Welfare effects of business cycles and monetary policies in a small open emerging economy

Jolan Mohimont*

February 15, 2018

Abstract

The measure of the cost of business cycles and the design of appropriate monetary policies received a lot of attention in advanced countries. However, studies in low and middle-income countries remains limited and the conclusions drawn for developed countries do not automatically apply because welfare measures and policy recommendations are likely to be affected by these countries specificities. This paper measures the cost associated to business cycle fluctuations in an estimated small open economy DSGE model tailored to an emerging economy and applied to South Africa. The model includes i) an interaction between labour market idiosyncratic risks and households imperfect integration in financial markets; ii) a commodity sector responsible for a large share of exports and iii) a significant exposure to the domestic and global business cycles. Moreover, this paper explores potential welfare gains from simple monetary policy rules to draw relevant policy recommendations.

JEL classifications: E3, E52, E32, C51

Keywords: Monetary Policy, Welfare, Emerging Markets, SOE, DSGE, Bayesian

---

*DeFiPP (CRED & CeReFiM)-University of Namur. Email to: jolan.mohimont@unamur.be. I am grateful to Romain Houssa, Christoph Gortz, John Fender, Kaushik Mitra and Pei Kuang for useful comments on this ongoing work.
1 Introduction

Business cycle fluctuations generate welfare loss to risk averse agents. To the extent that those fluctuations are relatively severe in developing and emerging economies, they could cause large welfare cost. Stabilizing macroeconomic fluctuations could therefore bring large welfare gains in these countries and should be a priority of policy makers.

The measure of the cost of business cycles and the design of appropriate monetary policies received a lot of attention in advanced countries. However, studies in low and middle-income countries remains limited and the conclusions drawn for developed countries do not automatically apply because welfare measures and policy recommendations are likely to be affected by these countries specificities. Abstracting from these specificities might lead to misguided policy recommendations which could generate substantial welfare losses.

This paper builds and estimates a small open economy New-Keynesian DSGE model tailored to an emerging economy and applied to South Africa. In this framework, it measures the cost associated to business cycle fluctuations and explores potential welfare gains from simple monetary policy rules to draw relevant policy recommendations.

New Keynesian DSGE models offer a natural framework for welfare and monetary policy analysis for various reasons. First, they incorporate a wide range of shocks - generating business cycle fluctuations - and frictions hampering a free optimal reallocation of resources. Second, in those models, monetary instruments can influence real variables at business cycle frequencies and can therefore have a role to play in fine-tuning the economy. Third, their micro foundations capture agents’ reactions to changes in policy, insulating them to the Lucas (1976) critique. Finally, their use of utility functions provides a consistent measure of policy evaluation based on agents’ welfare.

The model developed in this paper consists of two blocks: a domestic small open emerging economy (which is the primary focus of the paper) and a foreign economy (which captures the exposure of the domestic economy to development in the rest of the world and serves as a benchmark for comparison). The domestic economy builds on Adolfson et al. (2007) while the foreign economy draws on Smets and Wouters (2007). As such, the model includes a variety of aggregate shocks (such as domestic and foreign supply, demand and monetary shocks) as well as idiosyncratic risks (in the form of staggered contracts in the goods and labour markets).

The model is then extended in two dimensions that are particularly relevant to a typical small open emerging economy. First, it includes two categories of households in the domestic economy to capture key differences between financially included and excluded households. The later do not save or borrow and simply consume their entire labour income in every period as in Mankiw (2000). This feature captures the high relative volatility of aggregate consumption in emerging
countries and affects the efficiency of the traditional interest rate channel. In addition, it interacts with business cycle shocks and idiosyncratic labour market risks by hampering the ability of certain households to ensure against any uncertain outcome. In South Africa, only 70% of the population over 15 years has an account at a financial institution (compared to 94% in the US) and only 33% has some form of formal savings (54% in the US)\(^1\). Second, there are two different types of goods in the domestic and foreign economies: primary commodities and secondary products (e.g. Kose (2002)). Many developing and emerging countries are dependant on commodities. In South Africa, primary commodities represents about 45% of total exports\(^2\). Commodity prices, which are influenced by external factors, very volatile and highly persistent, could have strong welfare implications and cause particular challenges for domestic stabilisation policies.

The model is estimated with Bayesian methods on South African and US data. It is used to estimate the distribution of the model’s parameters and to identify a set of domestic and foreign shocks responsible for the business cycle fluctuations in South Africa. The identification of disturbances based on their origins is crucial since the monetary authority can exert a closer control on domestic than on foreign sources of fluctuations. It also clearly identifies shocks according to their nature, consisting of both demand and supply shocks. This distinction is key since it is well documented that supply shocks impose bitter trade off between monetary policy objectives than demand shocks.

In a first step, this paper measures the welfare cost of business cycles fluctuations in the emerging economy using a second order approximation of the model (Schmitt-Grohe and Uribe (2004)). Welfare cost of business cycles is defined along the lines of Lucas (1987) as the share of consumption that an agent would be ready to give up at every period to insulate the economy from all shocks and therefore eliminate aggregate fluctuations. This measure is provided for the two categories of domestic households and compared to the foreign economy serving as a benchmark. Confidence bands are constructed based on the distribution of estimated parameters. In addition, the relative importance of each business cycle shocks and frictions is evaluated by shutting down each of these channels one at a time.

In a second step, this paper explores potential welfare gains from alternative monetary policy rules. It focus on simple and implementable monetary rules (Schmitt-Grohe and Uribe (2007)). Those rules determine the response of policy variables as a function of a small number of easily observable macroeconomic indicators (such as inflation, output and exchange rate measures) and delivers uniqueness of the rational expectation equilibrium. As such, they include the rule advocated by Taylor (1993) as well as most modified versions proposed in the literature. Their

\(^1\)World Bank, Financial Inclusion Database, 2014.
\(^2\)UNCTAD, State of commodity dependence, 2014.
simplicity is of particular relevance for policy analysis in emerging countries and they have been widely used to describe monetary policy in South Africa (e.g. Ortiz and Sturzenegger (2007)). In addition, optimal simple rules often deliver a virtually identical level of welfare compared to the optimal Ramsey policy (e.g. Schmitt-Grohe and Uribe (2007) and Gali and Monacelli (2005)).

Since an accurate welfare measure heavily depends on an accurate description of households’ preferences and on the model complete characterisation of disturbances (a point noted for e.g. in Rotemberg and Woodford (1999)), this paper also describes the impact of alternative policy rules on the moments of key variables such as inflation, output and the exchange rate. These measures are less sensitive to approximations in the utility function but are of great importance to households and policy makers. In addition, those results allow to chose monetary policy rules based on considerations unrelated to welfare, such as central banks preference over inflation, output or exchange rate fluctuations.

This paper offers two contributions to the business cycle literature in emerging countries. First, it measures the relative cost of business cycle in an emerging economy compared to an advanced economy. Such measures have been proposed in Pallage and Robe (2003) and Houssa (2013) and suggest that the welfare cost of business cycles might be much larger in developing and emerging countries. This measure differs in the sense that it rely on a structural model whose parameters are estimated on a large set of observed variables. As argued by Otrok (2001), estimated models bring discipline to the choice of preferences and consumption process. In addition, the model developed in this paper captures key emerging markets characteristics allowing to consider their potential impact on the costs of business cycles. Second, it evaluates potential gains from simple monetary policy rules and draw policy recommendation. The inclusion of emerging countries specificities in the model and the focus on simple and implementable rules permits to design policies tailored to their specific needs. Hove et al. (2015) and Prasad and Zhang (2015) also considered optimal simple rules in model adapted to fit emerging countries specificities such as commodity exports or imperfect financial markets. In this paper, households labour market idiosyncratic risks interacts with these specificities and generate heterogeneity among financially excluded households. In addition, the inclusion of a large set of disturbances classified according to their nature and origin allows to consider both benefits and limitations for stabilisation policies. Moreover, the posterior distribution of parameters resulting from a Bayesian estimation is used to assess the robustness of policy recommendations to parameter uncertainty.
2 State of the art

2.1 Welfare measures in advanced economies

In a seminal work, Lucas (1987) measured welfare costs of aggregate fluctuations based on a specification of preferences and a time series representation of the aggregate consumption process. He assumed a standard CRRA utility function, a trend-stationary IID consumption process matching the mean and variance of US data and the existence of market where individuals can insure against idiosyncratic risks. He expressed a welfare loss as a fraction of consumption that agents would be willing to pay in order to avoid aggregate consumption fluctuations. Using US data over the 1947-2001 period, Lucas (2003) calculates that the welfare costs of business cycles would be as low as 0.05% of permanent consumption. The literature then produced different competing models whose welfare costs estimates range from minor to substantial in advanced countries.

Departures from Lucas assumptions include a variety of utility function and different process governing consumption flows. Obstfeld (1994) and Dolmas (1998) experiment with different consumption processes - including persistent and permanent disturbances - and with utility functions distinguishing between risk aversion and intertemporal elasticity of substitution - such as Epstein and Zin (1989) preferences (EZ) - leading to potentially larger welfare costs estimates. Storesletten et al. (2001) show that households consumption is persistent in US data, and that their variance is larger during aggregate downturns. They model aggregate TFP shocks that interact with heteroskedastic and persistent shocks to individual labour market productivity. They find that the costs associated to business cycle fluctuations are relatively large, even for moderate levels of risk aversion. Tallarini (2000) argues that, using EZ preferences in order to increase the risk aversion coefficient while holding the intertemporal elasticity of substitution constant, can improve the predictions of a stochastic growth model and also results in large welfare costs.

Other studies challenge agents homogeneity and the perfect market hypothesis allowing each individual households to insure against idiosyncratic risks. Imrohoruglu (1989) considers employment idiosyncratic risks together with imperfect credit markets in the form of liquidity constraints. Business cycles are characterised by good times (high employment probability) and bad times (low employment probability). When agents only have access to a simple storage technology (with no borrowing opportunities), business cycles generates costs that are four to five times larger compared to a perfect insurance market environment similar to Lucas assumption. However, when agents are allowed to borrow (at an exogenous rate set to 8%), the costs are substantially reduced and closer to Lucas experiment. In a similar framework, Atkeson and Phelan (1994)\(^3\) argue that

---

\(^3\)They consider wage and employment idiosyncratic risks and incomplete financial markets (where agents have access to risk-free bonds but no state-contingent assets) with endogenous interest rates.
counter-cyclical policies could directly reduce income risks faced by each individuals or simply reduce the correlation in earnings. In the later case, the only gain offered by stabilisation policies is to stabilise the interest rate (avoiding interest rate rises during downturns when more agents want to borrow). They find that welfare gains of such policies are extremely low, since interest rates fluctuations are small in US data. Building on these two studies, Krusell and Smith (1999) and Krusell et al. (2009) additionally consider unequal wealth distributions (generated using different discount factors), replicating the stylised fact that wealth is heavily concentrated in the US. In the presence of borrowing constraints, the welfare costs of business cycles are much larger for households with little wealth. Indeed, prolonged periods of unemployment quickly exhaust poor households wealth with large welfare effects. Eliminating business cycle fluctuations would generate substantial welfare gains for those households. In addition, in the stable environment, precautionary savings decrease, resulting in higher interest rates benefiting the very richest.

Lester et al. (2014) and Cho et al. (2015) extend the debate by arguing that macroeconomic volatility might have positive effects on welfare. Although households prefer smooth consumption plans, firms might use volatility by adjusting their use of inputs in good and bad times. Firms have the opportunity to use more inputs when those inputs have a large marginal productivity, which could increase their average productivity level. They show that in a RBC model, volatility can be welfare increasing, as long as the elasticity of factor supply is large. In addition, capital stock plays a key role in allowing households to substitute inter-temporally. These results indicate that welfare measures abstracting from production decisions might overestimate welfare costs of fluctuations by abstracting from this option effect. The literature also considers the potential impact of stabilisation policies on growth and their ability to "fill in the troughs without shaving off the peaks" leading to potentially different welfare estimates.

Discrepancies in welfare costs estimates and their dependence on the utility function call for a consistent treatment of its parameters. Otrok (2001) argues that an estimated business cycle model would bring discipline to the choice of preferences and consumption process. He evaluates welfare costs in a RBC model with parameters estimated on US consumption and investment data and finds that welfare costs are close to Lucas (1987) estimates. However, New Keynesian (NK) models usually attribute larger welfare costs to business cycle fluctuations. From the New-Keynesian perspective, recessions feature inefficient drops in output, below its natural level. In addition to

---

4Gomes et al. (2001) also find a similar results for TFP shocks in RBC model with search and matching in the labour market. In their model, workers have the option to accept or reject job offers and like fluctuations, since they have the option to reject bad offers resulting from employers low TFP draws.

5The impact of volatility on growth could be positive or negative (e.g. Jones et al. (1999) and Barlevy (2004b)).

6Stabilisation policies, by "filling in troughs without shaving off peaks" or by reducing the probability of extreme events such as the great depression, could have large beneficial effects by increasing the average level of consumption (e.g. DeLong and Summers (1988), Cohen (2000) and Chatterjee and Corbae (2007)).
aggregate consumption (and hours) fluctuations, those models generates additional welfare costs arising from prices (and wages) dispersions.

2.2 Stabilisation Policies in advanced economies

Considering the evidence reviewed in the previous subsection, it could be argued that business cycle fluctuations are relatively costly in economies facing persistent shocks, imperfect credit markets and supply-side rigidities; especially for liquidity-constrained households. It is therefore tempting to advocate for more aggressive stabilisation policies in order to offset those costs. However, benefits from stabilisation policies depends on the type of shocks responsible for macroeconomic fluctuations. Indeed, monetary and fiscal policies are well suited in order to stabilise fluctuations generated by demand (or nominal) shocks, but not in the case of supply (or real) shocks. In fact, although the gains from post-war stabilisation policies in the US was substantial, the literature has showed that they might be only minor gains from further stabilisation.7

Most advanced countries central banks recognise price stability as their primary objective. This is reflected in the gradual move, pioneered by New Zealand, towards a formal inflation targeting frameworks. The benefits, in term of welfare, of price stability is well recognised. Among other advantages, price stability ensures transparency of the price mechanism, reduces the inflation risk premia, prevent arbitrary redistribution of resources and ease contract formation. In addition, through its impact on aggregate demand, monetary policy can also influence output and employment. Some central banks such as the US Federal Reserve therefore explicitly aims at supporting activity. The objectives of monetary policy are therefore often expressed as price (or inflation) and output (or growth) stability and their volatility is a natural measures of monetary policy efficiency. Small open economies also often add exchange rate considerations to their policies which can be affected by monetary policy through the uncovered interest rate parity condition.

Estimated DSGE models have become the standard toolkit for macroeconomic policy analysis. Much is known about solving (e.g. Uhlig (1995); Klein (2000) and Sims (2002)) and estimating (e.g. DeJong et al. (2000); Schorfheide (2000) and Otrok (2001)) these models as well as what modelling features are most relevant for understanding observed macroeconomic fluctuations in these economies (e.g. Smets and Wouters (2007) and Adolfson et al. (2007)). Those models have also been used to devise optimal monetary (e.g. Gali and Monacelli (2005)) and fiscal policies (e.g. Ratto et al. (2009)). In fact, the central banks of most developed countries (e.g. the US Federal Reserve, the Bank of England, the European Central Bank) as well as policy institutions such as the International Monetary Fund all use estimated DSGE models for policy analysis.

7See for e.g. Barlevy (2004a) for a more detailed argumentation.
An early literature, mostly applied to the US, has produced various papers studying optimal monetary policy in simple models with price rigidities (e.g. Ireland (1997), Rotemberg and Woodford (1997, 1999) and Clarida et al. (1999)). These authors show that, since the Volcker era, the Fed adopted an implicit inflation targeting framework and successfully insulated the US economy from demand shocks, which increased welfare. This literature argues that central bank (credible) commitment to respond to inflation is enough for monetary policy to be efficient. Therefore, Taylor rules responding to inflation can stabilise the economy and backward-looking monetary policy rules are efficient as long as the private sector is forward-looking. However, these papers further argue that the Fed leaning against the wind policy was not optimal due to the identification of large supply disturbances. Optimal policy response to supply shocks requires the Fed to accommodate the change in output in order to stabilise prices.

