Labor Mobility in a Monetary Union

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February 2018

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Abstract

The optimal currency literature has stressed the importance of labor mobility as a precondition for the success of monetary unions. But only a few studies formally link labor mobility to macroeconomic adjustment and policy. In this paper, we study macroeconomic dynamics and monetary policy in an economy with cyclical labor flows across two distinct regions sharing trade links and a common monetary framework. In our New Keynesian DSGE model, migration flows are driven by fluctuations in the relative labor market performance across the monetary union. Whilst labor mobility can be an additional channel for cross-regional spillovers as well as a regional shock absorber, we find that a mobile labor force significantly reduces the welfare costs of joining a monetary union. Labor mobility does not affect the welfare ranking of a set of simple monetary policy rules. But it affects optimal monetary policy. In later versions of this paper, we plan to present results from a study of the optimal conduct of monetary policy in this environment.

Keywords: Labor Mobility, Monetary Policy, Monetary Union.

JEL Codes: E32, E52, F45.

1. Introduction

A mobile labor force may help cushion the effects of regional shocks when conventional stabilization mechanisms are unavailable. The optimal currency literature in the tradition of Mundell (1961) has therefore stressed the importance of labor mobility as a precondition for monetary unions (see e.g. Dellas and Tavlas (2009), for a survey). But only a few studies formally link labor mobility to macroeconomic adjustment and policy. In this paper, we seek to fill this gap in the literature by studying macroeconomic dynamics in an economy with cyclical labor flows across two distinct regions

\textsuperscript{☆}The views expressed are solely those of the authors and do not necessarily reflect those of the Bank of Canada or the Bank of England or its committees. This version of the paper is preliminary and incomplete.
sharing trade links and a common monetary policy. Our ultimate aim is to answer normative questions related to the conduct of monetary union when labor is mobile across borders.

In our New Keynesian DSGE model, workers may participate in the labor market of both regions as native and migrant workers, respectively. The share of unemployed workers crossing the regional border in search of a job is chosen optimally by households. Employment is then determined through a search and matching process (Diamond (1982), Mortensen and Pissarides (1999)) in the regional labor market. Hence, migration flows are driven by fluctuations in the relative labor market performance across the monetary unions as households allocate workers to equalize expected net gains from participation in the two regional labor markets. This key feature of the model is consistent with the empirical evidence, see e.g., Saks and Wozniak (2011) and Lkhagyasuren (2012).

Within this environment, we first study macroeconomic dynamics following a set of economic disturbances. We show that labor mobility can be an additional channel for cross-regional spillovers as well as a regional shock absorber. Following a regional shock to total factor productivity (TFP) for example, labor mobility reduces macroeconomic volatility in the region where the disturbance originates, facilitating the adjustment process in the labor market following technological change. But it also increases volatility in the rest of the monetary union, and effectively ‘exports’ the unemployment that would otherwise be associated with technological advances. From a welfare perspective, the flow of migrants is welcomed by workers in other regions, who work overtime as a consequence of the common monetary authority’s accommodating policy stance. Similarly, following an increase in regional job market efficiency, migrants flow out of the more efficient region as firms reap a large share of the efficiency gains in the form of lower real wages.

Second, we assess the impact of labor mobility on the welfare cost of joining a monetary union. In line with the optimal currency literature, a mobile labor force is found to significantly reduce the cost of joining a monetary union – defined as the difference in welfare losses associated with a given monetary policy rule in a monetary union and when both regions have access to independent monetary policies. For our benchmark calibration, assuming no home bias in consumption, labor mobility leads to a reduction in these costs of 30-80%, depending on the monetary policy in place. In line with Farhi and Werning (2014), we also find a strong interaction between trade integration and labor mobility. Specifically, relaxing our baseline assumption of no home bias leads to a more limited role of trade in absorbing region-specific shocks. The more pronounced asymmetric effects of these regional shocks therefore lead the union-wide monetary policy to face stronger trade offs. The cost of joining a monetary union with a fixed labor force thus significantly increases with some home bias in consumption, providing more room for a mobile labor force to reduce it. In that respect, reducing the role of interregional trade significantly strengthens the welfare-enhancing role of a mobile labor force.
Finally, we find that a mobile labor force does not affect the welfare ranking of a set of simple monetary policy rules in a monetary union. But with a mobile labor force, the standard distortions from price rigidities interact with an additional distortion stemming from inefficient migration decisions in the market equilibrium. The new inefficiency arises as households do not internalize the impact of their migration decisions on aggregate labor-market variables. A later version of this paper will discuss the implications of this interaction for the optimal conduct of monetary policy.

The paper is organized as follows. Section 2 lays out the model. The efficient allocation is characterized and compared to the market equilibrium in Section 3. Section 4 presents the calibration to U.S. data. Section 5 contains a description of macroeconomic dynamics following disturbances to supply, demand, and labor market efficiency. Welfare results are reported in Section 6, and concluding remarks in Section 7.

2. The Model

The model economy is composed of two regions $i$ and $j$ sharing a common monetary authority. In each region, households consume and enjoy leisure. A share of workers are employed either at home as native workers, or across the regional border as migrant workers. At the beginning of each period, households allocate those workers that are not currently in employment to the two labor markets. Here, wages are settled with firms through a wage bargaining process. Regional intermediate firms hire new native and migrant workers from the pool of job searchers before selling their products to retailers in a competitive wholesale market. Retailers turn intermediate products into marketable goods for final use in the economy. They operate under monopolistic competition and set prices subject to a nominal rigidity. The common monetary authority targets union-wide inflation and sets the nominal interest rates according to a simple instrument rule.

2.1. Households

The representative household in region $i$ consists of a continuum of family members of mass 1. During any given period $t$, individual members are employed or unemployed either in labor market $i$ ($E_i^{ui}$ and $U_i^{ui}$) or in labor market $j$ ($E_i^{uj}$ and $U_i^{uj}$) so that $E_i^{ui} + U_i^{ui} + E_i^{uj} + U_i^{uj} = 1$. When a period ends, a fraction $\rho_i$ of the employed in region $i$, and a fraction $\rho_j$ of those employed in region $j$, are separated from their jobs. At the beginning of each period the total mass of household $i$ members searching for a job is given by those members who spent the previous period as unemployed as well as those

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1 We consider four different rules: a simple Taylor rule targeting inflation, a simple Taylor rule targeting both inflation and unemployment, a Taylor rule with an interest rate smoother stabilizing inflation and output, and a regime of strict inflation targeting.
who have just lost their jobs, in either of the two labor markets:

\[ S_{H,t}^i = U_{t-1}^{ii} + \rho^i E_{t-1}^{ii} + U_{t-1}^{ij} + \rho^j E_{t-1}^{ij} \]

\[ = 1 - (1 - \rho^i) E_{t-1}^{ii} - (1 - \rho^j) E_{t-1}^{ij} \]  \hspace{1cm} (1)

We assume that the household allocates its pool of searchers to each of the two labor markets, regardless of their previous migration status. Specifically, taken labor market conditions as given, the household keeps a fraction \( \Omega_t^i \) of its searchers at home to look for jobs in region \( i \), \( S_{ii,t}^i = \Omega_t^i S_{H,t}^i \), while the remaining searchers are sent as migrants to seek employment in region \( j \), \( S_{ij,t}^i = (1 - \Omega_t^i) S_{H,t}^i \). Once the migration decision has been made, members can search in their designated regional labor market only. If they fail to find a job in that particular market, they will spend the period there in unemployment. Assuming instantaneous hiring, the evolution of employment of household \( i \) members as native workers in the home labor market becomes

\[ E_{t}^{ii} = (1 - \rho^i) E_{t-1}^{ii} + M_t^{ii} = (1 - \rho^i) E_{t-1}^{ii} + f_t^{ii} S_t^{ii} \]  \hspace{1cm} (2)

where \( M_t^{ii} \) denotes the number of successful matches of job searchers, \( S_t^{ii} \), to vacancies in region \( i \), and \( f_t^{ii} \equiv M_t^{ii}/S_t^{ii} \) is the job finding rate. Similarly, the evolution of employment of household \( i \) migrants in region \( j \) becomes

\[ E_{t}^{ij} = (1 - \rho^j) E_{t-1}^{ij} + f_t^{ij} S_t^{ij} \]  \hspace{1cm} (3)

with \( f_t^{ij} \equiv M_t^{ij}/S_t^{ij} \). Corresponding unemployment rates are

\[ U_t^{ii} \equiv S_t^{ii} - M_t^{ii} = (1 - f_t^{ii}) S_t^{ii}; \quad U_t^{ij} \equiv (1 - f_t^{ij}) S_t^{ij} \]  \hspace{1cm} (4)

For convenience, we assume that family members consume and enjoy leisure in the region of their origin and that idiosyncratic income risk is perfectly insured away within the family.\(^2\) In period \( t \), then, the household chooses consumption, \( C_t^i \), savings in the form of a portfolio of state-contingent zero-coupon nominal bonds, \( D_{t+1}^i \), as well as the share of searchers remaining in the home labor market, \( \Omega_t^i \), to maximize expected life-time utility, which we specify in the following form

\[ E_t \sum_{t=0}^{\infty} \beta^t U_t^i = E_t \sum_{t=0}^{\infty} \beta^t \left[ (C_t^i)^{\beta} (l_t^i)^{1-\beta} \right]^{1-\sigma} - 1, \]  \hspace{1cm} (5)

\(^2\)Considerations relative to the location and type of consumption of different workers, as discussed in Farhi and Werning (2014), are thus abstracted from.
with
\[
\tilde{l}_t^i = 1 - \tilde{L}_t^i, \\
\tilde{L}_t^i = \left[ (1 - \alpha_1)^{1/\nu_1} (\tilde{L}_t^i)^{\frac{\nu_1}{\nu_1-1}} + (\alpha_1)^{1/\nu_1} (\tilde{L}_t^j)^{\frac{\nu_1}{\nu_1-1}} \right]^\frac{\nu_1}{\nu_1-1}, \\
\tilde{L}^i_{ji} = E^i_{ti} + \zeta U^i_{ti}, \\
\tilde{L}^j_{ij} = E^j_{ti} + \zeta U^j_{ti}, \\
C^i_t = \left[ (1 - \alpha_2)^{1/\nu_2} (C^i _H,t)^{\frac{\nu_2}{\nu_2-1}} + (\alpha_2)^{1/\nu_2} (C^i _F,t)^{\frac{\nu_2}{\nu_2-1}} \right]^\frac{\nu_2}{\nu_2-1}.
\]

\(\tilde{L}_t^i\) captures the utility cost in terms of the time spent on labor market activities in either of the two labor markets. This disutility is assumed to be a CES aggregator of family members’ labor market activities in regions \(i\) and \(j\), where \(\nu_1 < 0\) measures the elasticity of substitution between being a native member of the labor force in region \(i\) and being a migrant member of the labor force in labor market \(j\), and \(\alpha_1\) captures differences in the disutility attached to working or searching in either of the two regions.\(^3\) Equations (8) and (9) further define the time intensity of the labor force in the two regions, where \(\zeta\) captures time spent on labor market activities by the unemployed, such as searching for jobs, developing skills or collecting unemployment benefits. Similarly, aggregate consumption is measured by a CES aggregator of domestic and imported consumption goods, each in turn given by a CES function,

\[
C^i_{H,t} = \left[ \int_0^1 C^i_{H,t}(l)^{\frac{\nu_1}{\nu_1-1}} dl \right]^{\frac{\nu_1}{\nu_1-1}}; \quad C^i_{F,t} = \left[ \int_0^1 C^i_{F,t}(l)^{\frac{\nu_2}{\nu_2-1}} dl \right]^{\frac{\nu_2}{\nu_2-1}}
\]

where \(l \in [0,1]\) denotes the consumption good variety, assuming that each country produces a continuum of differentiated final goods. The parameter \(\alpha_2\) reflects the openness of the regional economy and is inversely related to households degree of home bias in consumption, whilst \(\nu_2\) determines the elasticity of substitution between domestic and foreign goods.

Total consumption expenditures of the representative household in \(i\) is given by \(P^i_{H,t} C^i_{H,t} + P^i_{F,t} C^i_{F,t} = P^i_t C^i_t\), where \(P^i_{H,t}\) and \(P^i_{F,t}\) are indices of domestically produced and imported goods, respectively, and \(P^i_t\) is the consumer price index (CPI). Hence, households maximize utility subject to the following budget constraint

\[
P^i_t C^i_t + E_t \left( Q_{t,t+1} D^i_{t+1} \right) \leq D^i_t + W^i E^{ii}_t + P^i b^{ij} U^{ii}_t + W^{ij} E^{ij}_t + P^i b^{ij} U^{ij}_t + T^i_t.
\]

The employed earn \(W^i\) as natives and \(W^{ij}\) as migrants, and the unemployed are entitled to unemployment benefits specific to the labor market they are searching in

\(^3\)As shown in Section 4 \(\alpha_1\) pins down the steady-state share of migrant (un)employment to total (un)employment as well as the share of migrant searchers in the entire pool of searchers.
\( (b_{ii} \equiv B_{ii} / P_{it}^i \text{ and } b_{ij} \equiv B_{ij} / P_{jt}^j, \text{ respectively}). \) \( Q_{i,t+1}^i \) is the stochastic discount factor, and \( T_i^t \) are lump-sum taxes paid in region \( i. \)

The first-order conditions with respect to consumption, \( C_{it}^i \), and the zero-coupon bond, \( D_{it+1}^i \), yields a standard Euler equation\(^4\)

\[
\beta R_t E_t \left\{ \left( \frac{U_{it+1}^i}{U_{it}^i} \right) \left( \frac{1}{\Pi_{it+1}^i} \right) \right\} = 1,
\]

where \( \Pi_{it+1}^i \equiv P_{it+1}^i / P_{it}^i \) captures CPI inflation in region \( i \) and where \( U_{it}^i \) denotes the marginal utility of consumption of household \( i \) in period \( t. \) For the given utility specification (5) the marginal utility of consumption states as

\[
U_{it}^i = \left[ (C_t^i)^{b_i} (l_t^i)^{1-b_i} \right]^{-\sigma} b_i \left( \frac{C_t^i}{l_t^i} \right)^{b_i-1} Z_t^i.
\]

\( Z_t^i \) is a preference shock, where

\[
\log Z_t^i = \rho \log Z_{t-1}^i + \nu_{t,z}^i,
\]

and \( \nu_{t,z}^i \) is i.i.d with zero mean and variance \( \sigma_a^2. \) Under the assumption of complete markets for securities traded across the two geographical regions, equalizing stochastic discount factors gives a standard risk-sharing condition

\[
U_{it}^i Q_{it}^i = U_{jt}^j \vartheta
\]

where \( Q_{it}^i \equiv P_{it}^j / P_{it}^i \) is the real exchange rate, and \( \vartheta \) is a constant which depends on initial conditions.

The first-order condition with respect to \( \Omega_t^i \) – derived under the assumption that households take both job finding rates and real wages as given (see Appendix A for further details) – gives a condition determining the optimal migration flows:

\[
f_{ii}^i W_{it}^{ii} / P_{it}^i + (1 - f_{ii}^i) b_{ii}^i - U_{it}^i / U_{it}^{ii} \left[ \left( \frac{1 - \alpha_1 \bar{L}_t^i}{\bar{L}_t^i} \right) \right]^{1/\nu_1} \left[ f_{ii}^i + (1 - f_{ii}^i) \zeta \right] =
\]

\[
f_{ij}^j W_{jt}^{ij} Q_t^j + (1 - f_{ij}^j) b_{ij}^j - U_{jt}^j / U_{jt}^{ij} \left[ \left( \frac{\alpha_1 \bar{L}_t^j}{\bar{L}_t^j} \right) \right]^{1/\nu_1} \left[ f_{ij}^j + (1 - f_{ij}^j) \zeta \right].
\]

This condition states that the household allocates searchers to the two labor markets so that the respective expected net benefits from participating in them are equalized. With probability \( f_{ii}^i, \) a searcher in region \( i \) is matched to a vacancy and earns a real wage \( W_{it}^{ii} / P_{it}^i. \) With the complement probability, the searcher does not find a job and is

\(^4\text{By a no-arbitrage argument } E_t \{ Q_{i,t+1}^i \} = R_t^{-1}.\)
left with the unemployment benefit $b^{ii}$. The expected net benefit from sending an extra searcher to region $i$ is therefore the probability weighted sum of the real wage and the unemployment benefit in excess of the expected utility cost of labor market activities in region $i$. These activities require one unit of time if employed (with probability $f_{ii}^{ii}$) and $\zeta$ units if unemployed (with probability $1 - f_{ii}^{ii}$) and are evaluated at the household’s marginal rate of substitution between consumption and leisure. If this expected benefit on the left-hand side of (17) were not equal to the expected benefit from participation in labor market $j$ on the right-hand side, the household could increase its expected utility by sending more searchers to the labor market with the highest expected net benefit.

