Federal unemployment insurance: theory, and an application to Europe

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Abstract

This paper studies the optimal federal provision of unemployment insurance in a group of small economies. In each, the labor market is characterized by search and matching frictions, risk-averse workers, endogenous hiring and separation, and unobservable search effort. Countries are subject to idiosyncratic, persistent business cycle shocks amid demand externalities. International financial markets are incomplete. Analytically, we show how the optimal design of the federal unemployment insurance system depends on the extent to which member states (can) make optimal use of their respective labor-market policies over the business cycle. A calibration to the European Monetary Union shows that federal provision of UI will serve a meaningful role only if member states do not make optimal use of the policy instruments at their disposal, or if there are strong demand externalities. Otherwise, a federal unemployment insurance scheme can provide insurance only in the severest of circumstances.

Keywords: labor-market policy mix, fiscal federalism, search and matching

JEL-Codes: E32, E24, J64

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1 Introduction

The current paper discusses the optimal design of unemployment insurance (UI) in a federal union of member states. In federal unions, unemployment benefits can be financed locally, at the level of the member state. Or, they can be financed jointly, at the federal level. Federal financing has the benefit of automatically providing insurance against regional shocks. Nevertheless, a striking feature of actual UI systems is the different importance that they assign to the federal component of UI.\footnote{The U.S. gives autonomy to member states when it comes to labor-market regulation. The UI system mainly rests on states. A federal component is activated only after severe shocks, U.S. Department of Labor (2017). In line with this, the literature finds only a very limited role of the federal UI in smoothing regional shocks in the US, see Asdrubali et al. (1996) and Feyrer and Sacerdote (2013). At the different end of the spectrum is Germany, which provides generous federal UI. Indeed, UI is administered exclusively at the federal level. At the same time, labor-market policies and regulations are centralized as well.} The current paper seeks to provide guidance – both theoretical and quantitative – as to how the federal component of UI should be designed. The theory that we present is general. The quantitative application is targeted at the European Monetary Union (EMU).

The paper models a union of small member states that can jointly finance unemployment benefits. Member states are subject to country-specific business cycle shocks. International financial markets are incomplete and there are demand externalities (modelled following Krueger et al., 2016). This means that there is a case for state-contingent federal transfers, as in Farhi and Werning (forthcoming). Federal UI benefits provide insurance to the member states, and ameliorate the externalities associated with low aggregate demand. The central element that may restrict the scope for a federal UI scheme is regional free riding as in Persson and Tabellini (1996). Member states control their own labor-market policies (namely, hiring subsidies, layoff taxes, and their own UI system). Mortensen and Pissarides (1994) search and matching frictions mean that through labor-market policies, member states can affect local unemployment.

To start with, we derive analytical intuition in a tractable one-shot version of the model, building on Landais et al. (forthcoming). Countries can be in a boom or a recession. The social planner would perfectly smooth consumption across member states. When federal transfers need to be conditioned on unemployment, and member states can use some of their labor-market instruments (such as their own UI system) in their own interest, member states \emph{ex ante} will support only limited federal insurance. Federal UI would be provided only to member states that are at a point of the business cycle when those member states’ unemployment rate is rather irresponsive to benefits (a low general-equilibrium (macro) elasticity of unemployment). This means two things. First, whenever member states can optimally use their own labor-market policy mix to respond to a recession, the scope for federal UI is limited. Then, member states makes sure that jobs are never hard to come by, that is, that the general-equilibrium elasticity of unemployment never is small to start with so that federal...
UI would distort member states’ policies. Second, when member states cannot adjust the entire mix optimally, but only parts of it, federal UI should be paid only in severe-enough recessions. The reason is simple again. Insurance to the individual worker can be provided by the member state itself, without distorting member states’ incentives. This leaves a role for federal UI only insofar as it insures the region as a whole. Absent demand externalities, the benefits of this insurance are second-order, however, while the incentives to free-ride near the steady state would be of first-order importance. The optimal federal UI scheme, therefore, provides payments only far enough from steady state. Demand externalities weaken this reasoning because curbing demand externalities through federal transfers has beneficial first-order welfare effects. Demand externalities do not entirely overturn the limits on federal provision of UI, however. The reason is that member states could adjust their own policies (even if the set of policies is limited) to stimulate employment and demand. A federal UI scheme crowds out of member states’ efforts.

Next, we want to understand what the qualitative statements mean in practice. Toward this end, we provide a quantitative exploration in a dynamic business-cycle setting. Having a dynamic setting is important in three respects. First, because it allows us to discuss separately two dimensions of the member states’ response to a federal UI scheme: how the scheme alters the average level of member states’ policy instruments (think of a permanent change in policies) and how federal UI changes the member states’ policy response to cyclical fluctuations. Second, because it allows us to discuss schemes that explicitly index benefits to past unemployment rates in a member state, which may dampen the incentives to free ride. Third, because the calibration to the business cycle facts allows us to use the model to identify the size and movement over the cycle of the macro-elasticities of unemployment with respect to policy that shape the optimal provision of federal UI.

We provide a quantitative assessment for a stylized European Monetary Union (EMU). EMU is an important case to study for it is a union of sovereign member states and, to date, these control their own labor-market policies. The status quo is that UI is exclusively provided by member states. The limited extent of cross-state risk-sharing, both public and private (documented, for example, in Furceri and Zdzenicka, 2015), has prompted calls for further fiscal integration by, for example, Jean-Claude Juncker (2015) and European Commission (2017). Federal unemployment insurance is a natural starting point, for example Andor (2016).

EMU is also of particular interest because of a notable divergence in views in EMU as to which policies member states can implement or should be able to implement on their own (Brunnermeier et al., 2016). A case in point is how much member states can use cyclical labor-market policies to smooth their business cycle. Different readers may, thus, hold different views as to the primitive assumption of the exercise. The aim is to nest these views and to highlight how the conclusions with regard to federal UI are shaped by the very primitives.
The current paper, thus, seeks to spell out are the conditions under which federal UI is both desirable and quantitatively meaningful.

The results point to control over a broad range of labor-market policies as a central determinant of the optimal generosity of and role for federal UI. Federal provision of UI will serve a meaningful role in terms of improving welfare only if member states do not (or cannot) make optimal use of the policy instruments at their disposal, or if there are strong demand externalities. Otherwise, in our model EMU, a federal unemployment insurance scheme can provide insurance only in the severest of circumstances, and in terms of welfare it is close to irrelevant. The calibration entails considerable fluctuations in unemployment that arise from wage rigidity. The resulting fluctuations in employment are socially inefficient. In line with Jung and Kuester (2015), the optimal domestic response of member-state government’s absent a federal UI scheme would be to use the labor-market policy instruments with a view toward reducing the size of unemployment fluctuations. In theory, they could do so in a way that hardly alters unemployment benefits. Having done so, there still does remain scope for international insurance. In line with the intuition developed earlier, however, if implemented through a federal UI scheme, payments are close to zero for most of the state space, otherwise they would crowd out the member states’ efforts at stabilizing employment.

What is important to note is that this effect pertains to the cyclical response of labor-market policies as much as to the long-term response. A long-term response of policies to a federal UI scheme can be circumvented by indexing federal UI payments to a long-term unemployment rate in the member state. We show that this does not alleviate the incentives of the member state government to provide insufficient cyclical stabilization. This we consider an important finding of the current paper.

There is another important finding of the quantitative exercise. Namely, the desirability of federal UI depends on the entire labor-market policy mix. What our quantitative results show is that harmonizing the UI benefit scheme across member states (for example, by legislating a certain fixed generosity) will not on its own make a generous federal UI scheme more viable. Indeed, the other dimensions of the labor-market policy mix (in our simulations, the layoff taxes in particular) have an equal bearing on member states’ unemployment rates. A generous federal UI scheme is desirable and viable if member states do not control labor market policy.

Of course, the reader may doubt if member states would be able to adjust labor-market policies in an optimal manner to start with, and if so if this applies to the average level only or the cyclical component of policy. Indeed, one may see the present institutional heterogeneity in EMU as a sign that member states currently have the right, but not the political ability to set labor-market policies optimally. If this is so, there is a case for generous federal UI.

The paper is organized as follows: next, we review the related literature. Section 2 spells out the quantitative dynamic model and the member states’ and federal governments’ problems. Section 3 provides intuition as to the optimal design of federal UI using a one-period
simplification of the full model. Section 4 calibrates the full model to EMU and, then, shows the quantitative implications for the optimal federal unemployment policy. A final section concludes. An extensive appendix provides derivations as well as proofs to the propositions.

Related literature

The paper is linked to three streams of literature. The first stream of literature is concerned with the theory of fiscal federalism. The second with fiscal policy in currency unions. The third stream is concerned with the optimal setting of labor-market policies.

The paper builds on the seminal contribution to fiscal federalism of Persson and Tabellini (1996). They study the optimal insurance of aggregate risk in a federation of states, in which individual states retain authority over the provision of insurance against idiosyncratic shocks, taxation, as well as public investment programs. International risk pooling induces local governments to under-invest in programs alleviating local risk. We contribute to this strand of the literature by extending the analysis to a setting that helps to move to the discussion toward a very concrete set of federal and local labor-market policies and provides a quantitative exploration. Bordignon et al. (2001) analyze optimal fiscal redistribution across regions when the central government has limited information on regional shocks. In order to signal that a region truly had a bad shock it has to engage in costly policies, in their case raising taxes in bad times. The optimal federal UI schemes that we obtain shares this flavor. If member states can adjust their policies to take advantage of federal UI, federal UI is provided only in severe-enough circumstances. Celentani et al. (2004) study how market incompleteness may arise from decentralized fiscal policies, whereas we take incompleteness as a primitive. The optimal risk-sharing arrangements within a union of countries have also been studied in Bucovetsky (1998); Lockwood (1999) and Evers (2015), among others.

An influential literature has documented that fiscal risk sharing may be of particular importance for currency unions, Mundell (1961); McKinnon (1963); Kenen (1969). Farhi and Werning (forthcoming) characterize optimal fiscal transfers within a monetary union, when there are demand externalities. They point out large welfare gains from transfers when a lack of demand has first-order effects on welfare. We capture such demand externalities as in Krueger et al. (2016). While there is notable scope for fiscal transfers to be valuable, our paper highlights the quantitative limits that country-specific policies can bring to bear on the implementation of such transfers through an automatic UI-based system. Our paper is also linked to a stream of papers that argue in micro-founded open economy models that local fiscal policy has to complement a common area-wide monetary policy, Beetsma and Jensen (2005); Galí and Monacelli (2008); Ferrero (2009). In our model as well, local policies (labor-market policies) can contribute to stabilizing local economic conditions. We study how the scope of local policies shapes the optimal federal UI scheme.
Our paper also builds on a literature that studies optimal cyclical labor-market policies. We build on earlier work of ours, Jung and Kuester (2015), that has characterized analytically and numerically the optimal mix of unemployment benefits, hiring subsidies and layoff taxes, both over the business cycle and in steady state – in a closed economy. The current paper studies a federal setting with international market incompleteness. Member states alone can no longer implement the planner’s allocation. The question that we ask is: how much scope is there for a federal UI scheme? The current paper’s analytical intuition builds heavily on the one-shot model of Landais et al. (forthcoming). Their theory nests different views as to how changes in unemployment benefits in isolation transmit to the aggregate economy, for example, those of Chodorow-Reich and Karabarbounis (2016) and Hagedorn et al. (2013). We extend Landais et al.’s work in three important dimensions: to a federal union with self-interested member states, to a wide range of labor-market policies, and to accommodate demand externalities. We document the extent to which member states’ use of their own labor-market policies affects the shape of a welfare-improving federal UI scheme.

It is clear that EMU member states to date are characterized by considerable heterogeneity in labor-market institutions; a fact that we abstract from on purpose. We wish to explore the scope for federal insurance and its determinants when member states retain control over labor-market policies and labor-market regulation. Our main finding is that the scope for federal insurance may be large if member states cannot react to the federal policy, but may be limited if they retain the right and the ability to make decisions on labor-market policy. Allowing for heterogeneity and its complications likely will not weaken those incentives.

A number of insightful papers that start from the assumption that member states do not react to federal UI. Moyen et al. (2016) explore federal unemployment insurance amid permanently heterogenous member states. Making recourse to Landais et al. (forthcoming) and to simulations in a New Open Macroeconomics two-country model, they find considerable scope for a European UI scheme. Dolls et al. (2015) conduct micro simulations and find that a European UI scheme would have provided notable stabilization of disposable income in EMU member states in the latest recession. The maintained assumption is that the macroeconomic environment is not affected by the presence of federal UI. In a heterogeneous agent macro model, Ábrahám et al. (2017) account both for idiosyncratic heterogeneity of workers and permanent heterogeneity of labor markets across member states. The authors exert great effort to capture the heterogeneity of labor-market flow rates across the different member states. When considering the redesign of UI schemes, including federal components, these flow rates are treated as constant. The current paper seeks to provide quantitative guidance on the optimal design of a federal UI scheme and its desirability if, instead, labor-market policies do affect flow rates and if member states can adjust their policies in response to a federal scheme.

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2 Even focusing on only one dimension of labor-market policies, national UI systems, there is notable heterogeneity Esser et al. (2013). Also see the data presented in Ábrahám et al. (2017).
2 The model

There is a federal union that consists of a unit mass of atomistic member states. Member states are linked through a federal unemployment insurance system. The federal level raises taxes and provides federal UI transfers. There is no international borrowing and lending, and there also is no trade across borders. That is, international insurance only occurs through the centralized fiscal authority. These assumptions facilitate the exposition and they give a clear role for international insurance.\(^3\) We also abstract from international labor-mobility. Each member state may retain authority regarding domestic labor-market policy.

In the following, variables that pertain to member states are marked by subscript \(i \in [0, 1]\). At the level of the member state, the model and exposition closely follow Jung and Kuester (2015). For clarity we use round parentheses whenever we wish to highlight the arguments of a function. We use square brackets to gather terms.

We describe the model in three steps. First, we only describe the technological constraints faced by each member state. Then, we introduce the elements of the decentralized economy including member states’ policy instruments, the federal UI scheme and private-sector decisions. Last, we describe the policy problems of the member states and the federal level.

2.1 Technological constraints for each member state

Each member state is populated by a continuum of measure one of workers, an infinite mass of potential one-worker firms that produce labor services, and a unit mass of representative firms that use labor services to produce a final consumption good. Workers are homogeneous in regard to their \textit{ex ante} efficiency of working. Firms produce a homogeneous good that cannot be stored. Time is discrete.

2.1.1 Labor market flows

We denote the measure of workers who are employed in a particular member state at the beginning of period \(t\) by \(e_{it}\). Employment at the beginning of the next period evolves according to

\[
e_{i,t+1} = [1 - \xi_i^t] \cdot e_i^t + m_i^t,
\]

where \(m_i^t\) are new firm-worker matches. \(\xi_i^t\) is the rate of separation of existing firm-worker matches in period \(t\). The government cannot observe the search effort of workers. All workers who are not employed at the beginning of the period are counted as “unemployed:” \(u_i^t = 1 - e_i^t\).

\(^3\)That is, we abstract from international insurance through financial markets (no trade in state-contingent securities, and no self-insurance through borrowing and lending) and international trade. International financial markets clearly are not complete, and the European fiscal crisis has shown the limits to international borrowing and lending. To the extent that we wish to model federal insurance at business cycle frequency, our assumption on trade may be tenable as well. The reason is that, short-run trade elasticities tend to be small. In any case, clearly, these assumptions stack the cards in favor of federal insurance.
A worker can be recruited after posting a vacancy at resource cost $\kappa_v > 0$. New matches are created according to the matching function

$$m^i_t = A^i_t \left[ v^i_t \right]^\gamma \cdot \left[ \xi^i_t e^i_t + u^i_t s^i_t \right]^{1-\gamma}. \quad (2)$$

Here, $A^i_t$ are fluctuations in match efficiency. $\gamma \in (0,1)$ is the elasticity of matches with respect to the number of vacancies $v^i_t$ posted by firms. The last term, in turn, is explained as follows: The mass of workers who are potentially searching during period $t$ equals $\xi^i_t e^i_t + u^i_t$. That mass comprises the workers laid off at the beginning of the period, $\xi^i_t e^i_t$, and the mass of workers who entered the period unemployed, $u^i_t$. $s^i_t$ is the share of those workers who search for a job. Match efficiency follows an autoregressive process

$$\log(A^i_t/\chi) = \rho_A \log(A^i_{t-1}/\chi) + \varepsilon^i_A,t, \quad \rho_A \in [0,1), \ varepsilon^i_A,t \sim N(0,\sigma^2_A).$$

Parameter $\chi > 0$ governs the steady-state match efficiency. It is the heterogenous realizations of the member state-specific shocks that generate scope for federal insurance.

For subsequent use, we define labor-market tightness as $\theta^i_t := v^i_t / (\xi^i_t e^i_t + u^i_t)$, the job-finding rate as $f^i_t := m^i_t / (\xi^i_t e^i_t + u^i_t)$, and the job-filling rate as $q^i_t := m^i_t / v^i_t = A^i_t \left[ \theta^i_t \right]^{\gamma-1} = f^i_t / \theta^i_t$.

### 2.1.2 Consumption, value of the worker and search

Workers are risk-averse and have period utility functions $u(c) : \mathcal{R} \to \mathcal{R}$ that are twice continuously differentiable, strictly increasing and concave in the period’s consumption level.$^4$ $\beta \in (0,1)$ is the time-discount factor. Workers who are not employed enjoy an additive utility of leisure $\bar{h}$. Workers employed throughout period $t$ consume $c^i_{e,t}$. Workers who are employed at the beginning of $t$ but whose match is severed in $t$ consume $c^i_{e0,t}$. Workers who enter the period unemployed consume $c^i_{u,t}$.