The literature has built on those models by gradually increasing their complexity to gain in realism. Schmitt-Grohe and Uribe (2007) study optimal simple monetary and fiscal rules in a NK-DSGE model. The model includes price stickiness, capital accumulation and demand for money origination from firms’ working capital and households’ cash in advance constraints. They calibrate the model on US data and compare the welfare costs of alternative Taylor rules coefficients (with interest rate smoothing and inflation and output responses) relative to the time-invariant stochastic equilibrium allocation associated with the Ramsey policy. They find that welfare gains from increasing the value of the inflation coefficient beyond the Taylor principle and from interest rate smoothing are extremely low. In addition, responding to output generates a large welfare loss. Erceg et al. (2000) extend the baseline NK-DSGE model to consider price and wage stickiness. The volatility in aggregate wage inflation causes dispersion in individual wages, which generate inefficient fluctuations in individual hours worked. In this context, it is impossible for the central bank to stabilise the output-gap and price and wage inflation rates at the same time, imposing a trade-off to monetary policy. In addition, strict inflation targeting is found to be inefficient. Benigno and Woodford (2005) show that these results hold in the case of a distorted steady-state. However, Schmitt-Grohé and Uribe (2006) build a medium scale NK-DSGE model (including both capital and sticky wages) and find that the central bank should place more emphasis on price than on wage stability. The literature has also recently advocated for the inclusion of different types of agents in a context of imperfect financial markets (e.g. Bilbiie (2008), Nisticò (2016), Cúrdia and Woodford (2016) and Bilbiie and Ragot (2017)). These papers motivations include the impact of imperfect financial markets on the interest rate transmission channel; the role of monetary policy on financial stability; and the link between monetary policy and inequality.

The literature also offered results for small open economies. Gali and Monacelli (2005) develop a SOE version of the canonical NK-DSGE model. They find that a Taylor rule based on
domestic inflation is preferable (in term of welfare) to a similar Taylor rule based on CPI and to an exchange rate peg; and that the optimal rule implies *perfect* stabilisation of the domestic price index. Kollmann (2002) considers monopolistic distortions and finds that the optimal rule consists of a *strict* response to the domestic price index which tolerates large fluctuation in the exchange rate and minimal fluctuations in the price index. In addition, Leitemo and Soderstrom (2005) show that the gains of integrating the exchange rate in the Taylor rule are generally low using a variety of models. However, Engel (2011) brings the issue of currency misalignment (the fact that prices can differ between countries when compared in the same currency) in an otherwise simple framework. Violations of the low of one price occur due to local-currency pricing and are inefficient. If prices are sticky in the importer’s currency, then targeting CPI delivers the optimal policy.

### 2.3 Welfare effects of business cycles in developing and emerging economies

Business cycles properties in emerging and developing economies are well documented. Studies such as Agenor et al. (2000), Rand and Tarp (2002), Neumeyer and Perri (2005), Aguiar and Gopinath (2007) and Male (2010) highlight some stylised facts common to those countries. Two of these are particularly relevant to welfare analysis. First, macroeconomic variables such as output and inflation are more volatile in these countries than in advanced economies, on average. Second, in contrast to advanced economies, consumption tend to be more volatile than output.

Developing and emerging countries experience sharper consumption fluctuations provoked by larger domestic and foreign shocks exacerbated by weaker resilience. Domestic shocks comprise volatile fiscal or monetary policies as well as shocks in the production sector such as extreme weather conditions. External shocks include sudden stops of capital inflows or abrupt changes in their terms of trade. The relative importance of foreign shocks in developing and emerging countries ranges from small but economically meaningful (e.g. Raddatz (2007)) to important (e.g. Houssa et al. (2015)). However, those papers reach one similar conclusion: among foreign shocks, commodities are the most import driver of macroeconomic fluctuations in those countries. The high volatility and persistence of commodity prices generate large term of trade and exchange rate fluctuations (exchange rate fluctuations are on average three times larger in developing countries) which eventually affect prices and output. In addition, their capacity to cope with different types of shocks is limited by lower international risk-sharing opportunities, less efficient stabilisations policies and heavier microeconomic regulations (Loayza et al. (2007)). Their financial sectors are incomplete and therefore do not offer enough instruments to adequately insure against all shocks and additionally tend to dry up in times of crisis. Heavier microeconomic regulations tend to

---

8Recently, there has been a growing number of studies assessing the role of term of trade shocks (e.g. Schmitt-Grohé and Uribe (2015), Fernandez et al. (2015), Drechsel and Tenreyro (2017), and Fernández et al. (2017)).
hamper the reallocation of resources from low to high productive firms/sectors required to cope with shocks.

These stylised facts indicate that the costs associated to business cycle fluctuations might be more important than the estimates provided for advanced countries. Pallage and Robe (2003) compared the costs of business cycles in advanced and developing countries in different environments (using different consumption processes and contrasting CRRA with EZ utility functions). They find that the cost of business cycle fluctuations ranges from 10 to 30 times their estimated values for the United States. Houssa (2013) estimates those costs for developed and developing countries with Bayesian methods accounting for parameter uncertainty. He finds that those costs are on average two to four times larger in developing countries, that they compare with a 1% increase in long-term growth for many developing countries and that oil producing and politically unstable countries would benefit the most from successful stabilising policies.

Reviewing these evidences shows that business cycles might be relatively more costly in developing and emerging countries. In addition, those empirical measures might actually represent a lower bound for the relative cost of business fluctuations. Indeed, these papers have abstracted from agents heterogeneity and imperfect markets. Considering the fact that more households are excluded from financial markets in those countries, the relative costs of business cycles might actually be higher. In addition, these measures abstracted from production decisions. The succession of "good" and "bad times" allow firms to transfer inputs from the bad to the good environment. As long as the elasticity of factor supply is large, firms can successfully transfer resources from environment. However, if heavier microeconomic regulations in low and middle income countries prevent these reallocations, these potential benefits from macroeconomic fluctuations could be lower these countries.

2.4 Stabilisation policies for developing and emerging economies

These evidences call for the study of stabilisation policies in emerging and developing countries. Indeed, in contrast to advanced countries, there could still be much room for improving macroeconomic management. The evaluation of alternative policies requires a structural model adapted to those countries. The literature has initially used the exact same models proposed for advanced economies with few account for their specificities. However, it has also proposed some ingredients to adapt those models. These specificities include a dependence on commodity prices (e.g. Mendoza (1995)) and foreign intermediate inputs (e.g. Kose (2002)) as well as non-Ricardian households excluded from financial markets (e.g. Medina and Soto (2007)).

Monetary policy analysis in developing and emerging countries relies on models initially tai-
lored to advanced economies that are extended in order to take some of their specificities into account. One of the most important extension is the introduction of a commodity sector. Hove et al. (2015) study optimal monetary policy response to exogenous term of trade shocks in a small scale SOE-DSGE model with tradables and non-tradables. They calibrate their model to the South African economy and approximate welfare using loss functions with different weights on the variance of inflation, output-gap, interest rates and exchange rates. They find that a CPI inflation targeting performs better than non-traded inflation targeting. In addition, they find that exchange rate targeting has a detrimental aggregate welfare effect. Prasad and Zhang (2015) include heterogeneous households and incomplete financial markets in a two-sector (tradable and non-tradeable goods) SOE-DSGE model. Prices are sticky in the non-tradeable sector and flexible in the tradable sector. Workers in the non-tradeable sector are hand to mouth consumers. They calibrate the model to an emerging economy and also find that exchange rate targeting has a detrimental aggregate welfare impact. However, in the case of a positive productivity shock in the home tradeable goods sector (which implies an appreciation), households in this sector fare better under exchange rate management in the short run.

Stabilisation policies based on Taylor rules require a choice of the targeted price index. In some developing and emerging countries, this choice is particularly important since imported goods prices display stronger fluctuations driven by volatile exchange rates. Devereux et al. (2006) construct a two sectors SOE model representing an emerging economy. They introduce imperfect exchange rate pass-through and lending constraints on external credit for investment (labelled in foreign currency) in an otherwise basic NK model. They find that the optimal monetary rule depends on the degree of exchange rate pass-through. In a high pass-through environment, the central bank should target the domestic non-traded good price index, while in the low pass-through case, it is preferable to target the consumer price index. Other considerations specific to developing and emerging countries relate to the volatility of food prices and the management of volatile aid flows.

___

9. Anand et al. (2015) study the optimal inflation measure that a developing country central bank should target. They build a three sectors model: an informal food sector, a formal non-traded good sector and a formal traded-good sector (capturing the role of commodities). In a context of credit-constrained households primarily working in the food sector and spending a large portion of their income in food, they find that it is preferable to target headline inflation than core inflation. The optimal index includes food but excludes imported goods.

10. Adam et al. (2009) build a two sectors DSGE model with sticky prices in the non-traded sector, currency substitution and volatile aid flows. Monetary policy operates through foreign exchange reserves management and government securities transactions with the private sector. When increased aid flows translate into lower domestic financing requirements (i.e. when all aid is not immediately spent), the decrease in domestic money supply encourages domestic households to substitute foreign currency into domestic currency. This generate an appreciation which has destabilising impacts for both prices and output. These can be mitigated with net foreign assets accumulation at the central bank as a response to appreciation.
In practice, emerging and developing countries tend to respond to exchange rate fluctuations by "leaning against the wind", supporting the "fear of floating hypothesis" (e.g. Calvo and Reinhart (2002) and Mohanty and Klau (2004)). Even in countries officially operating under floating regimes, central banks often react to changes in the exchange rate. Their motivations include currency mismatches in balance sheets, the pass-through of exchange rate into prices, the impact of the exchange rate on net exports and large nominal or risk premium shocks, among others. Hausmann et al. (2001) find that countries with limited ability to borrow in domestic currency tend to tolerate less volatility in their exchange rate (relative to their interventions in foreign reserves and interest rates).

In particular, the South African Reserve Bank (SARB) introduced a formal inflation targeting framework in 2000. It targets an average rate of consumer price inflation between 3 and 6%. Prior to this period, Ortiz and Sturzenegger (2007) estimate a Taylor rule in a NK-DSGE model. They find that the South African Reserve Bank responded consistently to inflation fluctuations and attached a larger weight on output, but a lower on the exchange rate when compared to other emerging economies. In a similar exercise, Peters (2016) finds that the inflation coefficient is positive and significant, while the output gap and exchange rate coefficients are also positive but insignificant.

Considering this variety of practice in the conduct of monetary policy and the results from the literature, it would be interesting to examine the impact of monetary policy on the economy and on agents welfare. Although the benefits from inflation targeting are relatively well established, it is important to study the desirability of targeting a particular price index (e.g. consumer vs domestic price index). It would also be interesting to consider the role of output and exchange rate management as well as the desirability of more exotic policies such as a response to commodity price fluctuations.
3 Model

The model consists of two blocks: a domestic small open emerging economy and the rest of the world. The domestic block extends the SOE-DSGE model proposed by Adolfson et al. (2007), that build on the closed economy model originally developed by Christiano et al. (2005). It incorporates key emerging countries structural characteristics that are particularly relevant to the South African economy. These include: i) two categories of households to capture key differences between financially included and excluded households and ii) two different types of goods to account for the specific role of commodities in exports.

The working of the enlarged model is summarized as follows. The economy is populated by two types of households: savers and rule of thumbs. Households derive utility from consumption of a composite good (consisting of both domestic and imported goods) and leisure. Consumption preferences are subject to habit formation.

Savers accumulate wealth in the form of risk free domestic and foreign bonds and capital. Each of them supplies monopolistically a differentiated labour service and sets its own wage. Wages are sticky à-la Calvo (1983). They build capital which is sector specific and subject to investment adjustment costs. The investment basket is a composite of domestic and imported inputs.

Rule of thumb households are excluded from financial markets and unable to accumulate wealth. They consume their entire income in each period. They do not optimize but rather follow savers by setting the same wage whenever they receive a random signal.

There are two sorts of good: commodities and secondary products. Commodity goods are homogeneous and produced under perfect competition. Commodity producers combine capital, labour and land and sell their products in the world market.

Secondary goods are produced in a perfectly competitive environment (but distributed by firms enjoying market power). Secondary good producers combine capital and labour to produce an undifferentiated intermediate goods.

Distributors operate in three markets: domestic, import and export. The distribution sector is composed of intermediate and aggregating firms. Domestic distributors turn secondary goods into consumption and investment differentiated goods sold to households. Importing distributors turn foreign secondary goods into foreign consumption and investment goods sold to domestic households. Exporting distributors sell domestic secondary goods to foreign households. Each intermediate distributor is a monopoly supplier of its specific product and follows a price adjustment rule à-la Calvo (1983). Nominal rigidities in importing and exporting sectors allow for short-run incomplete exchange rate pass-through to both import and export prices\textsuperscript{11}.

\textsuperscript{11}Compared to the traded/non traded good literature, this framework offers more flexibility by incorporating one
The government collects pay-roll, labour income, consumption and capital gain taxes. It follows a simple rule that determines its level of public consumption. The central bank sets the interest rate which depends on inflation, output growth and the change in exchange rate.

The rest of the world is a closed version of the domestic economy based on Smets and Wouters (2007). It is extended with a commodity sector. Commodity prices are endogenously determined in the foreign economy bloc assuming that the small open domestic economy has no impact on commodity prices. The following subsections describe the model in details.

3.1 Households

The economy is populated by two types of households: savers and rule of thumb consumers. Any \( j \)th household attains utility from consumption \( C_j, t \) and dis-utility from labour efforts \( h_j, t \). Its lifetime utility is given by

\[
E_0^{\beta} \sum_{t=0}^{\infty} \beta^t \left[ \left( \frac{C_j, t}{\left( \bar{C}_{t-1} \right)^{1/\sigma_c}} \right)^{1-\sigma_c} - 1 \right] - A_h \left( \frac{h_j, t}{1 + \sigma_h} \right)^{1+\sigma_h},
\]

(3.1)

where \( E \) is the expectation operator and \( \beta \) is the discount factor. The parameters \( \sigma_c \) and \( \sigma_h \) denote the inverse of the inter-temporal elasticity of substitution for consumption and the inverse of the elasticity of work effort, respectively; \( A_h \) is the relative importance of labour in the utility; \( b \) an external multiplicative habits parameter\(^{12} \) and \( \bar{C}_{t-1} \) is the lagged average value of consumption of a reference group.

Households derive utility from the consumption of a composite good (consisting of both domestic and imported goods). Consumption \( C_j, t \) for any household \( j \) is given by the CES index of domestic and imported goods

\[
C_{j, t} = \left[ \left( 1 - \omega_c \right)^{1/\eta_c} \left( C_{j, t}^d \right)^{(\eta_c - 1)/\eta_c} + \left( \omega_c \right)^{1/\eta_c} \left( C_{j, t}^m \right)^{(\eta_c - 1)/\eta_c} \right]^{\eta_c / (\eta_c - 1)},
\]

(3.2)

where \( C_{j, t}^d \) and \( C_{j, t}^m \) denote consumption of the domestic and imported good, respectively, \( \omega_c \) is the steady-state share of imports in consumption, and \( \eta_c \) is the elasticity of substitution between domestic and foreign consumption goods.