2.2. Intermediate production

Intermediate firms in region $i$ produce intermediate goods according to the linear production function

$$X_{it} = A_{it} E_{it},$$

where employment is given as the sum of native and migrant workers in the region, $E_{it} = E_{ii}^{ii} + E_{ji}^{ji}$, and $A_{it}$ is an exogenous TFP process evolving according to

$$\log(A_{it}) = \rho_{i}^A \log(A_{i,t-1}) + \nu_{it},$$

where $\nu_{it}$ is an i.i.d shock with zero mean and standard deviation $\sigma_{it}^A$. To hire new workers, firms post $V_{ii}^{ii}$ vacancies for native workers and $V_{ji}^{ji}$ vacancies for migrant workers at a cost of $\kappa$ per vacancy. The number of successful hires or ‘matches’ is a function of searching workers and vacancies:

$$M_{ii}^{ii} = \omega_{it}^{i} (V_{ii}^{ii})^{\gamma} (S_{ii}^{ii})^{1-\gamma}; \quad M_{ji}^{ji} = \omega_{it}^{j} (V_{ji}^{ji})^{\gamma} (S_{ji}^{ji})^{1-\gamma}.$$  

(20)

where $1 - \gamma$ captures the elasticity of matches to searchers, and matching efficiency evolves according to

$$\log(\omega_{it}^{i}) = (1 - \rho_{i}^{i}) \log(\omega_{i,t-1}) + \rho_{i}^{i} \log(\omega_{i,t-1}) + \nu_{i,t}^{i},$$

(21)

where $\nu_{i,t}^{i}$ is an i.i.d shock with zero mean and standard deviation $\sigma_{i}^{i}$. We define labor market tightness in region $i$ for native and migrant workers, respectively, as

$$\theta_{it}^{ii} = V_{it}^{ii} / S_{it}^{ii}; \quad \theta_{it}^{ji} = V_{it}^{ji} / S_{it}^{ji};$$

(22)

Hence, the probabilities of filling a vacancy of each kind in period $t$ are given by

$$q_{it}^{ii} = \frac{M_{it}^{ii}}{V_{it}^{ii}} = \omega_{it}^{i} (\theta_{it}^{ii})^{\gamma-1}; \quad q_{it}^{ji} = \frac{M_{it}^{ji}}{V_{it}^{ji}} = \omega_{it}^{j} (\theta_{it}^{ji})^{\gamma-1}. $$

(23)
Notice that, by assumption, a filled vacancy becomes productive immediately. To derive the number of vacancies posted by intermediate firms, let $P_{x,t}^i$ denote the price of intermediate goods produced in region $i$. Since the intermediate firms operate under perfect competition, the marginal value of a filled vacancy of the two types of workers can be expressed in terms of the final consumption bundle as

$$V_{j,t}^{ii} = \frac{P_{x,t}^i}{P_t^i} A_t^i - \frac{W_{j,t}^{ii}}{P_t^i} + (1 - \rho^i)E_t\{Q_{t,t+1}^{ii}V_{j,t+1}^{ii}\} \tag{24}$$

$$V_{j,t}^{ji} = \frac{P_{x,t}^i}{P_t^j} A_t^j - \frac{W_{j,t}^{ji}}{P_t^j} + (1 - \rho^j)E_t\{Q_{t,t+1}^{ji}V_{j,t+1}^{ji}\} \tag{25}$$

The value of a filled vacancy to an intermediate producer is given by the marginal real revenue minus the real marginal cost (the real wage), plus the discounted continuation value from the match. With probability $(1 - \rho^i)$, the job remains active and earns the expected value; the job is destroyed with probability $\rho^i$ and thus has zero value.

With free entry, the expected real value of posting a vacancy (e.g. $q_{t}^{ii}V_{j,t}^{ii}$) must equal the cost ($\kappa P_{H,t}^i/P_t^i$). Hence, the marginal values of filled vacancies are constrained by the conditions

$$V_{j,t}^{ii} = \frac{\kappa}{q_t^{ii}} \frac{P_{H,t}^i}{P_t^i} ; \quad V_{j,t}^{ji} = \frac{\kappa}{q_t^{ji}} \frac{P_{H,t}^j}{P_t^j} \tag{26}$$

Combining equations (25) and (26) yields a job-creation condition determining the number of vacancies posted for each of the two types of workers:

$$\frac{\kappa}{q_t^{ii}} \frac{P_{H,t}^i}{P_t^i} = \frac{P_{x,t}^i}{P_t^i} A_t^i - \frac{W_{j,t}^{ii}}{P_t^i} + (1 - \rho^i)E_t\{Q_{t,t+1}^{ii} \frac{\kappa}{q_t^{ii}} \frac{P_{H,t+1}^i}{P_{t+1}^i}\} \tag{27}$$

$$\frac{\kappa}{q_t^{ji}} \frac{P_{H,t}^j}{P_t^j} = \frac{P_{x,t}^j}{P_t^j} A_t^j - \frac{W_{j,t}^{ji}}{P_t^j} + (1 - \rho^j)E_t\{Q_{t,t+1}^{ji} \frac{\kappa}{q_t^{ji}} \frac{P_{H,t+1}^j}{P_{t+1}^j}\} \tag{28}$$

2.3. Wage determination

Successful matches of searchers to vacancies generate an economic surplus given as the sum of the values of these matches to households and firms. For the employment of native workers in region $i$, we have $V_{j,t}^{ii} + V_{W,t}^{ii}$, where $V_{j,t}^{ii}$ is given by (25), and where the value of a native worker to household $i$, $V_{W,t}^{ii}$, reads

$$V_{W,t}^{ii} = \frac{W_{j,t}^{ii}}{P_t^i} - b^{ii} - (1 - \zeta) \frac{U_{t,t}^i}{U_{c,t}^i} \left[ \frac{(1 - \alpha_1)\tilde{L}_{t}^i}{\tilde{L}_{t}^{ii}} \right]^{1/\nu_i} + (1 - \rho^i)E_t\{Q_{t,t+1}^{ii}(1 - f_{t+1}^{ii})V_{W,t+1}^{ii}\} \tag{29}$$

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5 The value of a match to the firm can be derived as the partial derivative of the value of the firm to its owners, in turn given as the present discounted value of profit streams, with respect to employment.

6 The value of a match to the household can be derived as the partial derivative of the household’s value function with respect to employment. See Appendix A for details.
By this expression, a match generates a surplus for the household in period \( t \) by increasing real wage income at the expense of unemployment benefits and an additional disutility of working rather than being unemployed. In addition, with probability \((1 - \rho^i)\), the match has an expected discounted continuation value increasing in the expected difficulty of finding a job in the next period.

Wages are determined through a Nash bargaining process to distribute the aggregate surplus between workers and firms. Specifically, the wage is chosen to maximize the Nash product \((V_{ij}^i)^\eta (V_{ij}^i)^{1-\eta}\). The real wage chosen by the match, \(W_i^t/P_t^i\), satisfies the optimality condition

\[
\eta V_{ij}^i = (1 - \eta)V_{ij}^j \tag{30}
\]

where \(\eta\) governs the bargaining power of firms. Combining this Nash bargaining rule with the free-entry condition (26), the job-creation condition (28), and the definition of the marginal value of a native worker from equation (29), results in a wage equation for native workers in region \( i \)

\[
\frac{W_i^t}{P_t^i} = \eta \left[ b^i + (1 - \varsigma) \frac{U_{i,t}^j}{U_{e,t}^j} \left[ \frac{(1 - \alpha_1)\tilde{L}_t^i}{\tilde{L}_t^j} \right]^{1/\nu} \right] + (1 - \eta) \left[ \frac{P_{x,t}^i}{P_t^i} A_t^i + (1 - \rho^i)E_t \left( Q_{t,t+1}^i f_{t+1}^{ii} \kappa \theta_{t+1}^{ii} \frac{P_{H,t+1}^i}{P_{t+1}^i} \right) \right] \tag{31}
\]

When firms have no bargaining power \((\eta = 0)\), the wage is set to the upper bound of the wage bargaining set

\[
\frac{W_i^t}{P_t^i} = \frac{P_{x,t}^i}{P_t^i} A_t^i + (1 - \rho^i)E_t \left( Q_{t,t+1}^i f_{t+1}^{ii} \kappa \theta_{t+1}^{ii} \frac{P_{H,t+1}^i}{P_{t+1}^i} \right) \tag{32}
\]

and households earn their full marginal product in addition to a premium increasing in expected future labor market tightness. When households have no bargaining power, \((\eta = 1)\), the wage is set at the lower bound of the bargaining set:

\[
\frac{W_i^t}{P_t^i} = b^i + (1 - \varsigma) \frac{U_{i,t}^j}{U_{e,t}^j} \left[ \frac{(1 - \alpha_1)\tilde{L}_t^i}{\tilde{L}_t^j} \right]^{1/\nu} \tag{33}
\]

In this case, workers will have to accept a wage equal to the value of their time in addition to the unemployment benefit, i.e. their reservation wage. In intermediate cases, the surplus is split between workers and firms according to their respective bargaining power.
Similarly, the wage equation for migrant workers in region $i$ states as

$$\frac{W_{ti}^j}{P_t} = \left[ \frac{\eta}{\eta + Q_i^j(1 - \eta)} \right] \left[ b^j + Q_i^j(1 - \eta) \frac{U_{ti}^j}{U_{it}^j} \left( \frac{\alpha_1 L_t^j}{L_{ij}^j} \right)^{1/\nu_t} \right]$$

$$+ \left[ \frac{Q_i^j(1 - \eta)}{\eta + Q_i^j(1 - \eta)} \right] \left[ \frac{P_{x,t}^i}{P_t^i} A_t^i \right] + \left[ \frac{Q_i^j(1 - \eta)}{\eta + Q_i^j(1 - \eta)} \right] (1 - \rho^j) E_t \left\{ Q_{t,t+1}^i \frac{P_{H,t+1}^{ji}}{P_{t+1}^i} \left( 1 - \frac{Q_{t,t+1}^i}{Q_t^i (1 - f_{t+1}^{ji})} \right) \right\} \quad (34)$$

where, by (16), $\frac{Q_{t,t+1}^i}{Q_t^i} = \frac{Q_{t+1}^i}{Q_t^i}$.

### 2.4. Final Goods Producers

Final goods producers are retailers combining domestically produced intermediate goods to a final consumption good sold in region $i$ and exported to region $j$. Any firm $l \in [0, 1]$ produces a differentiated good using the common technology

$$Y_i^t(l) = (X_i^t(l))^{1 - \alpha} \quad (35)$$

in a regime of monopolistic competition. Aggregate production of final goods reads

$$Y_t^i = \left[ \int_0^1 (Y_i^t(l))^{\frac{1}{\alpha - 1}} dl \right]^{\frac{\alpha}{\alpha - 1}} = \left( \frac{X_t^i}{\Delta_t^i} \right)^{1 - \alpha} \quad (36)$$

where

$$\Delta_t^i = \int_0^1 \left[ \frac{P_t(i)}{P_t} \right]^{\frac{1}{\alpha - 1}} di \quad (37)$$

is a measure of price dispersion. Total costs of production of final retailer $l$ is then given by

$$TC_t^i = P_{x,t}^i X_t^i(l) = P_{x,t}^i (Y_t^i(l))^{\frac{1}{1 - \alpha}} \quad (38)$$

such that the nominal marginal costs are defined as

$$MC_t^i(l) = \frac{\partial TC_t^i}{\partial Y_i^t} = \frac{P_{x,t}^i (Y_t^i(l))^{\frac{\alpha}{1 - \alpha}}}{1 - \alpha} \quad (39)$$

The real marginal cost of production, defined in terms of domestic prices, $P_{H,t}^i$, then states as

$$RMC_t^i(l) = \frac{MC_t^i(l)}{P_{H,t}^i} = \frac{P_{x,t}^i (Y_t^i(l))^{\frac{\alpha}{1 - \alpha}}}{P_{H,t}^i} \frac{P_{x,t}^i P_t^i (Y_t^i(l))^{\frac{\alpha}{1 - \alpha}}}{P_t^i P_{H,t}^i} \frac{1}{1 - \alpha}$$

$$= \frac{P_{x,t}^i}{P_t^i} \left[ (1 - \alpha_2) + \alpha_2 (1 - \alpha_2) (Q_t^i)^{1 - \nu_t - 2 - \alpha_2} \right]^{\frac{1}{1 - \nu_t}} \frac{(Y_t^i(l))^{\frac{\alpha}{1 - \alpha}}}{1 - \alpha} \quad (40)$$
Final retailers face a downward-sloping demand function, capturing both consumption demand from households in both regions $i$ and $j$, and demand from intermediate producers in $i$ whose vacancy cost is denominated in the final consumption good

$$Y^i_t(l) = \left( \frac{P^i_{H,t}(l)}{P^i_{H,t}} \right)^{-\varepsilon} \left[ C^i_H + C^j_F + \kappa V^i_t \right]$$

(41)

$$= \left( \frac{P^i_{H,t}(l)}{P^i_{H,t}} \right)^{-\varepsilon} \left[ (1 - \alpha_2) \left( \frac{P^i_{F,t}}{P^i_t} \right)^{-\nu_2} C^i_t + \alpha_2 \left( \frac{P^j_{F,t}}{P^j_t} \right)^{-\nu_2} C^j_t + \kappa V^i_t \right]$$

Prices in the final sector are assumed to be sticky as in Calvo (1983). In any period, with probability $1 - \delta^i$ each firm has the chance to reset its price so as to maximize profits. With probability $\delta^i$ the firm sticks to the price charged in the previous period. From the optimal pricing problem, the log-linear Phillips curve of the final goods’ sector in region $i$ can be derived

$$\pi^i_{H,t} = \beta E_t \{ \pi^i_{H,t+1} \} + \lambda rmc^i_t$$

(42)

where $\pi^i_{H,t} = \log(\Pi^i_{H,t}) - \log(\Pi^i_{H})$, $\lambda^i \equiv \frac{(1 - \delta^i)(1 - \beta^i)}{\delta^i} \frac{1 - \alpha}{1 - \alpha + \alpha \varepsilon}$ and $rmc^i_t = \log(RMC^i_t) - \log(RMC^n)$, with $RMC^i = (Y^i)^{\alpha / (1 - \alpha)}$ denoting steady-state real marginal costs.

2.5. Market Clearing

Aggregate output in the retail sector in region $i$ is defined as

$$Y^i_t = \left[ \int_0^1 Y^i_t(l) \frac{1}{l} dl \right]^{\frac{1}{\varepsilon - 1}}$$

(43)

such that integrating the demand for good $l$, given by (41), yields a conventional aggregate resource constraint

$$Y^i_t = \left[ (1 - \alpha_2) + \alpha_2 \frac{(1 - \alpha_2)(Q^i_t)^{1 - \nu_2} - \alpha_2}{(1 - \alpha_2) - \alpha_2 (Q^i_t)^{1 - \nu_2}} \right]^{\frac{\nu_2}{1 - \nu_2}} \left[ (1 - \alpha_2)C^i_t + \alpha_2 (Q^i_t)^{\nu_2} C^j_t + \kappa V^i_t \right]$$

(44)

The aggregate production function results from combining the demand for final goods (41) with the respective production function (35)

$$Y^i_t = (X^i_t)^{1 - \alpha} (\Delta^i_t)^{\alpha - 1}.$$  

(45)

Finally, aggregate (un)employment are defined as follows

$$E^i_t = E^{ii}_t + E^{ji}_t \quad ; \quad E^j_t = E^{ij}_t + E^{jj}_t$$

(46)

$$U^i_t = U^{ii}_t + U^{ji}_t \quad ; \quad U^j_t = U^{ij}_t + U^{jj}_t$$

(47)
2.6. Monetary Policy

A specification of monetary policy is needed to close the model. We let the common monetary authority follow a simple interest rate rule taking the log-linear form

\[ r_t = \rho_r r_{t-1} + (1 - \rho_r) (\Gamma_\pi \pi_t - \Gamma_u u_t + \Gamma_y y_t) \]  

(48)

where \( \rho_r \in [0, 1] \) denotes the degree of interest rate smoothing, and \( \Gamma_\pi > 1 \), \( \Gamma_u \) and \( \Gamma_y \) represent the weights on union-wide inflation (\( \pi_t = \pi^i_{H,t} / 2 + \pi^j_{H,t} / 2 \)), unemployment (\( u_t = u^i_t / 2 + u^j_t / 2 \)) and output (\( y_t = y^i_t + y^j_t / 2 \)), respectively. In Section 5, we simply assume that monetary policy responds to inflation so that \( \Gamma_\pi = 1.5 \) and \( \rho_r = \Gamma_u = \Gamma_y = 0 \). Section 6 compares this simple rule (R1) to three alternative specifications in Table 1.

3. Efficient and natural equilibrium

The efficient equilibrium is defined as the solution to a social planner’s problem. A cooperative planner chooses a real allocation to maximize the (weighted) sum of representative utility functions in the two regions, subject to technology and resource constraints. In a symmetric monetary union, the problem states as

\[ \max E_0 \sum_{t=0}^{\infty} \beta^t (U^i_t + U^j_t) \]

subject to (2)-(4), (6)-(11), (18), (20), (35), and the aggregate resource constraint \( Y^i_t = C^i_{H,t} + C^j_{F,t} + \kappa V^i_{t,i} + \kappa V^j_{t,j} \) for region \( i \), as well as the corresponding relations for region \( j \).