**Value of an employed worker**

The value of an employed worker at the beginning of the period, before idiosyncratic shocks are realized, is

$$V^i_{e,t} = [1 - \xi^i_t] \cdot \left[ u(c^i_{e,t}) + \beta E_t V^i_{e,t+1} \right] + \xi^i_t V^i_{e0,t}. \quad (3)$$

If the match does not separate, the worker consumes $c^i_{e,t}$ and the match continues into $t + 1$. $E_t$ marks the expectation operator. $V^i_{e0,t}$ is the value of a worker who has just separated from a firm. Apart from the consumption stream in the first period, this has the same value as $V^i_{u,t}$, the value of a worker who enters the period unemployed: $V^i_{e0,t} = V^i_{u,t} + u(c^i_{e0,t}) - u(c^i_{u,t})$. The value $V^i_{u,t}$ will be explained in detail below. For future

$^4$Observe the difference between $u^i_t$ and $u_t$. $u^i_t$ marks the unemployment rate at the beginning of the period, whereas $u$ marks the utility function.
use, define the surplus of the currently employed worker from employment as \( \Delta^i_t := V^i_{e,t} - V^i_{u,t} \).

**Value of an unemployed worker and search**

Unemployed workers need to actively search in order to find a job. Search is a 0-1 decision. Workers are differentiated by their utility cost of search, \( \iota \sim F_\iota(0, \sigma^2_\iota) \). For tractability, these costs are independently and identically distributed both across workers and across time. \( F_\iota(0, \sigma^2_\iota) \) marks the logistic distribution with mean 0 and variance \( \sigma^2_\iota := \pi^2 \psi^2_3 \), where a lower-case \( \pi \) refers to the mathematical constant. All workers whose disutility of search falls below a certain cutoff value \( \iota^{s,i}_t \) do search for a job. For the worker who is just at the cutoff value, the utility cost of search just balances with the expected gain from search:

\[
\iota^{s,i}_t = f^i_t \beta E_t [\Delta^i_{t+1}] .
\]  

(4)

The gain from search is the discounted increase in utility when employed in the next period rather than unemployed multiplied by the probability, \( f^i_t \), that a searching worker will find a job. Using the properties of the logistic distribution, \( s^i_t \), the share of unemployed workers who search is given by

\[
s^i_t = \text{Prob}(\iota \leq \iota^{s,i}_t) = 1/[1 + \exp\{-\iota^{s,i}_t / \psi_s\}] .
\]

(5)

The value of an unemployed worker *ex ante*, that is, before the search preference shock has realized, is given by

\[
V^i_{u,t} = u(c^i_{u,t}) + \bar{h} + \int_{-\infty}^{\iota^{s,i}_t} [-t + f^i_t \beta E_t V^i_{e,t+1} + [1 - f^i_t] \beta E_t V^i_{u,t+1}] dF_\iota(t) + \int_{\iota^{s,i}_t}^{\infty} \beta E_t V^i_{u,t+1} dF_\iota(t)
\]

(6)

Regardless of the search decision, in the current period the unemployed worker receives consumption \( c^i_{u,t} \) and enjoys utility of leisure \( \bar{h} \). If the worker decides to search (second row), the utility cost is \( \iota_t \). With probability \( f^i_t \) the worker will find a job. In that case, the worker’s value at the beginning of the next period will be \( V^i_{e,t+1} \). With probability \( (1 - f^i_t) \) the worker remains unemployed, in which case the worker’s value at the beginning of the next period will be \( V^i_{u,t+1} \). If the worker does not search (third row), the worker will continue to be unemployed in the next period.

**2.1.3 Production and separation**

There are two sets of firms: “employment-services firms” and “final-goods firms.”

*Employment-services firms.*
There is an infinite mass of potential one-worker firms that produce employment services. A firm that enters the period matched to a worker can either produce or separate from the worker. Production entails a firm-specific resource cost, $\epsilon_j$. For analytical tractability, we specify this as a shock that is independently and identically distributed across both matches and time, $\epsilon_j \sim F(\mu_\epsilon, \sigma_\epsilon^2)$. $F(\cdot, \cdot)$ marks the logistic distribution with mean $\mu_\epsilon$ and variance $\sigma_\epsilon^2 = \pi^2/3$. The firm separates from the worker and avoids paying the resource cost whenever the idiosyncratic cost shock, $\epsilon_j$, is larger than a threshold $\epsilon_{i,t}^\xi$. Using the properties of the logistic distribution, conditional on the threshold, the separation rate can be expressed as

$$\xi_i^t = \frac{\text{Prob}(\epsilon_j \geq \epsilon_{i,t}^\xi)}{1 + \exp\{(\epsilon_{i,t}^\xi - \mu_\epsilon)/\psi\}}.$$  \hspace{1cm} (7)

Each firm-worker match, that does not separate, produces. Total production of labor services is given by

$$L_i^t = e_i^t(1 - \xi_i^t) \exp\{a_i^t\},$$  \hspace{1cm} (8)

where $e_i^t(1 - \xi_i^t)$ is the mass of existing matches that are not separated in $t$.

The exogenous component to aggregate productivity, $a_i^t$, evolves according to

$$a_i^t = \rho a_{i-1}^t + \epsilon_{a,t}^i, \quad \rho_a \in [0,1), \quad \epsilon_{a,t}^i \sim N(0, \sigma_a^2).$$

**Final-goods firms.**

Final goods are produced by a representative firm that uses employment services as an input. The final goods firm may operate under decreasing returns to scale, its output being

$$y_i^t = [L_i^t]^\alpha [e_i^t]^\varsigma, \quad \alpha \in (0,1], \varsigma \geq 0.$$  \hspace{1cm} (9)

We allow for decreasing returns to scale so as to be able to accommodate hiring freezes as in Michaillat (2012). In addition, we wish to allow for a demand-side channel, such that the provision of federal insurance can have stabilizing effects beyond the mere transfer. Toward this end, we follow Krueger et al. (2016) and assume that labor productivity is the product of two components: an endogenous component, and an exogenous shock to productivity. Productivity may depend non-negatively on aggregate consumption, $c_i^t := c_i^t(1 - \xi_i^t)\epsilon_{c,t}^i + e_i^\xi_i^t c_{0,t}^i + (1 - c_i^t)\epsilon_{u,t}^i$. Parameter $\varsigma$ captures the size of the spillovers in the member state between aggregate demand and productivity. The larger $\varsigma$, the stronger the demand-side effects.

**2.1.4 Resource constraint**

Each member state’s output is used for consumption, production costs, and vacancy posting. Additionally, by participating in a federal insurance scheme, the local authority has access to
net transfers. Let net transfers to member state $i$ be denoted by $\mathbb{B}_i$. These are given by

$$\mathbb{B}_i := B_{F,t}(u_i) - \tau_F. \quad (10)$$

Here $B_{F,t}(u_i)$ mark payments from the federal level to the individual member state. Payments are a function of $u_i$, that is, the mass of workers for which the member state pays unemployment benefits. Note that all member states, realistically, are subject to the same structure of the transfer scheme. $\tau_F$ marks a fixed payment from each member state to the federal level toward financing the federal insurance scheme.

With this notation at hand, the member state’s resource constraint is

$$y_i^t + \mathbb{B}_i = e_i^t c_{i,t} + u_i^t c_{u,t} + e_i^t \int_{-\infty}^{t} \epsilon dF_\epsilon(\epsilon) + \kappa \nu v_i^t. \quad (11)$$

### 2.2 Decentralized economy

The conditions spelled out above are technological constraints that would constrain the member state’s planning problem. We now discuss those parts of the model that pertain to the decentralized economy only. We start with the government.

#### 2.2.1 Member state government

The member state provides unemployment benefits and hiring subsidies in its constituency only. There, it also levels layoff taxes and production taxes. As documented in Jung and Kuester (2015) for the case with constant returns to scale and without the demand externality, this set of instruments allows the member state’s government to implement the autarkic constraint-efficient planner’s allocation. In addition, the local government receives net transfers from the central government. The member state government’s budget constraint is given by

$$e_i^t [1 - \xi_i] \tau_{J,t} + e_i^t \xi_i \tau_{\xi,t} + \mathbb{B}_i^t = u_i^t B_{i,t} + \tau_{v,t} v_i^t, \quad (12)$$

The left-hand side has revenue from the production and layoff taxes, and the net transfers $\mathbb{B}_i$ from the central government. The right-hand side has unemployment benefits and the vacancy subsidy. The tax and subsidy rules, $\tau_{J,t}$, $\tau_{\xi,t}$, $\tau_{v,t}$, and UI benefit payments, $B_{i,t}$ are specified further below.\(^5\)

#### 2.2.2 Consumption

Firms are owned in equal proportion by each worker located in the member state. Ownership of firms is not traded. $\Pi_i$ marks the dividends that the firms pay. Consumption of the worker

\(^5\) As long as the member state government can set its own unemployment benefits freely, it is entirely inconsequential if federal UI payments are channeled through the member state government’s budget constraint or are paid directly to unemployed workers.
is given by
\[
\begin{align*}
    c_{i,t}^e &:= w_i^t + \Pi_i^t & \text{if employed at the beginning of } t \text{ and working in } t, \\
    c_{0,t}^e &:= w_{eu,t}^i + \Pi_i^t & \text{if employed at the beginning of } t \text{ but laid off in } t, \\
    c_{u,t}^e &:= B_i^t + \Pi_i^t & \text{if unemployed at the beginning of } t.
\end{align*}
\] (13)

Here \( w_i^t \) marks the wage. \( w_{eu,t}^i \) marks severance payments from the firm to a worker who has just been laid off. In the following, we will assume that \( w_{eu,t}^i = w_i^t \), that is, the laid-off worker receives one-period’s worth of wages as a severance payment. In a period that the worker enters unemployed already, the worker receives an amount \( B_i^t \) of unemployment benefits.

### 2.2.3 Production and the value of the firm

Final goods firms purchase labor services in a competitive market at price \( x_i^t \). The first-order condition for hiring labor services then is
\[
\alpha(L_i^t) = x_i^t. \tag{14}
\]

Our notation already imposes that, in equilibrium demand for employment services needs to equal supply, with price \( x_i^t \) clearing the member states’ labor market.

The decisions made by employment-services firms are dynamic and involve discounting future profits. We assume that employment-services firms discount the future using discount factor \( Q_{i,t,t+s}^i \), where \( Q_{i,t,t+s}^i := \beta \lambda_i^{t+s} \), with \( \lambda_i^t \) being the weighted marginal utility of the firm’s owners:
\[
\lambda_i^t := \left[ \frac{e_i^t(1 - \xi_i^t)}{u'(c_{e,t}^i)} + \frac{\xi_i^t}{u'(c_{0,t}^i)} + \frac{\xi_i^t}{u'(c_{u,t}^i)} \right]^{-1}. \tag{15}
\]

Ex ante, namely, before the idiosyncratic shock \( \epsilon_j \) is realized, the value of a firm that has a worker is given by
\[
J_i^t = -\int_{\epsilon_{\xi,i}^t}^{\infty} \left[ \tau_{i,t}^\epsilon + w_{eu,t}^i \right] dF_i(\epsilon_j) + \int_{-\infty}^{\epsilon_{\xi,i}^t} \left[ x_i^t \exp\{a_i^t\} - \epsilon_j - w_i^t - \tau_{J,t}^\epsilon + E_t Q_{i,t,t+1} J_{i,t+1} \right] dF_i(\epsilon_j). \tag{16}
\]

The firm separates from the worker (first line) whenever the idiosyncratic cost shock, \( \epsilon_j \), is larger than a state-dependent threshold \( \epsilon_{\xi,i}^\epsilon \). The latter being given by the firm’s break-even condition:
\[
\epsilon_{\xi,i}^\epsilon = x_i^t \exp\{a_i^t\} - \tau_{i,t}^\epsilon - \tau_{J,t}^\epsilon + E_t Q_{i,t,t+1} J_{i,t+1}
\]

Doing so, it is mandated to pay layoff tax \( \tau_{i,t}^\epsilon \) to the government and a previously negotiated severance payment \( w_{eu,t}^i \) to the worker. The match will produce (second line) if \( \epsilon_j \) does not exceed the threshold. In that case, the firm produces \( \exp\{a_i^t\} \) units of labor services which it
sells at price \( x^i_t \), and the firm will pay wage \( w_t \) to the worker and a production tax \( \tau^i_{jt} \) to the government. A match that produces this period continues into the next.

### 2.2.4 Matching and vacancy posting

An employment services firm that does not have a worker can post a vacancy. In equilibrium, employment services firms post vacancies until the after-tax cost of posting a vacancy equals the prospective gains from hiring:

\[
\kappa_v - \tau^i_{v,t} = q^i E_t \left[ Q^i_{i,t+1} J^i_{t+1} \right],
\]

where \( q^i_t \) is the probability of filling a vacancy.

### 2.2.5 Wage setting

Wage setting is specified further below. For the simplified model, we will follow Landais et al. (forthcoming) and entertain a wage rule that nests a wide range of wage determination mechanisms in the literature. The quantitative exploration will be based on Nash bargaining.

### 2.2.6 Dividends

Aggregate profits in each member state are given by the sum of profits of final goods firms and employment services firms

\[
\Pi^i_t = \Pi^i_{F,t} + \Pi^i_{L,t}.
\]

These are distributed in equal amount as dividends to all workers in the economy. Profits by final-goods firms are given by

\[
\Pi^i_{F,t} = (1 - \alpha)L^\alpha_t c^\gamma_i.
\]

Profits of all employment-services firms aggregated are

\[
\Pi^i_{L,t} = e^i \left[ \int x^i_t \exp\{a^i_t\} - \epsilon - w^i_t - \tau^i_{jt} \right] dF_\epsilon(-) - \int e^i \left[ w^i_t + \tau^i_{\xi,t} \right] dF_\epsilon(=) - (\kappa_v - \tau^i_{v,t}) v^i_t.
\]

### 2.2.7 Federal government

We restrict our attention to equilibria in which the federal unemployment insurance scheme is implemented under full commitment by a central fiscal authority that is a Stackelberg leader. The federal government in each period has to balance the budget of the federal UI system, so that \( \int_0^1 B_t^i \, dt = 0 \).
Since we consider only country-specific shocks and countries are atomistic, the law of large numbers implies
\[ E \left[ B_{F,t} \left( u_i^t \right) \right] = \tau_F. \]

2.2.8 The member state’s problem

A central element of this paper is that we account for the optimal response of member states’ governments to the federal insurance system. Toward this end, in each member state, we consider a utilitarian Ramsey planner who gives equal weight to all workers in that member state. Since consumption in the period of separation, \( c_{i,0,t} \), does not affect the search incentives of a worker who was just laid off, the planner will provide such a worker with full insurance.

Using the assumptions laid out above, and using the properties of the logistic distribution, the member state planner’s objective can be written as

\[ \max_{\{\tau^i_{v,k}, \tau^i_{x,k}, \tau^i_{j,k}, B_i^k\}_{k=t}^{\infty}} E_t \sum_{k=t}^{\infty} \beta^k \left[ e^i_k u(c^i_{c,k}) + u^i_k u(c^i_{u,k}) + (e^i_k \xi^i_k + u^i_k)(\Psi_s(s^i_k) + \bar{h}) \right], \tag{21} \]

subject to the laws of motion of the economy, and taking the federal UI scheme as given.\(^6\)

The first term in the objective is the consumption-related utility of employed workers. The second term is the consumption-related utility of unemployed workers. The third term refers to the value of leisure and the utility costs of search.\(^7\)

2.2.9 The federal problem

The federal planner chooses the scheme \( B_F(u_t), \tau_F \) so as to maximize \textit{ex-ante} utilitarian welfare of the union’s constituents. In doing so, the federal planner anticipates the response of the member member states’ governments. The federal planner also ensures that the federal unemployment-insurance budget is balanced.

3 The main mechanisms – analytical results

The model will be used, in Section 4, to provide quantitative guidance. The current section aims to build intuition based on a one-period version of the model. This analysis extends the single-country analysis in Landais et al. (forthcoming) by the three ingredients that, in our view, are central for discussing federal unemployment insurance: a federal dimension, a range of self-interested local labor-market policies, and demand externalities.

The section has two takeaways. First, when member states can use their own labor-market policies optimally, the federal component of UI should be limited in scope: unemployment

\(^6\) We will also allow for the case in which the planner only controls a subset of the instruments.

\(^7\) Here \( \Psi_s(s_k) := -\psi_s \left[ (1 - s_k) \log(1 - s_k) + s_k \log(s_k) \right] \cdot \Psi_s(\xi_k) \), which is used further below, is defined in an analogous manner.
insurance mostly is left with the member state, and federal UI only pays in severe-enough recessions. Second, the entire mix of available labor-market policies – not only local unemployment benefits – matters for determining the optimal design of the federal UI scheme. Appendix A provides a detailed description of the one-period version of the model. Here, we focus only on those elements that change relative to the model of Section 2.1. We consider one period. At the beginning of the period, all workers are unemployed. They may be hired within the period and, then, will produce. Otherwise, they are unemployed. Time ends afterward. We abstract from an endogenous separation margin. Consequently, we also drop layoff taxes from the member state’s policy instruments. The wage is determined by a wage rule $w_i \equiv w(A^i, a^i, \theta^i, \Delta^i)$, which we assume to be twice continuously differentiable in all arguments. As in Landais et al. (forthcoming), this nests several cases discussed in the literature, such as the case of Nash bargaining, or the case of wage rigidities.

To keep the expressions tractable, when summarizing our results, we will refer to three elasticities. The definition follows exactly Landais et al. (forthcoming). Namely, the “elasticity of search effort with respect to the job finding rate” is defined as $e^s \equiv \frac{\partial s}{\partial f(\theta)} f(\theta)$.

3.1 The member state can only adjust UI benefits

We start by discussing the scope for federal UI when the member state can only adjust unemployment benefits, but not vacancy subsidies. We keep vacancy subsidies at zero throughout. Labor taxes balance the budget. First, we derive and discuss the member state’s optimal policy for a given federal UI scheme. Then, we discuss properties of the optimal federal UI scheme.

3.1.1 The member states’ optimal response

Define the replacement rate as $b^i := (B^i + \tau^i)/w^i$, that is the ratio to the wage of unemployment transfers plus the labor taxes that an unemployed worker saves.