---

\[ \text{non traded good: the final good sold by domestic distributors; and } \text{two traded goods: commodities (produced in a perfectly competitive environment) and final goods (sold by importing/exporting distributors). The traded and non traded final goods have different Phillips Curves.} \]

\(^{12}\)Multiplicative habits were introduced by Abel (1990) and Gali (1994).
3.1.1 Savers

There is a continuum of savers indexed by $j \in (0,1)$. The representative saver maximizes the inter-temporal utility by choosing her consumption and investment levels, as well as domestic and foreign bond holdings.\textsuperscript{13}

For any given period $t$, savers face the same budget constraint which is given, in nominal terms, by

$$B_{j,t+1} + S_t B^*_{j,t+1} + (1 + \tau^c) P^c_t C_{j,t} + P^i_t \left(I^p_t + I^f_t\right) = \left(1 - \tau^k\right) \left(R_t^{k,p} \tilde{R}^p_{j,t} + R_t^{k,f} \tilde{R}^f_{j,t}\right) + \frac{1 - \tau^v}{1 + \tau^w} W_t h_{j,t} + TR^s_t + SCS_{j,t} + \varepsilon_{b,t-1} R_{t-1} B_{j,t} + \varepsilon_{b,t-1} R_{t-1}^* \Phi(A_{t-1}, \tilde{\phi}_{t-1}) S_t B^*_{j,t} \tag{3.3}$$

where the subscript $j$ indicator denotes the household’s choice variables, whereas the upper-case variables, without the subscript, are the economy-wide aggregates. $B_t$ denotes the value of nominal domestic bonds, $S_t$ is the nominal exchange rate representing the amount of local currency per unit of foreign currency and $B^*_t$ the value of foreign bonds (in foreign currency). $R_t$ and $R^*_t$ are gross interest rates on domestic and foreign bonds, respectively. The exogenous process $\varepsilon_{b,t-1}$ creates a wedge between policy and private interest rates. $P^c_t$ is the consumer price index and $P^i_t$ the investment good price index. Households invest $I^f_t$ in private capital $\tilde{R}^f_t$ used in the secondary sector and $I^p_t$ in private capital $\tilde{R}^p_t$ used in the primary sector. $R^{k,p}_t$ and $R^{k,f}_t$ are the returns of capital in the primary and final sectors, respectively. The term $W_t$ is the wage rate, $TR^s_t$ are transfers from the government and the firms, $SCS_{j,t}$ is the household’s net cash income from participating in state contingent securities at time $t$. The government collects various taxes: $\tau^c$ is consumption tax, $\tau^w$ is pay-roll tax, $\tau^v$ is labour-income tax, $\tau^k$ is capital-income tax.

Country risk premium In equation (3.3) the term $R^*_{t-1} \Phi(A_{t-1}, \tilde{\phi}_{t-1})$ represents the risk-adjusted gross interest rate paid by foreign bonds (in foreign currency). The function $\Phi(.)$ captures the country risk premium function of the real aggregate net foreign asset position $A_t \equiv \frac{S_t B^*_{t+1}}{P_t}$ and a time varying shock to the risk premium $\tilde{\phi}_t$.\textsuperscript{14}

This function illustrates the imperfect integration in the international financial markets of the domestic economy and induces stationarity of the model.\textsuperscript{15} Therefore, domestic households are

\textsuperscript{13} The domestic financial markets are assumed to be complete, thus each financially included household can insure against any type of idiosyncratic risk through the purchase of the appropriate portfolio of state contingent securities. This prevents the frictions from causing these households to become heterogeneous, so the representative agent framework is still valid for this type of households.

\textsuperscript{14} The function $\Phi(A_t, \tilde{\phi}_t) = \exp(-\tilde{\phi}_t (A_t - \bar{A}) + \tilde{\phi}_t)$ is strictly decreasing in $A_t$ and satisfies $\Phi(\bar{A},0) = 1$. In particular, Adolfson et al. (2007) set $\bar{A} = 0$.

\textsuperscript{15} see Schmitt-Grohé (2003).
charged a premium over the (exogenous) foreign interest rate $R^*_t$ if the domestic economy is a net borrower ($B^*_t < 0$), and receive a lower remuneration on their savings if the domestic economy is a net lender ($B^*_t > 0$).

**Capital accumulation**  Capital and investment are sector specific. The capital accumulation rule is subject to investment adjustment costs and follows

$$\tilde{K}^q_{t+1} = (1 - \delta)\tilde{K}^q_t + Y_t F(I_t^q, I_{t-1}^q),$$  
(3.4)

where $q \in (p, f)$ represents the primary or secondary sector and $\delta$ is the depreciation rate. The variable $Y_t$ is a stationary investment-specific technology shock common to both sectors and $F(I_t, I_{t-1})$ represents a function which turns investment into physical capital. The $F(I_t, I_{t-1})$ function is specified following Christiano et al. (2005) as:

$$F(I_t, I_{t-1}) = (1 - \tilde{S}(I_t/I_{t-1}))I_t,$$  
(3.5)

where the function $\tilde{S}(I_t/I_{t-1})$ is defined by

$$\tilde{S}(I_t/I_{t-1}) = \phi^i 3 \left\{ \exp \left( \frac{I_t}{I_{t-1}} - 1 \right) + \exp \left( - \frac{I_t}{I_{t-1}} + 1 \right) - 2 \right\},$$  
(3.6)

with $\tilde{S}(1) = \tilde{S}'(1) = 0$ and $\tilde{S}''(1) \equiv \tilde{S}'' = 2\phi^i > 0$.

**Investment basket**  The investment good ($I_t^q$) with $q \in (p, f)$ is given by a CES aggregate of domestic ($I_t^{d,q}$) and imported investment inputs ($I_t^{m,q}$)

$$I_t^q = \left[ (1 - \omega_i)^{1/\eta_i} (I_t^{d,q})^{(\eta_i - 1)/\eta_i} + (\omega_i)^{1/\eta_i} (I_t^{m,q})^{(\eta_i - 1)/\eta_i} \right]^{\eta_i/(\eta_i - 1)},$$  
(3.7)

where $\omega_i$ is the steady-state share of imports in investment and $\eta_i$ is the elasticity of substitution between domestic and imported investment goods.

**Wage setting**  Every household is a monopoly supplier of a differentiated labour service and sets its own wage $W_{j,t}$ with an adjustment rule à la Calvo. Households have a probability $(1 - \xi_w)$ to be allowed to re-optimize their wages. Households that cannot re-optimize their wage follow an indexation mechanism described by

$$W_{j,t+1} = (\pi_t^c)^{\kappa_w} (\bar{\pi}_{W,j+1}^{\kappa_w})^{1-\kappa_w} W_{j,t},$$
such that they index their wages to last period consumer price inflation $\pi_t^c = \frac{P_t^c}{P_{t-1}^c}$ and to the inflation target $\bar{\pi}$. The wage-indexation parameter $\kappa_w$ determines the relative importance of past consumer price inflation in the indexation process. The exogenous process $\epsilon_{w,t+1}$ is a wage-indexation shock.

Each household sells its labour $(h_{j,t})$ to a labour union which transforms it into a homogeneous labour input $H^S_t$ using the following production function:

$$H^S_t = \left[ \int_0^1 (h_{j,t})^{\epsilon_{w-1}} \frac{d j}{\epsilon_{w-1}} \right]^{\epsilon_w}, \quad 1 < \epsilon_w < \infty, \quad (3.8)$$

where $\epsilon_w$ is a labour demand elasticity. This labour union takes the input price of the $j^{th}$ differentiated labour input as given, as well as the price of the homogeneous labour services. Consequently, the relative labour demand for labour type $j$ is given by

$$h_{j,t} = \left( \frac{W_{j,t}}{W_t} \right)^{-\epsilon_w} H^S_t \quad (3.9)$$

where $W_{j,t}$ is household $j$ wage rate and $W_t$ is the wage index defined by

$$W_t = \left[ \int_0^1 W_{j,t}^{1-\epsilon_w} d j \right]^{1/{1-\epsilon_w}} \quad (3.10)$$

**Savers aggregate utility** The aggregate utility level for savers is given by

$$U^S_t = \int_0^1 \left[ \left( C^S_{j,t}/(C_{t-1}^S)^b \right)^{1-\sigma_c} - 1 \right] \left[ A_h (h_{j,t})^{1+\sigma_h} \right] \frac{d j}{1-\sigma_c} \quad (3.11)$$

Their access to complete domestic financial markets (implying that all savers have the same level of consumption at each period) together with equation (3.9), implies that

$$U^S_t = \frac{(C^S_t/(C_{t-1}^S)^b)^{1-\sigma_c} - 1}{1-\sigma_c} - \frac{A_h (H^S_t)^{1+\sigma_h}}{1+\sigma_h} \nu^w_t \quad (3.12)$$

where

$$\nu^w_t = \int_0^1 \left( \frac{W_{j,t}}{W_t} \right)^{-\epsilon_w(1+\sigma_l)} d j \quad (3.13)$$

where the term $\nu^w_t \geq 1$ is a measure of wage dispersion which generates a waste of labour resources resulting in a utility loss.
3.1.2 Rule of thumb households

There is a continuum of rule of thumb households of mass 1 indexed by \( j \in (0, 1) \) with preferences given by (3.1). These households do not have access to credit and capital markets. They consume their entire labour income in every period. Their budget constraint is given by

\[
(1 + \tau^c) P_t^c C_{j,t} = \frac{1 - \tau^y}{1 + \tau^w} W_{j,t} h_{j,t} + TR^r_{j,t},
\]

where \( TR^r \) are government transfers. This specification relates to the hand-to-mouth consumers considered in Prasad and Zhang (2015) that study optimal monetary policy in an emerging economy, but differs since it allows for heterogeneity in households wages.

**Wage setting** Those households mimic savers in setting their wage. Rule of thumbs that are allowed to reset their wages set them to the level of re-optimising savers. Others simply index their wages following equation (3.10).

**Rule of thumb’s aggregate utility** These households have no access to the financial market and have therefore no opportunity to insure against labour market idiosyncratic risks. Their aggregate level of utility is therefore given by

\[
U^R_t = \int_0^1 \left[ \left( \frac{C_{j,t}}{(C^R_{t-1})^b} \right)^{1 - \sigma_c} - A_{h,r} \left( \frac{h_{j,t}}{1 + \sigma_h} \right)^{1 + \sigma_h} \right] dj.
\]

which, using equations (3.9) and (3.14), simplifies to

\[
U^R_t = \frac{(C^R_t / (C^R_{t-1})^b)}{1 - \sigma_c} \psi_t^c - A_{h,r} \left( H^R_t \right)^{1 + \sigma_h} \psi_t^w
\]

where

\[
\psi_t^c = \int_0^1 \left( \frac{W_{j,t}}{W_t} \right)^{(1 - \epsilon_w)(1 - \sigma_c)} dj
\]

where the term \( \psi_t^c \) is a measure of consumption dispersion generated by (uninsured) labour income dispersion. This dispersion generates a loss to risk-averse individuals. In addition, the term \( \psi_t^w \geq 1 \) also captures the waste in labour resources presented in equation (3.13).

---

\(^{16}\)Rule of thumb households were introduced in Coenen and Straub (2005), Erceg et al. (2006) and Galí et al. (2007). These types of households have been introduced in DSGE models applied to developing countries by Medina and Soto (2007) and Céspedes et al. (2012).
3.1.3 Labour Unions

Labour unions dispatch the workers (savers and rule of thumb households) to the firms.

**Hours aggregate** Hours worked by savers or rule of thumb households are perfect substitutes. Therefore, the aggregate labour effort available to the unions is simply given by

\[ H_t = H_t^S + H_t^R \] (3.18)

**Labour mobility** Labour mobility is imperfect similarly to Horvath (2000) and Dagher et al. (2010). Labour unions allocate labour between primary and secondary sectors. Total labour effort is given by a CES aggregation of hours worked in the primary and secondary sectors:

\[ H_t = \left[ (1 - \omega_h)^{-1/\eta_h} (H_t^f)^{(1+\eta_h)/\eta_h} + \omega_h^{-1/\eta_h} (H_t^p)^{(1+\eta_h)/\eta_h} \right]^{\eta_h/(1+\eta_h)}, \] (3.19)

where \( H_t^p \) and \( H_t^f \) denote labour effort in the primary and final sectors, respectively, \( \omega_h \) is the steady-state share of primary sector employment in total employment, and \( \eta_h \) is the elasticity of substitution between labour services provided in the two sectors. The intuition behind this specification is that there are costs associated to labour mobility such as sector specific skills.

3.2 Firms

There are two categories of goods in this model: primary commodities and secondary products.

3.2.1 Commodity sector

Primary commodities are produced in the domestic and foreign blocks. While the rest of the model section focuses exclusively on the domestic economy, this subsection also describes the commodity market in the foreign economy in order to highlight how commodity prices are endogenously determined in the world market.

**Domestic commodity producers** The commodity good is produced under perfect competition in the domestic economy. Firms combine capital, labour and land to produce a commodity input \( Y_t^p \) and sell their production on the world market. The production function for primary producers
is given by

\[ Y_p^t = Y_0^p e_t^p \left( \frac{e_{k,t} K_t^p}{K_0^p} \right)^{\alpha_p} \left( L_0^p \right)^{\beta_p} \left( \frac{H_t^p}{H_0^p} \right)^{(1-\alpha_p-\beta_p)}, \]  

(3.20)

where the terms \( Y_0^p, K_0^p, L_0^p \) and \( H_0^p \) are normalising constants and only represent choices of units. The term \( e_t^p \) is a pure commodity supply shock; \( e_{k,t} \) is a capital augmenting technology shock common to the primary and secondary sectors, \( K_t^p \) is capital used in the mining sector, \( L_0^p \) is a fixed stock of land and \( H_t^p \) labour services. The income shares of capital and land are given by \( \alpha_p \) and \( \beta_p \), respectively.

The commodity sector captures the impact of world commodity price on the domestic economy. Previous studies include for example Medina and Soto (2007) and Céspedes et al. (2012). This model departs from those works by assuming that commodity production is not exogenously given and require some inputs. With this respect, this specification is very similar to Kose (2002) who studies primary sector price shocks in a RBC model. It also relates to Hove et al. (2015) which examine monetary policy responses to commodity price fluctuations in a small calibrated NK DSGE model.

**Foreign commodity market**  
World commodity price \( P_t^* \) is determined endogenously through the confrontation of the foreign supply and foreign demand for commodities. Foreign commodity supply is modelled as an exogenous AR(1) process

\[ Y_t^p = \rho Y_{t-1}^p + \varepsilon_{p,t}, \]  

(3.21)

where \( \varepsilon_{p,t} \) is the foreign commodity supply shock which is assumed to be an IID process.\(^\text{17}\)

The foreign demand for commodity is determined by the foreign secondary good sector where it served as inputs. Two steps are involved in the production of foreign secondary goods. In the first step, foreign firms combine capital and labour to produce a foreign intermediate using a Cobb-Douglas technology

\[ N_t^* = N_0^* \left( \frac{e_{k,t} K_t^*}{K_0^*} \right)^{\alpha^*} \left( \frac{H_t^*}{H_0^*} \right)^{(1-\alpha^*)}, \]  

(3.22)

where \( H_t^* \) is hours worked, \( K_t^* \) is capital and \( e_{k,t}^* \) is a capital efficiency shock. In the second step, foreign firms combine this intermediate good with their demand for commodities to obtain

\(^{17}\)This shock could be also interpreted as pure commodity price shocks hitting the world commodity price unrelated to fluctuations in world commodity prices that stem from world commodity demand.
secondary foreign goods

\[ Y_t^* = Y_0^* \left[ \beta^* \left( \frac{Y_t^{p*}}{Y_0^{p*}} \right)^{\sigma_p^{p-1}} + (1 - \beta^*) \left( \frac{N_t^*}{N_0^*} \right)^{\sigma_p^{p-1}} \right]^{\sigma_p^*}, \quad (3.23) \]

where \( Y_0^{p*} \) and \( N_0^* \) are normalizing constants; \( \beta^* \) is the (income) share of commodities in foreign secondary goods sector; and \( \sigma_p^* \) is the elasticity of substitution between commodities and foreign intermediate good. Equation (3.23) shows how foreign (supply or demand) shocks could be transmitted to the domestic economy through commodity prices. Typically, whenever foreign firms want to increase production, demand for commodity increases which is transmitted to commodity prices. The elasticity \( \sigma_p^* \) is a key parameter that determines the strength of commodity price responses to changes in foreign demand for commodity.

### 3.2.2 Secondary sector

Domestic and foreign secondary goods are used for domestic and foreign consumption and investment as imperfect substitutes. Imperfect competition is introduced in three steeps: i) production of an un-differentiated secondary good, ii) its differentiation with brand-naming technology and finally iii) its aggregation into a consumption or investment good. Step one is performed by secondary good producers while steps two and three depends on intermediate and final distributors operating in the domestic, import and export markets.

**Secondary good producers** The secondary good is produced under perfect competition. Firms use capital \( K^f \) and hire labour \( H^f \) to produce an undifferentiated secondary good denoted \( Y^f \). The production function for the secondary good is given by

\[ Y_t^f = Y_0^f \left( \frac{\varepsilon_{k^f}K_t^f}{K_0^f} \right)^{\alpha_f} \left( \frac{H_t^f}{H_0^f} \right)^{(1-\alpha_f)}, \quad (3.24) \]

where the terms \( Y_0^f, K_0^f \) and \( H_0^f \) are normalising constants and only represent choices of units. The term \( \varepsilon_{k^f} \) represents a capital augmenting technology shock common to the primary and secondary sectors and \( \alpha_f \) is the capital income share.

