{1+\varphi}
\]

where \(\sigma\) is the inverse of the intertemporal elasticity of substitution (IES), \(\varphi\) is the inverse of the Frisch elasticity of labour supply to wages and \(\chi_0 > 0\).

Swanson (2012) shows that the connection between coefficient of relative risk-aversion (CRRA) and parameter \(\alpha\) of the recursive utility in equation (2) is\(^3\):

\[
CRRA \simeq \frac{\sigma}{1+\frac{\sigma}{\varphi}} + \frac{\alpha(1-\sigma)}{1+\frac{\sigma-1}{1+\varphi}}.
\]

Households possess two types of assets: shares in a mutual fund of firms and government bonds. Let \(x_t\) denote the share in the mutual fund of firms entering period \(t\). In each period the mutual fund pays the representative household a total profit (in units of currency) of all firms that produce in that period, \(P_tN_id_t\). In period \(t\) the representative household purchases \(x_{t+1}\) shares in a mutual fund of \(N_{H,t} \equiv N_t + N_{E,t}\) firms where the first term refer to firms already operating at time \(t\) while the second term stands for the new entrants. Only \(N_{t+1} = (1-\delta)N_{H,t}\) firms will produce and pay dividends at time \(t + 1\). As the household does not know the share of firms induced to leave the market due to the exogenous exit shock \(\delta\) at the end of period \(t\), it finances the continuing operation of all preexisting firms and all new entrants during period \(t\). The nominal price of a claim to the future profit stream of the mutual fund of \(N_{H,t}\) firms at time \(t\) equals to \(V_{t}^{firm} = P_t^{firm};x_{t+1}\).

At time \(t\) the representative household holds nominal bonds and a share \(x_t\) in the mutual fund. It receives labour income \((W_tL_t)\), interest income \(i_{t-1}\) on nominal bonds and dividend income (in nominal terms) on mutual fund share holdings \((D_t \equiv P_id_t)\) and the value of selling its initial share position \((V_{t}^{firm})\).

Therefore, the period budget constraint of the representative household (in units of currency) can be written as:

\[
B_{N,t+1} + V_{t}^{firm}N_{H,t}x_{t+1} + P_tC_t = (1+i_{t-1})B_{N,t} + (D_t + V_{t}^{firm})N_t x_t + W_t L_t + T_L
\]

where \(D_t\) stands for the nominal value of dividends \((D_t \equiv P_id_t)\), \(1+i_t\) is the gross nominal interest rate and \(T_L\) are lump-sum taxes in nominal terms.

\(^2\)Note that this felicity function is slightly different from the one of RS mainly because we abstract from deterministic growth in line with Bilbiie et al. (2007, 2012).

\(^3\)Note that this formula applies only when the utility function in equation (3) is used.
1.3 Equilibrium

In the symmetric equilibrium all firms make identical choices so that \( p_t(\omega) = p_t \), \( d_t(\omega) = d_t \), \( y_t(\omega) = y_t \), \( v_t^{firm}(\omega) = v_t^{firm} \), \( l_t(\omega) = l_t \), \( \mu_t(\omega) = \mu \) and \( pac_t(\omega) = pac_t \).

The labour market clearing is given by:

\[
L_t = N_t l_t + N_{E,t} \frac{f_{E,t}}{Z_t}
\]  

(4)

where the firm term on the RHS denotes the amount of labour used in production while the second term stands for the amount of labour employed to set up new firms. One can use equation (4) to back out \( N_{E,t} \) in equation (1).

The aggregate output of the consumption basket (\( Y^C_t \)) is used for private (\( C_t \)) and public consumption (\( G_t \)) and to pay price adjustment costs:

\[
Y^C_t = C_t + G_t + PAC_t \\
= N_t \rho_t y_t \\
= N_t \rho_t Z_t l_t
\]

The previous accounting identity says that total absorption (the first row) equals to total production (second row). The last row made use of the production function.