As shown in Appendix C, the first-order conditions can be reduced to a set of efficiency conditions for labor market outcomes. In particular, efficiency in the market for domestic employment in region \( i \) demands

\[
\frac{(1 - \alpha_i)Y_t^i}{E_t^i} - \frac{U^i_{c,t}}{U^i_{c,t}} \left( \frac{(1 - \alpha_1)^2 L^i_t / L^i_t}{1 - \alpha_2} \right) \left[ \left( 1 - \alpha_1 \right) \tilde{L}_t^i / \tilde{L}_t^i \right]^{1/\nu_1} = \frac{\kappa^i}{\gamma q_{it}} - \frac{U^i_{c,t}}{U^i_{c,t}} \left( \frac{(1 - \alpha_1)^2 C^i_t / C^i_{H,t}}{(1 - \alpha_2) C^i_t / C^i_{H,t}} \right) \left[ \left( 1 - \alpha_1 \right) \tilde{L}_t^i / \tilde{L}_t^i \right]^{1/\nu_2}
\]

\[
- \beta (1 - \rho_i) E_t \left\{ \frac{U^i_{c,t+1}}{U^i_{c,t}} \left[ \left( 1 - \alpha_2^j / C_{H,t+1} \right) \gamma \nu_1 \right] \left[ \frac{\kappa^i}{\gamma q_{it+1}} \right] - \frac{1 - \gamma}{\gamma} \kappa^i \theta_{it+1} \left\} \right.
\]

(49)

Similar conditions hold for the markets for domestic labor in region \( j \), and foreign labor in \( i \) as well as \( j \). In addition, efficiency restricts relative labor market tightness by the condition

\[
\frac{U^i_{c,t}}{U^i_{c,t}} \left\{ \left[ \left( 1 - \alpha_2^j / C_{H,t+1} \right) \gamma \nu_1 \right] \left[ \frac{\kappa^i}{\gamma q_{it+1}} \right] - \frac{1 - \gamma}{\gamma} \kappa^j \theta_{it+1} \right\}
\]

(50)

\[ \text{For the full set of equilibrium conditions see Appendix B.} \]
As in the standard model with a Walrasian labor market, the social planner seeks to increase employment up until the point where the marginal product of labor equals the marginal rate of substitution, the rate at which households are willing to substitute leisure for consumption by working in labor market \( i \), as reflected in the first terms on the left-hand side of (49). But subject to a matching technology, the planner also seeks to balance marginal matching costs for firms, given by \( \kappa^i / \gamma q_i^i \), and the time costs of searching for households, given by \( \zeta \) times the marginal rate of substitution, as reflected in the first two terms on the right-hand side of (49). Moreover, as a fraction \( (1 - \rho^j) \) of matches continue into the next period, the planner solves a trade-off between net marginal matching costs today and in the future, as reflected in the final forward-looking term in (49). Finally, according to condition (50), the planner determines the relative labor market tightness in the two labor markets so as to balance relative vacancy cost with relative disutilities of search activities.

We define the natural equilibrium as the market outcome for the special case in which prices are fully flexible and the distortion from monopolistic competition in the zero-inflation steady state has been eliminated by an appropriate production subsidy. The natural equilibrium is thus one in which both the average market in the final goods sector, \( \mu_i^t \equiv (1 - \alpha) (Y_i^t)^{\alpha/(1 - \alpha)} P_{x,t}^i / P_i^t \), and price dispersion, \( \Delta_i^t \), are equal to one. Further assuming away unemployment benefits (so that \( b_i^t = b_i^j = 0 \)), the job creation condition (28) and the wage equation (31) can be combined to give the flexible-price natural equilibrium condition for the market for domestic labor in region \( i \)

\[
\frac{(1 - \alpha) Y_i^t}{E_i^t} - \frac{U_i^t}{U_{c,t}} \left[ \frac{(1 - \alpha_1) \tilde{L}_i^t / \tilde{L}_i^t}{(1 - \alpha_2) C_i^t / C_{H,t}} \right]^{1/\nu_1} = \frac{\kappa^i}{\eta q_i^i} - \frac{U_i^t}{U_{c,t}} \left[ \frac{(1 - \alpha_2) C_i^t / C_{H,t}}{(1 - \alpha_2) C_i^t / C_{H,t}} \right]^{1/\nu_2} - \beta (1 - \rho_i^t) \mathbb{E}_t \left\{ \frac{U_i^t}{U_{c,t}} \left[ \frac{(1 - \alpha_2) C_i^t / C_{H,t+1}}{(1 - \alpha_2) C_i^t / C_{H,t+1}} \right]^{1/\nu_2} \right\} \left[ \frac{\kappa^i}{\eta q_i^i} - \frac{1 - \eta}{\eta} \kappa^i \theta^{ij} \right] \right) \quad (51)
\]

The optimal migration decision (17), can be rewritten as

\[
\frac{U_i^t}{U_{c,t}} \left\{ \frac{\alpha_1 \tilde{L}_i^t}{\tilde{L}_i^t} \right\}^{1/\nu_1} - \left[ \frac{(1 - \alpha_2) \tilde{L}_i^t}{\tilde{L}_i^t} \right]^{1/\nu_1} = \frac{\alpha_2 C_i^t}{C_{H,t}} \kappa^i \theta^{ij} - \frac{\alpha_2 C_i^t}{C_{F,t}} \kappa^i \theta^{ij} \right) \quad (52)
\]

The natural labor market equilibrium (51) coincides with (49) whenever the Hosios (1990) condition \( \eta = \gamma \) holds. This condition thus eliminates all distortions associated with the search and matching process in the labor market. Specifically, considering
an increase in the number of vacancies the social planner solves a trade-off between
the congestion-search externality – a negative externality for firms associated with
increased competition for workers to fill these vacancies – and the thick-market ex-
ternality – a positive externality for workers associated with decreasing competition
among workers for a given vacancy. For \( \eta = \gamma \) the private marginal value of an addi-
tional vacancy coincides with its social value and the two externalities perfectly offset
each other. In the benchmark model without mobility searchers are predetermined so
that an efficient number of vacancies leads to efficiency in labor-market tightness.

When labor is mobile, however, the Hosios condition is not sufficient to render the
flexible-price equilibrium efficient out of steady state. The Hosios condition is suffi-
cient to equalize the private and social marginal value of vacancies, but not of searchers.
Specifically, unless all employment contracts are terminated at the end of every period
so that \( \rho = 0 \) (or households fully discount the future so that \( \beta = 0 \)), \( \eta = \gamma \) does not
ensure that the optimal migration decision in the market equilibrium coincides with
the efficiency condition (50). Effectively, by taking wages as given, households take
too strong a signal from expected future labor market outcomes when making their
migration decisions.

Throughout, we assume that the Hosios condition holds, that a production subsidy
offsets the distortion from monopolistic competition, and, unless otherwise stated, that
unemployment benefits are unavailable. In the full model, therefore, standard distort-
ions from price rigidities interact only with the remaining distortion from inefficient
migration decisions in the market equilibrium.

4. Calibration

We focus on a zero-inflation non-stochastic symmetric steady state and calibrate
the model in a three step procedure. First, we set basic parameters to conventional
values from the literature. Second, we chose a number of parameters affecting labor
market outcomes to match a set of empirical targets for steady state relations in the
model. And third, we find standard deviations to shocks in the model by matching a
set of moments in the data. Parameter values are summarized in Table 2, while the
standard deviations of all three shocks are reported in Table 4.

Following the business-cycle literature, we set the preference discount factor to
\( \beta = 0.99 \); the elasticity of substitution between consumption goods to \( \epsilon = 11 \); price
stickiness to \( \delta^i = \delta^j = \delta = 0.75 \); and marginal returns to labor in production to
\( \alpha = 1/3 \). In our benchmark calibration, we set \( \nu_2 = 1.5 \), within the empirically
relevant range of \([1.5, 3.5]\) for interregional trade elasticities reported by Bilgic et al.
(2002). The job-separation rate \( \rho^i = \rho^j = \rho = 0.1 \) is chosen so that jobs last on average
for about 10 quarters (see e.g. Shimer (2005)). The bargaining power of firms is set
to the conventional value of \( \eta = 0.4 \) in line with estimates in Flinn (2006). Hence,
from the Hosios (1990) condition, the elasticity of matches to vacancies is \( \gamma = \eta = 0.4 \).
This value for \( \gamma \) corresponds to the midpoint of values typically used in the literature
(see e.g. Gertler and Trigari (2009)). Parameter \( \zeta \in (0, 1) \) captures the fraction of
leisure time foregone by the unemployed relative to the employed, as the unemployed search for jobs, develop skills or collect unemployment benefits. We assume that this cost is the same for native and migrant searchers, as the extra cost of searching in the foreign labor market is captured by parameters $\alpha_1$ and $\nu_1$. Similar to Campolmi and Gnocchi (2016) $\zeta$ is calibrated to data from the American Time Use Survey (ATUS) for the period 2003–2006 as reported by Krueger and Mueller (2012). Table 3 shows time devoted to leisure, depending on the employment status, and the implied value for $\zeta$.

In the symmetric steady state, $\alpha_1$ defines the shares of migrant employment to total employment, migrant unemployment to total unemployment, and migrant searchers to total searchers, i.e. $\alpha_1 = E^{ij}/E = U^{ij}/U = S^{ij}/S$. $\alpha_1$ is thus calibrated to the average share of gross migration flows to total employment over all U.S. states, as observed in the IRS data over the 1976–2016 period. Firms’ search cost, $\kappa$, the unemployment benefit, $b_U$, the utility parameter $b$, and the matching efficiency, $\omega$, are set simultaneously to target (i) an employment rate of $E = 0.94$, computed using quarter U.S. data from FRED II over the period 1976:Q1–2016:Q4, (ii) a job-filling rate of $q = 2/3$, (iii) a replacement rate of $b_U/(W/P) = 0.4$, and (iv) a vacancy cost per filled job as a fraction of the real wage $(\kappa/\bar{q})/(W/P) = 0.045$ (see Gali (2010)).

Finally, the standard deviations of regional technology, matching-efficiency and preference shocks as well as the elasticity of labor substitution ($\nu_1$) are calibrated to minimize the average distance of simulated unconditional moments from their empirical counterparts with the serial correlation of all shocks fixed at 0.88. The targeted data moments are the standard deviation of output, the relative standard deviation of employment, the correlation between output and employment, and (for the model with mobility) the relative volatility of gross migration flows. Under the model assumption that regions are symmetric, we match regional moments in the model to aggregate moments in quarterly U.S. data from the FRED II database for the period 1976:Q1 – 2016:Q4. Moments are calculated after applying a standard HP filter. Output volatility is expressed in terms of percentage standard deviations. The relative volatility of gross migration flows corresponds to the median over relative standard deviations of state-to-state labor flows as measured in IRS data.

Table 4 reports empirical and simulated moments for the models, both with and without labor mobility. In both cases, the model matches the targeted moments quite closely. Output volatility in the model and the data are near-identical, whilst the model somewhat overstates the relative volatility of employment, a common feature of business cycle models. The correlation between output and employment is positive in the model as in the data, even if it is somewhat smaller in the model with mobility than in the model without it. Consequently, the model does well in capturing the negative correlations between output and unemployment, and employment and unemployment, a key feature of labor market dynamics. The reduced correlation between output and employment allows the model with mobility to better match the relative unemployment volatility. Finally, the model with mobility closely tracks relative gross migration
volatility in the data as well as the correlations of migration flows with key aggregate variables. Overall, we consider the empirical fit of the model to be satisfactory for our welfare analysis.

5. Migration dynamics and labor market variables

We consider macroeconomic dynamics in the monetary union with the benchmark calibration summarized in Table 2 under simple inflation targeting, i.e. where the policymaker follows rule R1 in Table 1. We compare responses with labor mobility (red lines with circles) to a version of the model without it, that is where we fix $\Omega^t_i = 1$ for all $t$ (blue lines with stars), and to the efficient allocation with labor mobility (black lines with diamonds). We consider a region $i$-specific shock to total factor productivity, (19), to matching efficiency for domestic workers, (21), and to a shock to the marginal utility of consumption, (15).

5.1. Technology shock

Figure 1 shows responses to an idiosyncratic TFP shock in region $i$. The improvement in technology reduces marginal costs, putting downward pressure on domestic prices. Since prices are sticky, however, firms do not reduce them fully to reflect the fall in costs. The increase in output required to meet the extra demand can therefore be obtained whilst employment falls. The reduction in employment is brought about by a decline in vacancies posted by firms; as consumer prices remain relatively high, the real value of a match falls for firms in the region despite the increase in TFP. A conventional monetary policy response – a moderate reduction in union-wide interest rates following the fall in inflation – is thus insufficiently stimulatory to avoid an increase in unemployment when technology improves.\footnote{See e.g. Galí and Rabanal (2004), Balleer (2012), and Mandelman and Zanetti (2014).}

In the benchmark model without labor mobility (blue stars in Figure 1), the number of searchers in region $i$’s labor market is predetermined by the number of workers currently out of employment. With a fixed pool of searchers and a decline in vacancies, labor market tightness falls, and the number of matches drops. As firms value matches less and leisure time becomes less dear to households, the wage bargaining set shifts down and workers have to accept lower real wages despite the pick-up in productivity.

Both the fall in prices in region $i$ and the union-wide monetary policy stimulus result in a union-wide aggregate demand increase. The value of a match to firms in region $j$ therefore increases, and more vacancies are posted there. Hence, there is a stimulatory spillover of the TFP shock in $i$ to region $j$, where the labor market tightens, and both employment and production increase. With an upward shift in the wage bargaining set, workers in $j$ are able to negotiate higher real wages.

In the full model with labor mobility (red circles in Figure 1), households are able to respond to these diverging labor market performances in the two regions. Whilst
its pool of searchers is fixed initially, households in region $i$ can immediately relocate unmatched workers from the home labor market to the relatively better performing labor market in region $j$. Similarly, households in region $j$ can keep more of their searchers at home. Consistent with the empirical evidence e.g. in Hauser (2014), and Saks and Wozniak (2011), labor mobility allows households to reallocate searchers towards the best performing labor market. Consequently, labor market tightness falls substantially less in region $i$, and it increases substantially less in region $j$, when labor is mobile. Hence, labor mobility reduces the volatility of key labor-market variables, consistent with the intuition in Mundell (1961).

Moreover, the net flows of unmatched searchers out of region $i$ and into region $j$ work to improve welfare in the monetary union by allowing region $i$ to ‘export’ unemployment during the adjustment process to a temporary improvement in TFP. Excess leisure in region $i$ and overtime in region $j$, both detrimental to welfare, are moderated. This improvement occurs despite an amplification from labor mobility of the inefficient responses caused by price rigidity. When prices are sticky, labor flows towards the less productive region $j$ rather than towards the more productive region $i$ as in the efficient allocation (black diamonds in Figure 1). The ‘misdirected’ flows amplify the contraction in employment in $i$ so that the expansion in output is limited. In region $j$ labor mobility amplifies the spillover effects so that both employment and output increase by more. With perfect risk sharing and an absence of home bias in consumption, however, aggregate consumption in the two regions are not affected by the relocation of labor.

5.2. Matching efficiency shock

Figure 2 shows responses to an innovation to matching efficiency in labor market $i$ (for both native and migrant workers). When matching technology improves temporarily, the desired number of workers can be hired with fewer vacancies, so that firms in region $i$ post fewer of them. When hiring is cheap, the value of a match to the firm falls so that workers have to accept a lower real wage. This fall in marginal costs – although rather small – allows firms to reduce prices. Monetary policy responds by reducing interest rates, stimulating consumption. The shock mostly affects relative labor market performance, with a limited effect on consumption and output.

In a model without labor mobility (blue stars in Figure 2), the number of searchers is fixed initially. Fewer vacancies therefore translate directly into looser labor-market conditions in region $i$. At the same time, the temporary rise in matching efficiency increases job finding rates in a hump-shaped fashion. As the number of matches goes up, employment and output increase to meet the extra demand. In region $j$, where matching efficiency is unaffected, the monetary stimulus increases aggregate demand. To meet this demand, firms increase employment and output by posting more vacancies. Labor market conditions in region $j$ unambiguously improve: both the job finding rate and real wages increase.

With labor mobility (red circles in Figure 2), households respond to divergent labor
market conditions – higher job finding rates but lower real wages in $i$ as opposed to both better job finding prospects and higher real wages before migration flows in region $j$. Consequently, despite the improved matching efficiency in region $i$, households reallocate searchers towards region $j$’s labor market. In equilibrium, labor market prospects equalize across the two regions. While real wages still increase, the job finding rate declines following the inflow of migrants. Unemployment shifts from region $i$ to region $j$.

As for the TFP shock considered above, monetary policy is insufficiently accommodating to allow aggregate demand to keep up with the efficient level of output in region $i$ (black diamonds in Figure 2). The social planner would have firms in region $i$ post more vacancies to expand employment and output at this time of low hiring costs. But in a market economy with sticky prices, such an expansion requires a corresponding increase in aggregate demand. A moderate monetary policy response fails to bring this demand about.

Unlike the TFP shock, however, the regional matching efficiency induces migration flows in the right direction from the perspective of the social planner. In the efficient allocation, despite an increase in employment in region $i$ (where it is relatively cheap to bring about) and a decline in region $j$ (where it is relatively expensive), searchers are relocated towards the less effective labor market in region $j$ to equalize labor market conditions across the union. These migration flows work to reduce labor market tightness in region $j$ as well as in region $i$. In the market economy migration flows work to the same effect. An inflow of searchers to region $j$ reduces labor market tightness. As a result, fewer matches are made, and employment and output fall more than in the case without labor mobility. In region $i$, by contrast, firms respond to a somewhat smaller fall in labor market tightness by increasing vacancies and so employment and output further.