**Domestic distributors** There are two types of domestic distributors: intermediate and final. There is a continuum of intermediate distributors, indexed by \( i \in [0,1] \). Each intermediate distributor buys a homogeneous secondary good \( Y^f \); turns it into a differentiated intermediate good
(using a brand naming technology) and then sells it to a final distributor at price $P_{i,t}$. Every intermediate distributor is assumed to be a price taker in the secondary good markets (it purchases secondary goods at their marginal costs) and a monopoly supplier of its own variety (it sets its own price).

The intermediate distributor follows a price adjustment rule à-la Calvo (1983). Every period $t$, with probability $(1 - \xi_d)$, any intermediate distributor $i$ is allowed to re-optimize its price by choosing the optimal price $P_{i,new}$. With probability $\xi_d$, it cannot re-optimize, and it simply indexes its price for period $t + 1$ according to the following rule:

$$P_{i,t+1} = (\pi_t)^{\kappa_d} (\epsilon_{\pi,t+1} \pi)^{1-\kappa_d} P_t,$$

where $\pi_t = \frac{P_t}{P_{t-1}}$ is last period’s inflation, $\pi$ is the inflation target; $\kappa_d$ is an indexation parameter and $\epsilon_{\pi,t+1}$ is a shock to the price indexation process.

The final distributor is an aggregator which uses a continuum of differentiated intermediate goods to produce the final homogeneous good, which is then used for consumption and investment by domestic households and sold at price $P_t$. The final distributor is assumed to have the following CES production function:

$$J_d^t = \left[ \int_0^1 (J_i^d)^{\frac{\varepsilon_d-1}{\varepsilon_d}} d\varepsilon \right]^{\frac{\varepsilon_d}{\varepsilon_d-1}}, \quad 1 < \varepsilon_d,$$

where $J \in (C,I)$ refers to the consumption or investment good and $\varepsilon_d$ is the elasticity of substitution between intermediate inputs. Consequently, the relative demand for input type $i$ is given by

$$J_{d,i}^t = \left( \frac{P_{i,t}}{P_t} \right)^{-\varepsilon_d} J_{i}^d$$

where $P_t$ is the price index defined by

$$P_t = \left[ \int_0^1 P_i^{1-\varepsilon_d} di \right]^{1-\varepsilon_d}$$

Exporting distributors  The intermediate exporting firm buys a homogeneous domestic good $Y^f$ to domestic secondary producers, turns them into a type specific differentiated good using a brand naming technology and then sells it in the foreign market to an aggregator at price $P_{i,t}^x$ expressed in foreign currency.

---

18 Since all distributors allowed to reset their prices are virtually identical and will always choose the same price the index $i$ is dropped to simplify the notation.
Domestic intermediate exporting firms follow a Calvo price setting rule and can optimally change their price only when they receive a random signal. In any period \( t \), each exporting firm has a probability \( (1 - \xi_x) \) to re-optimize its price by choosing \( P_{x, new,t} \). With probability \( \xi_x \) the importing firm cannot re-optimize at time \( t \) and, instead, it indexes its price according to the following scheme: \( P_{x,t}^{x+1} = (\pi_x^t)^{\kappa_x} (\bar{P})^{1-\kappa_x} P_x^t \) where \( \pi_x^t = \frac{P_x^t}{\bar{P}_{t-1}} \). This foreign currency price stickiness assumption implies short run incomplete exchange rate pass-through to the export price.

The aggregator produces final exported consumption and investment goods sold at price \( P_x^t \) to foreign households. The final, composite, exported good aggregates a continuum of \( i \) differentiated exported goods, each supplied by a different firm, according to

\[
X^f_t = \left[ \int_0^1 (\bar{X}_{i,t})^{\varepsilon_{x,i}-1} \varepsilon_{x,i} \right]^{\varepsilon_{x}} d\varepsilon_{x}, \quad 1 < \varepsilon_{x}. \tag{3.28}
\]

where \( \varepsilon_{x} \) is the elasticity of substitution between intermediate inputs.

Assuming that aggregate foreign consumption and investment follow CES functions, foreign demand for the aggregate final exported good is defined by

\[
X^f_t = \left( \frac{P_x^t}{P^*_t} \right)^{-\eta_f} X^*_t, \tag{3.29}
\]

where \( P^*_t \) is the price of the foreign good in foreign currency, \( P_x^t \) is the export price (denominated in export market currency) and \( X^*_t \) is foreign demand function of foreign consumption and investment. The coefficient \( \eta_f \) is the foreign elasticity of substitution between foreign and domestic goods and allows for short run deviations from the law of one price.

**Importing distributors** The (foreign owned) intermediate importing firm buys a homogeneous foreign good in the world market. It turns it into a type specific good using a differentiating technology (brand naming) and then sells it in the domestic market to an aggregator at price \( P_{m,i}^t \).

Foreign intermediate importing firms follow a Calvo price setting rule and can optimally change their price only when they receive a random signal. In any period \( t \), each importing firm has a probability \( (1 - \xi_m) \) to re-optimize its price by choosing \( P_{m, new,t}^{m} \). With probability \( \xi_m \) the importing firm cannot re-optimize at time \( t \) and, instead, it indexes its price according to the following scheme: \( P_{m,t}^{m+1} = (\pi_m^t)^{\kappa_m} (\bar{P})^{1-\kappa_m} P_m^t \) where \( \pi_m^t = \frac{P_m^t}{\bar{P}_{t-1}} \). This local currency price stickiness assumption implies incomplete exchange rate pass-through to the consumption and investment import prices.

The aggregator produces final imported consumption and investment goods sold at price \( P_i^m \)

---

\(^{19}\) All importing firms that are allowed to re-optimize their price, in a given period, will choose the same price, therefore it is not necessary to use a firm index.
to households. The final imported consumption and investment goods are aggregated using a continuum of differentiated imported goods. Each are supplied according to

\[ J_t^m = \left[ \int_0^l (J_{t,i}^m)^{\varepsilon_{m-1}} d \varepsilon_i \right]^{\varepsilon_m}, 1 < \varepsilon_m, \tag{3.30} \]

where \( \varepsilon_m \) is the elasticity of substitution between intermediates in the imported consumption and investment sectors.

### 3.3 Public authorities

The public sector comprises a central bank and a government.

**Central bank** The monetary authority is assumed to follow a simple Taylor-type rule

\[ R_t = \rho_r R_{t-1} + (1 - \rho_r) \left( R + \tau_\pi \pi_t^* + \tau_{\Delta y} \left( \frac{Y_t - Y_{t-1}}{Y_{t-1}} \right) + \tau_{\Delta s} \left( \frac{S_t}{S_{t-1}} \right) \right) + \varepsilon_{R,t}, \tag{3.31} \]

where \( \rho_r \) is the interest rate smoothing parameter, \( \tau_\pi \) is the response to current consumer price inflation, \( \tau_{\Delta y} \) to (real) GDP growth and \( \tau_{\Delta s} \) to the change in the nominal exchange rate. The exogenous process \( \varepsilon_{R,t} \) is a monetary policy shock. Similar Taylor rules include Lubik and Schorfheide (2007), Ortiz and Sturzenegger (2007), Liu et al. (2009), Alpanda et al. (2011) and Hove et al. (2015) for models applied to South Africa and is consistent with the adoption in February 2000 of a formal inflation targeting framework.

**Government** The government collects taxes on consumption, labour and capital and follows a simple public consumption rule

\[ G_t = \rho_g G_{t-1} + (1 - \rho_g) \tilde{G} + \varepsilon_{g,t}, \tag{3.32} \]

where \( \tilde{G} \) is the steady-state value of government spendings and \( \varepsilon_{g,t} \) is a government spending shock. Government consumption is composed of domestic goods only.

---

20 This assumption departs from Adolffson et al. (2007) by assuming that the imported good price is the same for both investment and consumption. In addition, importing firms buy the foreign input at their marginal costs to foreign producers (instead of foreign distributors).
3.4 Closing market conditions and definitions

In equilibrium the goods, labour and bonds markets have to clear. The final goods market is in equilibrium when the demand from domestic households, the government and the foreign households equals the production of the final good. This aggregate resource constraint reads:

\[
\left( C^d_t + I^d_t + G_t \right) \nu^d_t + X^f_t \nu^x_t \leq Y^f_t. \tag{3.33}
\]

where

\[
\nu^d_t = \int_0^1 \left( \frac{P_{i,t}}{P_t} \right)^{-\epsilon_d} di \tag{3.34}
\]

is a measure of domestic price dispersion resulting to an input loss in the domestic distribution process (3.25) and

\[
\nu^x_t = \int_0^1 \left( \frac{P^x_{i,t}}{P^x_t} \right)^{-\epsilon_x} di \tag{3.35}
\]

is a measure of export price dispersion resulting to an input loss in the export distribution process (3.28). Those two price dispersion measures are bounded from below one. They imply that price dispersion increases the amount of inputs \( Y^f \) required to produce domestically consumed goods \( C^d, I^d \) or \( G \) and exported goods \( X^f \).

The domestic bond market clears when the demand for liquidity from households equals the monetary injection by the central bank. Since the central bank money supply is perfectly inelastic at its policy rate it is not necessary to define it. The foreign bond market clears when the positions of the exporting and importing firms equal the households’ choice of foreign bond holdings. Foreign assets evolve according to:

\[
S_t B^*_t+1 = R^*_t \Phi \left( A_{t-1}, \tilde{\Phi}_{t-1} \right) S_t B^*_t + S_t \left( P^{x} X^f_t + P^{x} p Y^p_t \right) - P^m_t \left( C^m_t + I^m_t \right), \tag{3.36}
\]

Finally, the GDP identity is defined by

\[
Y_t = C_t + I_t + G_t + X_t - M_t, \tag{3.37}
\]

where \( I_t = I^p_t + I^f_t \); \( X_t = X^f_t + X^p_t \) and \( M_t = C^m_t + I^m_t \).
4 Empirical Methodology

This section presents the methodology. First, the different shocks introduced in the model are classified according to their types in order to reflect their potential implications for optimal monetary policy and to ease the exposition of results. Second, it describes the value of calibrated parameters and identifies some parameters values that should involve robustness checks. Third, it outlines the estimation methods and its dataset. Finally, it defines the welfare measures used in the results section.

4.1 Structural shocks

Table 1 reports the innovations analysed in this paper. There are 19 shocks considered in this paper. In order to summarize their implication for monetary policy and to ease the exposition of key results, these shocks are classified: first according to their origins and then according to their types.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Process</th>
<th>Group</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wedge</td>
<td>$\varepsilon_{b,t}$</td>
<td>AR(1)</td>
<td>AD*</td>
</tr>
<tr>
<td>Government demand</td>
<td>$\varepsilon_{g,t}$</td>
<td>AR(1)</td>
<td>AD*</td>
</tr>
<tr>
<td>Monetary policy</td>
<td>$\varepsilon_{R,t}$</td>
<td>IID</td>
<td>AD*</td>
</tr>
<tr>
<td>Productivity</td>
<td>$\varepsilon_{k,t}$</td>
<td>AR(1)</td>
<td>(R)AS*</td>
</tr>
<tr>
<td>Investment specific</td>
<td>$\Upsilon_{t}$</td>
<td>AR(1)</td>
<td>(R)AS*</td>
</tr>
<tr>
<td>Commodity supply</td>
<td>$\varepsilon_{p,t}$</td>
<td>IID</td>
<td>(R)AS*</td>
</tr>
<tr>
<td>Price indexation</td>
<td>$\varepsilon_{\pi,t}$</td>
<td>IID</td>
<td>(N)AS*</td>
</tr>
<tr>
<td>Wage indexation</td>
<td>$\varepsilon_{w,t}$</td>
<td>IID</td>
<td>(N)AS*</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Process</th>
<th>Group</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Country risk premium</td>
<td>$\tilde{\phi}_{t}$</td>
<td>AR(1)</td>
<td>UIP</td>
</tr>
</tbody>
</table>

There are three broad categories of shocks defined according to their origins: domestic, foreign, and SOE shocks. Domestic and foreign shocks are disturbances that are unambiguously identified from domestic and foreign origins, respectively. SOE shocks, on the other hand, are disturbances
that may have both domestic and foreign origins. In particular, the country risk premium shock could be explained by a change in domestic country risk (beyond what is captured by the net foreign asset position) or by a change in foreign risk aversion leading to a revision of the price of exchange rate risks. It is therefore labelled as a SOE shock.

Domestic and foreign shocks are classified in 2 sub-groups. On the one hand, aggregate demand shocks (AD) include wedge shocks\(^{21}\), government consumption shocks and monetary policy shocks. On the other hand, aggregate supply shocks (AS) consist of three real (R) technology shocks: capital productivity, commodity supply and investment specific shocks as well as two nominal (N) disturbances: price indexation and wage indexation shocks.

### 4.2 Calibration

Table 2 shows the value of calibrated parameters. Most of them are set in order to fit the empirical mean observed in the data.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h )</td>
<td>Hours devoted to work</td>
<td>0.3000</td>
</tr>
<tr>
<td>( \bar{\pi} )</td>
<td>Mean inflation rate - Inflation target</td>
<td>1.0150</td>
</tr>
<tr>
<td>( R )</td>
<td>Mean risk-free rate</td>
<td>1.0250</td>
</tr>
<tr>
<td>( \tau_k )</td>
<td>Capital gain taxes</td>
<td>0.2000</td>
</tr>
<tr>
<td>( \tau_w )</td>
<td>Pay-roll tax</td>
<td>0.0500</td>
</tr>
<tr>
<td>( \tau_y )</td>
<td>Labour income taxes</td>
<td>0.0300</td>
</tr>
<tr>
<td>( \tau_c )</td>
<td>Value added tax</td>
<td>0.1400</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Capital depreciation rate</td>
<td>0.0200</td>
</tr>
<tr>
<td>( \alpha_d )</td>
<td>Capital income share in final good sector</td>
<td>0.3333</td>
</tr>
<tr>
<td>( \alpha_p )</td>
<td>Capital income share in primary sector</td>
<td>0.3333</td>
</tr>
<tr>
<td>( \varepsilon_d )</td>
<td>input demand elasticity</td>
<td>10.0000</td>
</tr>
<tr>
<td>( \varepsilon_w )</td>
<td>labour demand elasticity</td>
<td>10.0000</td>
</tr>
<tr>
<td>( \gamma_c )</td>
<td>Share of mining sector in GDP</td>
<td>0.1000</td>
</tr>
<tr>
<td>( \omega_h )</td>
<td>Share of mining sector in employment</td>
<td>0.0600</td>
</tr>
<tr>
<td>( \omega_e )</td>
<td>share of imports in consumption</td>
<td>0.2500</td>
</tr>
<tr>
<td>( \omega_i )</td>
<td>share of imports in investment</td>
<td>0.4000</td>
</tr>
<tr>
<td>( a_y )</td>
<td>Foreign Debt to quarterly GDP ratio</td>
<td>0.0000</td>
</tr>
<tr>
<td>( a_y )</td>
<td>Government consumption to GDP ratio</td>
<td>0.1950</td>
</tr>
<tr>
<td>( \phi_a )</td>
<td>Debt-elastic foreign interest rate</td>
<td>0.0010</td>
</tr>
</tbody>
</table>

In particular, \( A_L \) is calibrated such that agents devote 30% of their time to labour activities (and only represents a choice of units). The mean annual inflation rate is set to 6% and corresponds

---

\(^{21}\)The wedge shock is usually considered as a financial shocks. Considering that its main impact is on consumption and investment demands, it qualifies as a demand shock.
to the upper band of the South African Reserve Bank inflation target and is close to its empirical mean. The discount factor $\beta$ is set to 0.99 in order to match the average observed real risk-free rate.

The depreciation rate is set to 0.02 in order to match the investment to GDP ratio. The capital income share is set to 1/3 in both sectors. The land share $\beta_p$ is calibrated to ... to ensure that the mining sector represents 10% of GDP while households devote 6% of their labour efforts to this sector on average. The share of imports in consumption $\omega_c$ and investment $\omega_i$ are set to 0.25 and 0.4, respectively. Those values imply an import to GDP ratio of about 28% as observed in the data. The net foreign asset position is set to 0 at steady-state for simplicity. The government consumption to GDP ratio is set to 19.5%.