**Proposition 1** Suppose there is a federal government that provides unemployment-based transfers $B^i$. Consider a member state that chooses the level of benefits for its unemployed constituents so as to maximize its own constituents’ welfare. Then the optimal replacement
rate is given by

\[ b^i = R^i \cdot \frac{1 - \Upsilon^i}{1 - \Upsilon^i \cdot \epsilon_{m,i}^{M,i}} - \Upsilon^i \cdot \frac{\epsilon_{M,i}^{M,i}}{\epsilon_{m,i}^{M,i}} \]

\[ + \left( 1 - \frac{\epsilon_{M,i}^{M,i}}{\epsilon_{m,i}^{M,i}} \right) \frac{1}{1 + \epsilon_{s,i}^{M,i}} \left( \frac{\Delta^i}{\psi^i w^i} + b^i (1 + \epsilon_{s,i}^{M,i}) - \frac{\gamma}{1 - \gamma q(\theta^i) w^i} \right) \frac{1}{1 - \Upsilon^i \epsilon_{m,i}^{M,i}} \]

\[ + \frac{dB^i(u^i)}{du^i} \frac{1}{w^i} \cdot \frac{\epsilon_{M,i}^{M,i}}{\epsilon_{m,i}^{M,i}} \frac{1}{1 - \Upsilon^i \epsilon_{m,i}^{M,i}} \]

\[ (22) \]

with \( R^i := \frac{e^i}{\epsilon_{m,i}^{M,i}} \frac{\Delta^i}{w^i} \left( \frac{1}{u_c(c^i_h)} - \frac{1}{u_c(c^i_u)} \right), \]

\[ \frac{1}{\psi^i} := \left( \frac{e^i}{u_c(c^i_h)} + \frac{w^i}{u_c(c^i_u)} \right) (1 - \Upsilon^i) > 0, \]

and \( \Upsilon^i := \varsigma \exp(a^i) \left[ e^i \right]^{\varsigma - 1} \left[ e^i \right]^\alpha \)

The payroll tax balances the member state’s budget.

**Proof.** See Appendix B.1. \( \blacksquare \)

If there were no demand externalities \( (\Upsilon^i = 0) \), the first two rows would perfectly resemble the expressions in Landais et al. (forthcoming). The member state would provide the higher a replacement rate the smaller the effect of benefits is on unemployment duration at the micro level.\(^8\) In addition, the member state would provide a higher replacement rate whenever the general-equilibrium elasticity of unemployment \( \epsilon_{M,i}^{M,i} \) is smaller than the microeconomic elasticity \( \epsilon_{m,i}^{M,i} \), that is, whenever jobs are hard to come by. This is the correction for the state of the business cycle that Landais et al. emphasize.

Relative to the existing literature, there are two novel elements in Proposition 1. First, the term on the third line (the term starting with \( \frac{dB^i}{du^i} \)). This has the following interpretation. All things equal, that is for a given gain from work \( \Delta^i \) and given market tightness \( \theta^i \), the optimal replacement rate \( b^i \) that member state \( i \) sets depends on the payment from the federal level for the marginal unemployed worker. The member state will set the higher a replacement rate the greater is the marginal payment from the federal UI scheme. The following proposition formalizes the impact of the federal transfer scheme on unemployment.

**Proposition 2** Consider the same assumptions as in Proposition 1. Consider a change in the slope of federal insurance payouts \( \frac{dB^i}{du^i} \) such that the aggregate resources in a given country remain unchanged. The utility gain from work that the local government chooses, decreases

\( ^8 \)The term \( R^i \) is a version of the Baily-Chetty formula under which, absent macroeconomic effects of benefits, benefits are provided up to the point where the marginal utility gain for the unemployed equals the marginal utility loss of the employed, bearing in mind that higher benefits will increase unemployment duration.
with the slope of the federal payouts. Formally,

\[
\frac{d\Delta^i}{d\bar{B}^i} \sim -\epsilon^{M,i} \frac{\phi^i}{\Delta^i u^i} \leq 0
\]

**Proof.** See Appendix B.2. ■

Proposition 2 indicates that the utility gain from employment (and, thereby search intensity) shrinks with the generosity of the federal insurance provided. The intuition is the following. The local government would like to equalize the consumption of unemployed and employed workers. The unobservable search effort, however, forces the local government to keep a positive utility gain from employment. When a member state receives funds per unemployed worker, this reduces the fiscal costs of unemployment. The member state will, therefore, reduce the utility cost of unemployment.

What is noteworthy is that the response of the member state to federal UI will be the more significant the greater is the macroeconomic elasticity of unemployment with respect to UI, \(\epsilon^{M,i}\). To understand this relationship, consider the extreme case of \(\epsilon^{M,i} = 0\). Then the local government cannot influence the level of unemployment and the provision of resources by the federal level does not distort the member state’s incentives. The more control the member state has over unemployment, that is, the greater is \(\epsilon^{M,i}\) the more resources the local government is able to extract from the federal level by shrinking the gain from work by a fixed amount.

The second novel element in Proposition 1 is the effect that the demand externality has on the replacement rate and how this interacts with both the federal UI scheme and the business cycle. Suppose first, that there are no federal UI benefits. Whenever macro and micro-elasticity of unemployment coincide, the member state will unambiguously choose a lower replacement rate in the presence of demand externalities (first row). The reason is that a lower replacement rate stimulates search and thereby employment and production, which in turn reduces the demand externality. What is important is that the lower the macro elasticity is relative to the micro elasticity, the smaller the incentives to reduce the replacement rate. The reason is that a larger cut in the replacement rate (and thus a reduction in insurance for the individual worker) is needed to stimulate employment and demand.

Next, suppose that there are federal UI benefits. The demand externality will unambiguously raise the member state’s replacement rate further. Increasing transfers increase demand and, thereby, economic activity. A lower macroelasticity dampens this channel.9

### 3.1.2 The optimal federal UI scheme

Next, we turn to a description of the optimal federal UI scheme. In order to have a tractable representation, we make two further simplifying assumptions. First, we assume that there

---

9At the same time, observe that the correction term \(\Upsilon^i\) itself is cyclical.
are only two shock states, a state that in autarky would be associated with a “boom” (the H-state, high output) and a state associated with a “recession” (the L-state, low output), reflected in commensurate values of \(A^i\) and \(a^i\). With probability \(\pi_H\) a country will be in a boom, with probability \(\pi_L\) in a recession. Second, we restrict ourselves to linear federal UI schemes, in which \(B(u^i) = B \cdot u^i - \tau_F\), \(B > 0\) being a constant parameter.

Anticipating the response of member states (detailed in Proposition 1), the federal government chooses the generosity of the federal UI system (slope \(B\)) and federal taxes \(\tau_F\) so as to maximize the \textit{ex-ante welfare} of the union while balancing the federal budget:

\[
\max_{B, \tau_F} \pi_H W(A^H, a^H, \theta^H, \Delta^H; B) + \pi_L W(A^L, a^L, \theta^L, \Delta^L; B),
\]

where \(W\) is the value function of the local Ramsey planner, as defined in equation (27).

**Proposition 3** Consider the same assumptions as in Proposition 1. In addition, apply the assumptions spelled out in Section 3.1.2 (linear UI scheme and two states). Let \(\frac{1}{\phi_H^u} = \left(\frac{e^H}{u^H(c^H)} + \frac{1-e^H}{u^H(c^H)}\right)(1 - \Upsilon^H)\) and \(\frac{1}{\phi_L^u}\) be defined analogously. Then the optimal federal insurance scheme satisfies

\[
B = \pi_H \cdot \pi_L \cdot \left[e^H - e^L\right] \cdot \left[\frac{\phi^L - \phi^H}{\pi_H \phi^H + \pi_L \phi^L}\right] \cdot \left[\frac{\pi_H}{d^u^H dB} + \frac{\pi_L}{d^u^L dB}\right]^{-1}
\]

**Proof.** See Appendix B.3. ■

The following elements shape the optimal level of the federal UI benefits per unemployed worker, \(B\): the bigger the employment difference \(e^H - e^L\) between boom and bust countries, the bigger the transfers. The next term, \(\frac{\phi^L - \phi^H}{\pi_H \phi^H + (1 - \pi) \phi^L}\), captures the difference between marginal social values of resources in boom and bust country. Absent incentive considerations, a federal planner would transfer resources from boom to bust countries up to the point where the marginal value of resources is equalized within the federal union. The desire to equalize marginal utilities of consumption is traded off, however, against the need to mitigate the free-riding incentives of member states.

Member states’ moral hazard is captured by the last term. By increasing \(B\), the federal UI scheme induces local governments to raise the replacement rate, and – thus – it shrinks the utility gains from employment, compare (22). That is, the terms \(\left[\frac{d_u^H}{dB}\right]\) are positive (or, at least, non-negative). The more responsive unemployment rates are to federal benefits, that is, the larger the moral hazard, the lower are the federal benefits provided.

It is instructive to dissect the moral-hazard term further:

\[
\left[\pi_H \frac{du^H}{dB} + \pi_L \frac{du^L}{dB}\right] = \pi_H \left[-\frac{d\Delta^H}{dB}\right] u^H \epsilon^{M,H} + \pi_L \left[-\frac{d\Delta^L}{dB}\right] u^L \epsilon^{M,L}
\]

The larger this term is, the less generous is the federal insurance scheme. The magnitude of
the term, again, depends on the impact of federal UI benefits on equilibrium unemployment, $\epsilon^{M,i}$. If the local government can hardly affect the unemployment rate, that is, $\epsilon^{M,i}$ is small, benefits can be generous.

Ideally, the federal scheme would make benefit payments to those member states only that should be net recipients of transfers. That is, ideally, unemployment insurance is provided – first and foremost – by the member states. On top of this, the federal government should implement UI based transfers to member states in recessions. Transfers should be the highest, the more recessions come with policy-insensitive unemployment rates to start with. This is formalized in the following proposition.

**Proposition 4** Consider the same assumptions as in Proposition 3. Assume further that the payout function is piecewise linear, such that

$$B(u^i) = \begin{cases} u^i \cdot B^H - \tau^F & \text{if } u^i < \bar{u}^L \\ u^i \cdot B^L - \tau^F & \text{if } u^i \geq \bar{u}^L \end{cases},$$

where $B^H, B^L \geq 0$. Let $\bar{u}^L$ denote the level of unemployment prevailing in the country with the low shock, absent the international insurance scheme. Then, the optimal federal UI scheme satisfies

$$B^H = 0$$

and

$$B^L = \min \left\{ \frac{u^L \cdot \pi_H}{\pi_H \phi^H + \pi_L \phi^L} \left[ -\frac{d\Delta L}{dB^L} \cdot \frac{u^L}{\Delta L} \cdot \epsilon^{M,L} \right]^{-1}, B^L \right\} > 0,$$

where $B^L$ is the upper bound on admissible generosity of the federal transfers so as to ensure that the boom country does not choose an unemployment level that qualifies for transfers $W^H(\Delta^H, \theta^H; 0) > W^H(\Delta^L, \theta^L; B^L)$, where welfare is as in equation (27).

**Proof.** See appendix B.4

### 3.2 The member state can adjust the labor-market policy mix

A central point above was the following. Federal UI benefits are an imperfect means of insuring member states against member state-specific shocks. Rather, the provision of federal UI will distort local governments’ labor-market policies. Distortions are smallest if the macro elasticity of unemployment with respect to benefits is small. Federal UI would, thus, be provided when labor markets are characterized by a low macroelasticity.

This reasoning took the elasticity as largely outside the control of the local government. The contribution of this section, instead, is to highlight that once member states can adjust the labor-market policy mix more generally, the macroelasticity of unemployment may never need
to be small to start with. This, reduces the scope for federal UI as a mechanism to insure member states against member state-specific shocks.

Suppose now that the member state controls both the generosity of its own UI benefit system and the vacancy (hiring) subsidies. For a given federal UI scheme, Proposition 5 characterizes the member states’ optimal setting of the mix of its own labor-market instruments.

**Proposition 5** Suppose there is a federal government that provides unemployment-based transfers \( B(u^i) \). Consider a member state that chooses the level of benefits and the level of vacancy subsidies so as to maximize welfare of its own constituents. Then the optimal policy mix set by the member state can be characterized as follows:

The optimal replacement rate is characterized by

\[
\begin{align*}
    b^i &= R^i - \frac{\Upsilon^i}{1 - \Upsilon^i} - \frac{\tau^i}{w^i q^i (1 - \Upsilon^i)} + \frac{dR^i(u^i)}{du^i} \frac{1}{w^i} \frac{1}{1 - \Upsilon^i}.
\end{align*}
\]

(23)

The optimal hiring subsidy satisfies

\[
\begin{align*}
    \frac{\tau^i_v}{q^i} &= \left( \frac{y^i}{e^i} - (c^i_e - c^i_u)(1 - \Upsilon^i) - \frac{dR^i(u^i)}{du^i} \right) \frac{1 + \phi^i R^i w^i}{\Delta} \left( 1 - \Upsilon^i \right) \left( \epsilon^{s,i} + 1 \right) - \frac{y^i}{e^i} + w
\end{align*}
\]

(24)

\( R^i, \Upsilon^i \) and \( \phi^i \) take the same form as in Proposition 1.

**Proof.** See appendix B.5

When comparing Proposition 5 here with Proposition 1 earlier, what is most striking is what is not in Proposition 5. Namely, there no longer appear any terms related to the gap between micro and macro elasticities of unemployment. The reason is as simple as it is important: if the member-state’s government provides for the optimal labor-market policy mix, it will make sure that jobs will never be hard to come by. Similarly, the formula for the hiring subsidies does not contain the macro elasticity, see (24). The incentives to free-ride on the federal government’s UI scheme will, therefore, never be smaller than usual (compare the last term for benefits here and in the other proposition). This also means that the demand externality will not affect the member states’ incentives differently in booms and recessions. These findings extend our earlier results in Jung and Kuester (2015) to the case with demand externalities and a federal UI scheme.

### 3.2.1 The optimal federal UI scheme

If the member states can alter their labor-market policy mix in its entirety, the optimal (linear) policy of the federal government changes markedly.

**Proposition 6** Consider the same assumptions as in Proposition 5. In addition, apply the assumptions spelled out in Section 3.1.2 (linear UI scheme and two states). Then the optimal
federal insurance system satisfies:

\[
\mathcal{B} = \pi_H \cdot \pi_L \cdot \left[ e^H - e^L \right] \left[ \frac{\phi^L - \phi^H}{\pi_H \phi^H + \pi_L \phi^L} \right].
\]

\[
\cdot \left[ \pi_H \left[ - \frac{d \Delta^H u^H}{d \beta} e^{m,H} \right] - \frac{d \theta^H e^H}{d \theta^H} (1 + \epsilon_{s,H}) \gamma \right] + \pi_L \left[ - \frac{d \Delta^L u^L}{d \beta} e^{m,L} - \frac{d \theta^L e^L}{d \theta^L} (1 + \epsilon_{s,L}) \gamma \right]^{-1}
\]

only micro-economic elasticities

Proof. See appendix B.6

The terms on the first row of the formula for \( \mathcal{B} \) are analogous to those present in Proposition 3. The availability of the hiring subsidy alters the term on the second row, namely, the term related to member states’ moral hazard. Compare this to the last term in Proposition 3. Again, what is absent from the formula is the macro-elasticity of unemployment. When the member-state government optimally uses the labor-market policy mix, the extent to which it can affect unemployment is given by the micro elasticities, always. The member state can control market tightness directly, rather than only indirectly through the transmission of unemployment benefits on on wages. What this means for federal UI is more severe moral-hazard distortions and, hence, a less generous federal insurance scheme than if benefits are the only instrument that the local government can change.

3.3 Summary

We have modeled a union of member states that are subject to uninsurable member state-specific shocks. A first-best federal policy would transfer resources from member states that have experienced favorable shocks to those suffering from adverse shocks. In practice, conditioning transfer schemes on exogenous shocks is difficult, however. Federal UI provides one way to design such a state-contingent transfer. It does so in an imperfect way, however. By construction, it conditions transfers on an endogenous variable, unemployment, that member states can affect through the policies that they implement. Optimal federal UI, therefore, has to identify circumstances under which the shock occurred and the member states cannot or do not wish to influence the variable that drives the magnitude of transfers. Optimal federal UI provides resources only in these circumstances.

The current section holds three main lessons. First, subsidiarity: the optimal federal UI scheme leaves unemployment insurance first and foremost to the member states. Federal UI is provided to recession states only. Second, federal UI can be the more generous, the harder it is for member states to affect local employment. This depends on the entire labor-market policy mix that member states can implement. Third, the presence of demand externalities strengthens the welfare gains from a first-best transfer mechanism. Demand externalities need not imply that the optimal federal UI scheme is more generous. The reason being that federal UI is not a first-best transfer mechanism.
4 A quantitative assessment for EMU

While the theory presented above is general, the application will be specific. The remainder of the paper will be concerned with the design of an optimal federal UI system for the European Monetary Union. Toward this end, we calibrate the dynamic model of Section 2. Having a dynamic setting is important in three respects. First, because it allows us to discuss separately two dimensions of the member states’ response to a federal UI scheme: how the scheme alters the average level of member states’ policy instruments (think of a permanent change in policies) and how federal UI changes the member states’ policy response to cyclical fluctuations. Second, because it allows us to discuss schemes that explicitly index benefits to past unemployment rates in a member state, which may dampen the incentives to free ride. Third, because the calibration to the business cycle facts allows us to use the model to identify the size and movement over the cycle of the macro-elasticities that shape the optimal provision of federal UI.

4.1 Calibration

Our strategy is to calibrate the model to resemble a union of generic euro area member states. The baseline is resembles the current status quo in which there is no federal UI. Member states set their labor-market policy mix, that is the replacement rate, layoff tax and vacancy-posting subsidy so as to maximize welfare in their constituency. The instruments are fixed at the level that decentralizes the constrained-efficient steady state absent federal UI. We calibrate parameters to match euro area averages and “typical” cyclical fluctuations of the labor market. We do not currently spell out the degree to which shocks originate domestically or in the rest of the euro area. We assign all fluctuations to country-specific shocks. The parameters that emerge from the calibration, we treat as structural parameters in the policy experiments that we conduct later.