As discussed in Section 3, however, the competitive equilibrium in the mobility model is characterized by an additional inefficiency, related to households’ migration decisions. As the welfare costs of matching-efficiency shocks overall are rather small, the welfare losses related to this inefficiency increase the welfare costs of suboptimal policies when labor is mobile.

5.3. Preference shock

Figure 3 shows responses to a temporary increase in the marginal utility of consumption in region $i$. In response to a higher marginal utility, households in region $i$ increase their current consumption through intertemporal substitution. Firms respond to this increase in aggregate demand by increasing prices.

Despite affecting households in region $i$ only, for our benchmark calibration the shock induces identical dynamics in most of the aggregate variables in the two regions. The only difference is in the response of consumption (not shown), which goes up in region $i$, and falls in region $j$. In turn, monetary policymakers raise interest rates, which moderates the increase in aggregate demand in region $i$, and contracts consumption.
in region $j$. Since both regions' consumption bundles consist of domestic and foreign goods in equal measures under our baseline calibration, aggregate demand increases equally for all firms across the union. As the relative productive capacity is unaffected by the shock, and as firms do not discriminate between native and migrant workers, output and employment in the two regions must move together.

In both the model with and without labor mobility (red circles and blue stars in Figure 3, respectively), marginal rates of substitution are equalized across the regions entirely through diverging consumption responses. With identical labor market performance and labor supply, net migration is zero in the mobility model. In both models, as firms post more vacancies and labor markets tighten in response to the increase in demand, workers are able to negotiate higher wages. In region $i$, the increase in consumption works to contract labor supply. But this effect on wage demands is moderated by an increase in the value of time for a given level of consumption. In region $j$, a similar moderation is brought about by an increase in the value of leisure through lower consumption.

Again, because of sticky prices interacting with monetary policy, the market equilibrium does not coincide with the efficient one (black diamonds in Figure 3). Since some firms are prevented from increasing prices in response to the demand shock, output and employment expand more in both regions than then social planner would prescribe. But in a symmetric monetary union subject to a regional demand shock, labor mobility is inconsequential for the welfare costs of standard price distortions.

This result, however, hinges crucially on the absence of home bias in consumption. With home bias, the extra demand from region $i$ is biased towards locally produced goods. Similarly, the contraction in demand in region $j$ (brought about by the contractionary monetary policy response) affects that region’s firms more than region $i$’s firms. Hence, for $\alpha_2 < 0.5$, the regional preference shock is also associated with a potentially sizable reallocation of aggregate demand from goods produced in region $j$ to those produced in $i$. As shown in Figure 4, a regional preference shock contributes to a further overheating in region $i$ the larger the degree of home bias, whilst output in region $j$ may fall below its efficient level for sufficiently small values of $\alpha_2$. In this case, a mobile labor force reallocates workers toward region $i$, limiting the increase in unemployment and the fall in labor market tightness in region $j$, and easing the tightening of region $i$’s labor market. In this sense, labor mobility eases the trade-off for the common monetary policy in responding to divergent developments in regional labor markets. But with looser (tighter) labor-market conditions in region $i$ ($j$), firms in region $i$ ($j$) hire more (fewer) workers. As a result, employment and output move further away from their efficient levels when home bias is sizable. Labor mobility therefore has an ambiguous effect on the welfare costs of standard price distortions in this case.
6. Labor mobility, monetary policy and the cost of joining a monetary union

In this section we use a conventional second-order perturbation method for the calibrated version of the model where the steady state is efficient\(^9\) in order to assess the impact of labor mobility on the cost of joining a monetary union and in order to rank alternative simple rules. Shocks are calibrated according to Table 4.

6.1. Cost of joining a monetary union

In this section we study the impact of a mobile labor force on the cost of relinquishing an independent monetary policy by joining a monetary union. This cost is defined as the difference between the welfare losses associated to a given policy in a monetary union and the same policy when both regions have access to an independent monetary policy.

Assuming that monetary policy is conducted according to a suboptimal rule – as e.g. a simple Taylor rule (R1) – a mobile labor force significantly reduces the cost of joining a monetary union. For our benchmark calibration this reduction is in the order of 30\% (see Table 5). For all other monetary policies under consideration a mobile labor force leads to a reduction in the cost of giving up an independent monetary policy lying between 30–80\%.

Following a linear-quadratic approach, it can be shown that in the benchmark model without mobility strict inflation targeting (R2) implements the Pareto-efficient equilibrium and is thus optimal. According to the discussion in Section 3 there is an additional inefficiency related to households’ migration decisions so that R2 does not implement the efficient allocation in the full model with mobility. As this additional inefficiency is present both under independent monetary policies and in a monetary union, it does not affect the cost of joining a monetary union. For our benchmark calibration this cost is reduced by roughly 40\% when labor is mobile across regions (see Table 6). However, while for R1 labor mobility simultaneously reduces welfare costs in the case of independent monetary policies and when the two regions form a monetary union, for R2 monetary policy cannot neutralize the inefficiency related to workers’ migration decisions. For policies that are close to implementing the efficient allocation – as e.g. R2 – the welfare costs of the inefficiency related to households’ migration decisions exceeds the benefits of a mobile labor force. The welfare costs associated with R2 in a monetary union with labor mobility are therefore larger compared to the benchmark model with a fixed labor force.\(^{10}\)

\(^9\)This is the case if the unemployment benefit is set to zero and production of final goods is subsidized to offset the monopolistic distortion.

\(^{10}\)Welfare losses of a given rule are compared against different efficient equilibria in the models with and without mobility. As welfare levels in the mobility model are higher compared to the benchmark model, bigger welfare losses in the mobility model – or said in other words, bigger disparities between the competitive equilibrium and the efficient allocation – do not necessarily imply that labor mobility is reducing welfare levels.
Relaxing the assumption of no home bias strengthens the welfare-enhancing role of labor mobility. Home bias leads to more pronounced effects of region-specific shocks, as the interregional transmission of the latter through trade is more limited. A union-wide monetary policy therefore faces stronger trade-offs. The cost of joining a monetary union with a fixed labor force thus significantly increases with some home bias in consumption, while a mobile labor force reduces this same cost by much more compared to the benchmark case where \( \alpha_2 = 0.5 \) (see e.g. Table 7).

6.2. Labor mobility and different monetary policy rules

Table 8 compares welfare losses associated with different policy rules and the implied standard deviations of macroeconomic variables. While labor mobility does not affect the welfare ranking of the different policies under consideration, some policies entail smaller welfare losses without mobility (e.g. R2 and R3) and others are associated with smaller welfare losses when labor is allowed to flow across regions. For policies implementing an allocation that is close to efficiency and thus entailing relatively small welfare costs, the welfare losses associated with the additional inefficiency in the full model with labor mobility outweighs the benefits of having a mobile labor force. When departing from the assumption of no home bias a mobile labor force reduces the welfare losses of all rules so that mobility unambiguously improves welfare.

7. Conclusion

For a given monetary policy, we have found that labor mobility can be an additional channel for cross-regional spillovers as well as a regional shock absorber. But labor mobility is found to significantly reduce the welfare cost of joining a monetary union, in particular if households have a home bias in consumption. We do not find evidence to suggest that the welfare ranking of simple monetary policy rules should be affected by labor mobility in a monetary union. But optimal monetary policy will be affected. In future versions of this paper, however, we plan to compare policy prescriptions from these simple rules with optimal monetary policy in our monetary union with labor market mobility.
### Tables and Figures

<table>
<thead>
<tr>
<th>Rule</th>
<th>Description</th>
<th>$\rho_r$</th>
<th>$\Gamma_\pi$</th>
<th>$\Gamma_u$</th>
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<td>R1</td>
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<td>R4</td>
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Table 1: Monetary policy rules
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<td>Discount Factor</td>
<td>$\beta$</td>
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<td>4% avg. real return</td>
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<tr>
<td>Elasticity of substitution</td>
<td>$\epsilon$</td>
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<td>10% price markup</td>
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<td>Calvo price stickiness</td>
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<td>1 year exp. duration</td>
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<td>Returns to scale in production</td>
<td>$\alpha$</td>
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<td>Galí (2015)</td>
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<td>Trade Elasticity</td>
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<td>Bilgic et al. (2002)</td>
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<td>Job-separation Rate</td>
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<td>Shimer (2005)</td>
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<td>Firms’ bargaining power</td>
<td>$\eta$</td>
<td>0.40</td>
<td>Flinn (2006)</td>
</tr>
<tr>
<td>Elasticity of matches to vacancies</td>
<td>$\gamma$</td>
<td>0.40</td>
<td>Hosios condition</td>
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<tr>
<td>Unemployment disutility</td>
<td>$\zeta$</td>
<td>0.735</td>
<td>ATUS data</td>
</tr>
</tbody>
</table>

| Home bias in labor                            | $\alpha_1$ | 0.0779  | IRS data              |
| Elasticity of labor substitution              | $\nu_1$    | -9.5    | IRS data              |
| Vacancy cost                                  | $\kappa$  | 0.03    | $\frac{V}{W/P} = 0.4$|
| Unemployment benefit                          | $b^U$      | 0.40    | 0.94% employment rate |
| Matching efficiency                           | $\omega$  | 0.64    | 2/3 job-filling rate  |
| Utility parameter                             | $b$        | 0.97    | $\frac{\kappa}{q} \frac{W}{P} = 0.045$ |

Table 2: Benchmark calibration
Table 3: Time allocated to leisure time (in minutes per average weekday). Data are from the ATUS and were collected over the period 2003–2006. The maximum leisure time is computed as total available time (24 hours times 60 minutes) minus average time spent on sleeping, personal care and eating, averaged over employed and unemployed persons. Definition 1 for leisure time accounts for the following activities: *Leisure and socializing; voluntary, religious and civic activities; sport; travel*. Definition 2 is more restrictive and encompasses time spent on *leisure and socializing* only. $\zeta$ is defined as the difference between the maximum leisure time and the leisure time of unemployed relative to the difference between the maximum leisure time and the leisure time of employed, e.g. for definition 1 $\zeta = (827.5 - 450)/(827.5 - 293) = 0.71$. For our benchmark calibration we set $\zeta$ to the average value over the two definitions of maximum leisure time, i.e. $\zeta = 0.735$.

<table>
<thead>
<tr>
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<td>Maximum</td>
<td>827.5</td>
<td>827.5</td>
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<tr>
<td>Unemployed</td>
<td>450</td>
<td>344</td>
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<tr>
<td>Employed</td>
<td>293</td>
<td>186</td>
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<td>Search cost $\zeta$</td>
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<td>0.76</td>
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<td>Unconditional Moments</td>
<td>Data</td>
<td>Mobility</td>
</tr>
<tr>
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<td>--------</td>
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<td>Output volatility</td>
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<td>Correlation output and wages</td>
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<td>Relative unemployment rate volatility</td>
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<tr>
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Calibrated Parameters

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<td>$\sigma_{me}$</td>
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<td>$\sigma_z$</td>
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<td>$\nu_1$</td>
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Table 4: Selected unconditional moments.
Table 5: Unconditional moments and welfare losses in the mobility model and the benchmark model without labor mobility for flexible inflation targeting (R1) in a monetary union (average of domestic inflation rates) vs. independent monetary policies in each region (targeting of region-specific domestic inflation rate).
Table 6: Unconditional moments and welfare losses in the mobility model and the benchmark model without labor mobility for strict inflation targeting (R2) in a monetary union (average of domestic inflation rates) vs. independent monetary policies in each region (targeting of region-specific domestic inflation rate).

<table>
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<tr>
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<td>Efficient Allocation</td>
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<tr>
<td>$\sigma$</td>
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<td>0.0008</td>
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<tr>
<td>$\sigma / \sigma_y$</td>
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<td>$\sigma_u / \sigma_y$</td>
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<td>$\sigma_l / \sigma_y$</td>
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<td>5.10</td>
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<td>$\sigma_h / \sigma_y$</td>
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<table>
<thead>
<tr>
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<th>No Mobility</th>
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<tr>
<td>Welfare Losses</td>
<td>Efficient Allocation</td>
<td>Regions $i$ and $j$</td>
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<tr>
<td>$\sigma$</td>
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<td>1.09</td>
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<tr>
<td>$\sigma / \sigma_y$</td>
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<td>0.58</td>
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<tr>
<td>$\sigma_u / \sigma_y$</td>
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<td>9.31</td>
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<tr>
<td>$\sigma_l / \sigma_y$</td>
<td>7.57</td>
<td>8.21</td>
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<tr>
<td>$\sigma_h / \sigma_y$</td>
<td>42.60</td>
<td>47.49</td>
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Table 7: Unconditional moments and welfare losses in the mobility model and the benchmark model without labor mobility for flexible inflation targeting (R1) in a monetary union (average of domestic inflation rates) vs. independent monetary policies in each region (targeting of region-specific domestic inflation rate), with home bias in consumption ($\alpha_2 = 0.35$).

<table>
<thead>
<tr>
<th>Mobility</th>
<th>Flexible Inflation Targeting (R1)</th>
<th>Efficient Allocation</th>
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<th>Regions $i$ and $j$ Independent MP</th>
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<table>
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<th>Regions $i$ and $j$ Independent MP</th>
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<tr>
<td>$\sigma_\theta/\sigma_y$</td>
<td>42.52</td>
<td>39.65</td>
<td>39.16</td>
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Table 8: Unconditional moments and welfare losses for region i and j in the mobility model and the benchmark model without labor mobility under four different monetary policy rules.
Figure 1: Impulse responses of selected variables to a one-percent positive TFP shock in region $i$. 
Figure 2: Impulse responses of selected variables to a one-percent positive matching-efficiency shock in region $i$. 
Figure 3: Impulse responses of selected variables to a one-percent positive preference shock in region $i$. 
Figure 4: Impact responses of selected variables to a one-percent positive preference shock in region i for different degrees of home bias ($\alpha_2 < 0.5$).


Appendix A. Households’ value function

The Bellman equation associated to the optimization problem of the representative household in region \( i \) states as follows

\[
V^*_i(E^i_{t-1}, E^i_{t-1}) = \max_{\Omega_i, E^i_t, E^i_t} \left\{ U^i_t \left[ C^i_t(.), l^i_t(.) \right] + \beta E_t \{ V^*_i \} \right\} \quad (A.1)
\]

The constraints the household takes into account are

\[
\begin{align*}
I^i_t &= 1 - \left( 1 - \alpha_1 \right)^{1/\nu_i} \left( E^i_t + \zeta U^i_t \right)^{1/\nu_i} + (\alpha_1)^{1/\nu_i} \left( E^i_t + \zeta U^i_t \right)^{1/\nu_i} \\
E^i_{t-1} &= (1 - \rho_i)(1 - f^i_{t-1}\Omega^i_{t-1})E^i_{t-1} + f^i_{t-1}\Omega^i_{t-1} \left( 1 - (1 - \rho_i)E^i_{t-1} \right) \\
E^{i,j}_{t-1} &= (1 - \rho_i)(1 - f^{i,j}_{t-1}(1 - \Omega^i_{t-1}))E^{i,j}_{t-1} + f^{i,j}_{t-1}(1 - \Omega^i_{t-1}) \left( 1 - (1 - \rho^j_i)E^{i,j}_{t-1} \right) \\
U^i_t &= (1 - f^i_{t-1}\Omega^i_{t-1} \left[ 1 - (1 - \rho_i)E^i_{t-1} - (1 - \rho^j_i)E^{i,j}_{t-1} \right] \\
U^{i,j}_t &= (1 - f^{i,j}_{t-1})(1 - \Omega^i_{t-1} \left[ 1 - (1 - \rho_i)E^i_{t-1} - (1 - \rho^j_i)E^{i,j}_{t-1} \right] \\
C^i_t &= \frac{D^i_{t-1} P^i_{t-1}}{P^i_{t-1}} - R^i_1 D^i_t \frac{P^i_{t-1}}{P^i_t} E^i_t + W^{i,j}_{t-1} E^i_t + b^i_t U^i_t + \frac{W^{i,j}_t}{P^i_t} E^i_t + \frac{P^i_{t}}{P^i_t} b^{i,j}_t U^{i,j}_t + T^i_t \quad (A.7)
\end{align*}
\]

Notice, that equations (A.3), (A.4), (A.5), (A.6), and (A.8) make a system of 5 equations and 5 variables (\( E^i_t, E^{i,j}_t, U^i_t, U^{i,j}_t \) and \( \Omega^i_t \)), where the fifth equation is implied by the first four. For example, summing up (A.3), (A.4), (A.5), and (A.6) is equal to 1 and thus yields (A.8). In what follows, equation (A.8) is thus not explicitly taken into account in households’ maximization problem.