However, some parameters such as the debt-elastic interest rate $\phi_a$ could not be calibrated precisely based on macro data\(^{22}\). In addition, estimating this parameter could bring an identification issue. The robustness of key results to changes in the value of $\phi_a$ is assessed for values in the range of 0.01 to 0.0001. Similarly, labour and input demand elasticities $\varepsilon_d$ and $\varepsilon_w$ are set to 10 which implies that distributors impose a 11% mark-up on their marginal costs. Although profits data could be used to pin down this parameter, it is interesting to test other plausible values (usually in the range of 5 to 10 in most DSGE).

4.3 Bayesian Estimation

The model is estimated with Bayesian methods using 11 domestic and 8 foreign variables. In the baseline model, foreign parameters are estimated separately solely based on foreign variables, and then domestic parameters are estimated on the full dataset, calibrating foreign parameters at their mode. In a robustness exercise, the estimation of domestic and foreign parameters is performed jointly.

The data set includes domestic GDP; private consumption; investment; government consumption and commodity exports; employment; consumer price index; import price index; aggregate wage index; risk free rate; and nominal effective exchange rate. Commodity exports are proxied by sales in the mining sector (about 70% is exported). Foreign variables include GDP; private consumption; investment; government consumption, consumer price index; wages; risk free rate; hours worked and commodity price. US data are used in the baseline estimation and G7 data as a robustness check. Commodity prices are measured as an average of world coal, platinum, silver, gold and aluminium prices. It includes important South African commodities. Since there are

\(^{22}\)Adolfson et al. (2007) calibrate this parameter to 0.01. However, Brzoza-Brzezina and Kotlowski (2016) find that its value could be much lower and argue that it depends on the debt-level itself.
more observed variables than shocks, calibrated measurement errors are introduced on the domes-
tic price and wage indexes as well as on the domestic and foreign GDPs.

Data range from 1994Q1 to 2016Q1 in order to exclude the apartheid period in South Africa
(characterised by instability and low trade levels). I also experiment stopping in 2010Q1 in order
to avoid most of the zero lower bound period in the US (which is difficult to capture with a simple
Taylor rule) as well as starting in 2000Q1 which corresponds to the introduction of formal inflation
targeting in South Africa.

4.4 Welfare measures

Welfare cost of business cycle This paper measures the welfare cost of business cycles fluctua-
tions in the emerging economy using a second order approximation of the model (Schmitt-Grohe
and Uribe (2004))\(^{23}\). Welfare cost of business cycles is defined along the lines of Lucas (1987) as
the share of consumption that an agent would be ready to give up at every period to insulate the
economy from all shocks and therefore eliminate aggregate fluctuations. This measure is provided
for the two categories of domestic households and compared to the foreign economy serving as a
benchmark.

A second order approximation implies that different values of the variance of shocks can have
an impact on the mean of endogenous variables. This could make the volatile environment arti-
ficially attractive if, for example, precautionary motives would encourage capital accumulation.
Conditional welfare measures are therefore computed. They solve this issue by imposing that all
simulations (i.e. with or without shocks) start from the same initial point (including the same value
of capital). In this paper, this common starting point is the deterministic steady-state.

Comparison of simple policy rules This paper also explores potential welfare gains from alter-
native monetary policy rules. It focus on simple and implementable monetary rules (Schmitt-Grohe
and Uribe (2007)). Those rules determine the response of policy variables as a function of a small
number of easily observable macroeconomic indicators (such as inflation, output and exchange rate
measures) and delivers uniqueness of the rational expectation equilibrium. As such, they include
the rule advocated by Taylor (1993) as well as most modified versions proposed in the literature.
A simple evaluation of conditional aggregate welfare measures would be enough in order to rank
alternative policy rules. However, conditional compensation measures are also provided in order
to assess the magnitude of the difference in welfare. It is defined as the fraction of consumption

\(^{23}\)The literature has also used the linear-quadratic approach of Benigno and Woodford (2005) and Benigno and
Woodford (2012). Although this approach delivers a closed form (as a function of deep parameters) solution to the
welfare loss function, the complexity of this model would make computation difficult.
that an agent would be ready to give up in a benchmark economy in order to be equally well off in that economy than in an alternative economy. The details to compute these welfare measures are provided in appendix C.

5 Empirical Results

This section presents the results. In a first step, it describes the value of estimated parameters. In particular, it focuses on the persistence and magnitude of estimated business cycle shocks, on the importance of nominal and real frictions and on the estimated monetary policy rule followed by the central bank. In a second step, this section reviews the relative importance of domestic and foreign disturbances as well as their decomposition into demand and supply shocks. In a third step, welfare costs of business cycles in the domestic economy are computed and compared to the foreign economy. In a fourth and final step, this section presents the impact of alternative simple monetary policy rules on welfare.

5.1 Estimated parameters

The distribution of some key estimated parameters are reported in Figure 1 and 2 below. In this subsection, some domestic estimated parameters with specific welfare implications are compared to their foreign counterparts.

Shocks magnitude and persistence Large and persistent shocks are known to generate larger welfare effects. Figure 1 shows the size and persistence of each shocks and indicates that domestic shocks tend to be larger but less persistent, on average. This is particularly true for all technology shocks (capital specific TFP, investment specific TFP and commodity sector TFP). The volatility of these shocks is much larger, but they also tend to be less persistent. Demand shocks do not display such a difference, although the wedge shock seems slightly more volatile and more persistent. Nominal supply shocks (cost and wage-push shocks) are similar in the domestic and foreign economies. Take together, those shocks replicate the stylised facts that domestic variables are more volatile than foreign variables and that consumption tend to be more volatile than output in emerging economies.

Nominal rigidities Nominal price and wage rigidities are crucial since they reduce firms and households ability to adjust their prices and wages to business cycle fluctuations. They generate price and wage dispersions that are costly in term of welfare. Domestic prices and wages seem to
be less rigid in South Africa, when compared to the US. It is therefore unlikely that this amount of nominal rigidities could amplify the costs of business cycles fluctuations when compared to advanced countries. However, incomplete exchange rate pass-through seems important and implies potentially inefficient deviations from the low of one price.

**Real rigidities** Real rigidities are also important since they can impair the reallocation of resources across time or sectors. Investment adjustment costs ($\tilde{S}'\nu$) are larger in the domestic economy. In addition, rigidities in the labour market are also estimated to be important: labour mobility across sectors is small ($\eta_h<0.5$)\textsuperscript{24}. These relatively important real rigidities might exacerbate the costs of business cycle fluctuations.

\textsuperscript{24} $\eta_h$ is calibrated to one in Horvath (2000) and Dagher et al. (2010). There is no equivalent in the foreign economy since commodity supply is assumed to be exogenous for simplicity.
Figure 2: Prior and posterior densities - coefficients

Utility parameters  The aggregate labour supply elasticity ($\sigma_h$)\textsuperscript{25} is not well identified in the domestic and foreign economies. It means that the choice of an appropriate prior is key. In this case, values in the range of 1 to 3 are all plausible. The coefficient of relative risk aversion ($\sigma_c$) and external habits ($b$) are both relatively high (although they are relatively smaller for the domestic economy).

Taylor Rule  The coefficients of the estimated Taylor rule show that the SARB responded to inflation fluctuations ($\tau_{\pi}=1.85$), which is consistent with its mandate and the introduction of inflation targeting. In addition, changes in interest rate were smooth ($\rho_r=0.87$) and the SARB responded to output growth ($\tau_{\Delta y}=0.38$) and to the change in the nominal exchange rate ($\tau_{\Delta s}=0.1$). These results are relatively close to the literature and will serve as a benchmark for comparing the costs of business cycles under different monetary policy rules. Note that the Taylor rule in the foreign economy is calibrated in order to avoid an estimation on the zero lower bound period.

\textsuperscript{25} $\sigma_h$ is the inverse of the labour supply elasticity
5.2 Sources of business cycle fluctuations

This section explores the sources of business cycle fluctuation in the domestic economy. The sources of business cycle fluctuations have important implications for welfare and monetary policies. Indeed, domestic monetary and fiscal policies are especially well armed to deal with demand shocks. Therefore, an economy mainly driven by domestic demand shocks could benefit the most from stabilisation policies. Supply disturbance on the other hand impose harder trade-off (in term of inflation and output) on the monetary authority. This implies that monetary policies face limitation in a context of large supply-driven fluctuations. Similarly, foreign sources of disturbances can impose trade-off (in term of inflation, output and exchange rate stabilisation) to the domestic monetary policy. This section therefore presents variance decomposition in order to assess the relative importance of each type of shocks to the domestic economy. In addition, it describes impulse response functions (IRF) to the most important shocks in order to describe the (potential) trade-offs imposed on monetary policy responses.

Table 3: Variance decomposition

<table>
<thead>
<tr>
<th></th>
<th>AD*</th>
<th>AS*</th>
<th>AD</th>
<th>R-AS</th>
<th>N-AS</th>
<th>SOE</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>4.71</td>
<td>9.32</td>
<td>24.63</td>
<td>33.59</td>
<td>13.28</td>
<td>0.58</td>
</tr>
<tr>
<td>Employment</td>
<td>7.19</td>
<td>15.33</td>
<td>36.11</td>
<td>23.71</td>
<td>17.25</td>
<td>0.42</td>
</tr>
<tr>
<td>Consumption</td>
<td>2.68</td>
<td>23.56</td>
<td>44.18</td>
<td>8.22</td>
<td>17.49</td>
<td>3.86</td>
</tr>
<tr>
<td>Investment</td>
<td>1.77</td>
<td>15.44</td>
<td>7.78</td>
<td>62.78</td>
<td>1.51</td>
<td>10.73</td>
</tr>
<tr>
<td>Exports</td>
<td>10.18</td>
<td>4.95</td>
<td>1.63</td>
<td>74.91</td>
<td>2.95</td>
<td>5.36</td>
</tr>
<tr>
<td>Imports</td>
<td>3.33</td>
<td>37.29</td>
<td>16.16</td>
<td>22.54</td>
<td>2.16</td>
<td>18.53</td>
</tr>
<tr>
<td>Mining exports</td>
<td>1.09</td>
<td>3.84</td>
<td>0.95</td>
<td>92.29</td>
<td>0.35</td>
<td>1.48</td>
</tr>
<tr>
<td>CPI</td>
<td>1.98</td>
<td>6.82</td>
<td>57.72</td>
<td>7.65</td>
<td>16.08</td>
<td>9.10</td>
</tr>
<tr>
<td>Wage</td>
<td>2.29</td>
<td>8.55</td>
<td>31.22</td>
<td>2.11</td>
<td>25.39</td>
<td>2.37</td>
</tr>
<tr>
<td>Risk-free rate</td>
<td>1.94</td>
<td>4.79</td>
<td>73.57</td>
<td>5.22</td>
<td>4.80</td>
<td>9.68</td>
</tr>
<tr>
<td>NEER</td>
<td>6.09</td>
<td>5.27</td>
<td>14.96</td>
<td>0.79</td>
<td>0.34</td>
<td>72.55</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>AD</th>
<th>R-AS</th>
<th>N-AS</th>
<th>SOE</th>
</tr>
</thead>
<tbody>
<tr>
<td>US GDP</td>
<td>42.73</td>
<td>50.88</td>
<td></td>
<td></td>
</tr>
<tr>
<td>US Consumption</td>
<td>52.89</td>
<td>47.12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>US Investment</td>
<td>23.35</td>
<td>76.65</td>
<td></td>
<td></td>
</tr>
<tr>
<td>US Hours</td>
<td>50.15</td>
<td>49.84</td>
<td></td>
<td></td>
</tr>
<tr>
<td>US CPI</td>
<td>29.76</td>
<td>70.23</td>
<td></td>
<td></td>
</tr>
<tr>
<td>US Wage</td>
<td>26.98</td>
<td>73.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>US Risk-free rate</td>
<td>55.97</td>
<td>44.02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Commodity Price</td>
<td>17.51</td>
<td>82.48</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Most variables are expressed in year on year growth rates. Risk free rate in level and NEER in quarter to quarter growth rate.
Table 3 shows the variance decomposition of all shocks, classified according to their type (AD vs AS) and origin (domestic vs foreign). This table shows that foreign shocks are important drivers of economics fluctuations in the domestic economy. All together, they explain about 14% of the fluctuations in South African GDP and a larger share of consumption and investment fluctuations. They might therefore generate important welfare costs and cause challenge to domestic stabilisation policies. In addition, both demand and supply shocks are important. This indicates that there is both room and limitation for stabilising monetary policies.

### 5.3 Welfare costs of business cycles

Table 4 presents the costs of business cycles fluctuations for savers and rule-of-thumb households in the domestic economy, expressed in percentage of consumption equivalents. It is compared to the same values obtained for foreign households.

<table>
<thead>
<tr>
<th>Baseline</th>
<th>Cdt cost</th>
<th>Relative cost</th>
<th>Uncdt cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saver</td>
<td>0.898</td>
<td>1.892</td>
<td>-0.712</td>
</tr>
<tr>
<td></td>
<td>(0.553 ; 1.637)</td>
<td>(1.039 ; 3.499)</td>
<td>(-1.302 ; -0.155)</td>
</tr>
<tr>
<td>Hand to mouth</td>
<td>2.741</td>
<td>5.743</td>
<td>1.602</td>
</tr>
<tr>
<td></td>
<td>(1.793 ; 4.154)</td>
<td>(2.834 ; 9.658)</td>
<td>(0.72 ; 2.902)</td>
</tr>
<tr>
<td>Foreign</td>
<td>0.459</td>
<td>1</td>
<td>0.586</td>
</tr>
<tr>
<td></td>
<td>(0.315 ; 0.931)</td>
<td>-</td>
<td>(0.395 ; 1.199)</td>
</tr>
</tbody>
</table>

Note: Conditional and unconditional welfare costs are expressed in percentage points. Numbers in parenthesis correspond to a 90% confidence band. The second column show domestic households costs relative to foreign households.

Table 4 shows that savers would be ready to give up 0.9% of their steady-state level of consumption in order to avoid macroeconomic fluctuations while hand-to-mouth households would be ready to give up 2.7%. Those costs are substantially larger than in the foreign economy: foreign households would be ready to give-up 0.5%. The welfare costs of business cycles are significantly larger for domestic households at a 90% confidence interval. They are particularly very high, in relative term, for domestic hand to mouth households: they would be ready to give-up between 2.8 and 9.7 times as much, as a fraction of steady-state consumption, than foreign households, in order to avoid macroeconomic fluctuations.