Data. One period in the model is a month. We calibrate the model to the period 1991M1 to 2015M12.\textsuperscript{10} The sample period above includes the deep recession that ensued after the financial and debt crises. Where applicable, the data series are seasonally adjusted. All data are reported at quarterly frequency. Where necessary, we take quarterly averages of monthly data. The business cycle properties of the data are reported in Table 1. The table reports log deviations from an HP trend with a smoothing parameter of 1600. The main data source is the ECB’s area-wide-model data set (AWM). Output $y$ in the model is taken to be real gross domestic product. Labor productivity, $\frac{y}{e(1-\xi)}$, is measured as output per employed worker. Employment and the unemployment rate are the respective equivalents of the database. Our measure of the wage, $w$, is the ratio of the total compensation of employees deflated with the

\textsuperscript{10}The initial date is dictated by the availability of internationally comparable OECD Harmonized Unemployment data that we use to construct time series of the labor market tightness in the euro area. 2015Q4 constitutes the last period available in the current release of area-wide-model database.
GDP deflator to the number of employed workers. The data set does not report time series for vacancies and labor-market flow rates. For a euro-area series of vacancies, we resort to the OECD (stocks of unfilled vacancies from the “Short–Term Labour Situation Database”). The statistics in Table 1 are derived after aggregating the vacancies for all those member states for which there are observations. To the best of our knowledge, there does not exist a data set that provides high-frequency job-finding and separation rates for a sizable number euro area member states. For the sake of comparison, the labor market flow rates presented in the table refer to the German economy alone. The data are from Hartung et al. (2016). For consistency, the corresponding entries in the correlation matrix report the correlation with the corresponding German series from Eurostat.

Table 1: Business cycle properties of the data

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>Standard deviation</td>
<td>1.98</td>
<td>0.91</td>
<td>11.04</td>
<td>23.85</td>
<td>0.97</td>
<td>24.63</td>
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<tr>
<td>Autocorrelation</td>
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<td>0.97</td>
<td>0.95</td>
<td>0.91</td>
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<td>0.18</td>
<td>0.03</td>
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<td>-0.16</td>
<td>0.22</td>
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<td>1.00</td>
<td>-0.16</td>
<td>-0.06</td>
<td>-0.27</td>
</tr>
</tbody>
</table>

Notes:
The table reports summary statistics of the data. The sample is 1991Q1 to 2015Q4. $Lprod$ is labor productivity per worker. $urate$ is the unemployment rate. All data are quarterly aggregates, in logs, HP(1600) filtered and multiplied by 100 and, hence, can be interpreted as the percent deviation from the steady state. The first row reports the standard deviation. The next row reports the autocorrelation. The following rows report the contemporaneous correlation matrix.

Parameters. Table 2 summarizes the calibrated parameters. The monthly discount factor $\beta$ equals .996. We set the value of leisure to $\Psi_s(s) + h = 0.369$, in order to match an average unemployment rate of 9.5 percent. We set $\psi_s = .118$ with a view to matching the micro-elasticity of unemployment with respect to benefits. The value chosen here implies an elasticity of the average duration of unemployment with respect to UI benefits of 0.8, in line with micro estimates such as Meyer (1990). The coefficient of relative risk aversion is set to $\sigma = 1$, implying log utility. We set a vacancy posting cost of $\kappa_v = 0.95$ so as to obtain an average value of the monthly job finding rate of 0.7 percent, the euro area average in Elsby et al. (2013a). This results in an average cost per hire net of hiring subsidy, $\frac{\nu(\kappa_v - \tau_v)}{m}$ of one monthly wage, in line with a broader notion of recruiting costs, (Silva and Toledo, 2009). We
Table 2: Parameters for baseline

<table>
<thead>
<tr>
<th>Preferences</th>
<th></th>
</tr>
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<tbody>
<tr>
<td>( \beta )</td>
<td>time–discount factor. 0.996</td>
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<tr>
<td>( \Psi_s(s) + \bar{h} )</td>
<td>value of leisure. 0.369</td>
</tr>
<tr>
<td>( \psi_s )</td>
<td>scaling parameter dispersion utility cost of search. 0.118</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>relative risk aversion. 1</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Vacancies, matching and bargaining</th>
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</thead>
<tbody>
<tr>
<td>( \kappa_v )</td>
<td>vacancy posting cost. 1.14</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>match elasticity with respect to vacancies. 0.3</td>
</tr>
<tr>
<td>( \chi )</td>
<td>scaling parameter for match-efficiency. 0.103</td>
</tr>
<tr>
<td>( \eta )</td>
<td>steady–state bargaining power of firm. 0.3</td>
</tr>
<tr>
<td>( \gamma_w )</td>
<td>degree of cyclicality of bargaining power of worker. 19.3</td>
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<table>
<thead>
<tr>
<th>Production and layoffs</th>
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<tbody>
<tr>
<td>( \mu_\epsilon )</td>
<td>mean idiosyncratic cost. 0.4</td>
</tr>
<tr>
<td>( \psi_\epsilon )</td>
<td>scaling parameter dispersion idiosyncratic cost shock. 2.22</td>
</tr>
<tr>
<td>( \rho_a )</td>
<td>autocorrelation of the aggregate productivity. 0.98</td>
</tr>
<tr>
<td>( \sigma_a \cdot 100 )</td>
<td>std. dev. of innovation to aggregate productivity. 0.182</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>returns to scale 1</td>
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<tr>
<td>( \varsigma )</td>
<td>demand externality 0</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Labor market policy</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( b )</td>
<td>replacement rate. 0.526</td>
</tr>
<tr>
<td>( \tau_v )</td>
<td>vacancy posting subsidy. 0.67</td>
</tr>
<tr>
<td>( \tau_\xi )</td>
<td>layoff tax. 7.23</td>
</tr>
</tbody>
</table>

Notes: The table reports the calibrated parameter values in the baseline economy.

set the elasticity of the matching function with respect to vacancies to \( \gamma = .3 \), within the range of estimates deemed reasonable by Pissarides and Petrongolo (2001). The wage-setting protocol follows Jung and Kuester (2015). Namely, wages are determined through Nash bargaining. The bargaining power of firms, denoted by \( \eta_t \), is procyclical. We set the firm’s bargaining power in steady state to \( \eta = \gamma = .3 \) so that, absent risk aversion, in the steady state the Hosios (1990) condition would be satisfied without any government intervention. We view this as a natural – and customary – choice. In order to replicate the cyclical volatility of the labor market, we employ a mechanism that attenuates wage fluctuations and, thus, increases variability of the labor market. Specifically, the bargaining power evolves as \( \eta_t = \eta \cdot \exp\{\gamma_w \cdot a_{t-1}\}, \gamma_w \geq 0 \). Note that related assumptions are common in the literature.\(^{11}\)

We choose \( \gamma_w = 19.33 \) to generates an amount of volatility in the unemployment rate, \( u_t \), that is comparable to the data summarized in Table 1. As a result, for a 1 percent negative productivity shock the bargaining power of firms falls by 19 percent, from a steady state value of .3 to .24.

\(^{11}\)Landais et al. (2010) directly specify that \( w_t = \pi \exp\{\rho a_t\} \), with \( \rho = 0.5 \), as an exogenous wage rule.
The matching-efficiency parameter is set to $\chi = .103$ so as to match a quarterly job-filling rate of 71 percent, as in den Haan et al. (2000).

We calibrate the average idiosyncratic cost of retaining a match to $\mu = .4$. We interpret this as a reduced form of capturing the cost of capital. As regards job-finding and separation rates, Elsby et al. (2013b) provide annual estimates for monthly job-finding and separation rates for selected OECD countries. Among their sample are the euro area countries Austria, Finland, France, Germany, Ireland, Italy, Portugal, and Spain. Our calibration strategy targets the relative volatility of job-finding and job-separation rates documented by Elsby et al. (2013b).

Toward this end, we calibrate the dispersion parameter for the idiosyncratic cost shock to $\psi = 2.22$.

We set the serial correlation of the productivity shock to $\rho = 0.96$ and the standard deviation of the shock to $\sigma = 0.0018$. With these values, the model replicates the volatility and persistence of measured labor productivity in the data.

The layoff tax, the vacancy–posting subsidy, and the replacement rate are fixed at those values that decentralize the constraint-efficient steady-state allocation in autarky. The optimal vacancy subsidy amounts to 1 monthly wage. The optimal layoff tax equals approximately 11 monthly wages, in line with the extended average duration of unemployment spells in the euro area. The replacement rate is 52 percent, a reasonable value for the euro area, compare Christoffel et al. (2009).

Table 3 reports business cycle statistics in the baseline model based on a first-order approximation of the model. The calibrated model does a reasonably job of replicating the fluctuations in the data. Unemployment and vacancies are considerably more volatile than productivity and so are the job-finding and separation rates. Vacancies and unemployment are negatively correlated, the Beveridge-curve relationship. For completeness, we also report moments for the job-finding rate, which is procyclical, and the separation rate, which is countercyclical.

From the perspective of the quantitative exercise, an important statistic is the responsiveness of the labor market to the member states’ policy instruments. The more responsive the labor market is to the benefits and taxes, the smaller is the scope for the federal UI scheme. The following table 4 presents the steady-state elasticities of equilibrium employment with respect to the policy instruments. We calculate the macroeconomic elasticity of unemployment with respect UI, $\epsilon^M$ as defined in Section 3, Definition 2. In our baseline calibration, the macroelasticity amounts to $\epsilon^M = .93$.\(^\text{12}\)

**Aggregate demand externality.** For the baseline model, we assume constant returns to scale in aggregate production $\alpha = 1$ and we also abstract from demand externalities, setting $\varsigma = 0$. In an extension, we also consider a different baseline calibration, with demand externalities. Following Krueger et al. (2016), we set $\varsigma = .3$. We then calibrate the model in the same way as we do in the baseline case, replicate the same moments, and target the same

\(^{12}\text{Landais et al. (forthcoming) report macroelasticities in the a range of 0.08 to 0.32.}\)
Table 3: Business cycle properties of the model

<table>
<thead>
<tr>
<th></th>
<th>y</th>
<th>Lprod</th>
<th>urate</th>
<th>v</th>
<th>w</th>
<th>θ</th>
<th>f</th>
<th>ξ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard deviation</td>
<td>2.09</td>
<td>0.91</td>
<td>11.04</td>
<td>23.31</td>
<td>0.66</td>
<td>32.14</td>
<td>9.64</td>
<td>3.96</td>
</tr>
<tr>
<td>Autocorrelation</td>
<td>0.99</td>
<td>0.96</td>
<td>1.00</td>
<td>0.93</td>
<td>0.99</td>
<td>0.96</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y</td>
<td>1.00</td>
<td>0.95</td>
<td>-0.97</td>
<td>0.87</td>
<td>0.92</td>
<td>0.96</td>
<td>0.96</td>
<td>-0.99</td>
</tr>
<tr>
<td>Lprod</td>
<td>-</td>
<td>1.00</td>
<td>-0.86</td>
<td>0.98</td>
<td>0.75</td>
<td>1.00</td>
<td>1.00</td>
<td>-0.99</td>
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<tr>
<td>urate</td>
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<td>-</td>
<td>1.00</td>
<td>-0.73</td>
<td>-0.98</td>
<td>-0.87</td>
<td>-0.87</td>
<td>0.92</td>
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<tr>
<td>Correlation</td>
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<td></td>
<td></td>
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<td></td>
</tr>
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<td>v</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.00</td>
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<td>0.97</td>
<td>0.97</td>
<td>-0.94</td>
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<td>f</td>
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<td>-</td>
<td>-</td>
<td></td>
<td>0.77</td>
<td>1.00</td>
<td>1.00</td>
<td>-0.99</td>
</tr>
<tr>
<td>ξ</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-0.84</td>
<td>-0.99</td>
<td>-</td>
<td>1.00</td>
</tr>
<tr>
<td>w</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<td>1.00</td>
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<tr>
<td>θ</td>
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<td>-</td>
<td>-</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Notes: The table reports second moments in the model. \( Lprod \) is labor productivity per worker. \( urate \) is the unemployment rate. All data are quarterly aggregates, in logs and multiplied by 100 in order to express them in percent deviation from the steady state. We report unconditional standard deviations from the model. The first row reports the standard deviation. The next row reports the autocorrelation. The following rows report the contemporaneous correlation matrix. Table 1 reports the corresponding business cycle statistics in the data.

Table 4: Steady-state elasticities of equilibrium unemployment with respect to policies

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Baseline</th>
<th>Demand Externalities</th>
</tr>
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<tbody>
<tr>
<td>( b )</td>
<td>6.19</td>
<td>6.55</td>
</tr>
<tr>
<td>( \tau_v )</td>
<td>0.35</td>
<td>0.33</td>
</tr>
<tr>
<td>( \tau_\xi )</td>
<td>-3.29</td>
<td>-3.45</td>
</tr>
</tbody>
</table>

Notes: The elasticity is defined as \( \frac{d \log \bar{u}}{d \log x} \), where \( x \in b, \tau_v, \tau_\xi \) and \( \bar{u} \) denotes the steady state unemployment.

steady state values. We choose the value of the replacement rate so that the steady state employment chosen by the Ramsey planner in the demand externality economy is equal to the employment prevailing in the steady state in the baseline environment.

4.2 The shape of federal UI

We resort to simple, implementable federal UI schemes. We parameterize the form of the federal UI scheme, and choose the parameters to maximize ex-ante welfare of the member states. Payouts from the federal UI scheme are parameterized as follows:

\[
B_{F,t}(u; \nu, \omega, B) = \frac{\exp (\nu \cdot (u - u_{aut} - \omega))}{1 + \exp (\nu \cdot (u - u_{aut} - \omega))} \cdot B \cdot u, \tag{25}
\]
where $\nu > 0$, $B \geq 0$ and $\omega \in \mathbb{R}$. $u_{\text{aut}}$ is the autarkic steady state unemployment rate. This functional form allows federal UI benefits to have the properties identified as optimal in section 3. Namely, first, subsidiarity: the federal UI system does not pay UI benefits in the vicinity of the steady state. Second, contingency: federal UI may be paid only if member states suffer from large-enough shocks. The functional form above can be parameterized to be flat on some of the domain and increasing at higher values of unemployment. The numerical results reported below are based on a second-order perturbation of the model.

4.3 European unemployment insurance - scenarios

The following figures plots deviations of GDP from the steady state on the y-axis and the net transfers provided by the optimized federal UI scheme on the x-axis. In order to set the stage and highlight the full scope for a federal UI system in the baseline model, we plot what an optimal federal UI system would look like if member states would not be able to react to the introduction of the federal scheme whatsoever. The optimal parameterization of the federal UI scheme reads $\omega = -u_{\text{aut}}$, $B = 1.5$. Note that $\omega^* = -u_{\text{aut}}$ implies a linear contract, without any threshold. To interpret the value of $B = 1.5$, note that the ratio of the standard deviation of employment to the standard deviation of GDP is approximately 1.5. This means that, on average, a unit drop in employment is associated with a 1.5 percent drop in GDP. The federal UI scheme implements transfers that almost fully compensate for the loss in consumption. Subsequently, and for comparability, we fix

Figure 1: Optimal federal UI - net federal benefits absent moral hazard

Notes: The payouts from the European UI under the contract that is optimal given that the local government cannot adjust local labor market policy in response. Federal transfers are expressed in terms percent of the GDP prevailing absent the insurance scheme.
parameter $B = 1.5$ as was found optimal for the current case of entirely constant instruments. We optimize then optimize over $\nu$ and $\omega$. The costs of business cycles absent the federal scheme amount to 0.09 percent of steady-state consumption. The federal UI scheme reduces the costs of fluctuation by a third. Even the optimal federal UI scheme, for constant domestic instruments, cannot eliminate all the welfare costs of business cycles, but not all. The reason is that constant domestic labor-market instruments mean that the model federal union as a whole uses its resources inefficiently: there is too much employment in boom countries, and too little in recession countries.

4.3.1 Accounting for a response by member states

A distinguishing feature of the European setup to date is that member states retain full sovereignty over their own labor-market policies. An important question, thus, is to what extent a welfare-improving federal UI scheme can be implemented if that sovereignty is left untouched. The current section, therefore, asks an optimal federal UI scheme would look like if member states were able to adjust their labor-market policies once, keeping policies constant ever after (and in particular, not responding to the business cycle).

Table 5 summarizes how the member states’ change in policies affects the steady state if a linear federal UI scheme similar to the one in Figure 3 were implemented. Table 5

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>autarky $B = 0$</th>
<th>Federal insurance $B = .1$</th>
<th>$B = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>output.</td>
<td>0.895</td>
<td>0.809</td>
<td>0.310</td>
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<tr>
<td>$u$</td>
<td>unemployment rate.</td>
<td>0.095</td>
<td>0.171</td>
<td>0.625</td>
</tr>
<tr>
<td>$v$</td>
<td>vacancies.</td>
<td>0.017</td>
<td>0.025</td>
<td>0.034</td>
</tr>
<tr>
<td>$f$</td>
<td>job–finding rate.</td>
<td>0.062</td>
<td>0.059</td>
<td>0.046</td>
</tr>
<tr>
<td>$s$</td>
<td>fraction of job seekers.</td>
<td>0.899</td>
<td>0.873</td>
<td>0.748</td>
</tr>
<tr>
<td>$\xi$</td>
<td>layoff rate.</td>
<td>0.006</td>
<td>0.012</td>
<td>0.070</td>
</tr>
<tr>
<td>$b$</td>
<td>replacement rate</td>
<td>0.526</td>
<td>0.532</td>
<td>0.512</td>
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<tr>
<td>$\tau_\epsilon$</td>
<td>separation tax.</td>
<td>7.228</td>
<td>6.094</td>
<td>3.702</td>
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<td>$\tau_v$</td>
<td>vacancy posting subsidy.</td>
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<td>0.645</td>
<td>0.632</td>
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<tr>
<td>$\tau_J$</td>
<td>lump–sum production tax.</td>
<td>0.003</td>
<td>0.023</td>
<td>0.343</td>
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<tr>
<td>$\tau_F$</td>
<td>lump–sum contribution to the EUI.</td>
<td>0.000</td>
<td>0.018</td>
<td>0.333</td>
</tr>
</tbody>
</table>

Notes: The table reports the steady-state values of selected endogenous variables when member states conduct Ramsey-optimal policy in response to a linear federal UI scheme. The first column refers to the steady state absent a federal UI scheme. Column two and three refer to the federal UI scheme of Figure 3 but with lower levels of generosity, indexed by $B = 0.1$ and $B = 1$ (the scheme underlying Figure 3 featured $B = 1.5$).

shows that once countries are allowed to react to a linear scheme, the steady state changes markedly. We report results for less generous schemes than discussed earlier, with slope
$B = .1$ or $B = 1$ instead of 1.5. Even for these less generous schemes the results are large. Employment falls by 10 percent (for $B = 0.1$) or 65 (!) percent (for $B = 1$). The main culprit of this is not the replacement rate. Indeed, virtually identical results would emerge if the member states were not allowed (or able) to adjust benefits after the introduction of federal UI. Rather, what makes the difference is that member states can change the other labor-market policies. In particular, member states no longer bear the full marginal fiscal cost of unemployment. As a result, they are inclined to reduce layoff taxes, in particular, which in turn raises unemployment. The burden that the countries impose on each other leads to a significant drop of economic activity. In other words, in order to make generous cross-country insurance implementable through federal UI benefits, the above suggests that it is not sufficient – nor even necessary – to restrict only the member state’s benefit policies. Rather, member states would need to be unable to adjust a wide range of other labor-market policies.