1.4 Monetary and Fiscal Policy

Monetary Policy. The New Keynesian model is closed by a monetary policy rule (a so-called Taylor rule):

\[
dR_t = \rho_t dR_{t-1} + (1-\rho)[\bar{R} + \log \bar{\Pi}_t + g_\pi (\log \bar{\Pi}_t - \log \Pi^*_t) + g_y (Y_t - \bar{Y})/\bar{Y}] + \epsilon^i_t,
\]  

(5)

where \( dR_t \) is the deviation of the policy rate from its steady-state, \( \bar{\Pi}_t \) is a four-quarter moving average of inflation (defined below), and \( \bar{Y} \) denotes the steady-state level of \( Y_t \). \( \bar{R} \) is the steady-state gross interest rate which equals to \( \log(\Pi^*/\beta) \). \( \Pi^*_t \) is the target rate of inflation, and \( \epsilon^i_t \) is an iid shock with mean zero and variance \( \sigma^2_i \). We use the baseline version of the RS model with no ambiguity about long-run inflation target and thus the inflation target is not stochastic but constant (\( \Pi^*_t = \bar{\Pi}^* \) for all \( t \)).

The four-quarter moving average of inflation (\( \bar{\Pi}_t \)) can be approximated by a geometric moving average of inflation:

\[
\log \bar{\Pi}_t = \theta_\pi \log \bar{\Pi}_{t-1} + (1-\theta_\pi) \log \Pi_t,
\]  

(6)
where \( \theta_r = 0.7 \) ensures that the geometric average in equation (6) has an effective duration of approximately four quarters.

**Fiscal Policy.** The government spending follows the process:

\[
\log(G_t / \bar{G}) = \rho_G \log(G_{t-1} / \bar{G}) + \varepsilon^G_t, \quad 0 < \rho_G < 1,
\]

(7)

where \( \bar{G} \) is the steady-state level of \( G_t \), and \( \varepsilon^G_t \) is an iid shock with mean zero and variance \( \sigma^2_G \). \(^4\) We assume that the government budget is balanced in each period through income taxes as in Kaszab and Marsal (2013) who have shown that income taxation helps raising the nominal term premium. As a sensitivity check we consider a richer fiscal setup where we allow for time-varying debt (for more details see the appendix).

Further details about the full list of equilibrium conditions and calculation of the steady-state can be found in chapter three of Kaszab (2014).

\[\text{2 Bond and equity pricing}\]

The price of a default-free \( n \)-period zero coupon bond that pays $1 at maturity can be described with a recursive formula:

\[
p_t^{(n)} = E_t \{ m_{t+1} p_{t+1}^{(n-1)} \},
\]

where \( m_{t+1} \equiv m_{t,t+1} \) is the stochastic discount factor. \( p_t^{(n)} \) denotes the price of the bond at time \( t \) with maturity \( n \), and \( p_t^{(0)} \equiv 1 \), i.e. the time \( t \) price of $1 delivered at time \( t \) is $1. To calculate the term premium we need the bond-price expected by the so-called risk-neutral investor who is rolling over a one-period investment for 10 years (in case a bond with 10-year maturity). The risk neutral bond price can be expressed through the expectations hypothesis of the term structure:

\[
\hat{p}_t^{(n)} = e^{-nt} E_t \hat{p}_{t+1}^{(n-1)}
\]

(8)

where again \( \hat{p}_t^{(0)} \equiv 1 \). Equation (8) is another recursion which states that the current period price of the bond is the present discounted value of the next period bond’s price and where the discount factor is the risk-free rate rather than the stochastic discount factor.

\(^4\)In a companion paper (see Horvath et al. (2017)) we also explore the implications of spending reversals (i.e. government spending is lowered as debt-to-GDP rises) on the term structure of interest rates.
The continuously compounded yield to maturity of the \( n \)-period zero-coupon bond is defined as:

\[
i_t^{(n)} = -\frac{1}{n} \log p_t^{(n)}.
\]

The implied term premium is defined as the difference between the yield expected by the risk-averse investor \( i_t^{(n)} \) minus the yield awaited by the risk-neutral investor \( i_t^{(n)} \):

\[
TP_t^{(n)} = i_t^{(n)} - i_t^{(n)}.
\]

We also report two imperfect but frequently used measures of the risk of nominal bonds. The first one is the slope of the term structure which is defined as the difference between the yield with maturity \( n \) and the short yield (3-month yield). The second alternative riskiness indicator is the excess holding period return which can be written as:

\[
x_t^{(n)} = \frac{p_t^{(n-1)}}{p_t^{(n)}} - i_{t-1}.
\]  

In equation (9) \( p_t^{(n-1)} \) is the period \( t \) price of a bond that matures in \( n - 1 \) quarters and \( i_{t-1} \) is 3-month yield in period \( t - 1 \). \( p_t^{(n)} \) is the period \( t - 1 \) price of a bond that matures in \( n \) quarters.