Table 4 assesses the relative importance of each categories of shocks in the welfare costs of business cycles in the domestic economy. Domestic aggregate demand shocks are an important source of welfare costs in the domestic economy. Financially included households would be ready
Table 5: Decomposition of the WC of BC

<table>
<thead>
<tr>
<th></th>
<th>Cdt cost</th>
<th>Relative cost</th>
<th>Premium</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Baseline:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Saver</td>
<td>0.9943</td>
<td>2.0823</td>
<td></td>
</tr>
<tr>
<td>Hand to mouth</td>
<td>2.9733</td>
<td>6.2270</td>
<td></td>
</tr>
<tr>
<td><strong>No AD:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Saver</td>
<td>0.5159</td>
<td>1.0082</td>
<td>0.4814</td>
</tr>
<tr>
<td>Hand to mouth</td>
<td>1.8370</td>
<td>3.5900</td>
<td>1.1590</td>
</tr>
<tr>
<td><strong>No NAS:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Saver</td>
<td>0.7252</td>
<td>1.4619</td>
<td>0.2715</td>
</tr>
<tr>
<td>Hand to mouth</td>
<td>2.2352</td>
<td>4.5057</td>
<td>0.7564</td>
</tr>
<tr>
<td><strong>No RAS:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Saver</td>
<td>0.8291</td>
<td>1.6203</td>
<td>0.1670</td>
</tr>
<tr>
<td>Hand to mouth</td>
<td>2.7415</td>
<td>5.3577</td>
<td>0.2391</td>
</tr>
<tr>
<td><strong>No SOE:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Saver</td>
<td>1.1280</td>
<td>2.2840</td>
<td>-0.1359</td>
</tr>
<tr>
<td>Hand to mouth</td>
<td>2.9719</td>
<td>6.0178</td>
<td>0.0015</td>
</tr>
<tr>
<td><strong>No foreign:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Saver</td>
<td>0.4205</td>
<td>0.5773</td>
<td></td>
</tr>
<tr>
<td>Hand to mouth</td>
<td>1.8084</td>
<td>1.1884</td>
<td></td>
</tr>
<tr>
<td><em><em>No CS</em>:</em>*</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Saver</td>
<td>1.0089</td>
<td>2.0356</td>
<td>-0.0148</td>
</tr>
<tr>
<td>Hand to mouth</td>
<td>2.7962</td>
<td>5.6418</td>
<td>0.1828</td>
</tr>
</tbody>
</table>

Note: Conditional welfare costs are expressed in percentage points. The second column show domestic households costs relative to foreign households (except when those shocks are set to zero). The premium correspond to the price, in consumption equivalent, that an individual would be ready to pay to edge against a particular set of shocks.

to give up about 0.5% of their steady-state level of consumption in order to avoid fluctuations caused by these shocks while financially excluded households would be willing to pay as much as 1.2%. This indicates that the potential for monetary policy improvement could be substantial. Nominal and real aggregate supply shocks generate lower costs but are economically relevant. This table also indicates that SOE shocks (shocks to the UIP condition) might be welfare increasing for financially included households. These households have the opportunity to use the implied fluctuations in the exchange rate to purchase more imported investment goods when the term of trade is in their favour. This option effect seems to dominate the negative impact of the implied volatility. Surprisingly, foreign shocks have large impact on welfare which compare, in magnitude, to the costs implied by domestic demand shocks. Among foreign shocks, commodity supply shocks would be a natural suspect in order to explain these large welfare costs. Foreign commodity supply shocks, through their impact on commodity prices, have large repercussions on the domestic economy and generate important consumption and investment fluctuations. However, the impact of these shocks is limited for financially excluded households, and could even be welfare increasing to financially included households. Commodity price fluctuations offer option effects, in the sense that firms can
transfer production from low commodity price periods to high commodity price periods. These
options effects could dominate the adverse welfare effects of consumption fluctuations as long as
the elasticity of labour supply is large enough and if households have a mean to transfer wealth
across periods in the form of capital or foreign bonds.

Other important issue could be tackled in this framework. In particular, it would also be in-
teresting to identify which frictions are responsible for those large estimate. Assessing the role of
each frictions could help to understand the role of monetary policy. Indeed, costly price or wage
rigidities would call for more aggressive response to price or wage fluctuations. Similarly, welfare
decreasing imperfect exchange rate pass-through would justify the use of an exchange rate target.
Finally, real frictions would justify the use of some real activity target.

5.4 Welfare effects of monetary policy

This section compares, in term of welfare, the impact of different simple rules compared to the
baseline case where the Taylor rule is calibrated at the mode of its estimated parameters. First,
It considers a muted response to output by imposing that \( \tau_{\Delta y} = 0 \). Second, It considers a muted
response to the exchange rate by imposing that \( \tau_{\Delta s} = 0 \). Third, It simulates a pure CPI-inflation tar-
getting rule where \( \tau_{\pi} = 3 \) and \( \tau_{\Delta y} = \tau_{\Delta s} = 0 \).\(^{26}\) Finally, It simulates a pure domestic-inflation targeting
rule with the same parameters values.

<table>
<thead>
<tr>
<th></th>
<th>Cdt cost</th>
<th>Relative cost</th>
<th>Premium</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Baseline:</strong> Saver</td>
<td>0.9943</td>
<td>2.0823</td>
<td></td>
</tr>
<tr>
<td>Hand to mouth</td>
<td>2.9733</td>
<td>6.2270</td>
<td></td>
</tr>
<tr>
<td><strong>Pure CPI targeting:</strong> Saver</td>
<td>0.7854</td>
<td>1.6448</td>
<td>0.2110</td>
</tr>
<tr>
<td>Hand to mouth</td>
<td>2.3224</td>
<td>4.8637</td>
<td>0.6679</td>
</tr>
<tr>
<td><strong>Pure PPI targeting:</strong> Saver</td>
<td>0.7520</td>
<td>1.5750</td>
<td>0.2446</td>
</tr>
<tr>
<td>Hand to mouth</td>
<td>2.1058</td>
<td>4.4101</td>
<td>0.8881</td>
</tr>
<tr>
<td><strong>Muted response to output:</strong> Saver</td>
<td>0.9916</td>
<td>2.0768</td>
<td>0.0027</td>
</tr>
<tr>
<td>Hand to mouth</td>
<td>2.9762</td>
<td>6.2330</td>
<td>-0.0030</td>
</tr>
<tr>
<td><strong>Muted response to NEER:</strong> Saver</td>
<td>1.0113</td>
<td>2.1179</td>
<td>-0.0173</td>
</tr>
<tr>
<td>Hand to mouth</td>
<td>3.0190</td>
<td>6.3228</td>
<td>-0.0473</td>
</tr>
</tbody>
</table>

**Muted response to output** Had the central bank muted its response to output, welfare costs
of business cycles would have been slightly larger for hand-to-mouth consumer, but lower for

\(^{26}\)Schmitt-Grohe and Uribe (2007) argue that policy coefficients larger than 3 or negative would be difficult to
communicate to policy-makers or the public.
financially included households. This result show that monetary policy could have different impact on different types of agents but should be considered with caution since no confidence bands are reported here. In addition, the magnitude of these results are small, indicating that the welfare effects of responding to output growth could be small.

**Muted response to the exchange rate** Had the central bank muted its response to the nominal exchange rate, welfare costs of business cycles would have been slightly larger, although the magnitude of the change in these costs is very small.

**Pure CPI-inflation targeting** The model is then simulated with a much larger weight on CPI-inflation and no weight on output or the exchange rate. This experiment could be interpreted as a pure CPI-inflation targeting exercise. In this case, welfare gains for both types of households are substantial. This would indicate that there is more room for improvement through perusing a more aggressive inflation targeting policy.

**Pure domestic-inflation targeting** The last experiment considers pure domestic inflation targeting. In this case, welfare costs of business cycles are also reduced by a large amount, and the gain are larger than with pure CPI targeting. Interestingly, hand to mouth households gain relatively more from PPI-inflation targeting.

### 6 Conclusion

This paper measures the cost associated to business cycle fluctuations and explore potential gains from appropriate monetary policies in an estimated small open economy DSGE model tailored to an emerging economy and applied to South Africa. It captures the key characteristics of a typical small open emerging economy: a large exposure to developments in world commodity markets and the imperfect nature of its financial markets, limiting the ability of some households to smooth consumption and to insure against idiosyncratic labour market risks.

In this context, preliminary results indicate that the welfare costs of business cycles might be substantial in these economies. Among casual suspects, the volatility of supply shocks, the importance of real rigidities and the imperfect nature of financial markets are potential factors driving those results. In particular, although the costs of business cycles are large for each categories of households in the emerging economy, they are much larger for households excluded from the financial markets. The importance of domestic demand shocks in driving macroeconomic variables
fluctuations; and their important welfare effects indicates that there might be important gains from further stabilisation through more appropriate monetary policies.

This paper then explores potential welfare gains from simple monetary policy rules to draw relevant policy recommendations. By experimenting with different Taylor rules, preliminary results show that the welfare gains from a more aggressive anti-inflation stance might be very important while changes in the output growth or exchange rate fluctuations would generate modest welfare effects. Although financially excluded households gain more from appropriate monetary policies, the welfare costs of business cycles remains substantially larger for this category of agents.
References


A Complete set of equilibrium conditions

This appendix provides the reader with the details on how to derive the complete set of equilibrium conditions for households and firms written in their stationary form. In addition, for some equations, it shows how they are implemented in Dynare. Those equations have been expressed following the convention that lower case variables denote the stationarized equivalent of their upper case counterparts. In particular, some variables have a nominal trend because of the positive inflation rate target.

A.1 Households

The consumption demand functions for the domestic and the imported good are given by maximising the consumption basket under a fixed consumption spendings:

\[ C_d^t = (1 - \omega_c) \left[ \frac{P_t}{P_t^C} \right]^{-\eta_c} C_t, \quad (A.1) \]

\[ c_m^t = \omega_c \left[ \frac{P_m^t}{P_t^C} \right]^{-\eta_c} c_t, \quad (A.2) \]

where \( P_t \) is the domestic good price, \( P_m^t \) the imported good price and \( P_t^C \) represents the Consumer price index (CPI) and is given by:

\[ P_t^C = \left[ (1 - \omega_c)(P_t)^{1-\eta_c} + \omega_c(P_m^t)^{1-\eta_c} \right]^{1/(1-\eta_c)} \]

which is made stationary as follows:

\[ (\pi_t^c)^{1-\eta_c} = (1 - \omega_c) \left( \pi_t \frac{P_{t-1}}{P_{t-1}^C} \right)^{1-\eta_c} + \omega_c \left( \pi_t^m \frac{P_{t-1}^m}{P_t^m} \right)^{1-\eta_c} \quad (A.3) \]

where gross inflation rates are defined as: \( \pi_t^c = \frac{P_t^C}{P_t^{C^t}} \); \( \pi_t = \frac{P_t}{P_{t-1}} \) and \( \pi_t^m = \frac{P_m^t}{P_{t-1}^m} \). Note that some price ratios are explicitly defined in order to save on notations. In Dynare, it requires to define a variable for each price ratios.

A.1.1 Savers

Savers maximise their utility with respect to domestic and foreign bonds holding and consumption. The first order conditions associated to savers with shadow value \( \psi_t \) on their budget constraint (3.3) are given by:
\[ \text{w.r.t. } C_t^S : \left( C_t^S \right)^{-\sigma_c} \left( C_{t-1}^S \right)^{-b(1-\sigma_c)} = \psi_t \frac{P_t}{P_{t-1}} (1 + \tau_t^c) \]  
(A.4)

\[ \text{w.r.t. } B_{t+1} : \psi_t = \beta E_t \frac{\psi_{t+1}}{\pi_{t+1}} e_{b,t} R_t \]  
(A.5)

\[ \text{w.r.t. } B_{t+1}^* : \psi_t = \beta E_t \frac{\psi_{t+1}}{\pi_{t+1}} S_{t+1} e_{b,t} R_t^* \Phi(A_t, \tilde{\phi}_t) \]  
(A.6)

These variables are stationarized following Altig et al. (2003) such that \( x_t = \frac{X_t}{P_t} \) for nominal variables while the Lagrange multiplier is redefined as \( \psi_t = \nu_t P_t \).

Savers also maximise their utility with respect to the capital stock and investment in each sector \( q \in (p, f) \) subject to their budget constraint (3.3) and capital accumulation rule (3.4):

\[ \text{w.r.t. } K_{t+1}^q : \psi_t \frac{P_{t+1}^k}{P_t} \equiv K_{t+1}^q = \beta \psi_{t+1} \left( 1 - \tau_k \right) r_{t+1}^k + \left( 1 - \delta \right) \frac{P_{t+1}^k}{P_{t+1}} \]  
(A.7)

\[ \text{w.r.t. } I_t^q : -\psi_t \frac{P_t}{P_{t+1}} + \psi_t \frac{P_{t+1}^k}{P_t} Y_t \left( 1 - \tilde{S} \frac{I_t^q}{I_t^q} - \tilde{S}' \frac{I_t^q}{I_t^q} \right) I_t^q \]  
\[ + \beta E_t \left( \frac{P_{t+1}^k}{P_t} \psi_{t+1} Y_{t+1} \tilde{S}' \left( \frac{I_{t+1}^q}{I_t^q} \right) \left( \frac{I_{t+1}^q}{I_{t+1}^q} \right)^2 \right) = 0 \]  
(A.8)

where \( r_k^q \equiv \frac{P_{t+1}^k}{P_t} \) is the rental rate of capital corresponding to marginal productivity of capital and \( \frac{P_{t+1}^k}{P_t} \) is the real price of the capital good (introduced for convenience).

**Country risk premium** Combining the FOC with respect to domestic and foreign bonds gives the uncovered interest rate parity (UIP)

\[ R_t = R_t^* \Phi(A_t, \tilde{\phi}_t) E_t \frac{S_{t+1}}{S_t} \]

This last equality shows that the spread between domestic and foreign nominal risk free rates depends on the anticipated domestic currency depreciation, the country-wide foreign debt and an UIP shock.

**Capital Accumulation** The capital accumulation rule reads:

\[ K_{t+1}^q = (1 - \delta)K_t^q + Y_t (1 - \tilde{S}(I_t/I_{t-1}))I_t \]  
(A.9)
**Investment Basket**  The two investment demand functions are given by maximising the investment basket under a fixed investment spending:

\[ I_{t}^{d,q} = (1 - \omega_{i}) \left[ \frac{P_{i}}{P_{t}} \right]^{-\eta_{i}} I_{t}^{q}, \]  

(A.10)

\[ I_{t}^{m,q} = \omega_{i} \left[ \frac{P_{m}}{P_{t}} \right]^{-\eta_{i}} I_{t}^{q}, \]  

(A.11)

where \( P_{t}^{i} \) is the aggregate investment price given by:

\[ P_{t}^{i} = \left[ (1 - \omega_{i})(P_{t})^{1-\eta_{i}} + \omega_{i}(P_{m}^{n})^{1-\eta_{i}} \right]^{1/(1-\eta_{i})} \]

which is made stationary as follows:

\[ (\pi_{t}^{i})^{1-\eta_{i}} = (1 - \omega_{i}) \left( \frac{\pi_{t-1}}{\pi_{t-1}} \right)^{1-\eta_{i}} + \omega_{i} \left( \frac{\pi_{t-1}}{\pi_{t-1}} \right)^{1-\eta_{i}} \]  

(A.12)

where \( \pi_{t}^{i} = \frac{P_{t}^{i}}{P_{t-1}^{i}} \).

**Wage setting**  Households are represented by a labour union on the labour market. Each household has a probability \((1 - \xi_{w})\) to be allowed to optimally reset the nominal wage. Otherwise, wage is indexed on previous period consumer price inflation \(\pi_{c,t-1}\) and the Central Bank inflation target \(\bar{\pi}\). Households that can re optimize their wage maximize

\[ \sum_{s=0}^{\infty} (\beta \xi_{w})^{s} \left( \nu_{t+s} \frac{1 - \tau^{v}}{1 + \tau^{w}} W_{j,t+s} h_{j,t+s} - A_{h} (h_{j,t+s})^{1+\sigma_{h}} \right) \]

where

\[ W_{j,t+s} = W_{j,t}^{new} (\pi_{t}^{c} \ldots \pi_{t+s-1}^{c})^{\kappa_{w} \bar{\pi}^{(1-\kappa_{w})s}} \]

\[ h_{j,t+s} = \frac{W_{j,t+s}^{new} (\pi_{t}^{c} \ldots \pi_{t+s-1}^{c})^{\kappa_{w} \bar{\pi}^{(1-\kappa_{w})s}}}{W_{t+s}^{new}} \]

\[ = \frac{W_{j,t+s}^{new} (\pi_{t}^{c} \ldots \pi_{t+s-1}^{c})^{\kappa_{w} \bar{\pi}^{(1-\kappa_{w})s}}}{W_{t+s}^{new}} \]

\[ H_{t+s}^{S} \]

48
with respect to the new wage $W_{t+s}\new$. Note that $\nu_{t+s}$ is the Lagrange multiplier in the household optimisation problem and that it is also useful to define

$$\Pi_{t,s+1} = (\pi_{t+1}^c \ldots \pi_{t+s-1}^c)$$

$$\Pi_{t+1} = (\pi_{t+1}^c \ldots \pi_{t+s})$$

Rearranging using the above equations gives:

$$\sum_{s=0}^{\infty} (\beta \xi_w) s^n \nu_{t+s} \frac{1 - \tau^y}{1 + \tau^y} W_{t,s}^\new \left( \frac{\Pi_{t,s+1}^c}{\Pi_{t+1,s+1}^c} \right) \frac{\kappa_w}{\tilde{\pi}(1 - \kappa_w)} \left( \frac{W_{t,s}^\new \left( \frac{\Pi_{t,s+1}^c}{\Pi_{t+1,s+1}^c} \right) \kappa_w}{\tilde{\pi}(1 - \kappa_w)} \right) - \varepsilon_w H_{t+s}^S$$