Appendix A.0.1. Optimal Migration Decision

Households’ first-order condition with respect to \( \Omega^i_t \), i.e., the optimal migration decision implies

\[
\begin{align*}
\frac{\partial V^i_t}{\partial E^i_t} S^i_{H,t} f^i_t + \frac{\partial V^i_t}{\partial U^i_t} S^i_{t,H} (1 - f^i_t) &= \frac{\partial V^i_t}{\partial E^i_t} S^i_{H,t} f^{i,j}_t + \frac{\partial V^i_t}{\partial U^{i,j}_t} S^i_{t,H} (1 - f^{i,j}_t) \\
\frac{\partial V^i_t}{\partial E^i_t} f^i_t + \frac{\partial V^i_t}{\partial U^i_t} (1 - f^i_t) &= \frac{\partial V^i_t}{\partial E^{i,j}_t} f^{i,j}_t + \frac{\partial V^i_t}{\partial U^{i,j}_t} (1 - f^{i,j}_t) \\
\frac{\partial U^i_t}{\partial E^i_t} f^i_t + \frac{\partial U^i_t}{\partial U^i_t} (1 - f^i_t) &= \frac{\partial U^i_t}{\partial E^{i,j}_t} f^{i,j}_t + \frac{\partial U^i_t}{\partial U^{i,j}_t} (1 - f^{i,j}_t)
\end{align*}
\]

which can be rewritten as

\[
\begin{align*}
U^i_{c,t} \left\{ \begin{bmatrix} \frac{\partial C^i_t}{\partial E^i_t} + \frac{U^i_{t,t}}{E^i_t} \frac{\partial l^i_t}{\partial E^i_t} \end{bmatrix} f^i_t + \begin{bmatrix} \frac{\partial C^i_t}{\partial U^i_t} + \frac{U^i_{t,t}}{U^i_t} \frac{\partial l^i_t}{\partial U^i_t} \end{bmatrix} \right( 1 - f^i_t) \right\} = \\
U^{i,j}_{c,t} \left\{ \begin{bmatrix} \frac{\partial C^i_t}{\partial E^{i,j}_t} + \frac{U^i_{t,t}}{E^{i,j}_t} \frac{\partial l^i_t}{\partial E^{i,j}_t} \end{bmatrix} f^{i,j}_t + \begin{bmatrix} \frac{\partial C^i_t}{\partial U^{i,j}_t} + \frac{U^i_{t,t}}{U^{i,j}_t} \frac{\partial l^i_t}{\partial U^{i,j}_t} \end{bmatrix} \right( 1 - f^{i,j}_t) \right\}
\end{align*}
\]

36
and further states as follows

\[ f_t^{ii} \left[ \frac{W_t^{ii}}{P_t} - \frac{U_{i,t}}{U_{i,c,t}} (1 - \alpha_1)^{1/\nu_t} \left( \frac{E_t^{ii} + \zeta U_t^{ii}}{1 - l_t} \right)^{-1/\nu_t} \right] + \\
(1 - f_t^{ii}) \left[ b^{ii} - \zeta \frac{U_{i,t}}{U_{i,c,t}} (1 - \alpha_1)^{1/\nu_t} \left( \frac{E_t^{ii} + \zeta U_t^{ii}}{1 - l_t} \right)^{-1/\nu_t} \right] = \]

\[ f_t^{ij} \left[ \frac{W_t^{ij}}{P_t} Q_t - \frac{U_{i,t}}{U_{i,c,t}} (1 - \alpha_1)^{1/\nu_t} \left( \frac{E_t^{ij} + \zeta U_t^{ij}}{1 - l_t} \right)^{-1/\nu_t} \right] + \\
(1 - f_t^{ij}) \left[ Q_t b^{ij} - \zeta \frac{U_{i,t}}{U_{i,c,t}} (1 - \alpha_1)^{1/\nu_t} \left( \frac{E_t^{ij} + \zeta U_t^{ij}}{1 - l_t} \right)^{-1/\nu_t} \right] \tag{A.9} \]

The same optimality condition results from a setup where households are choosing the searchers to be sent to either of the two labor markets (\(S_t^{ii}\) and \(S_t^{ij}\)) directly. Specifically, given the pool of searching members, \(S_{H,t}\), instead of choosing \(\Omega_t\), households would optimally decide on \(S_t^{ii}\), which in turn pins down \(S_t^{ij}\).

**Appendix A.0.2. Value of native workers**

The first-order condition with respect to \(E_t^{ii}\) states

\[ \frac{\partial V_t}{\partial E_t^{ii}} = U_{i,c,t} \frac{\partial C_t^i}{\partial E_t^{ii}} + \frac{\partial l_t}{\partial E_t^{ii}} = U_{i,c,t} \left[ \frac{W_t^{ii}}{P_t} - \frac{U_{i,t}}{U_{i,c,t}} (1 - \alpha_1)^{1/\nu_t} \left( \frac{E_t^{ii} + \zeta U_t^{ii}}{1 - l_t} \right)^{-1/\nu_t} \right] \tag{A.10} \]

\[ \text{disutility of working as } E_t^{ii} \]

The current-period value of being native employed (in terms of consumption) is defined as the difference between the value of being matched to a firm and gaining wage \(W_t^{ii}/P_t\) and the disutility attached to working as a native in labor market \(i\). The first-order condition with respect to \(E_t^{ii-1}\)

\[ \frac{\partial V_t^{ii-1}}{\partial E_t^{ii-1}} = \frac{\partial V_t^{ii-1}}{\partial E_t^{ii}} \frac{\partial E_t^{ii}}{\partial E_t^{ii-1}} + \frac{\partial V_t^{ii-1}}{\partial E_t^{ij}} \frac{\partial E_t^{ij}}{\partial E_t^{ii-1}} + \frac{\partial V_t^{ii-1}}{\partial U_t^{ii}} \frac{\partial U_t^{ii}}{\partial E_t^{ii-1}} + \frac{\partial V_t^{ii-1}}{\partial U_t^{ij}} \frac{\partial U_t^{ij}}{\partial E_t^{ii-1}} = \]

\[ \frac{\partial U_t}{\partial E_t^{ii}} \frac{\partial E_t^{ii}}{\partial E_t^{ii-1}} + \frac{\partial U_t}{\partial E_t^{ij}} \frac{\partial E_t^{ij}}{\partial E_t^{ii-1}} + \frac{\partial U_t^{ii}}{\partial E_t^{ii}} \frac{\partial E_t^{ii}}{\partial E_t^{ii-1}} + \frac{\partial U_t^{ii}}{\partial E_t^{ij}} \frac{\partial E_t^{ij}}{\partial E_t^{ii-1}} = \]

\[ U_{i,c,t} \left[ \frac{\partial C_t^i}{\partial E_t^{ii}} + \frac{U_{i,t}}{U_{i,c,t}} \frac{\partial l_t}{\partial E_t^{ii}} \right] \frac{\partial E_t^{ii}}{\partial E_t^{ii-1}} + U_{i,c,t} \left[ \frac{\partial C_t^i}{\partial E_t^{ij}} + \frac{U_{i,t}}{U_{i,c,t}} \frac{\partial l_t}{\partial E_t^{ij}} \right] \frac{\partial E_t^{ij}}{\partial E_t^{ii-1}} + \]

\[ U_{i,c,t} \left[ \frac{\partial C_t^i}{\partial U_t^{ii}} + \frac{U_{i,t}}{U_{i,c,t}} \frac{\partial l_t}{\partial U_t^{ii}} \right] \frac{\partial U_t^{ii}}{\partial E_t^{ii-1}} + U_{i,c,t} \left[ \frac{\partial C_t^i}{\partial U_t^{ij}} + \frac{U_{i,t}}{U_{i,c,t}} \frac{\partial l_t}{\partial U_t^{ij}} \right] \frac{\partial U_t^{ij}}{\partial E_t^{ii-1}} = \]
\[ U_{c,t}^i (1 - \rho^i) \left\{ \left[ \frac{W_{ii}^t}{P_{it}} - \frac{U_{ii}^t}{U_{c,t}^i} (1 - \alpha_1)^{1/\nu_1} \left( \frac{E_{ii}^t + \zeta U_{ii}^t}{1 - l_t} \right) \right]^{-1/\nu_1} (1 - f_{ii}^t \Omega_t^i) - \right. \]
\[ \left. \left[ \frac{W_{ij}^t}{P_{it}} Q_{it}^i - \frac{U_{ij}^t}{U_{c,t}^i} (\alpha_1)^{1/\nu_1} \left( \frac{E_{ij}^t + \zeta U_{ij}^t}{1 - l_t} \right) \right]^{-1/\nu_1} f_{ij}^t (1 - \Omega_t^i) - \right. \]
\[ \left[ b_{ii}^i - \zeta \frac{U_{ii}^t}{U_{c,t}^i} (1 - \alpha_1)^{1/\nu_1} \left( \frac{E_{ii}^t + \zeta U_{ii}^t}{1 - l_t} \right) \right]^{-1/\nu_1} (1 - f_{ii}^t) \Omega_t^i - \right. \]
\[ \left. \left[ b_{ij}^i Q_{it}^i - \zeta \frac{U_{ij}^t}{U_{c,t}^i} (\alpha_1)^{1/\nu_1} \left( \frac{E_{ij}^t + \zeta U_{ij}^t}{1 - l_t} \right) \right]^{-1/\nu_1} (1 - f_{ij}^t) (1 - \Omega_t^i) \right\} \]  

(A.11)

The continuation value of being a native worker is the sum of the respective values for each of the five possible states a native worker in \( t \) might end up in period \( t + 1 \):

1. employed as native worker, gaining wage \( W_{ii}^{t+1}/P_{it}^{t+1} \) and suffering disutility from working
2. unemployed searcher in labor market \( i \), finding a new job in period \( t \) \( (f_{ii}^t \Omega_t^i) \)
3. unemployed searcher in labor market \( i \), remaining unmatched at the end of \( t \) \( ((1 - f_{ii}^t) \Omega_t^i) \)
4. unemployed searcher in labor market \( j \), finding a new job in period \( t \) \( (f_{ij}^t (1 - \Omega_t^i)) \)
5. unemployed searcher in labor market \( j \), remaining unmatched at the end of \( t \) \( ((1 - f_{ij}^t)(1 - \Omega_t^i)) \)

The effect of an additional native worker on households’ value function is given by\(^{11}\)

\[ \frac{\partial V_{t}^i}{\partial E_{ii}^t} = \frac{\partial U_{ii}^t}{\partial E_{ii}^t} + \beta \frac{\partial V_{t+1}^i}{\partial E_{ii}^t} = 0 \]

\(^{11}\)Notice, that the second equality in (A.11) follows from the fact

\[ \frac{\partial V_{t}^i}{\partial E_{ii}^{t-1}} = \frac{\partial U_{ii}^t}{\partial E_{ii}^{t-1}} + \beta \frac{\partial V_{t+1}^i}{\partial E_{ii}^{t-1}} = \frac{\partial U_{ii}^t}{\partial E_{ii}^{t-1}} \]

(A.12)
\[
\frac{\partial V_i^t}{\partial E_{ii}^t} = U_{i,c,t} \left[ \frac{W_{ii}^t}{P_t} - \frac{U_{i,t,1}^t}{U_{c,t}^t} (1 - \alpha_1)^{1/\nu_1} \left( \frac{E_{ii}^t + \zeta U_{ii}^t}{1 - \hat{q}_t^i} \right)^{-1/\nu_1} \right] + \beta (1 - \rho^j) U_{i,c,t}^{t+1} \\
\left\{ (1 - f_{ii}^t \Omega_{t+1}^i) \left[ \frac{W_{ii}^{t+1}}{P_{t+1}} - \frac{U_{i,t+1,1}^t}{U_{i,c,t+1}^t} (1 - \alpha_1)^{1/\nu_1} \left( \frac{E_{ii}^{t+1} + \zeta U_{ii}^{t+1}}{1 - \hat{q}_{t+1}^i} \right)^{-1/\nu_1} \right] - \\
(1 - \Omega_{t+1}^i f_{ij}^t) \left[ \frac{W_{ij}^{t+1}}{P_{t+1}} Q_{t+1}^i - \frac{U_{i,t+1,1}^t}{U_{i,c,t+1}^t} (1 - \alpha_1)^{1/\nu_1} \left( \frac{E_{ij}^{t+1} + \zeta U_{ij}^{t+1}}{1 - \hat{q}_{t+1}^i} \right)^{-1/\nu_1} \right] - \\
\Omega_{t+1}^i (1 - f_{ii}^t) \left[ b^i - \zeta \frac{U_{i,c,t+1}^t}{U_{i,c,t}^t} (1 - \alpha_1)^{1/\nu_1} \left( \frac{E_{ii}^{t+1} + \zeta U_{ii}^{t+1}}{1 - \hat{q}_{t+1}^i} \right)^{-1/\nu_1} \right] - \\
(1 - \Omega_{t+1}^i)(1 - f_{ij}^t) \left[ b^j Q_{t+1}^i - \zeta \frac{U_{i,c,t+1}^t}{U_{i,c,t}^t} (1 - \alpha_1)^{1/\nu_1} \left( \frac{E_{ij}^{t+1} + \zeta U_{ij}^{t+1}}{1 - \hat{q}_{t+1}^i} \right)^{-1/\nu_1} \right] \right\} 
\]

The effect of one additional native unemployed on households’ value function is

\[
\frac{\partial V_i^t}{\partial U_{ii}^t} = \frac{\partial U_i^t}{\partial E_{ii}^t} + \beta \frac{\partial V_i^t}{\partial U_{ii}^t} = \frac{\partial U_i^t}{\partial U_{ii}^t} = 0 \\
= U_{i,c,t} \left\{ b^i - \zeta \frac{U_{i,c,t}}{U_{i,c,t+1}^t} (1 - \alpha_1)^{1/\nu_1} \left( \frac{E_{ii}^t + \zeta U_{ii}^t}{1 - \hat{q}_t^i} \right)^{-1/\nu_1} \right\} 
\]

A native worker’s surplus is then defined as the difference between the value of native employment and the value of native unemployment.\(^\text{12}\)

\[
\tilde{V}^t_{ii} \equiv \frac{\partial V_i^t}{\partial E_{ii}^t} - \frac{\partial V_i^t}{\partial U_{ii}^t} = \frac{\partial U_i^t}{\partial E_{ii}^t} - \frac{\partial U_i^t}{\partial U_{ii}^t} + \beta \frac{\partial V_i^t}{\partial E_{ii}^t} = 0
\]

\(^\text{12}\)To be precise, the surplus of employing one additional native worker is computed as the difference between the value of native employment and the value of native unemployment, by keeping constant the fraction of searching members that are sent to labor markets \(i\) and \(j\), i.e. for \(\Omega_t^i = \Omega_t^j\). As one can show the first-order conditions with respect to native and migrant (un)employment are however unaffected by fixing \(\Omega_t^i = \Omega_t^j\).
In order to find a recursive expression of native workers’ surplus, consider again the envelope condition for $E_t^{ii}$, equation (A.11)

$$\frac{\partial V_t^{ii}}{\partial E_t^{ii}} = \frac{\partial V_t^{ii}}{\partial E_{t+1}^{ii}} \frac{\partial E_{t+1}^{ii}}{\partial E_t^{ii}} + \frac{\partial V_t^{ii}}{\partial U_{t+1}^{ii}} \frac{\partial U_{t+1}^{ii}}{\partial E_t^{ii}} + \frac{\partial V_t^{ij}}{\partial E_{t+1}^{ij}} \frac{\partial E_{t+1}^{ij}}{\partial E_t^{ii}} + \frac{\partial V_t^{ij}}{\partial U_{t+1}^{ij}} \frac{\partial U_{t+1}^{ij}}{\partial E_t^{ii}}$$

$$= \left(1 - \rho^i\right) \left(1 - f_t^{ii}\right) \frac{\partial V_t^{ii}}{\partial E_{t+1}^{ii}} \left(1 - \Omega_{t+1}^i\right) + \frac{\partial V_t^{ii}}{\partial U_{t+1}^{ii}} \frac{\partial U_{t+1}^{ii}}{\partial E_t^{ii}} + \frac{\partial V_t^{ij}}{\partial E_{t+1}^{ij}} \frac{\partial E_{t+1}^{ij}}{\partial E_t^{ii}} + \frac{\partial V_t^{ij}}{\partial U_{t+1}^{ij}} \frac{\partial U_{t+1}^{ij}}{\partial E_t^{ii}}$$

$$= (1 - \rho^i) \left(1 - f_t^{ii}\right) \frac{\partial V_t^{ii}}{\partial E_{t+1}^{ii}} - \frac{\partial V_t^{ii}}{\partial U_{t+1}^{ii}} \frac{\partial U_{t+1}^{ii}}{\partial E_t^{ii}} - \frac{\partial V_t^{ij}}{\partial E_{t+1}^{ij}} \frac{\partial E_{t+1}^{ij}}{\partial E_t^{ii}} - \frac{\partial V_t^{ij}}{\partial U_{t+1}^{ij}} \frac{\partial U_{t+1}^{ij}}{\partial E_t^{ii}}$$

$$= (1 - \rho^i) \left(1 - f_t^{ii}\right) \left(1 - \Omega_{t+1}^i\right) + \frac{\partial V_t^{ii}}{\partial E_{t+1}^{ii}} - \frac{\partial V_t^{ii}}{\partial U_{t+1}^{ii}} \frac{\partial U_{t+1}^{ii}}{\partial E_t^{ii}} - \frac{\partial V_t^{ij}}{\partial E_{t+1}^{ij}} \frac{\partial E_{t+1}^{ij}}{\partial E_t^{ii}} - \frac{\partial V_t^{ij}}{\partial U_{t+1}^{ij}} \frac{\partial U_{t+1}^{ij}}{\partial E_t^{ii}}$$

$$= (1 - \rho^i) \left(1 - f_t^{ii}\right) \left(1 - \Omega_{t+1}^i\right) + \frac{\partial V_t^{ii}}{\partial E_{t+1}^{ii}} - \frac{\partial V_t^{ii}}{\partial U_{t+1}^{ii}} \frac{\partial U_{t+1}^{ii}}{\partial E_t^{ii}} - \frac{\partial V_t^{ij}}{\partial E_{t+1}^{ij}} \frac{\partial E_{t+1}^{ij}}{\partial E_t^{ii}} - \frac{\partial V_t^{ij}}{\partial U_{t+1}^{ij}} \frac{\partial U_{t+1}^{ij}}{\partial E_t^{ii}}$$

where the last line follows from the first-order condition with respect to $\Omega_t^i$, equation (A.9). More precisely,

$$\frac{\partial V_t^{ii}}{\partial E_t^{ii}} = (1 - \rho^i) \left(1 - f_t^{ii}\right) \frac{\partial V_t^{ii}}{\partial U_{t+1}^{ii}} \frac{\partial U_{t+1}^{ii}}{\partial E_t^{ii}} = (1 - \rho^i)(1 - f_t^{ii}) \frac{\partial V_t^{ii}}{\partial W_{t+1}^{ii}}$$

$$\Omega_t^i = \Omega_t^i$$

(A.17)
Therefore, in order to define the surplus of native employment recursively, we need to condition on households’ optimal migration decision, such that

$$\tilde{V}_{W,t}^i \equiv \left. \frac{\partial U_i^t}{\partial E_{ii}^t} - \frac{\partial U_i^t}{\partial U_{ii}^t} + \beta \frac{\partial V_{t+1}^{i+1}}{\partial E_{ii}^t} \right|_{\Omega_t^i = \Omega_t^i}$$  \hspace{1cm} (A.18)

$$\tilde{V}_{W,t}^i = U_{c,t} \left\{ \frac{W_{ii}^t}{P_t} - b^{ii} - (1 - \zeta) \frac{U_{lt}^t}{U_{ct}^t} (1 - \alpha_1)^{1/\nu_1} \left( \frac{E_{ii}^t + \zeta U_{ii}^t}{1 - l_t^i} \right)^{-1/\nu_1} \right\} + \beta (1 - \rho^i) (1 - f_{t+1}^{ii}) \tilde{V}_{W,t+1}^{i+1}$$  \hspace{1cm} (A.19)