Member states’ ability to respond to the introduction of federal UI, therefore, notably changes the shape of the optimal federal UI scheme. This is shown in Figure 2. The optimal federal UI scheme (in the class that we consider) resembles a scheme with a threshold. Close to the steady state, there are no payouts. Only once a member state goes into a severe-enough recession, the federal level does provide UI based transfers. The figure shows the transfers implied by the optimal federal UI scheme for three different cases. The green dashed line marks the case in which the member state can only adjust unemployment benefits in response to the scheme (with production taxes balancing the budget), but not the other labor-market instruments. The cutoff is at a 2 percent drop in output, corresponding to a one standard deviation TFP shock. The purple dashed-dotted line gives the implied transfers if the member state can only adjust the vacancy subsidies. The threshold falls to a recession commensurate with a one percent drop in GDP. The red dotted line, instead, shows what happens to the payouts under the optimal federal insurance scheme if member states can adjust layoff taxes in response to the federal scheme only. Then, the cutoff moves toward a 1.5 standard deviation fall in output (a drop in output of 3 percent). These figures underscore two implications: first, even with moral hazard at the member state level, there remains scope for federal insurance. Second, the more control over labor-market policies member states retain, the smaller the scope for implementing federal transfers by means of a federal UI scheme. The welfare gains from the introduction of the optimal federal UI schemes are reduced notably as well. When only the hiring subsidy reacts, the welfare costs of business cycles fall by a tenth. With a response of benefits, the gain is halved again. Once layoff taxes are reduced, the welfare gains become negligible. In sum, the welfare gains that could be reaped from a federal UI system heavily depend on the policy options left to member states.
Figure 2: Optimal federal UI - gross payouts allowing for a response of long-run policies

Notes: The payouts from the European UI under the contract that is optimal given that the local government adjusts local labor market policy in response but only in the long term. Federal transfers are expressed in terms percent of the GDP prevailing absent the insurance scheme. Each line represents a setup with different restriction on which local instruments can be adjusted in response to the federal insurance.
4.3.2 Optimal federal UI and business-cycle stabilization policy

The previous section analyzed the scope for a federal UI scheme if member states adjusted long-run policies optimally in response. However, one may wonder if federal UI only affects long-run labor-market policies, or if – equally well – federal provision of UI may affect the member states’ incentives to engage in cyclical stabilization policy. The current section, therefore, asks how the optimal federal UI scheme would look like if member states could optimize their labor-market policy over the business cycle. Of course, one may have the view that member states simply cannot engage in cyclical stabilization policy. Then, the results of Section 4.3.1 highlight the scope for federal UI. Alternatively, one might consider that member states might have some scope for stabilization policy, in line with the fact that various member states of the euro area did implement labor-market stabilization policies in the recent recession (such as the German short-term work program). Here we wish to highlight how a federal UI scheme should be designed that accounts for such efforts.

Figure 3 shows the same three cases as before (in which only benefits, only the hiring subsidy, or only the layoff tax react), and a case in which all instruments react optimally. The difference to Figure 2 is that member states now can adjust their cyclical responses as much as their long-run policies. What is noteworthy is that the threshold of the federal UI system moves markedly to the right. Federal UI now only pays in truly large recessions, when output falls by at least 4.5 percent (when only hiring subsidy adjust), or by at least 9 percent (when all instruments are allowed to adjust). More flexibility of member-state governments severely reduces the scope for federal UI to implement a welfare-improving transfer scheme. The reason is that federal insurance will affect member states’ incentives to smooth their own business cycles, recall Proposition 6.

The distortions induced by a federal UI scheme can be decomposed into two channels: (i) the incentives to increase the average unemployment rate above the autarkic level and (ii) the reduced incentives to provide optimal stabilization policy. In order to focus only on the second channel, we next study a federal insurance scheme in which transfers depend on the deviations of current unemployment from the country-specific long-term average unemployment rate, rather than unemployment directly.

Formally, let $u_{i,avg}^t = \delta + (1-\delta)u_{i,avg}^{t-1}$ denote the average unemployment rate. We set $\delta = 1/120$ so as to mimic the average unemployment rate over last 10 years, or 120 months. Consider the following payout rule

$$\mathbb{B}(u^i; \nu, \omega, B) \equiv B_{F.i}(u^i_t - u_{i,avg}^t; \nu, \omega, B) - \tau_F,$$

The payout now depends on the deviation of current unemployment $u^i_t$ from the country-specific average unemployment $u_{i,avg}^t$.

Figure 4 shows the results. When federal UI is allowed to condition transfers on the average
Figure 3: Optimal federal UI - gross payouts allowing for a response of short-run policies

Notes: The payouts from the European UI under the contract that is optimal given that the local government adjusts local labor market policy in response. Federal transfers are expressed in terms percent of the GDP prevailing absent the insurance scheme. Each line represents a setup with different restriction on which local instruments can be adjusted in response to the federal insurance.
Figure 4: Optimal federal UI with indexation to past unemployment - gross payouts allowing for a response of short-run policies

Notes: The payouts from the European UI under the optimal contract under the assumption that the payout is conditioned on the deviation of the current unemployment from the long-term average. The local government adjusts all the available instruments optimally. Federal transfers are expressed in terms percent of the GDP prevailing absent the insurance scheme. One caveat of interpreting the above figure is that the realized insurance payouts will depend on the precise value of $u_{t,avg}^i$. The figure delineates the response of the payouts to the shock on impact, i.e. such that the average unemployment rate is not yet affected, $u_{t,avg}^i = \bar{u}^i$.

past unemployment rate in a given member state, federal transfers implemented through federal UI can be provided already in more moderate recessions. Still, when the member states can adjust all instruments optimally over the business cycle, even an indexation of benefits to average unemployment in the member state will not ensure that notable transfers can be provided. Payouts will still start only in a 3- standard deviation recession. This is an important point that has, so far, been overlooked in the literature. Federal UI does not only affect member states’ long-run behavior, but it also affects the incentives of member states to stabilize their own business cycles in the first place. The simulations shown here suggest that these effects may be quantitatively large.
5 Sensitivity analysis

This section presents sensitivity analysis. First, we derive the optimal federal UI schemes in a calibration that has a smaller macro elasticity of unemployment with respect to benefits. Then, we provide results for a calibration with demand externalities.

5.1 Sensitivity: lower macro elasticity of unemployment

To do...

5.2 Sensitivity: demand externalities

In this section we study how the optimal scope of international insurance changes in an economy that exhibits larger costs of business cycles. Demand externalities are a prominent case in point. Such demand externalities mean that business cycles can have first-order welfare effects. They, therefore, could be one reason for implementing a fiscal union in the first place, compare Farhi and Werning (forthcoming). With demand externalities, consumption stabilization provided by a federal UI scheme not only stabilizes consumption, but it also limits a costly (endogenous) further drop in labor productivity in a recession, which exacerbates the initial recessionary impulse.

The optimal insurance contract under demand externalities preserves the threshold-like structure that alleviates free-riding motive. The threshold moves towards shallower recessions, however. Nevertheless, a central result of the paper stands: if member states can adjust their labor-market policies in response to the introduction of a federal UI scheme, the optimal federal UI scheme has to account for these responses. The optimal federal UI schemes will provide transfers only in exceptionally rare recessions.
Figure 5: Optimal federal UI with demand externalities - gross payouts allowing for a response of short-run policies

Notes: The payouts from the European UI under the optimal contract in the economy with demand externalities that is optimal given that the local government adjusts local labor market policy in response. Federal transfers are expressed in terms percent of the GDP prevailing absent the insurance scheme.
6 Conclusions

How shall one design a federal unemployment insurance scheme in a union of member states that retain sovereignty? In the current paper, sovereignty was taken to be the member states’ room for manoeuvre with respect to local labor-market policies. The paper has provided intuition based on pencil-and-paper propositions and it has provided a quantitative exploration for a stylized euro area. The model was stylized in that it did not specify any scope for federal delegation. Optimally, the federal level would coordinate all policies. Indeed, in the model, if this were so, a federal UI scheme would insure countries against idiosyncratic business cycle risks and provide notable welfare gains.

Relative to this baseline, we find that member states’ sovereignty puts severe limits on the extent to which a federal UI scheme can provide regional insurance. There are three central results. First, there is subsidiarity: as long as labor-market policies are decided at the level of the member state, so should unemployment insurance. Federal unemployment insurance should not take over administration of national UI schemes. Unless that is, member states give up their sovereignty with respect to a wider range of labor-market policies. For implementing a generous federal UI system, it will not be enough to restrict/regulate local unemployment benefit systems only. Rather, all local labor-market instruments would need to be restricted. Second, federal benefits are to be paid only if member states suffer from severe recessions. Beyond such a threshold, federal UI can be generous. Quantitatively, we found that the thresholds were high, implying that a federal UI system provided insurance only exceptionally rare cases.

In closing, we wish to emphasize that our results have abstracted from a number of dimensions that may make implementability of a federal UI scheme in Europe either more desirable or more complicated. On the one hand, we have modeled a group of atomistic economies. To the extent that actual economies have some weight, the incentives to free ride would be smaller since countries share in the costs of the schemes. Still, for small countries or if there are regions within countries, the general mechanism should stand up to scrutiny. On the other hand, we have abstracted from heterogeneity across countries. What our paper shows is that, then, implementing a generous federal UI scheme may require member states to cede some sovereignty as regards to labor-market policies. Last, and perhaps most important, we have abstracted from productivity-enhancing effects that a federal component of UI may have. For example, a federal scheme would automatically imply some portability of benefits, which could foster mobility, and enhance productivity. These effects may well be of first-order importance. Still, even if we miss some of the microeconomic long-term benefits of a joint EUI system/welfare system, we believe our central result will stand: unless member states cede sovereignty or are severely impaired in regulating their labor markets to start with, the benefits of implementing a federal UI scheme will be rather limited.
References


A Simplified one-period model

This section describes the one-period version of the model. Consider one period. At the beginning of the period, all workers are unemployed. They may be hired within the period and, then, will produce. Otherwise, they are unemployed. Time ends afterward. For the sake of exposition, we abstract from an endogenous separation margin. Consequently, we also drop layoff taxes from the member state’s policy instruments.

A.0.1 Labor market

Since all workers are unemployed to start with, the matching function takes the form

\[ m^i = A^i [v^i]^{1-\gamma} [s^i]^\gamma. \]

Employment evolves according to

\[ e^i = s^i f(\theta^i). \] (26)

A.0.2 Firms

The representative firm produces output according to

\[ y^i = \exp(a^i) [c^i]^{\varsigma} [e^i]^\alpha, \]

\( e^i \) marks the number of workers that the firm seeks to employ. Throughout, we shall assume that \( \varsigma \) is chosen such that \( \frac{\partial y^i}{\partial e^i} \in [0, 1) \) always. The firms’ profits are given by \( \Pi^i = y^i - w^i e^i - (\kappa - \tau^i v^i) v^i \), reflecting that the local government may subsidize hiring. Profits can be rewritten as\(^{13}\)

\[ \Pi^i = y^i - w^i e^i - (\kappa - \tau^i v^i) \frac{e^i}{q^i}. \]

Firms will post vacancies until marginal profits are zero. We abstract from a possible interaction of the number of workers with the wage outcome. The vacancy posting first-order condition, then, is

\[ \exp(a^i) [c^i]^{\varsigma} \alpha |e^i|^{\alpha-1} = w^i + (\kappa - \tau^i v^i) \frac{1}{q^i}. \]

Bearing this in mind, the dividends that firms will rebate to the households, in equilibrium, will be:

\[ \Pi^i = (1 - \alpha) \exp(a^i) [c^i]^{\varsigma} [e^i]^\alpha. \]

\(^{13}v = \theta s = sf(\theta)/f(\theta) = s f(\theta)/q(\theta) = e/q(\theta), \) where the last step follows from the labor-flow equation.
A.0.3 Workers

The expected utility of a worker reads

\[ W^i = s^i f(\theta^i) u(c^e_i) + (1 - s^i f(\theta^i)) u(c^u_i) - \Psi(s^i), \]  

(27)

The first term captures that upon exerting effort \( s^i \), the probability of finding a job is \( s^i f(\theta^i) \). The second term takes into account that, with the opposite probability the worker will not find a job, in which case the worker has the consumption level of an unemployed worker. The final term is the utility cost from search. We assume \( \Psi \) is increasing, convex and twice continuously differentiable.

The budget constraint of the employed is

\[ c^e_i = w^i + \Pi^i - \tau^i, \]

where \( \tau \) are taxes that the government levels to finance benefits and subsidies. The budget constraint of the unemployed is given by

\[ c^u_i = B^i + \Pi^i. \]

Here, \( B \) are the unemployment benefits provided by the local government.

The optimal search effort exerted by all workers satisfies

\[ \Psi_s(s^i) = f(\theta^i) \Delta^i, \]  

(28)

where \( \Delta^i = u(c^e_i) - u(c^u_i) \) marks the gain from search. As the gain from search increases, so does the search effort. Similarly, a tighter labor market (tight from the perspective of firms) all else equal encourages search.

A.0.4 Wages

The wage is determined by wage rule \( w^i \equiv w(A^i, a^i, \eta^i, \theta^i, \Delta^i) \). We assume the wage rule is twice continuously differentiable in all arguments.

A.0.5 State-level government

In setting the level of unemployment benefits, vacancy subsidies, and taxes, the state-level government has to balance its budget so that

\[ \tau^i \cdot e^i + B^i = B^i \cdot [1 - e^i] + \tau^i \frac{e^i}{q^i}. \]

\footnote{In the simple model we tax workers rather than firms. Due to the static setup, taxes on firms would be identical to negative hiring subsidies.}
On the right-hand side are state-level unemployment benefits, and vacancy subsidies \( \tau^i_v \cdot v^i = \tau^i_v \cdot \frac{v^i}{q^i} \). On the left-hand side are the taxes raised from the employed \( \tau^i \cdot e^i \) and the net transfers that the local government receives from the federal level as a function of the local unemployment rate.

A.0.6 Federal government

The federal government administers the federal unemployment insurance system under a balanced-budget constraint:

\[
\int_0^1 B^i \, di = 0,
\]

where \( B \) marks the net transfers paid to country \( i \).

A.0.7 Equilibrium

In equilibrium, the resource constraint has to be satisfied in each member state:

\[
y^i + B^i = e^i c^i + \left[ 1 - e^i \right] \cdot c^i_u + \kappa \frac{e^i}{q(\theta^i)}. \tag{29}
\]

B Derivations and intermediate results

For the sake of notation, whenever it is clear from the context, we suppress the member state superscript \( i \). Let \( \tilde{y}(e) \) denote \( \exp(a)e^\alpha \), i.e. the output of the representative firm absent the demand externality.

B.1 Proof of Proposition 1)

Proof.

The proof rests on the following three lemmas.