Expressing it in term of real wage and simplifying gives:

$$\sum_{s=0}^{\infty} (\beta \xi_w) s^n \psi_{t+s} \frac{1 - \tau^y}{1 + \tau^w} \left( \frac{W_{t,s}^\new \left( \frac{\Pi_{t,s+1}^c}{\Pi_{t+1,s+1}^c} \right) \kappa_w}{\tilde{\pi}(1 - \kappa_w)} \right) - \varepsilon_w H_{t+s}^S$$

The FOC is now easy to derive and reads:

$$(\varepsilon_w - 1) W_{t,s+1}^\new \sum_{s=0}^{\infty} (\beta \xi_w) s^n \psi_{t+s} \frac{1 - \tau^y}{1 + \tau^w} \left( \frac{\Pi_{t,s+1}^c}{\Pi_{t+1,s+1}^c} \right) \frac{\kappa_w}{\tilde{\pi}(1 - \kappa_w)} - \varepsilon_w H_{t+s}^S$$

$$= \varepsilon_w W_{t,s+1}^\new \frac{(\Pi_{t,s+1}^c)}{\psi_{t+s} \Pi_{t+1,s+1}^c} \frac{\frac{(\Pi_{t,s+1}^c)}{\psi_{t+s} \Pi_{t+1,s+1}^c} \frac{\kappa_w}{\tilde{\pi}(1 - \kappa_w)} - (1 + \sigma_h) \varepsilon_w H_{t+s}^S}{1 + \sigma_h}$$

49
which simplifies to:

\[(w_t^{new})^{1+\sigma_h}e_w = \frac{\varepsilon_w}{\varepsilon_w - 1} \sum_{s=0}^{\infty} (\beta \xi_w)^s A_h \left( \frac{(\pi_{t+s+1}^i)^{\kappa_w} \pi_{t+s+1}^{(1-\kappa_w)}}{w_{t+s+1} H_{t+s+1}} \right)^{-(1+\sigma_h)\varepsilon_w} \frac{(H_t^S)^{1+\sigma_h}}{(w_{t+s+1} H_{t+s+1})^{1-\varepsilon_w}} \]

since all re-optimising households set the same wage. This last equation is the wage-Phillips curve with partial indexation. In Dynare, the infinite sum can be rewritten as a set of three equations:

\[(w_t^{new})^{1+\sigma_h}e_w = \left( \frac{\varepsilon_w}{\varepsilon_w - 1} \right) \frac{X_{1,t}^H}{X_{2,t}^H} \]

\[X_{1,t}^H = A_h w_t^{(1+\sigma_h)e_w}(H_t^S)^{1+\sigma_h} + \beta \xi_w \left( \frac{(\pi_{t+1}^c)^{\kappa_w} \pi_{t+1}^{(1-\kappa_w)}}{\pi_{t+1}} \right)^{-(1+\sigma_h)\varepsilon_w} \]

\[X_{2,t}^H = \frac{1 - \tau^y}{1 + \tau^w} w_t^{\varepsilon_w} H_t^S + \beta \xi_w \left( \frac{(\pi_{t+1}^c)^{\kappa_w} \pi_{t+1}^{(1-\kappa_w)}}{\pi_{t+1}} \right)^{-(1+\sigma_h)\varepsilon_w} \]

The real wage index evolves according to

\[w_t^{1-\varepsilon_w} = (1 - \xi_w)(w_t^{new})^{1-\varepsilon_w} + \xi_w \left( \frac{(\pi_{t-1}^c)^{\kappa_w} \pi_{t-1}^{(1-\kappa_w)} W_{t-1}}{\pi_t} \right)^{1-\varepsilon_w} \]

**Savers aggregate utility** Recall the aggregate utility level for savers is given by

\[U_t^S = \frac{C_t^S}{C_t^S_{t-1}}^{1-\sigma_c} - 1 - A_h (H_t^S)^{1+\sigma_h} \frac{1 - \sigma_c}{1 + \sigma_c} \]

where

\[v_t^w = \int_0^1 \left( \frac{W_{j,t}}{W_t} \right)^{-\varepsilon_w(1+\sigma_j)} d j \]

The later term can be re-expressed as a function of aggregate variables only. By the Calvo assumption, it implies that

\[v_t^w = (1 - \xi_w) \left( \frac{W_t^{new}}{W_t} \right)^{-\varepsilon_w(1+\sigma_j)} + \int_{1-\xi_w}^1 \left( \frac{(\pi_{t-1}^c)^{\kappa_w} \pi_{t-1}^{(1-\kappa_w)} W_{j,t-1}}{W_t} \right)^{-\varepsilon_w(1+\sigma_j)} d j \]

50
Rewriting this expression gives:

\[ \psi_t^w = (1 - \xi_w) \left( \frac{w_t^{new}}{w_t} \right)^{-\epsilon_w(1+\sigma_t)} + \int_{1-\xi_w}^1 \left( \frac{w_{t-1}^\pi}{w_t \pi_t} \right) d j \]

which simplifies to:

\[ \psi_t^w = (1 - \xi_w) \left( \frac{w_t^{new}}{w_t} \right)^{-\epsilon_w(1+\sigma_t)} + \xi_w \left( \frac{w_{t-1}^\pi}{w_t \pi_t} \right) \psi_{t-1}^w \]

and then to:

\[ \psi_t^w = (1 - \xi_w) \left( \frac{w_t^{new}}{w_t} \right)^{-\epsilon_w(1+\sigma_t)} + \xi_w \left( \frac{w_{t-1}^\pi}{w_t \pi_t} \right) \psi_{t-1}^w \quad (A.18) \]

A.1.2 Rule-of-thumb households

The rule of thumb households aggregate stationary budget constraint is given by

\[ \int_0^1 (1 + \tau_c) P_t^c C_{j,t} d j = \int_0^1 \left( \frac{1 - \gamma^y}{1 + \tau^w} W_{j,t} h_{j,t} + TR_{j,t}^r \right) d j , \]

which, using labour demand (3.9) and the definition of aggregate consumption simplifies to

\[ (1 + \tau_c) P_t^c C_{j,t} = \left( \frac{1 - \gamma^y}{1 + \tau^w} \right) W_t^{\epsilon_w} H_t^R \int_0^1 (W_{j,t})^{1-\epsilon_w} d j + TR_{j,t}^r , \]

and using the wage index (3.10) yields

\[ (1 + \tau_c) P_t^c C_{j,t} = \frac{1 - \gamma^y}{1 + \tau^w} W_t H_t^R + \tau^r \]

where \( w_t = \frac{W_t}{P_t} \)

Rule of thumb’s aggregate utility

Recall that their aggregate level of utility is given by

\[ U_t^R = \frac{(C_t^R / (C_{t-1}^R)^{\sigma_c})^{1-\sigma_c}}{1 - \sigma_c} \psi_t^c - 1 = \frac{A_{h,r}(H_t^R)^{1+\sigma_h}}{1 + \sigma_h} \psi_t^w \quad (A.20) \]
where

\[ 
\psi^c_t = \int_0^1 \left( \frac{W_{j,t}}{W_t} \right)^{(1-\epsilon_w)(1-\sigma_c)} d j
\]

This later term can also be written as a function of aggregate variables only using a similar procedure as above:

\[ 
\psi^c_t = (1 - \xi_w) \left( \frac{w_t}{w_t} \right)^{(1-\epsilon_w)(1+\sigma_t)} + \xi_w \left( \frac{w_{t-1} (\pi_{t-1}^c)^{\kappa w} (\overline{\pi})^{1-\kappa_w}}{w_t \pi_t} \right)^{(1-\epsilon_w)(1+\sigma_t)} \psi^c_{t-1} \quad (A.21)
\]

### A.1.3 Labour Unions

The labour allocation problem gives primary and secondary sectors wages function of the relative demand for labour in each sectors:

\[ 
w_t^f = \left[ \frac{H_f}{(1 - \omega_h) H_t} \right]^{1/\eta_h} w_t, \quad (A.22)
\]

\[ 
\omega_p \left[ \frac{H_p}{\omega_h H_t} \right]^{1/\eta_h} w_t, \quad (A.23)
\]

where \( w_t^p = \frac{w_t^p}{\pi_t^p} \) and \( w_t^f = \frac{w_t^f}{\pi_t^f} \)

### A.2 Firms

Here is the profit maximisation problem of the firms in the commodity and manufacturing sectors.

#### A.2.1 Commodity sector

**Domestic commodity producers** Commodity producers combine capital \( K_t^p \), labour \( H_t^p \) and land \( L_t^p \) to produce a commodity input. It gives the capital to labour ratio:

\[ 
\frac{K_t^p}{H_t^p} = \left( \frac{\alpha_p}{1 - \alpha_p - \beta_p} \right) \frac{w_t^p}{\pi_t^p}, \quad (A.24)
\]
and note that we could also find the capital to land ratio (if we were interested in the rental rate of land $R_t^L$):

$$\frac{K_t^p}{L_0^p} = \left( \frac{\beta_p}{1 - \alpha_p - \beta_p} \right) \frac{r_t^L}{r_t^{k,p}}$$

And since commodity producers operate in a perfectly competitive environment, the marginal cost of labour must equalise its productivity:

$$w_t^p = (1 - \alpha_p - \beta_p) \frac{Y_t^p}{H_t^p} \frac{S_t P_t^{sp}}{P_t}$$  \hspace{1cm} (A.25)

**Foreign commodity market**  Foreign demand for commodities come from their use as inputs in the foreign production process (while supply is exogenous and therefore do not requires to compute any FOCs). In particular, foreign firms combine a labour-capital aggregate to the commodity input in a CES function. The FOCs for this cost minimisation problem give:

$$P_t^p = \beta^* \lambda_t^* P_t^* Y_0^* \left[ \beta^* \left( \frac{Y_t^{p*}}{Y_0^{p*}} \right)^{\frac{\sigma_p}{\sigma_p - 1}} + (1 - \beta^*) \left( \frac{N_t^*}{N_0^*} \right)^{\frac{\sigma_{p-1}}{\sigma_p}} \right] \frac{\sigma_p}{\sigma_p - 1} \left( \frac{Y_t^{p*}}{Y_0^{p*}} \right)^{\frac{\sigma_p - 1}{\sigma_p - 1}} \frac{1}{Y_t^{p*}}$$

$$\bar{MC}_t^* = (1 - \beta^*) \lambda_t^* P_t^* Y_0^* \left[ \beta^* \left( \frac{Y_t^{p*}}{Y_0^{p*}} \right)^{\frac{\sigma_p}{\sigma_p - 1}} + (1 - \beta^*) \left( \frac{N_t^*}{N_0^*} \right)^{\frac{\sigma_{p-1}}{\sigma_p}} \right] \frac{\sigma_p}{\sigma_p - 1} \left( \frac{Y_t^{p*}}{Y_0^{p*}} \right)^{\frac{\sigma_p - 1}{\sigma_p - 1}} \frac{1}{N_t^*}$$

where $P_t^{p*}$ is the commodity price in foreign currency, $\bar{MC}_t^*$ is the unit marginal cost of the labour-capital aggregate $N_t^*$ and $\lambda_t^* P_t$ is the nominal unit production cost of the final good $Y_t^*$. These FOCs give the commodity ratio:

$$\frac{Y_t^{p*}}{N_t^*} = \left( \frac{\bar{MC}_t^*}{P_t^{p*}} \frac{\beta^*}{1 - \beta^*} \right) \frac{\sigma_p}{\sigma_p - 1} \left( \frac{N_0^*}{Y_0^{p*}} \right)^{\sigma_{p-1}}$$

where we can see that commodity demand depends on its price relative to the labour-capital input (where $\sigma_p^*$ is the elasticity of substitution of commodities) and to the intensity of labour and capital used in the foreign economy. It also give the final good marginal cost:

$$\lambda_t^* = \frac{1}{P_t^* Y_0^*} \left( (\beta^*)^{\frac{\sigma_p}{\sigma_p}} \left( P_t^{p*} Y_0^{p*} \right)^{1 - \frac{1}{\sigma_p}} + (1 - \beta^*)^{\frac{\sigma_p}{\sigma_p}} \left( \bar{MC}_t^* N_0^* \right)^{1 - \frac{1}{\sigma_p}} \right)^{-\frac{1}{1 - \sigma_p}}$$
A.2.2 Secondary sector

Secondary good producers  Cost minimization problem for the intermediate firm $i$ in period $t$ is given by

$$
\min_{K^i_t, H^i_t} W^f_i H^f_i + R^{k.f}_t K^f_i + \lambda_t P_t Y^f_i
$$

where the Lagrange multiplier $\lambda_t$ represents the nominal cost of producing an additional unit of the domestic good and $\lambda_t$ is the real marginal cost.

The first order conditions, with respect to $H^f_i$ and $K^f_i$, for firm’s $i$ cost minimization problem are given by:

$$
W^f_i = (1-\alpha)\lambda_t P^f_0 \left( \frac{\varepsilon_t K^f_i}{K^f_0} \right)^{\alpha_f} \left( \frac{H^f_i}{H^f_0} \right)^{(1-\alpha_f)} \frac{1}{H^f_i}
$$

$$
R^{k.f}_t = \alpha \lambda_t P^f_0 \left( \frac{\varepsilon_t K^f_i}{K^f_0} \right)^{\alpha_f} \left( \frac{H^f_i}{H^f_0} \right)^{(1-\alpha_f)} \frac{1}{K^f_i}
$$

From those equations we can find the capital to labour ratio:

$$
\frac{K^f_i}{H^f_i} = \left( \frac{\alpha_f}{1-\alpha_f} \right) \frac{w^f_i}{r^{k.f}_t}, \quad (A.26)
$$

As well as the equilibrium real marginal cost of the domestic input $mc_t$:

$$
mc_t \equiv \lambda_t = \frac{1}{Y^f_0} \left( \frac{H^f_0}{1-\alpha_f} \right)^{1-\alpha_f} \left( \frac{K^f_0}{\alpha_f} \right)^{\alpha_f} \left( \frac{r^{k.f}_t}{\varepsilon_t} \right)^{\alpha_f} \left( \frac{w^f_i}{1-\alpha_f} \right)^{1-\alpha_f}, \quad (A.27)
$$

Domestic Distributors  The optimization problem faced by the intermediate distributor $i$ when setting its price at time $t$ taking aggregator’s demand as given reads:

$$
\max_{P^d_t} \sum_{s=0}^{\infty} (\beta \xi_d)^s v_{t+s} (P^d_{i,t+s} - MC_{t+s}) Y^d_{i,t+s}
$$
where

\[
P_{t,t+s} = (\pi_t \cdots \pi_{t+s-1})^{K_d(\bar{\pi})^{(1-K_d)s} p_{t}^{new}}
\]

\[
Y_{t,t+s} = \left( \frac{(\pi_t \cdots \pi_{t+s-1})^{K_d(\bar{\pi})^{(1-K_d)s} p_{t}^{new}}}{P_{t+s}} \right)^{-\epsilon_d} Y_{t+s}^d
\]

\[
Y_{t+s}^d = C_{t+s}^d + l_{t+s}^d
\]

These expressions can be used to rewrite the maximisation problem as:

\[
E_t \sum_{s=0}^{\infty} (\beta \xi_d)^s \Psi_{t+s} \left( \frac{(\Pi_{t,t+s-1})^{K_d(\bar{\pi})^{(1-K_d)s} p_{t}^{new}}}{\Pi_{t+1,t+s}} - mc_{t+s} \right) \left( \frac{(\Pi_{t,t+s-1})^{K_d(\bar{\pi})^{(1-K_d)s} p_{t}^{new}}}{\Pi_{t+1,t+s}} \right)^{-\epsilon_d} Y_{t+s}^d
\]

where \( p_{t}^{new} = \frac{p_{t}^{new}}{P_{t}} \). Distributing for convenience gives:

\[
E_t \sum_{s=0}^{\infty} (\beta \xi_d)^s \Psi_{t+s} \left( \left( \frac{(\Pi_{t,t+s-1})^{K_d(\bar{\pi})^{(1-K_d)s} p_{t}^{new}}}{\Pi_{t+1,t+s}} \right)^{1-\epsilon_d} \frac{(\Pi_{t,t+s-1})^{K_d(\bar{\pi})^{(1-K_d)s} p_{t}^{new}}}{\Pi_{t+1,t+s}} \right) \left( \frac{(\Pi_{t,t+s-1})^{K_d(\bar{\pi})^{(1-K_d)s} p_{t}^{new}}}{\Pi_{t+1,t+s}} \right)^{-\epsilon_d} Y_{t+s}^d
\]