Finally, define the surplus from native employment to household $i$ in terms of current consumption of final goods, $V_{W,t}^i = \tilde{V}_{W,t}^i / U_{c,t}^i$, such that

$$V_{W,t}^i = \frac{W_{ii}^t}{P_t} - b^{ii} - (1 - \zeta) \frac{U_{lt}^t}{U_{ct}^t} (1 - \alpha_1)^{1/\nu_1} \left( \frac{E_{ii}^t + \zeta U_{ii}^t}{1 - l_t^i} \right)^{-1/\nu_1} + (1 - \rho^i) E_t \{ Q_{t,t+1} (1 - f_{t+1}^{ii}) V_{W,t+1}^{i+1} \}$$  \hspace{1cm} (A.20)

Equation (A.20) is a recursive expression of native workers’ surplus, expressed in terms of the final consumption basket of household $i$ and conditional on household $i$’s optimal migration decision.\(^\text{13}\)

**Appendix A.0.3. Value of migrant workers**

Households’ first-order condition with respect to $E_{t}^{ij}$ states

$$\frac{\partial U_i^t}{\partial E_{ij}^t} = U_{c,t} \frac{\partial C_{ij}^t}{\partial E_{ij}^t} + U_{t}^{ij} \frac{\partial l_t^i}{\partial E_{ij}^t} = U_{c,t} \left[ \frac{W_{ij}^t}{P_t} Q_t - \frac{U_{lt}^t}{U_{ct}^t} (\alpha_1)^{1/\nu_1} \left( \frac{E_{ij}^t + \zeta U_{ij}^t}{1 - l_t^i} \right)^{-1/\nu_1} \right]$$  \hspace{1cm} (A.21)

\(^\text{13}\)The very same solution results from the problem where household $i$ takes the participation of its members in the foreign labor market, $L_{ij}^t$, as given. By choosing $\Omega_t^i$ the household decides upon the pool of searchers in both labor markets, $S_{ii}^t$ and $S_{ij}^t$, which directly defines $L_{ii}^t$ and $L_{ij}^t$. The search and matching process taking place in both labor markets then defines the composition of both $L_{ii}^t = E_{ii}^t + U_{ii}^t$ and $L_{ij}^t = E_{ij}^t + U_{ij}^t$.
The effect of one additional migrant worker on households’ value function is given by

\[
\frac{\partial V_t^i}{\partial E_t^{ij}} = \frac{\partial U_t^i}{\partial E_t^{ij}} + \frac{\partial U_t^i}{\partial E_t^{ij}} + \frac{\partial V_t^i}{\partial U_t^{ij}} \frac{\partial E_t^{ij}}{\partial E_t^{ij}} + \frac{\partial V_t^i}{\partial U_t^{ij}} \frac{\partial E_t^{ij}}{\partial E_t^{ij}} = 0
\]

and the first-order condition with respect to \( E_{t-1}^{ij} \)

\[
\frac{\partial V_t^i}{\partial E_t^{ij}} = \frac{\partial U_t^i}{\partial E_t^{ij}} + \frac{\partial U_t^i}{\partial E_t^{ij}} + \frac{\partial V_t^i}{\partial U_t^{ij}} \frac{\partial E_t^{ij}}{\partial E_t^{ij}} + \frac{\partial V_t^i}{\partial U_t^{ij}} \frac{\partial E_t^{ij}}{\partial E_t^{ij}} = 0
\]

\[
U_{c,t}^i \left[ \frac{\partial C_t^i}{\partial E_t^{ij}} + \frac{U_t^i}{U_{c,t}^i} \frac{\partial l_t^i}{\partial E_t^{ij}} \right] \frac{\partial E_t^{ij}}{\partial E_t^{ij}} + U_{c,t}^i \left[ \frac{\partial C_t^i}{\partial E_t^{ij}} + \frac{U_t^i}{U_{c,t}^i} \frac{\partial l_t^i}{\partial E_t^{ij}} \right] \frac{\partial U_t^{ij}}{\partial E_t^{ij}} + U_{c,t}^i \left[ \frac{\partial C_t^i}{\partial U_t^{ij}} + \frac{U_t^i}{U_{c,t}^i} \frac{\partial l_t^i}{\partial U_t^{ij}} \right] \frac{\partial E_t^{ij}}{\partial E_t^{ij}} = 0
\]

\[
U_{c,t}^i(1 - \rho^j) \left\{ \left[ W_t^{ij} Q_t^i - \frac{U_t^i}{U_{c,t}^i} (\alpha_1)^{1/\nu_1} \left( \frac{E_t^{ij} + \zeta U_t^{ij}}{1 - l_t^i} \right)^{-1/\nu_1} \right] (1 - f_t^{ij}(1 - \Omega_t)) - \left[ \frac{W_t^{ij}}{P_t^j} - \frac{U_t^i}{U_{c,t}^i} (1 - \alpha_1)^{1/\nu_1} \left( \frac{E_t^{ij} + \zeta U_t^{ij}}{1 - l_t^i} \right)^{-1/\nu_1} \right] f_t^{ij} \Omega_t^i - \left[ b^{ij} Q_t^i - \frac{U_t^i}{U_{c,t}^i} (1 - \alpha_1)^{1/\nu_1} \left( \frac{E_t^{ij} + \zeta U_t^{ij}}{1 - l_t^i} \right)^{-1/\nu_1} \right] (1 - f_t^{ij})(1 - \Omega_t) - \left[ b^{ij} - \frac{U_t^i}{U_{c,t}^i} (1 - \alpha_1)^{1/\nu_1} \left( \frac{E_t^{ij} + \zeta U_t^{ij}}{1 - l_t^i} \right)^{-1/\nu_1} \right] (1 - f_t^{ij}) \Omega_t^i \right\}
\]

(A.22)

The effect of one additional migrant worker on households’ value function is given by

\[
\frac{\partial V_t^i}{\partial E_t^{ij}} = \frac{\partial U_t^i}{\partial E_t^{ij}} + \beta \frac{\partial V_t^{i+1}}{\partial E_t^{ij}} = 0
\]
\[
\frac{\partial V_i^j}{\partial E_{ij}^t} = U_{c,t}^i \left[ \frac{W_{ij}^t}{P_{ij}^t} Q_{ij}^t - \frac{U_{ij}^t}{U_{c,t}^i} (\alpha_1)^{1/\nu_1} \left( \frac{E_{ij}^t + \zeta U_{ij}^t}{1 - l_{ij}^t} \right)^{-1/\nu_1} \right] + \beta (1 - \rho^j) U_{c,t+1}^i
\]

\[
\begin{align*}
\left(1 - f_{ij}^t (1 - \Omega_{ij}^t)\right) & \left[ \frac{W_{ij}^t}{P_{ij}^t} - \frac{U_{ij}^t}{U_{c,t}^i} (1 - \alpha_1)^{1/\nu_1} \left( \frac{E_{ij}^t + \zeta U_{ij}^t}{1 - l_{ij}^t} \right)^{-1/\nu_1} \right] - \\
\Omega_{ij}^t f_{ij}^t & \left[ b_{ij}^t Q_{ij}^t - \zeta U_{ij}^t \left( \frac{E_{ij}^t + \zeta U_{ij}^t}{1 - l_{ij}^t} \right)^{-1/\nu_1} \right] - \\
\Omega_{ij}^t & \left[ \frac{b_{ij}^t}{U_{c,t}^i} (1 - \alpha_1)^{1/\nu_1} \left( \frac{E_{ij}^t + \zeta U_{ij}^t}{1 - l_{ij}^t} \right)^{-1/\nu_1} \right]
\end{align*}
\]  

(A.23)

The effect of one additional migrant unemployed on households’ value function is

\[
\frac{\partial V_i^j}{\partial U_{ij}^t} = \frac{\partial U_i^t}{\partial U_{ij}^t} + \beta \frac{\partial V_{ij}^t}{\partial U_{ij}^t} = 0
\]

\[
= U_{c,t}^i \left\{ b_{ij}^t Q_{ij}^t - \zeta U_{ij}^t (1 - \alpha_1)^{1/\nu_1} \left( \frac{E_{ij}^t + \zeta U_{ij}^t}{1 - l_{ij}^t} \right)^{-1/\nu_1} \right\} 
\]  

(A.24)

As for native workers the surplus of migrant workers is computed as the difference between the value of migrant employment and migrant unemployment.

\[
\tilde{V}_{ij}^t_{W,t} \equiv \frac{\partial V_i^j}{\partial E_{ij}^t} - \frac{\partial V_i^j}{\partial U_{ij}^t} = \frac{\partial U_i^t}{\partial E_{ij}^t} + \frac{\partial U_i^t}{\partial U_{ij}^t} + \beta \frac{\partial V_{ij}^t}{\partial E_{ij}^t} + \beta \frac{\partial V_{ij}^t}{\partial U_{ij}^t}
\]

43
Therefore, in order to define the surplus of migrant employment recursively, we need to condition on households’ optimal migration decision

\[
\begin{align*}
\beta(1 - ho^j) & \left\{ \frac{W_{ij}^t}{P_i^t} Q_t^i - b^j Q_t^i - (1 - \zeta) \frac{U_{ij}^t}{U_{ct}^t} (\alpha_1)^{1/\nu_i} \left( \frac{E_{ij}^t + \zeta U_{ij}^t}{1 - l_t^i} \right)^{-1/\nu_i} \right\} + \\
(1 - \Omega^i_{t+1}) & \left\{ \frac{W_{ij}^{t+1}}{P_i^{t+1}} Q_{t+1}^i - \frac{U_{ij}^{t+1}}{U_{ct}^{t+1}} (\alpha_1)^{1/\nu_i} \left( \frac{E_{ij}^{t+1} + \zeta U_{ij}^{t+1}}{1 - l_{t+1}^i} \right)^{-1/\nu_i} \right\} - \\
(1 - \Omega^i_{t+1})(1 - f_{ij}^{t+1}) & \left\{ b^i Q_{t+1}^i - \zeta \frac{U_{ij}^{t+1}}{U_{ct}^{t+1}} (\alpha_1)^{1/\nu_i} \left( \frac{E_{ij}^{t+1} + \zeta U_{ij}^{t+1}}{1 - l_{t+1}^i} \right)^{-1/\nu_i} \right\} - \\
\Omega^i_{t+1} f_{ij}^{t+1} & \left\{ \frac{W_{ij}^{t+1}}{P_i^{t+1}} - \frac{U_{ij}^{t+1}}{U_{ct}^{t+1}} (1 - \alpha_1)^{1/\nu_i} \left( \frac{E_{ij}^{t+1} + \zeta U_{ij}^{t+1}}{1 - l_{t+1}^i} \right)^{-1/\nu_i} \right\} + \\
\Omega^i_{t+1}(1 - f_{ij}^{t+1}) & \left\{ b^i - \zeta \frac{U_{ij}^{t+1}}{U_{ct}^{t+1}} (1 - \alpha_1)^{1/\nu_i} \left( \frac{E_{ij}^{t+1} + \zeta U_{ij}^{t+1}}{1 - l_{t+1}^i} \right)^{-1/\nu_i} \right\} = 0
\end{align*}
\]

As for native workers, conditional on the migration decision being optimal, the envelope condition for \( E_{ij}^t \), equation (A.22), can be expressed in terms of the future surplus of migrant workers

\[
\frac{\partial V_{ij}^t}{\partial E_{ij}^t} = (1 - \rho^j)(1 - f_{ij}^{t+1}) V_{W,t+1}^{ij} \iff \Omega_i^* = \Omega_i^t
\]

Therefore, in order to define the surplus of migrant employment recursively, we need to condition on households’ optimal migration decision

\[
V_{W,t}^{ij} = U_{ct}^i \left\{ \frac{W_{ij}^t}{P_i^t} Q_t^i - b^j Q_t^i - (1 - \zeta) \frac{U_{ij}^t}{U_{ct}^t} (\alpha_1)^{1/\nu_i} \left( \frac{E_{ij}^t + \zeta U_{ij}^t}{1 - l_t^i} \right)^{-1/\nu_i} \right\} + \beta(1 - \rho^j)(1 - f_{ij}^{t+1}) V_{W,t+1}^{ij}
\]

Finally, define the surplus from migrant employment to household \( i \) in terms of current consumption of final goods, \( V_{W,t}^{ij} = V_{W,t}^{ij} / U_{ct}^i \), such that

\[
V_{W,t}^{ij} = \frac{W_{ij}^t}{P_i^t} Q_t^i - b^j Q_t^i - (1 - \zeta) \frac{U_{ij}^t}{U_{ct}^t} (\alpha_1)^{1/\nu_i} \left( \frac{E_{ij}^t + \zeta U_{ij}^t}{1 - l_t^i} \right)^{-1/\nu_i} + \left\{ Q_{t+1}^i (1 - f_{ij}^{t+1}) V_{W,t+1}^{ij} \right\}
\]
Equation (A.29) is a recursive expression of migrant workers’ surplus, expressed in terms of the final consumption basket of household $i$ and conditional on household $i$’s optimal migration decision.