Lemma 7 The partial derivative of welfare with respect to the market tightness reads

\[
\frac{\partial W}{\partial \theta} = e \gamma \phi w \left( \frac{\Delta}{\phi w} + (1 + \epsilon^s) \left( b - \frac{1}{w} \frac{d \bar{B}}{d(1 - e)} \right) \right) - \frac{1 - \gamma}{\gamma} \frac{\kappa}{q(\theta)w} + (1 + \epsilon^s) \Upsilon(1 - b)
\]

Lemma 8 The effect of the generosity of the UI scheme \( \Delta \) on the market tightness is the same as in the economy without insurance.

\[
\frac{d \theta}{d \Delta} = - \frac{\theta}{\Delta} \frac{1 - e - 1}{\epsilon^m} \frac{\epsilon^m}{\gamma \left( 1 + \epsilon^s \right)} \left( 1 - \frac{\epsilon^M}{\epsilon^m} \right)
\]
Lemma 9 The partial derivative of welfare with respect to the generosity of the UI scheme \( \Delta \) is given by

\[
\frac{\partial W}{\partial \Delta} = (1-e)\phi w e^m \left\{ b - \frac{dB}{d(1-e)} \frac{1}{w} - \frac{e \Delta}{e^m w} \left( \frac{1}{u_c(c_e)} - \frac{1}{u_c(c_u)} \right) + \Upsilon \left[ (1-b) + \frac{e \Delta}{e^m w} \left( \frac{1}{u_c(c_e)} - \frac{1}{u_c(c_u)} \right) \right] \right\}
\]

Given the above lemmas, the proof of the propositions proceeds as follows. The first order condition of the Ramsey problem is

\[
\frac{\partial W}{\partial \theta} \frac{\partial \theta}{\partial \Delta} + \frac{\partial W}{\partial \Delta} = 0
\]

The three expressions on the left-hand side are characterized in turn by the three lemmas. Plugging the formulas delivers

\[
\frac{\partial W}{\partial \theta} \frac{\partial \theta}{\partial \Delta} = \frac{e}{\theta} \gamma w \left( \frac{\Delta}{\phi w} + (1+\epsilon) \left( b - \frac{1}{w} \frac{dB}{d(1-e)} \right) - \frac{1 - \gamma \kappa}{q(\theta)w} + (1+\epsilon^s) \Upsilon (1-b) \right) \cdot \left( - \frac{\theta}{e} \frac{1}{\gamma} \frac{e^m}{1+\epsilon^s} \left( 1 - \frac{e^M}{e^m} \right) \right)
\]

Which is the formula for the optimal replacement rate.
In order to back out the payroll tax observe that balanced budget requires

\[ e\tau + B = (1 - e)B \]
\[ e\tau + \tau - B = (1 - e)B \]
\[ \tau = (1 - e)(B + \tau) - B \]
\[ \tau = (1 - e)bw - B \]

**Proof.** (Lemma 7)

It follows that

\[ \frac{\partial W}{\partial \theta} = \frac{\partial s}{\partial f(\theta)} f'(A, \theta) f(\theta) \Delta + sf'(\theta) \Delta + u_c(c_u) \frac{\partial c_u}{\partial \theta} - \Psi'(s) \frac{\partial s}{\partial f(\theta)} f'(A, \theta) \]

Note that the envelope theorem implies that \( \frac{\partial s}{\partial f(\theta)} = 0 \) when it comes to the direct impact on welfare. Furthermore, \( sf'(\theta) \Delta = \frac{sf}{\theta} \theta' (\Delta) = \frac{e}{\theta} \gamma \Delta \). Lastly, we need to calculate \( \frac{\partial c_u}{\partial \theta} \) such that the change in consumption is consistent with the budget constraint. To this end, we implicitly differentiate the balanced budget requirement (29). Using \( c_e = u^{-1}(u(c_u) + \Delta) \), the constraint can be rewritten as

\[ \tilde{y}(\theta, \Delta) C(\theta, \Delta)^{\gamma} - \frac{e(\theta, \Delta)}{q} + B(1 - e) = e(\theta, \Delta)u^{-1}(u(c_u(\theta, \Delta)) + \Delta) + (1 - e(\theta, \Delta))c_u(\theta, \Delta) \]

Note that \( \frac{\partial \tilde{y}}{\partial \theta} = \frac{u_c(c_u)}{u_c(c_e)} \). Therefore, the differentiation of the demand externality delivers

\[ \frac{\partial C^s}{\partial \theta} = \gamma C^{s-1} \left( \frac{\partial e}{\partial \theta} (c_e - c_u) + \frac{\partial c_u}{\partial \theta} u_e(c_u) \left( \frac{e}{u_e(c_e)} + \frac{1 - e}{u_e(c_u)} \right) \right) + \kappa q' \left( \frac{e}{q} \right) \frac{1 - e}{q} \frac{\partial e}{\partial \theta} - \frac{dB}{d(1 - e)} \frac{\partial e}{\partial \theta} \]

Let \( Y = \tilde{y}(e, \Delta)^{s-1} \). The implicit differentiation of budget constraint wrt \( \theta \) yields

\[ \frac{\partial e}{\partial \theta} C^s \frac{\partial e}{\partial \theta} + Y \left( \frac{\partial e}{\partial \theta} (c_e - c_u) + \frac{\partial c_u}{\partial \theta} u_e(c_u) \left( \frac{e}{u_e(c_e)} + \frac{1 - e}{u_e(c_u)} \right) \right) + \kappa q' \left( \frac{e}{q} \right) \frac{1 - e}{q} \frac{\partial e}{\partial \theta} - \frac{dB}{d(1 - e)} \frac{\partial e}{\partial \theta} \]

\[ \frac{\partial e}{\partial \theta} \left( \frac{\partial e}{\partial \theta} C^s - \frac{\kappa}{q} c_e - c_u - \frac{dB}{d(1 - e)} + Y(c_e - c_u) \right) - \frac{1 - \gamma e}{q} \frac{\partial e}{\partial \theta} \]

\[ = \left( \frac{e}{u_e(c_e)} + \frac{1 - e}{u_e(c_u)} \right) u_e(c_u) \frac{\partial c_u}{\partial \theta} - Y \frac{\partial c_u}{\partial \theta} \left( \frac{e}{u_e(c_e)} + \frac{1 - e}{u_e(c_u)} \right) - \frac{\kappa}{q} \frac{1 - \gamma e}{q} \frac{\partial e}{\partial \theta} \]

\[ \frac{e}{\theta} \gamma (1 + e^s) \left( w - c_e + c_u - \frac{dB}{d(1 - e)} + Y(c_e - c_u) \right) - \kappa \frac{e}{\theta} \frac{1 - \gamma e}{q} \left( \frac{e}{u_e(c_e)} + \frac{1 - e}{u_e(c_u)} \right) (1 - Y) u_e(c_u) \frac{\partial c_u}{\partial \theta} , \]
where we used \( \epsilon^s = \frac{f(\theta)}{s} \frac{\partial s}{\partial f(\theta)} \), \(-\frac{q'(A,\theta)}{(q)^2} = -\frac{1}{\partial q} \frac{\partial q'(A,\theta)}{q} = \frac{1-\gamma}{\partial q} \) and

\[
\frac{\partial c}{\partial \theta} = \frac{\partial (s f(\theta))}{\partial \theta} = \frac{\partial s}{\partial f(\theta)} f'(A,\theta) f + s f'(A,\theta) \left( \frac{f(\theta)}{s} \frac{\partial s}{\partial f(\theta)} + 1 \right) = \frac{e}{\theta} (1 + \epsilon^s) \gamma.
\]

Let us define

\[
\frac{1}{\phi} = \left( \frac{e}{\psi(c_e)} + \frac{1-e}{\psi(c_u)} \right) (1-\Upsilon)
\]

and \( \Delta_c = c_e - c_u \). Furthermore, please note that the replacement rate satisfies \( bw = w - \Delta_c \).

With all the above results at hand, we have

\[
u_c(c_u) \frac{\partial c_u}{\partial \theta} = \phi e \gamma (1+\epsilon^s) \left( bw - \frac{dB}{(1-e)} + \Upsilon \Delta_c \right) - \phi e \frac{e}{\theta} \gamma
\]

Therefore, the partial derivative of welfare wrt the market tightness expands as follows.

\[
\frac{\partial W}{\partial \theta} = \frac{e}{\theta} \gamma \Delta + \phi e \gamma (1+\epsilon^s) \left( bw - \frac{dB}{(1-e)} + \Upsilon \Delta_c \right) - \phi e \frac{e}{\theta} \gamma
\]

where we used \( \Delta_w = 1-b \).

**Proof.** (Lemma 8)

Note that

\[
\epsilon^M = \epsilon^m + \frac{\theta}{1-e} \frac{\partial e}{\partial \theta} \frac{\Delta}{d \Delta} = \epsilon^m + \left[ \frac{e}{1-e} (1+\epsilon^s) \gamma \right] \frac{\Delta}{d \Delta}
\]

Rearranging the terms and dividing by \( \epsilon^m \) yields the desired result

\[
\frac{\Delta}{\theta} \frac{d \theta}{d \Delta} = - \left( 1 - \frac{\epsilon^M}{\epsilon^m} \right) \frac{1-e}{e} \frac{\epsilon^m}{1+\epsilon^s} \frac{1}{\gamma}
\]

**Proof.** (Lemma 9)

We proceed as in the proof of lemma 7.

Observe that

\[
\frac{\partial W}{\partial \Delta} = e + u_c(c_u) \frac{\partial c_u}{\partial \Delta}
\]

We need to derive the change \( \frac{\partial c_u}{\partial \Delta} \) that is implied by the balanced budget requirement. Recall that \( c_e = u^{-1} (u(c_u) + \Delta) \) and \( \frac{\partial c_e}{\partial \Delta} = \frac{1}{u_c(c_e)} \left( 1 + u_c(c_u) \frac{\partial c_u}{\partial \Delta} \right) \). Therefore, the differentiation of
the demand externality delivers
\[
\frac{\partial C^e}{\partial \Delta} = \varsigma C^{e-1} \left( \frac{\partial e}{\partial \Delta} c_e + e \left( \frac{1}{u_c(c_e)} \left( 1 + u_c(c_u) \frac{\partial c_u}{\partial \Delta} \right) \right) - \frac{\partial e}{\partial \Delta} c_u + (1 - e) \frac{\partial c_u}{\partial \Delta} \right)
\]
\[
= \varsigma C^{e-1} \left( \frac{\partial e}{\partial \Delta} (c_e - c_u) + \frac{\partial c_u}{\partial \Delta} u_c(c_u) \left( \frac{e}{u_c(c_u)} + \frac{1 - e}{u_c(c_u)} \right) + e \right)
\]

The implicit differentiation of the resource constraint yields
\[
\hat{y}'(e) \frac{\partial e}{\partial \Delta} C^e + \hat{y}(e) \varsigma C^{e-1} \left( \frac{\partial e}{\partial \Delta} (c_e - c_u) + \frac{\partial c_u}{\partial \Delta} u_c(c_u) \left( \frac{e}{u_c(c_u)} + \frac{1 - e}{u_c(c_u)} \right) + e \right)
\]
\[
- \frac{\kappa \frac{\partial e}{\partial \Delta}}{q} - \frac{\frac{d B}{\partial (1 - e)}}{\partial \Delta} + e \frac{1}{u_c(c_e)} \left( 1 + u_c(c_u) \frac{\partial c_u}{\partial \Delta} \right) + (1 - e) \frac{\partial c_u}{\partial \Delta} - \hat{y}(e) \varsigma C^{e-1} \left( \frac{\partial c_u}{\partial \Delta} u_c(c_u) \left( \frac{e}{u_c(c_u)} + \frac{1 - e}{u_c(c_u)} \right) + e \right)
\]
\[
= \frac{\partial e}{\partial \Delta} c_e + e \frac{\partial c_e}{\partial \Delta} - \frac{\partial e}{\partial \Delta} c_u + (1 - e) \frac{\partial c_u}{\partial \Delta}
\]

Which can be rearranged as follows
\[
\frac{\partial e}{\partial \Delta} \left( \hat{y}'(e) C^e - \frac{\kappa}{q} - c_e + c_u - \frac{\frac{d B}{\partial (1 - e)}}{\partial \Delta} + \hat{y}(e) \varsigma C^{e-1} (c_e - c_u) \right)
\]
\[
= e \frac{1}{u_c(c_e)} \left( 1 + u_c(c_u) \frac{\partial c_u}{\partial \Delta} \right) + (1 - e) \frac{\partial c_u}{\partial \Delta} - \hat{y}(e) \varsigma C^{e-1} \left( \frac{\partial c_u}{\partial \Delta} u_c(c_u) \left( \frac{e}{u_c(c_u)} + \frac{1 - e}{u_c(c_u)} \right) + e \right)
\]
\[
\frac{\partial e}{\partial \Delta} \left( w - \Delta_c - \frac{\frac{d B}{\partial (1 - e)}}{\partial \Delta} + \hat{y}(e) \varsigma C^{e-1} \Delta_c \right) - \frac{e}{u_c(c_e)} + \hat{y}(e) \varsigma C^{e-1} \frac{e}{u_c(c_e)}
\]
\[
= \left( \frac{e}{u_c(c_e)} + \frac{1 - e}{u_c(c_u)} - \hat{y}(e) \varsigma C^{e-1} \left( \frac{e}{u_c(c_u)} + \frac{1 - e}{u_c(c_u)} \right) \right) u_c(c_u) \frac{\partial c_u}{\partial \Delta}
\]
\[
\epsilon^m \frac{1 - e}{\Delta} \left( bw - \Delta_c - \frac{\frac{d B}{\partial (1 - e)}}{\partial \Delta} + \hat{y}(e) \varsigma C^{e-1} \Delta_c \right) - \frac{e}{u_c(c_e)} (1 - \hat{y}(e) \varsigma C^{e-1})
\]
\[
= \left( \frac{e}{u_c(c_e)} + \frac{1 - e}{u_c(c_u)} \right) (1 - \hat{y}(e) \varsigma C^{e-1}) \right) u_c(c_u) \frac{\partial c_u}{\partial \Delta}
\]

Were we used \( \frac{\partial e}{\partial \Delta} = \epsilon^m \frac{1 - e}{\Delta} \) and \( w - \Delta_c = bw \). Using the definition of marginal value of resources
\[
\frac{1}{\phi} = \left( \frac{e}{u_c(c_e)} + \frac{1 - e}{u_c(c_u)} \right) (1 - \hat{y}(e) \varsigma C^{e-1})
\]
and plugging the above formula for \( u_e(c_u) \frac{\partial c_u}{\partial \Delta} \) to the expression for \( \frac{\partial W}{\partial \Delta} \) we obtain

\[
\frac{\partial W}{\partial \Delta} = e + \phi \left( e^m \frac{1 - e}{\Delta} \left( b \left( \frac{d\mathcal{B}}{d(1-e)} \right) \right) + \tilde{y}(e) \xi C^\alpha - 1 \Delta c \right) - \frac{e}{u_e(c_e)} \left( 1 - \tilde{y}(e) \xi C^\alpha - 1 \right)
\]

\[
= (1 - e) \phi \Delta \epsilon^m \left( b - \frac{d\mathcal{B}}{d(1-e)} \frac{1}{w} + \frac{\Delta c}{w} \tilde{y}(e) \xi C^\alpha - 1 + \frac{\epsilon}{w^m} \frac{1}{1 - e} \left( \frac{1}{\phi} - \frac{1}{u_e(c_e)} \left( 1 - \tilde{y}(e) \xi C^\alpha - 1 \right) \right) \right)
\]

\[
= (1 - e) \phi \Delta \epsilon^m \left( b - \frac{d\mathcal{B}}{d(1-e)} \frac{1}{w} + (1 - b) \tilde{y}(e) \xi C^\alpha - 1 - (1 - \tilde{y}(e) \xi C^\alpha - 1) \frac{\epsilon}{w^m} \frac{1}{1 - e} \left( \frac{1}{\phi} - \frac{1}{u_e(c_e)} \left( 1 - \tilde{y}(e) \xi C^\alpha - 1 \right) \right) \right)
\]

\[
= (1 - e) \phi \Delta \epsilon^m \left( b - \frac{d\mathcal{B}}{d(1-e)} \frac{1}{w} - \frac{\epsilon}{w^m} \right) \left( \frac{1}{u_e(c_e)} - \frac{1}{u_e(c_u)} \right) + \Upsilon \left[ (1 - b) + \frac{\epsilon}{w^m} \left( \frac{1}{u_e(c_e)} - \frac{1}{u_e(c_u)} \right) \right]
\]

Where the equality follows from \( \frac{1}{\phi} - \frac{1}{u_e(c_e)} = -(1 - e) \left( \frac{1 - \tilde{y}(e) \xi C^\alpha - 1}{u_e(c_e)} - \frac{1 - \tilde{y}(e) \xi C^\alpha - 1}{u_e(c_u)} \right) \), \( \Delta \epsilon = 1 - b \), and \( \Upsilon = \tilde{y}(e) \xi C^\alpha - 1 \).

B.2 Proof of Proposition 2

Proof.

For the sake of this proof let us abuse notation and write \( W \left( \Delta \left( \frac{d\mathcal{B}}{d(1-e)} \right), \frac{d\mathcal{B}}{d(1-e)} \right) \) for the aggregate welfare in a typical member state. The first order condition of the government can be written as \( W_1 = 0 \), where the subscript denotes the partial derivative with respect to the first argument, \( \Delta \).

Fix some function \( \mathcal{B} \) and the implied policy \( \Delta \). These choices pin down some equilibrium. Consider now an infinitesimal change of the slope of the international UI function at the equilibrium point \( e(\Delta, \mathcal{B}) \) compensated so as to ensure that the aggregate resources available to the government do not change. The implicit differentiation of the FOC, \( \therefore \), delivers

\[
0 = \frac{dW_1}{d(1-e)} = W_{11} \frac{d\Delta}{d(1-e)} + W_{12} \Rightarrow \frac{d\Delta}{d(1-e)} = -W_{11}
\]

Due to the concavity of the problem, it follows that \( -W_{11} > 0 \).