The FOC with respect to \( p_{t}^{new} \) reads:

\[
(\epsilon_d - 1) \left( p_{t}^{new} \right)^{-\epsilon_d} E_t \sum_{s=0}^{\infty} (\beta \xi_d)^s \Psi_{t+s} \left( \frac{(\Pi_{t,t+s-1})^{K_d(\bar{\pi})^{(1-K_d)s} p_{t}^{new}}}{\Pi_{t+1,t+s}} \right)^{1-\epsilon_d} Y_{t+s}^d
\]

\[
= \epsilon_d \left( p_{t}^{new} \right)^{-\epsilon_d - 1} E_t \sum_{s=0}^{\infty} (\beta \xi_d)^s \Psi_{t+s} \left( \frac{(\Pi_{t,t+s-1})^{K_d(\bar{\pi})^{(1-K_d)s} p_{t}^{new}}}{\Pi_{t+1,t+s}} \right)^{-\epsilon_d} mc_{t+s} Y_{t+s}^d
\]

and can be rewritten as:

\[
p_{t}^{new} = \frac{\epsilon_d}{\epsilon_d - 1} \frac{E_t \sum_{s=0}^{\infty} (\beta \xi_d)^s \Psi_{t+s} \left( \frac{(\Pi_{t,t+s-1})^{K_d(\bar{\pi})^{(1-K_d)s} p_{t}^{new}}}{\Pi_{t+1,t+s}} \right)^{-\epsilon_d} mc_{t+s} Y_{t+s}^d}{E_t \sum_{s=0}^{\infty} (\beta \xi_d)^s \Psi_{t+s} \left( \frac{(\Pi_{t,t+s-1})^{K_d(\bar{\pi})^{(1-K_d)s} p_{t}^{new}}}{\Pi_{t+1,t+s}} \right)^{1-\epsilon_d} Y_{t+s}^d}
\]

which can also be rewritten as a set of three equations:

\[
p_{t}^{new} = \frac{\epsilon_d}{\epsilon_d - 1} \frac{X_{1,t}^D}{X_{2,t}^D} \tag{A.28}
\]

\[
X_{1,t}^D = \psi_t mc_t (C_t^d + l_t^d) + \beta \xi_d \left( \frac{\pi_t^{K_d} \bar{\pi}^{1-K_d}}{\pi_{t+1}} \right)^{-\epsilon_d} E_t X_{1,t+1}^D \tag{A.29}
\]

\[
55
\]
\[
X_{2,t}^D = \psi_t(C_t^d + I_t^d) + \beta \xi_d \left( \frac{\pi_t^{\kappa_d} \bar{\pi}_t^{1-k_d}}{\pi_{t+1}} \right)^{1-\epsilon_d} E_t X_{2,t+1}^D \tag{A.30}
\]

In addition, the domestic price index evolves according to:

\[
1 = \xi_d \left( \frac{\pi_t^{\kappa_d} \bar{\pi}_t^{1-k_d}}{\pi_t} \right)^{1-\epsilon_d} + (1 - \xi_d)(p_t^{new})^{1-\epsilon_d} \tag{A.31}
\]

Finally, the price dispersion measure

\[
\nu_t^d = \int_0^1 \left( \frac{P_{1,t}}{P_t} \right)^{-\epsilon_d} di
\]

can be written as:

\[
\nu_t^d = (1 - \xi_d) \left( \frac{p_t^{new}}{P_t} \right)^{-\epsilon_d} + \int_{1-\xi_d}^1 \left( \frac{\left(\pi_{t-1}\right)^{\kappa_d} \left(\bar{\pi}\right)^{1-k_d} P_{t-1,t-1}}{P_t} \right)^{-\epsilon_d} di
\]

which simplifies to:

\[
\nu_t^d = (1 - \xi_d) (p_t^{new})^{-\epsilon_d} + \int_{1-\xi_d}^1 \left( \frac{\left(\pi_{t-1}\right)^{\kappa_d} \left(\bar{\pi}\right)^{1-k_d} P_{t-1,t-1}}{\pi_t} \right)^{-\epsilon_d} di
\]

then to:

\[
\nu_t^d = (1 - \xi_d) (p_t^{new})^{-\epsilon_d} + \xi_d \left( \frac{\left(\pi_{t-1}\right)^{\kappa_d} \left(\bar{\pi}\right)^{1-k_d}}{\pi_t} \right)^{-\epsilon_d} \nu_{t-1}^d \tag{A.32}
\]

which is a function of aggregate variables only.

**Importing and exporting distributors** Optimisation in the importing and exporting distributors price setting problem is similar to the domestic good price setting problem presented above. The difference is that importing and exporting firms face the following marginal cost to sale price ratio: \(\frac{S_{MC_t}^*}{P_t^{new}}\) and \(\frac{MC_t}{S_t^*P_t^*}\), respectively. The difference with Adolfson et al. (2007) is that the importing firm consider \(MC_t^*\) instead of \(P_t^*\) for its nominal marginal costs, and that exporting firms consider \(MC_t\) instead of \(P_t\).
B Steady state

Here are the details on the computation of steady-state for the domestic economy.

**Calibration and choice of units** First some variables are calibrated to some values reflecting some freedom in the choice of units:

\[
\begin{align*}
Y^f &= Y^f_0 = 1 \\
P_0 &= P^*_0 = 1 \\
h^r &= h^s = 1/3
\end{align*}
\]

where \(Y^f_0\) and \(P_0\) are free choice of units and \(h_j = 1/3\) ensures that agents devote on average 1/3 of their time to labour activities and just imposes to calibrate \(A_h\) accordingly. It implies that total hours worked by savers and rule of thumb consumers is given by \(H^r = H^s = 1/3\); that \(H = H^r + H^s\) and that the time they spend working in each sectors is given by \(H^p = \omega_h H\) and \(H^f = (1 - \omega_h)H\).

The primary commodity sector’s share in GDP is calibrated to \(\omega_p\) to match its empirical counterpart. it implies that

\[
\begin{align*}
Y &= \frac{Y^f}{1 - \omega_p} \\
Y^p &= Y^p_0 = \omega_p Y
\end{align*}
\]

Assuming that inflation and and the risk-free rates are the same in the domestic and foreign economies:

\[
\begin{align*}
\pi &= \pi^* \\
R &= R^*
\end{align*}
\]

where \(\pi\) and \(R\) are calibrated to match the empirical mean of domestic variables. It implies that \(dS = 1\) (through the UIP condition). As a consequence, all inflations rates are equal to \(\pi\). By carefully calibrating mark-ups\(^{27}\) for each distributors, all relative prices \(\gamma\) equalise to one at steady-state.

\(^{27}\)by assuming that mark-ups in the import and foreign distribution sectors are identical; and that mark-ups in the export and domestic distribution sectors are identical
**Households**  Turning to patient households FOCs, the assumptions presented above implies that:

\[ \beta = \frac{\pi}{R} \]

and pins down the real price of capital and its rental rate to

\[ p_k' = \frac{p_k}{P} = \frac{p^i}{P} = 1 \]
\[ r^k = \frac{p_k'(1 - (1 - \delta)\beta)}{(1 - \tau^k)\beta} \]

where the real price of capital and its rental rate are the same in both sectors at steady-state.

**Final good sector**  Turning to final good distributors, the marginal costs are given by:

\[ mc = mc^x = \frac{\varepsilon_d - 1}{\varepsilon_d} \]
\[ mc^m = mc^* = \frac{\varepsilon_* - 1}{\varepsilon_*} \]

In addition, the use of a normalised production function in the final good sector allows to write

\[ MC_iY^f_i = R^k_jK^f_i + W^f_iH^f_i \]

where the capital and labour income shares at steady-state are given (in their stationary form) by

\[ w^f_H^f = (1 - \alpha)mcY^f \]
\[ r^{k,f}K^f = \alpha mcY^f \]

which implies that

\[ w^f = \frac{(1 - \alpha)Y^f mc}{H^f} \]
\[ K^f = \frac{\alpha Y^f mc}{r^k} \]

It also implies that the value of investment is:

\[ I^f = \delta K^f \]
Primary good sector  Since wages are equal in both sectors at steady-state, \( \bar{w} = \bar{w}^p = \bar{w}^f \). In addition, using once again a Normalised production function implies that

\[
S_t P^p_t Y_t^p = R_t^k P_t^p K_t^p + W_t^p H_t^p + R_t^L L_0^p
\]

where \( R_t^L \) is the rental rate of the land input. It implies that

\[
\begin{align*}
w^p H^p &= (1 - \alpha_p - \beta_p) Y^p \\
r^{k,p}K^p &= \alpha_p Y^p
\end{align*}
\]

which, since \( H^p \) is calibrated, imposes to set

\[
\begin{align*}
\beta_p &= 1 - \alpha_p - \frac{w^p H^p}{Y^p} \\
K^p &= \frac{\alpha_p Y^p}{r^k}
\end{align*}
\]

where \( \frac{w^p H^p}{Y^p} \) is the labour income share in the primary sector. Therefore,

\[
I^p = \delta K^p
\]

Aggregate resource constraints and definitions  Total, imported and domestic investments are given by

\[
\begin{align*}
I &= I^f + I^p \\
I^m &= \omega_i I \\
I^d &= (1 - \omega_i) I
\end{align*}
\]

The aggregate resource constraint evaluated at steady state reads

\[
Y^f - G = C^d + I^d + X^f
\]

Plugging steady state domestic consumption values from households yields

\[
Y^f - G = (1 - \omega_c) C + I^d + X^f
\]
The net foreign assets accumulation rule gives

\[ C^m + I^m = Y^p + X^f + \left( \frac{R}{\pi} - 1 \right) A \]

Plugging steady state value of imported consumption we have,

\[ \omega_c C + I^m = Y^p + X^f + \left( \frac{R}{\pi} - 1 \right) A \]

Assuming the net foreign asset position\(^{28}\) is calibrated, these are two equations with only \(x^f\) and \(c\) unknown. Solving yields

\[ C = Y^f - (I^m + I^d + G) + Y^p + \left( \frac{R}{\pi} - 1 \right) A \]

\[ X^f = Y^f - G - C^d - I^d \]

It implies that \(C^m = \omega_c C\) and \(C^d = (1 - \omega_c)C\). The consumption of rule of thumbs households is given by \(C^R = \frac{1 - \tau_w}{(1 + \tau_w)(1 + \tau_c)} wH^r + tr^r\) where \(tr^r\) can be set in order to attain any objective on \(C^R\) including \(C^R = C/2\) or set to zero for simplicity. The consumption level of savers is then simply given by \(C^S = C - C^r\).

\(^{28}\) Any net foreign asset position can be made consistent with steady state by setting the parameter \(\bar{A} = A\) in \(\Phi(.)\).
C Welfare cost measures

Conditional compensation measures are defined as the fraction of consumption that an agent \( j \) would be ready to give up in the economy \( l \) in order to be equally well off in that economy than in an alternative economy \( v \), when both economies state variables are initially identical. This appendix shows how to compute conditional compensation for a domestic rule of thumbs household. The method to derive this measure for savers or foreign households is similar.

In order to clarify notations, let \( W \left( (1 - \lambda)C_{j}^{R,l}, h_{j}^{R,l} \right) \) denotes welfare of a rule-of-thumb household \( j \) in economy \( l \) where this household renounces to a fraction \( \lambda \) of its consumption flow \( C_{j}^{R,l} \) at every period. Similarly, let \( W \left( C_{j}^{R,v}, h_{j}^{R,v} \right) \) be its welfare in economy \( v \).

The maximum fraction \( \lambda_{R}^{R} \) of consumption that household \( j \) would be ready to give up in economy \( l \) in order to be as well off as under economy \( v \) should satisfy

\[
E_{0}W \left( (1 - \lambda_{R}^{R})C_{j}^{R,l}, h_{j}^{R,l} \right) = E_{0}W \left( C_{j}^{R,v}, h_{j}^{R,v} \right)
\]  
(C.1)

in order to make this household indifferent between the two environments. In this expression, the conditional expectation operator \( E_{0} \) conditions on the initial state of the economy (assumed to be its steady-state) and integrates over the probability density of aggregate disturbances (business cycle shocks presented in Table 1) and idiosyncratic shocks (Calvo price and wage stickiness affecting each households differently). It is therefore convenient to rewrite this expression as

\[
E_{0} \int_{0}^{1} W \left( (1 - \lambda_{R}^{R})C_{j}^{R,l}, h_{j}^{R,l} \right) \, dj = E_{0} \int_{0}^{1} W \left( C_{j}^{R,v}, h_{j}^{R,v} \right) \, dj
\]  
(C.2)

where the conditional expectation operator \( E_{0} \) still conditions on the initial state of the economy but now only integrates over the probability density of aggregate disturbances while the integral explicitly integrates over households idiosyncratic shocks.

Expanding on the left-hand side of the above equation yields

\[
E_{0} \int_{0}^{1} \sum_{t=0}^{\infty} \beta^{t} \left\{ \frac{\left( (1 - \lambda_{R}^{R})C_{j,t}^{R,l} / \left( C_{t-1}^{R,l} \right)^{b} \right)^{1-\sigma_{c}}}{1 - \sigma_{c}} - \frac{A_{h}(h_{j,t}^{R,l})^{1+\sigma_{s}}}{1 + \sigma_{h}} \right\} \, dj = E_{0} \int_{0}^{1} W \left( C_{j}^{R,v}, h_{j}^{R,v} \right) \, dj
\]  
(C.3)
Solving for the integrals in the left and right-hand sides gives

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left( \frac{1 - \lambda_R^R}{(C_t^R)} \right) b \left( \frac{h_t^R}{(C_{t-1}^R)} \right) \frac{1 - \sigma_c}{1 - \sigma_c} \psi_t^c - 1 \right\} - \frac{A_h(h_t^R)^{1+\sigma_h}}{1+\sigma_h} \psi_t^v = \mathbb{E}_0 W(C_t^{R,v}, h_t^{R,v})$$

(C.4)

This expression can be solve for $\lambda_R^u$. Indeed, rearranging the terms gives

$$\lambda_R^u = 1 - \left( \frac{\mathbb{E}_0 \left( C_t^{R,v}, h_t^{R,v} \right) - \mathbb{E}_0 \left( h_t^{R,l} \right) + \frac{1}{(1-\beta)(1-\sigma_c)}}{\mathbb{E}_0 \left( C_t^{R,l} \right) + \frac{1}{(1-\beta)(1-\sigma_c)}}\right)^{1-\sigma_c}$$

(C.5)

which can be used in order to measure welfare costs. This expression for $\lambda_R^u$ can be evaluated provided that we can evaluate the functions $\mathbb{E}_0 W(\cdot)$. These functions are given by

$$\mathbb{E}_0 \left( C_t^{R,v}, h_t^{R,v} \right) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left( \frac{C_t^{R,v}}{(C_{t-1}^{R,v})} \right) b \left( \frac{h_t^R}{(C_{t-1}^{R,l})} \right) \frac{1 - \sigma_c}{1 - \sigma_c} \psi_t^c - 1 \right\} - \frac{A_h(h_t^R)^{1+\sigma_h}}{1+\sigma_h} \psi_t^v$$

(C.6)

$$\mathbb{E}_0 \left( h_t^{R,l} \right) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ -\frac{A_h(h_t^R)^{1+\sigma_h}}{1+\sigma_h} \psi_t^v \right\}$$

(C.7)

$$\mathbb{E}_0 \left( C_t^{R,l} \right) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left( \frac{C_t^{R,l}}{(C_{t-1}^{R,l})} \right) b \left( \frac{h_t^R}{(C_{t-1}^{R,l})} \right) \frac{1 - \sigma_c}{1 - \sigma_c} \psi_t^c - 1 \right\}$$

(C.8)

and can be evaluated by simulations. It requires to condition the initial state of the economy to be its steady-state, to generate shocks from their probability density and to simulate the model.

In Dynare, the trick is simply to rewrite these sums as

$$W_{t=0}(\cdot) = U_{t=0}(\cdot) + \beta E_{t=0} W_{t=1}(\cdot)$$

(C.9)

where $E_{t=0}$ is the expectation operator conditional on information available at time $t = 0$; and to simulate the model in order to recover the values of $W_{t=0}(\cdot)$. By averaging over enough simulations (and therefore averaging over different realisations of the shocks) the law of large number ensures convergence to $E_{0} W_{t=0}(\cdot)$.