**Appendix B. Equilibrium Condition**

This section defines the equilibrium for the mobility model in Section 2 of the main text.

$$\beta R_t E_t \left\{ \left( \frac{U^i_{c,t+1}}{U^i_{c,t}} \right) \left( \frac{1}{\Pi^i_{t+1}} \right) \right\} = 1 \quad (B.1)$$

$$U^i_{c,t} Q^i_t = U^i_{c,t} \theta \quad (B.2)$$

$$U^i_{c,t} = b(C^i_t)^{b(1-\sigma)-1}(l^i_t)^{1-b(1-\sigma)}Z^i_t \quad (B.3)$$

$$U^j_{c,t} = b(C^j_t)^{b(1-\sigma)-1}(l^j_t)^{1-b(1-\sigma)}Z^j_t \quad (B.4)$$

$$U^i_{l,t} = (1-b)(C^i_t)^{b(1-\sigma)}(l^i_t)^{1-b(1-\sigma)-1} \quad (B.5)$$

$$U^j_{l,t} = (1-b)(C^j_t)^{b(1-\sigma)}(l^j_t)^{1-b(1-\sigma)-1} \quad (B.6)$$

$$1 - l^i_t = \left[ (1 - \alpha_1)^{1/\nu_1} (E^{ii}_t + \zeta U^{ii}_t)^{\frac{\nu_1-1}{\nu_1}} + (\alpha_1)^{1/\nu_1} (E^{ij}_t + \zeta U^{ij}_t)^{\frac{\nu_1-1}{\nu_1}} \right]^{\frac{\nu_1}{\nu_1-1}} \quad (B.7)$$

$$1 - l^j_t = \left[ (1 - \alpha_1)^{1/\nu_1} (E^{ij}_t + \zeta U^{ij}_t)^{\frac{\nu_1-1}{\nu_1}} + (\alpha_1)^{1/\nu_1} (E^{ji}_t + \zeta U^{ji}_t)^{\frac{\nu_1-1}{\nu_1}} \right]^{\frac{\nu_1}{\nu_1-1}} \quad (B.8)$$

$$E^{ii}_t = (1 - \rho^i)E^{ii}_{t-1} + f^{ii}_t S^{ii}_t \quad (B.9)$$

$$E^{jj}_t = (1 - \rho^j)E^{jj}_{t-1} + f^{jj}_t S^{jj}_t \quad (B.10)$$

$$E^{jj}_t = (1 - \rho^j)E^{jj}_{t-1} + f^{jj}_t S^{jj}_t \quad (B.11)$$
E_{ij}^t = (1 - \rho^i)E_{t-1}^{ii} + f_{t}^{ij}S_{t}^{ij} \quad (B.12)

S_{t}^{ii} = \Omega_{t}^{i} \left[ 1 - (1 - \rho^i)E_{t-1}^{ii} - (1 - \rho^i)E_{t-1}^{ji} \right] \quad (B.13)

S_{t}^{ij} = (1 - \Omega_{t}^{i}) \left[ 1 - (1 - \rho^i)E_{t-1}^{ii} - (1 - \rho^i)E_{t-1}^{ji} \right] \quad (B.14)

S_{t}^{jj} = \Omega_{t}^{j} \left[ 1 - (1 - \rho^j)E_{t-1}^{jj} - (1 - \rho^j)E_{t-1}^{ij} \right] \quad (B.15)

S_{t}^{ji} = (1 - \Omega_{t}^{j}) \left[ 1 - (1 - \rho^j)E_{t-1}^{jj} - (1 - \rho^j)E_{t-1}^{ij} \right] \quad (B.16)

U_{t}^{ii} = (1 - f_{t}^{ii})\Omega_{t}^{i} \left[ 1 - (1 - \rho^i)E_{t-1}^{ii} - (1 - \rho^i)E_{t-1}^{ji} \right] \quad (B.17)

U_{t}^{jj} = (1 - f_{t}^{jj})\Omega_{t}^{j} \left[ 1 - (1 - \rho^j)E_{t-1}^{jj} - (1 - \rho^j)E_{t-1}^{ij} \right] \quad (B.18)

U_{t}^{ij} = (1 - f_{t}^{ij})(1 - \Omega_{t}^{i}) \left[ 1 - (1 - \rho^i)E_{t-1}^{ii} - (1 - \rho^i)E_{t-1}^{ji} \right] \quad (B.19)

U_{t}^{ji} = (1 - f_{t}^{ji})(1 - \Omega_{t}^{j}) \left[ 1 - (1 - \rho^j)E_{t-1}^{jj} - (1 - \rho^j)E_{t-1}^{ij} \right] \quad (B.20)

E_{i}^{i} = E_{t}^{ii} + E_{t}^{ji} \quad (B.21)

E_{i}^{j} = E_{t}^{jj} + E_{t}^{ij} \quad (B.22)

U_{i}^{i} = U_{t}^{ii} + U_{t}^{ji} \quad (B.23)

U_{i}^{j} = U_{t}^{jj} + U_{t}^{ij} \quad (B.24)

S_{i}^{i} = S_{t}^{ii} + S_{t}^{ji} \quad (B.25)

S_{i}^{j} = S_{t}^{jj} + S_{t}^{ij} \quad (B.26)

V_{i}^{i} = V_{t}^{ii} + V_{t}^{ij} \quad (B.27)

V_{i}^{j} = V_{t}^{jj} + V_{t}^{ij} \quad (B.28)
Optimal migration decision of household $i$ (Foc w.r. to $\Omega^i_t$)

\[
\begin{align*}
    f^i_t & \left[ \frac{W^{ii}_t}{P^i_t} - \frac{U^{ii}_{i,t}}{U^i_{c,t}} (1 - \alpha_1)^{1/\nu_1} \left( \frac{E^{ii}_t + \zeta U^{ii}_t}{1 - l^i_t} \right)^{-1/\nu_1} \right] + \\
    (1 - f^i_t) & \left[ b^{ii} - \zeta \frac{U^{ii}_{i,t}}{U^i_{c,t}} (1 - \alpha_1)^{1/\nu_1} \left( \frac{E^{ii}_t + \zeta U^{ii}_t}{1 - l^i_t} \right)^{-1/\nu_1} \right] = \\
    f^{ij}_t & \left[ \frac{W^{ij}_t}{P^j_t} Q^i_t - \frac{U^{ij}_{j,t}}{U^j_{c,t}} (\alpha_1)^{1/\nu_1} \left( \frac{E^{ij}_t + \zeta U^{ij}_t}{1 - l^j_t} \right)^{-1/\nu_1} \right] + \\
    (1 - f^{ij}_t) & \left[ b^{ij} - \zeta \frac{U^{ij}_{j,t}}{U^j_{c,t}} (\alpha_1)^{1/\nu_1} \left( \frac{E^{ij}_t + \zeta U^{ij}_t}{1 - l^j_t} \right)^{-1/\nu_1} \right] 
\end{align*}
\] (B.29)

Optimal migration decision of household $j$ (Foc w.r. to $\Omega^j_t$)

\[
\begin{align*}
    f^{ij}_t & \left[ \frac{W^{ij}_t}{P^j_t} - \frac{U^{ij}_{j,t}}{U^j_{c,t}} (1 - \alpha_1)^{1/\nu_1} \left( \frac{E^{ij}_t + \zeta U^{ij}_t}{1 - l^j_t} \right)^{-1/\nu_1} \right] + \\
    (1 - f^{ij}_t) & \left[ b^{ij} - \zeta \frac{U^{ij}_{j,t}}{U^j_{c,t}} (1 - \alpha_1)^{1/\nu_1} \left( \frac{E^{ij}_t + \zeta U^{ij}_t}{1 - l^j_t} \right)^{-1/\nu_1} \right] = \\
    f^{ji}_t & \left[ \frac{W^{ji}_t}{P^i_t} Q^j_t - \frac{U^{ji}_{i,t}}{U^i_{c,t}} (\alpha_1)^{1/\nu_1} \left( \frac{E^{ji}_t + \zeta U^{ji}_t}{1 - l^i_t} \right)^{-1/\nu_1} \right] + \\
    (1 - f^{ji}_t) & \left[ b^{ji} - \zeta \frac{U^{ji}_{i,t}}{U^i_{c,t}} (\alpha_1)^{1/\nu_1} \left( \frac{E^{ji}_t + \zeta U^{ji}_t}{1 - l^i_t} \right)^{-1/\nu_1} \right] 
\end{align*}
\] (B.30)

\[
\begin{align*}
    q^i_t &= \omega^i (\theta^i_t)^{\gamma - 1} \\
    q^j_t &= \omega^j (\theta^j_t)^{\gamma - 1} \\
    q_t^{ii} &= \omega^{ii} (\theta^{ii}_t)^{\gamma - 1} \\
    q_t^{ij} &= \omega^{ij} (\theta^{ij}_t)^{\gamma - 1} \\
    q_t^{ji} &= \omega^{ji} (\theta^{ji}_t)^{\gamma - 1} \\
    q_t^{jj} &= \omega^{jj} (\theta^{jj}_t)^{\gamma - 1} 
\end{align*}
\] (B.31, B.32, B.33, B.34, B.35, B.36)
\[ f^i_t = \omega^i (\theta^i_t)^\gamma \] (B.37)
\[ f^j_t = \omega^j (\theta^j_t)^\gamma \] (B.38)
\[ f^{ii}_t = \omega^{ii} (\theta^{ii}_t)^\gamma \] (B.39)
\[ f^{ji}_t = \omega^{ji} (\theta^{ji}_t)^\gamma \] (B.40)
\[ f^{jj}_t = \omega^{jj} (\theta^{jj}_t)^\gamma \] (B.41)
\[ f^{ij}_t = \omega^{ij} (\theta^{ij}_t)^\gamma \] (B.42)

\[ \theta^i_t = \frac{V^i_t}{S^i_t} \] (B.43)
\[ \theta^j_t = \frac{V^j_t}{S^j_t} \] (B.44)
\[ \theta^{ii}_t = \frac{V^{ii}_t}{S^{ii}_t} \] (B.45)
\[ \theta^{ji}_t = \frac{V^{ji}_t}{S^{ji}_t} \] (B.46)
\[ \theta^{jj}_t = \frac{V^{jj}_t}{S^{jj}_t} \] (B.47)
\[ \theta^{ij}_t = \frac{V^{ij}_t}{S^{ij}_t} \] (B.48)

Native job-creation condition in region \(i\)

\[
\kappa \frac{P^i_{H,t}}{q^i_t} \frac{P^i_t}{P^i_t} = \frac{P^i_{x,t}}{P^i_t} A^i_t - \frac{W^{ii}_t}{P^i_t} + (1 - \rho^i)E^t \left\{ \beta \frac{U^{i}_{c,t+1}}{U^{i}_{c,t}} \kappa \frac{P^i_{H,t+1}}{q^i_{t+1}} \right\} \] (B.49)

Migrant job-creation condition in region \(i\)

\[
\kappa \frac{P^i_{H,t}}{q^i_t} \frac{P^i_t}{P^i_t} = \frac{P^i_{x,t}}{P^i_t} A^i_t - \frac{W^{ji}_t}{P^i_t} + (1 - \rho^i)E^t \left\{ \beta \frac{U^{j}_{c,t+1}}{U^{j}_{c,t}} \kappa \frac{P^i_{H,t+1}}{q^i_{t+1}} \right\} \] (B.50)
Native job-creation condition in region $j$

$$\frac{\kappa}{q_{ij}^j} \frac{P_{H,t}^j}{P_t^j} = \frac{P_{x,t}^j}{P_t^j} A_i^j - W_{ij}^j + (1 - \rho^j) E_t \left\{ \frac{\beta}{U_{c,t}^j} \frac{U_{c,t}^j}{q_{ij}^j} \frac{P_{H,t+1}^j}{P_{t+1}^j} \right\}$$

(B.51)

Migrant job-creation condition in region $j$

$$\frac{\kappa}{q_{ij}^j} \frac{P_{H,t}^j}{P_t^j} = \frac{P_{x,t}^j}{P_t^j} A_i^j - W_{ij}^j + (1 - \rho^j) E_t \left\{ \frac{\beta}{U_{c,t}^j} \frac{U_{c,t}^j}{q_{ij}^j} \frac{P_{H,t+1}^j}{P_{t+1}^j} \right\}$$

(B.52)

Wage equations for native workers in region $i$

$$W_{i}^{ii} = \eta \left[ b^{ii} + (1 - \zeta) \frac{U_{i,t}^{ii}}{U_{i,t}^{ij}} (1 - \alpha_1)^{1/v_1} \left( \frac{E_t^{ii} + \zeta U_t^{ii}}{1 - l_t^{ii}} \right) \right]^{1/v_1}$$

$$+ (1 - \eta) \left[ \frac{P_{x,t}^{ii}}{P_t^{ii}} A_i^i + (1 - \rho^i) \beta E_t \left\{ \frac{U_{c,t+1}^i}{U_{c,t}^i} \frac{\kappa}{q_{it+1}^i} \frac{P_{H,t+1}^i}{P_{t+1}^i} \right\} \right]$$

(B.53)

Wage equations for migrant workers in region $i$

$$W_{i}^{ji} = \frac{\eta}{\eta + Q_t^i(1 - \eta)} \left[ b^{ji} + Q_t^i(1 - \zeta) \frac{U_{i,t}^{ji}}{U_{i,t}^{ij}} (1 - \alpha_1)^{1/v_1} \left( \frac{E_t^{ji} + \zeta U_t^{ji}}{1 - l_t^{ji}} \right) \right]^{1/v_1}$$

$$+ \left[ \frac{Q_t^i(1 - \eta)}{\eta + Q_t^i(1 - \eta)} \right] \left[ \frac{P_{x,t}^{ji}}{P_t^{ji}} A_i^i + (1 - \rho^i) \beta E_t \left\{ \frac{U_{c,t+1}^i}{U_{c,t}^i} \frac{\kappa}{q_{it+1}^i} \frac{P_{H,t+1}^i}{P_{t+1}^i} \right\} \right]$$

(B.54)

Wage equation for native workers in region $j$

$$W_{j}^{jj} = \eta \left[ b^{jj} + (1 - \zeta) \frac{U_{j,t}^{jj}}{U_{j,t}^{ij}} (1 - \alpha_1)^{1/v_1} \left( \frac{E_t^{jj} + \zeta U_t^{jj}}{1 - l_t^{jj}} \right) \right]^{1/v_1}$$

$$+ (1 - \eta) \left[ \frac{P_{x,t}^{jj}}{P_t^{jj}} A_i^j + (1 - \rho^j) \beta E_t \left\{ \frac{U_{c,t+1}^j}{U_{c,t}^j} \frac{\kappa}{q_{jt+1}^j} \frac{P_{H,t+1}^j}{P_{t+1}^j} \right\} \right]$$

(B.55)

Wage equation for migrant workers in region $j$

$$W_{j}^{ij} = \frac{\eta Q_t^j}{\eta Q_t^j + (1 - \eta)} \left[ b^{ij} + \frac{1}{Q_t^j} (1 - \zeta) \frac{U_{j,t}^{ij}}{U_{j,t}^{ij}} (1 - \alpha_1)^{1/v_1} \left( \frac{E_t^{ij} + \zeta U_t^{ij}}{1 - l_t^{ij}} \right) \right]^{1/v_1}$$

$$+ \left[ \frac{(1 - \eta)}{\eta Q_t^j + (1 - \eta)} \right] \left[ \frac{P_{x,t}^{ij}}{P_t^{ij}} A_i^i + (1 - \rho^j) \beta E_t \left\{ \frac{U_{c,t+1}^j}{U_{c,t}^j} \frac{\kappa}{q_{jt+1}^j} \frac{P_{H,t+1}^j}{P_{t+1}^j} \right\} \right]$$

(B.56)

$$X_t^i = A_i^i E_t^i \quad (B.57)$$
\[
X_t^j = A_t^j E_t^j
\]  
(B.58)

\[
RMC_i^j = \left[ \frac{P_i^{x,t}}{P_i^t} \left( 1 - \alpha_i^2 \right) + \alpha_i^2 \frac{(1 - \alpha_i^2)(Q_i^t)^{1-\nu_2} - \alpha_i^j}{(1 - \alpha_i^2) - \alpha_i^2 (Q_i^t)^{1-\nu_2}} \right]^{\frac{1}{1-\nu_2}} \left( Y_i^j \right)^{\frac{\alpha}{1-\alpha}} \frac{1}{1 - \alpha}
\]  
(B.59)

\[
RMC_i^j = \left[ \frac{P_j^{x,t}}{P_j^t} \left( 1 - \alpha_j^2 \right) + \alpha_j^2 \frac{(1 - \alpha_j^2)(Q_j^t)^{1-\nu_2} - \alpha_j^i}{(1 - \alpha_j^2) - \alpha_j^2 (Q_j^t)^{1-\nu_2}} \right]^{\frac{1}{1-\nu_2}} \left( Y_j^i \right)^{\frac{\alpha}{1-\alpha}} \frac{1}{1 - \alpha}
\]  
(B.60)

\[
\pi_{H,t}^i = \beta E_t \{ \pi_{H,t+1}^i \} + \lambda^i \hat{mc}_{i,t}^i
\]  
(B.61)

\[
\pi_{H,t}^j = \beta E_t \{ \pi_{H,t+1}^j \} + \lambda^j \hat{mc}_{j,t}^j
\]  
(B.62)

\[
Y_i^i = (X_i^i)^{1-\alpha} (\Delta_i^i)^{\alpha-1}
\]  
(B.63)

\[
Y_i^j = (X_j^i)^{1-\alpha} (\Delta_j^i)^{\alpha-1}
\]  
(B.64)

\[
Y_i^i = \left[ \frac{P_i^{x,t}}{P_i^t} \left( 1 - \alpha_i^2 \right) + \alpha_i^2 \frac{(1 - \alpha_i^2)(Q_i^t)^{1-\nu_2} - \alpha_i^j}{(1 - \alpha_i^2) - \alpha_i^2 (Q_i^t)^{1-\nu_2}} \right]^{\frac{\nu_2}{1-\nu_2}} \left[ (1 - \alpha_i^2) C_i^i + \alpha_i^2 \frac{1}{n} (Q_i^t)^{\nu_2} C_j^j \right] + \kappa V_i^i
\]  
(B.65)

\[
Y_i^j = \left[ \frac{P_j^{x,t}}{P_j^t} \left( 1 - \alpha_j^2 \right) + \alpha_j^2 \frac{(1 - \alpha_j^2)(Q_j^t)^{1-\nu_2} - \alpha_j^i}{(1 - \alpha_j^2) - \alpha_j^2 (Q_j^t)^{1-\nu_2}} \right]^{\frac{\nu_2}{1-\nu_2}} \left[ (1 - \alpha_j^2) C_j^j + \alpha_j^2 \frac{n}{1-n} (Q_j^t)^{\nu_2} C_i^i \right] + \kappa V_j^i
\]  
(B.66)

\[
Q_i^t = \frac{\Pi_i^t}{\Pi_{i-1}^t} Q_{i-1}^t
\]  
(B.67)
The model is closed with a definition of monetary policy and the law of motion for all three exogenous shock.