The equity premium is calculated as the return on the risky asset minus the return on the risk-free government bond.

## 3 Data and GMM Estimation

To discipline the choice of model parameters we estimate our models (with either constant of time-varying debt) with the GMM toolbox of Andreasen et al. (forthcoming) using the following quarterly US time series 1961-2007: i) the per capita consumption growth, \( dC_t \) (\( d \) denotes the temporal difference operator), ii) the one-quarter nominal interest rate, \( i_t \) iii) the per capita hours growth, \( dL_t \), iv) the growth rate of real wage \( d(W_t/P_t) \), v) inflation, \( \Pi_t \), vi) slope of the term structure proxied by the difference between the 10-year nominal interest rate, \( i_t^{(40)} \) and the one-quarter nominal interest rate, \( i_t \), and vii) the 10-year nominal term premium from Adrian et al. (2013), viii)
growth rate of the labor tax revenue per GDP \((d(\tau_t W_t L_t / Y_t))\). The appendix provides more information about the data used in the estimation.

Similarly to Andreasen et al. (forthcoming) and Bretscher et al. (2016) we consider three types of unconditional moments for the GMM estimation: i) sample means \(m_1(y_t) = y_t\), contemporaneous covariances \(m_2(y_t) = \text{vech}(y_t y'_t)\), and the own autocovariances, \(m_3(y_t) = \{y_{i,t} y_{i,t-k}\}^{n_{y_t}}_{t=1}\) for \(k = 1\) and \(k = 5\). The total set of moments used in the estimation are, therefore, given by \(m(y_t) = [m_1(y_t) \ m_2(y_t) \ m_3(y_t)]'\).

Letting \(\theta\) denote the structural parameters, the GMM estimator is given by:

\[
\arg \min_{\theta \in \Theta} \left( \frac{1}{T} \sum_{t=1}^{T} q_t - E(q_t(\theta)) \right)' W \left( \frac{1}{T} \sum_{t=1}^{T} q_t - E(q_t(\theta)) \right).
\]  

In equation (10), \(W\) is a positive definite weighing matrix, \(\frac{1}{T} \sum_{t=1}^{T} q_t\) are data moments and \(E(q_t(\theta))\) are moments computed from the model. We follow a conventional two-step procedure to implement GMM. In the first step, we set \(W_T = \text{diag}(\hat{S}^{-1})\) to obtain \(\hat{\theta}^{(1)}\) where \(\hat{S}\) denotes the long-run variance-covariance matrix of \(\frac{1}{T} \sum_{t=1}^{T} q_t\) when centered around its sample mean. In the second (final) step we obtain \(\hat{\theta}^{(2)}\) using the optimal weighting matrix \(W_T = \hat{S}^{-1}_{\hat{\theta}^{(1)}}\) where the diagonal of \(\hat{S}^{-1}_{\hat{\theta}^{(1)}}\) contains long-run variance of our moments re-centered around \(E\left(q_t(\hat{\theta}^{(1)})\right)\). The long-run variances in both steps are produced with the Newey-West estimator using five lags and our results are robust to the inclusion of e.g. ten lags.

Parameters estimated by GMM can be found in Table (1). The estimated coefficients in the tax rule and the government purchases are close to Leeper et al. (2010) and Zubairy (2014). Importantly, both models estimate lower relative risk-aversion coefficient (see the implied CRRA of 40 and 30 for the time-varying and constant debt models respectively) than earlier papers (see RS for value of 110 and Andreaesen (2012) with the number of 168). Regarding our other estimates of the parameters they are in line with the ones in Andreasen et al. (forthcoming) and Bretscher et al. (2016) who used somewhat different model setups. Similarly to the findings of Andreasen et al. (forthcoming) and Bretscher et al. (2016) the curvature parameter of recursive preferences is estimated rather unprecisely. The estimate of the technology shock is close to the GMM estimates of Andreasen (2012).