Next, we will derive \( W_{12} \). Recall that the values of \( c_u(\Delta), c_u(\Delta) \) are determined as a solution of the system of equations consisting of budget constraint and the definition of gain from employment, \( \Delta = u(c_e) - u(c_u) \). The assumption that the budget constraint remains intact ensures that, for a given \( \Delta \), the change in the slope of federal UI does not affect consumption levels. Likewise, for a given \( \Delta \), all other equilibrium allocations are not directly affected by a change in \( \frac{d\mathcal{B}}{d(1-e)} \).
Recall from the proof of Proposition 1 that the FOC, \( W_1 \), reads

\[
0 = -\frac{(1-e)\phi w e^m}{\Delta(1+e^s)} \left( \frac{\Delta}{\phi w} + (1+e^s) \left( b - \frac{1}{w} \frac{dB}{d(1-e)} \right) - \frac{1-\gamma}{\gamma} \frac{\kappa}{q(\theta)w} + (1+e^s)\Upsilon(1-b) \right) \left( 1 - \frac{e^M}{e^m} \right) \\
+ (1-e)\phi \frac{w}{\Delta} e^m \left\{ b - \frac{1}{d(1-e)} \frac{d\mathbb{E}}{d(1-e)} - \frac{e}{e^m w} \left( \frac{1}{u_c(c_e)} - \frac{1}{u_c(c_u)} \right) \right\} + \Upsilon \left[ (1-b) + \frac{e}{e^m w} \left( \frac{1}{u_c(c_e)} - \frac{1}{u_c(c_u)} \right) \right]\]

The derivative of the FOC with respect to \( \frac{d\mathbb{E}}{d(1-e)} \), \( W_{12} \) in our notation, delivers

\[
W_{12} = \frac{e}{\theta} \phi w \frac{e^m}{\Delta} + \frac{1-e}{\Delta} \frac{e^m}{\gamma(1+e^s)} \left( 1 - \frac{e^M}{e^m} \right) + (1-e)\phi \frac{w}{\Delta} e^m (-1/w)
\]

\[
= \phi \frac{1}{\Delta} (1-e) e^m \left( 1 - \frac{e^M}{e^m} \right) - (1-e)\phi \frac{1}{\Delta} e^m
\]

\[
= \frac{\phi}{\Delta} (1-e) e^m \left( 1 - \frac{e^M}{e^m} - 1 \right)
\]

\[
= -\frac{\phi}{\Delta} (1-e) e^M \leq 0
\]

Therefore, we have

\[
\frac{d\Delta}{d\frac{d\mathbb{E}}{d(1-e)}} = -\frac{\phi}{\Delta} (1-e) e^M \leq 0
\]

\[\blacksquare\]

**B.3 Proof of Proposition ??**

**Proof.**

Let us define the social welfare in a boom country as

\[
W^H(\Delta^H, \theta^H, \Delta^H; \mathbb{E}) = s^H \cdot f^H \cdot \Delta^H + u(c^H_u) - \Psi(s^H).
\]

\( W(A^L, \theta^L, \Delta^L; \mathbb{E}) \) is defined analogously.
The social welfare for a union as a whole can be written as

\[ V(B) = \pi \max_{\Delta^H, c^H_u} \left\{ W(A^H, \theta(\Delta^H), \Delta^H; B) \right. \]

\[ + \phi^H \left[ y(e^H) - \kappa \frac{e^H}{q^H} - e^H u^{-1}(u(c^H_u) + \Delta^H) - (1 - e^H) c^H_u + B (1 - e^H) \right] \}

\[ + (1 - \pi) \max_{\Delta^L, c^L_u} \left\{ W(A^L, \theta(\Delta^L), \Delta^L; B) \right. \]

\[ + \phi^L \left[ y(e^L) - \kappa \frac{e^L}{q^L} - e^L u^{-1}(u(c^L_u) + \Delta^L) - (1 - e^L) c^L_u + B (1 - e^L) \right] \}

\[ = \pi \phi^H (1 - e^H) + (1 - \pi) \phi^L (1 - e^L) - \left( \pi \phi^H + (1 - \pi) \phi^L \right) \]

Optimal insurance in class of linear contracts \( B(1 - e) \equiv (1 - e)B - \tau_F \) solves

\[ \max_B V(B) \]

s.t.

\[ \Delta^I(B) \text{ given by Posposition 1} \]

\[ \tau_F(B) = B \left[ \pi(1 - e^H) + (1 - \pi)(1 - e^L) \right] \]

FOC is

\[ 0 = \phi^H \pi \left( 1 - e^H - \frac{d\tau}{dB} \right) + (1 - \pi) \phi^L \left( 1 - e^L - \frac{d\tau}{dB} \right) \]

\[ = \pi \phi^H (1 - e^H) + (1 - \pi) \phi^L (1 - e^L) - \frac{d\tau}{dB} \left( \pi \phi^H + (1 - \pi) \phi^L \right) \]

Note

\[ \frac{d\tau_F}{dB} = \pi(1 - e^H) + (1 - \pi)(1 - e^L) \]

\[ + B \left( -\pi \frac{d\Delta^H}{dB} \left[ \frac{\partial e^H}{\partial \theta^H} \frac{\partial \theta^H}{\partial \Delta^H} + \frac{\partial e^H}{\partial \Delta^H} \right] - (1 - \pi) \frac{d\Delta^L}{dB} \left[ \frac{\partial e^L}{\partial \theta^L} \frac{\partial \theta^L}{\partial \Delta^L} + \frac{\partial e^L}{\partial \Delta^L} \right] \right) \]

It holds that

\[ \frac{\partial e}{\partial \theta} \frac{d\theta}{d\Delta} + \frac{\partial e}{\partial \Delta} = \frac{1 - e}{\Delta} \epsilon^M \]

Hence

\[ \frac{d\tau_F}{dB} = \pi(1 - e^H) + (1 - \pi)(1 - e^L) \]

\[ - B \left( \pi \frac{d\Delta^H}{dB} \frac{1 - e^H}{\Delta^H} \epsilon^M, H + (1 - \pi) \frac{d\Delta^L}{dB} \frac{1 - e^L}{\Delta^L} \epsilon^M, L \right) \]
So that the FOC is
\[
\frac{\pi \phi^H (1 - e^H) + (1 - \pi) \phi^L (1 - e^L)}{\pi \phi^H + (1 - \pi) \phi^L} = \pi (1 - e^H) + (1 - \pi) (1 - e^L) \\
- B \left( \pi \frac{d\Delta^H}{dB} \frac{1 - e^H}{\Delta^H} \epsilon^{M,H} + (1 - \pi) \frac{d\Delta^L}{dB} \frac{1 - e^L}{\Delta^L} \epsilon^{M,L} \right)
\]

Solving for optimal generosity \( B \) delivers
\[
B = \pi (1 - \pi) (e^H - e^L) \left( \frac{\phi^L - \phi^H}{\pi \phi^H + (1 - \pi) \phi^L} \right) \left( -\pi \frac{d\Delta^H}{dB} \frac{1 - e^H}{\Delta^H} \epsilon^{M,H} - (1 - \pi) \frac{d\Delta^L}{dB} \frac{1 - e^L}{\Delta^L} \epsilon^{M,L} \right)^{-1}
\]

\[\blacksquare\]

### B.4 Derivation of the Optimal Federal UI in Case of Piecewise Linear Payout Function. Proof of Proposition 4

Consider a federal planner choosing two different slopes, \( B^H, B^L \), for boom and bust states, respectively. Let \( e^L \) be the level of employment that would prevail in recession country absent federal insurance.

We proceed as in the proof of the Proposition 3. The aggregate welfare is given by equation (30). The benefit function becomes

\[
\mathbb{B}(e^i) = \begin{cases} (1 - e^i)B^H - \tau_F & \text{if } e^i \geq e^L \\ (1 - e^i)B^L - \tau_F & \text{if } e^i < e^L \end{cases}
\]

The incentive compatibility constraint for the poor country will never bind as it is clearly suboptimal to set very high employment in order to receive lower transfers \( B^H \).

For the rich country, we assume that the high-state productivity \( \exp(a^H) \) is large enough so that the drop of output resulting from setting \( e^H < \bar{e}^L \) is not compensated by transfer increase \( B^L - B^H \). This puts an upper bound on \( B^L \).

The problem reads

\[
\max_{B^H, B^L} V(B^H, B^L) \\
\text{s.t.} \\
\Delta^i(B^i) \text{ given by Popenosition 1, for } i = H, L \\
\tau_F(B^H, B^L) = B^H \pi (1 - e^H) + B^L (1 - \pi) (1 - e^L) \\
-B^L \leq 0, \quad -B^H \leq 0
\]

Let \( \mu^H, \mu^L \) denote the Kuhn-Tucker multipliers attached to the non-negativity constraints.
The first-order condition with respect to the slope in the boom country, $B^H$, reads
\[
0 = \pi \phi^H \left[ 1 - e^H - \frac{\partial \tau_F}{\partial B^H} \right] - (1 - \pi) \phi^L \frac{\partial \tau_F}{\partial B^H} + \mu^H
\]
\[
\Rightarrow 0 = -\frac{\partial \tau_F}{\partial B^H} (\pi \phi^H + (1 - \pi) \phi^L) + (1 - e^H) \pi \phi^H + \mu^H
\]

Similarly for $B^L$,
\[
-\frac{\partial \tau_F}{\partial B^L} (\pi \phi^H + (1 - \pi) \phi^L) + (1 - \pi) \phi^L (1 - e^L) + \mu^L = 0
\]

Furthermore,
\[
\frac{\partial \tau_F}{\partial B^H} = \pi (1 - e^H) - \pi B^H \frac{d e^H}{d \Delta^H} \frac{d \Delta^H}{d B^H}
\]

and
\[
\frac{\partial \tau_F}{\partial B^L} = (1 - \pi) (1 - e^L) - (1 - \pi) B^L \frac{d e^L}{d \Delta^L} \frac{d \Delta^L}{d B^L}
\]

So that FOCs become
\[
\left[ \pi (1 - e^H) - \pi B^H \frac{d e^H}{d \Delta^H} \frac{d \Delta^H}{d B^H} \right] (-\pi \phi^H - (1 - \pi) \phi^L) + \pi \phi^H (1 - e^H) + \mu^H = 0
\]

and
\[
\left[ (1 - \pi) (1 - e^L) - (1 - \pi) B^L \frac{d e^L}{d \Delta^L} \frac{d \Delta^L}{d B^L} \right] (-\pi \phi^H - (1 - \pi) \phi^L) + (1 - \pi) \phi^L (1 - e^L) + \mu^L = 0
\]

We can solve for the optimal slopes
\[
0 = \left[ -(1 - e^H) + B^H \frac{d e^H}{d \Delta^H} \frac{d \Delta^H}{d B^H} \right] \left( \pi \phi^H + (1 - \pi) \phi^L \right) + \phi^H (1 - e^H) + \mu^H \frac{1}{\pi}
\]
\[
\Rightarrow 0 = B^H \frac{1 - e^H}{\Delta^H - \epsilon^{M,H}} \frac{d \Delta^H}{d B^H} + \left( 1 - e^H \right) \left( \frac{\pi \phi^H + (1 - \pi) \phi^L}{\pi \phi^H + (1 - \pi) \phi^L} \right) + \mu^H \frac{1}{\pi} \left( \frac{\pi \phi^H + (1 - \pi) \phi^L}{\pi \phi^H + (1 - \pi) \phi^L} \right)
\]
\[
\Rightarrow B^H = \frac{(1 - e^H) (1 - \pi) (\phi^H - \phi^L)}{\pi \phi^H + (1 - \pi) \phi^L} \frac{\Delta^H}{1 - e^H \epsilon^{M,H}} \frac{1}{\left( \frac{d \Delta^H}{d B^H} \right)^{-1}} + \mu^H \frac{\Delta^H}{1 - e^H \epsilon^{M,H}} \frac{\left( -\frac{d \Delta^H}{d B^H} \right)^{-1}}{\pi} \left( \frac{\pi \phi^H + (1 - \pi) \phi^L}{\pi \phi^H + (1 - \pi) \phi^L} \right)
\]

The above equation implies that the optimal slope for the rich country, $B^H$, is zero. To see this, assume to the contrary that $B^H > 0$. Then $\mu^H = 0$ and
\[
B^H = \frac{(1 - e^H) (1 - \pi) (\phi^H - \phi^L)}{\pi \phi^H + (1 - \pi) \phi^L} \frac{\Delta^H}{1 - e^H \epsilon^{M,H}} \frac{1}{\left( \frac{d \Delta^H}{d B^H} \right)^{-1}} < 0,
\]

which is a contradiction. The inequality follows from the fact that all terms on the right-hand side are nonnegative, with the exception of $\phi^H - \phi^L < 0$. Indeed, due to the concavity of the
utility function, the boom country has a smaller marginal value of resources.

For completeness, let us note that in the corner solution we have

$$
\frac{(1 - e^H)(1 - \pi)(\phi^H - \phi^L)}{\pi \phi^H + (1 - \pi) \phi^L} \frac{\Delta^H}{1 - e^H} \frac{1}{\epsilon^{M,H}} \left(-\frac{d\Delta^H}{dB^H}\right)^{-1} = -\mu^H \frac{\Delta^H}{\pi \phi^H + (1 - \pi) \phi^L}
$$

Left-hand side is negative, as we noted before. This is consistent with the requirement for the Kuhn-Tucker multiplier to be nonnegative, $-\mu^H \leq 0$. Therefore, both sides of the equation are consistent.

We can derive the formula for $B^L$ in a similar way. We conjecture that $B^L > 0, \mu^L = 0$. The first-order condition becomes

$$
B^L = \frac{(1 - e^L)\pi (\phi^L - \phi^H)}{\pi \phi^H + (1 - \pi) \phi^L} \frac{\Delta^L}{1 - e^L} \frac{1}{\epsilon^{M,L}} \left(-\frac{d\Delta^L}{dB^L}\right)^{-1}
$$

Here all the terms, including $\phi^L - \phi^H$, are nonnegative. Similarly, assuming $B^L = 0, \mu^L \geq 0$, leads to a contradiction, since $\phi^L - \phi^H > 0$.

It is easy to see that, if $\epsilon^{M,L} = 0$, then the first-order condition implies $\phi^L = \phi^H$, which implements the first-best insurance.

The last step is to find the upper bound on $B^L$ such that the member state with high productivity will never want to set unemployment rate qualifying for the high transfer $B^L$. It is straightforward in the special case of constant utility cost of search. Then, we need only to consider the difference in the government’s budget constraint.

Let $u^H$ be an unemployment chosen by high-productivity state in response the the federal UI. We need to show that $u^H > \bar{u}^L$, that is the level of unemployment chosen by the boom country is above the autarkic unemployment of the recession country. Note that, since $B^H = 0$, the boom country will either set $u^H = \bar{u}^H$ or $u^H \leq \bar{u}^L$. To exclude the latter, we need to set an upper bound on $B^L$. It follows that

$$
y(1 - \bar{u}^H) - \kappa \frac{1 - \bar{u}^H}{q(\bar{\theta}^H)} > y(1 - \bar{u}^L) - \kappa \frac{1 - \bar{u}^L}{q(\bar{\theta}^L)} + \bar{u}^L B^L
$$

$$
\Rightarrow B^L \bar{u}^L < y(1 - \bar{u}^H) - y(1 - \bar{u}^L) - \kappa \left(\frac{1 - \bar{u}^H}{q(\bar{\theta}^H)} - \frac{1 - \bar{u}^L}{q(\bar{\theta}^L)}\right)
$$

B.5 Proof of Proposition 5

Proof.

Let the notational wage be $w = \tilde{y}'C^\zeta - \frac{\xi}{q} + \frac{\kappa \tau_v}{q}$.

Let us write the aggregate welfare as $W(\Delta, \theta(\Delta, \tau_v))$.

The optimal hiring subsidy solves $\frac{\partial W}{\partial \theta} \frac{d\theta}{d\tau_v} = 0$. Since $\frac{d\theta}{d\tau_v} > 0$, it follows that $\frac{\partial W}{\partial \theta} = 0$. The
optimal gain from employment solves $\frac{\partial W}{\partial \Delta} + \frac{\partial W}{\partial \theta} \frac{d\theta}{d\Delta} = 0$. Since $\frac{\partial W}{\partial \theta} = 0$ in the optimum, the FOC reduces to $\frac{\partial W}{\partial \Delta} = 0$. These partial derivatives are characterized by the formulas analogous to those in Lemmas 9 and 7, with the adjustment for the fact that notational wage involves hiring subsidy. We have

**Lemma 10** The partial derivative of welfare with respect to the market tightness reads

$$\frac{\partial W}{\partial \theta} = e \gamma \phi \left( \frac{\Delta}{\phi} + (1 + \epsilon^s) \left( \frac{\partial y'}{\partial e} C^\alpha - \frac{\kappa}{q} - \Delta c(1 - \gamma) - \frac{d\beta}{d(1 - e)} \right) - \frac{1 - \gamma}{\gamma} \frac{\kappa}{q} \right)$$

**Lemma 11** The partial derivative of welfare with respect to the generosity of the UI scheme $\Delta$ is given by

$$\frac{\partial W}{\partial \Delta} = (1 - e) \frac{1}{\Delta} e^m \left( \frac{\partial y'}{\partial e} C^\alpha - \frac{\kappa}{q} - \Delta c(1 - \gamma) - \frac{d\beta}{d(1 - e)} \Delta \left( \frac{1}{u_c(c_e)} - \frac{1}{u_c(c_u)} \right) \right)$$

Given the above lemmas, one can solve for the optimal allocation as follows. Therefore

$$0 = \frac{\partial W}{\partial \Delta} = \frac{\partial y'}{\partial e} C^\alpha - \frac{\kappa}{q} - \Delta c(1 - \gamma) - \frac{d\beta}{d(1 - e)} - \frac{e \gamma}{\epsilon m} \Delta \left( \frac{1}{u_c(c_e)} - \frac{1}{u_c(c_u)} \right) (1 - \gamma)$$

$$\Rightarrow \Delta = \frac{e \gamma}{\epsilon m} \left( \frac{1}{u_c(c_e)} - \frac{1}{u_c(c_u)} \right) (1 - \gamma)$$

and

$$0 = \frac{\partial W}{\partial \theta} = \Delta + \phi (1 + \epsilon^s) \left( \frac{\partial y'}{\partial e} C^\alpha - \frac{\kappa}{q} - \Delta c(1 - \gamma) - \frac{d\beta}{d(1 - e)} \right) - \frac{1 - \gamma}{\gamma} \frac{\kappa}{q} \phi$$

$$= \frac{e \gamma}{\epsilon m} \left( \frac{1}{u_c(c_e)} - \frac{1}{u_c(c_u)} \right) (1 - \gamma) \phi (1 + \epsilon^s) \left( \frac{\partial y'}{\partial e} C^\alpha - \frac{\kappa}{q} - \Delta c(1 - \gamma) - \frac{d\beta}{d(1 - e)} \right) - \frac{1 - \gamma}{\gamma} \frac{\kappa}{q} \phi$$

$$= \left( \frac{\partial y'}{\partial e} C^\alpha - \frac{\kappa}{q} - \Delta c(1 - \gamma) - \frac{d\beta}{d(1 - e)} \right) \left( \frac{e \gamma}{\epsilon m} \left( \frac{1}{u_c(c_e)} - \frac{1}{u_c(c_u)} \right) (1 - \gamma) \phi (1 + \epsilon^s) \right) - \frac{1 - \gamma}{\gamma} \frac{\kappa}{q} \phi$$

$$= -\frac{k}{q} \left( \frac{1}{u_c(c_e)} - \frac{1}{u_c(c_u)} \right) (1 - \gamma) \phi (1 + \epsilon^s) + \frac{1 - \gamma}{\gamma} \phi$$

$$+ \left( \frac{\partial y'}{\partial e} C^\alpha - \Delta c(1 - \gamma) - \frac{d\beta}{d(1 - e)} \right) \left( \frac{e \gamma}{\epsilon m} \left( \frac{1}{u_c(c_e)} - \frac{1}{u_c(c_u)} \right) (1 - \gamma) \phi (1 + \epsilon^s) \right)$$

thus

$$\frac{k}{q} = \left( \frac{\partial y'}{\partial e} C^\alpha - \Delta c(1 - \gamma) - \frac{d\beta}{d(1 - e)} \right) \left( \frac{e \gamma}{\epsilon m} \left( \frac{1}{u_c(c_e)} - \frac{1}{u_c(c_u)} \right) (1 - \gamma) \phi (1 + \epsilon^s) \right) + \phi (1 + \epsilon^s) + \frac{1 - \gamma}{\gamma} \phi$$
Now observe that in equilibrium \( \frac{k}{q} = \dot{y}'(e) - w + \tau_v/q \) and hence

\[
\tau_v/q = \left( \dot{y}'(e)C^s - \Delta_c(1 - \Upsilon) - \frac{d\mathbb{B}}{d(1-e)} \right) \frac{1}{\tau_v \left( \frac{1}{u_c(c_e)} - \frac{1}{u_c(c_u)} \right)} (1 - \Upsilon) + \frac{1}{\tau_v \left( \frac{1}{u_c(c_e)} - \frac{1}{u_c(c_u)} \right)} + \phi(1 + \epsilon^\ast) + \frac{1 - \gamma}{\gamma} \phi - \dot{y}'(e)C^s + w
\]