Appendix C. Efficient Allocation

The cooperative planner thus maximizes

\[ E_0 \sum_{t=0}^{\infty} \beta^t \left[ U^i_t (C^i, l^i_t) + U^j_t (C^j, l^j_t) \right] \]  (C.1)

subject to

\[ C^i_t = \left\{ (1 - \alpha^i_2)^{\frac{\nu_2}{\nu_2 - 1}} (C^i_{H,t})^{\frac{\nu_2 - 1}{\nu_2}} + (\alpha^i_2)^{\frac{1}{\nu_2}} (C^i_{F,t})^{\frac{\nu_2 - 1}{\nu_2}} \right\}^{\frac{1}{\nu_2}} \]  (C.2)

\[ C^j_t = \left\{ (1 - \alpha^j_2)^{\frac{\nu_2}{\nu_2 - 1}} (C^j_{H,t})^{\frac{\nu_2 - 1}{\nu_2}} + (\alpha^j_2)^{\frac{1}{\nu_2}} (C^j_{F,t})^{\frac{\nu_2 - 1}{\nu_2}} \right\}^{\frac{1}{\nu_2}} \]  (C.3)

\[ l^i_t = 1 - \left[ (1 - \alpha_1)^{1/\mu_1} (E^{ii}_t + \zeta U^{ii}_t)^{\frac{\mu_1 - 1}{\mu_1}} + (\alpha_1)^{1/\mu_1} (E^{ij}_t + \zeta U^{ij}_t)^{\frac{\mu_1 - 1}{\mu_1}} \right]^{\frac{1}{\mu_1}} \]  (C.4)

\[ l^j_t = 1 - \left[ (1 - \alpha_1)^{1/\mu_1} (E^{jj}_t + \zeta U^{jj}_t)^{\frac{\mu_1 - 1}{\mu_1}} + (\alpha_1)^{1/\mu_1} (E^{ji}_t + \zeta U^{ji}_t)^{\frac{\mu_1 - 1}{\mu_1}} \right]^{\frac{1}{\mu_1}} \]  (C.5)

\[ E^{ii}_t = (1 - \rho^i) E^{ii}_{t-1} + \omega^{ii} (\theta^{ii}_t)^{\gamma} S^{ii}_t \]  (C.6)

\[ E^{jj}_t = (1 - \rho^j) E^{jj}_{t-1} + \omega^{jj} (\theta^{jj}_t)^{\gamma} S^{jj}_t \]  (C.7)

\[ E^{ij}_t = (1 - \rho^j) E^{ij}_{t-1} + \omega^{ij} (\theta^{ij}_t)^{\gamma} S^{ij}_t \]  (C.8)

\[ E^{ji}_t = (1 - \rho^i) E^{ji}_{t-1} + \omega^{ji} (\theta^{ji}_t)^{\gamma} S^{ji}_t \]  (C.9)

\[ U^{ii}_t = [1 - \omega^{ii} (\theta^{ii}_t)^{\gamma}] S^{ii}_t \]  (C.10)

\[ U^{jj}_t = [1 - \omega^{jj} (\theta^{jj}_t)^{\gamma}] S^{jj}_t \]  (C.11)

\[ U^{ij}_t = [1 - \omega^{ij} (\theta^{ij}_t)^{\gamma}] S^{ij}_t \]  (C.12)

\[ U^{ji}_t = [1 - \omega^{ji} (\theta^{ji}_t)^{\gamma}] S^{ji}_t \]  (C.13)
\[ Y_t^i = \left[ A_t^i (E_t^{ii} + E_t^{ij}) \right]^{(1-\alpha)} \]  \hspace{1cm} (C.14) \\
\[ Y_t^j = \left[ A_t^j (E_t^{jj} + E_t^{ij}) \right]^{(1-\alpha)} \]  \hspace{1cm} (C.15) \\
\[ Y_t^i = nC_{H,t}^i + (1-n)C_{F,t}^j + \kappa^{ii} \theta_t^i S_t^{ii} + \kappa^{ij} \theta_t^j S_t^{ij} \]  \hspace{1cm} (C.16) \\
\[ Y_t^j = (1-n)C_{H,t}^j + nC_{F,t}^i + \kappa^{ji} \theta_t^j S_t^{jj} + \kappa^{ij} \theta_t^i S_t^{ij} \]  \hspace{1cm} (C.17)

Notice that the sum of constraints (C.6), (C.9), (C.10) and (C.13) yields

\[ E_t^{ii} + E_t^{ij} + U_t^{ii} + U_t^{ij} = (1 - \rho^j) E_{t-1}^{ii} + S_t^{ii} + (1 - \rho^j) E_{t-1}^{ij} + S_t^{ij} \]

\[ (1 - \rho^j) E_{t-1}^{ii} + \Omega_t S_{H,t}^i + (1 - \rho^j) E_{t-1}^{ij} + (1 - \Omega_t) S_{H,t}^j = 1 \]

Furthermore, given that \( S_t^{ii} + S_t^{ij} = S_{H,t}^i \), with \( S_{H,t}^i \) being predetermined, choosing \( S_t^{ii} \) determines \( S_t^{ij} \):

\[ S_t^{ij} = 1 - (1 - \rho^j) E_{t-1}^{ii} - (1 - \rho^j) E_{t-1}^{ij} - S_t^{ii} \]  \hspace{1cm} (C.18)

such that (C.7), (C.9), (C.11) and (C.13) become, respectively

\[ E_t^{ij} = (1 - \rho^j) E_{t-1}^{ij} + \omega^{ij} ( \theta_t^{ij} ) \gamma \left[ 1 - (1 - \rho^j) E_{t-1}^{ii} - (1 - \rho^j) E_{t-1}^{ij} - S_t^{ii} \right] \]  \hspace{1cm} (C.19) \\
\[ E_t^{ji} = (1 - \rho^i) E_{t-1}^{ji} + \omega^{ji} ( \theta_t^{ji} ) \gamma \left[ 1 - (1 - \rho^i) E_{t-1}^{jj} - (1 - \rho^i) E_{t-1}^{ji} - S_t^{jj} \right] \]  \hspace{1cm} (C.20) \\
\[ U_t^{ij} = [1 - \omega^{ij} ( \theta_t^{ij} ) \gamma] \left[ 1 - (1 - \rho^j) E_{t-1}^{ii} - (1 - \rho^j) E_{t-1}^{ij} - S_t^{ii} \right] \]  \hspace{1cm} (C.21) \\
\[ U_t^{ji} = [1 - \omega^{ji} ( \theta_t^{ji} ) \gamma] \left[ 1 - (1 - \rho^i) E_{t-1}^{jj} - (1 - \rho^i) E_{t-1}^{ji} - S_t^{jj} \right] \]  \hspace{1cm} (C.22)

Finally, use constraints (C.10), (C.12), (C.21) and (C.22) to substitute out all four types of unemployment in (C.4) and (C.5), and combining the resource constraints, (C.14) and (C.16), and (C.15) and (C.17), respectively. The cooperative planner thus maximizes

\[ E_0 \sum_{t=0}^{\infty} \beta^t \left[ U_t^i \left( C_t^i, l_t^i \right) + U_t^j \left( C_t^j, l_t^j \right) \right] = \]

\[ E_0 \sum_{t=0}^{\infty} \left[ \frac{[(C_t^i Z_t)^h(l_t^{1-b})^{1-b}]^{1-\sigma} - 1}{1 - \sigma} + \frac{[(C_t^j Z_t)^h(l_t^{1-b})^{1-b}]^{1-\sigma} - 1}{1 - \sigma} \right] \]
subject to the following constraints

\[
C_i^i = \left\{ (1 - \alpha_2^i)^{\frac{1}{2}} C_{H,i}^{\frac{\nu_2 - 1}{2}} + (\alpha_2^i)^{\frac{1}{2}} C_{F,i}^{\frac{\nu_2 - 1}{2}} \right\}^{\frac{\nu_2}{\nu_2 - 1}}
\]

\[
C_i^j = \left\{ (1 - \alpha_2^j)^{\frac{1}{2}} C_{H,j}^{\frac{\nu_2 - 1}{2}} + (\alpha_2^j)^{\frac{1}{2}} C_{F,j}^{\frac{\nu_2 - 1}{2}} \right\}^{\frac{\nu_2}{\nu_2 - 1}}
\]

\[
l_i^i = 1 - \left\{ (1 - \alpha_1)^{1/\nu_1} \left[ E_t^{ij} + \zeta (1 - \omega^i (\theta_t^i)^\gamma), S_t^{ij} \right]^{\frac{\nu_1 - 1}{\nu_1}} + \right. \]

\[
(\alpha_1)^{1/\nu_1} \left[ E_t^{ij} + \zeta (1 - \omega^i (\theta_t^i)^\gamma)(1 - (1 - \rho^i) E_t^{ij} - (1 - \rho^i) E_t^{ij} - S_t^{ij}) \right]^{\frac{\nu_1 - 1}{\nu_1}} \right\}^{\frac{\nu_1 - 1}{\nu_1 - 1}}
\]

\[
E_t^{ij} = (1 - \rho^j) E_t^{ij} + \omega^i (\theta_t^i)^\gamma S_t^{ij}
\]

\[
E_t^{ij} = (1 - \rho^j) E_t^{ij} - \omega^i (\theta_t^i)^\gamma S_t^{ij}
\]

Define the Lagrange multiplier on the two resource constraints as \(\lambda_{t,RC}^i\) and \(\lambda_{t,RC}^j\) respectively, and the Lagrange multipliers for the law of motion of each of the four types of employment as \(\lambda_{E,t}^i\), \(\lambda_{E,t}^j\), \(\lambda_{E,t}^{ij}\), and \(\lambda_{E,t}^{ij}\), the first-order conditions with respect to domestic and imported consumption state as follows:

\[
[C_{H,i}^i; C_{F,i}^i] : \lambda_{RC,t}^i = U_{c,t}^i \left[ \frac{1 - \alpha_2^i}{C_{H,i}^i} \right]^{\frac{1}{\nu_2}} \left[ \frac{\alpha_2^i C_{F,i}^i}{C_{F,i}^i} \right]^{\frac{1}{\nu_2}} = U_{c,t}^i \left[ \frac{\alpha_2^i C_{F,i}^i}{C_{F,i}^i} \right]^{\frac{1}{\nu_2}} \quad (C.23)
\]

\[
[C_{F,i}^i; C_{H,i}^i] : \lambda_{RC,t}^i = U_{c,t}^i \left[ \frac{\alpha_2^i C_{F,i}^i}{C_{F,i}^i} \right]^{\frac{1}{\nu_2}} = U_{c,t}^i \left[ \frac{1 - \alpha_2^i}{C_{H,i}^i} \right]^{\frac{1}{\nu_2}} \quad (C.24)
\]

\[
[\theta_t^{ij}] : \frac{U_{c,t}^i}{\alpha_{L,t}^i} \left[ \frac{1 - \alpha_1^i}{\alpha_{L,t}^i} \right]^{\frac{1}{\nu_1}} \zeta \gamma^i \omega^i (\theta_t^{ij})^{\gamma - 1} = \frac{\lambda_{E,t}^i \gamma^i \omega^i (\theta_t^{ij})^{\gamma - 1}}{U_{c,t}^i} + \kappa_{ij} \lambda_{RC,t}^i \quad (C.25)
\]

\[
[\theta_t^{ij}] : \frac{U_{c,t}^i}{\alpha_{L,t}^i} \left[ \frac{1 - \alpha_1^i}{\alpha_{L,t}^i} \right]^{\frac{1}{\nu_1}} \zeta \gamma^i \omega^i (\theta_t^{ij})^{\gamma - 1} = \frac{\lambda_{E,t}^i \gamma^i \omega^i (\theta_t^{ij})^{\gamma - 1} + \kappa_{ij} \lambda_{RC,t}^i}{U_{c,t}^i} \quad (C.26)
\]

53
Rewriting the efficiency condition with respect to $S_{i}^{ij}$, (28), using (23), (24), (30) and (31) results in

$$n \frac{U_{i,t}}{U_{c,t}^{i}} (1 - \zeta) \left[ \frac{(1 - \alpha_{1}) \tilde{L}_{t}^{i}}{L_{t}^{i}} \right]^{\frac{1}{\gamma_{i}}} = \left[ \frac{(1 - \alpha_{1}) \tilde{L}_{t}^{i}}{L_{t}^{i}} \right]^{\frac{1}{\gamma_{i}}} - \frac{\kappa_{ij}^{c}}{q_{ij}^{h}} \left[ \frac{(1 - \alpha_{2}) C_{t}^{i}}{C_{H,t}^{i}} \right]^{\frac{1}{\gamma_{i}}} + (1 - \rho_{j}) \frac{U_{i,t+1}^{c}}{U_{c,t}^{i}} ... \quad (32)$$

Combining (27) with (23), (30), and (31) results in

$$n \frac{U_{i,t}}{U_{c,t}^{i}} \zeta \left[ \frac{(1 - \alpha_{1}) \tilde{L}_{t}^{i}}{L_{t}^{i}} \right]^{\frac{1}{\gamma_{i}}} - \frac{1}{\gamma} \left[ \frac{(1 - \alpha_{1}) \tilde{L}_{t}^{i}}{L_{t}^{i}} \right]^{\frac{1}{\gamma}} + \kappa_{ij}^{c} \left[ \frac{(1 - \alpha_{2}) C_{t}^{i}}{C_{H,t}^{i}} \right]^{\frac{1}{\gamma}} + (1 - \rho_{j}) \frac{U_{i,t+1}^{c}}{U_{c,t}^{i}} ...$$

Rewriting the efficiency condition with respect to $S_{i}^{ij}$, (28), using (23), (24), (30) and (31) yields

$$n \frac{U_{i,t}}{U_{c,t}^{i}} \zeta \left[ \frac{(1 - \alpha_{1}) \tilde{L}_{t}^{i}}{L_{t}^{i}} \right]^{\frac{1}{\gamma}} - \frac{1}{\gamma} \left[ \frac{(1 - \alpha_{1}) \tilde{L}_{t}^{i}}{L_{t}^{i}} \right]^{\frac{1}{\gamma}} + \frac{\kappa_{ij}^{c}}{q_{ij}^{h}} \left[ \frac{(1 - \alpha_{2}) C_{t}^{i}}{C_{H,t}^{i}} \right]^{\frac{1}{\gamma}} + (1 - \rho_{j}) \frac{U_{i,t+1}^{c}}{U_{c,t}^{i}} ...$$
Combining the efficient migration decision, (C.33), with (C.32) leads to the efficiency condition for native workers

\[
\frac{U_{i,t}^{i}}{U_{c,t}^{i}} (1 - \zeta) \left[ \frac{(1 - \alpha_1)\tilde{L}_{i}^{i}}{L_{i}^{ii}} \right]^{\frac{1}{\nu_1}} = \frac{\left(1 - \alpha_2\right)C_{i,t}^{i}}{C_{H,t}^{i}} \left[ \frac{(1 - \alpha)\tilde{Y}_{i}^{i}}{E_{i}^{i}} - \frac{\kappa_{i}}{q_{i}^{ii} \gamma} \right] + \beta(1 - \rho^i) \frac{U_{c,t+1}^{i}}{U_{c,t}^{i}} \left[ \frac{(1 - \alpha_2)C_{i,t+1}^{i}}{C_{H,t+1}^{i}} \right]^{\frac{1}{\nu_2}} \frac{\kappa_{ii}}{q_{i+1}^{ii}} \frac{1 - (1 - \gamma)f_{i+1}^{ii}}{\gamma}
\]

where

\[
\beta(1 - \rho^i) \frac{U_{c,t+1}^{i}}{U_{c,t}^{i}} \left[ \frac{(1 - \alpha_2)C_{i,t+1}^{i}}{C_{H,t+1}^{i}} \right]^{\frac{1}{\nu_2}} \frac{\kappa_{i}}{q_{i+1}^{ii}} \frac{1 - (1 - \gamma)f_{i+1}^{ii}}{\gamma} = (1 - \rho^i)E_t \{ Q_{i,t+1}^{ii} V_{i,j,t+1}^{ii} \} + (1 - \rho^j)E_t Q_{j,t+1}^{ij} (1 - f_{j+1}^{ij}) V_{j,i,t+1}^{ij}
\]

Equivalently, for migrant workers combining (C.28) with (C.24), (C.30), and (C.31) results in

\[
\frac{U_{i,t}^{i}}{U_{c,t}^{i}} (1 - \zeta) \left[ \frac{\alpha_1\tilde{L}_{i}^{i}}{L_{i}^{ii}} \right]^{\frac{1}{\nu_1}} = \frac{\left(1 - \alpha_2\right)C_{i,t}^{i}}{C_{F,t}^{i}} \left[ \frac{(1 - \alpha)\tilde{Y}_{i}^{i}}{E_{i}^{i}} - \frac{\kappa_{i}}{q_{i}^{ij} \gamma} \right] + \beta(1 - \rho^j) \frac{U_{c,t+1}^{i}}{U_{c,t}^{i}} \left[ \frac{(1 - \alpha_2)C_{i,t+1}^{i}}{C_{F,t+1}^{i}} \right]^{\frac{1}{\nu_2}} \frac{\kappa_{ij}}{q_{j+1}^{ij}} \frac{1 - (1 - \gamma)f_{j+1}^{ij}}{\gamma}
\]

Finally, combining the first-order condition with respect to \( S_{i}^{ii} \) with equations (C.30) and (C.31) yields the following efficient migration decision

\[
\frac{U_{i,t}^{i}}{U_{c,t}^{i}} \zeta \left\{ \left[ \frac{(1 - \alpha_1)\tilde{L}_{i}^{i}}{L_{i}^{ii}} \right]^{\frac{1}{\nu_1}} - \left[ \frac{\alpha_1\tilde{L}_{i}^{i}}{L_{i}^{ii}} \right]^{\frac{1}{\nu_1}} \right\} = \frac{1 - \gamma}{\gamma} \left\{ f_{i+1}^{ii} V_{i,j,t}^{ii} - f_{j+1}^{ij} V_{j,i,t}^{ij} \right\}
\]