The estimated high risk-aversion is needed for matching the bond premium and is a feature of many recent macro-finance papers (see e.g. An-
There are number of possible explanations to justify the high risk-aversion. One of them is based on Barillas et al. (2009) who show that a model with Epstein-Zin preferences and high risk-aversion is “isomorphic to a model in which households have low risk aversion but a moderate degree of uncertainty about the economic environment.” (RS pp. 123). Another interpretation can be derived from Malloy et al. (2009) who find that consumption of stockholders has higher standard deviation than consumption of non-stockholders. Therefore, risk-aversion should be higher in a representative agent model like the RS model with/without entry than in a model which can distinguish between agents with different consumption smoothing behaviour. To put it differently, the DSGE models we use might understate the quantity of risks faced by households so that a higher risk-aversion is needed to match risk premiums in the data.

The estimate of the AR (1) term and the size of the shock for the government spending process are reasonably close to the estimates reported in the appendix. The estimates of the parameters in the Taylor rules as well as the monetary policy shock are in line with those of Rudebusch (2002) and Andreasen (2012). It is important to note that our models are successful in matching the NTP with lower CRRA not only because of the introduction of richer fiscal setup but also because the GMM estimates the technology and government spending shocks with higher autocorrelation and shock size parameters.

Some parameters and steady-state quantities are not estimated but calibrated as follows. $\varepsilon$ is the elasticity of substitution among intermediary goods and is calibrated to six. The $\gamma_b = 2.8$ is consistent with a yearly debt-to-GDP ratio of 70 per cent. The steady-state inflation rate is zero ($\Pi^* = 1$). The steady-state marginal cost ($\overline{m\pi}$) is the inverse of the markup: $(\varepsilon - 1)/\varepsilon$. The steady-state tax rate implied by the government budget constraint is 36 per cent.
Table 1: GMM estimates of the models

<table>
<thead>
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<th>Parameters and steady-states</th>
<th>Time-varying debt</th>
<th>Constant debt</th>
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<td><strong>Monetary Policy</strong></td>
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<tr>
<td>(\rho_i)</td>
<td>0.7305</td>
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<tr>
<td></td>
<td>0.33</td>
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<tr>
<td>(g_{\pi})</td>
<td>0.5298</td>
<td>0.5197</td>
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<tr>
<td></td>
<td>3.98</td>
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<td>(g_y)</td>
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<td><strong>Fiscal Policy</strong></td>
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<tr>
<td>(\rho_g)</td>
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<td>5.9</td>
<td>0.11</td>
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<tr>
<td>(\rho_{\tau})</td>
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<td>-</td>
</tr>
<tr>
<td></td>
<td>0.0055</td>
<td>-</td>
</tr>
<tr>
<td>(\rho_{tb})</td>
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</tr>
<tr>
<td></td>
<td>0.0021</td>
<td>-</td>
</tr>
<tr>
<td>(\rho_{ty})</td>
<td>0.9602</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>0.33</td>
<td>-</td>
</tr>
<tr>
<td>(\rho_a)</td>
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</tr>
<tr>
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<td>0.00097</td>
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<td><strong>Std. of Shocks</strong></td>
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<tr>
<td>(\sigma_a)</td>
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<td>(\sigma_g)</td>
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<tr>
<td>(\sigma_{\tau})</td>
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<td>0.0064</td>
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<tr>
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<tr>
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<td>0.0202</td>
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</table>

Notes: Numbers below the parameter estimates denote the standard deviation of the estimate in per cent. — indicates those parameters which do not appear in the constant debt model.
Table 2: Moments from the models