In the last step we substitute out \( \kappa/q \) in the expression for \( \Delta \), which delivers

\[
\Delta = \frac{1 - \gamma}{\gamma} \left( \dot{y}'(e)C^s - \Delta_c(1 - \Upsilon) - \frac{d\mathbb{B}}{d(1-e)} \right) (1 - \Upsilon) (1 + \epsilon^\ast + \frac{1 - \gamma}{\gamma})
\]

Alternatively, an implicit solution for instruments reads

\[
0 = \frac{\partial W}{\partial \theta} = \Delta \frac{b + \Upsilon(1-b)}{\tau_v/q} + \frac{d\mathbb{B}}{d(1-e)} \frac{1}{w(1 - \Upsilon)} + \frac{e}{\epsilon m} \Delta \left( \frac{1}{u_c(c_e)} - \frac{1}{u_c(c_u)} \right) - \frac{\tau_v}{wq(1 - \Upsilon)}
\]

and

\[
0 = \frac{\partial W}{\partial \theta} = \frac{\Delta}{\phi} + (1 + \epsilon^\ast) \left( b(1 - \Upsilon)w + \Upsilon w - \frac{d\mathbb{B}}{d(1-e)} + \frac{\tau_v}{q} \right) - \frac{1 - \gamma}{\gamma} \frac{\kappa}{q}
\]

Proof. (Lemma 10) We proceed as in the proof of Lemma 7 and use the same notation.

\[
\frac{\partial W}{\partial \theta} = \frac{e}{\theta} \gamma \Delta + u_c(c_u) \frac{\partial c_u}{\partial \theta}
\]
The implicit differentiation of budget constraint wrt $\theta$ yields

$$
\dot{y}' (e) C^\kappa \frac{\partial e}{\partial \theta} + \nu \left( \frac{\partial e}{\partial \theta} (c_e - c_u) + \frac{\partial c_u}{\partial \theta} u_c(c_u) \left( \frac{e}{u_c(c_e)} + \frac{1 - e}{u_c(c_u)} \right) \right) + \kappa \frac{q'}{q} e - \frac{1}{q} \frac{\partial e}{\partial \theta} - \frac{\partial u}{\partial \theta} \frac{\partial e}{\partial \theta}.
$$

$$
= \frac{\partial e}{\partial \theta} e + \nu \left( \frac{e}{u_c(c_e)} + \frac{1 - e}{u_c(c_u)} \right) \frac{\partial c_u}{\partial \theta} \frac{\partial e}{\partial \theta} - \frac{\partial e}{\partial \theta} e + (1 - e) \frac{\partial c_u}{\partial \theta}.
$$

$$
\frac{\partial e}{\partial \theta} \left( \dot{y}' (e) C^\kappa - \frac{\kappa}{q} c_e + c_u - \frac{d\nu}{d(1 - e)} + \nu (c_e - c_u) \right) - \kappa \frac{1 - e}{q} = \left( \frac{e}{u_c(c_e)} + \frac{1 - e}{u_c(c_u)} \right) (1 - \nu) u_c(c_u) \frac{\partial c_u}{\partial \theta}.
$$

We have

$$
u_c(c_u) \frac{\partial c_u}{\partial \theta} = \phi \frac{e}{\theta} (1 + e^\kappa) \left( \dot{y}' (e) C^\kappa - \frac{\kappa}{q} c_e - \frac{d\nu}{d(1 - e)} + \nu \Delta_e \right) - \phi^\kappa \frac{e}{\theta} \frac{1 - e}{q}.
$$

Therefore, the partial derivative of welfare wrt the market tightness expands as follows.

$$
\frac{\partial W}{\partial \Delta} = \frac{e}{\theta} \gamma \Delta + \phi \frac{e}{\theta} (1 + e^\kappa) \left( \dot{y}' (e) C^\kappa - \frac{\kappa}{q} c_e - \frac{d\nu}{d(1 - e)} + \nu \Delta_e \right) - \phi \frac{e}{\theta} \frac{1 - e}{q}.
$$

$$
= \frac{e}{\theta} \gamma \phi \left( \frac{\Delta}{\phi} + (1 + e^\kappa) \left( \dot{y}' (e) C^\kappa - \frac{\kappa}{q} c_e - \frac{d\nu}{d(1 - e)} + \nu \Delta_e \right) - \frac{1 - e}{q} \right).
$$

\begin{proof} (Lemma 11)

We proceed as in the proof of lemma 9.

$$
\frac{\partial W}{\partial \Delta} = \frac{e}{\theta} + u_c(c_u) \frac{\partial c_u}{\partial \Delta}.
$$

The implicit differentiation of the resource constraint yields

$$
\frac{\partial e}{\partial \Delta} \left( \dot{y}' (e) C^\kappa - \frac{\kappa}{q} c_e + c_u - \frac{d\nu}{d(1 - e)} + \nu (c_e - c_u) \right)
= e \frac{1}{u_c(c_e)} \left( 1 + u_c(c_u) \frac{\partial c_u}{\partial \Delta} \right) + (1 - e) \frac{\partial c_u}{\partial \Delta} - \nu \frac{c_u}{u_c(c_u)} \left( \frac{e}{u_c(c_e)} + \frac{1 - e}{u_c(c_u)} \right) + \frac{e}{u_c(c_e)}.
$$

$$
\frac{\partial e}{\partial \Delta} \left( w - \frac{\kappa \tau v}{q} c_e - \frac{\nu}{d(1 - e)} + \nu \Delta_e \right)
= \left( \frac{e}{u_c(c_e)} + \frac{1 - e}{u_c(c_u)} \right) \frac{\partial c_u}{\partial \Delta} - \nu \frac{c_u}{u_c(c_u)} \left( \frac{e}{u_c(c_e)} + \frac{1 - e}{u_c(c_u)} \right) + \nu \frac{c_u}{u_c(c_u)}.
$$

$$
\epsilon^m \frac{1 - e}{\Delta} \left( B w - \frac{\kappa \tau v}{q} - \frac{d\nu}{d(1 - e)} + \nu \Delta_e \right) - \nu \frac{c_u}{u_c(c_u)} (1 - \nu)
= \left[ \left( \frac{e}{u_c(c_e)} + \frac{1 - e}{u_c(c_u)} \right) (1 - \nu) \right] \frac{\partial c_u}{\partial \Delta}.
$$

\end{proof}
Plugging the above formula for $u_c(c_u)\partial \phi / \partial \Delta$ to the expression for $\partial W / \partial \Delta$ we obtain

$$\frac{\partial W}{\partial \Delta} = e + \phi \left( e^m \frac{1 - e}{\Delta} \left( y'(e)C^\kappa - \frac{\kappa}{q} \frac{d\bar{B}}{d(1 - e)} - \Delta_e (1 - \Upsilon) \right) - \frac{e}{u_c(c_e)} (1 - \Upsilon) \right)$$

$$= (1 - e) \phi \frac{1}{\Delta} e^m \left( y'(e)C^\kappa - \frac{\kappa}{q} \frac{d\bar{B}}{d(1 - e)} - \Delta_e (1 - \Upsilon) + \frac{\Delta}{e^m} \frac{e}{1 - e} \left( \frac{1}{\phi} - \frac{1}{u_c(c_e)} (1 - \Upsilon) \right) \right)$$

$$= (1 - e) \phi \frac{1}{\Delta} e^m \left( y'(e)C^\kappa - \frac{\kappa}{q} \frac{d\bar{B}}{d(1 - e)} - \Delta_e (1 - \Upsilon) - (1 - \Upsilon) \frac{e}{e^m} \Delta \left( \frac{1}{u_c(c_e)} - \frac{1}{u_c(c_u)} \right) \right)$$

\[\blacksquare\]

B.6 Proof of Proposition 6

Proof. We proceed as in the proof of Proposition 3 the only difference being that the federal principal needs to accommodate moral hazard distortions along both benefits and hiring subsidy margins. As a result $\frac{de}{d\bar{B}} = \frac{\partial e}{\partial \Delta} \frac{d\Delta}{d\bar{B}} + \frac{\partial e}{\partial \theta} \frac{d\theta}{d\bar{B}}$. The elasticities of the market tightness and gain from employment with respect to the federal transfers are given by the expressions in Proposition 5.

Formally, the optimal insurance in class of linear contracts $\mathbb{B}(1 - e) \equiv (1 - e)B - \tau_F$ solves

$$\max_{\mathcal{B}} V(\mathcal{B})$$

s.t.

$$\Delta^i(\mathcal{B}) \text{ given by Proposition 5}$$

$$\theta^i(\mathcal{B}) \text{ given by Proposition 5}$$

$$\tau_F(\mathcal{B}) = B \left[ \pi (1 - e^H) + (1 - \pi) (1 - e^L) \right],$$

where $V(\mathcal{B})$ is defined as in equation (30).

The proof is concluded by following the same steps as in the proof of Proposition 3, with the appropriately defined derivative of employment with respect to the federal transfers, i.e. $\frac{de}{d\bar{B}} = \frac{\partial e}{\partial \Delta} \frac{d\Delta}{d\bar{B}} + \frac{\partial e}{\partial \theta} \frac{d\theta}{d\bar{B}}$. \[\blacksquare\]

B.7 Demand Externality and UI Elasticities

Note that the labor demand reads

$$e^d = \left( \frac{w + \kappa / q}{\alpha \exp(a) C^\kappa} \right)^{\frac{1}{\alpha - 1}}$$
We have
\[
\frac{\partial e^d}{\partial \theta} \bigg|_\Delta = \frac{1}{\alpha \exp(a) (\alpha - 1)} \left( \frac{w + \kappa/q}{\alpha \exp(a) C^s} \right)^{\frac{1}{\alpha - 1}} \left\{ \left[ \frac{\partial w}{\partial \theta} - \frac{\kappa d'}{d^2} C^{-s} - \zeta C^{-s} \frac{\partial C}{\partial \theta} [w + \kappa/q] \right] \right\}
\]
\[
= \frac{1}{\alpha \exp(a) (\alpha - 1)} e^{-1} C^{-s} \left[ \frac{\partial w}{\partial \theta} + \kappa \frac{1 - \gamma}{\theta q} - \zeta C^{-1} \exp(a) \alpha e^{a - 1} C^s \frac{\partial C}{\partial \theta} \right]
\]
\[
= \frac{e^{\alpha - 1}}{y(e) \alpha (\alpha - 1)} \left[ \frac{\partial w}{\partial \theta} + \kappa \frac{1 - \gamma}{\theta q} - e \frac{\partial C}{\partial \theta} \right]
\]
Similarly
\[
\frac{\partial e^d}{\partial \Delta} \bigg|_\theta = \frac{e^{\alpha - 1}}{y(e) \alpha (\alpha - 1)} \left[ \frac{\partial w}{\partial \Delta} - e \frac{\partial C}{\partial \Delta} \right]
\]
So that the macro elasticity reads
\[
\epsilon^M = \frac{\Delta}{1 - e^\left( \frac{\partial e^d}{\partial \Delta} \bigg|_\theta + \frac{\partial e^d}{\partial \theta} \bigg|_\Delta \right)} = \frac{\Delta}{1 - e y(e) \alpha (\alpha - 1)} \left( \frac{\partial w}{\partial \Delta} - e \frac{\partial C}{\partial \Delta} + \left[ \frac{\partial w}{\partial \theta} + \kappa \frac{1 - \gamma}{\theta q} - e \frac{\partial C}{\partial \theta} \right] \theta \frac{1 - e}{\Delta} \frac{1 - 1 + e^s (\epsilon^m - \epsilon^M)}{1 + e^s (\epsilon^m - \epsilon^M)} \right)
\]
Recall from lemma 8 that
\[
\frac{d\theta}{d\Delta} = \frac{-\theta}{\frac{1 - e}{\Delta} \frac{1 - e}{\gamma 1 + e^s (\epsilon^m - \epsilon^M)}}
\]
Thus
\[
\epsilon^M = \frac{\Delta}{1 - e y(e) \alpha (\alpha - 1)} \left( \frac{\partial w}{\partial \Delta} - e \frac{\partial C}{\partial \Delta} + \left[ \frac{\partial w}{\partial \theta} + \kappa \frac{1 - \gamma}{\theta q} - e \frac{\partial C}{\partial \theta} \right] \theta \frac{1 - e}{\Delta} \frac{1}{1 + e^s (\epsilon^m - \epsilon^M)} \right)
\]
The elasticity crucially depends on the response of wages. For simplicity, consider first a fixed wage model as in Michaillat (2012). Furthermore, note that
\[
\frac{\partial C}{\partial \theta} = \frac{\partial e}{\partial \theta} \Delta e + \frac{\partial c_u}{\partial \theta} u_c(c_u) \left( \frac{e}{u_c(c_e)} + \frac{1 - e}{u_c(c_u)} \right)
\]
and
\[
\frac{\partial C}{\partial \Delta} = \frac{\partial e}{\partial \Delta} \Delta e + \frac{\partial c_u}{\partial \Delta} u_c(c_u) \left( \frac{e}{u_c(c_e)} + \frac{1 - e}{u_c(c_u)} \right) + \frac{e}{u_c(c_e)}
\]
Therefore
\[
\epsilon^M = \frac{\Delta}{1 - e y(e) \alpha (\alpha - 1)} \left( -e \frac{\partial C}{\partial \Delta} - \left[ \kappa \frac{1 - \gamma}{\theta q} - e \frac{\partial C}{\partial \theta} \right] \theta \frac{1 - e}{\Delta} \frac{1}{1 + e^s (\epsilon^m - \epsilon^M)} \right)
\]
\[
= \frac{e^{\alpha - 1}}{y(e) \alpha (1 - \alpha)} \frac{1 - \gamma}{\gamma q e} \frac{\kappa}{1 + e^s (\epsilon^m - \epsilon^M)} - B
\]
\[ B = e \Upsilon \left[ \frac{\partial C}{\partial \Delta} + \frac{\partial C}{\partial \theta} \frac{1 - e}{\Delta} \frac{1}{\gamma} \frac{1}{1 + \epsilon^s} \left( e^m - e^M \right) \right] \]

\[ = e \Upsilon \left[ \left\{ \frac{\partial e}{\partial \Delta} \Delta_c + \frac{\partial c_u}{\partial \Delta} u_c(c_u) \left( \frac{e}{u_c(c_e)} + \frac{1 - e}{u_c(c_u)} + \frac{e}{u_c(c_e)} \right) \right\} \right. \]

\[ - \left\{ \frac{\partial e}{\partial \theta} \Delta_c + \frac{\partial c_u}{\partial \theta} u_c(c_u) \left( \frac{e}{u_c(c_e)} + \frac{1 - e}{u_c(c_u)} \right) \right\} \theta \frac{1 - e}{\Delta} \frac{1}{\gamma} \frac{1}{1 + \epsilon^s} \left( e^m - e^M \right) \]

\[ = e \Upsilon \left[ \Delta_c \left( \frac{\partial e}{\partial \Delta} - \frac{\partial e}{\partial \theta} \frac{1 - e}{\Delta} \frac{1}{\gamma} \frac{1}{1 + \epsilon^s} \left( e^m - e^M \right) \right) \right. \]

\[ + \frac{u_c(c_u)}{\phi(1 - \Upsilon)} \left( \frac{\partial c_u}{\partial \Delta} - \frac{\partial c_u}{\partial \theta} \frac{1 - e}{\Delta} \frac{1}{\gamma} \frac{1}{1 + \epsilon^s} \left( e^m - e^M \right) \right) + \frac{e}{u_c(c_e)} \]

\[ = e \Upsilon \left[ \Delta_c \epsilon^M + \frac{u_c(c_u)}{\phi(1 - \Upsilon)} \left( \frac{\partial c_u}{\partial \Delta} - \frac{\partial c_u}{\partial \theta} \frac{1 - e}{\Delta} \frac{1}{\gamma} \frac{1}{1 + \epsilon^s} \left( e^m - e^M \right) \right) + \frac{e}{u_c(c_e)} \right] \]

Next, form Lemma 7

\[ u_c(c_u) \frac{1}{\phi(1 - \Upsilon)} \frac{\partial c_u}{\partial \theta} = \frac{1}{1 - \Upsilon} \frac{e}{\gamma} (1 + \epsilon^s) \left( bw - \frac{d \mathbb{B}}{d(1 - e)} + \Upsilon \Delta_c - \kappa \frac{1 - \gamma}{\gamma q(1 + \epsilon^s)} \right) \]

and

\[ u_c(c_u) \frac{1}{\phi(1 - \Upsilon)} \frac{\partial c_u}{\partial \Delta} = \frac{1}{\gamma} \frac{1}{1 + \epsilon^s} \left( e^m - e^M \right) = \left( bw - \frac{d \mathbb{B}}{d(1 - e)} + \Upsilon \Delta_c - \kappa \frac{1 - \gamma}{\gamma q(1 + \epsilon^s)} \right) \frac{1 - e}{\Delta} \left( e^m - e^M \right) \]

From Lemma 9 we have

\[ \frac{1}{\phi(1 - \Upsilon)} u_c(c_u) \frac{\partial c_u}{\partial \Delta} + \frac{e}{u_c(c_e)} = \frac{1}{\phi(1 - \Upsilon)} \epsilon^m \frac{1 - e}{\Delta} \left( bw - \frac{d \mathbb{B}}{d(1 - e)} + \Upsilon \Delta_c \right) \]

Taken together this yields

\[ B = e \Upsilon \left[ \Delta_c \epsilon^M + \right] \]