<table>
<thead>
<tr>
<th>Unconditional Moment</th>
<th>US data, 1961-2007</th>
<th>Time-varying debt</th>
<th>Constant debt</th>
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</thead>
<tbody>
<tr>
<td>SD($dC$)</td>
<td>2.69</td>
<td>2.76</td>
<td>2.65</td>
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<tr>
<td>SD($L$)</td>
<td>1.71</td>
<td>1.87</td>
<td>1.41</td>
</tr>
<tr>
<td>SD($W/P$)</td>
<td>0.82</td>
<td>0.98</td>
<td>1.43</td>
</tr>
<tr>
<td>SD($\pi$)</td>
<td>2.52</td>
<td>3.69</td>
<td>4.34</td>
</tr>
<tr>
<td>SD($R$)</td>
<td>2.71</td>
<td>3.59</td>
<td>4.21</td>
</tr>
<tr>
<td>SD($R_{\text{real}}$)</td>
<td>2.30</td>
<td>2.29</td>
<td>1.26</td>
</tr>
<tr>
<td>SD ($R^{(40)}$)</td>
<td>2.41</td>
<td>3.43</td>
<td>3.47</td>
</tr>
<tr>
<td>Mean($NTP^{(40)}$)</td>
<td>1.06</td>
<td>1.26</td>
<td>1.32</td>
</tr>
<tr>
<td>SD($NTP^{(40)}$)</td>
<td>0.54</td>
<td>0.46</td>
<td>0.43</td>
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<tr>
<td>Mean($R^{(40)} - R$)</td>
<td>1.43</td>
<td>1.62</td>
<td>1.52</td>
</tr>
<tr>
<td>SD($R^{(40)} - R$)</td>
<td>1.33</td>
<td>1.34</td>
<td>1.54</td>
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<tr>
<td>Mean($x^{(40)}$)</td>
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<td>2.64</td>
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<tr>
<td>SD($x^{(40)}$)</td>
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<td>Mean(EQPR)</td>
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<td>7.1</td>
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<td>SD(EQPR)</td>
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<td>24.52</td>
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<td>Sharpe Ratio</td>
<td>0.3</td>
<td>0.2731</td>
<td>0.2598</td>
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<tr>
<td>Corr($dC, \pi$)</td>
<td>-0.34</td>
<td>-0.26</td>
<td>-0.21</td>
</tr>
<tr>
<td>Corr($d(\tau WL)/Y, dY$)</td>
<td>0.63</td>
<td>0.48</td>
<td>0.25</td>
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<tr>
<td>SD($d(\tau WL)/Y$)</td>
<td>3.06</td>
<td>3.57</td>
<td>0.83</td>
</tr>
</tbody>
</table>

Notes: the Mean, SD, Corr and Autocorr denote the unconditional mean, standard deviation, correlation and first-order autocorrelations. $NTP^{(40)}$ = nominal term premium on a 40-quarter bond, $R^{(40)} - R$ is the slope and $x^{(40)}$ is the excess holding period return for a 10-year bond. Moments are calculated using parameters estimated with GMM on US data 1961-2007. The model equivalent of the real wage is the reservation wage defined above. — denotes statistics that are not available in case of the constant debt model. EQPR denotes the equity premium. The Sharpe Ratio is defined as the mean of the equity premium divided by the standard deviation of equity.
4 Results

4.1 Equity and bond premium

Table 2 presents selected macro and finance moments calculated from US data 1961-2007\(^5\). Beyond macro and finance variables the models’ fit is assessed on the basis of fiscal moments such as the unconditional correlation of the labor tax revenue and and first-order autocorrelation of labor tax revenue. The models with either constant or time-varying government debt exhibit modest to fit a series of macroeconomic and finance moments. The mean and standard deviation of the slope of the term structure and the excess holding period return are reported in the table because they are regarded as imperfect measures of the mean and standard deviations of the nominal term premium. Next we explain the mechanisms that help account for the mean and variability of bond and equity returns observed in the data.

The model generates the high mean of equity premium due to procyclical firm entry. To shed light on the mechanism consider a positive productivity shock which induces entry of new firms due to positive profit opportunities in the future. The strong positive wealth effect of the productivity boosts consumption, dividends and, thus, higher returns on equity. Hence, productivity shocks imply positive comovement between consumption and the return on equity resulting in high equity premium. To generate sufficient variation in the return on equity and to match the empirical level of the Sharpe ratio we introduce a shock to dividends. Now we turn to discuss the implications of the model for the nominal term premium.

The model produces the negative correlation between consumption growth and inflation which Rudebusch and Swanson (2012) consider as the key ingredient for generating high and volatile nominal term premium on long-term nominal bonds. The main source of the large nominal term premium (a measure of riskiness of the bond) in this paper and in Rudebusch and Swanson (2012) is attributed to temporary productivity shocks which induce the negative correlation between consumption growth and inflation. To explain why nominal bonds are risky consider a negative productivity shock which depresses consumption but raises inflation. In bad times (consumption is low due to negative realisations of the technology) nominal bonds yield low real return because of higher inflation and are considered to be poor hedge.

\(^5\)We focus on data before the great recession to avoid complications posed by the fact that the US policy rate reached its lower bound from the end of 2008.
4.2 Is competition effect important for our results?

Several papers of firm-entry feature countercyclical markups in response to firm entry (see e.g. the so-called 'competition effect' in Colciago and Etro (2010a) or Lewis and Winkler (2017)). We emphasize that countercyclical markups need not be introduced into our model to produce the procyclality of profits and thus the high equity premium. We also explored the properties our model with introducing competition effect in the form of translog preferences (see more in the Appendix) and we found that our main results hold with little negative effects on the model's ability of matching macro and finance moments jointly.

5 Conclusion

We show that a New Keynesian model with Epstein-Zin preferences and costly firm-entry is consistent with both bond and equity premium puzzles. We estimate the model on post-war US data and find that it can match a list of macro and finance moments quite well.

References


6 Appendix

To construct the following time series we follow the procedure in Christoefel et al. (2013) and Leeper et al. (2010):

- $PY$: Gross Domestic Product. Bureau of Economic Analysis (BEA). Nipa Table 1.1.5, line 1.
- $P$: GDP deflator personal consumption expenditures. Source: BEA, Nipa Table 1.1.4, line 2.
- $C$: Private Consumption. Source: BEA, Nipa Table 1.1.6, line 2.
- $L$: hours, measure of the labour input. This is computed as $L = H \times (1 - U/100)$, where $H$ and $U$ are the average over monthly series of hours and unemployment. Source: BLS, series LNU02033120 for hours and LNS14000000 for unemployment.
- $INT$: Net Interest Payments of Federal Government Debt. Source: BEA, Nipa Table 3.2 (line 29-line13).
- $G$: Government consumption is computed as current consumption expenditures (line 21)+gross government investment (line 42)+net purchases of non-produced assets (line 44)-consumption of fixed capital (line 45). Source: BEA, Nipa Table 3.2
- $WL$: labour income tax base. Source: Nipa Table 1.12 (line 3).
- $\tau$: average effective labour income tax rate as in Jones (2002) and Leeper et al. (2010). We follow the procedure in the appendix of Leeper et al. (2010) to construct $\tau_t$.
7 Not for publication appendix.

In this appendix we present that our results stay robust to three modifications of the baseline entry model (alternative fiscal setup, translog preferences that induce competition effect and different entry costs).

7.1 Alternative fiscal setups

In one of our fiscal scenarios the government can issue debt in each time $t$. The evolution of debt from time $t - 1$ to time $t$ is described by the government’s budget constraint:

$$b_t + \tau_t w_t L_t = \frac{\gamma^{-1} R_{t-1} b_{t-1}}{\Pi_t} + g_t,$$

(11)

where $b_t$ and $w_t$ represent de-trended real government debt and real wages, respectively. All quantities are expressed in real terms, except for the nominal interest rate ($R_t$). $R_{t-1} b_{t-1}$ denotes interest payments on the previous period’s debt.

Our second fiscal scenario is the case of balanced budget (with either positive or zero steady-state government debt). If one imposes a restriction $b_t = b_{t-1} = 0$ for all $t$, then expression (11) boils down to the balanced budget case ($g_t = \tau_t w_t L_t$ for all $t$ in the absence of steady-state debt $b = 0$). In both fiscal scenarios the government budget is consolidated through distortionary labor income tax revenue.

To observe the role of steady-state debt, we linearize equation (11) to the first order:

$$\dot{b}_t + \eta d \tau_t + \tilde{\tau} \eta \dot{w}_t + \tilde{\tau} \eta \dot{L}_t = \gamma^{-1} \gamma_b (dR_{t-1} - \tilde{R} \pi_t) + \gamma^{-1} \tilde{R} \dot{b}_{t-1} + \dot{g}_t,$$

(12)

where $\eta \equiv \bar{w} \bar{L}/\bar{y}$, $\dot{b}_t \equiv (b_t - \bar{b})/\bar{y}$, and $\gamma_b \equiv \bar{b}/\bar{y}$ is the government debt-to-GDP ratio. The rest of the variables are defined above. Note that the deviations of debt and government spending from their respective steady states are defined relative to the steady-state output. When steady-state debt is zero, i.e., $\gamma_b = 0$, the real interest rate ($dR_{t-1} - \tilde{R} \pi_t$) does not have a direct effect on taxes ($d\tau_t$). Positive and increasing $\gamma_b$ is shown to raise the nominal term premium (see the Results section below).

Our fiscal rule is motivated by the evidence in Romer and Romer (2010) who estimate the effects of exogenous tax changes on output and emphasize
that ignoring the influences of economic activity on tax policy leads to biased estimates of the macroeconomic effects of tax changes. To address these concerns, we allow the tax rate at time $t$ to respond to previous period output allowing for the long delays in legislation and also to react to previous period debt to prevent the build-up of large debt-to-GDP ratios:

$$d\tau_t = \rho_r d\tau_{t-1} + \rho_{rb} \hat{\delta}_{t-1} + \rho_{ry} \hat{y}_{t-1} + \varepsilon_t^\tau. \quad (13)$$

Our specification of the fiscal policy rule captures four main features suggested by Leeper et al. (2010) and Zubairy (2014). First, the response of taxes to the deviations of output from its steady-state captures some ‘automatic stabiliser’ component of fiscal policy (see parameter $\rho_{ry}$). Second, we allow the income tax rate to respond to the state of government debt (see parameter $\rho_{rb}$). Third, government spending and tax rates evolve persistently, which is allowed for in the form of autoregressive terms, $\rho_y$ and $\rho_r$, in equations (7) and (13), respectively. Fourth, unexpected movements in the tax rate is reflected by the tax shock, $\varepsilon_t^\tau$ which has a mean of zero and variance $\sigma_\tau^2$.

Finally, we note that goods and labor markets clear in equilibrium and that the transversality condition regarding bond holdings is satisfied.

7.2 Model with entry costs specified in consumption units

Our baseline model is the one where firm entry cost is specified in terms of wages. Alternatively, firm entry cost can be specified in consumption units. The latter framework better captures the effects of monetary policy shocks (see Bilbiie et al. (2007)). In general we find that our results are robust to the introduction of a different kind of firm entry cost.

7.3 Translog preferences

In the baseline model there is no competition effect i.e. the elasticity of substitution (ES) among goods is fixed and does not vary with the number of firms (the case of CES preferences). Instead, IES can be chosen to vary with the number of firms with translong preferences (see below).

Bilbiie et al. (2007) show that the basic New Keynesian model produces countercyclical markup in response to productivity shocks when i) variety
effect operates through a CES aggregator \( \rho(N_t) = N_t^{1/(\theta - 1)} \) and \( \theta \) is the constant elasticity of substitution among goods, ii) aggregate labour supply is inelastic and iii) the coefficient on the output gap in the Taylor rule is zero.

However, we found that the markup is not counter-cyclical anymore under a CES aggregator if labour supply is endogenous and there is a positive coefficient in the Taylor-rule on the output gap. In order to maintain the countercyclical markup the model has to feature the competition effect which makes the elasticity of substitution among goods \( (\theta(N)) \) rise with the appearance of new entrants and, hence, new varieties \( (N) \). In the literature there are several ways to induce competition effect. In particular, Colciago and Etro (2010a) induce strategic interactions among firms through quantity competition a la Cournot while Colciago and Etro (2010b) consider competition in prices a la Bertrand. Furthermore, Colciago and Etro (2010b) compare the response of the markup to a temporary technology shock under Cournot, Bertrand and translog preferences and conclude that competition effect is the strongest under translog preferences as in Bilbiie et al. (2012). Therefore, the markup which is an inverse function of \( \theta \) declines with an increase in the number of varieties. The variety effect \( (\rho(N)) \) and the markup \( (\mu(N_t)) \) under translog preferences can be written as:

\[
\rho(N_t) = \exp \left( -\frac{1}{2} \frac{\tilde{N} - N_t}{\varsigma \tilde{N} N_t} \right), \quad \tilde{N} \equiv \text{Mass}(\Omega)
\]

\[
\mu(N_t) = 1 + \frac{1}{\varsigma N_t}
\]

where \( \varsigma \) is chosen such that the steady-state number of firms under the CES and translog case is the same.